Problems of the day 2

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01/02/2018

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1 Toss of dices

The first problem asks for the probability of a single toss of two six-faced dices which the sum is at most 9. We could solve it by enumerating all the cases that a single toss of two six-faced dices have the sum of at most 9, but that is a lot of cases, much more than the cases where the sum is greater than 9. Have said that we could use the axioms of probability to help us.

$$p(E) = p$$
$$p(\neg E) = 1 - p$$

Using the above formula and enumerating the cases which the toss is greater than nine we can solve it easily.

Toss	Die 1	Die 2
1	4	6
2	5	6
3	6	6
4	5	5
5	6	5
6	6	4

As we can see we have 6 tosses that the sum is greater than nine, therefore there are 30 cases in which the sum is not greater than nine. If you are wondering why, think of the number of possible cases there are. Calling our probability, of having a single toss which the sum of the faces of the dices are not greater than nine, **e** we can solve:

$$p(e) = \frac{30}{36}$$

Hence, the probability of a single toss of two six-faced dices whose the sum of the two up-faced numbers are not greater than nine is

$$p(e) = \frac{5}{6}$$

2 More toss of dices

The second problem wants the probability of a single toss of two dices whose the sum of the up-faces are 6 and have different faces. Using almost the same strategy as the earlier question we can enumerate all the cases of the question:

Toss	Die 1	Die 2
1	2	4
2	4	2
3	5	1
4	1	5

As we have four cases we can calculate the probability directly

$$p(e) = \frac{4}{36} = \frac{1}{9}$$

3 Balls and urns

The third problem is a little bit more complicated as it is compound probability. The problem is described as follows:

> There are urns labeled X, Y, and Z.
>
> 1. Urn 1 contains 4 red balls and 3 black balls.
> 2. Urn 2 contains 5 red balls and 4 black balls.

3. Urn 3 contains 4 red balls and 4 black balls.

One ball is drawn from each of the 3 urns. What is the probability that, of the 3 balls drawn, 2 are red and 1 is black?

To solve this problem I adopted one strategy. Calculate each individual probability of each drawn ball for each urn. I call E the event of drawing a black ball from the urn and $\neg E$ the event of not drawing a red ball from the urn, that means, a red ball. Calculating every possibility:

$$P_x(E) = \frac{3}{7} \quad P_x(\neg E) = \frac{4}{7}$$
$$P_y(E) = \frac{4}{9} \quad P_y(\neg E) = \frac{5}{9}$$
$$P_z(E) = \frac{1}{2} \quad P_z(\neg E) = \frac{1}{2}$$

To calculate the final probability we have first to know how many possibilities of drawing 2 balls red and 1 black. We can use combinations for this purpose as the order of drawing the balls doesn't matter:

$$C_1^3 = \frac{3!}{1! * (3-1)!} = 3$$

Knowing that are 3 total possibilities we can calculate the final probability with the following formula.

$$P(e) = P_1(R \cap R \cap B) + P_2(R \cap B \cap R) + P_3(B \cap R \cap R) \tag{1}$$

$$P_1(R \cap R \cap B) = \tfrac{4}{7} * \tfrac{5}{9} * \tfrac{1}{2} \tag{2}$$

$$P_2(R \cap B \cap R) = \tfrac{4}{7} * \tfrac{4}{9} * \tfrac{1}{2} \tag{3}$$

$$P_2(B \cap R \cap R) = \tfrac{3}{7} * \tfrac{5}{9} * \tfrac{1}{2} \tag{4}$$

Sum everything up:

$$P(e) = P_1 + P_2 + P_3 \tag{5}$$

$$P(e) = \tfrac{20}{63*2} + \tfrac{8}{63} + \tfrac{15}{63*2} \tag{6}$$

$$P(e) = \frac{51}{126} = \frac{17}{42} \tag{7}$$

Hence, the probability of drawing two red and one black ball of the urns is $\frac{17}{52}$.