HOMEWORK 9
415G 001 COMBINATORICS AND GRAPH THEORY

DUE WEDNESDAY 12/9

Exercises

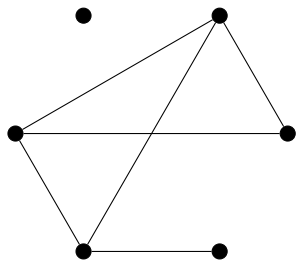
1. As in exercise 5.4, how many permutations of the set [14] can be written as a product of two 2-cycles, two 3-cycles and a 4-cycle. (Explain)

2. A Graph is *color critical* if the removal of any vertex decreases the graph's chromatic number. Show that every color critical graph with $\chi(G) = k$ has the following properties:
(a) G is connected.
(b) Every vertex of G has degree $\geq k - 1$.
(c) G has no vertex whose removal disconnects the graph.

3. The *line graph* $L(G)$ of a graph G is the graph that has a vertex for every edge of G and two vertices in $L(G)$ are adjacent if the corresponding edges in G share a common end vertex. Show that G can be properly edge-colored with k colors if and only if $L(G)$ can be properly vertex-colored with k colors.

4. In a round-robin tournament where each pair of 6 contestants plays each other, a major problem is scheduling the play over a minimal number of days (each contestant plays at most one match a day). What is the minimal number of days needed for such a tournament? (Hint: Restate the problem as an edge coloring problem).

5. Compute the chromatic polynomial of the following graph.



Suggested exercises

From the book. 5.1, 5.2, 5.3, 5.4, 5.5, 5.10, 5.12, 5.13, 5.18, 5.19