

HOMEWORK 11
415G 001 COMBINATORICS AND GRAPH THEORY

DUE FRIDAY 12/9

Exercises

1. In the early versions of the Enigma machine, used in Germany in the 1930's, the plug-board swapped six pairs of distinct letters of the alphabet. In how many ways can this be done (assuming 26 letters)?
2. For a sequence of numbers $\{a_n\}_{n \geq 0}$ we define its *exponential generating function (e.g.f.)* to be the formal power series

$$A(x) = \sum_{n \geq 0} a_n \frac{x^n}{n!}.$$

Let $B(n)$ be the n -th Bell number and $F(x) = \sum_{n \geq 0} B(n) \frac{x^n}{n!}$.

(A). Use the recursion for the Bell numbers, $B(0) = 1$ and

$$B(n) = \sum_{k \geq 0} \binom{n-1}{k} B(k) \quad n \geq 1,$$

to prove that

$$\frac{dF(x)}{dx} = e^x F(x).$$

(B). Use the differential equation above to prove that $F(x)$ is given by

$$F(x) = e^{e^x - 1}.$$

(C). Finally expand the formula in part (B) to give the explicit closed formula

$$B(n) = \frac{1}{e} \sum_{j \geq 0} \frac{j^n}{j!}.$$

Suggested exercises

From the book. 5.1, 5.2, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, 5.10