

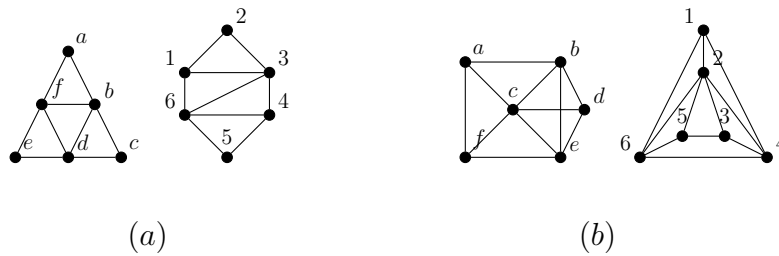
# HOMEWORK 6

## 415G 001 COMBINATORICS AND GRAPH THEORY

DUE FRIDAY 10/30

### Exercises

1. List all nonisomorphic (not necessarily simple) graphs with 4 vertices.
2. Are the following pairs of graphs isomorphic? Explain why.



3. Build six-vertex graphs with the following degrees of vertices, if possible. If not possible explain why not.
  - (a) Vertices of degrees 3, 3, 3, 1, 1, 1.
  - (b) Vertices of degrees 1, 2, 2, 3, 4, 5.
  - (c) Vertices of degrees 2, 2, 4, 4, 4, 4.
4. A graph is called *regular* if all vertices have the same degree. Find all nonisomorphic regular simple graphs with four and five vertices.
5. Suppose all vertices of a graph  $G$  have degree  $d$ , where  $d$  is an odd number. Show that the number of edges of  $G$  is a multiple of  $d$ .
6. Show that any planar graph can be drawn on the surface of a sphere without crossing edges and vice versa.
7. Suppose a planar graph is not connected but instead consists of several components. Find the appropriate modification of Euler's formula (and prove that your formula holds) for a planar graph with  $c$  components.
8. List all the spanning trees of the complete graph with four vertices  $K_4$ . How many are there?.
9. Exercise 3.13.

### Suggested exercises

**From the book.** 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 3.10, 3.11, 3.12, 3.15, 3.16