HOMEWORK 4
415G 001 COMBINATORICS AND GRAPH THEORY

DUE FRIDAY 10/02

Exercises

1. Find a recurrence relation for the number of ways to arrange cars in a row with n spaces if we can use compact cars, pick-up trucks or limousines. A compact car uses one parking spot, a pick-up truck uses two spots and a limousine uses four spots.

2. (a). Find a recurrence relation for the number R_n of regions created by n mutually overlapping circles on a piece of paper such that no three circles have a common intersection point.
(b). Find a closed formula for R_n.

3. Find a recurrence relation for the number Q_n of n-digit quaternary sequences (i.e. with digits 0, 1, 2 and 4) with at least one 1 and the first 1 occurring before the first 0 (possibly no 0's).

4. Find a recurrence relation for the number T_n of ways to pair off $2n$ players for tennis matches. Then prove that $T_n = (2n-1)!!$, where the number $n!!$ ("n double-factorial") is defined to be the product of all odd integers less or equal than n (for example $5!! = 1 \cdot 3 \cdot 5$ and $8!! = 1 \cdot 3 \cdot 5 \cdot 7$).

5. A *partition* of the set $[n]$ is a disjoint collection $\{B_1, B_2, \ldots, B_k\}$ of subsets $B_i \subseteq [n]$ (called the *blocks* of the partition) such that $[n] = \bigcup_{i=1}^{k} B_i$ (the union of all the blocks is $[n]$). Let $S(n,k)$ be the number of partitions of $[n]$ in exactly k blocks. Find a recurrence relation for $S(n,k)$ (Hint: the recurrence is in terms of $S(n-1,k-1)$ and $S(n-1,k)$).

6. Solve the recurrence

$$a_n - 5a_{n-1} + 8a_{n-2} - 4a_{n-3} = 0$$

with initial conditions $a_1 = 1$, $a_2 = 5$ and $a_3 = 17$.

Suggested exercises

From the book. 2.1, 2.2, 2.3, 2.5, 2.7, 2.9, 2.11, 2.14