

HOMEWORK 3
415G 001 COMBINATORICS AND GRAPH THEORY

DUE FRIDAY 9/18

Exercises

1. How many arrangements are there with n 0's and m 1's, with k runs of 0's? (A run is a group of consecutive letters that are repeated, for example the arrangement 00011011100 has three runs of 0's.)
2. How many different n th-order partial derivatives does a function $f(x_1, x_2, \dots, x_k)$ have?
3. How many ways are there to arrange the 26 letters of the alphabet so that no pair of vowels appears consecutively?
4. How many integer solutions are there to $x_1 + x_2 + x_3 = 0$ with $x_i \geq -5$ for $i \in [3]$?
5. Prove the following theorem:

Theorem. Let M be a *multiset* with n objects, λ_1 of type 1, λ_2 of type 2, ..., and λ_k of type k , where $\lambda_1 + \lambda_2 + \dots + \lambda_k = n$. Then the number of ordered arrangements (*multipermutations* or *multiset permutations*) of the objects in M is

$$\frac{n!}{\lambda_1! \lambda_2! \dots \lambda_k!}.$$

(This number is also denoted $\binom{n}{\lambda_1, \lambda_2, \dots, \lambda_k}$ and called the *multinomial coefficient*)

Suggested exercises

From the book. 1.5, 1.13, 1.14, 1.15, 1.16

Additional.

1. How many ways are there to pick a collection of 10 coins from piles of pennies, nickels, dimes and quarters?
2. How many ways are there to arrange the letters in MISSISSIPPI?