HOMEWORK 2
415G 001 COMBINATORICS AND GRAPH THEORY

DUE FRIDAY 9/11

Exercises

1. If 13 players are each dealt four cards from a 52-card deck, what is the probability that each player gets one card of each suit?

2. How many arrangements of the word "JUPITER" are there with the vowels occurring in alphabetical order?

3. For the identity

$$\sum_{r=0}^{k} \binom{n}{r}\binom{m}{k-r} = \binom{n+m}{k}.$$

(i) Give a proof counting a set in two different ways.
(ii) Give another proof using the binomial theorem.

4. For the identity

$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}.$$

(i) Give a proof counting a set in two different ways.
(ii) Give another proof using the binomial theorem.

5. According to Figure 1 find the number of lattice paths from the point A to the point B taking only EAST and NORTH steps and avoiding the point C knowing that the coordinates of A, B and C are:
(i) $A = (0, 0)$, $B = (10, 7)$ and $C = (7, 4)$.
(ii) $A = (0, 0)$, $B = (n, m)$ and $C = (k, r)$, with $0 \leq k \leq n$ and $0 \leq r \leq m$.

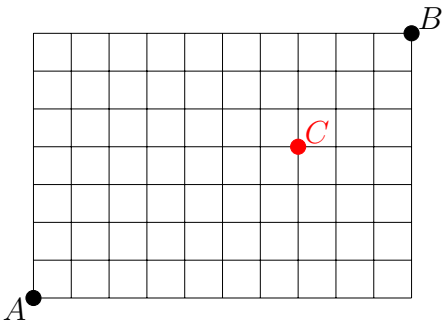

FIGURE 1

Suggested exercises. 1.1, 1.2, 1.7, 1.10, 1.17, 1.19