HOMEWORK 7
415G 001 COMBINATORICS AND GRAPH THEORY

DUE MONDAY 11/09

Exercises

1. Suppose all vertices of a graph G have degree p, where p is an odd number. Show that the number of edges in G is a multiple of p.

2. A graph G is *critical planar* if G is not planar but any subgraph obtained by removing a vertex is planar.
   (a) Are K_5, $K_{3,3}$ and the Petersen graph P critical planar?.
   (b) Prove that critical planar graphs must be connected and cannot have a vertex whose removal disconnects the graph.

3. Show that $K_{3,3}$ and K_5 can be drawn on the surface of a doughnut (torus) without crossing edges

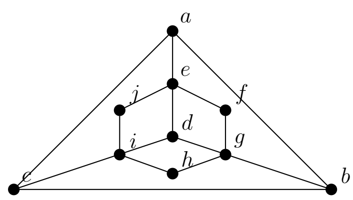
4. If a planar graph with n vertices all of degree 4 have 10 regions, determine n.

5. Suppose G is a graph whose vertices have degree at least k. Show that G contains a path of length at least k.

6. How many hamiltonian cycles are there in the complete graph K_n, $n \geq 3$?

7. For what values of n can we visit all the squares of an $n \times n$ chessboard in a cycle (the only square that is repeated is the first-last square) if we are only allowed to move to adjacent squares (front, back, left and right moves)? For those possible values of n exhibit such a cycle.

8. Show that the following graph is not hamiltonian.



9. Exercise 3.14.

Suggested exercises

From the book. 4.1, 4.2(a), 4.3(a)