PART2

Part f

```
rain_geo_rainfall2 <- as.geodata(
  cbind(sqrt(rain$rainfall),
        rain$x,
        rain$y,rain$altitude),
  data.col = 1,
  coords.col = 2:3,covar.col=4)

rain_geo_rainfall2
```

```
## $coords
##           Coord1    Coord2
##   [1,]  29.52739  80.71854
##   [2,]  33.77939  99.52954
##   [3,]  46.80639 102.58454
##   [4,]  48.71439 121.45354
##   [5,]  49.31639 113.65554
##   [6,]  53.21039  79.09954
##   [7,]  54.51039 106.87954
##   [8,]  60.31039 132.35054
##   [9,]  60.79839  75.62054
##  [10,]  60.81839  76.73254
##  [11,]  76.45039  97.59954
##  [12,]  77.70339 177.65154
##  [13,]  85.25839 179.76254
##  [14,]  86.30039  93.00554
##  [15,]  86.20739  30.73054
##  [16,]  94.39639 129.59654
##  [17,] 101.28339  68.35054
##  [18,] 106.13639  19.36854
##  [19,] 107.31339 131.66954
##  [20,] 123.54739  73.69754
##  [21,] 124.59039 110.38554
##  [22,] 127.39639 182.64354
##  [23,] 127.47739 193.76254
##  [24,] 135.85839  84.73554
##  [25,] 140.10639 169.22154
##  [26,] 141.56539 159.20654
##  [27,] 143.23239 193.66954
##  [28,] 144.67939 181.43154
##  [29,] 145.27339 143.62254
##  [30,] 146.01239 139.17254
##  [31,] 148.43039 179.19254
##  [32,] 150.38039  85.78154
##  [33,] 152.77239 120.24554
##  [34,] 152.80439 131.36454
##  [35,] 155.96239 183.61754
##  [36,] 156.42339  54.63054
```

```
##  [37,] 163.46639 173.59754
##  [38,] 170.24439 110.21254
##  [39,] 171.78039  60.17554
##  [40,] 176.31239 128.00754
##  [41,] 184.01239  79.09454
##  [42,] 183.96139 101.33354
##  [43,] 186.16839 130.24954
##  [44,] 191.40439 150.28154
##  [45,] 194.35439 168.08454
##  [46,] 195.72839 198.11254
##  [47,] 200.35939 172.56054
##  [48,] 201.25339 145.87754
##  [49,] 201.20639 154.77354
##  [50,] 202.29139  93.62054
##  [51,] 203.14039  78.05754
##  [52,] 204.13539  36.92054
##  [53,] 203.86439 217.05654
##  [54,] 207.62239 213.74454
##  [55,] 209.26739  76.98254
##  [56,] 211.37039 104.79654
##  [57,] 212.15639 207.10354
##  [58,] 214.78339 157.08454
##  [59,] 215.97839 197.12454
##  [60,] 217.71839  72.59554
##  [61,] 219.37639  57.04154
##  [62,] 219.44239 141.55354
##  [63,] 224.21539 198.30654
##  [64,] 226.55339  28.19254
##  [65,] 225.85239 183.86654
##  [66,] 225.84139 184.97854
##  [67,] 229.14839  78.25754
##  [68,] 233.57539  20.47754
##  [69,] 233.16239 202.84454
##  [70,] 235.70839 176.18354
##  [71,] 235.44639 199.53354
##  [72,] 236.24639 195.09454
##  [73,] 238.42839 136.18154
##  [74,] 241.21639  92.84454
##  [75,] 240.77439 192.92254
##  [76,] 241.91839 160.68754
##  [77,] 243.30239 170.71354
##  [78,] 248.28639  77.36354
##  [79,] 251.84939 151.92054
##  [80,] 251.95039 198.62854
##  [81,] 254.68439 111.92554
##  [82,] 255.53039 158.64454
##  [83,] 257.97139  95.29154
##  [84,] 256.87539 169.78554
##  [85,] 264.89139 186.59054
##  [86,] 266.71039 121.00454
##  [87,] 266.45739 136.57054
##  [88,] 268.17739  77.65554
##  [89,] 275.17839  71.10054
##  [90,] 277.06039  94.48954
```

```
##  [91,] 283.14239  95.71354
##  [92,] 281.62639 175.76254
##  [93,] 284.53639  62.37454
##  [94,] 290.88139  90.30354
##  [95,] 292.86039  68.09854
##  [96,] 294.99739 112.63254
##  [97,] 298.08639 110.47354
##  [98,] 312.06739  66.29654
##  [99,] 315.29239  59.69954
## [100,] 320.91139  49.82554
##
## $data
##   [1] 12.288206 15.968719  8.888194 13.820275 13.928388 18.275667 10.344080
##   [8] 17.204651 19.849433 19.849433 18.000000 10.246951 11.618950 24.186773
##  [15] 10.677078 18.275667 11.445523  8.831761 19.949937 11.874342 13.856406
##  [22] 12.288206 10.344080 12.041595 18.275667 18.083141 14.594520 18.193405
##  [29] 20.000000 18.083141 19.493589  9.695360 13.601471 15.459625 18.165902
##  [36]  5.477226 15.937377  7.280110  8.831761  8.426150  7.874008  8.426150
##  [43]  7.681146  7.745967 11.135529 12.369317  8.660254 11.704700  9.273618
##  [50] 11.357817 18.574176 21.000000 13.564660 11.000000 18.601075 16.431677
##  [57] 10.000000  6.708204 10.344080 18.947295 16.673332  8.485281 11.445523
##  [64]  9.486833 11.874342 11.445523 21.260292  4.000000 11.661904 11.401754
##  [71] 10.862780 10.440307 12.041595 15.937377 11.832160 12.328828  7.745967
##  [78] 16.822604 13.564660 11.269428 14.832397 13.341664 14.764823 11.704700
##  [85] 12.000000 15.165751 16.792856 11.357817  8.062258 13.784049 13.038405
##  [92] 12.489996 11.445523  3.162278  9.949874  9.591663  8.185353  4.242641
##  [99]  4.472136  7.416198
##
## $covariate
##         covar1
## 1   0.12916667
## 2   0.15397727
## 3   0.08257576
## 4   0.15776515
## 5   0.10965909
## 6   0.09943182
## 7   0.08200758
## 8   0.22140152
## 9   0.10378788
## 10  0.14848485
## 11  0.14867424
## 12  0.11325758
## 13  0.08219697
## 14  0.14261364
## 15  0.08825758
## 16  0.08238636
## 17  0.20776515
## 18  0.38901515
## 19  0.13219697
## 20  0.24337121
## 21  0.10208333
## 22  0.06325758
## 23  0.04696970
## 24  0.18958333
```

```
## 25  0.15132576
## 26  0.08863636
## 27  0.05681818
## 28  0.07821970
## 29  0.12386364
## 30  0.15606061
## 31  0.11287879
## 32  0.23049242
## 33  0.17916667
## 34  0.23162879
## 35  0.10852273
## 36  0.13731061
## 37  0.07348485
## 38  0.13068182
## 39  0.29848485
## 40  0.33522727
## 41  0.26117424
## 42  0.23106061
## 43  0.14450758
## 44  0.08257576
## 45  0.10700758
## 46  0.07708333
## 47  0.11155303
## 48  0.14204545
## 49  0.11534091
## 50  0.20662879
## 51  0.24848485
## 52  0.15265152
## 53  0.14318182
## 54  0.09299242
## 55  0.18863636
## 56  0.20435606
## 57  0.08011364
## 58  0.07689394
## 59  0.08768939
## 60  0.17859848
## 61  0.20719697
## 62  0.17121212
## 63  0.08825758
## 64  0.15681818
## 65  0.10340909
## 66  0.10075758
## 67  0.17727273
## 68  0.14583333
## 69  0.11818182
## 70  0.14488636
## 71  0.08598485
## 72  0.08371212
## 73  0.12443182
## 74  0.30416667
## 75  0.08200758
## 76  0.12253788
## 77  0.15965909
## 78  0.36287879
```

```
## 79  0.19166667
## 80  0.08977273
## 81  0.20549242
## 82  0.25151515
## 83  0.35397727
## 84  0.18106061
## 85  0.08693182
## 86  0.24147727
## 87  0.08806818
## 88  0.28958333
## 89  0.38636364
## 90  0.20378788
## 91  0.22765152
## 92  0.07537879
## 93  0.31571970
## 94  0.27140152
## 95  0.37670455
## 96  0.45795455
## 97  0.32575758
## 98  0.44166667
## 99  0.20037879
## 100 0.13844697
##
## attr(,"class")
## [1] "geodata"
```

```
fit_spatial <- likfit(
  rain_geo_rainfall2,
  trend =~covar1 ,
  cov.model = "exponential",
  ini.cov.pars = c(5, .01),
  fix.nugget=TRUE,
  nugget=0.0,nospatial=FALSE)
```

```
## kappa not used for the exponential correlation function
## ---------------------------------------------------------------
## likfit: likelihood maximisation using the function optimize.
## likfit: Use control() to pass additional
##          arguments for the maximisation function.
##         For further details see documentation for optimize.
## likfit: It is highly advisable to run this function several
##         times with different initial values for the parameters.
## likfit: WARNING: This step can be time demanding!
## ---------------------------------------------------------------
## likfit: end of numerical maximisation.
```

```
fit_NO_spatial <- likfit(
  rain_geo_rainfall2,
  trend =~covar1 ,
  cov.model = "exponential",
  ini.cov.pars = c(5, .01),
  fix.nugget=TRUE,
  nugget=0.0,nospatial=TRUE)
```

```
## kappa not used for the exponential correlation function
```

```
## ------------------------------------------------------------
## likfit: likelihood maximisation using the function optimize.
## likfit: Use control() to pass additional
##          arguments for the maximisation function.
##         For further details see documentation for optimize.
## likfit: It is highly advisable to run this function several
##         times with different initial values for the parameters.
## likfit: WARNING: This step can be time demanding!
## ------------------------------------------------------------
## likfit: end of numerical maximisation.
```

```
summary(fit_spatial)
```

```
## Summary of the parameter estimation
## ----------------------------------
## Estimation method: maximum likelihood
##
## Parameters of the mean component (trend):
##   beta0   beta1
## 11.5970  0.0288
##
## Parameters of the spatial component:
##    correlation function: exponential
##       (estimated) variance parameter sigmasq (partial sill) =  20.97
##       (estimated) cor. fct. parameter phi (range parameter)  =  42.41
##    anisotropy parameters:
##       (fixed) anisotropy angle = 0  ( 0 degrees )
##       (fixed) anisotropy ratio = 1
##
## Parameter of the error component:
##       (fixed) nugget = 0
##
## Transformation parameter:
##       (fixed) Box-Cox parameter = 1 (no transformation)
##
## Practical Range with cor=0.05 for asymptotic range: 127.0387
##
## Maximised Likelihood:
##    log.L n.params      AIC      BIC
## "-247.7"      "4"  "503.5"  "513.9"
##
## non spatial model:
##    log.L n.params      AIC      BIC
##   "NULL"   "NULL"   "NULL"   "NULL"
##
## Call:
## likfit(geodata = rain_geo_rainfall2, trend = ~covar1, ini.cov.pars = c(5,
##     0.01), fix.nugget = TRUE, nugget = 0, cov.model = "exponential",
##     nospatial = FALSE)
```

```
summary(fit_NO_spatial)
```

```
## Summary of the parameter estimation
## ----------------------------------
## Estimation method: maximum likelihood
```

```
##
## Parameters of the mean component (trend):
##   beta0   beta1
## 11.5970  0.0288
##
## Parameters of the spatial component:
##    correlation function: exponential
##       (estimated) variance parameter sigmasq (partial sill) =  20.97
##       (estimated) cor. fct. parameter phi (range parameter)  =  42.41
##    anisotropy parameters:
##       (fixed) anisotropy angle = 0  ( 0 degrees )
##       (fixed) anisotropy ratio = 1
##
## Parameter of the error component:
##       (fixed) nugget = 0
##
## Transformation parameter:
##       (fixed) Box-Cox parameter = 1 (no transformation)
##
## Practical Range with cor=0.05 for asymptotic range: 127.0387
##
## Maximised Likelihood:
##    log.L n.params      AIC      BIC
## "-247.7"      "4"  "503.5"  "513.9"
##
## non spatial model:
##    log.L n.params      AIC      BIC
## "-286.2"      "3"  "578.3"  "586.2"
##
## Call:
## likfit(geodata = rain_geo_rainfall2, trend = ~covar1, ini.cov.pars = c(5,
##     0.01), fix.nugget = TRUE, nugget = 0, cov.model = "exponential",
##     nospatial = TRUE)
```

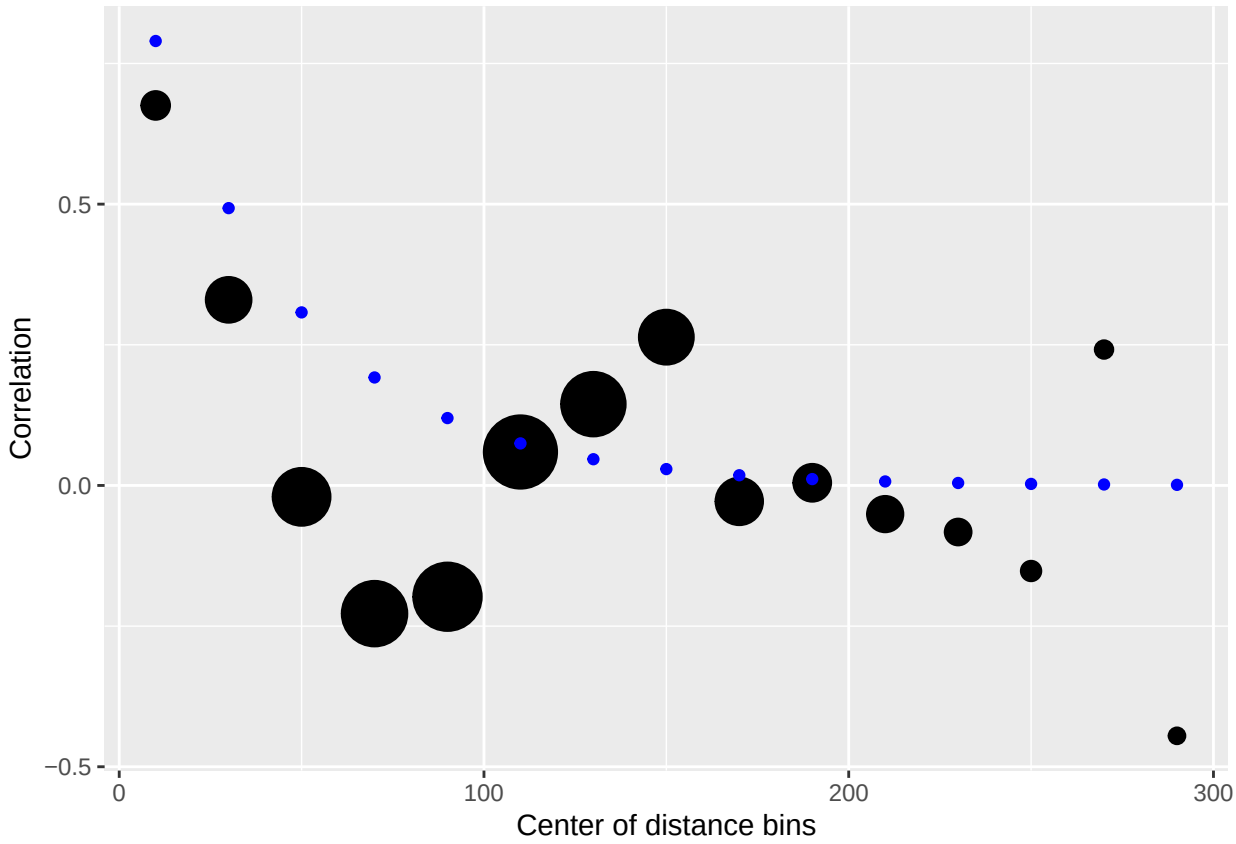
ANSWER

We have that $\beta_0 = 11.5970$, $\beta_1 = 0.0288$ and $\sigma^2 = 20.97$, $\phi = 42.41$

Part g

```
corr_func<-exp((-1/42.41)*c(10,30,50,70,90,110,130,150,170,190,210,230,250,270,290))
df3<-data.frame(centerBins=centers,corr=corr_func)


ggplot() + geom_point(data = df2, aes(x =centerBins , y = corr),cex=10*(counters_dist+min(counters_dist)
  labs(x = "Center of distance bins",y = "Correlation")+ geom_point(data = df3, aes(x =centerBins , y =
```



Part h

```
summary(fit_spatial)
```

```
## Summary of the parameter estimation
## -----------------------------------
## Estimation method: maximum likelihood
##
## Parameters of the mean component (trend):
##   beta0   beta1
## 11.5970  0.0288
##
## Parameters of the spatial component:
##    correlation function: exponential
##       (estimated) variance parameter sigmasq (partial sill) =  20.97
##       (estimated) cor. fct. parameter phi (range parameter)  =  42.41
##    anisotropy parameters:
##       (fixed) anisotropy angle = 0  ( 0 degrees )
##       (fixed) anisotropy ratio = 1
##
## Parameter of the error component:
##       (fixed) nugget = 0
##
## Transformation parameter:
##       (fixed) Box-Cox parameter = 1 (no transformation)
##
## Practical Range with cor=0.05 for asymptotic range: 127.0387
```

```
##
## Maximised Likelihood:
##    log.L n.params      AIC      BIC
## "-247.7"      "4"  "503.5"  "513.9"
##
## non spatial model:
##    log.L n.params      AIC      BIC
##   "NULL"   "NULL"   "NULL"   "NULL"
##
## Call:
## likfit(geodata = rain_geo_rainfall2, trend = ~covar1, ini.cov.pars = c(5,
##     0.01), fix.nugget = TRUE, nugget = 0, cov.model = "exponential",
##     nospatial = FALSE)
```

```
summary(fit_NO_spatial)
```

```
## Summary of the parameter estimation
## ----------------------------------
## Estimation method: maximum likelihood
##
## Parameters of the mean component (trend):
##   beta0   beta1
## 11.5970  0.0288
##
## Parameters of the spatial component:
##    correlation function: exponential
##       (estimated) variance parameter sigmasq (partial sill) =  20.97
##       (estimated) cor. fct. parameter phi (range parameter)  =  42.41
##    anisotropy parameters:
##       (fixed) anisotropy angle = 0  ( 0 degrees )
##       (fixed) anisotropy ratio = 1
##
## Parameter of the error component:
##       (fixed) nugget = 0
##
## Transformation parameter:
##       (fixed) Box-Cox parameter = 1 (no transformation)
##
## Practical Range with cor=0.05 for asymptotic range: 127.0387
##
## Maximised Likelihood:
##    log.L n.params      AIC      BIC
## "-247.7"      "4"  "503.5"  "513.9"
##
## non spatial model:
##    log.L n.params      AIC      BIC
## "-286.2"      "3"  "578.3"  "586.2"
##
## Call:
## likfit(geodata = rain_geo_rainfall2, trend = ~covar1, ini.cov.pars = c(5,
##     0.01), fix.nugget = TRUE, nugget = 0, cov.model = "exponential",
##     nospatial = TRUE)
```

ANSWER

Non spatial AIC = 578.3.

Spatial AIC =503.5

Therefore, the spatial model better fits the data

Part i

ANSWER

We need the convex hull of the set of data points to generate the grid for prediction locations.