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Advanced Predictive Models for Complex Data

Assignment 7 part 1

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a The matrix is sparse, because we have lots of zeros. Mostly all elements are zeros except for the diagonal elements. Just $1/6 \approx 16.66\%$ of the elements are different of zero.

The matrix is symmetric, because for all elements in the matrix with subindices $i, j \in \{1, 2, 3, 4, 5, 6\}$, we have that $a_{ij} = a_{ji}$. If we transpose the matrix , the matrix will be exactly the same $A = A^T$

b In this case all the diagonal elements are different of zero. So, we can notice that for a diagonal matrix when we do the matrix multiplication with a vector full of zeros except for the ith element, we will get a vector full of zeros except for the ith element

$$\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ v_i \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ v_i a_{ii} \\ \vdots \\ 0 \end{bmatrix} \tag{1}$$

If $v_i = 1$ then we get a vector full of zeros except for the ith entry with the ith diagonal element. Hence , we notice that

$$\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ a_{ii} \\ \vdots \\ 0 \end{bmatrix} = a_{ii} \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \tag{2}$$

So, a_{ii} is the eigenvalue of the vector $\begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$ with 1 at the ith element. So, applying this to the matrix of the homework, we easily notice that the eigenvalues are the diagonal elements $2, 1.5, -3, -1, 0.5, -0.5$.

And using the discussion above we can easily get the associated eigenvectors. For example, I show how to obtain the eigenvector associated to 2.

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

Hence, the eigenvector is $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

It is not necessary to find every single eigenvector because we just have to make a vector full of zeros and with a 1 in the ith element for the corresponding ith diagonal entry.

Therefore, we have that the eigenpairs are $(2, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix})$, $(1.5, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix})$, $(-3, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix})$, $(-1, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix})$, $(0.5, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix})$, $(-0.5, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix})$

c For the Frobenius norm we square every single element of the matrix, sum them and take the square root. Given that we have a diagonal matrix, we just have to take the sum of the square of the diagonal elements. Hence

$$||A||_F = \sqrt{2^2 + 1.5^2 + (-3)^2 + (-1)^2 + (0.5)^2 + (-0.5)^2} = 4.092676386 \tag{4}$$

Now, for the operator norm we see what is the greatest singular value, which is the same to see the largest eigenvalue in magnitude. Therefore,

$$||A||_{op} = 3 \tag{5}$$

Finally, nuclear norm is calculated by summing the singular values,

$$||A||_* = 2 + 1.5 + 3 + 1 + 0.5 + 0.5 = 8.5 \tag{6}$$

d The largest eigenvalue (2)does not match with the largest singular value 3. The reason is that the largest singular value is the largest eigenvalue in magnitude. So, the largest eigenvalue in magnitude is actually the -3.