```
import time
import numpy as np
import pandas as pd
from scipy.sparse import csr_matrix
```

→ Part a

```
dist_mat=pd.read_csv("/content/TexasCityDistanceMatrix.xls")
keys=pd.read_csv("/content/keys.csv").to_numpy()
#keys = pd.ExcelFile(r"/content/Keys.xls")
print(dist_mat.head())
#remove the index of the cities
dist mat2=dist_mat.drop(["Place"], axis=1)
ndist_mat2=dist_mat2.to_numpy()
n_dist_z=np.nan_to_num(ndist_mat2)
sdist mat=csr matrix(dist mat2.fillna(0))
print("Maximum distance", sdist_mat.max())
print("Number of elements greater than 99.99971353 are:",len(n dist z[n dist z>99.99971353]))
        Place
               V1
                           V2
                                V3
                                     V4
                                                V5
                                                            V6 V7 V8 V9
    0
              0.0
                                                    75.440971 Nan Nan Nan
           1
                          NaN
                               NaN
                                    NaN
                                               NaN
                                                                            . . .
    1
              NaN
                     0.000000
                               NaN
                                    NaN
                                         90.673319
                                                           Nan Nan Nan Nan
    2
              NaN
                          NaN
                               0.0
                                    NaN
                                               NaN
                                                           Nan Nan Nan Nan
                                                           NaN NaN NaN NaN
    3
                          NaN
                                               NaN
              NaN
                               NaN
                                    0.0
            4
                    90.673319
                                          0.000000
                                                           Nan Nan Nan Nan
    4
              NaN
                               NaN
                                    NaN
            V1743
                   V1744
                          V1745
                                 V1746
                                            V1747
                                                        V1748
                                                                   V1749
                                                                              V1750
    0
       85.953325
                                                                     NaN
                                                                                NaN
                     NaN
                           NaN
                                   NaN
                                              NaN
                                                         NaN
    1
             NaN
                     NaN
                            NaN
                                   NaN
                                              NaN
                                                          NaN
                                                                     NaN
                                                                                NaN
    2
                     NaN
                            NaN
                                              NaN
                                                          NaN
                                                                                NaN
                                   NaN
                                                                     NaN
                                   Nan 37.771591 70.901728 37.784807 15.815951
    3
             NaN
                     NaN
                            NaN
    4
             NaN
                     NaN
                            NaN
                                   NaN
                                              NaN
                                                          NaN
                                                                     NaN
                                                                                NaN
        V1751
              V1752
    0
         NaN
                 NaN
          NaN
                 NaN
    2
          NaN
                 NaN
    3
          NaN
                 NaN
          NaN
                 NaN
    [5 rows x 1753 columns]
    Maximum distance 99.99971353
    Number of elements greater than 99.99971353 are: 0
```

Answer

Maximum distance in the matrix: 99.99971353

▼ Part b

```
indexk=0
for i in range(1,keys.shape[0]):
    if "Katy" in str(keys[i]):
        indexk=i

max_dist_katy=np.argmax(np.nan_to_num(ndist_mat2[indexk]))
max_num_distance_katy=ndist_mat2[indexk][max_dist_katy]

print("Number of elements greater", max_num_distance_katy, "are", len(ndist_mat2[indexk][ndist_mat2[indexk]>max_num_distance_katy
print("Maximum Katy distance is to ", keys[max_dist_katy])
```

```
Nans_huge=np.nan_to_num(ndist_mat2,nan=10000)
Nans_huge[indexk][indexk]=10000
index min Nans huge=np.argmin(Nans huge[indexk])
print(index min Nans huge)
print("Closest place to Katy city is :" , keys[index_min_Nans_huge])
print("Closest distance to Katy city is :" , Nans_huge[indexk][index_min_Nans_huge])
index_fmt=0
for i in range(1,keys.shape[0]):
  if "Flower Mound town" in str(keys[i]):
    index fmt=i
\#\mbox{All} nan values made them huge , so that they are not converted to zero
# because we want all those values lower 25 miles
no_nan_fmd=np.nan_to_num(ndist_mat2[index_fmt],nan=1000)
print(no nan fmd)
print("Number of locations within 25 miles of Flower Mound town are ",len(no_nan_fmd[no_nan_fmd<=25])-1)
    Number of elements greater 99.60808349 are 0
    Maximum Katy distance is to ["1249,Port O'Connor CDP"]
    276
    Closest place to Katy city is : ['277,Cinco Ranch CDP']
    Closest distance to Katy city is : 5.337001509
    [ 79.67487195 1000.
                                  1000.
                                                                  1000.
                 1
    Number of locations within 25 miles of Flower Mound town are 75
```

Answer

Closest location to Katy city is to Cinco Ranch CDP

Number of locations within 25 miles of Flower Mound town are 75

- Part c

```
#Not sparse n_dist_z

#sparse sdist_mat

times_sum_all=[]
for i in range (1000):
    start = time.time()
    np.sum(n_dist_z)
    end = time.time()
    times_sum_all.append(end - start)

average_sum_all=sum(times_sum_all)/len(times_sum_all)
    print("Average time for summing all elements of the matrix ",average_sum_all )

Average time for summing all elements of the matrix 0.0023846786022186277
```

Answer

In order to get a more accurate result, I take the average over 1000 repetitions of summing all values of the matrix.

The average time for summing all elements of the matrix 0.0023846786022186277

→ Part d

```
dist mat d=pd.read csv("/content/TexasCityDistanceMatrix.xls")
ndist_mat_d=dist_mat_d.to_numpy()
n_dist_wNan=np.nan_to_num(ndist_mat_d)
times sum_all_d=[]
for i in range(n_dist_wNan.shape[0]):
  for j in range(n_dist_wNan.shape[1]):
    if n dist wNan[i][j]>50:
      n_dist_wNan[i][j]=0
for i in range (1000):
 start3 = time.time()
 np.sum(n_dist_wNan)
 end3 = time.time()
  times_sum_all_d.append(end3 - start3)
average_sum_all_d=sum(times_sum_all_d)/len(times_sum_all_d)
print("Average time for summing all elements of the matrix with values greater than 50 and nans set to 0 is ", average_sum_all_d
    Average time for summing all elements of the matrix with values greater than 50 and nans set to 0 is 0.0019653208255767823
print((1-0.0019653208255767823/0.0023846786022186277)*100)
    17.58550507610076
```

Answer

In order to get a more accurate result, I take the average over 1000 repetitions of summing all values of the matrix.

Average time for summing all elements of the matrix with values greater than 50 and nans set to 0 is 0.0019653208255767823

The time when setting all values greater than 50 and nans to zero was smaller than taking the sum of all values. Specifically it was 17.58550507610076% smaller than just summing all values. Making many elements as zero simplifies the computation, so resulting a faster operation

▼ Part e

```
dist_mat_e=pd.read_csv("/content/TexasCityDistanceMatrix.xls")
ndist_mat_e=dist_mat_e.to_numpy()
for i in range(ndist_mat_e.shape[0]):
  for j in range(ndist mat e.shape[1]):
    if ndist_mat_e[i][j]==0:
      ndist_mat_e[i][j]=-2
n_diste_wNan=np.nan_to_num(ndist_mat_e)
spartse_e=csr_matrix(n_diste_wNan)
times_sum_all_e=[]
for i in range (1000):
 start4 = time.time()
 np.sum(spartse_e)
 end4 = time.time()
  times_sum_all_e.append(end4 - start4)
average_sum_all_e=sum(times_sum_all_e)/len(times_sum_all_e)
print("Average time for summing all elements of the matrix ",average_sum_all_e )
```

```
Average time for summing all elements of the matrix 0.000845179796218872

p_smaller_d=(1- average_sum_all_e/average_sum_all_d)*100

p_smaller_c=(1- average_sum_all_e/average_sum_all)*100

print("Percentage smaller than d", p_smaller_d)

print("Percentage smaller than c", p_smaller_c)

Percentage smaller than d 56.99532690949689

Percentage smaller than c 64.55791587878785
```

Answer

In order to get a more accurate result, I take the average over 1000 repetitions of summing all values of the matrix.

Average time for summing all elements of the matrix not considering all Nan values and keeping all original zeros, was 0.000845179796218872

There is a huge difference compared to the last two results. Specifically comparing to the sum done in part c, the time for summing all values now was 64.55791587878785% and compared to part d was 64.55791587878785% smaller.

The sum is now much faster because we are ignoring a great amount of the matrix elements. Given that the matrix is sparse with many Nans elements, by ignoring them the computing time is consideribly faster.

→ Part f

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We can exploit symmetry by just storing elements a_{ij} such that j > i. By doing this we are counting just once the distances because we are storing the upper elements of the matrix. If we are interested in summing all elements of the matrix we just have to sum all stored a_{ij} such that j > i and multiply by two.

In the folling piece of code is shown that indeed we are getting the same sum if we just consider all values a_{ij} such that j > i. By this way we are saving just N(N-1)/2 elements instead of N^2

```
dist_mat_f=pd.read_csv("/content/TexasCityDistanceMatrix.xls")
ndist_mat_f=dist_mat_f.to_numpy()
original=np.nan_to_num(ndist_mat_f)

mod=np.nan_to_num(ndist_mat_f)

sum_mod=0
sum_original=0
for i in range(mod.shape[0]):
    for j in range(mod.shape[1]):
        sum_original=sum_original+1
        if j>i:
            sum_mod=sum_mod+1

print(sum_original)
print(sum_original)
```

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