Project Title: Adaptive Hybrid VEM-Deep Learning for Two-Dimensional Structural Problems with Uncertainty Estimation and PINN Comparison

Objective

This project aims to develop an **adaptive hybrid Virtual Element Method (VEM) - Deep Learning (DL) approach** for solving **two-dimensional structural mechanics problems under the Theory of Linear Elasticity**. The study will focus on implementing **uncertainty-aware adaptive refinement strategies**, extending the hybrid method to more complex **2D structural systems**, and **comparing the approach with Physics-Informed Neural Networks (PINNs)** to assess its advantages and limitations.

Motivation

Traditional numerical methods such as **Finite Element Method (FEM) and Virtual Element Method (VEM)** require computationally expensive meshing and numerical system solutions, especially for high-resolution problems. Deep Learning offers the potential for **fast inference**, but its accuracy and generalization depend on data availability and the training process. By combining these methods, the hybrid VEM-Deep Learning approach aims to **preserve the accuracy of VEM while leveraging the speed of deep learning inference**, making it particularly relevant for **adaptive mesh refinement strategies** and real-time structural analysis. Furthermore, **uncertainty estimation** can help refine the mesh where the neural network lacks confidence, improving computational efficiency.

Research Plan

The project will be divided into three interconnected phases:

Phase 1: Adaptive Refinement Strategies for Hybrid VEM-Deep Learning

Goal

Develop a self-adaptive hybrid VEM-Deep Learning framework that dynamically refines the mesh based on model uncertainty.

Key Steps

1. **Uncertainty Quantification:**

- Implement **Bayesian Deep Learning** (e.g., Monte Carlo Dropout) to estimate model uncertainty.

- Analyze confidence intervals in the **displacement field predictions**.

2. **Adaptive Mesh Refinement:**

- Define an **error metric** based on uncertainty estimates to refine the VEM mesh adaptively.

- Implement an **iterative refinement process**, where the mesh is refined in regions where the neural network has high uncertainty or large prediction errors.

3. **Performance Evaluation:**

- Compare **computational cost and accuracy** of the adaptive model vs. fixed-resolution hybrid and pure VEM approaches.
- Assess whether the uncertainty-aware refinement improves the model's **efficiency and robustness**.

Expected Outcome

A hybrid model that **balances computational efficiency and accuracy** by refining only where necessary, reducing the computational cost of high-resolution VEM simulations.