Time-resolved optical conductivity and Higgs oscillations in two-band dirty superconductors

Rafael Haenel,[1,2] Paul Froese,[1,2] Dirk Manske,[1] and Lukas Schwarz[1]

[1] *Max Planck Institute for Solid State Research, 70569 Stuttgart, Germany*
[2] *Quantum Matter Institute, University of British Columbia, Vancouver V6T 1Z4, Canada*

(Dated: October 16, 2020)

...

I. INTRODUCTION

- Ultrafast spectroscopy
- Collective modes in superconductors: Higgs, Goldstone (shifted to plasma energy due to Anderson-Higgs)
- In two-band superconductors: Additional out-of-phase Leggett mode, can couple to Higgs in nonequilibrium
- Difficulties excitation Higgs mode, in clean-limit only weak coupling
- In dirty superconductors: Coupling is enhanced
- This work: 1) Higgs oscillations in two-band sc. with bands in different limits, 2) Nonequilibrium optical conductivity, 3) Leggett mode in dirty-limit, 4) Prediction for MgB_2

II. MODEL

- In this section show only the final formula, all derivation into the appendix A as the equations are similar to the Murotani paper.
- Show Hamiltonian, gap equation, Mattis-Bardeen replacement, general approach for calculating time evolution...
- I suggest putting the (final) equations for current, optical conductivity, $\delta\Delta(t)$ into the respective section, but in principle we could also put all equations in this section and show only the results in the following sections
- Used parameters (take general parameters for section III and IV where no Leggett mode occurs, Leggett mode is discussed later separately and MgB_2 will be also discussed later. Show the used parameters in these section)
- Implementation details? Are there any subtle points?

$$H_{\mathrm{BCS}} = \sum_{i\mathbf{k}\sigma} \varepsilon_{i\mathbf{k}} c^\dagger_{i\mathbf{k}\sigma} c_{i\mathbf{k}\sigma} + \sum_{i\mathbf{k}} \left(\Delta_i c^\dagger_{i-\mathbf{k}\uparrow} c^\dagger_{i\mathbf{k}\downarrow} \right) , \quad (1)$$

where $\varepsilon_{i\mathbf{k}} = s_i \left(\mathbf{k}^2/2m_i - \varepsilon_{F_i} \right)$ and the superconducting order parameter is self-consistently determined by $\Delta_i = \sum_{j\mathbf{k}} U_{ij} \langle c_{j-\mathbf{k}\downarrow} c_{j\mathbf{k}\uparrow} \rangle$.

$$H_{\text{p-p}} = -\sum_{i\mathbf{k}\mathbf{k}'\sigma} \mathbf{J}_{i\mathbf{k}\mathbf{k}'} \cdot \mathbf{A}\, c^\dagger_{i\mathbf{k}\sigma} c_{i\mathbf{k}'\sigma} + \sum_{i\mathbf{k}\sigma} \frac{s_i e^2}{2m_i} \mathbf{A}^2\, c^\dagger_{i\mathbf{k}\sigma} c_{i\mathbf{k}\sigma}$$

$$\begin{aligned} \langle |\mathbf{e}\cdot\mathbf{J}_{i\mathbf{k}\mathbf{k}'}|^2 \rangle_{\mathrm{Av}} &= \int \frac{d\Omega_{\mathbf{k}}}{4\pi} \frac{d\Omega'_{\mathbf{k}}}{4\pi} |\mathbf{e}\cdot\mathbf{J}_{i\mathbf{k}\mathbf{k}'}|^2 \\ &\approx \frac{(ev_{F_i})^2}{3\pi N_i(0)} \frac{\gamma_i}{(\varepsilon-\varepsilon')^2+\gamma_i^2} \end{aligned}$$

Discussion of A, A^2.
The full Hamiltonian is given by $H = H_{BCS} + H_{p-p}$.

III. SINGLE-BAND SUPERCONDUCTIVITY

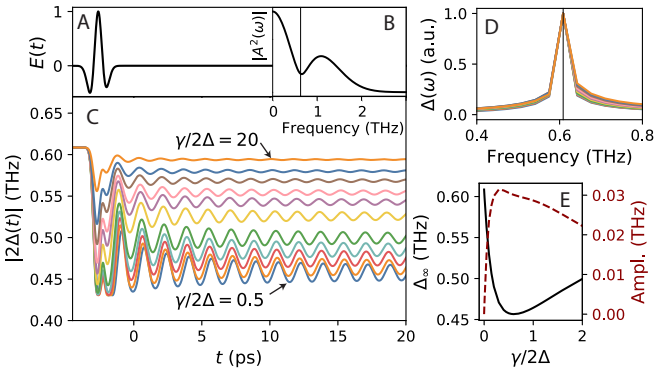

Figure 1. ABC

IV. HIGGS OSCILLATIONS

- Equation for Current
- Equation for optical conductivity
- Present and discuss equilibrium optical conductivity in Fig. 1 for the four different limit cases, used in the rest of the paper

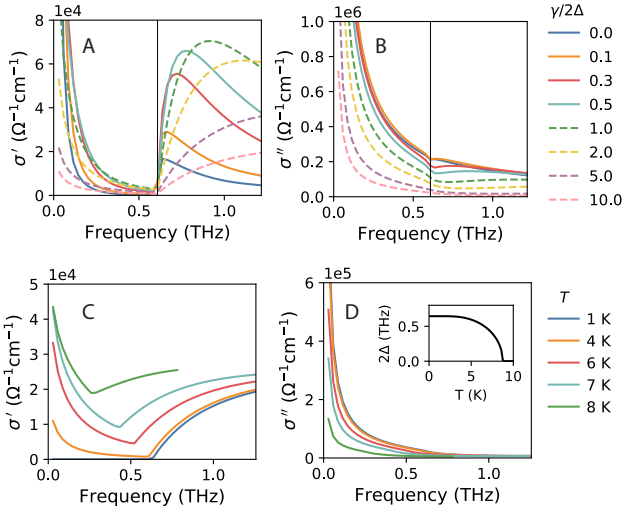

Figure 2. ABC

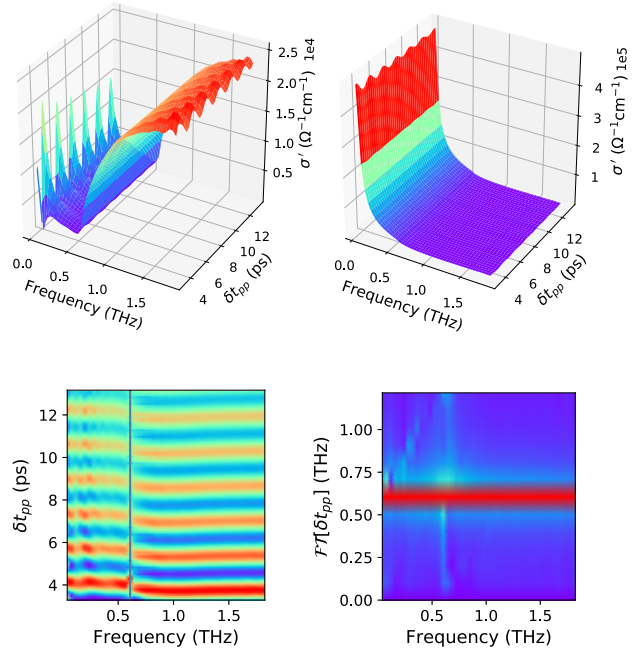

Figure 3. ABC

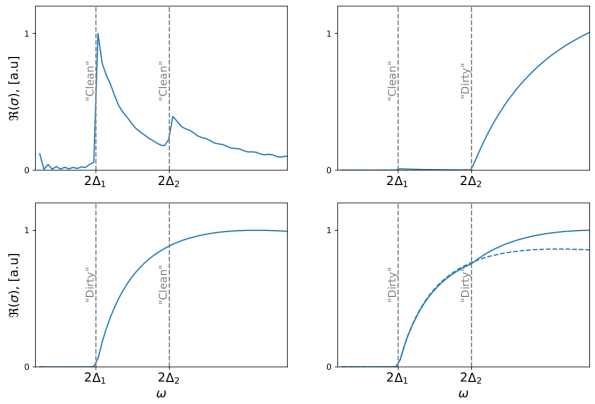

Figure 4. Equilibrium optical conductivity of a two-band superconductor with gaps in different impurity scattering limits (clean-clean case rescaled as response is much smaller?)

- Equation for $\delta\Delta(t)$
- Show and discuss Higgs oscillations in Fig. 2 of the four cases with a suitable pump pulse (refer to appendix B for details about the pump pulse)

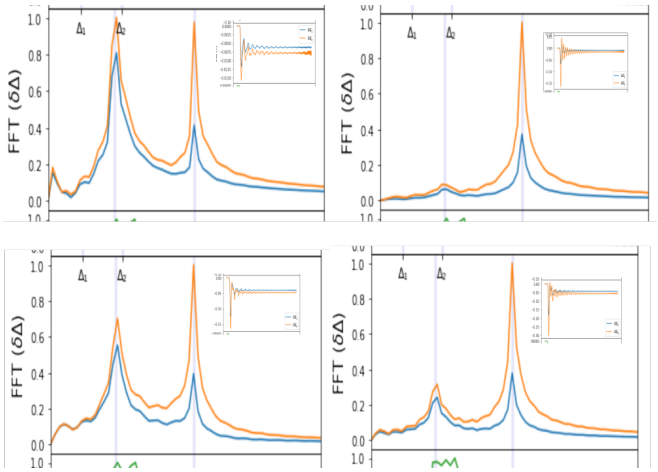

Figure 5. Higgs oscillations excited by a pump pulse with enough bandwidth to cover both gaps for the different impurity scattering limits shown in Fig. 4

V. NONEQUILIBRIUM OPTICAL CONDUCTIVITY

- Equation for nonequilibrium optical conductivity with two pulses. Maybe here more details (instead of appendix), as this is not covered by the Murotani paper
- Show and discuss nonequlibrium conductivity in Fig. 3.

- More figures of nonequilibrium conductivity? Suggestions: Imaginary part, maybe appendix. Nice 3d plot. Oscillations along cuts.

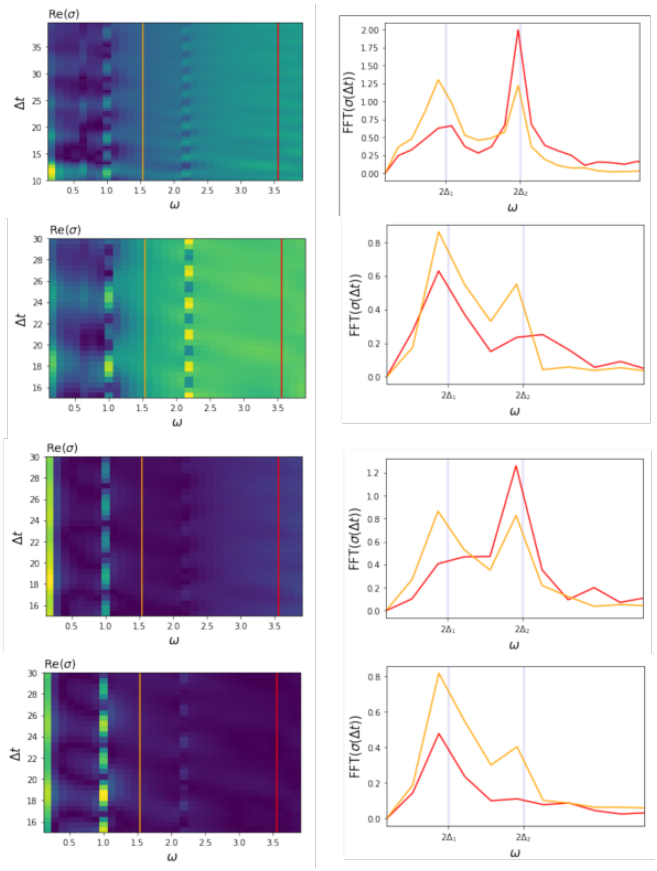

Figure 6. Nonequilibrium optical conductivity of a pump-probe experiment for the different impurity scattering limits shown in Fig. 4

VI. LEGGETT MODE

- Definition, equation of Leggett mode
- Parameters for Fig.4 where Leggett mode occurs
- Discuss Fig. 4
- Maybe rethink what exactly to show in Fig. 4
- New plot of time-resolved conductivity showing Leggett mode?

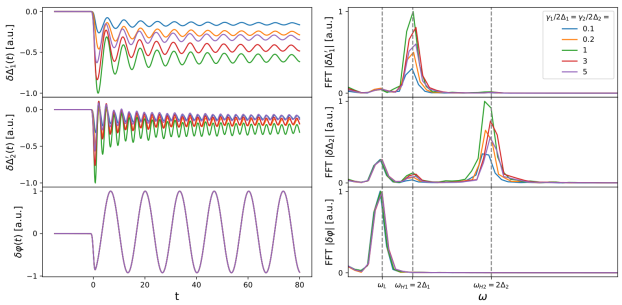

Figure 7. Induced amplitude (Higgs) and out-of-phase (Leggett) oscillations of a two-band superconductor for different impurity scattering rates

- Discuss Fig. 5 for varying coupling strength and compare with clean limit result

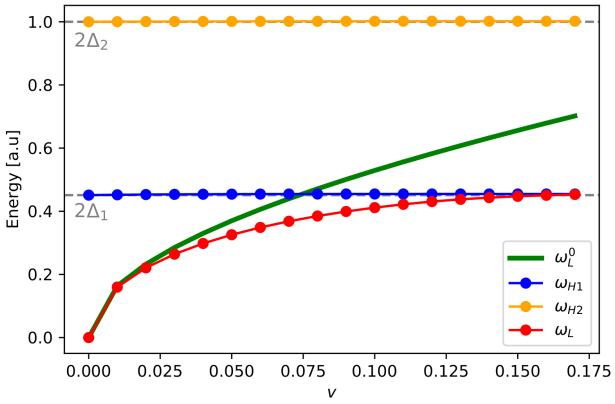

Figure 8. Frequency of Leggett mode as function of interband coupling strength in the dirty limit

VII. MgB_2

- Parameters which match MgB_s
- Show prediction in Fig. 6 of equilibrium conductivity, Higgs oscillations, nonequilibrium conductivity

TODO

Figure 9. Result with parameters for MgB_2 a) Equilibrium optical conductivity, b) Higgs oscillations, c) pump-probe conductivity and spectra

VIII. CONCLUSION

...

ACKNOWLEDGMENTS

...

Appendix A: Derivation of nonequilibrium optical conductivity

- Put here all equations and derivations of the main results

Appendix B: Influence of pump pulse frequency

- Discuss influence of pump pulse frequency and bandwith to excite only one or both Higgs mode

- Show result in Fig. 7

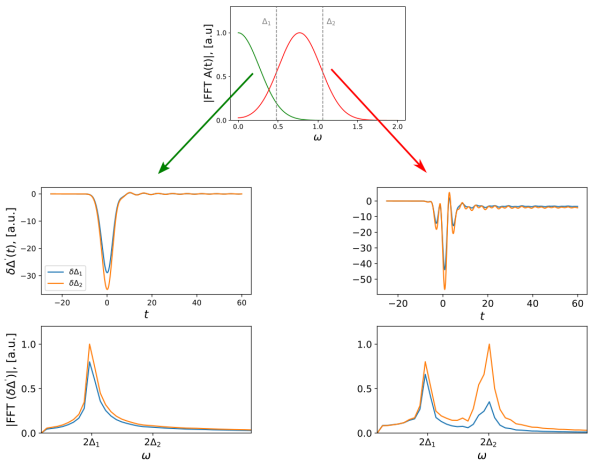

Figure 10. Influence of pump pulse frequency