Incoherent tunneling and topological superconductivity in twisted cuprate bilayers

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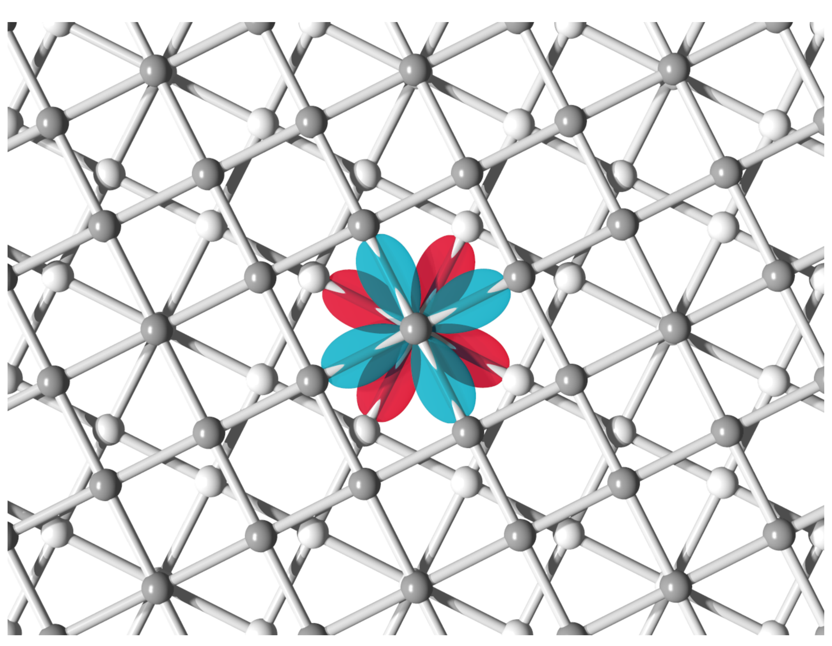
Summary

We assess the influence of incoherent tunneling on the phase diagram of a twisted cuprate bilayer, previously shown to engender a chiral topological state with spontaneously broken time-reversal symmetry $\mathcal{T}$ in the absence of incoherence. We show that the bilayer system continues to support a fully gapped topological phase, even in the limit of strong disorder-mediated interlayer coupling.

Introduction



A pair of cuprate monolayers, stacked and twisted to 45°, spontaneously breaks time-reversal symmetry $\mathcal{T}$ [1]. This can be understood from a Landau-Ginzburg theory [2] with two order parameters $\Psi_i e^{\pm i\varphi/2}$ and free energy

$$\mathcal{F}(\varphi) = -B\,\Psi_1^2\Psi_2\cos\varphi + C\,\Psi_1\Psi_2^2\cos 2\varphi\,. \tag{1}$$

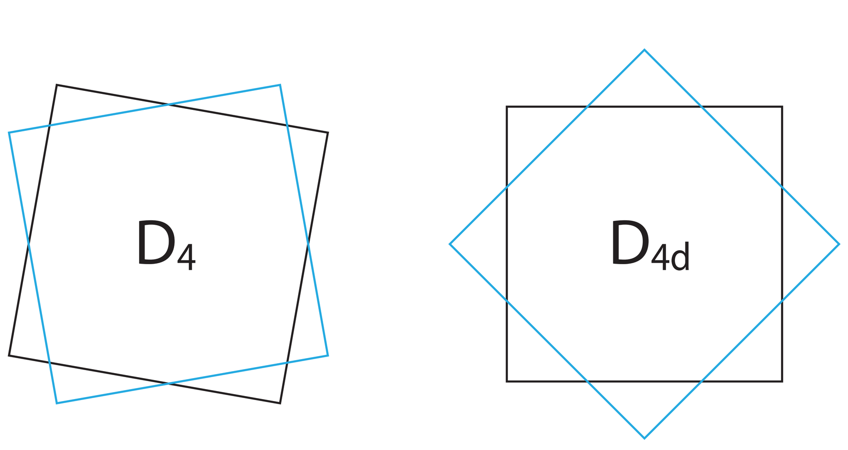
At 45° twist, the point group changes from D_4 to D_{4d} with the additional symmetry element in the d-wave E_2 irrep

$$S_8 : (\Psi_1, \Psi_2) \to (\Psi_2, -\Psi_1)$$

that only leaves the $\cos 2\varphi$ term in Eq. 1 invariant. Thus, the free energy becomes $\mathcal{F}(\varphi) = C\,\Psi_1^2\Psi_2^2\cos 2\varphi$ whose minima spontaneously break $\mathcal{T}$ for $C > 0$.

D_4	Basis functions	E	$2C_4$	C_2	$2C_2'$	$2C_2''$
A_1	1	1	1	1	1	1
A_2		1	1	1	-1	-1
B_1	x^2-y^2	1	-1	1	1	-1
B_2	xy	1	-1	1	-1	1
E	(x,y)	2	0	-2	0	0

D_{4d}	Basis functions	E	$2S_8$	$2C_4$	$2S_8^3$	C_2	$4C_2'$	$4\sigma_d$
A_1	1	1	1	1	1	1	1	1
A_2		1	1	1	1	1	-1	-1
B_1	$(x^2-y^2)^2-4x^2y^2$	1	-1	1	-1	1	1	-1
B_2	$xy(x^2-y^2)$	1	-1	1	-1	1	-1	1
E_1	(x,y)	2	$\sqrt{2}$	2	-$\sqrt{2}$	-2	0	0
E_2	(x^2-y^2, xy)	2	0	-2	0	2	0	0
E_3	$(y(3x^2-y^2), x(x^2-3y^2))$	2	-$\sqrt{2}$	0	$\sqrt{2}$	-2	0	0



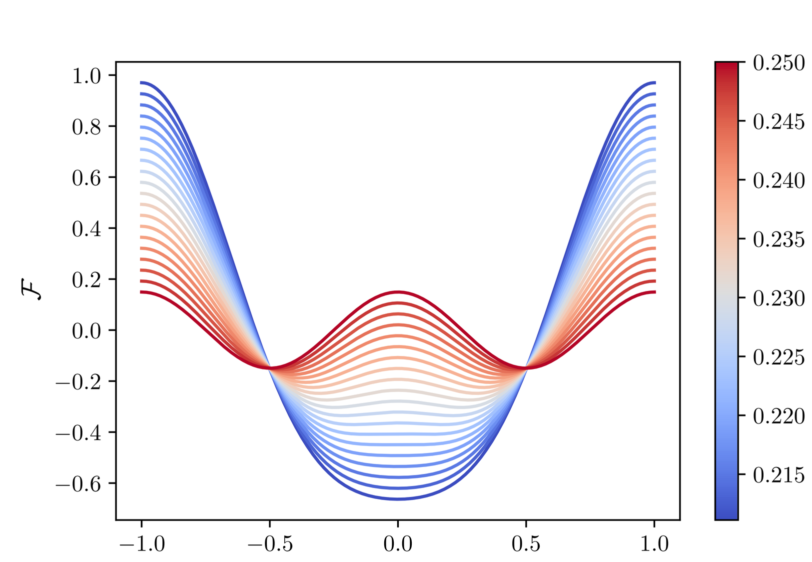


Figure 1: Free energy $\mathcal{F}(\varphi)$ as a function of twist angle θ.

Model

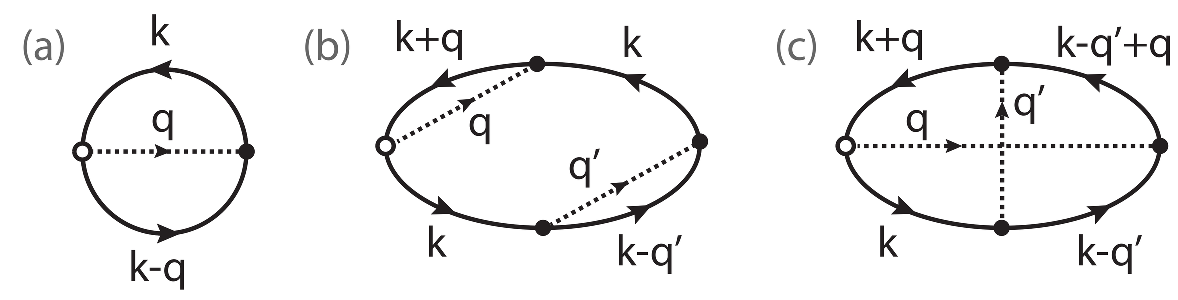


Figure 2: Diagrammatic expansion of the interlayer current (a) at order g^2 and (b-c) at order g^4. Full lines correspond to electronic propagators G_0, dashed lines correspond to impurity vertices paired by disorder average.

The Hamiltonian of the uncoupled layers is given by

$$\mathcal{H}_0 = \sum_{\mathbf{k}l} \Psi_{\mathbf{k}l}^\dagger \left(\xi_\mathbf{k}\sigma_z + \Delta'_{\mathbf{k}l}\sigma_x - \Delta''_{\mathbf{k}l}\sigma_y,\right)\Psi_{\mathbf{k}l} \tag{2}$$

with $\xi_\mathbf{k} = \mathbf{k}^2/2m - \mu$ and the superconducting gaps

$$\Delta_{\mathbf{k}1} = \Delta e^{i\varphi/2}\cos(2\alpha_\mathbf{k} - \theta)$$
$$\Delta_{\mathbf{k}2} = \Delta e^{-i\varphi/2}\cos(2\alpha_\mathbf{k} + \theta)\,. \tag{3}$$

Here, $\alpha_\mathbf{k}$ is the polar angle and the bilayers are twisted by θ. The interlayer coupling term is

$$\mathcal{H}' = \sum_{\mathbf{kq}} \gamma_\mathbf{q} c^\dagger_{\mathbf{k},1} c_{\mathbf{k}-\mathbf{q},2} + \text{h.c.} \tag{4}$$

where disorder is captured via a set of Gaussian-distributed random variables $\gamma_\mathbf{q}$ of average $\overline{\gamma_\mathbf{q}} = 0$ and variance given by

$$\overline{\gamma^*_\mathbf{q}\gamma_{\mathbf{q}+\mathbf{p}}} = \frac{1}{N}\frac{4\pi g^2}{3\Lambda^2}\delta_{\mathbf{p},0}e^{-\mathbf{q}^2/\Lambda^2}\,. \tag{5}$$

The scale Λ defines the momentum non-conservation of the interlayer tunneling. The model becomes coherent in the limit $\Lambda \to 0$. We compute the interlayer current

$$J = \sum_{\mathbf{kq}} ie^{i\varphi/2}\overline{\gamma_\mathbf{q}\langle c^\dagger_{\mathbf{k},1}c_{\mathbf{k}-\mathbf{q},2}\rangle} + \text{h.c.} = \text{Tr}\left[\overline{j_\mathbf{q} G(\mathbf{k},\mathbf{k}-\mathbf{q},\omega_n)}\right]$$

by a diagrammatic expansion up to fourth order in g. Here, $G(\mathbf{k},\mathbf{k}',\tau) = \langle T_\tau c_\mathbf{k}(\tau)c^\dagger_{\mathbf{k}'}(0)\rangle$ is the full imaginary time ordered Green's function of the disordered system The φ-dependence of the free energy can be extracted from the Josephson relation $J(\varphi) = 2\partial\mathcal{F}(\varphi)/\partial\varphi$ and is used to evaluate the phase diagram of the $\mathcal{T}$-broken phase.

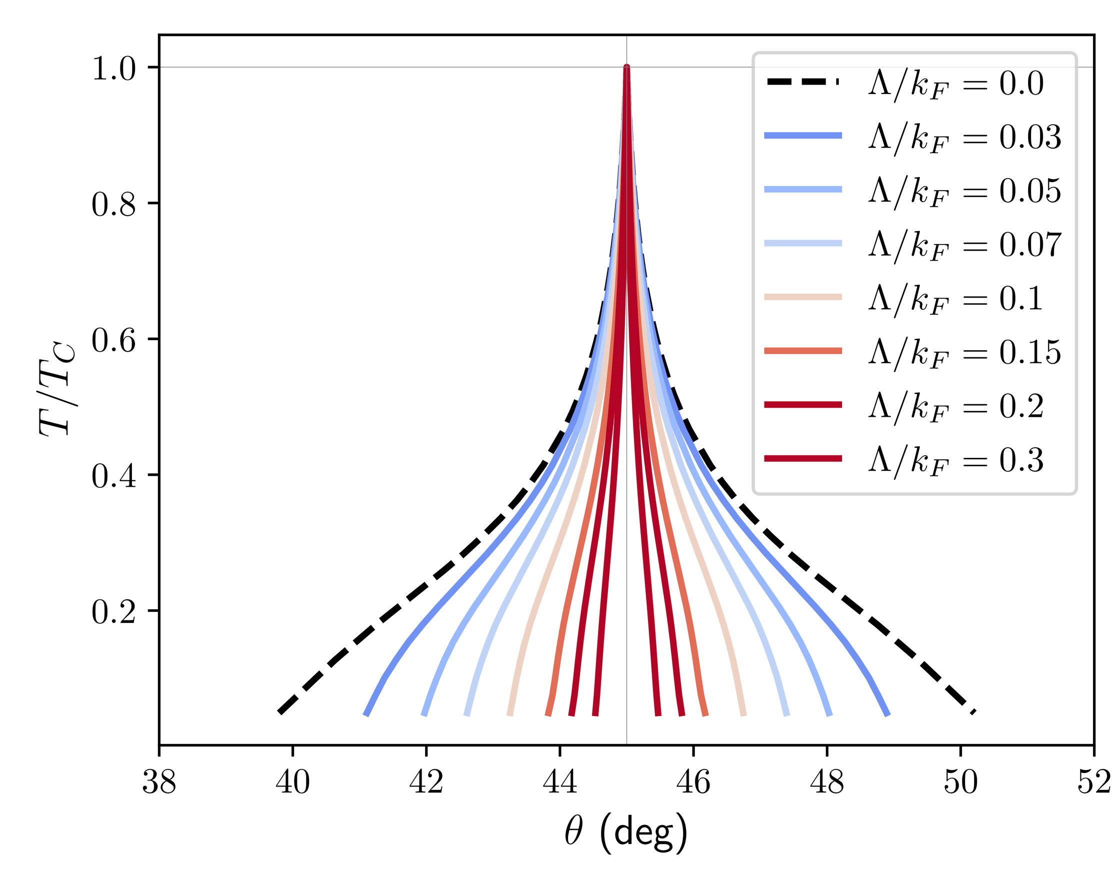


Figure 3: Phase diagram of incoherently coupled twisted bilayer cuprates. For a given Λ, the inside of the cone-shaped region breaks $\mathcal{T}$. Black-dashed lines mark the phase boundary in the clean limit, previously introduced in [1].

We confirmed the $\mathcal{T}$-breaking physics of the continuum model by exact diagonalization of a lattice model with open boundaries of part of the moire unit cell. This models a realistic electronic structure and also captures the effect of Brillouin zone folding.

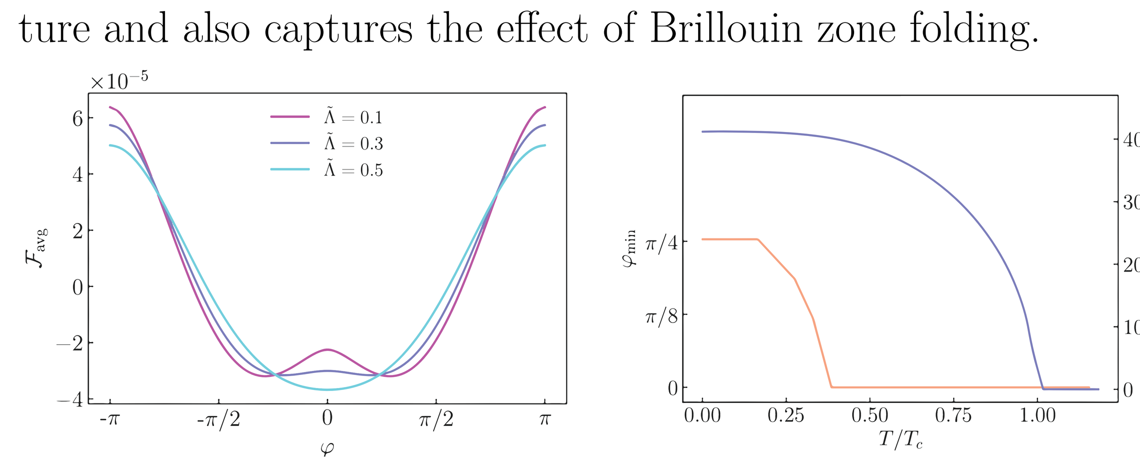


Figure 4: Disorder averaged free energy of the bilayer (left) at zero temperature as a function of the phase difference. The minima $\varphi_{\min}$ are situated away from zero at small disorder. Dependence of the order parameter amplitude and phase as a function of temperature for $= 0.2$ (right).

Topological superconductivity

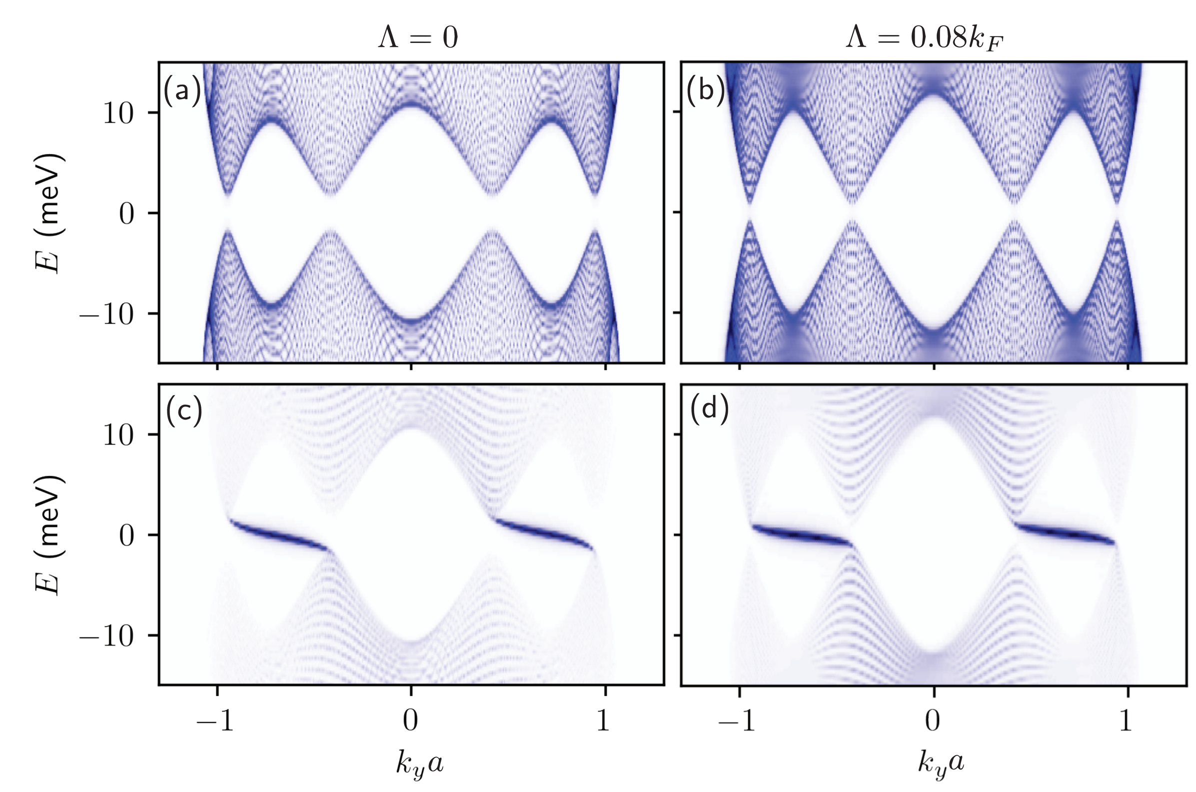


Figure 5: Bulk (a-b) and boundary (c-d) spectrum for incoherently coupled cuprate bilayers with $\Lambda = 0$ (left) and $\Lambda/k_F = 0.08$ (right) at 45° twist angle. The spectrum shows chiral edge modes traversing the bulk gap indicating a Chern number $\mathcal{C} = 4$.

To assess the topological character of the model, we examine the boundary physics with the surface Green function

$$G_B(x,k_y,\omega_n) = G(x,k_y) - G(x,k_y)T(k_y)G(-x,k_y)$$
$$T(k_y) = \left[\frac{1}{\sqrt{N}}\sum_{k_x} G(k_x,k_y)\right]^{-1}\,. \tag{6}$$

Here, G is the impurity averaged Green function of the coupled cuprate layers in the Born approximation. The surface spectral function clearly shows edge modes, indicative of a chiral phase with Chern number $\mathcal{C} = 4$.

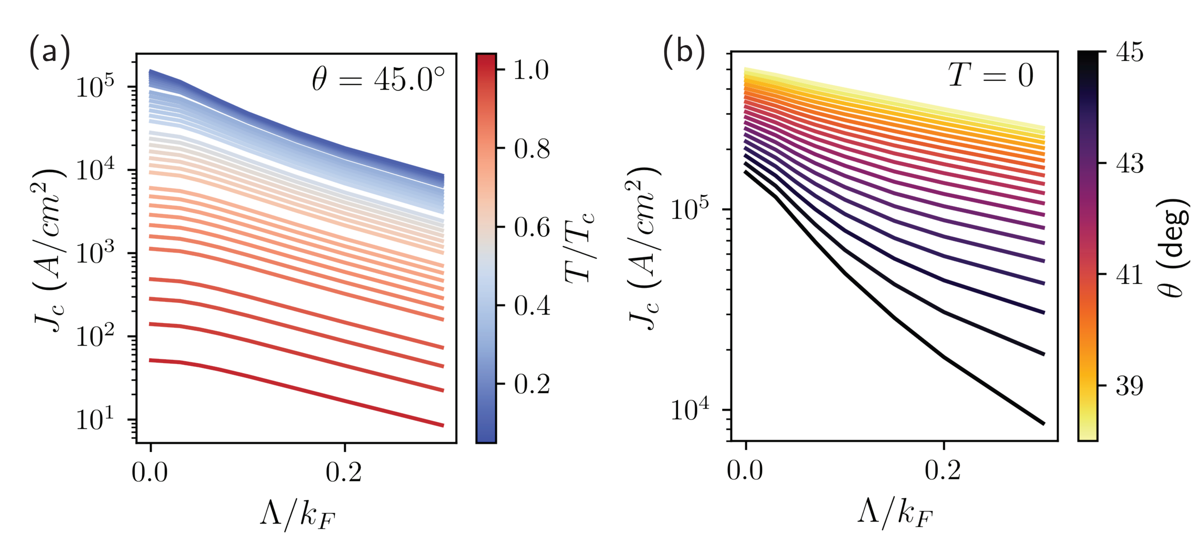


Figure 6: Critical current J_c of the twisted bilayer as a function as interlayer coherence scale Λ. Incoherence significantly reduces J_c. The color scale denotes temperature T in panel (a) and twist angle θ in (b).

Summary

We showed that a model of twisted cuprate bilayers with incoherent, impurity-mediated, interlayer tunneling processes gives rise to a broad topological chiral phase around a twist angle of 45° that spontaneously breaks time-reversal symmetry $\mathcal{T}$.

References

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