Lab 7: Amplitude Modulation and Complex Lowpass Signals

1 Introduction

Many channels either cannot be used to transmit baseband signals at all, or pass signal energy very inefficiently, except for a relatively narrow passband region at frequencies substantially higher than those contained in a baseband message signal. A well-known example is electromagnetic transmission of radio signals at a frequency f_c in free space which requires an antenna of length comparable to $\lambda_c/2$ for a dipole, or $\lambda_c/4$ for a monopole, where $\lambda_c = 3\times 10^8/f_c$ is the wavelength in meters corresponding to f_c in Hz. Thus, transmission at $f_c = 10$ kHz would require an antenna of length comparable to 15 km for a dipole, whereas at $f_c = 900$ MHz a length of 8.3 cm is enough for the monopole antenna of a cell phone.

1.1 Amplitude Modulation with Suppressed Carrier

The most straightforward way to shift a signal spectrum from baseband to a passband location with center frequency f_c is to make use of the frequency shift property of the Fourier transform (FT) which says that

$$m(t)\, e^{j(2\pi f_c t+\theta_c)} \qquad \Longleftrightarrow \qquad M(f-f_c)\, e^{j\theta_c} \,.$$

Thus, $A_c\, m(t)\, e^{j(2\pi f_c t+\theta_c)}$ is a complex-valued bandpass signal with amplitude A_c and center frequency f_c if $m(t)$ is a (bandlimited) baseband signal. To make this into a real bandpass signal $x(t)$, write

$$\begin{aligned} x(t) &= \mathrm{Re}\{A_c\, m(t)\, e^{j(2\pi f_c t+\theta_c)}\} = \mathrm{Re}\big\{A_c\, m(t)\, \big(\cos(2\pi f_c t+\theta_c) + j\sin(2\pi f_c t+\theta_c)\big)\big\} \\ &= A_c\, m(t)\, \cos(2\pi f_c t+\theta_c)\,, \end{aligned}$$

where the last equality assumes that $A_c\, m(t)$ is real-valued. The signal $x(t)$ obtained in this way is a **AM-DSB-SC** (amplitude modulation, double side-band, suppressed carrier) signal with **carrier frequency** f_c, **carrier phase** θ_c and Fourier transform

$$x(t) = A_c\, m(t)\, \cos(2\pi f_c t+\theta_c) \quad \Longleftrightarrow \quad X(f) = \frac{A_c}{2}\big[M(f-f_c)\, e^{j\theta_c} + M(f+f_c)\, e^{-j\theta_c}\big]\,.$$

Starting from

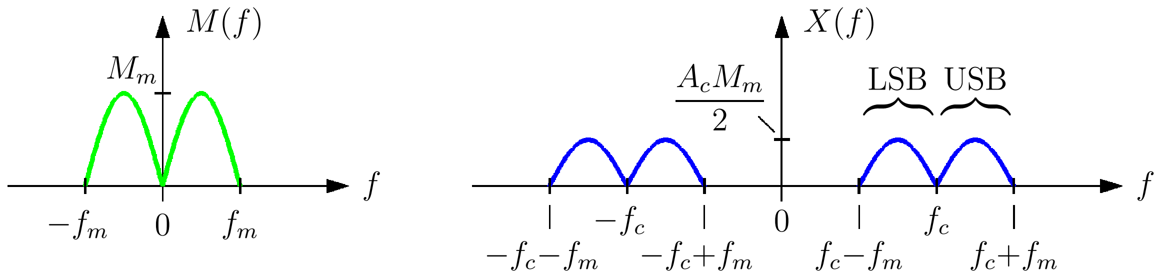
$$x(t) = \mathrm{Re}\{A_c\, m(t)\, e^{j(2\pi f_c t+\theta_c)}\} = A_c\, m(t)\, \frac{e^{j(2\pi f_c t+\theta_c)} + e^{-j(2\pi f_c t+\theta_c)}}{2}\,,$$

we could also have derived this as

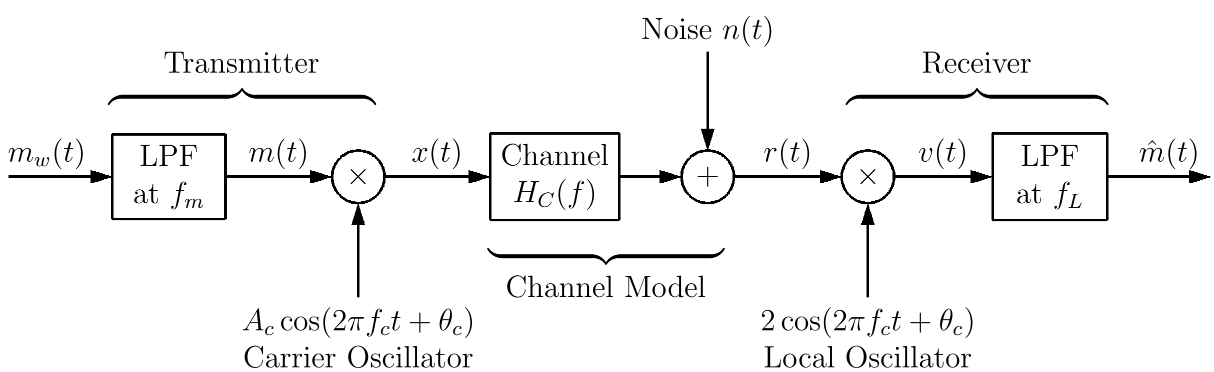
$$X(f) = A_c\,M(f) * \Big[\frac{\delta(f-f_c)\,e^{j\theta_c} + \delta(f+f_c)\,e^{-j\theta_c}}{2}\Big] = \frac{A_c}{2}\,\big[M(f-f_c)\,e^{j\theta_c} + M(f+f_c)\,e^{-j\theta_c}\big]\,.$$

What is important to note here is that taking the real (or imaginary) part of a signal in the time domain is an operation that has a well-defined and easy to evaluate counterpart in the frequency domain.

In the frequency domain $x(t) \Leftrightarrow X(f)$ can be visualized as follows (assuming $\theta_c = 0$ for simplicity)



From the figure it is evident that if the bandwidth of $m(t)$ is f_m, then the bandwidth of $x(t)$ is $2f_m$, which explains the "DSB" in AM-DSB-SC. It is also clear that if $m(t)$ has no dc component (which is the case for speech and music signals, for instance), then $x(t)$ has no component at the carrier frequency f_c, which is where the "SC" comes from. The portion of the spectrum of $x(t)$ for which $f_c - f_m \leq |f| < f_c$ is called the **lower side-band (LSB)**, whereas the portion for which $f_c < |f| \leq f_c + f_m$ is called the **upper side-band (USB)**. To recover $m(t)$ undistorted from $x(t)$, $f_c \geq f_m$ is required, but usually $f_c \gg f_m$ in practice. The block diagram of an AM-DSB-SC transmission system is shown in the following figure.



The transmitter consists of a LPF that bandlimits the wideband message signal $m_w(t)$ to $|f| \leq f_m$ and the modulator which multiplies the resulting message signal $m(t)$ with the output $A_c \cos(2\pi f_c t + \theta_c)$ of the carrier oscillator. The channel is modeled as a filter $H_C(f)$ with noise added at the output. In the receiver the incoming signal $r(t)$ is multiplied by the local oscillator signal $2\,\cos(2\pi f_c t + \theta_c)$ and then lowpass filtered at f_L. Assuming an ideal channel with attenuation γ and no noise such that $r(t) = \gamma\, x(t)$, the demodulation operation can be described as

$$v(t) = 2r(t)\cos(2\pi f_c t+\theta_c) = 2\gamma A_c m(t)\cos^2(2\pi f_c t+\theta_c) = \gamma A_c m(t)\big(1+\cos(4\pi f_c t+2\theta_c)\big)\ .$$

Assuming that $f_c \geq f_m$, the second term, which is a AM-DSB-SC signal with carrier frequency $2f_c$ and carrier phase $2\theta_c$, can be removed by lowpass filtering at $f_L = f_m$ and thus

$$\hat{m}(t) = \gamma A_c\, m(t)\ .$$

In the absence of noise and other channel impairments this is an exact replica of the transmitted message signal, scaled by γA_c.

If $m(t)$ is a wide-sense stationary process with autocorrelation function $R_m(\tau)$, then the autocorrelation function of the AM-DSB-SC signal $x(t)$ can be computed as

$$\begin{aligned} R_x(t_1,t_2) &= E\big[A_c\, m(t_1)\, \cos(2\pi f_c t_1+\theta_c)\, A_c^*\, m^*(t_2)\, \cos(2\pi f_c t_2+\theta_c)\big] \\ &= |A_c|^2\ \underbrace{E[m(t_1)\, m^*(t_2)]}_{=R_m(t_1-t_2)}\ \ \underbrace{\cos(2\pi f_c t_1+\theta_c)\, \cos(2\pi f_c t_2+\theta_c)}_{=\frac{1}{2}\big[\cos\big(2\pi f_c(t_1-t_2)\big)+\cos\big(2\pi f_c(t_1+t_2)+2\theta_c\big)\big]} \\ &= \frac{|A_c|^2}{2}\, R_m(t_1-t_2)\, \big[\cos\big(2\pi f_c(t_1-t_2)\big)+\cos\big(2\pi f_c(t_1+t_2)+2\theta_c\big)\big]\ . \end{aligned}$$

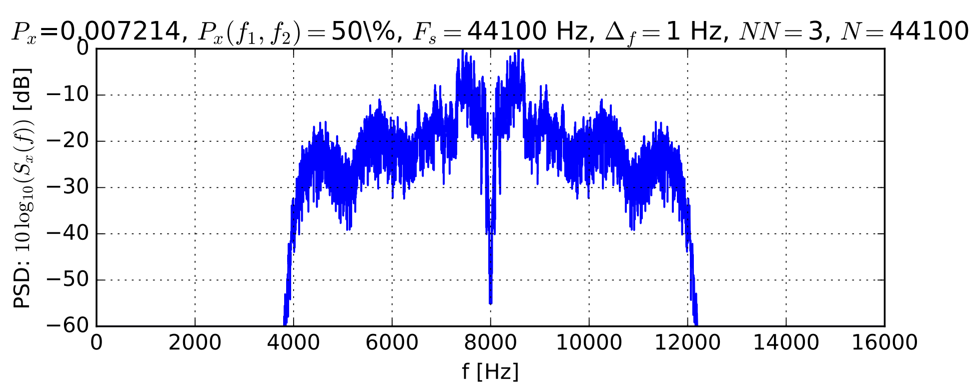
Note that $x(t)$ is a cyclostationary process with period $1/f_c$. The time-averaged autocorrelation function of $x(t)$ is

$$\bar{R}_x(\tau) = f_c\int_0^{1/f_c} R_x(t+\tau,t)\, dt = \frac{|A_c|^2}{2}\, R_m(\tau)\, \cos(2\pi f_c\tau)\ .$$

Thus, if $m(t)$ has PSD $S_m(f)$, then the PSD of the AM-DSB-SC signal $x(t)$ is

$$S_x(f) = \frac{|A_c|^2}{4}\, \big[S_m(f-f_c)+S_m(f+f_c)\big]\ .$$

The PSD of a speech signal after AM-DSB-SC modulation with $f_c = 8000$ Hz and $f_m = 4000$ Hz is shown in the following graph.



1.2 Coherent AM Reception

An idealizing assumption which is tacitly made in the AM-DSB-SC transmission system block diagram given earlier, is that the local oscillator at the receiver is synchronized with the carrier oscillator at the transmitter. To see why this synchronism between transmitter and receiver is important, assume that the local oscillator signal is $2\cos(2\pi f_c t)$, but the received AM-DSB-SC signal is $r(t) = \gamma A_c m(t)\cos(2\pi(f_c+f_e)t+\theta_e)$, i.e., there is a frequency error f_e and a phase error θ_e between transmitter and receiver. Now the receiver computes

$$\begin{aligned} v(t) &= 2\gamma A_c\, m(t)\, \cos(2\pi(f_c+f_e)t+\theta_e)\, \cos(2\pi f_c t) \\ &= \gamma A_c\, m(t)\, \big[\cos(2\pi f_e t+\theta_e) + \cos\big(2\pi(2f_c+f_e)t+\theta_e\big)\big] \,, \end{aligned}$$

and thus (for sufficiently small f_e)

$$\hat{m}(t) = \gamma A_c\, m(t)\, \cos(2\pi f_e t+\theta_e) \,,$$

after the LPF at $f_L = f_m$. When $f_e = 0$, a small phase error $|\theta_e| \ll \pi/2$ attenuates $m(t)$ by $\cos(\theta_e) \approx 1$, which presents no big problem, but a phase error close to $\pm\pi/2$ attenuates $m(t)$ substantially or even suppresses it altogether. If f_e is non-zero, then θ_e does not matter and $\hat{m}(t)$ changes periodically in intensity because of the multiplication with $\cos(2\pi f_e t)$, which is quite annoying.

On the positive side, however, the fact that $m(t)\, \cos(\theta_e) = 0$ for $\theta_e = \pm\pi/2$ means that two AM-DSB-SC signals, such as

$$x_i(t) = A_c m_i(t)\cos(2\pi f_c t) \,, \qquad \text{and} \qquad x_q(t) = A_c m_q(t)\cos(2\pi f_c t+\pi/2) \,,$$

can use the same carrier frequency f_c to transmit two independent message signals $m_i(t)$ and $m_q(t)$. This is known as **quadrature amplitude modulation (QAM)**, and $x_i(t)$ is called the **in-phase component** of the AM signal at f_c, whereas $x_q(t)$ is called the **quadrature component**. At any rate, it is crucial for the correct demodulation of AM signals with suppressed carrier, that the receiver is phase (and frequency) synchronized with the transmitter. Receivers of this type are called synchronous or **coherent receivers**. In practice the maintenance of exact phase synchronism between two oscillators in different physical locations is quite a non-trivial problem and requires a considerable amount of active hardware and/or software.

1.3 Complex-Valued Lowpass Signals

A QAM signal $x(t)$ is of the form

$$x(t) = x_i(t) + x_q(t) = A_c\, m_i(t)\, \cos(2\pi f_c t) + A_c\, m_q(t)\, \cos(2\pi f_c t+\pi/2) \,,$$

with Fourier transform

$$X(f) = \frac{A_c}{2}\big[M_i(f-f_c) + j\, M_q(f-f_c) + M_i(f+f_c) - j\, M_q(f+f_c)\big] \,.$$

The two baseband signals $m_i(t) \Leftrightarrow M_i(f)$ and $m_q(t) \Leftrightarrow M_q(f)$ are real-valued, bandlimited to f_m, and independent of each other. Since the overall signal $x(t)$ has bandwidth $2f_m$, using QAM is one way of avoiding the doubling of the bandwidth associated with amplitude modulation.

More generally, let

$$x_L(t) = m_i(t) + j\, m_q(t)$$

be a complex-valued lowpass signal with bandwidth f_m, made up from the real-valued signals $m_i(t)$ and $m_q(t)$. Then we can obtain a real-valued QAM bandpass signal in two steps as follows. In the first step $x_L(t)$ is multiplied by A_c and shifted right by f_c in the frequency domain to obtain the complex-valued signal $x_u(t)$ as

$$x_u(t) = A_c\, x_L(t)\, e^{j2\pi f_c t} \,.$$

In the second step the real-valued QAM signal $x(t)$ is obtained by

$$x(t) = \mathrm{Re}\{x_u(t)\} = \frac{x_u(t) + x_u^*(t)}{2} \,.$$

In the frequency domain this corresponds to

$$X(f) = \frac{X_u(f) + X_u^*(-f)}{2} = \frac{A_c}{2}\left[M_i(f-f_c) + j\, M_q(f-f_c) + M_i^*(-f-f_c) - j\, M_q^*(-f-f_c)\right] .$$

Since $m_i(t)$ and $m_q(t)$ are real-valued, we have $M_i(f) = M_i^*(-f)$ and $M_q(f) = M_q^*(-f)$ and therefore

$$X(f) = \frac{A_c}{2}\left[M_i(f-f_c) + j\, M_q(f-f_c) + M_i(f+f_c) - j\, M_q(f+f_c)\right] .$$

Thus, the $x(t) \Leftrightarrow X(f)$ obtained in this way is the same as the one we obtained before from $x(t) = x_i(t) + x_q(t)$.

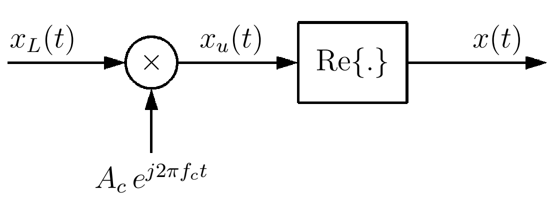
To demodulate the QAM signal $x(t)$ and recover $x_L(t)$ and therefore $m_i(t)$ and $m_q(t)$ as the real and imaginary parts of $x_L(t)$, we can again use the frequency shift property of the FT. We multiply $x(t)$ by $2\, e^{-j2\pi f_c t}$ to obtain

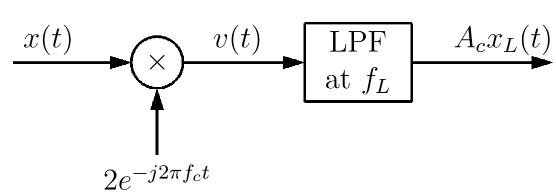
$$v(t) = x(t)\, 2\, e^{-j2\pi f_c t} = [x_u(t) + x_u^*(t)]\, e^{-j2\pi f_c t} = A_c\, [x_L(t) + x_L^*(t)\, e^{-j4\pi f_c t}] \,.$$

After lowpass filtering at $f_L = f_m$ this yields

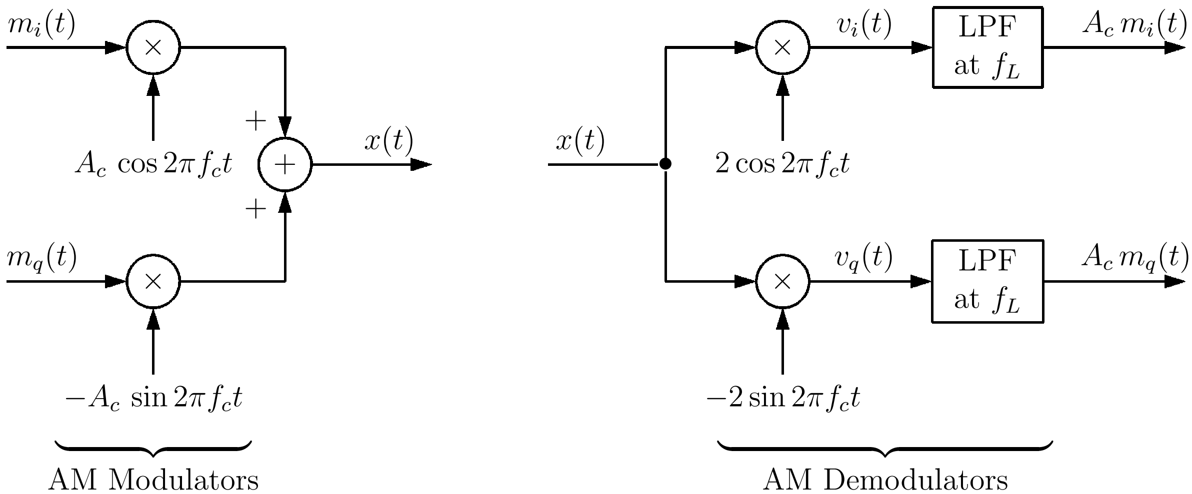
$$\hat{x}_L(t) = \mathrm{LPF}\{v(t)\} = A_c\, x_L(t) \,.$$

Graphically, QAM modulation and demodulation using complex-valued lowpass signals can be visualized as follows.





Using $x_L(t) = m_i(t) + j\, m_q(t)$ and $e^{\pm j2\pi f_c t} = \cos(2\pi f_c t) \pm j\, \sin(2\pi f_c t)$, this can also be implemented using only real-valued signals as shown in the next blockdiagram.



Note that $-\sin(2\pi f_c t) = \cos(2\pi f_c t + \pi/2)$.

1.4 Coherent AM Reception Revisited

Let $x_L(t) = m_i(t) + j\, m_q(t)$ be a complex-valued baseband signal with independent real-valued components $m_i(t)$ and $m_q(t)$, both bandlimited to f_m. Using QAM, the corresponding transmitted bandpass signal can be written in the time domain as

$$x(t) = \mathrm{Re}\{A_c\, x_L(t)\, e^{j2\pi f_x t}\} = \frac{A_c}{2}\big[x_L(t)\, e^{j2\pi f_x t} + x_L^*(t)\, e^{-j2\pi f_x t}\big] ,$$

with transmitter carrier frequency f_x. At the receiver, tuned to carrier frequency f_c, the QAM signal, attenuated by a factor γ, looks like this

$$r(t) = \frac{\gamma\, A_c}{2}\big[x_L(t)\, e^{j(2\pi(f_c+f_e)t+\theta_e)} + x_L^*(t)\, e^{-j(2\pi(f_c+f_e)t+\theta_e)}\big] ,$$

where f_e and θ_e represent the frequency and the phase errors between transmitter and receiver.

If the receiver uses a QAM demodulator that outputs complex-valued lowpass signals, then the spectrum of $r(t)$ is shifted left in the first step to obtain

$$v(t) = r(t)\, 2\, e^{-j2\pi f_c t} = \gamma\, A_c\, \big[x_L(t)\, e^{j(2\pi f_e t+\theta_e)} + x_L^*(t)\, e^{-j(2\pi(2f_c+f_e)t+\theta_e)}\big] .$$

After lowpass filtering at $f_L \approx f_m$ we thus have

$$\hat{x}_L(t) = \mathrm{LPF}\{v(t)\} = \gamma\, A_c\, x_L(t)\, e^{j(2\pi f_e t+\theta_e)} .$$

Suppose now that $\hat{x}_L(t)$ has some special properties from which f_e and θ_e can be estimated. Then it is possible to obtain the scaled, but otherwise error-free demodulated signal from the complex-valued QAM demodulator output $\hat{x}_L(t)$ by multiplying with $e^{-j(2\pi f_e t+\theta_e)}$

$$\hat{x}_L\, e^{-j(2\pi f_e t+\theta_e)} = \gamma\, A_c\, x_L(t)\ .$$

If, on the other hand, the receiver uses an entirely real-valued QAM demodulator implementation and $r(t)$ is correspondingly converted to

$$r(t) = \gamma\, A_c \left[m_i(t)\, \cos(2\pi(f_c+f_e)t+\theta_e) - m_q(t)\, \sin(2\pi(f_c+f_e)t+\theta_e)\right]\ ,$$

then

$$\begin{aligned} v_i(t) = r(t)\, 2\, \cos(2\pi f_c t) = \gamma\, A_c\big[m_i(t)\, \big(&\cos(2\pi f_e t+\theta_e) + \cos(2\pi(2f_c+f_e)t+\theta_e)\big)+ \\ &- m_q(t)\big(\sin(2\pi f_e t+\theta_e) + \sin(2\pi(2f_c+f_e)+\theta_e)\big)\big]\ , \end{aligned}$$

and

$$\begin{aligned} v_q(t) = -r(t)\, 2\, \sin(2\pi f_c t) = \gamma\, A_c\big[m_i(t)\, \big(&\sin(2\pi f_e t+\theta_e) - \sin(2\pi(2f_c+f_e)t+\theta_e)\big)+ \\ &+ m_q(t)\big(\cos(2\pi f_e t+\theta_e) - \cos(2\pi(2f_c+f_e)+\theta_e)\big)\big]\ . \end{aligned}$$

After lowpass filtering at $f_L \approx f_m$ the demodulated real-valued signals are

$$\hat{m}_i(t) = \mathrm{LPF}\{v_i(t)\} = \gamma\, A_c \left[m_i(t)\, \cos(2\pi f_e t+\theta_e) - m_q(t)\, \sin(2\pi f_e t+\theta_e)\right]\ ,$$

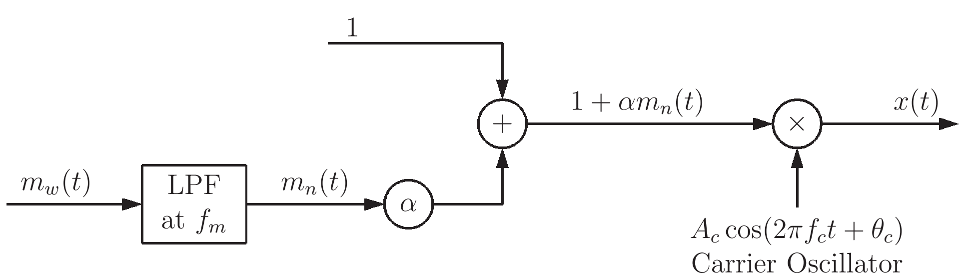
and

$$\hat{m}_q(t) = \mathrm{LPF}\{v_q(t)\} = \gamma\, A_c \left[m_q(t)\, \cos(2\pi f_e t+\theta_e) + m_i(t)\, \sin(2\pi f_e t+\theta_e)\right]\ .$$

In this case it is in general not possible to obtain scaled, but otherwise error-free demodulated signals from $\hat{m}_i(t)$ and $\hat{m}_q(t)$. Thus, the preferred way for (digital) signal processing in radio receivers is to use complex-valued lowpass signals for as long as possible and to convert to real-valued signals only after all other necessary processing has been done.

1.5 Amplitude Modulation with Carrier

An entirely different approach to solve the problem of synchronization between transmitter and receiver for real-valued message signals $m(t)$ is to add a sufficiently large dc term to $m(t)$ so that the carrier signal $\cos(2\pi f_c t+\theta_c)$ always gets multiplied by a non-negative number. The block diagram of a **AM-DSB-TC** (amplitude modulation, double side-band, transmitted carrier) transmitter is shown in the following figure.



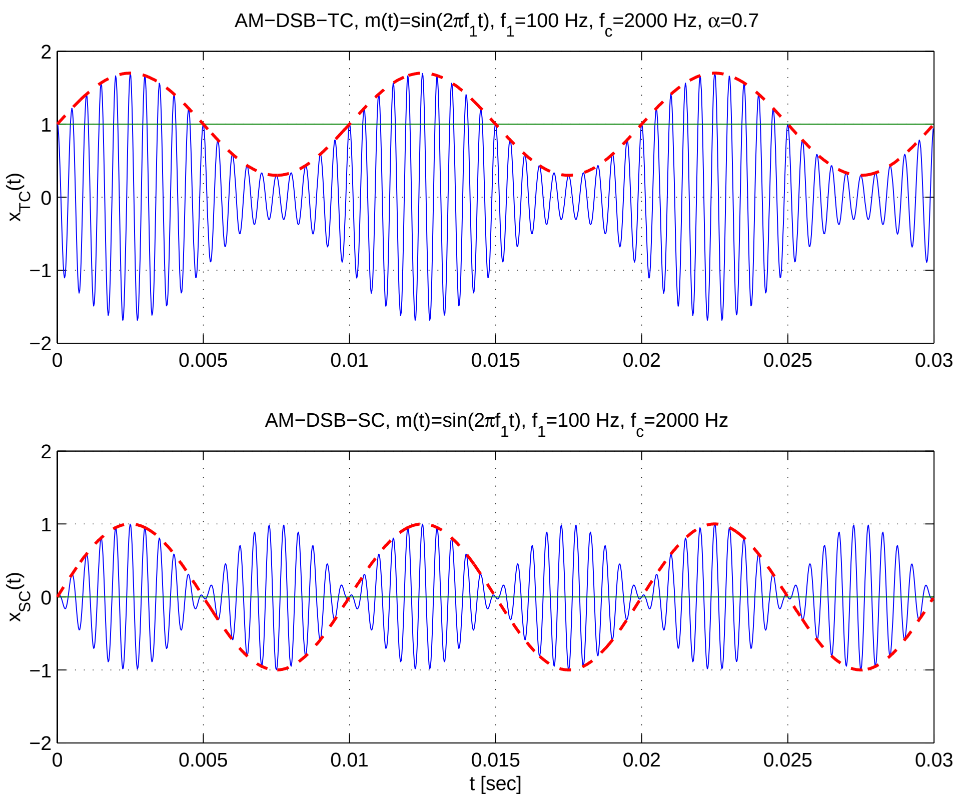
Written out explicitly, the general form of a AM-DSB-TC signal is

$$x(t) = A_c\,\big(1+\alpha m_n(t)\big)\,\cos(2\pi f_c t+\theta_c) = \underbrace{A_c\cos(2\pi f_c t+\theta_c)}_{\text{carrier term}} + \underbrace{A_c\alpha m_n(t)\cos(2\pi f_c t+\theta_c)}_{\text{AM-DSB-SC signal}}\,,$$

where $m_n(t)$ is the normalized message signal, obtained from the lowpass filtered wideband signal $m(t) = \text{LPF}\{m_w(t)\}$ as

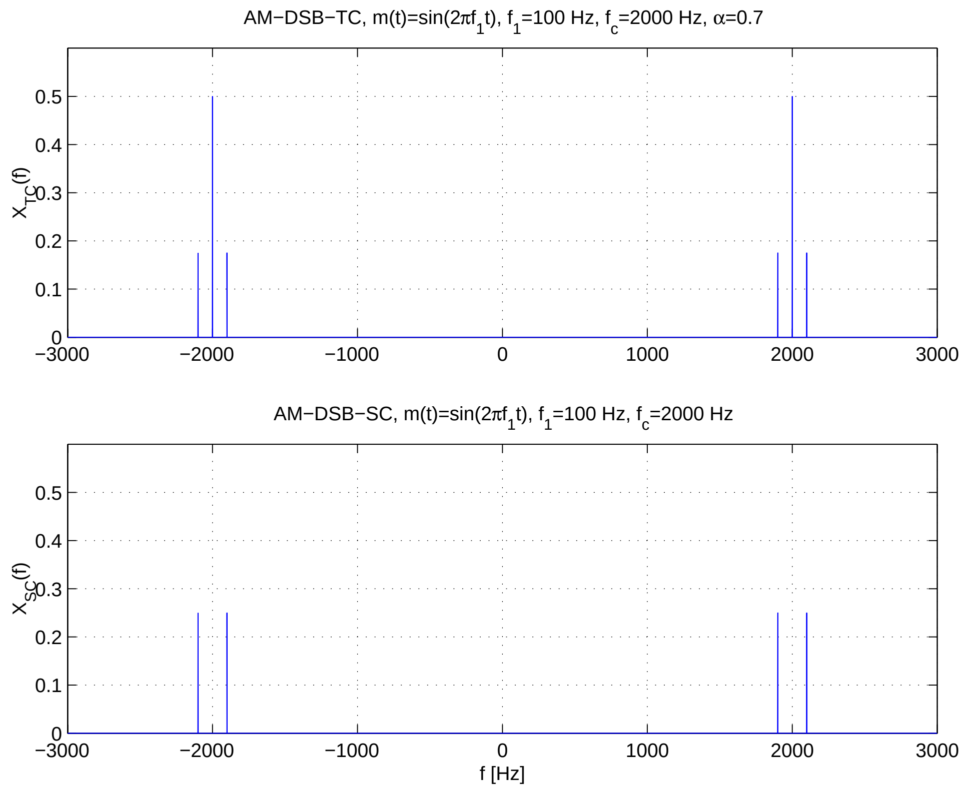
$$m_n(t) = \frac{m(t)}{\max_t |m(t)|}\,,$$

and $0 \le \alpha \le 1$ is the **modulation index** (often expressed in percent as $100\alpha\%$). Comparing this with AM-DSB-SC, the only difference is that instead of using $m(t)$ (or $m_n(t)$) directly, the offset version $1+\alpha m_n(t)$ is used to modulate the carrier amplitude. The following figure shows the AM-DSB-TC (upper graph) and the AM-DSB-SC (lower graph) signals that result from a sinusoidal message signal $m(t)$. The modulation index for the AM-DSB-TC signal is $\alpha = 0.7$



Note that the carrier (blue line) never changes phase in the AM-DSB-TC case since the message signal (red dashed line) is never negative due to the dc offset (green line at +1). For the AM-DSB-SC signal, however, the phase of the carrier (blue line) changes by 180° when the message signal (red dashed line) becomes negative because it has no dc offset (green line at 0). Thus, in contrast to AM-DSB-SC, an AM-DSB-TC signal can be demodulated using an **envelope detector** which only looks at the magnitude of the peaks of the received signal which are independent of changes in phase and frequency of the carrier signal.

In the frequency domain the AM-DSB-TC and the AM-DSB-SC signals for a sinusoidal message signal $m(t) = sin(2\pi f_1 t)$, $f_1 = 100$ Hz, look as follows.



Note that in the AM-DSB-TC case the carrier has always at least twice the amplitude of the sidebands. Since the carrier itself is unmodulated, only the sidebands carry information, and the efficiency η of AM-DSB-TC is therefore

$$\eta = \frac{\text{average power in sidebands}}{\text{total average power}} = \frac{\alpha^2 < m_n^2(t) >}{1 + \alpha^2 < m_n^2(t) >} \,,$$

where

$$< y(t) > = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} y(t)\, dt \,, \quad \text{and thus} \quad < m_n^2(t) > = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} m_n^2(t)\, dt \,,$$

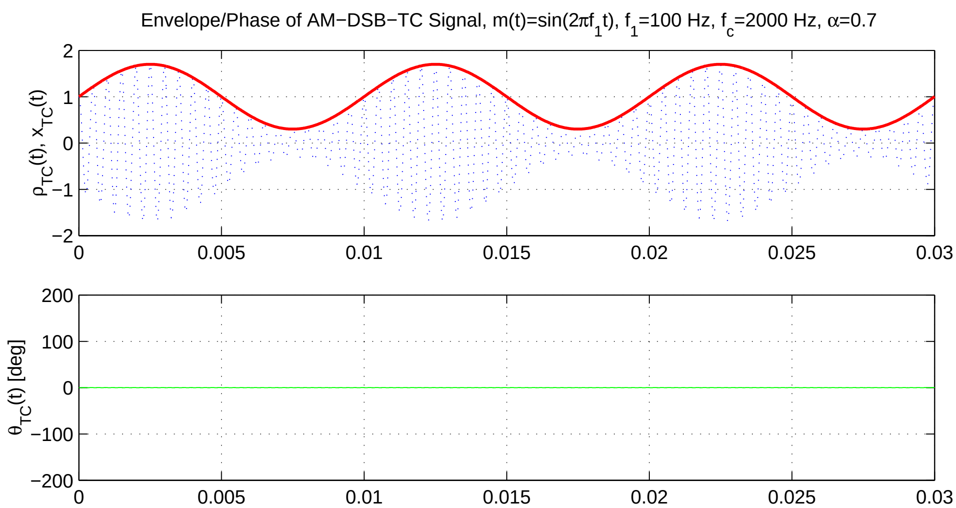
which is (typically much) less than the $\eta = 100\%$ value which is achieved by AM-DSB-SC.

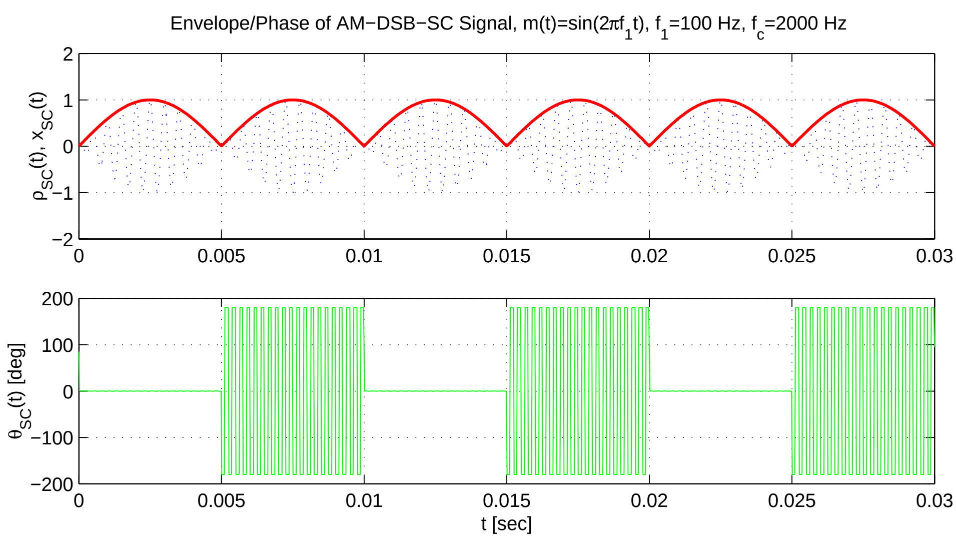
1.6 Non-Coherent Reception for AM-DSB-TC

A sinusoid with frequency f_c whose amplitude and phase vary over time can be written in the form

$$x(t) = \rho(t) \cos\big(2\pi f_c t + \theta(t)\big) \,, \qquad \rho(t) \geq 0 \,.$$

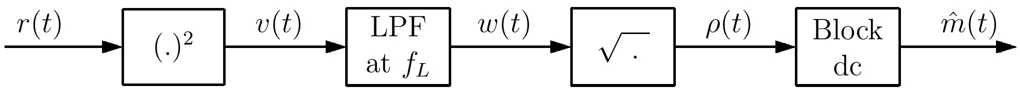
The quantity $\rho(t)$, which is non-negative by convention, is called the **envelope** of $x(t)$ and $\theta(t)$ is called the **phase** of $x(t)$. The following figure shows the envelopes (bold red line) and the phases (green line) of a AM-DSB-TC (upper graphs) and a AM-DSB-SC (lower graphs) signal when $m(t)$ is a sinusoid.





Quite clearly the envelope of the AM-DSB-TC signal has the same shape as $m(t)$, whereas the envelope of the AM-DSB-SC signal is the absolute value $|m(t)|$ of $m(t)$. For the AM-DSB-TC signal the phase is constant for all t, whereas for the AM-DSB-SC signal the phase jumps by $\pm 180°$ for those t where $m(t) < 0$. Thus, demodulation of a AM-DSB-SC signal requires both $\rho(t)$ and $\theta(t)$, but a received AM-DSB-TC signal $r(t)$ can be demodulated based on the envelope of $r(t)$ alone, without the need to synchronize to the phase (and precise frequency) of the carrier of $r(t)$. A receiver which does that is called a **non-coherent** receiver, whereas a receiver that needs to be precisely synchronized with the carrier oscillator at the transmitter is called a **coherent** receiver.

The following block diagram of a non-coherent "**squaring receiver**" for AM-DSB-TC is more complicated than the circuit that is actually used in most standard AM receivers, but it makes it very easy to show analytically why AM-DSB-TC does not need a phase (and frequency) synchronized circuit for demodulation.



Assume that the received signal is $r(t) = \gamma x(t)$, where γ is the attenuation factor of the transmission channel. Then, referring to the notation in the above block diagram,

$$\begin{aligned} v(t) = r^2(t) &= \gamma^2 A_c^2 \big(1{+}\alpha m_n(t)\big)^2 \cos^2(2\pi f_c t + \theta_c) \\ &= \frac{\gamma^2 A_c^2}{2} \big(1{+}\alpha m_n(t)\big)^2 \big(1 + \cos(4\pi f_c t + 2\theta_c)\big) \, . \end{aligned}$$

The LPF is designed to remove the AM signal at twice the carrier frequency, while passing $(1 + \alpha m_n(t))^2$ unchanged, so that

$$w(t) = \frac{\gamma^2 A_c^2}{2} \big(1 + \alpha m_n(t)\big)^2 \, .$$

Therefore, after taking the (positive) square root, the envelope of $r(t)$ is obtained as

$$\rho(t) = \frac{\gamma A_c}{\sqrt{2}} \big|1 + \alpha m_n(t)\big| = \frac{\gamma A_c}{\sqrt{2}} \big(1 + \alpha m_n(t)\big) \, .$$

The second equality follows from the fact that $(1 + \alpha m_n(t)) \geq 0$ if $0 \leq \alpha \leq 1$. Finally, removing the dc component from $\rho(t)$ yields the estimate

$$\hat{m}(t) = \frac{\gamma \alpha A_c}{\sqrt{2}} \, m_n(t) \, ,$$

of the transmitted message signal. In the absence of noise and channel distortion, this is an exact (but scaled) copy of the original message signal $m(t)$, independent of the exact value of f_c and independent of any knowledge of the phase θ_c of the carrier signal.

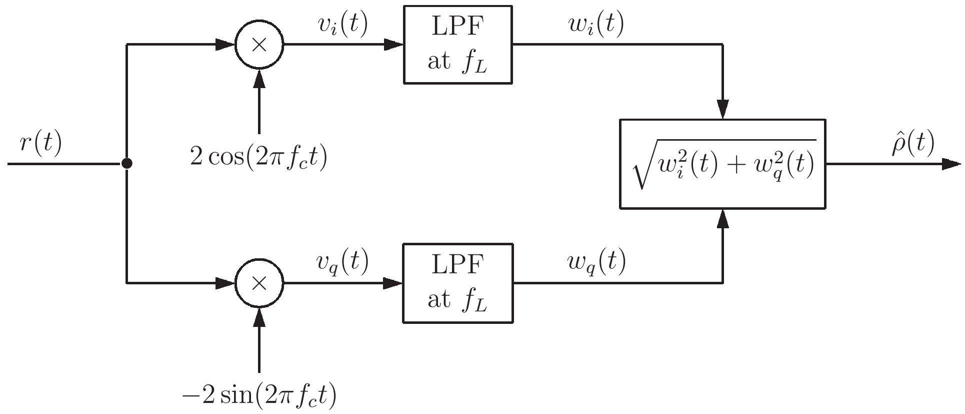
Using standard trigonometric identities, a sinusoidal signal $r(t)$ with envelope $\rho(t) \geq 0$, carrier frequency f_c, and phase $\theta(t)$ can be expressed as

$$r(t) = \rho(t) \, \cos\big(2\pi f_c t + \theta(t)\big) = \underbrace{\rho(t)\cos\theta(t)}_{= w_i(t)} \cos(2\pi f_c t) - \underbrace{\rho(t)\sin\theta(t)}_{= w_q(t)} \sin(2\pi f_c t) \, .$$

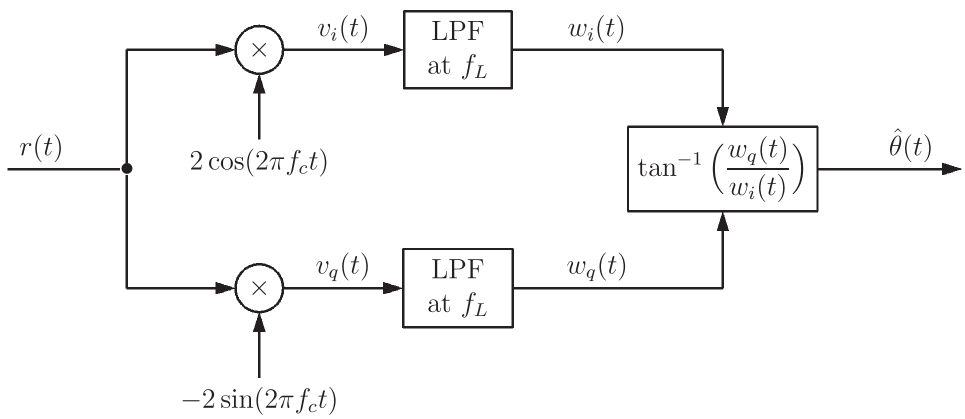
From this one easily obtains

$$\rho(t) = \sqrt{w_i^2(t) + w_q^2(t)} \, , \qquad \text{and} \qquad \theta(t) = \tan^{-1}\Big(\frac{w_q(t)}{w_i(t)}\Big) \, .$$

The equation for $\rho(t)$ leads to another, more sophisticated receiver for AM-DSB-TC, the I-Q **envelope detector** (or I-Q absolute value detector) shown in the following block diagram.



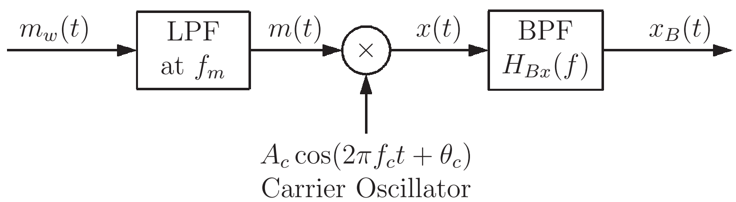
Similarly, the equation for $\theta(t)$ leads to the block diagram of a I-Q **phase detector** as shown next.



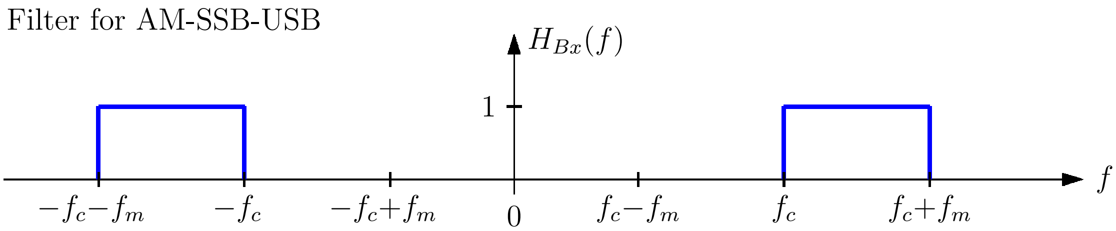
Finally, the (equivalent) circuit that is used in most AM receivers is the "**absolute value receiver**" shown below.

1.7 AM-SSB-SC and AM-VSB-SC

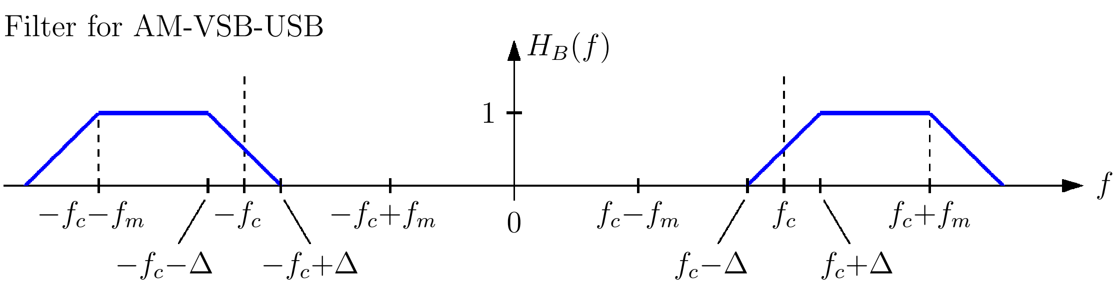
One of the disadvantages of AM-DSB-SC is that it occupies twice the bandwidth of the original message signal. One straightforward way to reduce the bandwidth to the original value is to only keep one of the sidebands of the AM signal and suppress the other one. The resulting AM signals are known as **AM-SSB-LSB** (amplitude modulation, single sideband, lower sideband) and as **AM-SSB-USB** (amplitude modulation, single sideband, upper sideband) depending on whether the lower or upper sideband is kept. To convert AM-DSB-SC to AM-SSB-SC (either LSB or USB), the AM-DSB-SC signal can be filtered with a bandpass filter (BPF) as shown in the following block diagram.



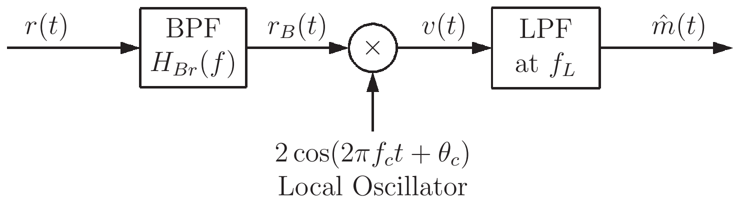
For AM-SSB-USB, for example, the transmitter filter $H_{Bx}(f)$ is chosen as shown in the following figure.



A problem with this filter are the sharp cutoffs needed near f_c, especially if $m(t)$ has a dc component (which is the case for analog TV broadcast signals, for instance). To alleviate this problem, **vestigial sideband (VSB)** modulation can be used. This is essentially a compromise between AM-DSB and AM-SSB, with a well controlled (usually linear) overall transition from the passband of $H_B(f)$ to the stopband near f_c, extending over a range of 2Δ around f_c. Depending on whether the lower or upper sideband is kept, the resulting AM signal is either called **AM-VSB-LSB** (amplitude modulation, vestigial sideband, lower sideband) or **AM-VSB-USB** (amplitude modulation, vestigial sideband, upper sideband). An example of a filter $H_B(f)$ that converts a AM-DSB-SC signal to a AM-VSB-USB-SC signal is shown in the following figure.



Demodulation of AM-SSB-SC signals and AM-VSB-SC signals is done in a similar fashion as for AM-DSB-SC by multiplying the received signal with the local oscillator signal $2\cos(2\pi f_c t + \theta_c)$, followed by lowpass filtering at f_m. To remove noise and/or interference from the unused (portion of the) sideband, a BPF should be used at the input of the receiver, as shown in the following blockdiagram.

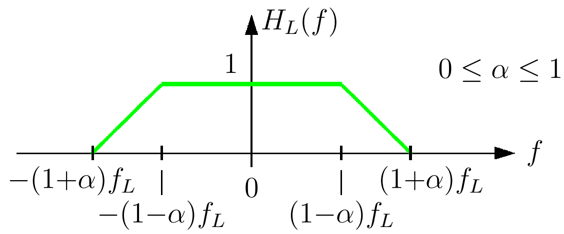


For AM-SSB-SC the same BPF can be used for both the transmitter and the receiver. For AM-VSB-SC the product $H_{Bx}(f)H_{Br}(f)$ of the frequency responses of the BPFs at the transmitter and receiver must be equal to $H_B(f)$ as shown above.

1.8 Bandpass Filters

Suppose you have a lowpass filter $h_L(t) \Leftrightarrow H_L(f)$, e.g., an LPF with trapezoidal frequency response and thus

$$h_L(t) = \frac{\sin(2\pi f_L t)}{\pi t}\,\frac{\sin(2\pi\alpha f_L t)}{2\pi\alpha f_L t} \qquad \Longleftrightarrow$$



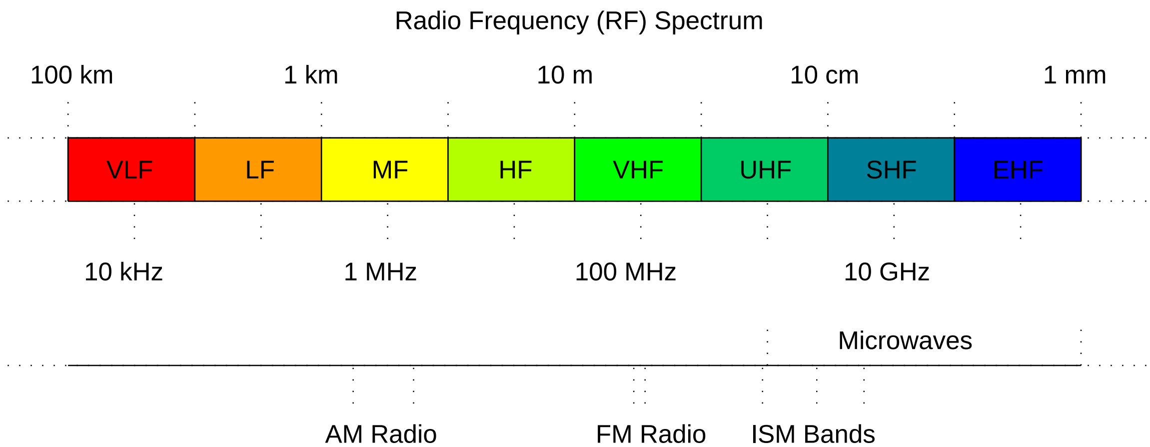
By making use of the frequency shift property of the FT, this LPF can be converted to a BPF $h_{BP}(t) \Leftrightarrow H_{BP}(f)$ which is symmetric about some center frequency $f_c \geq (1+\alpha)\,f_L$ (where $\alpha = 0$ for an ideal LPF) by

$$h_{BP}(t) = 2\,h_L(t)\,\cos(2\pi f_c t) \quad \Longleftrightarrow \quad H_{BP}(f) = H_L(f) * [\delta(f - fc) + \delta(f + f_c)]\,.$$

BPFs that are obtained from ideal LPFs (i.e., $\alpha \to 0$) are well suited for picking out one particular signal from several FDM (frequency division multiplexed) signals, or for generating SSB (single sideband) AM signals from DSB AM signals. BPFs that are obtained from LPFs with trapezoidal frequency response can be used for similar tasks, but in addition they can also be used to convert frequency to amplitude (in the transition region of the BPF) and to generate VSB (vestigial sideband) AM signals.

1.9 Frequency Division Multiplexing

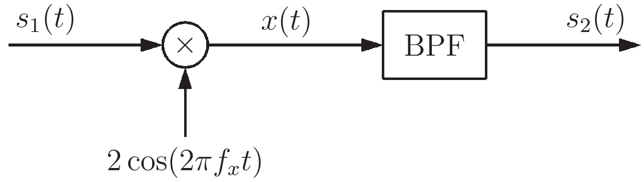
Multiplexing is key to using communication system resources efficiently and share them among many users. Time division multiplexing (TDM) assigns different time slots to different users. **Frequency division multiplexing (FDM)** uses the equivalent approach in the frequency domain by allocating different frequency bands to different users.



U.S. Frequency Allocations for Selected Radio Frequency Services		
Service	Frequency Allocation	Remarks
AM Radio	$535 \ldots 1605$ kHz	$f_c = 540 \ldots 1600$ kHz, spacing 10 kHz
FM Radio	$88 \ldots 108$ MHz	$f_c = 88.1 \ldots 107.9$ MHz, spacing 200 kHz
ISM Bands	915 ± 13 MHz	Cordless phones, speakers
	2450 ± 50 MHz	Bluetooth, IEEE 802.11b WLAN
	5800 ± 75 MHz	IEEE 802.11a WLAN
GPS	1575.42 MHz (L1)	Coarse/Acquisition & P Codes
	1227.60 MHz (L2)	P Code (encrypted) only
Satellite Radio	$2320 \ldots 2345$ MHz	XM, Sirius

1.10 Mixers

A **mixer** is a device that has two inputs which are multiplied together to obtain one output which contains the convolution of the spectra of the input signals. If one of the inputs is a sinusoid produced by a local oscillator, then the output consists of the input spectrum shifted by the local oscillator frequency f_x to the left and to the right. Usually only one of the shifted spectra is desired and thus a mixer is normally followed by a BPF (or sometimes an LPF), as shown in the following block diagram.



If $s_1(t)$ is an AM signal of the form $s_1(t) = v(t)\,\cos(2\pi f_{c1} t)$, where $v(t)$ could either be directly a message signal for AM-DSB-SC, or a normalized message signal plus a dc-component for AM-DSB-TC, then one easily finds that

$$\begin{aligned} x(t) &= 2\,s_1(t)\,\cos(2\pi f_x t) = 2\,v(t)\,\cos(2\pi f_{c1} t)\,\cos(2\pi f_x t) \\ &= v(t)\left[\cos\big(2\pi(f_{c1}+f_x)t\big) + \cos\big(2\pi(f_{c1}-f_x)t\big)\right]. \end{aligned}$$

Thus, the two logical choices for the center frequency of the BPF are either $f_{c2} = f_{c1} + f_x$ or $f_{c2} = |f_{c1} - f_x|$. Note that both $f_x \le f_{c1}$ and $f_x > f_{c1}$ are possible. In either case, the output is $s_2(t) = v(t)\,\cos(2\pi f_{c2} t)$, i.e., it is another AM signal with new carrier frequency f_{c2}. This is a feature that is used extensively in transmitters to produce a signal, e.g., using digital signal processing (DSP), at lower frequencies and then move it up to the actual transmit frequency which may be in the GHz range. Receivers then use the same feature in the opposite way to bring a signal down from the actual transmit frequency to a (much) lower frequency range where DSP can be used.

1.11 Carrier Frequency Extraction

Let $r(t)$ be a received noiseless AM-DSB-SC signal with attenuation γ, i.e.,

$$r(t) = \gamma\,x(t) = \gamma A_c\,m(t)\,\cos\big(2\pi(f_c+f_e)t + \theta_e\big)\,,$$

where f_e is the frequency error and θ_e is the phase error between the transmitter and the receiver. To obtain (an estimate of) the error signal $\psi(t) = 2\pi f_e t + \theta_e$ from $r(t)$, start from squaring $r(t)$ to obtain

$$r^2(t) = \gamma^2\,A_c^2\,m^2(t)\,\cos^2\big(2\pi(f_c+f_e)t + \theta_e\big) = \frac{\gamma^2\,A_c^2\,m^2(t)}{2}\left[1 + \cos\big(4\pi(f_c+f_e)t + 2\theta_e\big)\right].$$

Multiplying this by $2\cos(4\pi f_c t)$ yields

$$\begin{aligned} v_i(t) &= \gamma^2\,A_c^2\,m^2(t)\left[1 + \cos\big(4\pi(f_c+f_e)t + 2\theta_e\big)\right]\,\cos 4\pi f_c t \\ &= A(t)\left[2\cos 4\pi f_c t + \cos(4\pi f_e t + 2\theta_e) + \cos\big(4\pi(2f_c+f_e)t + 2\theta_e\big)\right], \end{aligned}$$

where $A(t) = \gamma^2 A_c^2\,m^2(t)/2$ is a time-varying amplitude. Simlarly, multiplying by -$2\sin 4\pi f_c t$ results in

$$\begin{aligned} v_q(t) &= -\gamma^2\,A_c^2\,m^2(t)\left[1 + \cos\big(4\pi(f_c+f_e)t + 2\theta_e\big)\right]\,\sin 4\pi f_c t \\ &= A(t)\left[-2\sin 4\pi f_c t + \sin(4\pi f_e t + 2\theta_e) - \sin\big(4\pi(2f_c+f_e)t + 2\theta_e\big)\right]. \end{aligned}$$

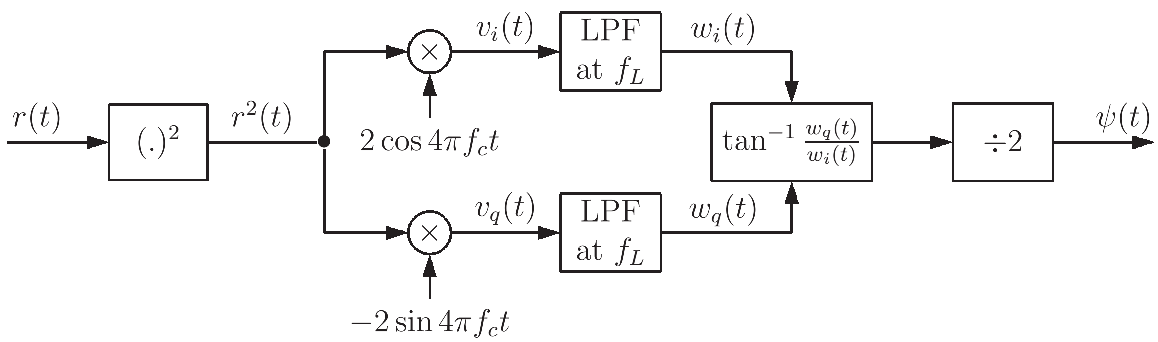
Thus, after lowpass filtering with $2f_e < f_L < f_c$,

$$w_i(t) = A(t)\,\cos(4\pi f_e t + 2\theta_e) \qquad \text{and} \qquad w_q(t) = A(t)\,\sin(4\pi f_e t + 2\theta_e)\,.$$

Finally, the error estimate $\psi(t)$ is obtained by taking an inverse tangent and dividing by 2 as follows

$$\psi(t) = \frac{1}{2}\,\tan^{-1}\left(\frac{w_q(t)}{w_i(t)}\right)\,.$$

This whole process is shown in blockdiagram form in the next figure.



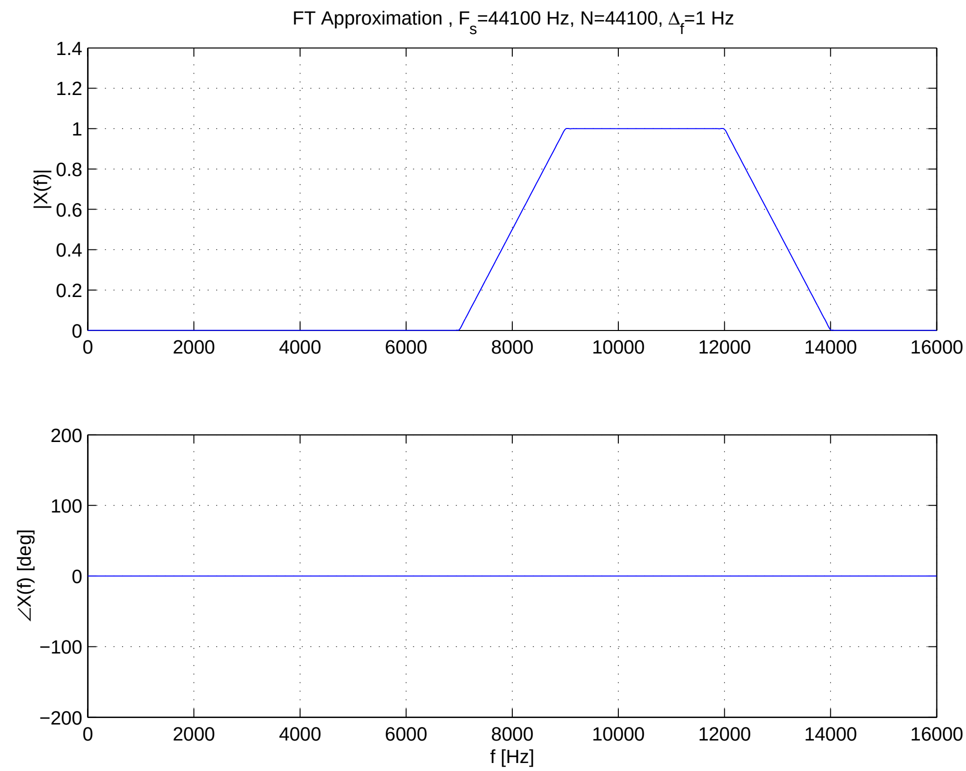
Note that, before the division by 2 to obtain $\psi(t)$, it is crucial that the phase (which is only resolved modulo 2π by the inverse tangent) is unwrapped. To demodulate the received AM-DSB-SC signal $r(t)$, the local oscillator term $2\cos(2\pi f_c t + \psi(t))$ is then used instead of the $2\cos(2\pi f_c t + \theta_c)$ term shown in an earlier blockdiagram.

2 Lab Experiments

E1. AM Transmitter/Receiver. (a) FIR LPF/BPF with Trapezoidal $H(f)$. Modify your `trapfilt` function in the `filtfun` module so that it can be used as either a lowpass or a bandpass filter with trapezoidal frequency response. The header of the extended function is shown below.

```
def trapfilt(sig_xt, fparms, k, alfa):
    """
    Delay compensated FIR LPF/BPF filter with trapezoidal
    frequency response.
    >>>>> sig_yt, n = trapfilt(sig_xt, fparms, k, alfa) <<<<<
    where  sig_yt: waveform from class sigWave
           sig_yt.signal():  filter output y(t), samp rate Fs
           n:    filter order
           sig_xt: waveform from class sigWave
           sig_xt.signal():  filter input x(t), samp rate Fs
           sig_xt.get_Fs():  sampling rate for x(t), y(t)
           fparms = fL       for LPF
           fL:   LPF cutoff frequency (-6 dB) in Hz
           fparms = [fBW, fc]  for BPF
           fBW:  BPF -6dB bandwidth in Hz
           fc:   BPF center frequency in Hz
           k:    h(t) is truncated to
                    |t| <= k/(2*fL) for LPF
                    |t| <= k/fBW  for BPF
           alfa: frequency rolloff parameter, linear
                 rolloff over range
                 (1-alfa)fL <= |f| <= (1+alfa)fL  for LPF
                 (1-alfa)fBW/2 <= |f| <= (1+alfa)fBW/2  for BPF
    """
```

To test your modified `trapfilt` function, estimate the parameters of the BPF whose frequency response is shown below and recreate $h(t) \Leftrightarrow H(f)$ with your `trapfilt` function.



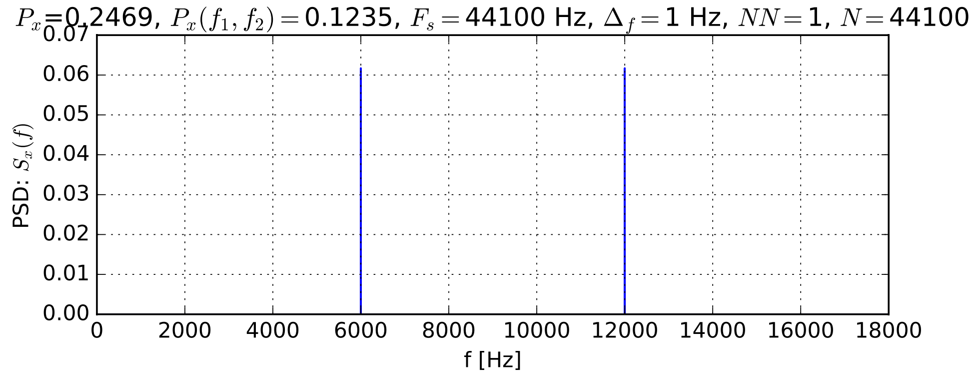
(b) Start a new Python module, called `amfun.py`, and write a function, called `amxmtr` which performs the tasks of an AM transmitter to produce AM-DSB-SC, AM-DSB-TC, AM-SSB, and AM-VSB signals for a real-valued (wideband) message signal $m(t)$. This function uses the extended `trapfilt` function to lowpass filter $m(t)$ to f_m and to bandpass filter the AM signal $x(t)$. The header of `amxmtr` looks as follows:

```
def amxmtr(sig_mt, xtype, fcparms, fmparms=[], fBparms=[]):
    """
    Amplitude Modulation Transmitter for suppressed ('sc')
    and transmitted ('tc') carrier AM signals
    >>>>> sig_xt = amxmtr(sig_mt, xtype, fcparms, fmparms, fBparms) <<<<<
    where  sig_xt: waveform from class sigWave
           sig_xt.signal():   transmitted AM signal
           sig_xt.timeAxis(): time axis for x(t)
           sig_mt: waveform from class sigWave
           sig_mt.signal():   modulating (wideband) message signal
           sig_mt.timeAxis(): time axis for m(t)
           xtype:  'sc' or 'tc' (suppressed or transmitted carrier)
           fcparms = [fc, thetac]  for 'sc'
           fcparms = [fc, thetac, alfa]  for 'tc'
           fc:     carrier frequency
           thetac: carrier phase in deg (0: cos, -90: sin)
           alfa:   modulation index 0 <= alfa <= 1
           fmparms = [fm, km, alfam]  LPF at fm parameters
                   no LPF at fm if fmparms = []
           fm:     highest message frequency
           km:     LPF h(t) truncation to |t| <= km/(2*fm)
           alfam:  LPF at fm frequency rolloff parameter, linear
                   rolloff over range 2*alfam*fm
           fBparms = [fBW, fcB, kB, alfaB]  BPF at fcB parameters
                   no BPF if fBparms = []
           fBW:    -6 dB BW of BPF
           fcB:    center freq of BPF
           kB:     BPF h(t) truncation to |t| <= kB/fBW
           alfaB:  BPF frequency rolloff parameter, linear
                   rolloff over range alfaB*fBW
    """
```

Test your transmitter using the message signal `sig_mt` generated below as input.

```
Fs = 44100                                # Sampling rate
tlen = 1.0                                # Duration
f0, f1 = 3000, 5000                       # Message frequencies
tt = np.arange(np.round(tlen*Fs))/float(Fs)  # Time axis
mt = np.cos(2*np.pi*f0*tt) + np.cos(2*np.pi*f1*tt) # Message signal
sig_mt = comsig.sigWave(mt, Fs, 0)        # Waveform from class sigWave
```

Set `xtype='sc'`, $f_c = 9000$ Hz, $\theta_c = 0°$, $f_m = 4000$, $k_m \approx 10 \ldots 20$, and $\alpha_m = 0.05$. The LPF at the transmitter should remove the frequency component at 5000 Hz. The 3000 Hz cosine should be moved to $f_c \pm 3000$ Hz so that the PSD looks as shown below.



How does the PSD change if you use the same parameters as above, except for setting `xtype='tc'` and `alfa=0.7`?

(c) Use the speech signal in `speech701.wav` and the music signal in `music701.wav` to generate AM-DSB-SC signals $x_1(t)$ and $x_2(t)$, respectively, with $f_c = 8000$ Hz, $f_m = 4000$ Hz, $k_m \approx 10\ldots20$, and $\alpha_m = 0.05$. Use $\theta_c = -90°$ for the speech signal and $\theta_c = 0°$ for the music signal. Adjust the carrier amplitude A_{c2} of $x_2(t)$ (modulated with the music signal) such that the average powers $P(x_1(t))$ and $P(x_2(t))$ of the AM-DSB-SC signals are approximately equal. Create a third signal $x_3(t) = (x_1(t) + x_2(t))/\sqrt{2}$. Save the three signals in `myam701.wav`, `myam702.wav`, and `myam703.wav`, respectively, for later use. Display the PSDs of each of the three signals and compare them. Does the bandwidth for $x_3(t)$, which contains two message signals, change? Display also the PSDs of the squared AM signals $x_1^2(t)$, $x_2^2(t)$, and $x_3^2(t)$ and analyze them in the vcinity of $2f_c$ (zoom-in to a range of approximately 15900 to 16100 Hz). Is there any useful information that you can get from the squared signals? If so, what is this information and for which of the three signals is it actually present?

(d) For the Python module `amfun.py`, write a function called `amrcvr` that demodulates a received AM signal $r(t)$ and produces an estimate $\hat{m}(t)$ of the transmitted message $m(t)$. Here is the header for this function

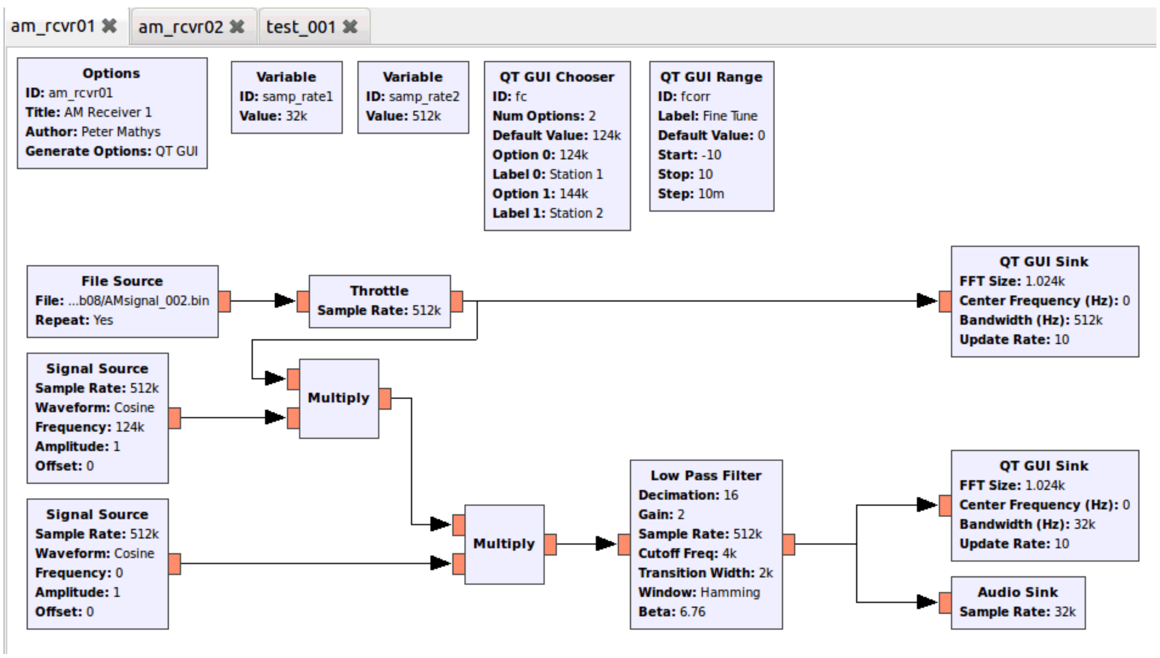
```
def amrcvr(sig_rt, rtype, fcparms, fmparms=[], fBparms=[], dcblock=False):
    """
    Amplitude Modulation Receiver for coherent ('coh') reception,
    or absolute value ('abs'), or squaring ('sqr') demodulation,
    or I-Q envelope ('iqabs') detection, or I-Q phase ('iqangle')
    detection.
    >>>>> sig_mthat = amrcvr(sig_rt, rtype, fcparms, fmparms,
                                                fBparms, dcblock) <<<<<
    where  sig_mthat: waveform from class sigWave
           sig_mthat.signal():   demodulated message signal
           sig_mthat.timeAxis(): time axis mhat(t)
           sig_rt: waveform from class sigWave
           sig_rt.signal():   received AM signal
           sig_rt.timeAxis(): time axis for r(t)
           rtype:  Receiver type from list
                   'abs' (absolute value envelope detector),
                   'coh' (coherent),
                   'iqangle' (I-Q rcvr, angle or phase),
                   'iqabs' (I-Q rcvr, absolute value or envelope),
                   'sqr' (squaring envelope detector)
           fcparms = [fc, thetac]
           fc:     carrier frequency
           thetac: carrier phase in deg (0: cos, -90: sin)
           fmparms = [fm, km, alfam]   LPF at fm parameters
                   no LPF at fm if fmparms = []
           fm:     highest message frequency
           km:     LPF h(t) truncation to |t| <= km/(2*fm)
           alfam:  LPF at fm frequency rolloff parameter, linear
                   rolloff over range 2*alfam*fm
           fBparms = [fBW, fcB, kB, alfaB]  BPF at fcB parameters
                   no BPF if fBparms = []
           fBW:    -6 dB BW of BPF
           fcB:    center freq of BPF
           kB:     BPF h(t) truncation to |t| <= kB/fBW
           alfaB:  BPF frequency rolloff parameter, linear
                   rolloff over range alfaB*fBW
           dcblock: remove dc component from mthat if true
    """
```

Test your receiver with the AM-DSB-SC signals that you produced in part (c). Use the same f_m, k_m and α_m as for the transmitter. Can you recover the speech and music signals from $x_3(t)$ without any interference between the two signals?

(e) Analyze and, if possible, demodulate the AM signals in the wav-files `amsig701.wav`, `amsig702.wav`, `amsig703.wav`, and `amsig704.wav`. Look at the signals in the frequency domain and listen to the demodulated signals (make a wav file in Python and then use a music player for listening). Try different demodulation methods (coherent, non-coherent, I-Q envelope detection, etc). Interpret the graphs and the different demodulation methods and relate your findings to how the demodulated signals sound.

(f) Repeat (e) for the AM signals in the wav-files `amsig705.wav`, `amsig706.wav`, and `amsig707.wav`.

(g) Real-valued AM demodulator for AM-DSB-SC signals in GNU Radio. Build the GNU Radio flowgraph shown below to demodulate the two AM-DSB-SC signals in the file `AMsignal_002.bin`. The file was recorded using a sampling rate of 512 kHz and each sample is a 32-bit (real) floating point number.



The nominal carrier frequencies of the two signals are $f_{c1} = 124$ kHz and $f_{c2} = 144$ kHz, but the transmitters were off a little bit (within ± 10 Hz) from the nominal values. The receiver attempts to demodulate the signals with the nominal carrier frequency values, followed by fine tuning in the range from -10 to +10 Hz. The goal of this experiment is to find out how successful that strategy is when working with real-valued signal processing and to discuss its advantages and shortcomings.

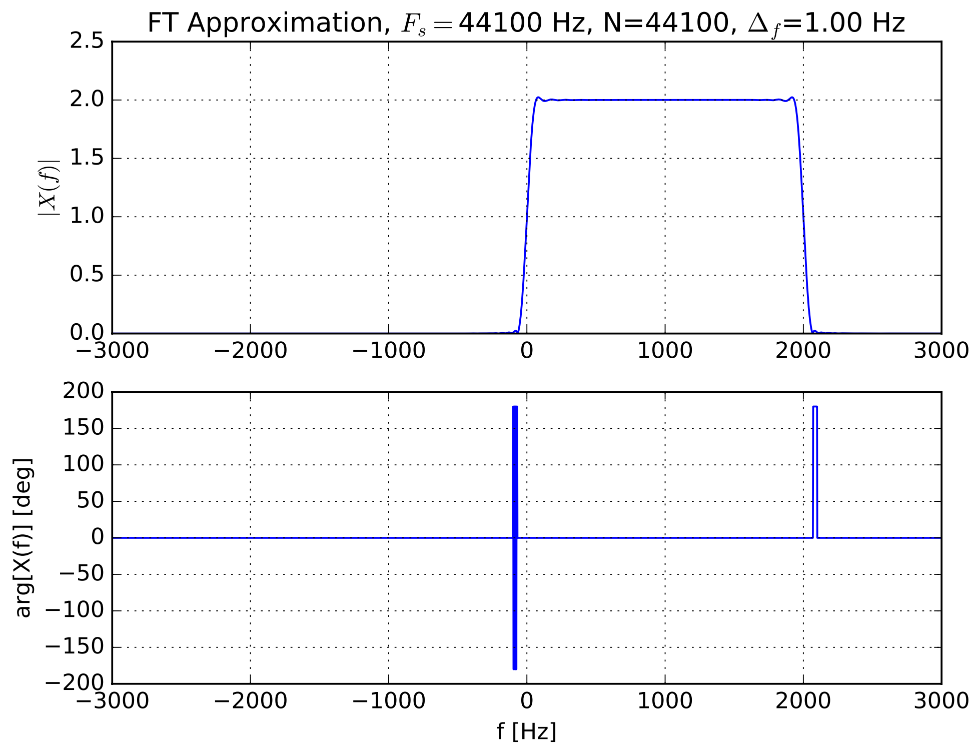
E2. QAM Transmitter/Receiver. (a) FIR LPF/BPF with Complex-Valued Filter Coefficients. If the LPF/BPF with trapezoidal frequency response is modified such that

$$h_{BP}(t) = 2\, h_L(t)\, e^{j2\pi f_c t} \qquad \Longleftrightarrow \qquad H_{BP}(f) = H_L(f) * \delta(f - f_c)\,,$$

then we obtain a filter with conplex-valued filter coefficients that can be used for such things as generating AM-SSB and AM-VSB signals at baseband. The header of this complex-valued version of `trapfilt`, called `trapfilt_cc`, is shown below.

```
def trapfilt_cc(sig_xt, fparms, k, alfa):
    """
    Delay compensated FIR LPF/BPF filter with trapezoidal
    frequency response, complex-valued input/output and
    complex-valued filter coefficients.
    >>>>> sig_yt, n = trapfilt_cc(sig_xt, fparms, k, alfa) <<<<<
    where  sig_yt: waveform from class sigWave
           sig_yt.signal():   complex filter output y(t), samp rate Fs
           n:    filter order
           sig_xt: waveform from class sigWave
           sig_xt.signal():   complex filter input x(t), samp rate Fs
           sig_xt.get_Fs():   sampling rate for x(t), y(t)
           fparms = fL         for LPF
           fL:   LPF cutoff frequency (-6 dB) in Hz
           fparms = [fBW, fBc]  for BPF
           fBW:  BPF -6dB bandwidth in Hz
           fBc:   BPF center frequency (pos/neg) in Hz
           k:    h(t) is truncated to
                    |t| <= k/(2*fL) for LPF
                    |t| <= k/fBW  for BPF
           alfa: frequency rolloff parameter, linear
                 rolloff over range
                 (1-alfa)*fL <= |f| <= (1+alfa)*fL  for LPF
                 (1-alfa)*fBW/2 <= |f| <= (1+alfa)*fBW/2  for BPF
    """
```

Test your `trapfilt_cc` function by recreating the filter with $h(t) \Leftrightarrow H(f)$ shown below. In your solution include a time domain plot (real and imaginary part) of $h(t)$.



(b) Complex-Valued QAM Modulator. In the Python module `amfun` add the function `qamxmtr`, whose header is shown below, for QAM modulation of complex-valued message signals (of the form $m(t) = m_i(t) + j\, m_q(t)$).

```
def qamxmtr(sig_mt, fcparms, fmparms=[]):
    """
    Quadrature Amplitude Modulation (QAM) Transmitter with
    complex-valued input and real-valued output signals
    >>>>> sig_xt = qamxmtr(sig_mt, fcparms, fmparms) <<<<<
    where  sig_xt: waveform from class sigWave
           sig_xt.signal():   real-valued QAM signal
           sig_xt.timeAxis(): time axis for x(t)
           sig_mt.signal():   complex-valued (wideband) message signal
           sig_mt.timeAxis(): time axis for m(t)
           fcparms = [fc, thetaci, thetacq]
           fc:      carrier frequency
           thetaci: in-phase (cos) carrier phase in deg
           thetacq: quadrature (sin) carrier phase in deg
           fmparms = [fm, km, alfam]  for LPF at fm parameters
                    no LPF/BPF at fm if fmparms = []
           fm:      highest message frequency (-6dB)
           km:      h(t) is truncated to |t| <= km/(2*fm)
           alfam:   frequency rolloff parameter, linear
                    rolloff over range (1-alfam)*fm <= |f| <= (1+alfam)*fm
    """
```

Test your `qamxmtr` function by recreating the `myam703.wav` QAM signal described in E1c. To test both `qamxmtr` and `trapfilt_cc`, use the `speech701.wav` signal to generate a AM-SSB-LSB signal with bandwidth ≈ 4000 Hz using complex-valued lowpass signal processing followed by QAM modulation at $f_c = 8000$ Hz and $\theta_c = 0°$. Save this signal in `myam701ssb.wav` for later use.

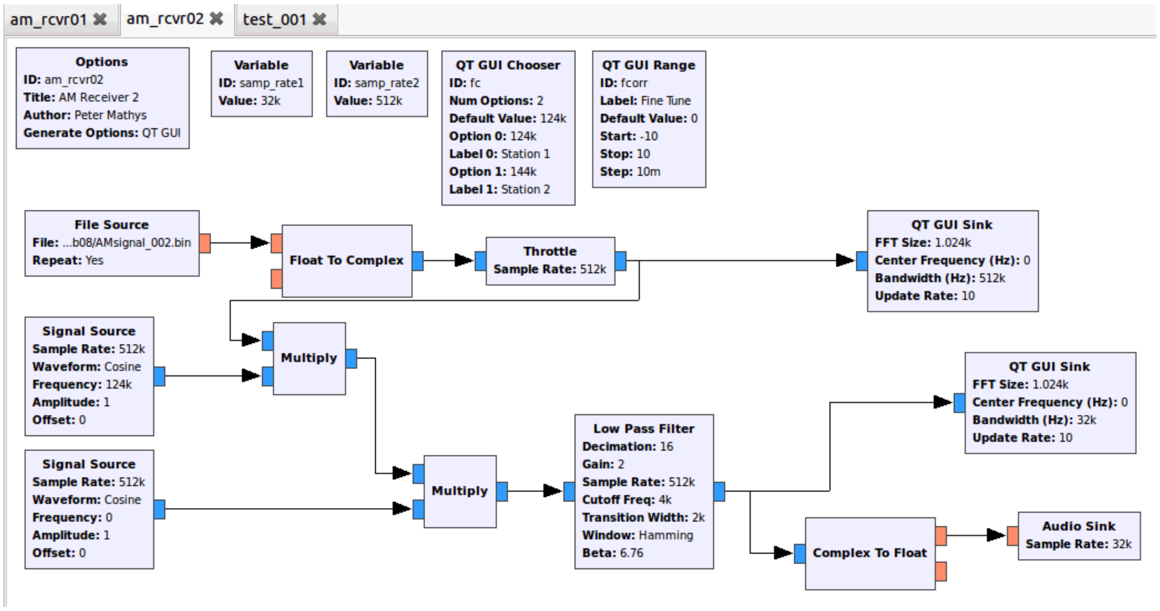
(c) The counterpart to the `qamxmtr` function is the QAM receiver function `qamrcvr` which uses complex-valued signal processing. Add this function whose header is shown below to the `amfun` module.

```
def qamrcvr(sig_rt, fcparms, fmparms=[]):
    """
    Quadrature Amplitude Modulation (QAM) Receiver with
    real-valued input and complex-valued output signals
    >>>>> sig_mthat = qamrcvr(sig_rt, fcparms, fmparms) <<<<<
    where  sig_mthat: waveform from class sigWave
           sig_mthat.signal():   complex-valued demodulated message signal
           sig_mthat.timeAxis(): time axis for mhat(t)
           sig_rt: waveform from class sigWave
           sig_rt.signal():   received QAM signal (real-valued)
           sig_rt.timeAxis(): time axis for r(t)
           fcparms = [fc, thetaci, thetacq]
           fc:      carrier frequency
           thetaci: in-phase (cos) carrier phase in deg
           thetacq: quadrature (sin) carrier phase in deg
           fmparms = [fm, km, alfam]   for LPF at fm parameters
                    no LPF at fm if fmparms = []
           fm:      highest message frequency (-6 dB)
           km:      h(t) is truncated to |t| <= km/(2*fm)
           alfam:   frequency rolloff parameter, linear
                    rolloff over range (1-alfam)*fm <= |f| <= (1+alfam)*fm
    """
```

Test your receiver with the signals that you produced in part (b) and in E1c. What happens if you remove one of the sidebands of an AM-DSB-SC signal, frequency shift the resulting (complex-valued) baseband signal, e.g., by 100 Hz, then take the real part and listen to it?

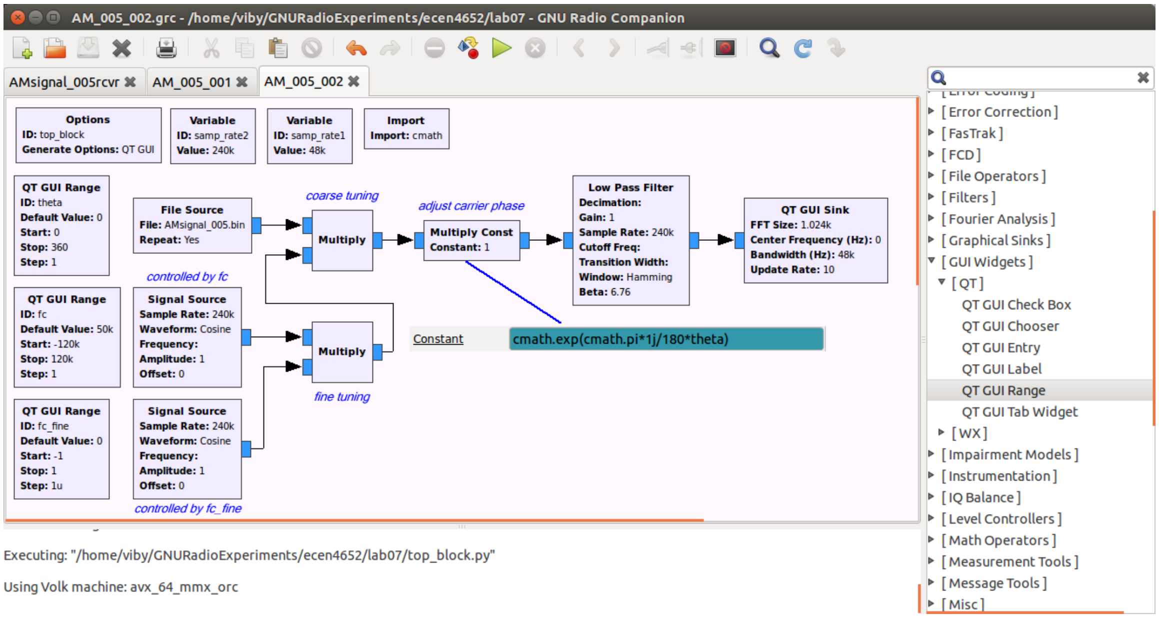
(d) Look at the AM signals in E1e and E1f (`amsig701.wav`...`amsig707.wav` again. Can you improve the quality of any of the demodulated signals using complex lowpass signal processing operations, e.g., by removing one of the sidebands?

(e) Complex-valued AM demodulator for AM-DSB-SC signals in GNU Radio. Build the GNU Radio flowgraph shown below to demodulate the two AM-DSB-SC signals in the file `AMsignal_002.bin`. The file was recorded using a sampling rate of 512 kHz and each sample is a 32-bit (real) floating point number.



The nominal carrier frequencies of the two signals are $f_{c1} = 124$ kHz and $f_{c2} = 144$ kHz, but the transmitters were off a little bit (within ± 10 Hz) from the nominal values. The receiver attempts to demodulate the signals with the nominal carrier frequency values, followed by fine tuning in the range from -10 to +10 Hz. The goal of this experiment is to find out how successful that strategy is when working with complex-valued signal processing and to discuss its advantages and shortcomings. Compare also to E1g.

(f) The file `AMsignal_005.bin` is a binary file that contains the I and Q components of several radio signals in the frequency range from 0 to 120 kHz. The sampling rate of the file is $F_s = 240$ kHz and the bandwidth allowed for each station is 10 kHz. Use this file as input from a File Source in the GNU Radio Companion (GRC). Build a flowgraph in the GRC for tuning to and demodulating AM-DSB-SC and, more generally QAM signals (i.e., the sum of two AM-DSB-SC signals at the same carrier frequency, one with a cosine and one with a sine carrier). Find all radio signals in `AMsignal_005.bin` and characterize their properties, such as f_c, θ_c, AM-DSB vs QAM, stability of f_c, interference between different stations, etc. Try to demodulate the signals as cleanly as possible. Here is an example of a flowgraph that can be used to analyze the different signals.



Note that some parameters are left blank and you have to decide (and make the case) for the best (or at least a good) choice. In the QT GUI Sink consider looking at the Constellation Display in addition to the Frequency and Time Domain Displays to distinguish between AM-DSB and QAM signals (why?).

E3. Analysis and Demodulation of AM Signals with Impairments. (a) Analyze, characterize and, if possible, demodulate the AM signals in the wav-files `amsig708.wav` ... `amsig713` with as little impairment as possible. Explain your strategy for choosing the best demodulation technique for each signal.

(b) The two AM signals in `amsig714.wav` and in `amsig715.wav` contain the same message signals, but using different variants of AM modulation. Determine how the two signals were modulated and try to demodulate the message signals as independently as possible (one is a pure music signal and the other is a pure speech signal). Explain your decoding strategy.

(c) The wav-file in `amsig720.wav` contains 5 different "radio stations" with carrier frequencies of $f_{c1} = 2000$ Hz, $f_{c2} = 6000$ Hz, $f_{c3} = 10000$ Hz, $f_{c4} = 14000$ Hz, and $f_{c5} = 18000$ Hz. Each of the 5 radio stations uses AM-VSB-USB-SC with a linear attenuation transition band from $f_c - 1000$ Hz to $f_c + 1000$ Hz. Write a Python script to demodulate each of the 5 radio stations independently.