

Optimizing Investment Portfolios in the Brazilian Market with Integrated Neural Networks: CNN, LSTM, and GRU

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Summary

In a contemporary scenario characterized by an expansive flow of data, artificial intelligence (AI) technologies are emerging as crucial elements in the strategic decision-making process, especially in the financial sphere. This study delved into the analysis of investment portfolio optimization, adopting an innovative approach using hybrid models. These models combine

the advanced capabilities of Convolutional Neural Networks (CNN), Long-Short Term Memory (LSTM), and Gated Recurrent Unit (GRU) networks, aiming for a more accurate and comprehensive market analysis. A diverse set of variables was compiled to feed these models, ranging from technical and fundamental indicators to macroeconomic elements, providing a holistic view of investment dynamics. One significant revelation of this study was identifying a notable discrepancy between the variables influencing overall portfolio performance versus those affecting specific stocks, highlighting the complexity of asset management. In addition, the study explored asset allocation strategies, including equal-weighted allocation and a more sophisticated weighted approach using the Hierarchical Risk Parity (HRP) model. The hybrid methodology, especially with HRP, demonstrated a capital appreciation potential of 88.28% over approximately 11 months, disregarding transaction costs. When these costs were considered, the return adjusted to 53.97%, underlining the critical importance of accounting for operating costs in investment strategies. This contrast accentuates the need for careful analysis of transaction costs to avoid overly optimistic or pessimistic assessments of actual performance, reinforcing the relevance of well-founded strategies and AI technologies for more efficient and profitable portfolio management.

Keywords: Investment strategies; Deep learning; Asset allocation; Portfolio performance; Hybrid models

1. Introduction

The financial markets generate a large volume of data daily, and analyzing it has become an increasingly complex challenge (Aithal et al., 2023). Many academics are investigating machine learning algorithms and deep learning as methods for transforming data into useful information for decision-making in the stock market (Khan et al., 2022).

Machine learning and deep learning are subfields of artificial intelligence (AI), a constantly evolving area that has become indispensable for financial market participants. One of its most significant contributions lies in its ability to analyze vast volumes of data from multiple sources in real-time. This agility and breadth of analysis, carried out in fractions of a second, gives operators a significant competitive advantage when making strategic decisions (Cohen, 2022).

Kim et al. (2023) point out that the use of AI in financial markets has become a universal phenomenon. Fund managers seek to improve returns with AI, and financial institutions strive to improve work efficiency through AI.

The stock market acts as an accurate barometer of economic health, with its fluctuations directly reflecting the adversities of the economic system (Khalil & Bakar, 2023). The mechanism underlying stock price formation is complex and influenced by many variable factors. This complexity is exacerbated by non-linear behavior, making traditional analytical methods unsuitable for effective forecasting (Yang et al., 2022).

One of the algorithms that has gained prominence on the market in recent years is artificial neural networks. They are adaptive statistical models developed in analogy to the human brain. Neurons are linked by solid or weak connections created through the learning process (D'Amato et al., 2022).

The evolution of neural networks to Recurrent Neural Networks (RNN) is noted as being particularly effective in time series analysis. However, traditional RNNs often face technical problems such as the explosion and disappearance of gradients, especially when dealing with long-term dependencies in the data. To mitigate these limitations, the *Long-Short Term Memory (LSTM)* architecture was introduced by Hochreiter and Schmidhuber (1997).

Similarly, the *Gated Recurrent Unit (GRU)*, proposed by Cho et al. (2014), is considered a more simplified but equally effective alternative, demonstrating the ability to generate results comparable to those of LSTM.

Convolutional Neural Networks (CNN) also find their place in this context. Two articles published by Lecun et al. (1989) and Lecun et al. (1998) laid the foundations for CNNs. They introduced the LeNet-5 architecture, one of the first CNNs to recognize handwritten characters in postal codes. Today, they are also used for object and image recognition, including facial recognition. Although not considered an RNN, this architecture has been used in the financial market to recognize patterns in financial market data (Matuozzo et al., 2023).

Many studies have recognized the power of neural networks in work related to the capital market context. In addition, some authors are going beyond using isolated models and have started to study combined models; for example, Yang et al. (2022) suggest using combined models to achieve good results.

Song and Choi (2023) used the hybrid neural network models CNN-LSTM and GRUCNN and the decision tree-based models XGBoost and Random Forest to predict the return of the DAX, DOW, and S&P500 indices. The neural network combinations generated superior results. Friday et al. (2022) used the GRU, LSTM, and CNN models in isolation to predict the returns of Tesla and Google shares. In all forecasts, the GRU model outperformed the other two because, according to the authors, it has better retention when dealing with time series data sets. Gülmez (2023) used different RNN variations, including LSTM-ARO, ANN, LSTM1D, LSTM2D, and LSTM3D, to forecast the stock prices of 30 large companies listed on the Dow Jones Industrial Average (DJIA) index. The best results were achieved using LSTM-ARO. Prokhrel et al. (2022) tested the LSTM, GRU, and CNN models to predict the Nepal Stock Exchange [NEPSE] index. The authors achieved superior results using LSTM. Lawi et al. (2022) tested the LSTM and GRU algorithms to obtain the next-day closing values of four stocks (AMZN, GOOGL, BLL, and QCOM) listed on American stock exchanges. The results showed that GRU is more accurate in all the indicators evaluated and that LSTM, despite having a lower result than GRU, has a more consistent result, i.e., with fewer deviations from the average.

Hanauer and Kalsbach (2023) point out that most studies on machine learning approaches in the financial market are applied in developed countries. As such, the authors noted a lack of studies for emerging countries that contribute approximately 60% of the world's GDP.

Given the above, the question arises: Are recurrent neural networks combined with convolutional neural networks effective for selecting stocks to create investment portfolios in the Brazilian market? This study aims to create investment portfolios using hybrid neural network models that integrate LSTM, GRU, and CNN.

2. Models and related literature

Within the broad field of machine learning and deep learning models, methods can be categorized into two main groups (which are not the only ones): supervised and unsupervised. For the former, explanatory variables are inserted, and the behavior of an explained variable (or response variable, "target" or simply "Y") is analyzed. The unsupervised models analyze how specific individuals are grouped based on their characteristics. Given this, the algorithm used in this study fits into the supervised models since the aim is to insert explanatory variables, predict the behavior of stock returns, and create portfolios based on the predicted behavior. It should be noted that Python was used in all stages of the work.

According to Friday et al. (2022), given the characteristics of the financial market, traditional models such as linear regression and support vector machines (SVM) are less accurate and less used. Deep learning approaches such as RNN, LSTM, GRU, and CNN have grown in this sense. In addition, there has been a substantial increase in research related to these approaches applied to the financial market, especially in high-impact journals.

CNNs, generally used for image classification, can also classify time series (Kim et al., 2023). This type of network comprises several layers, including the convolutional layer, which is responsible for inserting weights into each feature weighting them; the clustering layer, which transforms these inputs into a conceptual form in its hidden memory cells; the flattening layer, which converts the multidimensional data into a onedimensional matrix; and the fully connected layer, made up of several neurons that learn all the features of the previous layers. Using these layers together, CNNs can provide accurate and efficient classifications for various data, including time series (Friday et al., 2022).

LSTM and GRU are two RNN variations that allow long-term memories. In addition, while the RNN has only one activation function in the intermediate layers, the LSTM and GRU allow for several activation functions, with complex operations performed in several gates (Lawi et al., 2022).

LSTM was initially conceived to address the complexity inherent in retaining long-term memories. This technique effectively solves problems related to learning and memory in recurrent neural networks, especially in contexts that require the maintenance of temporal relationships between distinct entities (Friday et al., 2022). It uses three gates; the input gate (i_t) adds new information and can be represented by eq. (1):

$$i_t = \sigma(w_i \cdot [h_{t-1}, x_t] + b_i) \quad (1)$$

The second is the forgetting gate (f_t), responsible for the charge that will not be retained. It can be represented by eq. (2):

$$f_t = \sigma(w_f \cdot [h_{t-1}, x_t] + b_f) \quad (2)$$

The third and final gate is the output gate, which provides the processed information, according to eq. (3):

$$o_t = \sigma(w_o \cdot [h_{t-1}, x_t] + b_o) \quad (3)$$

GRU works similarly to LSTM but with two gates. These gates determine how much knowledge should be passed on. The update gate (u_t) determines what information should be retained. The restart gate (r_t) determines which parts of the information in the previous hidden state should be combined with the current input to develop a new hidden state that determines which information to forget, i.e., which information is not considered relevant for learning the model. The GRU gates are represented by eq. (4) and (5):

$$r_t = \sigma(x_t \cdot w_r + h_{t-1} \cdot u_r) \quad (4)$$

$$u_t = \sigma(x_t \cdot w_u + h_{t-1} \cdot u_u) \quad (5)$$

The architectures above have been used in the same order in which they were presented. This order is significant because each layer learns different representations of the data. The Conv1D layer can capture local information and patterns in the data, while the LSTM and GRU layers are designed to learn long-term dependencies and capture sequential information. Three or more activations are required in a target variable with three or more categories (Anese et al., 2023). These activations are passed to the Softmax function, which normalizes the activations to obtain the distribution of probabilities between the classes and determine the conditional probability of a class x_i , with $i=1,2,...,k$.

The Softmax function generates values between 0 and 1, and the sum of the probabilities of all the classes is 1. In this study's initial phase, the function's use to classify assets into three categories was tested: negative return, zero return, and positive return. Given the unsatisfactory results, the function was changed to predict only positive returns, which thus fits into a binary classification. Therefore, only one activation function is required, and, in this case, the sigmoid activation function was used for the output layer, according to eq. (6):

$$\text{Sigmoid}(x) = \frac{1}{1 + \exp(-x)} \quad (6)$$

Where \exp is the exponential function, and x is the input value.

The sigmoid function, also known as the logistic function, returns a value between 0 and 1, which in the context of this study can be translated as the probability of an asset's future return being positive.

Based on the model's output, the assets with the highest probability of positive returns were considered for daily allocation as long as this probability was above a certain cut-off point. The number of assets in the portfolio and the cut-off point were set using a backtesting that varied from one portfolio to another. Two forms of allocation were then used: (1) naive allocation, i.e., with equal weights for all assets; (2) optimized allocation using the *Hierarchical Risk Parity* (HRP) model.

Zhao et al. (2023) argue that portfolio optimization refers to distributing capital among various financial assets, such as stocks, bonds, currencies, or cryptocurrencies, and continuously adjusting the weights based on performance and correlation with other assets. The aim is to diversify investment risks and maximize expected returns.

HRP models aim to solve the three main concerns of quadratic optimization models: instability, concentration, and poor performance. It applies modern mathematics (graph theory and machine learning techniques) to develop a diversified portfolio based on the information in a covariance matrix (López de Prado, 2016; Lohre et al., 2020).

As described by Lohre et al. (2020), HRP takes advantage of hierarchical grouping to assign asset position sizes based on the risk characteristics of the subgroups. HRP seeks to infer the hierarchical relationship between assets used for portfolio diversification. Many traditional allocation systems require the inversion of the covariance matrix, a step that is avoided in HRP. This gives the model an additional advantage, as it avoids significantly increasing errors (Jain & Jain, 2019; Burggraf, 2021; Raffinot, 2017).

According to Shahbazi and Byun (2022) and Jain and Jain (2019), HRP is operationalized in three stages. The first involves determining the hierarchical relationship between the assets that make up the portfolio using a recursive cluster formation scheme. Groups are formed using correlations to identify similar groups of assets that are successively merged until they form a large group. The second stage involves quasi-diagonalization, in

which the correlation matrix generated in the first stage is rearranged. Quasi-diagonalization allows similar assets to be kept as close as possible and different assets as far apart as possible. In the third stage, the weights of each asset are distributed using the inverse variance allocation between subgroups obtained by recursive bisection.

Al-Alawi and Alaali (2023) argue that making predictions in the stock market is a significant challenge that has attracted the attention of many researchers. Accurate forecasts allow investors to make better buying or selling decisions. In a literature review produced by the authors, they concluded that forecasts using LSTM and GRU are more accurate in the context of the stock market. For Cui et al. (2023), the trading process can be described as a process in which returns are maximized by restricting risk.

Table 1 shows the main articles that influenced the development of this study. There are three opportunities to be explored: (1) studies in the Brazilian context are rare; (2) a greater focus is placed on selection models, and little attention is paid to allocation optimization; (3) no study has used the triple combination of CNN-LSTM-GRU in the stock market.

Table 1
Survey of articles

Author/year	Sample	Model(s)
BREITUNG, (2023).	18,973 companies from various countries	Random Forest
TANG; TANG; YU, 2023	Main Chinese stock indices	Decision Tree, XGBoost, Random Forest (RF), Support Vector Machine (SVM), Linear SVM and Long Short-Term Memory (LSTM)
(Gülmez, 2023)	30 large companies listed on the Dow Jones Industrial Average (DJIA) index	LSTM-ARO, ANN, LSTM1D, LSTM2D and LSTM3D
(Sriman et al. 2023)	Bitcoin and Ethereum	CNN, LSTM, and GRU
(Dahal et al. 2023)	Shares in major American companies and stock indices from various countries	LSTM and GRU
(Xu et al. 2023)	Chinese stock market	BiGRU, BiLSTM and GMDH
(PANDEY et al. 2023)	Companies listed on the National Stock Exchange (NSE).	ARIMA and LSTM
(EGGEBRECHT; LÜTKEBOHMERT, 2023)	US large-cap stocks	CNN and LSTM
(Deshpande et al. 2023)	Tata Motors Limited	LSTM
(W. Tang et al. 2023)	Gold and Bitcoin	LSTM
(Matuozzo et al. 2023)	SXXP and DAX	CNN
(VERMA; SAHU; SAHU, 2023)	S&P500, NASDAQ and RUSSELL2000	LSTM, CNN, and CNN-LSTM
(LI, 2022)	Shanghai Stock Exchange	RNN, LSTM and LSTM+Attention
(Usmani & Shamsi, 2023)	Pakistan Stock Exchange (PSX)	LSTM
(Singh et al. 2022)	ICICI Bank Limited	RNN, GRU, and LSTM
(Liang et al. 2023)	Three indices of the Chinese market	BGRU AND DB-BLSTM

(Sornavalli et al. 2022)	100 largest companies in the S&P 500	GAN and XGBoost
(Souza et al. 2022)	Bovespa Index	RNN and LSTM and VADER
(Chang & Zhang, 2023)	American stocks	CNN and NLP
(Salemi Mottaghi & Haghir Chehreghani, 2023)	8 Iranian companies	PCA and LSTM
(Gupta et al., 2023)	Companies listed on NIFTY-50	PCA and DBSCAN
(Ku et al. 2023)	100 shares on the Malaysian Stock Exchange	LSTM
(Mu et al. 2023)	of 6 assets from different sectors of the Chinese market	LSTM, CNN, and RNN

Notes: Ordered Weighted Average (OWA), AutoRegressive Integrated Moving Average (ARIMA), Convolutional Neural Network (CNN), Long Short-Term Memory (LSTM), Gated Recurrent Unit (GRU), Recurrent Neural Network (RNN), Bidirectional Gated Recurrent Unit (BGRU), Daubechies Bidirectional Long Short-Term Memory (DB-BLSTM), Generative Adversarial Network (GAN), Extreme Gradient Boosting (XGBoost), Valence Aware Dictionary and Sentiment Reasoner (VADER), Natural Language Processing (NLP), Principal Component Analysis (PCA) and Density-Based Spatial Clustering of Applications with Noise (DBSCAN).

Source: Original research data

Some of the terms in Table 1 already have equivalents in Portuguese, such as "Principal Components Analysis," but not all translations sound natural. To maintain consistency, we chose to use the original terms in English. The selection of articles in the table came from a search on the Scopus database using specific queries for the period 2022 to 2024, focused on "machine learning," "neural network," "random forest," "xgboost," "catboost," and "deep learning" related to the stock market. The search for "neural network" generated the most results. The table does not include all the articles published but those most relevant to our study. Considering evidence that hybrid models outperform stand-alone models, the study focused on these models without testing individual approaches.

The diagram in Figure 1 summarizes the general structure of the article. More details on the processing are given below.

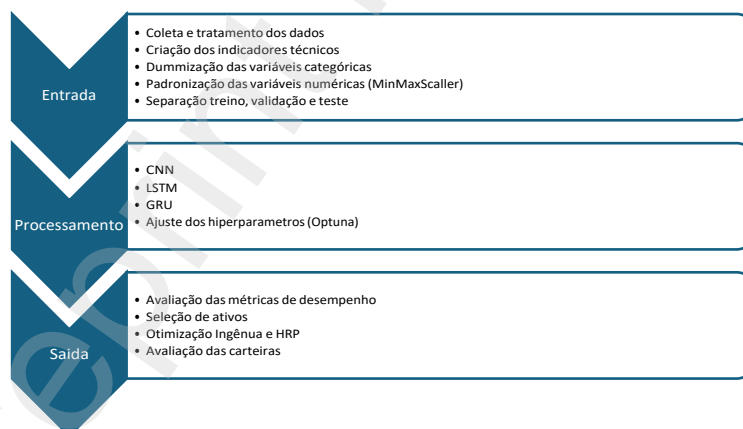


Figure 1 *Workflow used in the research*

Source: Original study data

In general, the core of the work is the creation and training of neural networks. The architecture begins with an input layer that receives the training data. Next, a Conv1D layer is created to extract essential features from the data, followed by a *dropout* layer to avoid *overfitting* and a layer called GlobalMaxPooling1D to reduce dimensionality.

In addition, there are two LSTM layers with *dropout* for learning complex time sequences. A GRU layer also captures long-term relationships in the input data. *Dropout* is applied again to avoid *overfitting*.

Subsequently, the outputs of the Conv1D, LSTM, and GRU layers are concatenated into a single layer. This concatenation allows the network to capture different aspects of the data. In the final part, the output model for the classification was defined. A *dense* layer was used to create a fully connected layer (or dense layer) with an output unit. The sigmoid activation function was applied to the output of this layer.

The Adam optimizer was used, a variation of the stochastic gradient descent (SGD) that adapts the learning rate over time. It is known for its efficiency and good results in many machine-learning problems. The loss function is defined as *binary_crossentropy*, which is the loss commonly used in binary classification problems.

In the research, the hyperparameter values were carefully selected to optimize the model's performance. The filters were set to 131, with a kernel size of 5. The batch size was set to 32, and the learning rate was set to 0.001985519. Specific dropout rates were applied for different layers: 0.001721488 for the convolutional layer, 0.391008828 for the LSTM, and 0.095231996 for the GRU. In addition, the number of units was adjusted to 140 in the LSTM and 208 in the GRU to maximize the model's effectiveness in prediction tasks.

During training, *callback* functionsⁱⁱ were used to save the model with the best results based on the validation loss. The training was stopped early if there was no improvement in validation loss after a specific number of epochs.

The period from 01/01/2000 to 31/12/2021 was used as a training period. Not all assets have data for the entire period; in this case, the maximum period available was used. The period from 01/01/2022 to 30/06/2022 was used as the adjustment validation period. The data was divided into 60-day windows, i.e., asset returns were forecast based on the last 60 days. The remainder of the sample was used for testing from 01/07/2022 to 30/06/2023.

The *Accuracy*, *Area Under the ROC Curve (AUC)*, *Precision*, *Recall*, and *F1-score* metrics were generated from the test data. In addition, the main metrics and graphs for evaluating investment portfolios were presented.

3. Data collection and processing

The data was collected via the Yahoo Finance platform using the *yfinance* API. Daily data was used for the maximum period available (starting in 2000) for all companies listed on the Brazil 100 Index (IBrX100). For some companies, no data is available for the entire period, so the maximum period available was used.

The IBrX100 comprises the 100 most tradable assets with at least 95% presence in trading sessions. Shares and *units of shares* are listed, which cannot be *penny stocks* and are weighted by the market value of the *free float*. The index is released every four months on the first Monday of January, May, and September. In this work, the May 2023 portfolio was used.

In addition to shares, to increase the diversification of the portfolios in different contexts, information on *Exchange Traded Funds (ETFs)* was collected for inclusion in the portfolios, as shown in Table 2.

Table 2*ETFs included in the sample*

Ticker	Description
BOVA11	It seeks to replicate Ibovespa, the Brazilian market.
SMAL11	Seeks to replicate SMLL index
BRAX11	It seeks to replicate the IBRX100
PIBB11	Search replica IBRX50
IVVB11	It seeks to replicate the S&P 500, USA.
NASD11	It seeks to replicate the NASDAQ 500, USA.
ECOO11	Aims to replicate the S&P/B3 Brazil ESG index
GOVE11	Seeks to replicate the Corporate Governance Trade Index (IGCT)
ISUS11	It seeks to replicate the Dow Jones Sustainability Index Emerging Markets
HASH11	Seeking to replicate the Nasdaq Crypto Index
BITH11	It seeks to replicate Bitcoin.
ETHE11	It seeks to replicate Ethereum.

Source: Original research data

For all the assets, adjusted information was collected on the opening, maximum, minimum, and closing prices and the volume traded. This information created technical indicators and was used as explanatory variables in the neural network.

Parida et al. (2023) used technical indicators (William (%R), Stochastic Oscillator (%K), Moving Average Convergence/Divergence (MACD), Relative Strength Index (RSI), and Moving Average) as input in a hybrid CNN model integrated with LSTM to forecast the stock returns of two Indian banks. These indicators make it possible to capture different market behaviors.

Srivinay et al. (2023) used 20 technical indicators to classify the highs and lows of the share prices of companies listed on the CNX Nifty (the benchmark index of the Indian stock exchange). From the 20 indicators, the authors used the Boruta algorithm in conjunction with *Random Forest* to select the most relevant indicators. From the filtered data, samples were inserted into the H₂ platform with between 74% and 89% accuracy for five companies.

For Ullah et al. (2022), the unpredictability and volatility of the stock market make it challenging to make a substantial profit using any generalized methodology. They argue that few authors focus on finding the best characteristics (*features* or variables) for specific periods, i.e., for them, the characteristics must be dynamic and selected according to each time sample. Features, characteristics, and variables are synonymous in supervised machine learning models. The ones used in this study are shown in Tables 3 and 4.

Table 3

Quotation variables were collected from Yahoo Finance, categorical variables indicated the days of the week and return, target variables were created from quotations, and technical variables were created using the TA-Lib library.

Var.	Description
open	Opening price of the day (adjusted)
high	Highest price of the day (adjusted)
low	Lowest price of the day (adjusted)

close	Closing price for the day (adjusted)
volume	Turnover
week	Dummies day of the week (1 to 5)
R	Daily return calculated on closing (lagged from 1 to 21)
r_fwd	Daily return calculated on d+1 closing
Target	If $r_fwd > 0$, then the target receives the value 1, otherwise it receives the value 0

Var.	Description (Original name)	Translated term
IHR	Relative Strength Index	Relative Strength Index
WILLR	Williams %R	Williams Percentage
BBH	Bollinger Bands High	Upper Bollinger Band
BBL	Bollinger Bands Low	Lower Bollinger Band
NATR	Normalized True Range	Normalized True Range
ATR	Average True Range	Medium True Range
PPO	Percentage Price Oscillator	Percentage Price Oscillator
MACD	Moving Average Convergence Divergence	Moving Average Convergence and Divergence
MOM	Momentum	Moment
WMA	Weighted Moving Average	Weighted Moving Average
EMA	Exponential Moving Average	Exponential Moving Average
JRC	Commodity Channel Index	Commodity Channel Index
CMO	Chande Momentum Oscillator	Chande Momentum Oscillator
ROC	Rate of Change	Rate of Change
ADOS	Accumulation Distribution Oscillator	Accumulation Distribution Oscillator
ADX	Average Directional Index	Average Directional Index

Source: Original research data

Based on the technical indicators shown in Table 3, 6 to 21 days were implemented to capture different behaviors over time. Five *dummies* representing the five days of the *week* were created for the week variable. This resulted in 273 characteristics. In addition to these variables, we also used macroeconomic variables collected from the Institute for Applied Economic Research (Ipea) and fundamentalist variables obtained from the Comdinheiro platform, which are detailed in Table 4.

Table 4
Macroeconomic and fundamental variables

Macroeconomic variables	
Name	Description
GM366_ERC366	Exchange rate - R\$ / US\$ - commercial - buy - average
GM366_EREURO366	Euro Zone - exchange rate - euro / US\$ - average
GM366_EUROC366	Euro Zone - exchange rate - R\$ / euro - buy - average
EIA366_PWTI366	Price - crude oil - WTI (FOB)
GM366_DOW366	Stock index - Dow Jones - closing
SGS366_NASDAQ366	Stock index - NASDAQ - closing
VALUE366_FEDFUND366	Basic interest rate - Federal Funds - set by the FOMC
BMF366_FUT1DI1366	Interest rate - 1-day DI - future: maturity t+1

BMF366_FUT3DI1366	Interest rate - 1-day DI - future: maturity t+3
BMF366_FUT3DI1V366	Interest rate - 1-day DI - future: maturity t+3 - volatility
BMF366_FUT6DI1366	Interest rate - 1-day DI - future: maturity t+6
ANBIMA366_TJTLN12366	Fixed interest rate - term structure - LTN - 12 months
ANBIMA366_TJTLN1366	Fixed interest rate - term structure - LTN - 1 month
ANBIMA366_TJTLN3366	Fixed interest rate - term structure - LTN - 3 months
ANBIMA366_TJTLN6366	Fixed interest rate - term structure - LTN - 6 months
BM366_TJOVER366	Interest rate - Selic - set by Copom
GM366_IBVSP366	Stock index - Ibovespa - closing
GM366_IBVSPV366	Stock index - Ibovespa - closing - volatility
JPM366_EMBI366	EMBI + Brazil Risk

Fundamental variables

Name	Description
P/L	price over profit
EV	Enterprise Value
EV/EBITDA	Enterprise Value over EBITDA
P/VPA	price over book value per share

Note: For transparency, the codes used in IPEADATA have been used as names for the macroeconomic variables. **Source:** Original research data

In order to reduce the dimensions of the database, we tested the principal component analysis (PCA) approaches (Salemi et al., 2023), recursive feature elimination (RFE), and the Boruta algorithm, as suggested by Srivinyan et al. (2023). They all resulted in gains in training performance (disregarding the dimensionality reduction time). However, there were no gains in the model evaluation metrics (Accuracy, AUC, Precision, Recall and F1score), i.e., despite making neural network training less costly, the above methods could not eliminate noise that could compromise accuracy and other metrics. We, therefore, opted to use the data without dimensionality reduction.

In addition to trying to eliminate noise and improve model performance, a crucial issue when using ML models is to avoid *data leakage*, especially in the stages of dividing the data into training, validation, and testing, which is nothing more than ensuring that the information is allocated at the right time for the predictions. (Hanauer and Kalsbach, 2023). Care must also be taken with information leakage in the MinMaxScaler module, which was used to place the data for all the metric variables on a scale between 0 and 1. Adjusting the data to the same size prevents the model from being affected by significant differences or large variations in the data. At this stage, it is essential to perform the MinMaxScaler only on the training base and then apply the transformations to the training and test sets. It is expected to find studies in which the procedure is carried out on the entire *dataset*, compromising the analysis of the results (Ku et al., 2023). In the financial market, information leaks are common when data that has been republished is collected, i.e., a specific point in time is evaluated with data that did not exist.

The article's next section is dedicated to presenting and discussing the results obtained during this study. It will detail the main findings, their interpretation in the light of the proposed objective, and the relevance of these results to the field of study in question.

4. Results and discussion

This study aimed to create stock investment portfolios using hybrid neural network models. In addition, the naive allocation (1/N) and the optimized allocation using the HRP method were tested. Table 5 shows the descriptive analysis of the macroeconomic and fundamental variables used in the study. Due to the large number of technical variables, they have not been included in the table.

Table 1
Descriptive statistics of the variables used

variable	Obs	average	Std. Dev.	min.	max.
open	379645	13316.17	238755.27	-93.86	4989083.00
high	379645	13325.83	238783.76	-94.99	4989083.00
low	379645	13299.25	238625.29	-86.00	4989083.00
close	379645	13308.53	238652.02	-87.64	4989083.00
volume	379645	14348206.47	179619442.08	0.00	17560000000.00
r01	379536	0.00	0.41	-1.01	241.60
weekday_0	379645				
GM366_ERC366	379645	3.24	1.30	1.53	5.94
GM366_EREURO366	379645	0.84	0.10	0.63	1.20
GM366_EUROC366	379645	3.79	1.34	1.56	6.94
EIA366_PWTI366	379645	67.40	24.50	-36.98	145.31
GM366_DOW366	379645	19358.92	8596.28	6547.05	36799.65
SGS366_NASDAQ366	379645	5944.54	4058.19	1114.11	16057.44
VALOR366_FEDFUND366	379645	1.36	1.68	0.13	6.50
TJTLN12366	379645	11.31	4.95	2.23	35.40
TJTLN1366	379645	44.23	1969.46	1.92	116258.00
TJTLN3366	379645	10.93	4.78	1.90	28.82
TJTLN6366	379645	11.05	4.89	1.88	33.41

Table 5

Cont.

variable	Obs.	average	Std. Dev.	min.	max.
BMF366_FUT1DI1366	379645	10.81	4.74	1.89	26.75
BMF366_FUT3DI1366	379645	10.87	4.75	1.87	27.56
BMF366_FUT6DI1366	379645	11.03	4.79	0.00	27.64
BM366_TJOVER366	379645	10.87	4.66	2.00	26.50
GM366_IBVSP366	379645	67875.85	30323.65	8370.88	130776.27
GM366_IBVSPV366	379645	1.55	0.77	0.57	8.15
JPM366_EMBI366	379645	330.18	227.46	136.00	2443.00
P/L	287271	-236.68	18203.19	-1348571.32	60039.59
EV	329257	31500.51	79357.50	-567145.63	779757.70
EV/EBITDA	243329	35.19	334.09	-8088.13	13394.81
P/VPA	287338	8183.19	1657500.66	-788.89	344965486.89

Note: Besides the abovementioned variables, technical variables were used, totaling 273

Source: Original survey results

Although Table 5 shows a large gap between the minimum and maximum values due to price adjustments over the sample period, the interquartile values (omitted due to space limitations) showed values compatible with the stock price. An alternative approach could eliminate the extreme values (*outliers*), but this was not the case in this study.

In this study, various modeling strategies were explored. Model 1 was trained using all available variables. For Model 2, it was decided to exclude fundamental variables, emphasizing only technical and macroeconomic variables. Model 3, in turn, incorporated both technical and fundamental variables. Finally, Model 4 was developed using exclusively technical variables. It is essential to highlight a limitation related to the use of fundamental variables: these were not available throughout the period studied, resulting in a reduced number of observations in the models that used them.

The models were used to classify whether the subsequent return for certain stocks would be greater than zero or less than or equal to zero. The sigmoid activation function was used for the model outputs, allowing the results to be expressed in probability. Based on these probabilities, investment portfolios were built, prioritizing the stocks with the highest probability of generating positive returns in the future. Specifically, the stocks with the highest probabilities of positive returns were selected as long as these probabilities were higher than a certain cut-off point defined in the *backtest*.

This study aimed to create two types of stock portfolios, both designed with the joint contribution of the CNN-LSTM-GRU algorithms. In the first portfolio, CNNLSTM-GRU, a naive allocation model was used, in which all the stocks were allocated equal weights. In the second portfolio, CNN-LSTM-GRU-HRP, the shares were allocated with weighted weights, according to the HRP method, to minimize the associated risk. In this way, two portfolios were created for each of the four models. Table 6 shows the statistics of the CNN-LSTM-GRU and CNN-LSTM-GRU-HRP portfolios compared to the IBOV results created from Model 1.

Table 6
Results of portfolios created using Model 1 (26/07/2022 to 29/06/2023)

Statistics	IBOV	CNN-LSTM-GRU	CNN-LSTM-GRU-HRP
Total Return	18.65%	24.03%	20.29%
Sharpe Daily	0.97	0.96	1.05
Daily Sortino	1.74	1.82	1.87
CAGR	20.30%	26.20%	22.10%
Maximum loss (<i>drawdown</i>)	-18.35%	-16.82%	-14.40%
<u>Calm Index</u>	<u>1.11</u>	<u>1.56</u>	<u>1.53</u>

Note: CAGR (Compound Annual Growth Rate)

Source: Original survey results

The results generated by Model 1 were very similar. Although the CNN-LSTMGRU portfolio generated a higher total return, the risk-adjusted return was better for the CNNLSTM-GRU-HRP portfolio. Figure 2 shows the graphs for evaluating the portfolios.

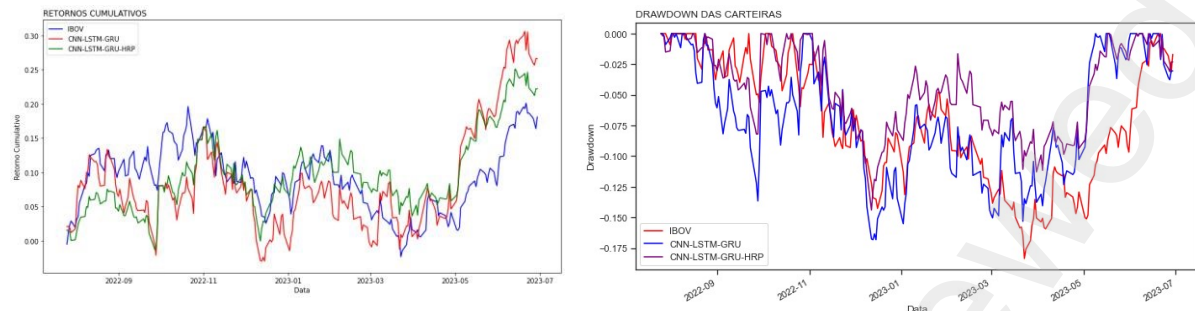


Figure 2 *Graphs of cumulative returns and maximum drawdown*
Source: Original survey results

Table 7 shows the statistics for the CNN-LSTM-GRU and CNN-LSTM-GRU-HRP portfolios compared to the IBOV results. As mentioned above, Model 2 considered macroeconomic and technical variables, i.e., fundamental variables were not part of the model. The idea of excluding the fundamental variables was based on three factors: 1) the smaller number of observations in the sample, both due to the lower availability of data and the impossibility of including ETFs; 2) the possibility that the fundamental values are already explained in the price and consequently in the technical indicators. (Chang and Zhang, 2023) 3) the overall result of the "SHAP" graph showed that fundamental variables are not among the most important (omitted due to space restrictions).

Table 7
Results of the portfolios created using Model 2 (26/07/2022 to 29/06/2023)

Statistics	IBOV	CNN-LSTM-GRU	CNN-LSTM-GRU-HRP
Total Return	18.57%	32.30%	21.08%
Sharpe Daily	0.97	1.28	1.15
Daily Sortino	1.74	2.52	2.18
CAGR	20.21%	35.32%	22.97%
Maximum loss (<i>drawdown</i>)	-18.35%	-12.98%	-9.02%
Calm Index	1.1	2.72	2.55

Note: CAGR (Compound Annual Growth Rate)

Source: Original survey results

As can be seen in Table 7, the exclusion of fundamental variables improved portfolio returns. Figure 3 shows the graphs for analyzing the results.

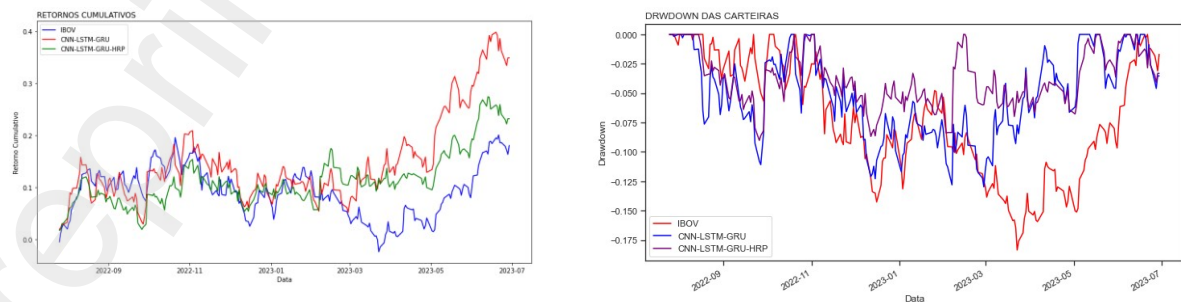


Figure 3 *Graphs of cumulative returns and maximum drawdown*
Source: Original survey results

For Model 3, whose results are shown in Table 8, we tried to follow the same logic of eliminating less relevant variables. However, we eliminated the macroeconomic variables in this model and kept the fundamentalist ones.

Table 8

Results of the portfolios created using Model 3 (26/07/2022 to 29/06/2023)

Statistics	IBOV	CNN-LSTM-GRU	CNN-LSTM-GRU-HRP
Total Return	18.69%	15.47%	15.56%
Sharpe Daily	0.98	0.7	0.85
Daily Sortino	1.75	1.27	1.47
CAGR	20.34%	16.82%	16.91%
Maximum loss (<i>drawdown</i>)	-18.32%	-17.94%	-15.31%
Calm Index	1.11	0.94	1.1

Note: CAGR (Compound Annual Growth Rate)

Source: Original survey results

Excluding the macroeconomic variables while keeping the fundamentalist ones deteriorated the model, generating lower returns than the IBOV. The decision to include or exclude certain sets of variables could give rise to an extensive debate, which exceeds the scope of this study. Therefore, the aim here is merely to assess the different impacts of specific groups of variables. Figure 4 shows the graphs for analyzing the portfolios.

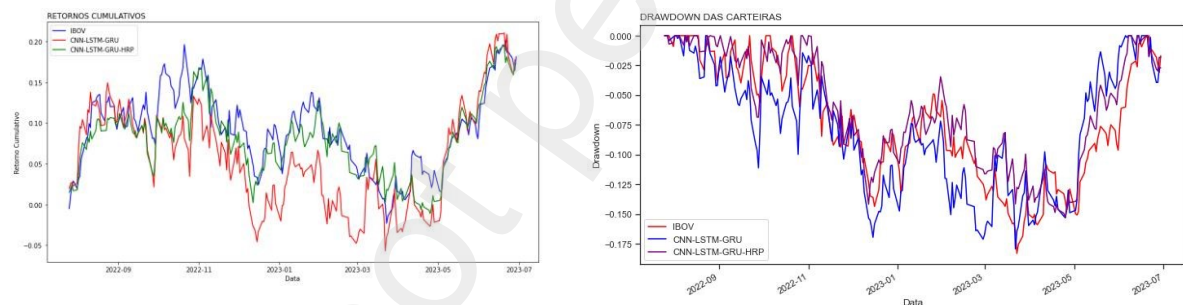


Figure 4 *Graphs of cumulative returns and maximum drawdown*

Source: Original survey results

Finally, the decision was made to use prices and the technical indicators derived from them exclusively. The premise behind this choice is that macroeconomic and fundamentalist information is already embedded in asset prices. Some of this information is incorporated in advance, based on expectations and projections, while the rest is assimilated later, reflecting the events that occurred. Similar approaches have been adopted and validated by authors such as Parida et al. (2023), Srivinyan et al. (2023), and Breiung (2023), who obtained positive results using only technical indicators. The results for Model 4 are shown in Table 9.

Table 9

Results of the portfolios created using Model 4 (26/07/2022 to 29/06/2023)

Statistics	IBOV	CNN-LSTM-GRU	CNN-LSTM-GRU-HRP
Total Return	18.65%	66.90%	85.91%
Sharpe Daily	0.97	1.68	2.11
Daily Sortino	1.74	3.43	4.32
CAGR	20.30%	73.93%	95.44%

Maximum loss (<i>drawdown</i>)	-18.35%	-23.33%	-22.61%
Calm Index	1.11	3.17	4.22

Note: CAGR (Compound Annual Growth Rate)

Source: Original survey results

As shown by the results presented in Table 10, adopting a strategy that maintains exclusively technical variables was validated as successful. This success suggests that including other variables could introduce noise, considering that the price already encapsulates all available information. These findings contradict the claims made by Salemi Mottaghi and Haghiri Chehreghani (2023), who argue that the isolated use of prices and technical indicators does not produce satisfactory results. Therefore, It is suggested that each context's specificities should be assessed independently.

Given that the best return was achieved using Model 4, as seen in Table 9, the following evaluations will be made in this context. Table 10 shows the values of the classification metrics used.

Table 10

Metric	Value
<i>Metric results and daily hit rates for Model 4</i>	
Accuracy	0.51
AUC (Area Under the Curve)	0.45
F1 score	0.68
Daily Win Ratio IBOV	0.53
Daily Win Ratio CNN-LSTM-GRU	0.53
Daily Win Ratio CNN-LSTM-GRU-HRP	0.56

Source: Original survey results

Although the overall result indicates an average accuracy of 0.51, a sample of the individual results in Table 11 reveals that this metric can fluctuate significantly depending on the asset.

Table 11

Accuracy per ticker (Model 4)

ticker	accuracy	Ticker	accuracy	ticker	accuracy
CIEL3	55.60	RDOR3	50.86	IRBR3	47.41
BRKM5	54.31	WEGE3	50.86	NTCO3	47.41
EMBR3	50.86	ENBR3	47.41	IVVB11	37.93

Source: Original survey results

As shown for the other models, Figure 5 below shows the graphs for the latest portfolios.

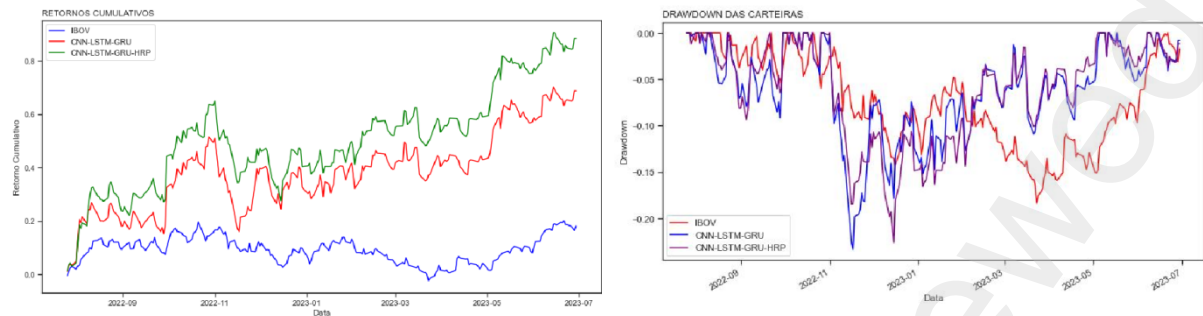


Figure 5 Cumulative returns and maximum drawdown
Source: Original survey results

The cumulative returns shown in Figure 5 show that the portfolios outperformed the IBOV from the outset and ran somewhat in parallel. Although the three portfolios had similar *drawdowns* until January 2023, the portfolios formulated in the study stabilized after that, while the IBOV fell again.

As shown in Table 12, the differences in the averages of the returns obtained were not statistically significant. This suggests that although the CNN-LSTM-GRU-HRP portfolio showed an accumulated return of 85.91%, in contrast to the 18.65% of the IBOV, this performance could result from chance.

Table 12

Shapiro-Wilk normality test and t and Mann-Whitney (U) tests for differences in means

Shapiro-Wilk	statistic	p-value
IBOV	0,98850	0.060451626777648926
CNN-LSTM-GRU	0,95614	1.6210457260967814e-06
CNN-LSTM-GRU-HRP	0,94379	8.47941521442408e-08
Portfolio	t-statistic	p-value
IBOV versus CNN-LSTM-GRU	-0,964520	0,335290
IBOV versus CNN-LSTM-GRU-HRP	-1,275787	0,202672
CNN-LSTM-GRU versus CNN-LSTM-GRU-HRP	-0,206570	0,83436
Portfolio	U-statistic	p-value
IBOV versus CNN-LSTM-GRU	26742,00	0,90660
IBOV versus CNN-LSTM-GRU-HRP	25866,00	0,46910
CNN-LSTM-GRU versus CNN-LSTM-GRU-HRP	26286,00	0,66490

Source: Original survey results

It is imperative to note that these results specifically reflect the period analyzed and could vary considerably if other periods were isolated, during which the portfolios demonstrated significantly different behaviors. However, the aim of this study was not to present a selectively favorable view of the market but rather to provide comprehensive *insights* that can contribute to the advancement of both the market and future research.

In order to gain a deeper understanding of the variables that influenced these returns, Figure 7 shows the graphs generated using the *SHapley Additive exPlanations* (SHAP) library.

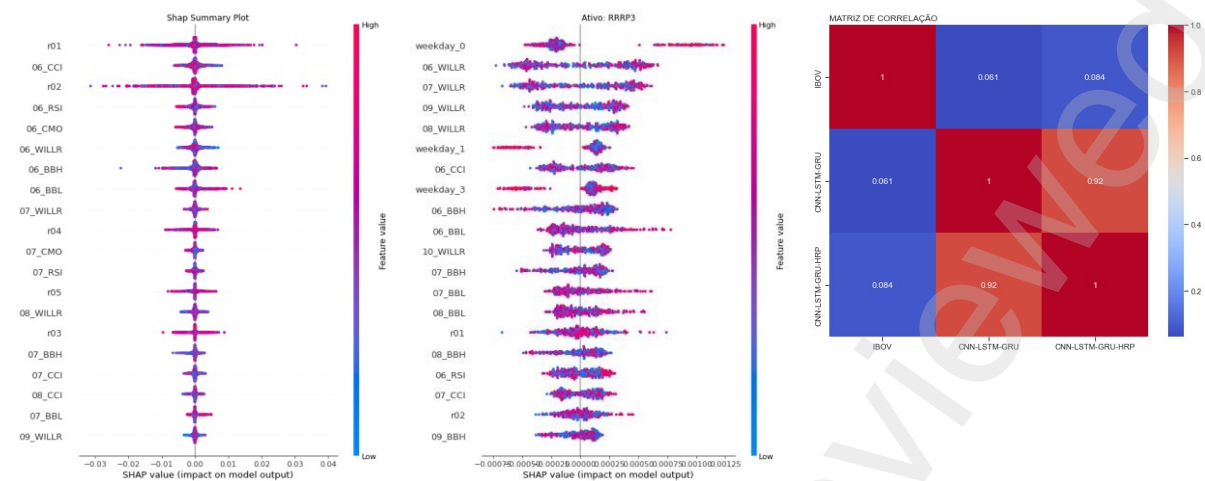


Figure 6 Summary charts of all assets, an isolated asset (RRRP3), and correlation matrix of the portfolios
Source: Original survey results

The graphs shown in Figure 6 illustrate the average behavior of the variables in the model, highlighting the variable "r01", which indicates the return lagged by one period, as the most influential, according to the *Summary plot*. It can be seen that the impact of the variables can vary significantly between different assets, as demonstrated by the specific behavior of the RRRP3 asset in the bottom right graph. Due to space limitations, this document has not included a more detailed analysis exploring the different behaviors of the main variables for each asset. This complementary analysis, which reveals a diversity of influences on the model and contributes to a deeper understanding of market dynamics, is documented separately and can be provided upon direct request to the authors. Finally, there was a low correlation between the portfolios and the IBOV and a high correlation between the portfolios. At this point, we can see how the financial market is dynamic and challenging to predict. Ullah et al. (2022) pointed out in their article that the unpredictability and volatility of the stock market make it challenging to make a substantial profit using any generalized methodology. They argue that few authors focus on finding the best characteristics for specific periods, i.e., for them, the characteristics must be dynamic and selected according to each sample.

4.1 Investment simulation

Table 13 shows the result of an investment simulation using the two strategies proposed for portfolio formation, i.e., naive allocation using CNN-LSTM-GRU and weighted allocation using CNN-LSTM-GRU-HRP. As many brokers have zero brokerage plans, these were disregarded. Therefore, settlement fees, emoluments, and income tax were considered.

For the sake of simplicity, investment balances are shown with and without transaction costs. However, it should be noted that other factors must be considered in the investment process, such as risk tolerance.

Table 13
Investment simulation from 26/07/2022 to 29/06/2023

Portfolios with taxes and IR	opening balance	fee	IR	final balance	growth
CNN-LSTM-GRU	100000.00	18843.10	7192.96	137288.78	37.29%

CNN-LSTM-GRU- HRP	100000.00	18202.07	9180.77	153974.07	53. 97%
Portfolios with taxes and IR	opening balance	fee	IR	final balance	growth
CNN-LSTM-GRU	100000.00			168723.00	68.72%
CNN-LSTM-GRU- HRP	100000.00			188284.00	88.28%
Outros	opening balance	fee	IR	final balance	growth
Poupança	100000.00			107661.10	7.66%
IBOV (BOVA11)	100000.00	64.98	2926.50	116518.52	16.52%
CDB (120% CDI)	100000.00		3033.18	112132.74	12.13%

Source: Original research results, B3 (2023) and BC (2023)

The fees and taxes above assume all positions were open and closed daily. This does not reflect what happens in the real market, where you may carry a position for several days as long as it is part of the strategy. Given this, with a smaller flow of entries and exits, the costs could be lower, but following an idea of conservatism, we have chosen to simulate the "worst" scenario.

Analyzing transaction costs cannot be underestimated since these costs can significantly impact an investment strategy's profitability. It is crucial to note that these costs are not uniform and can vary considerably from one country to another, from one financial asset to another, and even between different brokers. A study by Chavalle and Chavez-Bedoya (2019) found that transaction costs for retail investors in Peru are 14 times higher than those faced by their American counterparts when trading the same portfolio.

Busse et al. (2013) highlight the importance of paying attention to the *trade-off* between turnover, position size, and transaction costs. A strategy that seems profitable without considering transaction costs can become unprofitable once these costs are considered. Therefore, it is crucial to incorporate a realistic estimate of transaction costs into any backtesting to obtain an accurate assessment of the strategy's performance.

To Hsieh (2022), many point out that a robust investment strategy generates returns regardless of the market's direction and uses return as a robustness metric. However, many authors fail to show that a large part of the return can disappear when considering transaction costs.

Olivares-Nadal and DeMiguel (2018) approached transaction costs from a different perspective. They inserted transaction costs into the optimization process by considering these costs as a smoothing term. This idea is noteworthy because it allows the model to calibrate transaction costs in such a way as to minimize their impact while optimizing the portfolio. Figure 8 shows the accumulated portfolio balances with and without transaction costs.

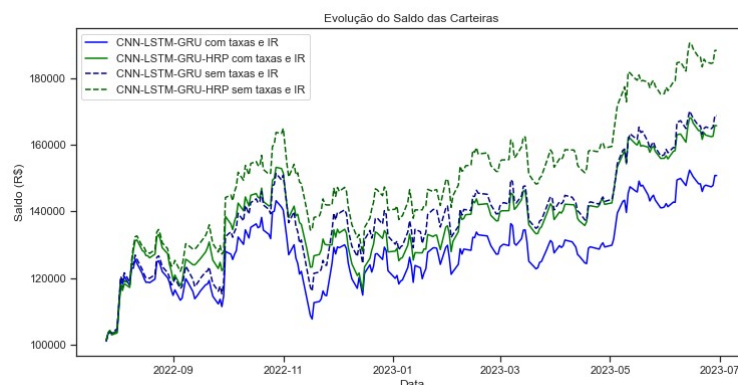


Figure 7 *Accumulated balance of portfolios with fees and IR and without fees and IR*

Source: Original survey results

This study has shown that it is possible to create stock investment portfolios using hybrid neural network models. It was observed that many characteristics and their combinations can influence the results differently. It was also shown that the impact of fees on daily transactions cannot be disregarded and that, depending on the strategy, the investor can make a "profit" without the fees and a loss after them.

Three main limitations were identified during this study. Firstly, a survivorship bias was observed, considering the data collected pertains exclusively to currently active companies. Therefore, the generalizability of the results may be affected since companies that have gone bankrupt or been acquired are not represented in the sample.

Secondly, the absence of fundamental data for specific periods has been noted, which may compromise the robustness of the conclusions reached. This gap in the data can introduce uncertainties in the analysis and the models used.

Finally, machine processing limitations were faced, restricting the complexity of the models that could be used. It was observed that more complex models took several days to complete training, thus impacting the efficiency and depth of the research.

For future work, the inclusion of inactive companies in new databases is suggested as a way of mitigating survival bias. In addition, portfolio diversification, including other types of assets, such as commodities, is recommended.

5. Final considerations

This article is one of the first contributions to the literature that integrates three neural network architectures (CNN-LSTM-GRU) to forecast the Brazilian equity market. The study revealed that technical, macroeconomic, and fundamental variables exert different degrees of influence on returns, with technical variables having the most significant impact. In addition, it was observed that the effect of these variables varies significantly when analyzed from the point of view of individual assets compared to an entire portfolio. The work also showed that optimization strategies effectively maximize gains and minimize risks. It is important to note that omitting transaction costs, even for simplification purposes, can result in inaccurate estimates of portfolio performance. Given the increasing accessibility of machine learning tools, the results of this study have practical implications for retail investors and fund managers.

i The word *backtest* is commonly used in finance and data analysis contexts to test a strategy or model based on historical data. The term is often kept in its original Portuguese English form, especially in technical and academic literature.

ii In training neural networks, *callbacks* are functions that are called at specific points during a model's training. They are used to monitor the model's performance, save the model's state, change the learning rate, or even stop training if certain conditions are met. Regarding translation into Portuguese, the term *callback* is often kept in its original form in technical literature and documentation due to its specificity.

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