Assignment 1

Algorithms, Spring 2024

Honor code: *Work on this assignment alone or with one partner. Between different teams, collaboration is at level 1 (verbal collaboration only). It is not allowed to search online for the specific problems in this assignment—doing so this violates academic honesty for the class.*

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1. **Finding the 1's in a Matrix:** Consider a 2D-array (matrix) A in which all the elements have one of two values, 1 or 2. Assume that the number of rows and columns in A is equal and denote it by n. Furthermore, suppose that each row in A is so that the following condition holds: in any row, all the 1's come before any 2's. The task is to come up with an algorithm which, given a matrix A with these properties, finds the row of A that contains the most 1's.

   (a) Describe a straightforward algorithm that runs in $O(n^2)$ time, and justify that it is correct (ie the row that it finds is indeed the row with the largest number of 1's).

   Note: The justification will be along the lines of "it is correct because it explores all rows and does not miss any row...". The point here is that you get used to thinking about correctness.

   (b) Describe an improved algorithm running in $O(n)$ time.
   We expect: Pseudocode and a brief English description of the idea of your algorithm.

   (c) Let $n = 10$; show an array that triggers worst case for your algorithm.

   (d) Let $n = 10$; show an array that triggers best case for your algorithm.

   (e) Argue why your algorithm from (b) runs in $O(n)$ time.

   Note: You do not need to show that the algorithm from (b) is correct because you would need to use induction which we do not cover in this class. The idea is the following. Let's assume, just for the sake of an example, that the algorithm has a loop that goes through all the rows, one by one. We would formulate a statement for iteration i of the loop (called a loop invariant) which, when $i = n$, would imply that the algorithm is correct. For example, this could be something of the form "in iteration i the max row we computed so far represents the row above i with the largest number of 1's" (note that when $i = n$ the statement essentially says that the algorithm is correct). Then we would prove the statement by induction on the iteration i.

2. **Finding min and max:** Let A be an array and denote its size by n.

   (a) Describe an algorithm for finding both the minimum and the maximum element in A using fewer than $3n/2$ comparisons in total.

   We expect: Pseudocode and a brief English description of the idea of your algorithm.

   (b) Analyize your algorithm and show that it performs fewer than $3n/2$comparisons.

   Hint: Start by counting how many comparisons it takes to find the min, and the max, and go from there.

3. **Breaking eggs:** Suppose you have an n-stories high building, and a bunch of eggs. An egg has a certain level l at which , if thrown from any level $\geq l$, it breaks. For example, an egg might have $l = 7$ meaning you can safely throw the egg down from levels 1 through 6, and it will not break; but if you through the egg from a level 7 or higher, it breaks.

   You are given a building and a bunch of eggs (all identical) and your goal is to find out the level l of the eggs. While you think about the problem, you can assume $n = 100$ (i.e. 100-level high building). But when you describe your solutions below, you need to do it in terms of n.

   (a) Describe an approach that only breaks one egg to find out l. How many throws does it do, worst-case (express in terms of n, use $O()$)?

   We expect: The rationale of the algorithm/high-level pseudocode.

   (b) Describe an approach that minimizes the number of throws. How many eggs might it break (express in terms of n, use $O()$)?

   We expect: the rationale of the algorithm/high-level pseudocode.

   (c) Assume now you have two eggs. Describe an approach that minimizes the number of throws.

   We expect: the rationale of the algorithm/high-level pseudocode. Analysis in terms of n.

   (d) (Optional) For $n = 100$, minimize the number of throws to find the level. What's the best you can do?

   (e) (Optional) Assume now you have three eggs. Describe an approach that minimizes the number of throws.

Evaluation

The assignment will be evaluated along three criteria: correctness, justification and style.
Style guidelines:

- Space: Write each problem on a separate page or leave plenty of space between problems so that we can write comments.
- Neat and legible: We expect your answers to be clearly written and easily human-readable, with complete sentences and well-organized logic, and should definitely not be your first draft. Ideally answers should be typed.
- Clear: Try to put yourself in the position of the reader. If you hadn't been thinking of this problem for 3 hours, would your answers make sense to you? Try to finish the assignment early, then step away for a day or two, and then come back to it and read it again. Chances are you'll find something you can write more clearly.