CSCI 2210: Theory of Computation

Problem Set 1 (due 09/16)

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Problem 1. Let A be the set $\{a, b, c\}$ and B be the set $\{a, c\}$.

(a) Is A a subset of B, that is, is $A \subseteq B$? Is $B \subseteq A$?

No, element b in A is not in B

Yes, all elements of B are in A. $\diamondsuit$

(b) What is $A \cup B$ and $A \cap B$?

$A \cup B : \{a, b, c\}$

$A \cap B : \{a, c\}$ $\diamondsuit$

(c) What is the power set of B?

$\{\{a, c\}, \{a\}, \{c\}, \emptyset\}$ $\diamondsuit$

(d) What is $A \times B$? How many elements are in $A \times B$?

$A \times B = \{\{a, a\}, \{a, c\}, \{b, a\}, \{b, c\}$
$\{c, a\}, \{c, c\}\}$

$|A \times B| = 6$ $\diamondsuit$

(e) In general, if a set A had n elements and B had m elements, how many elements would be in $A \times B$? Explain your answer.

$|A \times B| = n \cdot m$ because every element in A forms m tuples with elements from set B.

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Problem 2. Determine the type (one-one, onto, or bijection) of the following functions. Briefly explain your answer.

(a) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = x^2 + 3$

Not injective because $f(3) = 12$ and $f(-3) = 12$ and $3 \neq -3$

Not surjective because the x that satisfies $0 = x^2 + 3$ is not in $\mathbb{R}$

♢

(b) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = 3x + 5$

Injective because if you draw a horizontal line on the graph of $f(x)$, it will only ever intersect $f(x)$ once.

Surjective because for a b in $\mathbb{R}$, $\frac{b-5}{3}$ is in $\mathbb{R}$

Bijective because it is injective and surjective

♢

(c) $f : \mathbb{N} \to \mathbb{R}$ where $f(x) = \frac{1}{x+5}$

Injective because $f(x)$ only outputs one answer for each input

Not surjective because no natural numbers inputted to $f(x)$ outputs -1

♢

Problem 3. Consider the following proof that $2 = 1$. Find the error in this proof, i.e., find the invalid step that leads to a conclusion that is not true.

Consider numbers a and b and the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a + b)(a - b) = b(a - b)$, and divide each side by $(a - b)$ to get $a + b = b$. Finally, let a and b equal 1, which shows that $2 = 1$.

If $a, b = 1$ then $(a - b) = 0$. The step where you divide by $(a - b)$ is invalid.

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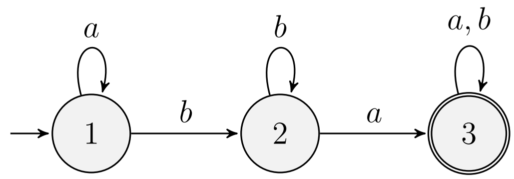
Problem 4. List each string of length 3 or less that is accepted by the following DFA:



b a

a b a

b a a

b a b

b b a

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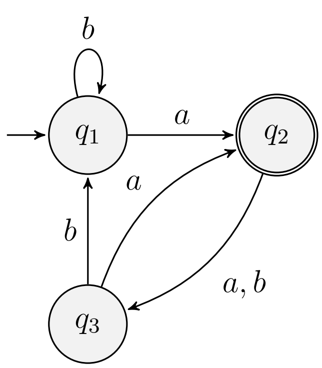
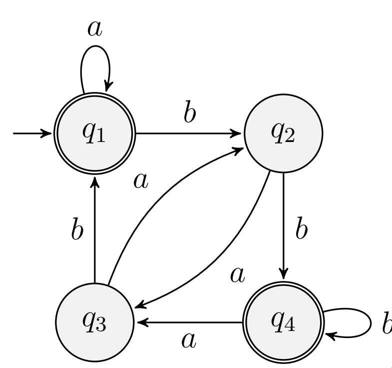
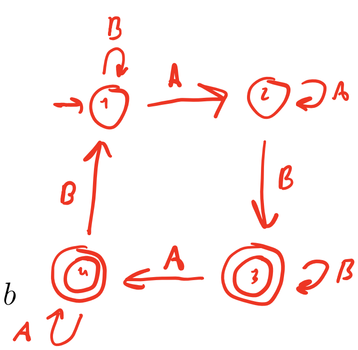
Problem 5. Consider the DFAs M_1 and M_2 in Figure 1.



(a) M_1



(b) M_2



(c) M_3

Figure 1: DFAs for Problem 5

(a) Are the following strings accepted or rejected by each M_1 and M_2? 1 ε, 2 $aabb$, 3 $babaa$, 4 $ababba$

	M_1	M_2	M_3
1	✗	✓	✗
2	✗	✓	✓
3	✗	✓	✓
4	✓	✗	✗

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(b) Describe in English the languages of M_1 and M_2.

M1: Ends in an odd # of consecutive A's

M2: ?

M 3: The substring AB must occur an odd # of times

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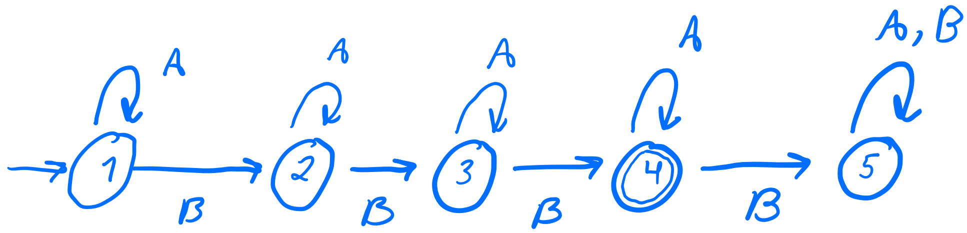
Problem 6. Design a finite state machine to recognize each of the following languages. In all cases the alphabet is $\Sigma = \{a, b\}$.

(a) $L_a = \{w \mid w \text{ contains exactly three } b\text{'s}\}$.



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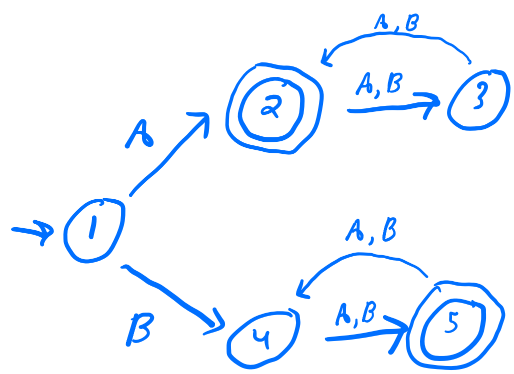
(b) $L_b = \{w \mid w \text{ starts with } a \text{ and has odd length, or starts with } b \text{ and has even length}\}$.



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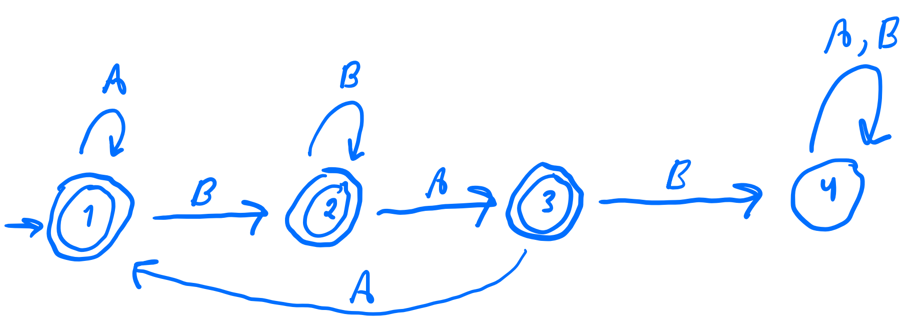
(c) $L_c = \{w \mid w \text{ does not contain the substring } bab\}$.



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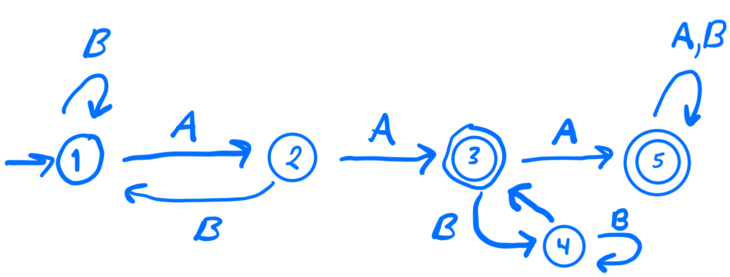
(d) $L_d = \{w \mid w \text{ contains at least two } a\text{'s that are not immediately followed by } b\}$.



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