CSCI 2210: Theory of Computation

Problem Set 4 (due 10/21)

Student: Rafael *Collaborators:* Cal

Problem 1. Let L be the language of the regular expression $(a+b)^*abaa(a+b)^*$. Argue that any DFA for L must have at least 3 states.

$a \not\equiv_L b$ $a \not\equiv_L ab$ $b \not\equiv_L ab$

$z = baa$ $z = aa$ $z = aa$

1. $a\ baa$ ✓ 1. $a\ a\ a$ X 1. $b\ a\ a$ X

2. $b\ baa$ X 2. $ab\ a\ a$ ✓ 2. $ab\ a\ a$ ✓

There are at least 3 equivalence classes as demonstrated above with the 3 distinguishable strings. Therefore a minimum of three states is needed for the DFA for L

Problem 2. Show that the following languages are not regular using the Myhill-Nerode theorem. Consider $\Sigma = \{a, b\}$.

(a) $L = \{a^n b^m \mid n, m \geq 0, n \neq m\}$

$I = \{a^n b \mid n > 1\}$

Pick two strings: $a^i b$ and $a^j b \mid i \neq j$

$z = b^{i-1}$

$a^i b b^{i-1}$ is out of L while $a^j b b^{i-1}$ is in L
because $|a^i| = |b b^{i-1}|$ and $|a^j| \neq |b b^{i-1}|$

(b) $L = \{w \mid$ number of a's is smaller or equal the number of b's in $w\}$

$I = \{b^n \mid n \geq 0\}$

Pick two strings: b^i and $b^j \mid i < j$

$z = a^j$

$b^i a^j$ is out of L while $b^j a^j$ is in L
because in $b^i a^j$, the # of a's is greater than the # of b's while in $b^j a^j$, they are equal.

Problem 3. Consider the context-free grammar G below.

$$S \to TT \mid U$$
$$T \to 0T \mid T0 \mid \#$$
$$U \to 0U00 \mid \#$$

(a) List three strings that are in $L(G)$, and three strings not in $L(G)$.

In $L(G)$

$S \to U \to \#$

$S \to TT \to \#T \to \#\#$

$S \to U \to 0U00 \to 0\#00$

Out of L → no # present

0

00

000

(b) Give a description of language $L(G)$.

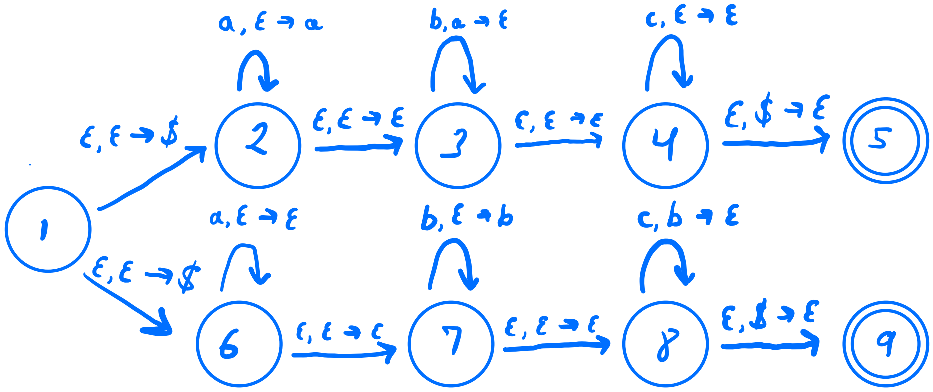
1. A sequence of any number of 0's with exactly two # anywhere

or

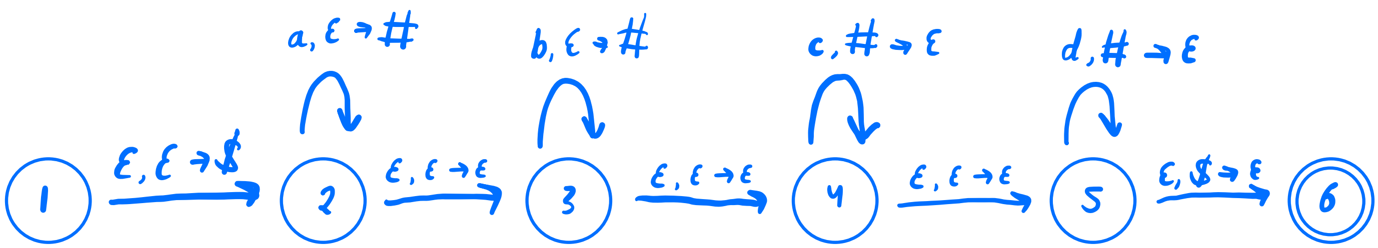
2. A sequence of zeros with a hashtag after 1/3 of the zeros.

Problem 4. Give state diagrams of pushdown automata for the following languages.

(a) $L_a = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$



(b) $L_b = \{a^m b^n c^p d^q \mid m, n, p, q \geq 0 \text{ and } m + n = p + q\}$



Problem 5. Give context-free grammars for the following language. Consider $\Sigma = \{a, b\}$.

(a) $L = \{a^n b^m \mid n > m \geq 0\}$

$S \to aS \mid aSb \mid a$

(b) $L = \{wc^k \mid w \in \Sigma^* \text{ and the number of } a\text{'s or the number of } b\text{'s in } w \text{ is } k\}$

$S \to U \mid V \mid \varepsilon$

$U \to aUc \mid bU \mid \varepsilon$

$V \to bVc \mid aV \mid \varepsilon$

Problem 6. Consider the following construction that attempts to prove that the class of context-free languages is closed under the Kleene star operation.

Let L be a context-free language that is generated by the context-free grammar $G = (V, \Sigma, R, S)$. By adding the new rule $S \to SS$ to G, we obtain the grammar G', and we have $L(G') = L^*$.

Give a counterexample to show that this construction doesn't work, i.e., the construction doesn't always define a grammar that generates L^*.

Given the below context free language

$S \to a$

Adding the below rule to it

$S \to SS$

Although it would allow for the generation of any number of A's, because the original language does not include the empty string as a terminal, it is not sufficient to simply add the above rule to get the Kleene star. The Kleene star contains the empty string, but the above language, even with the new rule, is still not able to generate the empty string