CSCI 2210: Theory of Computation

Problem Set 5 (due 10/28)

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Problem 1. Consider the language $L = \{a^n b^m c^n d^m \mid n, m \geq 0\}$. Show that L is decidable showing these is a halting Turing machine M such that $L(M) = L$. You can draw the state diagram for the Turing machine or provide a high-level description.

Solution.

M = "on input string *w*:

1. Mark the end of the string with a special character.
2. Sweep from the leftmost to the end character of the string until it has no *a*'s and *b*'s.
    1. If you see an *a*, delete it and jump forward to delete the first *c*.
    2. Repeat step *2.1* for *b* and *d*.
3. If the string does not contain all blanks, **reject.**
4. **Accept.**"

Problem 2. Draw the diagram of or describe on a high-level a Turing machine to compute the functions below. Consider $\Sigma = \{1\}$.

(a) $f(1^n) = 1^{2n}$

(b) $f(1^n) = 1^{\lfloor n/2 \rfloor}$

Solution.

Ma = "on input string *w*:

1. If you see a *1*, write a 0.
2. Jump past the rightmost character of the tape and write a *2.*
3. Jump to the leftmost *1.*
4. Repeat steps *1*, *2,* and *3* until there are no *1*'s left.
5. Replace all *0*'s *and 2*'s with a *1*."

Since for every original *1*, another character is written, the final tape is of length 2n. Replacing all the characters in the tape with a *1*, ensures there are 2n *1*'s.

Mb = "on input string *w*:

1. If you see a *1*, delete it.
2. Jump to the rightmost *1* and replace it with a *0*.
3. Jump to the leftmost character.
4. Repeat steps *1, 2,* and *3* until step *1* or *2* fails.
5. Replace all *0*'s with a *1*.

Since for every original *1*, it is deleted and another *1* is replaced with a *0*. There will be n/2 *0*'s at the end. The length will be exactly n/2 rounded down because the above machine first deletes and then tries to replace with a *0*.

Problem 3. A Turing machine with stay put instead of left is similar to an ordinary Turing machine, but the transition function has the form

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, S\}$$

At each point, the machine can move its head right or let it stay in the same position but it cannot move left. Does such a Turing machine variant recognize the same class of languages as an ordinary Turing machine? Explain your answer.

Solution.

A stay put Turing machine does not recognize the same class of languages as an ordinary Turing machine. The inability to move left means the stay put variant has severe memory limitations. It would be no better than a DFA since it cannot go back in any way to see what it wrote, meaning it cannot even recognize context free languages.

Problem 4. Show that decidable languages are closed under complement and intersection.

Solution.

Intersection

$$
\begin{aligned}
M_1 &= (Q_1, \Sigma, \Gamma_1, \delta_1, q_1^0, q_1^a, q_1^r) \\
M_2 &= (Q_2, \Sigma, \Gamma_2, \delta_2, q_2^0, q_2^a, q_2^r) \\
M_\cap &= (Q_\cap = Q_1 \times Q_2, \\
&\Sigma, \\
&\Gamma_\cap = \Gamma_1 \times \Gamma_2, \\
&\delta_\cap = (Q_1 \times Q_2) \times \Gamma^- \rightarrow (Q_1 \times Q_2) \times \\
&\Gamma_\cap \times \{R, L\},\ where\ \Gamma^- = \Gamma_\cap - \{q_\cap^a, q_\cap^r\}, \\
&q_\cap^0 = (q_1^0, q_2^0) \\
&q_\cap^a = (q_1^a, q_2^a) \\
&q_\cap^r = (q_1^r, q_2^r))
\end{aligned}
$$

Complement

$$
\begin{aligned}
M &= (Q, \Sigma, \Gamma, \delta, q_0, q_a, q_r) \\
\bar{M} &= (\bar{Q} = Q, \\
&\Sigma, \\
&\bar{\Gamma} = \Gamma, \\
&\bar{\delta} = \delta, \\
&\bar{q_0} = q_0, \\
&\bar{q_a} = q_r, \\
&\bar{q_r} = q_a)
\end{aligned}
$$