

STA Homework

Z-Transform

4. a) $x(k) = \left(\frac{1}{2}\right)^k$

$$Z\{x(k)\} = \sum_{k=0}^{\infty} x(k) \cdot z^{-k} \Rightarrow Z\left\{\left(\frac{1}{2}\right)^k\right\} = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \cdot z^{-k}$$

$$= 1 + \frac{1}{2z} + \frac{1}{(2z)^2} + \dots \quad (\text{geometric series with } a=1, r=\frac{1}{2z})$$

$$\Rightarrow Z\{x(k)\} = \frac{1}{1 - \frac{1}{2z}} = \frac{1}{\frac{2z-1}{2z}} = \frac{2z}{2z-1}$$

Inverse Z-Transform

5. $y(k+1) = (1+r)y(k)$ with $r=0.01$ and $y(0)=100$

$$y(k+1) = (1+r)y(k)$$

$$Z\{y(k+1)\} = Z\{a \cdot y(k)\} \Rightarrow Y(z) = \frac{z}{z-a} y_0$$

$$y(k) = Z^{-1}\left\{\frac{z}{z-a} y_0\right\} = y_0 Z^{-1}\left\{\frac{z}{z-a}\right\} = y_0 Z^{-1}\left\{\frac{1}{1-az^{-1}}\right\} = y_0 \cdot a^k$$

$$\Rightarrow y(k) = 100 \cdot (1.01)^k$$