Double Pendulum

In[21]:=
```
n = 2; tmax = 40; g = -10; MomentInertia = 1000;
Do[(m[v] = 1; length[v] = v; mi[v] = 1), {v, 1, n}];
m[1] = 1;
m[2] = 10;
(*x[n_][t_]:=Sum[length[v]*Sin[θ[1][t]]*Cos[θ[2][t]],{v,1,n}];
y[n_][t_]:=Sum[length[v]*Sin[θ[1][t]]*Sin[θ[2][t]],{v,1,n}];*)
x[n_][t_] := Sum[length[v] * Sin[θ[v][t]], {v, 1, n}];
y[n_][t_] := Sum[length[v] * Cos[θ[v][t]], {v, 1, n}];
L = .5 (m[1] + m[2]) * (length[1]) ^2 * (θ[1]'[t]) ^2 +
     .5 * m[2] * length[2] ^2 * (θ[2]'[t]) ^2 + m[2] * length[1] * length[2] * θ[1]'[t] *
      θ[2]'[t] * Cos[θ[1][t] - θ[2][t]] + (m[1] + m[2]) * g * length[1] * Cos[θ[1][t]] +
     m[2] * g * length[2] * Cos[θ[2][t]] // Simplify;
TexForm[L = .5 (m[1] + m[2]) * (length[1]) ^2 * (θ[1]'[t]) ^2 +
      .5 * m[2] * length[2] ^2 * (θ[2]'[t]) ^2 + m[2] * length[1] * length[2] * θ[1]'[t] *
       θ[2]'[t] * Cos[θ[1][t] - θ[2][t]] + (m[1] + m[2]) * g * length[1] * Cos[θ[1][t]] +
      m[2] * g * length[2] * Cos[θ[2][t]] // Simplify;]
(*eqns1=ParallelTable[D[L,ϕ[v][t]]==D[D[L,ϕ[v]'[t]],t],{v,1,n}]//FullSimplify;*)
eqns2 =
  ParallelTable[D[L, θ[v][t]] == D[D[L, θ[v]'[t]], t], {v, 1, n}] // FullSimplify;
(*vars=Table[ϕ[i][t],{i,1,n}];
ics=Join[Thread[(D[vars,t]/.{t→0})==ParallelTable[π Random[],{n}]],
  Thread[(vars/.{t→0})==ParallelTable[π Random[ ],{n}]]];*)
vars2 = ParallelTable[θ[i][t], {i, 1, n}];
ics2 = Join[Thread[(D[vars2, t] /. {t → 0}) == ParallelTable[π Random[], {n}]],
    Thread[(vars2 /. {t → 0}) == ParallelTable[π Random[ ], {n}]]];
```

Out[28]= `TexForm[Null]`

In[12]:=
```
L // TraditionalForm
```

Out[12]//TraditionalForm=

$$5.5\,(\theta(1)'(t))^2 + 20.\,(\theta(2)'(t))^2 + 20.\,\theta(2)'(t)\,\theta(1)'(t)\cos(\theta(1)(t) - \theta(2)(t)) - 110.\cos(\theta(1)(t)) - 200.\cos(\theta(2)(t))$$

In[32]:=
```
Parameters = Join[eqns2, ics2];
```

In[33]:=
```
Equations = vars2;
```

In[34]:=
```
soln = NDSolve[Parameters, Equations, {t, 1, tmax}, AccuracyGoal → 6,
     MaxSteps → Infinity, Method -> {"EquationSimplification" -> "Solve"}][[1]];
```

In[35]:=
```
trajectory[v_][t1_] := {x[v][t], y[v][t]} /. soln /. {t → t1}
```

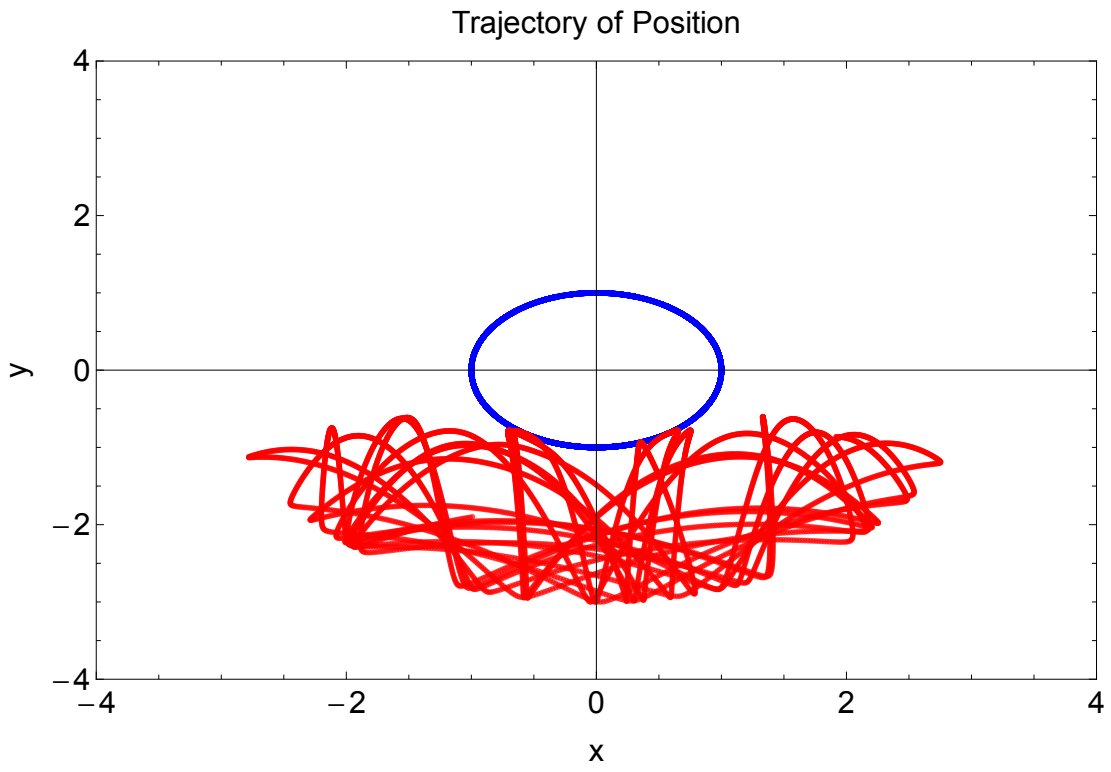
In[36]:=
```
trajectories[t1_] := ParallelTable[{trajectory[i][t1]}, {i, 1, n}]
```

In[37]:=
```
Do[data[v] = Flatten[ParallelTable[{trajectory[v][t]}, {t, 1, tmax, .004}], 1],
  {v, 1, n}];
```

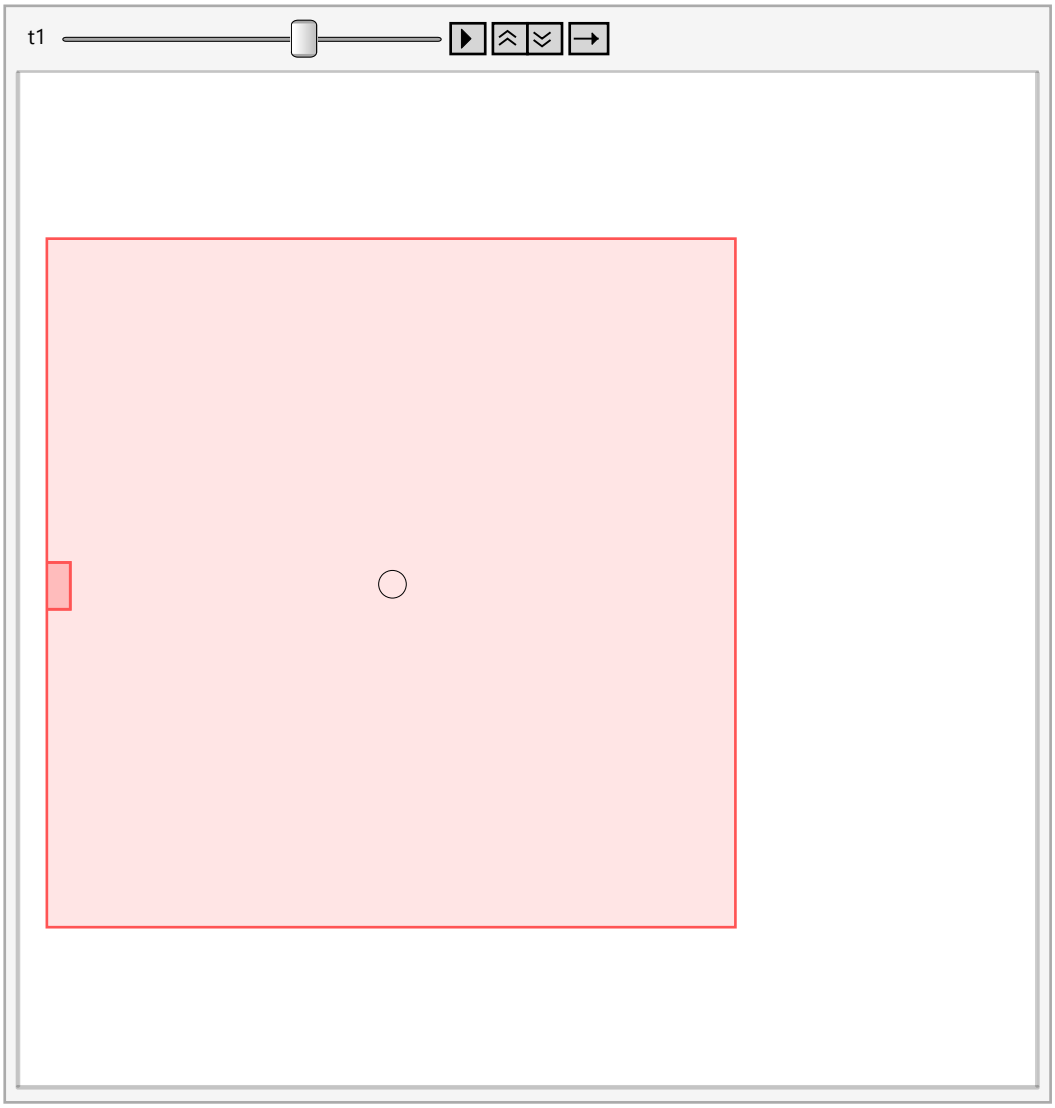
In[38]:=
```
Datas[t1_] := Table[{data[v]}, {v, 1, n}];
```

In[39]:=
```
ListPlot[{data[1], data[2]}, PlotStyle → {{Opacity[1, Blue]},
    {Opacity[.7], Red}, {Opacity[.6], Green}, {Opacity[.8], Purple}}, PlotLabel →
  Style["Trajectory of Position", FontFamily → "Helvetica", FontSize → 16],
 Frame → True, FrameTicks → Automatic, FrameLabel → {"x", "y"},
 LabelStyle → {FontFamily → "Helvetica", FontSize → 16},
 ImageSize → Large, PlotRange → {{-4, 4}, {-4, 4}}]
```

Out[39]=



```
Animate[Graphics[{Black, Circle[{0, 0}, .2], Blue,
    Circle[trajectory[1][t1], .3], Red, Circle[trajectory[2][t1], .3],
    Green, Line[{{0, 0}, trajectory[1][t1], trajectory[2][t1]}, .03]},
  PlotRange → {{-5, 5}, {-5, 5}}], {t1, 0, 10}]
```



Phase Space Diagrams

In[22]:=
```
PhaseEquations = {θ[1][t], θ[1]'[t]};
```

In[21]:=
```
PhaseEquations2 = {θ[2][t], θ[2]'[t]};
```

In[23]:=
```
phasesoln = NDSolve[Parameters, PhaseEquations, {t, 1, tmax}, AccuracyGoal → 10,
     MaxSteps → Infinity, Method -> {"EquationSimplification" -> "Solve"}][[1]];
```

In[24]:=
```
phasesoln2 = NDSolve[Parameters, PhaseEquations2, {t, 1, tmax}, AccuracyGoal → 10,
     MaxSteps → Infinity, Method -> {"EquationSimplification" -> "Solve"}][[1]];
```

In[25]:=
```
phasespace[1][t1_] := {θ[1][t], θ[1]'[t]} /. phasesoln /. {t → t1}
```

In[26]:=
```
phasespace[2][t1_] := {θ[2][t], θ[2]'[t]} /. phasesoln2 /. {t → t1}
```

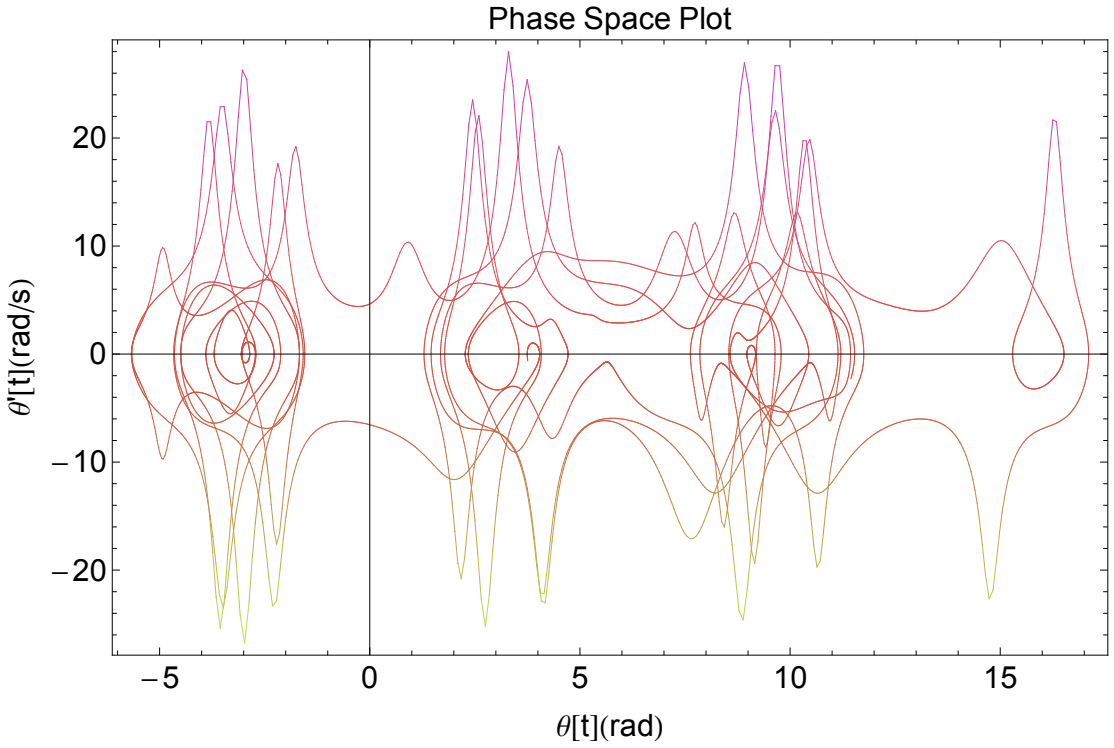
In[27]:=
```
phasedata[1] = Flatten[ParallelTable[{phasespace[1][t]}, {t, 1, tmax, .004}], 1];
```

In[28]:=
```
phasedata[2] = Flatten[ParallelTable[{phasespace[2][t]}, {t, 1, tmax, .004}], 1];
```

In[29]:=
```
PhaseDatas[t1_] := Table[{phasedata[v]}, {v, 1, n}];
```

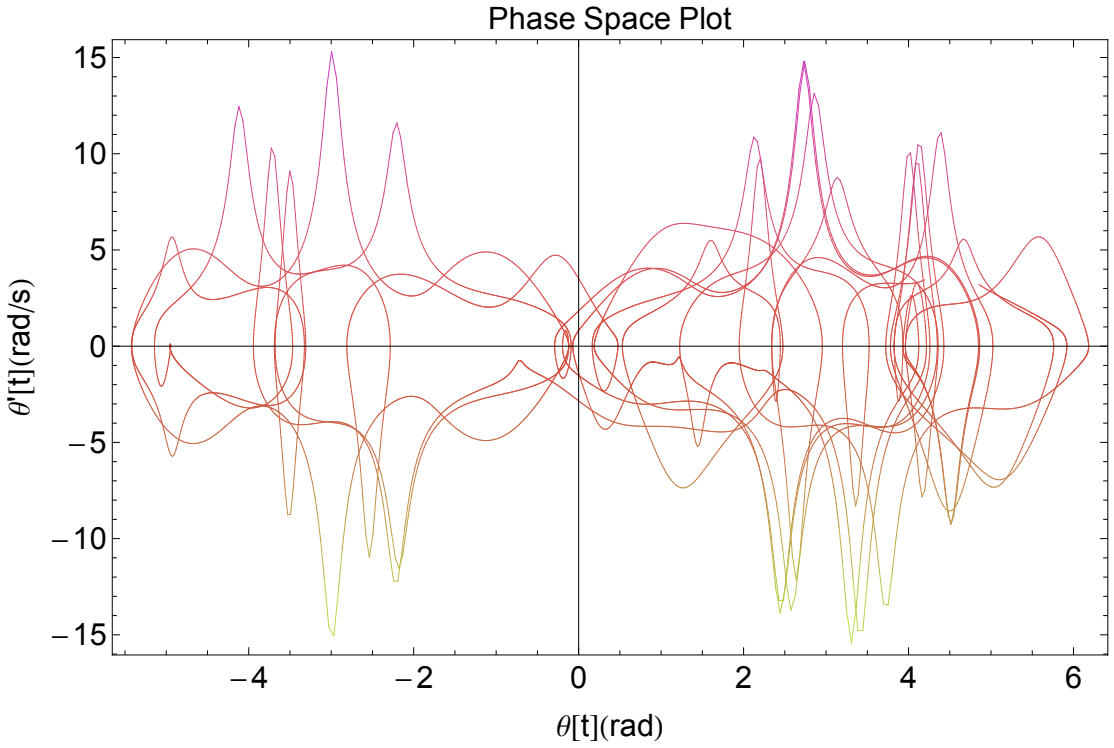
In[30]:=
```
ListPlot[{phasedata[1]},
 PlotStyle → {{Opacity[1, Blue]}}, ImageSize → Large, Joined → True,
 PlotLabel → Style["Phase Space Plot", FontFamily → "Helvetica", FontSize → 16],
 Frame → True, FrameTicks → Automatic, FrameLabel → {"θ[t](rad)", "θ'[t](rad/s)"},
 LabelStyle → {FontFamily → "Helvetica", FontSize → 16},
 ColorFunction → Function[{t, y}, ColorData["NeonColors"][y]]]
```

Out[30]=



In[31]:=
```
ListPlot[{phasedata[2]},
 PlotStyle → {{Opacity[1, Blue]}}, ImageSize → Large, Joined → True,
 PlotLabel → Style["Phase Space Plot", FontFamily → "Helvetica", FontSize → 16],
 Frame → True, FrameTicks → Automatic, FrameLabel → {"θ[t](rad)", "θ'[t](rad/s)"},
 LabelStyle → {FontFamily → "Helvetica", FontSize → 16},
 ColorFunction → Function[{t, y}, ColorData["NeonColors"][y]]]
```

Out[31]=



Bifurcation Analysis

In[1]:=
```
CloseKernels[];
LaunchKernels[4];
```

In[3]:=
```
Do[(result[v] = {}), {v, 1, 10}]
```

In[4]:=
```
Timing[Do[n = 2; tmax = 30; g = -10;
  m[1] = z * .1;
  m[2] = 10 * m[1];
  length[1] = 1;
  length[2] = 1;
  x[n_][t_] := Sum[length[v] * Sin[θ[v][t]], {v, 1, n}];
  y[n_][t_] := Sum[length[v] * Cos[θ[v][t]], {v, 1, n}];
  L = .5 (m[1] + m[2]) * (length[1])^2 * (θ[1]'[t])^2 +
      .5 * m[2] * length[2]^2 * (θ[2]'[t])^2 + m[2] * length[1] * length[2] * θ[1]'[t] *
       θ[2]'[t] * Cos[θ[1][t] - θ[2][t]] + (m[1] + m[2]) * g * length[1] * Cos[θ[1][t]] +
      m[2] * g * length[2] * Cos[θ[2][t]] // Simplify;
  eqns = Table[D[L, θ[v][t]] == D[D[L, θ[v]'[t]], t], {v, 1, n}] // FullSimplify;
  vars = Table[θ[i][t], {i, 1, n}];
  Equations2 = vars;

 PhaseEquations2 = {θ[2][t], θ[2]'[t]};
  Do[
   ics = Join[Thread[(D[vars, t] /. {t → 0}) == Table[π (2 * Random[] - 1), {n}]],
     Thread[(vars /. {t → 0}) == Table[π (2 * Random[ ] - 1), {n}]]];
    Parameters = Join[eqns, ics];
   phasesoln2 = NDSolve[Parameters, PhaseEquations2, {t, 1, tmax}, AccuracyGoal → 10,
      MaxSteps → Infinity, Method -> {"EquationSimplification" -> "Solve"}][[1]];
 bifurcate[t1_] := {θ[2][t], θ[2]'[t]} /. phasesoln2 /. {t → t1};
 bifurcatetable[w] = Flatten[Table[{bifurcate[t]}, {t, 1, tmax, .1}], 1];
   bitranspose[w] = Transpose[bifurcatetable[w]][[1]];
 index[w] = Position[bitranspose[w], _?(Abs[#] < 3 &)];
 If[TrueQ[index[w] == {}], Print["Noindex"], AppendTo[result[z],
     Transpose[bifurcatetable[w]][[2]][[Flatten[index[w][[1]]]]]]];
   Clear[phasesoln2];, {w, 1, 70}], {z, 1, 200}]]
```

Out[4]= {1271.500000, Null}

```
Flatten[Reap[For[i = 1, i < 140, i++, Sow[{1}]; Sow[result[1][[2 ;;]][[i + 1]]];]][[
    2, 1]]] ~Partition~ 2;
```

In[8]:=
```
s = Table[Flatten[
      Reap[For[i = 1, i < 63, i++, Sow[{x}]; Sow[result[x][[2 ;;]][[i + 1]]];]][[
       2, 1]]] ~ Partition ~ 2, {x, 1, 199}];
```

Part::partw : Part 62 of
{{−1.53026}, {−0.643518}, {−3.09313}, {−2.47001}, {5.19067}, {−3.85635}, ≪39≫, {−1.25126}, {−6.54208}, {1.30263}, {0.823417}, {−4.51541}, ≪11≫} does not exist. ≫

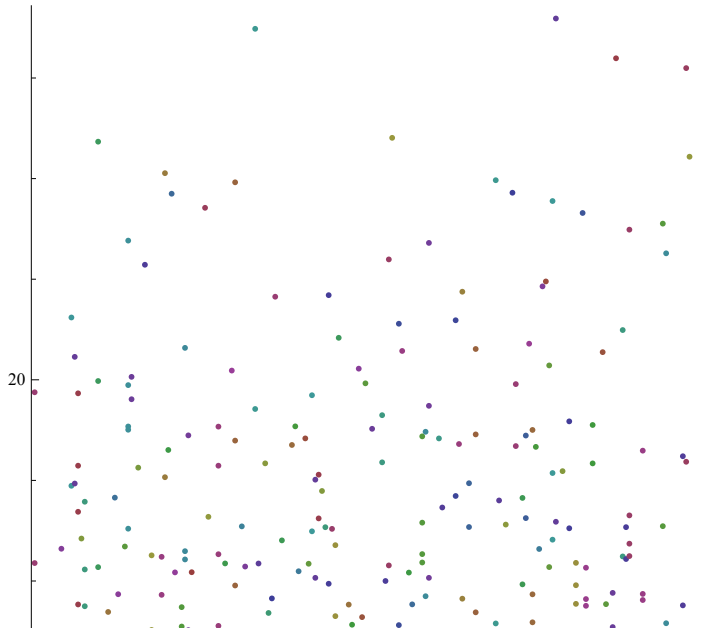
Part::partw : Part 63 of
{{−1.53026}, {−0.643518}, {−3.09313}, {−2.47001}, {5.19067}, {−3.85635}, ≪39≫, {−1.25126}, {−6.54208}, {1.30263}, {0.823417}, {−4.51541}, ≪11≫} does not exist. ≫

Part::partw : Part 63 of
result[{−4.57178}, {3.66335}, {−5.70765}, {4.44668}, {−1.21239}, {1.56353}, ≪40≫, {−4.7524}, {3.95264}, {−7.42343}, {6.5565}, ≪12≫] does not exist. ≫

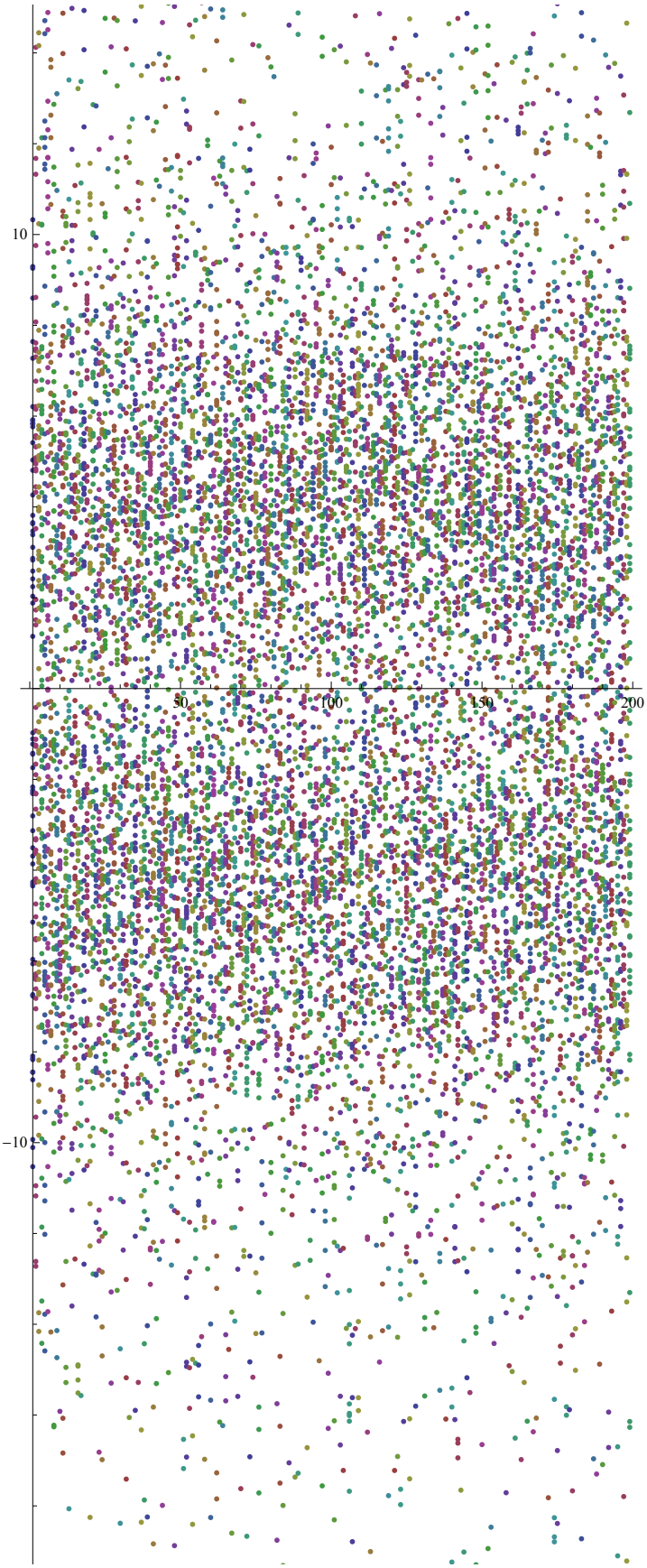
General::stop : Further output of Part::partw will be suppressed during this calculation. ≫

Out[9]=
```
{{100, 6.29985}, {100, 1.47871}, {100, 3.76868}, {100, 7.18192}, {100, 1.14167},
 {100, -5.90108}, {100, -7.7835}, {100, -3.0741}, {100, -6.26377}, {100, 3.09381},
 {100, 10.2141}, {100, 9.87641}, {100, 3.93093}, {100, 2.10086}, {100, -4.09499},
 {100, -6.77014}, {100, 4.76139}, {100, 8.17746}, {100, 5.26546}, {100, 6.62347},
 {100, -4.41956}, {100, 15.2819}, {100, 5.45177}, {100, 7.78153}, {100, -2.11846},
 {100, -5.75901}, {100, 5.43238}, {100, 5.78707}, {100, -7.02146}, {100, 5.59564},
 {100, 11.0995}, {100, -6.58928}, {100, 1.46833}, {100, -6.99246}, {100, 5.97964},
 {100, -9.2501}, {100, -5.17018}, {100, -3.51141}, {100, -2.28285},
 {100, -2.24335}, {100, -3.74617}, {100, 3.80707}, {100, 8.28921},
 {100, -5.03327}, {100, -0.41226}, {100, -0.0364799}, {100, 11.9234},
 {100, -4.0426}, {100, -7.00474}, {100, 6.83363}, {100, -3.85125}, {100, 5.32012},
 {100, 6.64908}, {100, -6.76512}, {100, 6.90769}, {100, -4.2929}, {100, -6.99556},
 {100, -6.6189}, {100, 5.2551}, {100, 5.27894}, {100, -9.47888}, {100, -8.16667}}
```

In[10]:=
```
ListPlot[s, ImageSize → Large, PlotRange → All, AspectRatio → 4]
```



Out[10]=



Parallelization

In[7]:=
```
Do[Pause[1]; f[i], {i, 4}] // AbsoluteTiming
ParallelDo[Pause[1]; f[i], {i, 4}] // AbsoluteTiming
```

Out[7]= {4.005672, Null}

Out[8]= {2.952973, Null}

In[23]:=
```
Do[n = 2; tmax = 30; g = -10;
  m[1] = z * .1;
  m[2] = 10 * m[1];
  length[1] = 1;
  length[2] = 1;
  x[n_][t_] := Sum[length[v] * Sin[θ[v][t]], {v, 1, n}];
  y[n_][t_] := Sum[length[v] * Cos[θ[v][t]], {v, 1, n}];
  L = .5 (m[1] + m[2]) * (length[1])^2 * (θ[1]'[t])^2 +
      .5 * m[2] * length[2]^2 * (θ[2]'[t])^2 + m[2] * length[1] * length[2] * θ[1]'[t] *
       θ[2]'[t] * Cos[θ[1][t] - θ[2][t]] + (m[1] + m[2]) * g * length[1] * Cos[θ[1][t]] +
      m[2] * g * length[2] * Cos[θ[2][t]] // Simplify;
  eqns = Table[D[L, θ[v][t]] == D[D[L, θ[v]'[t]], t], {v, 1, n}] // FullSimplify;
  vars = Table[θ[i][t], {i, 1, n}];
  Equations2 = vars;

 PhaseEquations2 = {θ[2][t], θ[2]'[t]};
  Do[
   ics = Join[Thread[(D[vars, t] /. {t → 0}) == Table[π (2 * Random[] - 1), {n}]],
     Thread[(vars /. {t → 0}) == Table[π (2 * Random[ ] - 1), {n}]]];
    Parameters = Join[eqns, ics];
   phasesoln2 = NDSolve[Parameters, PhaseEquations2, {t, 1, tmax}, AccuracyGoal → 10,
      MaxSteps → Infinity, Method -> {"EquationSimplification" -> "Solve"}][[1]];
 bifurcate[t1_] := {θ[2][t], θ[2]'[t]} /. phasesoln2 /. {t → t1};
 bifurcatetable[w] = Flatten[Table[{bifurcate[t]}, {t, 1, tmax, .1}], 1];
   bitranspose[w] = Transpose[bifurcatetable[w]][[1]];
 index[w] = Position[bitranspose[w], _?(Abs[#] < 3 &)];
 If[TrueQ[index[w] == {}], Print["Noindex"], AppendTo[result[z],
     Transpose[bifurcatetable[w]][[2]][[Flatten[index[w][[1]]]]]];
   Clear[phasesoln2];, {w, 1, 70}], {z, 1, 8}] // AbsoluteTiming
```

Out[23]=
```
{54.632450, Null}
```

In[24]:=
```
CloseKernels[];
LaunchKernels[2];
```

In[26]:=
```
ParallelDo[n = 2; tmax = 30; g = -10;
  m[1] = z * .1;
  m[2] = 10 * m[1];
  length[1] = 1;
  length[2] = 1;
  x[n_][t_] := Sum[length[v] * Sin[θ[v][t]], {v, 1, n}];
  y[n_][t_] := Sum[length[v] * Cos[θ[v][t]], {v, 1, n}];
  L = .5 (m[1] + m[2]) * (length[1])^2 * (θ[1]'[t])^2 +
      .5 * m[2] * length[2]^2 * (θ[2]'[t])^2 + m[2] * length[1] * length[2] * θ[1]'[t] *
       θ[2]'[t] * Cos[θ[1][t] - θ[2][t]] + (m[1] + m[2]) * g * length[1] * Cos[θ[1][t]] +
      m[2] * g * length[2] * Cos[θ[2][t]] // Simplify;
  eqns = Table[D[L, θ[v][t]] == D[D[L, θ[v]'[t]], t], {v, 1, n}] // FullSimplify;
  vars = Table[θ[i][t], {i, 1, n}];
  Equations2 = vars;

 PhaseEquations2 = {θ[2][t], θ[2]'[t]};
  Do[
   ics = Join[Thread[(D[vars, t] /. {t → 0}) == Table[π (2 * Random[] - 1), {n}]],
     Thread[(vars /. {t → 0}) == Table[π (2 * Random[ ] - 1), {n}]]];
    Parameters = Join[eqns, ics];
   phasesoln2 = NDSolve[Parameters, PhaseEquations2, {t, 1, tmax}, AccuracyGoal → 10,
      MaxSteps → Infinity, Method -> {"EquationSimplification" -> "Solve"}][[1]];
bifurcate[t1_] := {θ[2][t], θ[2]'[t]} /. phasesoln2 /. {t → t1};
bifurcatetable[w] = Flatten[Table[{bifurcate[t]}, {t, 1, tmax, .1}], 1];
   bitranspose[w] = Transpose[bifurcatetable[w]][[1]];
index[w] = Position[bitranspose[w], _?(Abs[#] < 3 &)];
If[TrueQ[index[w] == {}], Print["Noindex"], AppendTo[result[z],
     Transpose[bifurcatetable[w]][[2]][[Flatten[index[w][[1]]]]]];
   Clear[phasesoln2];, {w, 1, 70}], {z, 1, 8}] // AbsoluteTiming

1{, Null}
```

In[33]:=
```
CloseKernels[];
LaunchKernels[4];
```

In[37]:=
```
ParallelDo[n = 2; tmax = 30; g = -10;
  m[1] = z * .1;
  m[2] = 10 * m[1];
  length[1] = 1;
  length[2] = 1;
  x[n_][t_] := Sum[length[v] * Sin[θ[v][t]], {v, 1, n}];
  y[n_][t_] := Sum[length[v] * Cos[θ[v][t]], {v, 1, n}];
  L = .5 (m[1] + m[2]) * (length[1])^2 * (θ[1]'[t])^2 +
      .5 * m[2] * length[2]^2 * (θ[2]'[t])^2 + m[2] * length[1] * length[2] * θ[1]'[t] *
       θ[2]'[t] * Cos[θ[1][t] - θ[2][t]] + (m[1] + m[2]) * g * length[1] * Cos[θ[1][t]] +
     m[2] * g * length[2] * Cos[θ[2][t]] // Simplify;
  eqns = Table[D[L, θ[v][t]] == D[D[L, θ[v]'[t]], t], {v, 1, n}] // FullSimplify;
  vars = Table[θ[i][t], {i, 1, n}];
  Equations2 = vars;

 PhaseEquations2 = {θ[2][t], θ[2]'[t]};
  Do[
   ics = Join[Thread[(D[vars, t] /. {t → 0}) == Table[π (2 * Random[] - 1), {n}]],
     Thread[(vars /. {t → 0}) == Table[π (2 * Random[ ] - 1), {n}]]];
    Parameters = Join[eqns, ics];
   phasesoln2 = NDSolve[Parameters, PhaseEquations2, {t, 1, tmax}, AccuracyGoal → 10,
      MaxSteps → Infinity, Method -> {"EquationSimplification" -> "Solve"}][[1]];
 bifurcate[t1_] := {θ[2][t], θ[2]'[t]} /. phasesoln2 /. {t → t1};
 bifurcatetable[w] = Flatten[Table[{bifurcate[t]}, {t, 1, tmax, .1}], 1];
   bitranspose[w] = Transpose[bifurcatetable[w]][[1]];
 index[w] = Position[bitranspose[w], _?(Abs[#] < 3 &)];
 If[TrueQ[index[w] == {}], Print["Noindex"], AppendTo[result[z],
     Transpose[bifurcatetable[w]][[2]][[Flatten[index[w][[1]]]]]];
   Clear[phasesoln2];, {w, 1, 70}], {z, 1, 8}] // AbsoluteTiming
```

Out[37]= {22.684136, Null}

In[41]:=
```
CloseKernels[];
LaunchKernels[8];
```

```
ParallelDo[n = 2; tmax = 30; g = -10;
  m[1] = z * .1;
  m[2] = 10 * m[1];
  length[1] = 1;
  length[2] = 1;
  x[n_][t_] := Sum[length[v] * Sin[θ[v][t]], {v, 1, n}];
  y[n_][t_] := Sum[length[v] * Cos[θ[v][t]], {v, 1, n}];
  L = .5 (m[1] + m[2]) * (length[1])^2 * (θ[1]'[t])^2 +
      .5 * m[2] * length[2]^2 * (θ[2]'[t])^2 + m[2] * length[1] * length[2] * θ[1]'[t] *
       θ[2]'[t] * Cos[θ[1][t] - θ[2][t]] + (m[1] + m[2]) * g * length[1] * Cos[θ[1][t]] +
      m[2] * g * length[2] * Cos[θ[2][t]] // Simplify;
  eqns = Table[D[L, θ[v][t]] == D[D[L, θ[v]'[t]], t], {v, 1, n}] // FullSimplify;
  vars = Table[θ[i][t], {i, 1, n}];
  Equations2 = vars;

 PhaseEquations2 = {θ[2][t], θ[2]'[t]};
  Do[
   ics = Join[Thread[(D[vars, t] /. {t → 0}) == Table[π (2 * Random[] - 1), {n}]],
     Thread[(vars /. {t → 0}) == Table[π (2 * Random[ ] - 1), {n}]]];
    Parameters = Join[eqns, ics];
   phasesoln2 = NDSolve[Parameters, PhaseEquations2, {t, 1, tmax}, AccuracyGoal → 10,
      MaxSteps → Infinity, Method -> {"EquationSimplification" -> "Solve"}][[1]];
bifurcate[t1_] := {θ[2][t], θ[2]'[t]} /. phasesoln2 /. {t → t1};
bifurcatetable[w] = Flatten[Table[{bifurcate[t]}, {t, 1, tmax, .1}], 1];
   bitranspose[w] = Transpose[bifurcatetable[w]][[1]];
index[w] = Position[bitranspose[w], _?(Abs[#] < 3 &)];
If[TrueQ[index[w] == {}], Print["Noindex"], AppendTo[result[z],
     Transpose[bifurcatetable[w]][[2]][[Flatten[index[w][[1]]]]]]];
   Clear[phasesoln2];, {w, 1, 70}], {z, 1, 8}] // AbsoluteTiming
```

```
{122.82122340000000093596099759452044963837`7.378938835641728, Null}
```

```
maxKernel = 4;
data = Table[CloseKernels[];
     LaunchKernels[n];
     ParallelDo[n = 2; tmax = 30; g = -10;
       m[1] = z * .1;
       m[2] = 10 * m[1];
       length[1] = 1;
       length[2] = 1;
       x[n_][t_] := Sum[length[v] * Sin[θ[v][t]], {v, 1, n}];
       y[n_][t_] := Sum[length[v] * Cos[θ[v][t]], {v, 1, n}];
       L = .5 (m[1] + m[2]) * (length[1])^2 * (θ[1]'[t])^2 +
           .5 * m[2] * length[2]^2 * (θ[2]'[t])^2 + m[2] * length[1] * length[2] *
            θ[1]'[t] * θ[2]'[t] * Cos[θ[1][t] - θ[2][t]] + (m[1] + m[2]) * g * length[1] *
            Cos[θ[1][t]] + m[2] * g * length[2] * Cos[θ[2][t]] // Simplify;
       eqns = Table[D[L, θ[v][t]] == D[D[L, θ[v]'[t]], t], {v, 1, n}] // FullSimplify;
       vars = Table[θ[i][t], {i, 1, n}];
       Equations2 = vars;

  PhaseEquations2 = {θ[2][t], θ[2]'[t]};
       Do[
        ics = Join[Thread[(D[vars, t] /. {t → 0}) == Table[π (2 * Random[] - 1), {n}]],
          Thread[(vars /. {t → 0}) == Table[π (2 * Random[ ] - 1), {n}]]];
         Parameters = Join[eqns, ics];
        phasesoln2 = NDSolve[Parameters, PhaseEquations2,
            {t, 1, tmax}, AccuracyGoal → 10, MaxSteps → Infinity,
            Method -> {"EquationSimplification" -> "Solve"}][[1]];
bifurcate[t1_] := {θ[2][t], θ[2]'[t]} /. phasesoln2 /. {t → t1};
bifurcatetable[w] = Flatten[Table[{bifurcate[t]}, {t, 1, tmax, .1}], 1];
       bitranspose[w] = Transpose[bifurcatetable[w]][[1]];
index[w] = Position[bitranspose[w], _?(Abs[#] < 3 &)];
If[TrueQ[index[w] == {}], None, AppendTo[result[z],
          Transpose[bifurcatetable[w]][[2]][[Flatten[index[w][[1]]]]]]];
       Clear[phasesoln2];, {w, 1, 70}], {z, 1, 8}] // AbsoluteTiming; //
     AbsoluteTiming // First, {n, 0, maxKernel}]
```

ParallelDo::nopar : No parallel kernels available; proceeding with sequential evaluation. ≫

In[52]:=
```
data = {46.400959600000000193631422007456421852 11`7.6871268754007875,
  31.846244800000000907402863958850502967 83`7.52365814245374,
  24.426295499999998384055288624949753284 45`7.408457520003329,
  22.821223400000000935960997594520449638 37`7.378938835641728}
```

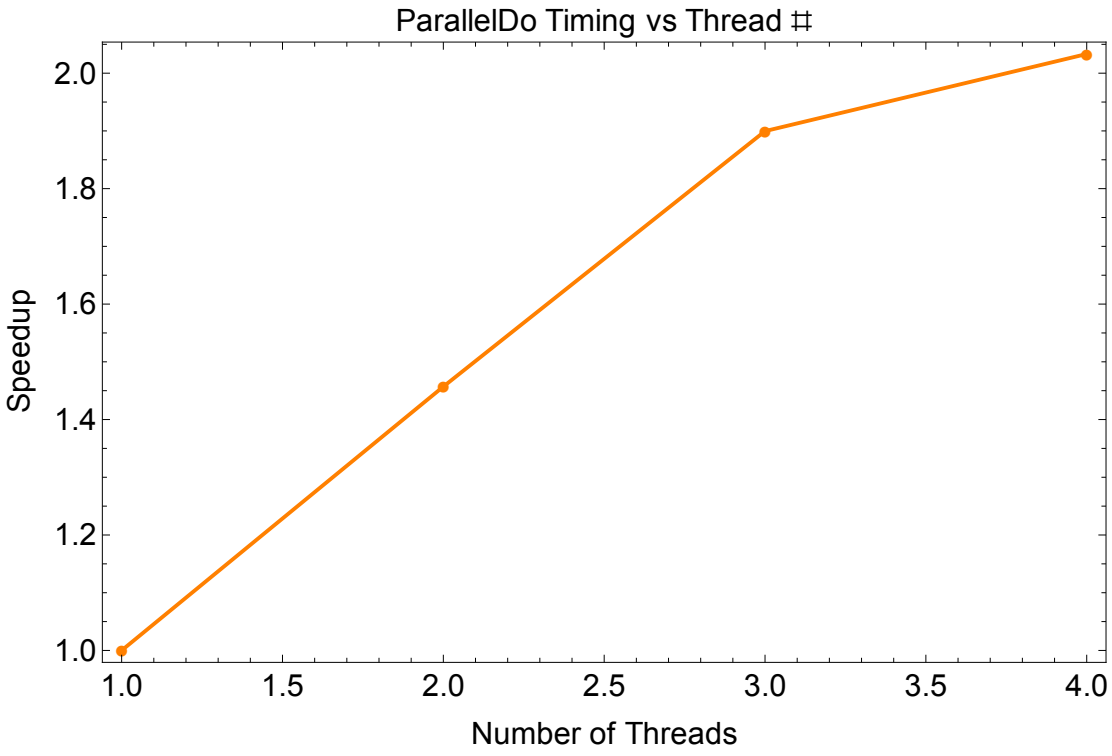
Out[52]= {46.400960, 31.846245, 24.426295, 22.821223}

In[56]:=
```
SpeedupData =
 Function[x, 46.400959600000000193631422007456421852l1`7.6871268754007875 / x] /@
  {46.400959600000000193631422007456421852l1`7.6871268754007875,
   31.846244800000000907402863958850502967 83`7.52365814245374,
   24.426295499999998384055288624949753284 45`7.408457520003329,
   22.821223400000000935960997594520449638 37`7.378938835641728}
```

Out[56]=
```
{1.0000000, 1.4570308, 1.899631, 2.033237}
```

In[87]:=
```
Plot1 = ListPlot[{{1, SpeedupData[[1]]}, {2, SpeedupData[[2]]},
    {3, SpeedupData[[3]]}, {4, SpeedupData[[4]]}}, ImageSize → Large,
  Joined → True, PlotLabel → Style["ParallelDo Timing vs Thread #",
    FontFamily → "Helvetica", FontSize → 16], Frame → True,
  FrameTicks → Automatic, FrameLabel → {"Number of Threads", "Speedup"},
  LabelStyle → {FontFamily → "Helvetica", FontSize → 16},
  PlotStyle -> {Orange, Thick}, PlotMarkers → Automatic]
```

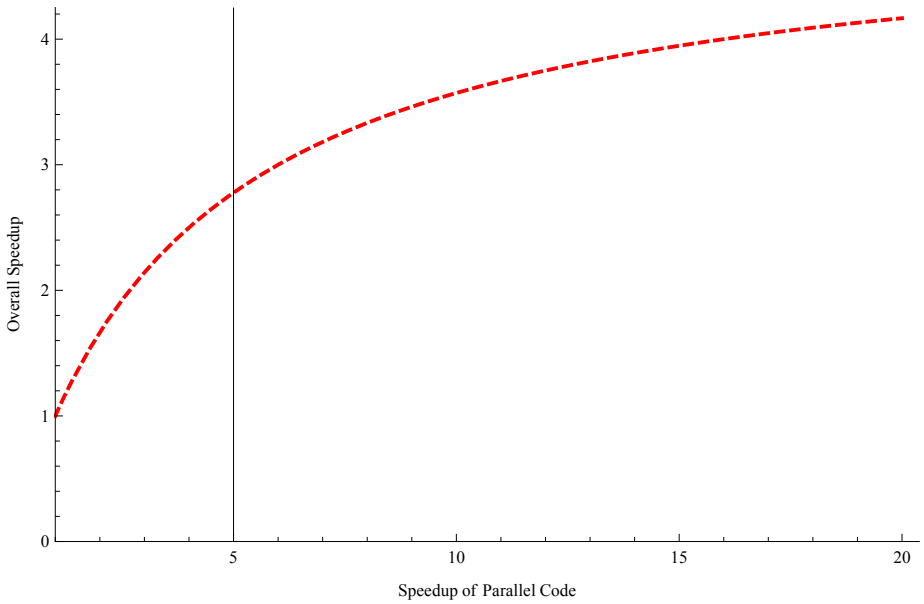
Out[87]=




```
Amdahl[f_, p_] := 1 / ((1 - f) + (f / p))
```

In[89]:=
```
Plot2 = Plot[Amdahl[.8, pro], {pro, 1.00, 20},
  PlotRange → {{1, Automatic}, {0, Automatic}},
  PlotLabel → Text@Style["Amdahl's Law .9 Parallelized", "Label", 14],
  FrameLabel → {"Speedup of Parallel Code", "Overall Speedup"},
  ImageSize → {540, 400}, ImagePadding → {{60, 30}, {35, 50}},
  Frame → {True, True, False, False}, PlotStyle → {Red, Dashed, Thick}]
```

Out[89]=



In[90]:=
```
Show[Plot1, Plot2 , PlotRange → {{1, 6}, {1, 5}}]
```

Out[90]=