Roll equations for high-powered rockets

Bruno Abdulklech Sorban, Mateus Stano Junqueira

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Nomenclature

$\overline{Y_t}$ Mean Aerodynamic Chord

$(C_{N\alpha})_0$ Normal force coefficient derivative of a 2D airfoil

$(C_{N\alpha})_1$ Normal force coefficient derivative of one fin

δ Fin cant angle

ω Angular velocity

$\overline{q}$ Dynamic pressure

ξ Distance to rotation axis

A_{ref} Reference area

C_r Root chord

C_t Tip Chord

F Force

L_{ref} Reference length, rocket diameter

M Moment

N Number of fins

r_t Reference radius at fins position

s Span

v_0 Rocket speed in relation to the wind

1 Derivation of coefficients

According to the equation formulated by Barrowman 1967, the rotational moment around the rockets axis is governed by two main forces:

1.1 Roll Forcing

Roll forcing is the moment that causes the rocket to rotate around its axis. Assuming $\omega = 0$ e $\delta \neq 0$.

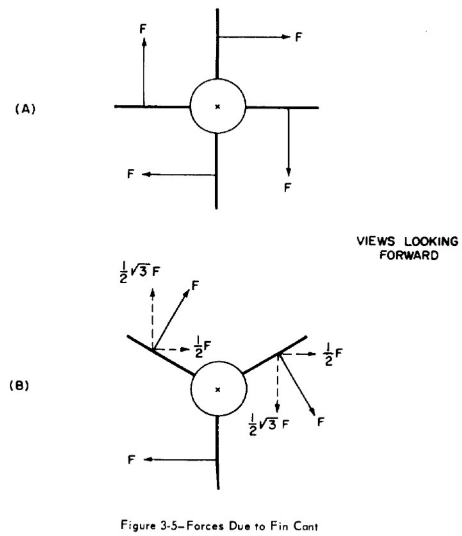

Figure 1: Forces due to fin cant. Source: Barrowman 1967

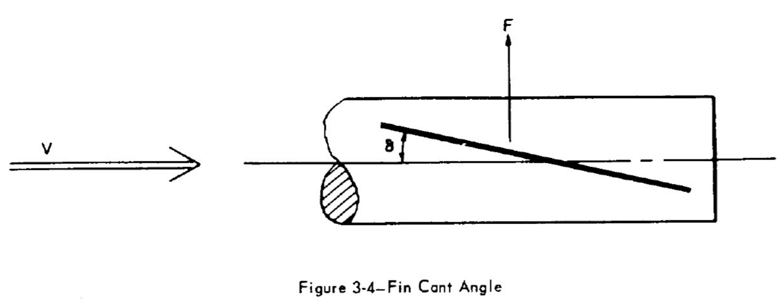

Figure 2: Fin cant angle. Source: Barrowman 1967

Due to the symmetry of the fins - as can be seen in Figure 1 - the forces cancel each other out so that the resulting force F_r is equal to zero. However, the resulting moment $M_r \neq 0$, that is, it constitutes a gyroscope binary.

According to Barrowman 1967, equation (3-31), the roll moment is given by:

$$M = N\overline{Y_t}(C_{N\alpha})_1 \delta \overline{q} A_r \tag{1}$$

The author defines the roll momentum coefficient as:

$$C_{lf} = \frac{M}{\overline{q} A_r L_r} \tag{2}$$

The letter "f" has been added to the name to differentiate *Forcing* from *Damping*. Note the similarity with the definition of drag coefficient ($C_d = \frac{2F_{Drag}}{\rho V^2 A_{ref}}$). Finally, you can also write C_l as:

$$C_{lf} = \frac{N\overline{Y_t}(C_{N\alpha})_1 \delta}{L_r} \tag{3}$$

And its derivative as:

$$C_{lf\delta} = \frac{N\overline{Y_t}(C_{N\alpha})_1}{L_r} \tag{4}$$

1.2 Roll Damping

While roll forcing causes the rotation movement, the roll damping force is what counteracts this movement. It is a force that scales with angular velocity and acts in the opposite direction. $\omega \neq 0$ and $\delta = 0$ are assumed.

It is defined in the same way as roll forcing:

$$C_{ld} = \frac{M}{\overline{q} A_r L_r} \tag{5}$$

Due to greater clarity, the Niskanen 2013 formulation was used. The author defines the *roll damping coefficient* as:

$$C_{ld} = \frac{N(C_{N\alpha})_0 \omega}{A_{ref} d v_0} \sum_i c_i \xi^2 \Delta\xi \tag{6}$$

According to the author, the sum is a constant that depends on the fin geometry. For trapezoidal fins, it is defined as:

$$\sum_i c_i \xi^2 \Delta\xi = \frac{C_r + C_t}{2} r_t^2 s + \frac{C_r + 2C_t}{3} r_t s^2 + \frac{C_r + 3C_t}{12} s^3 \tag{7}$$

And for ellipsoidal fins:

$$\sum_i c_i \xi^2 \Delta\xi = C_r\left(\frac{\pi}{4} r_t^2 s + \frac{2}{3} r_t s^2 + \frac{\pi}{16} s^3\right) \tag{8}$$

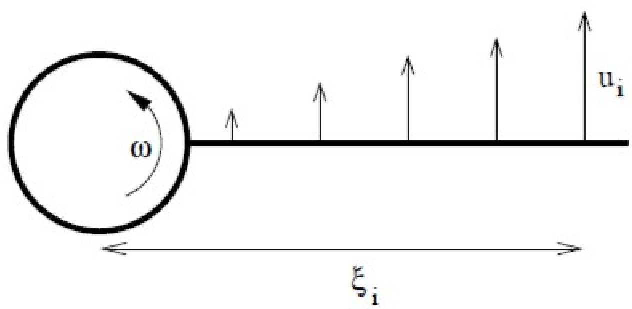

Figure 3: Radial velocity along the different positions of a fin. Source: Niskanen 2013

The initial hypothesis assumes that, for the roll damping calculation, the deflection is $\delta = 0$. This implies a larger cross-sectional area than is actually acting against the movement (analogous to flow passing through a surface). As a result, the term $Cos(\delta)$ was added to the original formulation:

$$C_{ld} = \frac{N(C_{N\alpha})_0\omega}{A_{ref}L_{ref}V_0} Cos(\delta) \sum_i c_i \xi^2 \Delta\xi \tag{9}$$

2 Comments

Roll moment is expected to increase linearly with velocity. This relationship can be verified in the rotation frequency equilibrium equation, described by Niskanen 2013 in equation (3.73), and again stated below:

$$f_{eq} = \frac{\omega}{2\pi} = \frac{A_{ref}\beta\overline{Y_t}(C_{N\alpha})_1}{4\pi^2 \sum_i c_i \xi^2 \Delta\xi} \delta V_0 \tag{10}$$

The auxiliary value β is defined as: $\beta = \sqrt{|1 - M|}$, where M is the speed of the rocket in Mach.

References

Barrowman, James S. (1967). "The practical calculation of the aerodynamic characteristics of slender finned vehicles". In.

Niskanen, S. (2013). "OpenRocket technical documentation". In: *Development of an Open Source model rocket simulation software.*