On the Calibration of Reduced-Order Models to Describe the Viscoelasticity in Steady-State Rolling Tires

Master's Thesis Presentation

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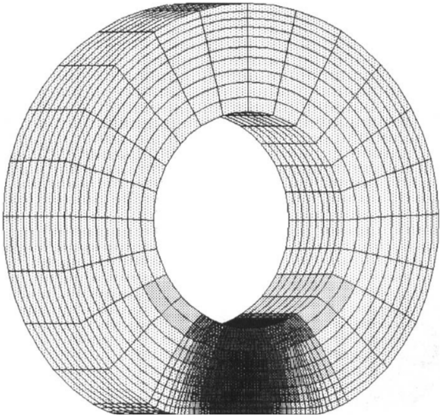
Modeling a steady-state rolling tire

- The finite element method is widely adopted in the tire industry:
- Equations describing the steady-state rolling motion;
- Set of constitutive laws describing the viscoelastic material;
- Internal variables are governed by nonlinear differential equations:

Time consuming.

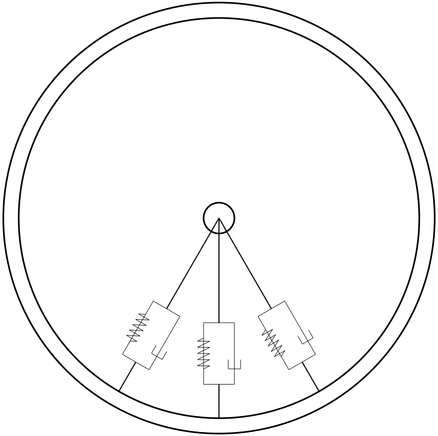


Finite element approximation of a steady-state rolling tire.

P. Le Tallec & C. Rahler. Numerical models of steady rolling for non-linear viscoelastic structures in finite deformations. International Journal for Numerical Methods in Engineering, Wiley 37 (7) (**1994**), pp. 1159–1186.

Reduced-order modeling

- A reduced-order model is based on physical simplifications of the full-order model;
- Unlike the surrogate models that are based on mathematics only, the **reduced-order model** preserves the physical description;
- A compromise between model complexity, computational costs and model accuracy.



An example of reduced-order tire model.

Problem statement

- This work suggests: a **reduced-order model** to alternatively describe viscoelastic internal variables of the finite element approximation of a steady-state rolling tire;
- and a **model calibration procedure** combining sensitivity analysis, optimization and statistical inference;

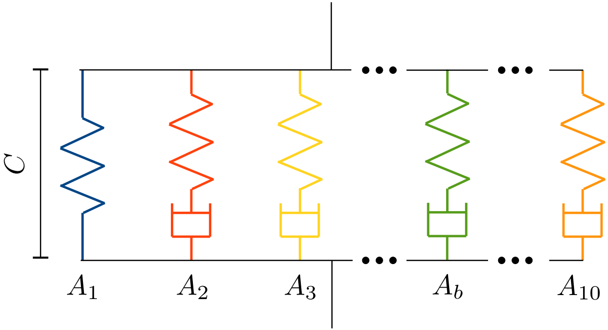
Objectives of this thesis:

- To **formulate** the reduced-order model;
- To implement the model **calibration strategy**;
- To discuss the **results** obtained from this calibration strategy.

Outline

Generalized Maxwell model

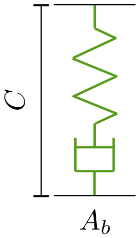
- Description of the tire viscoelastic material;
- Rheological representation:

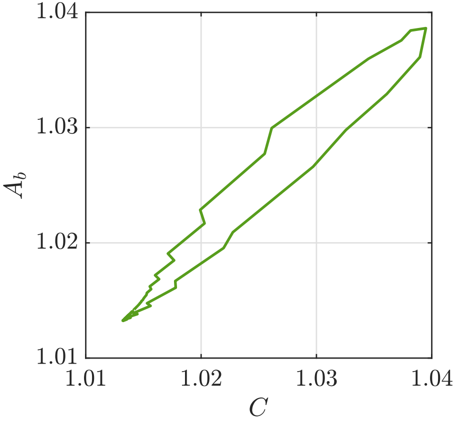


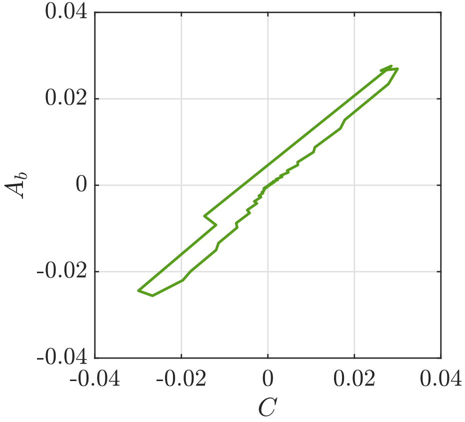
- C: Component of the right Cauchy-Green tensor;
- A_b: Viscoelastic internal variable in Branch b.

Hysteretic behavior

- Examples of hysteresis loop in a Branch b:







Hysteretic behavior between A_b and C.

The suggested reduced-order model

- Based on the Bouc-Wen hysteresis model (phenomenological):

$$\begin{cases} \dot{A}_b(t) = \dfrac{1}{c_b}\left(C(t) - k_b\, A_b(t) - \mathcal{Z}_b(A_b, \dot{A}_b)\right) \\ \dot{\mathcal{Z}}_b = \alpha_b\, \dot{A}_b - \gamma_b\, |\dot{A}_b| |\mathcal{Z}_b|^{\nu_b - 1} \mathcal{Z}_b - \delta_b\, \dot{A}_b |\mathcal{Z}_b|^{\nu_b} \end{cases}$$

- A nonlinear, first-order differential equations system where:
- c_b: Linear damping coefficient from the dashpot;
- k_b: Linear coefficient of elasticity from the spring;
- $\mathcal{Z}_b$: Hysteretic output from the Bouc-Wen model;
- α_b, γ_b, δ_b and ν_b: Bouc-Wen model parameters in Branch b;
- Set of parameters: $\boldsymbol{\theta} = \{c, k, \alpha, \gamma, \delta, \nu\}$.

Mohammed Ismail & Fayçal Ikhouane & José Rodellar. The Hysteresis Bouc-Wen Model, a Survey. Archives of Computational Methods in Engineering, Springer Science and Business Media LLC 16 (2) (**2009**), pp. 161–188.

Prior knowledge about the ROM parameters

- Through the Bouc-Wen model literature review and numerical simulations:

c	k	α	γ	δ	ν
$c > 0$	$k \geq 0$ and $k \approx 1$	$\alpha > 0$	$\gamma > 0$	$\gamma + \delta > 0$ and $\gamma - \delta \geq 0$	$\nu \geq 1$

These conditions ensure physical and mathematical consistency of the Bouc-Wen model.

Mohammed Ismail & Fayçal Ikhouane & José Rodellar. The Hysteresis Bouc-Wen Model, a Survey. Archives of Computational Methods in Engineering, Springer Science and Business Media LLC 16 (2) (**2009**), pp. 161–188.

Michelin's dataset

- Data from a finite element approximation of a steady-state rolling tire:
- Right Cauchy-Green deformation tensors;
- Viscoelastic internal variables (rank 2 symmetric positive-definite tensors);
- Calibration using this dataset: minor loss of information.

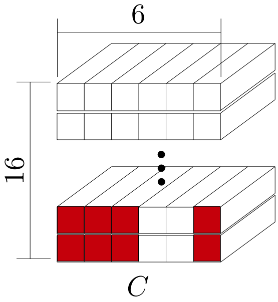
Data selection procedure

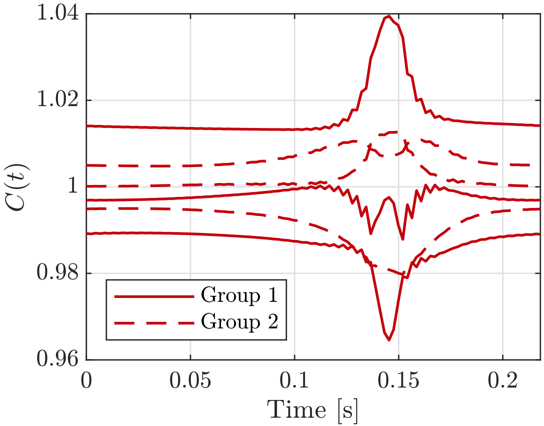
An initial study was conducted to understand:

- The dataset structure and;
- Relations between components of the right Cauchy-Green tensors and viscoelastic internal variables.

P. Le Tallec & C. Rahler. Numerical models of steady rolling for non-linear viscoelastic structures in finite deformations. International Journal for Numerical Methods in Engineering, Wiley 37 (7) (**1994**), pp. 1159–1186.

Components of the right Cauchy-Green tensor



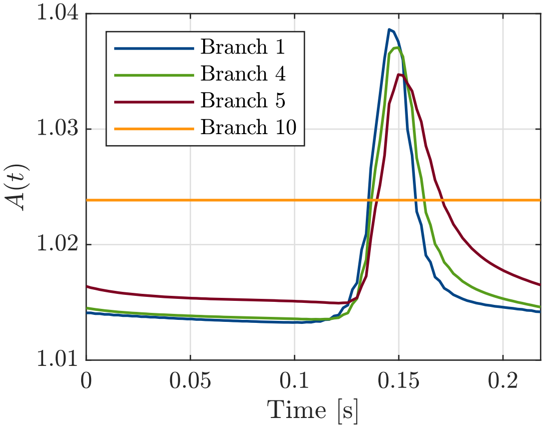


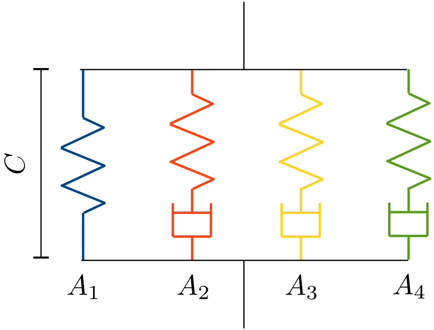
Visual identification and selection of a limited number of inputs C that are supposed to be representative.

16 inputs divided into Group 1 and Group 2: better simulation results.

Viscoelastic internal variables

- Examples of viscoelastic internal variables in Group 1:

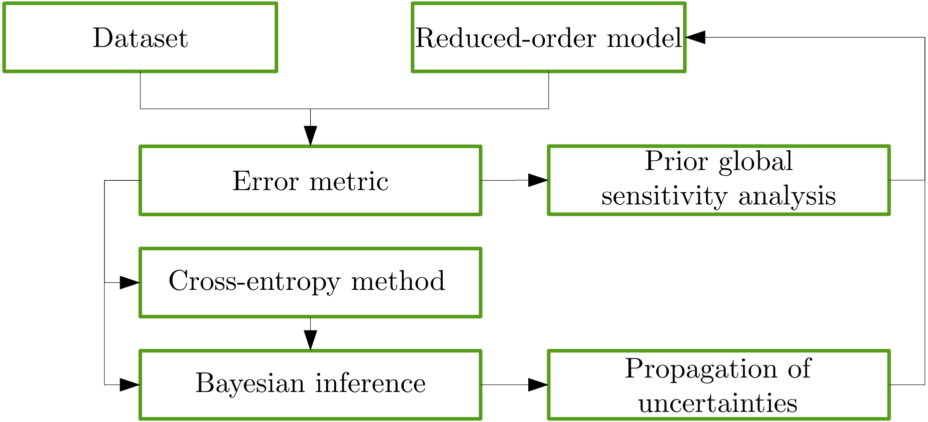




Branches of interest: dynamic behavior.

2 reduced-order models per branch of interest $\Rightarrow$ 8 reduced-order models in total.

A model calibration strategy



An error metric

- How to measure the accuracy of the ROM responses?
- Definition of the average of the mean absolute scaled error (MASE):

$$\boxed{\bar{E}(\boldsymbol{\theta}) = \frac{1}{N_{\text{in}} \cdot N_{\text{out}}} \sum_{i=1}^{N_{\text{in}}} \sum_{j=1}^{N_{\text{out}}} \frac{\left|\left|A_{ij}^{\mathsf{DS}} - A_{ij}(\boldsymbol{\theta})\right|\right|}{\frac{1}{N_{\text{out}}-1} \sum_{j=2}^{N_{\text{out}}} \left|\left|A_{ij}^{\mathsf{DS}} - A_{i(j-1)}^{\mathsf{DS}}\right|\right|}}$$

- Where:
- N_{out}: Number of outputs of the time series;
- N_{in}: Number of selected inputs;
- A_{ij}^{DS}: A selected viscoelastic internal variable from the dataset;
- $A_{ij}(\boldsymbol{\theta})$: The corresponding ROM response.

Sobol' indices

$$\boldsymbol{X} \Rightarrow \boxed{\mathcal{M}} \Rightarrow Y$$

$$X_i \sim \mathcal{U}(0,1)$$

- The Sobol' decomposition of $\mathcal{M}$ into summands of increasing dimension:

$$Y = \mathcal{M}_0 + \sum_{i=1}^{n} \mathcal{M}_i(X_i) + \sum_{1 \le i < j \le n} \mathcal{M}_{i,j}(X_i, X_j) + \cdots + \mathcal{M}_{1,2,\ldots,n}(X_1, \ldots, X_n)$$

- These summands are orthogonal to each other. By evaluating the variance of the Sobol' decomposition:

- First-order Sobol' indices:

$$S_i = Var(\mathcal{M}_i(X_i)) \Big/ Var(\mathcal{M}(\boldsymbol{X}))$$

- Total Sobol' indices:

$$S_i^{\mathrm{T}} = 1 - S_{\sim i}$$

I. M. Sobol. Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates. Mathematics and Computers in Simulation 55 (1-3) (**2001**), pp. 271–280.

Polynomial chaos expansion

- Polynomial chaos expansion (PCE) is defined as:

$$Y = \mathcal{M}(\boldsymbol{X}) \approx \sum_{\alpha \in \mathcal{A}} y_\alpha \cdot \psi_\alpha(\boldsymbol{X})$$

- y_α: Deterministic coefficients to be determined;
- ψ_α: Multivariate orthogonal polynomial bases.

- First-order Sobol' indices:

$$\boxed{\tilde{S}_i = \sum_{\substack{\alpha \in \mathcal{A}_i \\ \alpha \neq 0}} y_\alpha^2 \Big/ \sum_{\substack{\alpha \in \mathcal{A} \\ \alpha \neq 0}} y_\alpha^2}$$

- Total Sobol' indices:

$$\boxed{\tilde{S}_i^{\mathrm{T}} = \sum_{\substack{\alpha \in \mathcal{A}_i^{\mathrm{T}} \\ \alpha \neq 0}} y_\alpha^2 \Big/ \sum_{\substack{\alpha \in \mathcal{A} \\ \alpha \neq 0}} y_\alpha^2}$$

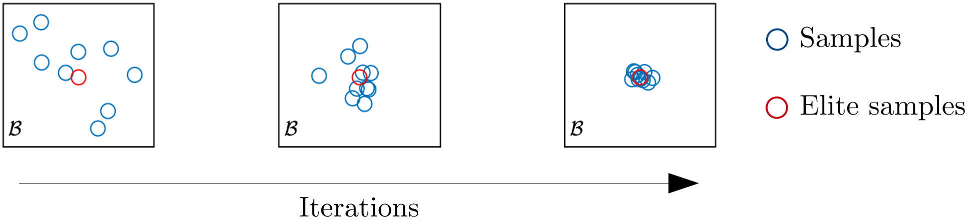
B. Sudret. Global sensitivity analysis using polynomial chaos expansions. Reliability Engineering and System Safety 93 (**2008**), pp. 964–979.

The cross-entropy method

- The cross-entropy (CE) method solves optimization problems based on importance sampling technique;
- The error metric constitute an objective function in which:

$$\boxed{\hat{\boldsymbol{\theta}} = \arg\min_{\mathcal{B}} \bar{E}(\boldsymbol{\theta})} \quad \star$$

- $\hat{E} = E(\hat{\boldsymbol{\theta}})$ is the optimal error metric;
- Geometric idea of the CE method:



Computational algorithm

Algorithm CE method (computational algorithm)

1. Set the **numbers of samples** and **elite samples**. Define a convergence **tolerance** and a family of **probability distributions**. Set the maximum of iteration levels and initialize the counter;
2. Update the level counter;
3. Generate **samples** from the distribution;
4. Evaluate each sample, sort the results and get the **elite samples**;
5. Update the **estimators** with the elite samples (and apply a smooth updating schema);
6. Repeat lines from 2 to 5 until reach a **stopping criterion**.

Americo Cunha Jr. Enhancing the performance of a bistable energy harvesting device via the cross-entropy method. Nonlinear Dynamics, Springer Verlag 103 (**2021**), pp. 137–155.

Bayesian inference: Bayes' theorem

- Bayesian inference is based on the Bayes's theorem:

$$\pi(\boldsymbol{\theta}|\hat{E}) = \frac{\pi(\hat{E}|\boldsymbol{\theta})\,\pi(\boldsymbol{\theta})}{\pi(\hat{E})}$$

- Where:
- $\pi(\boldsymbol{\theta})$ is the prior distribution of parameters;
- $\pi(\hat{E}|\boldsymbol{\theta})$ is the likelihood;
- $\pi(\boldsymbol{\theta}|\hat{E})$ is the posterior distribution;
- And what about $\pi(\hat{E})$? It is a normalized constant. So:

$$\pi(\boldsymbol{\theta}|\hat{E}) \propto \pi(\hat{E}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

Bayesian inference: the prior distribution

How to infer the posterior distribution $\pi(\boldsymbol{\theta}|\hat{E})$ of the set of parameters $\boldsymbol{\theta}$ given the optimal error metric $\hat{E}$?

- The prior distribution gathers the prior knowledge about the set of parameters $\boldsymbol{\theta}$ before any evidence;
- In the case of Uniform prior distribution:

$$\pi(\boldsymbol{\theta}|\hat{E}) \propto \pi(\hat{E}|\boldsymbol{\theta})$$

Bayesian inference: the likelihood function

- The likelihood is the probability of observing the optimal error metric $\hat{E}$ given a set of parameters $\boldsymbol{\theta}$;
- Assuming an additive discrepancy: $\hat{E} = \bar{E}(\boldsymbol{\theta}) + \boldsymbol{\epsilon}$;
- In the case of an independent and normally distributed discrepancy:

$$\pi(\hat{E}|\boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi\sigma_{\boldsymbol{\epsilon}}^2}} \exp\left(-\frac{1}{2}\frac{\left|\left|\hat{E} - \bar{E}(\boldsymbol{\theta})\right|\right|^2}{\sigma_{\boldsymbol{\epsilon}}^2}\right)$$

If unknown, the variance $\sigma_{\boldsymbol{\epsilon}}^2$ can be inferred: $\boldsymbol{x} = \{\boldsymbol{\theta}, \sigma_{\boldsymbol{\epsilon}}^2\}$

Bayesian inference: the posterior distribution

- The posterior distribution is the probability of the set of parameters $\boldsymbol{\theta}$ given the optimal error metric $\hat{E}$. It updates the prior knowledge through the evidence accumulated during the process;
- After the previous assumptions, the posterior distribution is proportional to the likelihood:

$$\boxed{\pi(\boldsymbol{x}|\hat{E}) \propto \frac{1}{\sqrt{2\pi\sigma_{\epsilon}^{2}}} \exp\left(-\frac{1}{2}\frac{\left|\left|\hat{E}-\bar{E}(\boldsymbol{\theta})\right|\right|^{2}}{\sigma_{\epsilon}^{2}}\right)}$$

Markov chain Monte Carlo (MCMC) methods to infer the posterior distribution $\pi(\boldsymbol{x}|\hat{E})$

Metropolis-Hastings algorithm

Algorithm Metropolis-Hastings algorithm (random walk Metropolis)

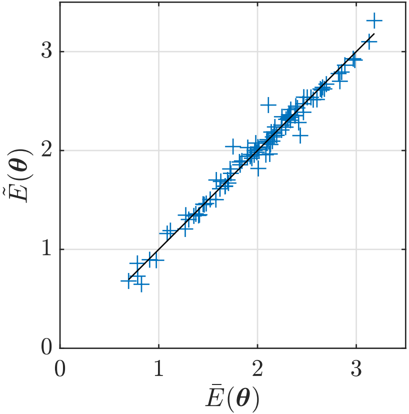
1. Set the **total number of samples** and the **random walk step size** σ;
2. Initialize the counter and assign the initial values $\boldsymbol{x}^{(0)}$;
3. Generate a **candidate** $\boldsymbol{x}^* \sim \mathcal{N}(\boldsymbol{x}^{(k)}, \sigma^2)$;
4. Compute the **acceptance** probability $a(\boldsymbol{x}^*, \boldsymbol{x}^{(k)}) = \min\{1, \pi(\boldsymbol{x}^*|\hat{E}) / \pi(\boldsymbol{x}^{(k)}|\hat{E})\}$;
5. Generate a **random number** $u \sim \mathcal{U}(0,1)$;
6. Does $u < a(\boldsymbol{x}^*, \boldsymbol{x}^{(k)})$?
7. Yes $\rightarrow$ Accept the candidate and $\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^*$;
8. No $\rightarrow$ Reject the candidate and $\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)}$;
9. Increment k and repeat lines from 3 to 9 until the **total number of samples**.

Prior global sensitivity analysis: ROM parameters distributions

- Uniform prior distribution: ROM parameters.

$\mathcal{U}(a, b)$	c	k	α	γ	δ	ν
a	0	0.999	0	1,000	-1,000	1
b	0.01	1.001	1	10,000	1,000	3

Prior global sensitivity analysis: PCE-based surrogate model validation

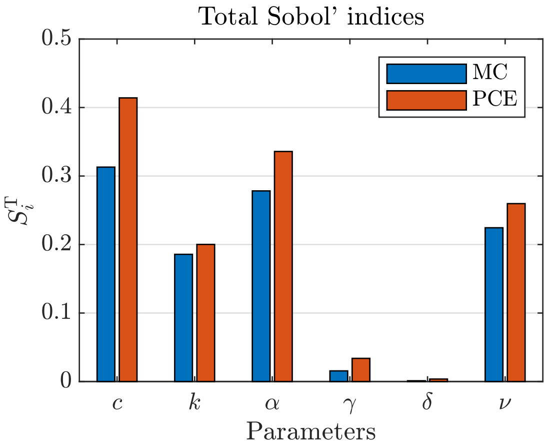


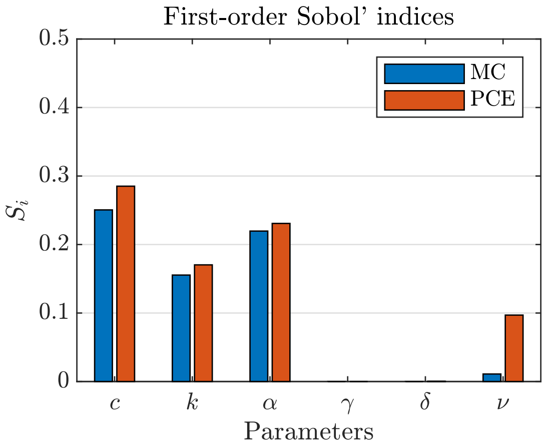
PCE degree	Exp. Design	LOO error*
13	1,000	$7.97 \cdot 10^{-3}$

Elapsed time: 25 min.

Branch 1, Group 1. *Leave-one-out cross-validation error.

Prior sensitivity analysis: Sobol' indices





Parameters γ and δ Sobol' indices can be neglected. In the sequence, $\gamma = 1,000$ and $\delta = 1,000$.

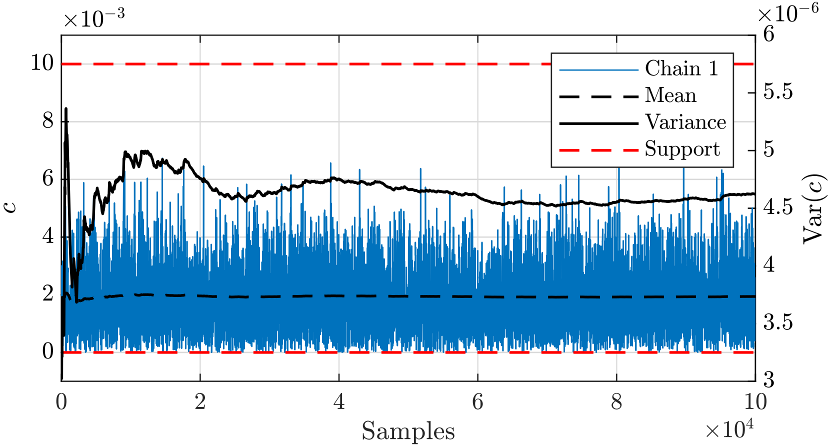
Bayesian inference: prior distribution

- Uniform prior distribution: influential ROM parameters and variance of the discrepancy:

$\mathcal{U}(a,b)$	c	k	α	ν	σ_ϵ^2
a	0	0.999	0	1	0
b	0.01	1.001	1	3	0.1

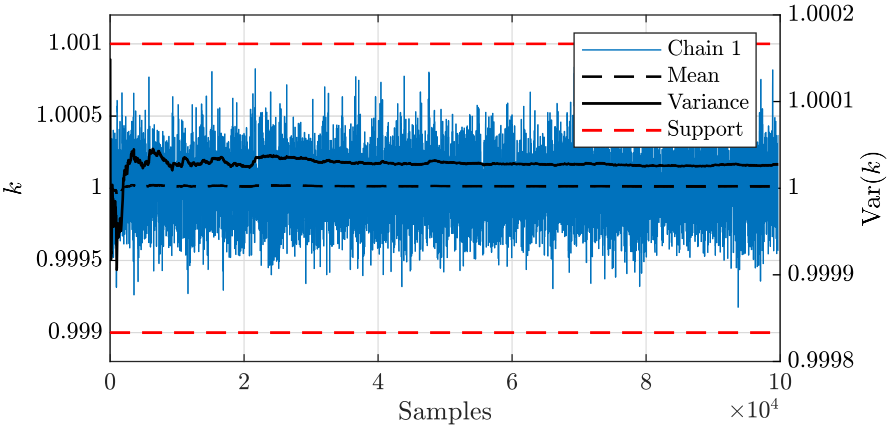
For Branch 4: $b = 0.03$ and $b = 2$ for parameters c and α, respectively.

Markov chain convergence analysis: parameter c



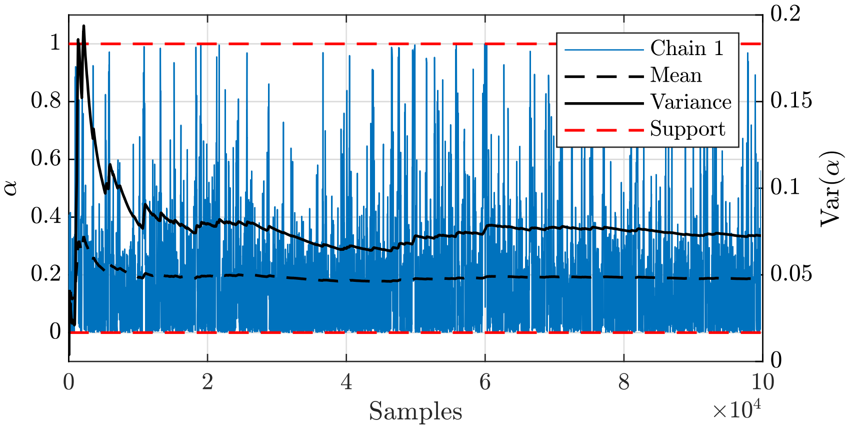
Branch 1, Group 1: 1×10^5 samples and $\bar{a} \approx 43\%$.

Markov chain convergence analysis: parameter k



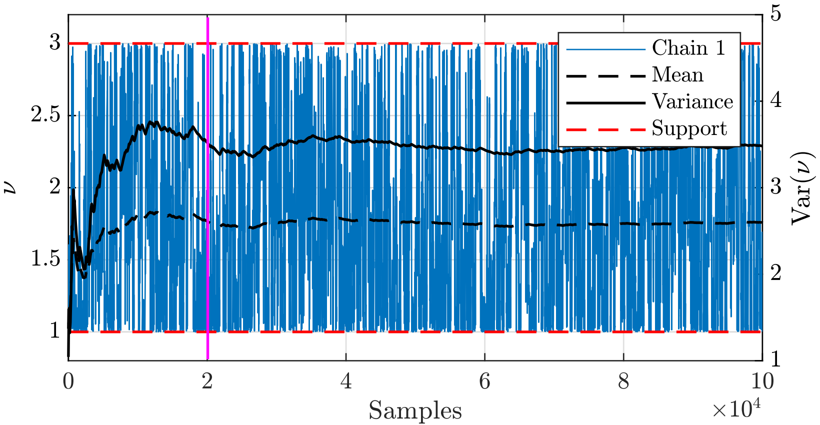
Branch 1, Group 1: 1×10^5 samples and $\bar{a} \approx 43\%$.

Markov chain convergence analysis: parameter α



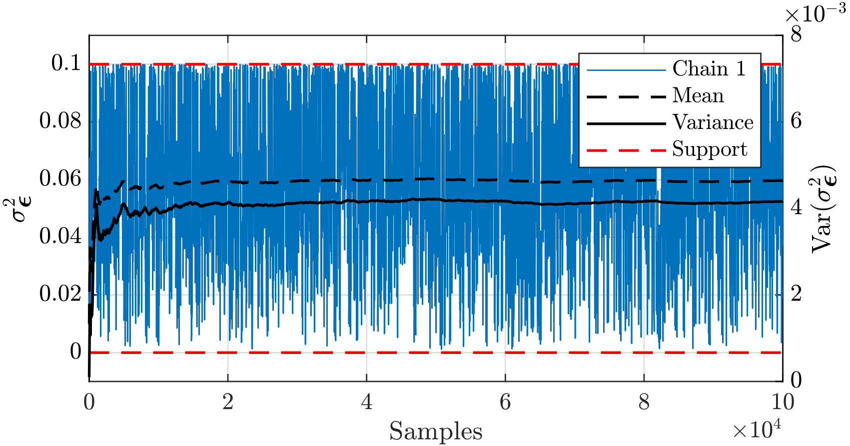
Branch 1, Group 1: 1×10^5 samples and $\bar{a} \approx 43\%$.

Markov chain convergence analysis: parameter ν



Branch 1, Group 1: 1×10^5 samples and $\bar{a} \approx 43\%$.
Minimum number of samples: 2×10^4 samples.

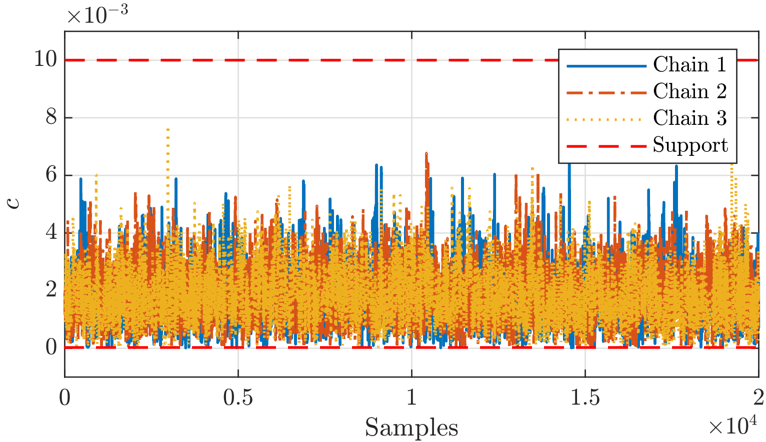
Markov chain convergence analysis: variance σ_ϵ^2



Branch 1, Group 1: 1×10^5 samples and $\bar{a} \approx 43\%$.

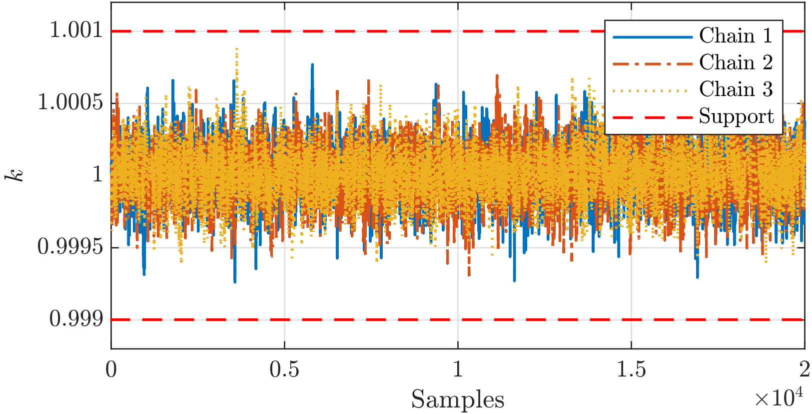
The Markov chain converged.

Markov chain stability analysis: parameter c



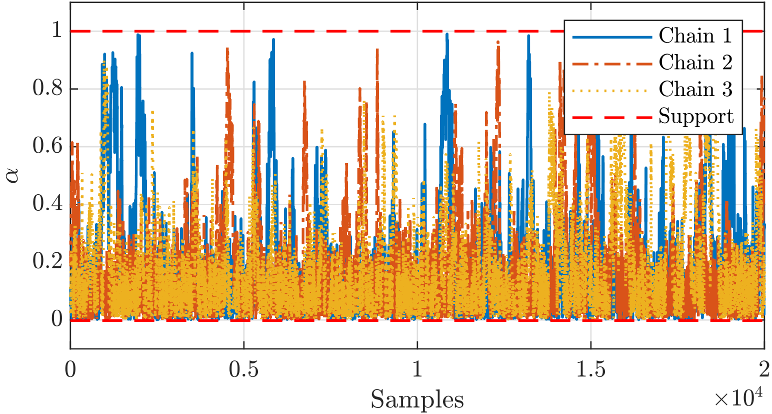
Branch 1, Group 1: 2×10^4 samples. $\bar{a} \approx 42\%$, $\bar{a}_2 \approx 45\%$ and $\bar{a}_3 \approx 44\%$.

Markov chain stability analysis: parameter k



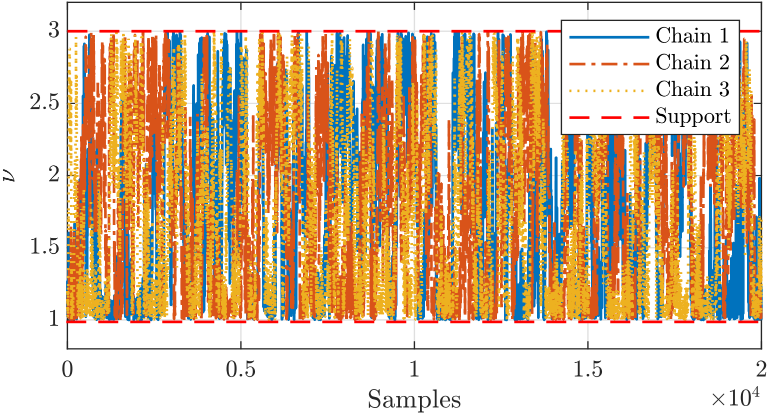
Branch 1, Group 1: 2×10^4 samples. $\bar{a} \approx 42\%$, $\bar{a}_2 \approx 45\%$ and $\bar{a}_3 \approx 44\%$.

Markov chain stability analysis: parameter α



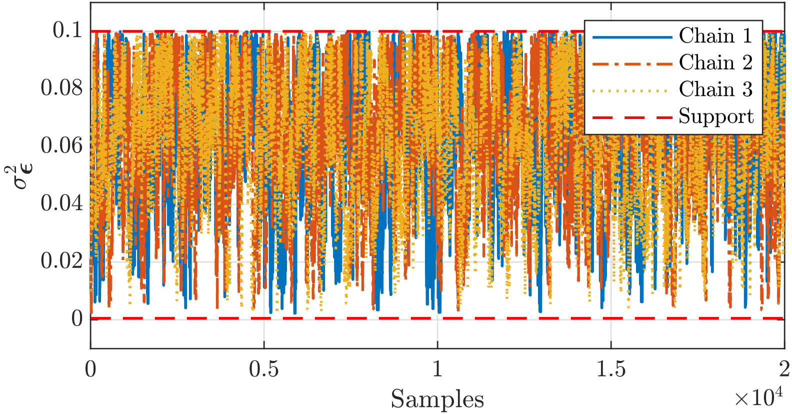
Branch 1, Group 1: 2×10^4 samples. $\bar{a} \approx 42\%$, $\bar{a}_2 \approx 45\%$ and $\bar{a}_3 \approx 44\%$.

Markov chain stability analysis: parameter ν



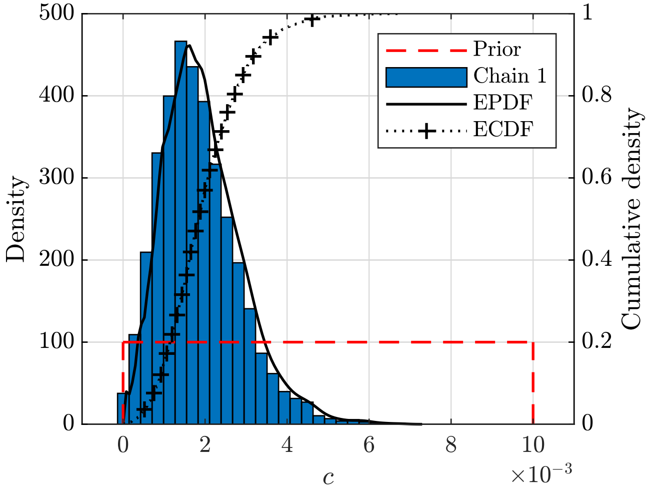
Branch 1, Group 1: 2×10^4 samples. $\bar{a} \approx 42\%$, $\bar{a}_2 \approx 45\%$ and $\bar{a}_3 \approx 44\%$.

Markov chain stability analysis: variance σ_ϵ^2



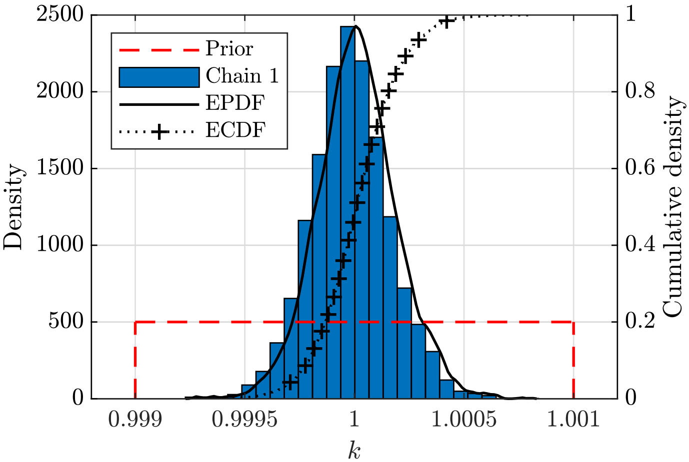
Branch 1, Group 1: 2×10^4 samples. $\bar{a} \approx 42\%$, $\bar{a}_2 \approx 45\%$ and $\bar{a}_3 \approx 44\%$.
The Markov chain converged and is stable.

Parameter c distribution



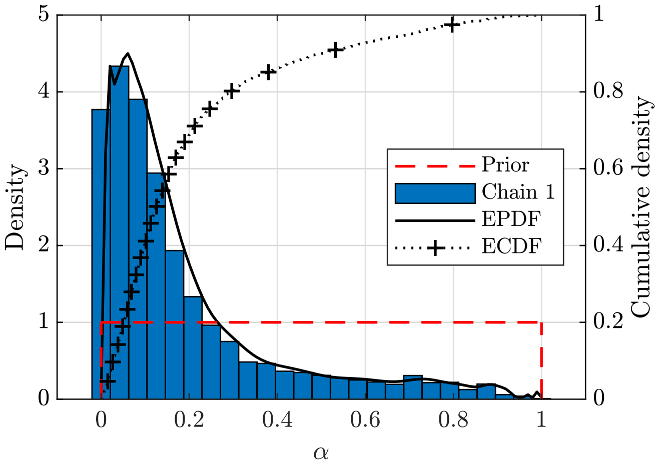
Branch 1, Group 1: 2×10^4 samples and $\bar{a} \approx 42\%$. Mean: $\bar{c} = 0.0020$.

Parameter k distribution



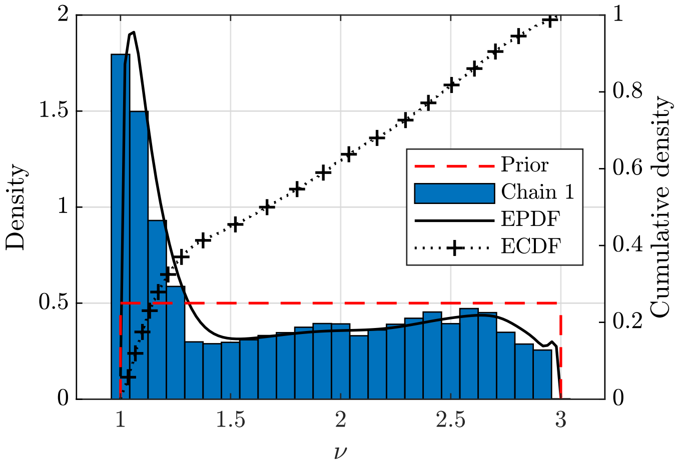
Branch 1, Group 1: 2×10^4 samples and $\bar{a} \approx 42\%$. Mean: $\bar{k} = 1.0000$.

Parameter α distribution



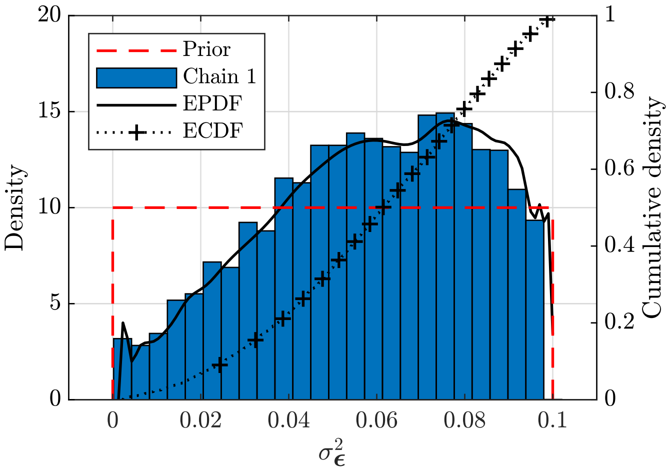
Branch 1, Group 1: 2×10^4 samples and $\bar{a} \approx 42\%$. Mean: $\bar{\alpha} = 0.1964$.

Parameter ν distribution



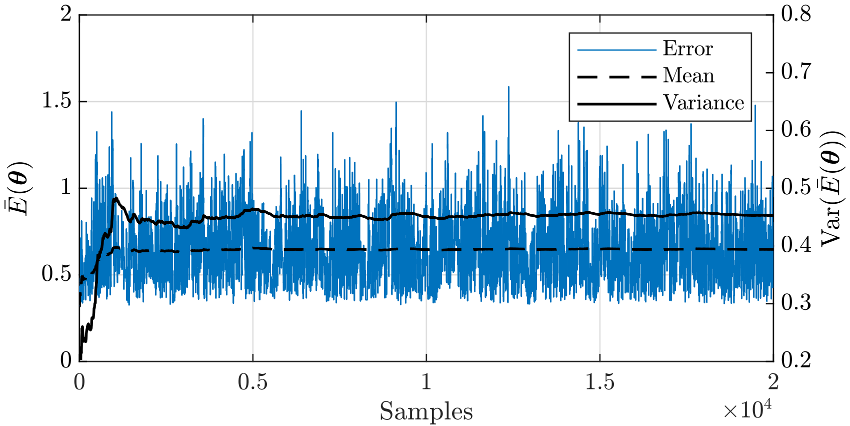
Branch 1, Group 1: 2×10^4 samples and $\bar{a} \approx 42\%$. Mean: $\bar{\nu} = 1.7664$.

Variance σ_ϵ^2 distribution



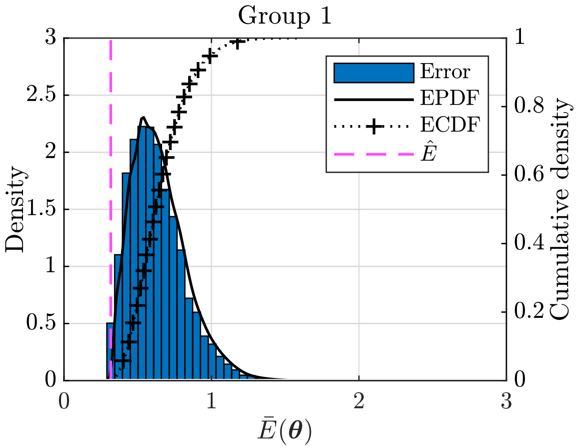
Branch 1, Group 1: 2×10^4 samples and $\bar{a} \approx 42\%$. Mean: $\bar{\sigma_\epsilon^2} = 0.0597$.

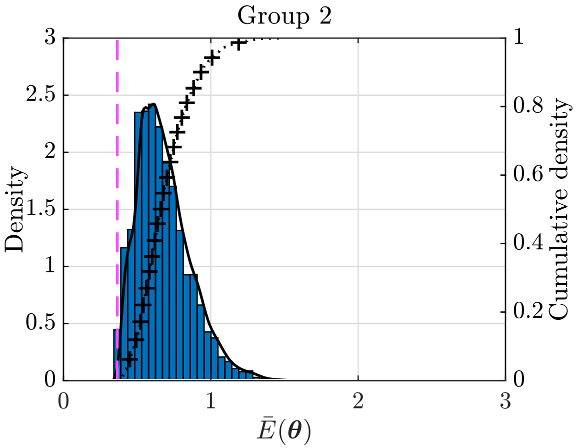
Error metric convergence analysis



Branch 1, Group 1: 2×10^4 samples and $\bar{a} \approx 42\%$.

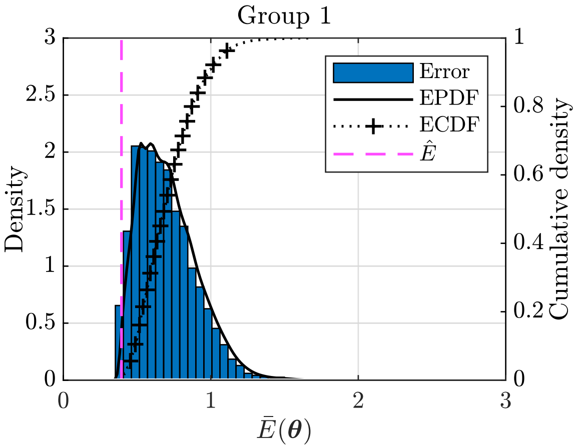
Error metric distributions: Branch 1

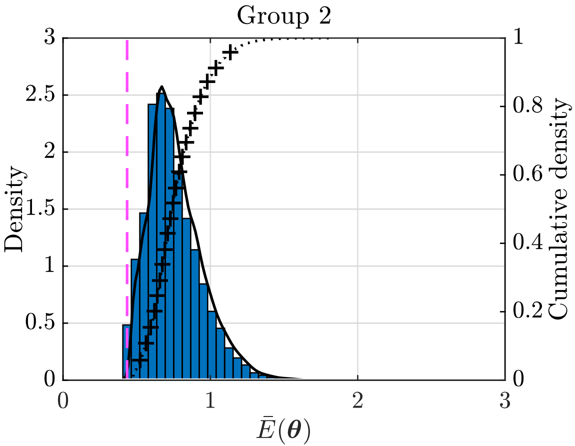




Branch 1, Group 1: $\hat{E} = 0.3171$. Group 2: $\hat{E} = 0.3655$.

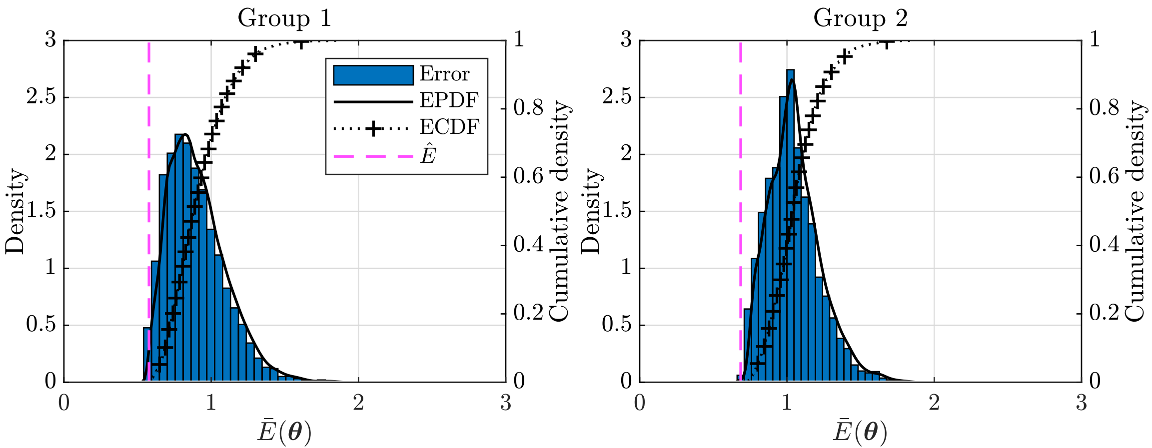
Error metric distributions: Branch 2





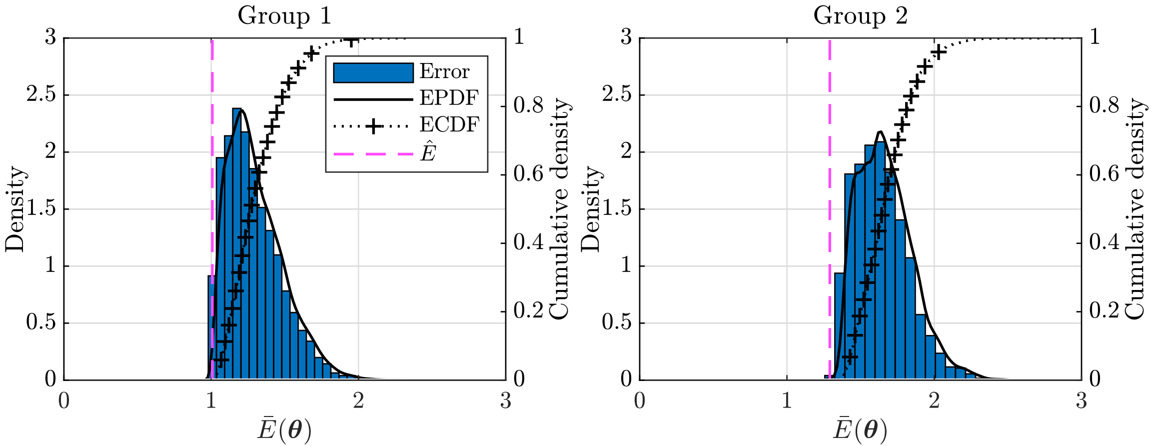
Branch 2, Group 1: $\hat{E} = 0.3942$. Group 2: $\hat{E} = 0.4347$.

Error metric distributions: Branch 3



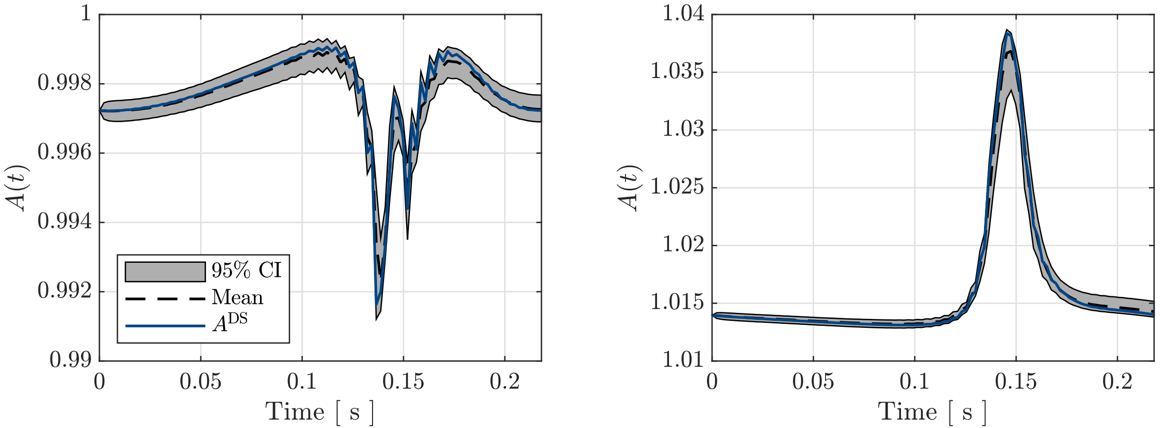
Branch 3, Group 1: $\hat{E} = 0.5776$. Group 2: $\hat{E} = 0.6857$.

Error metric distributions: Branch 4



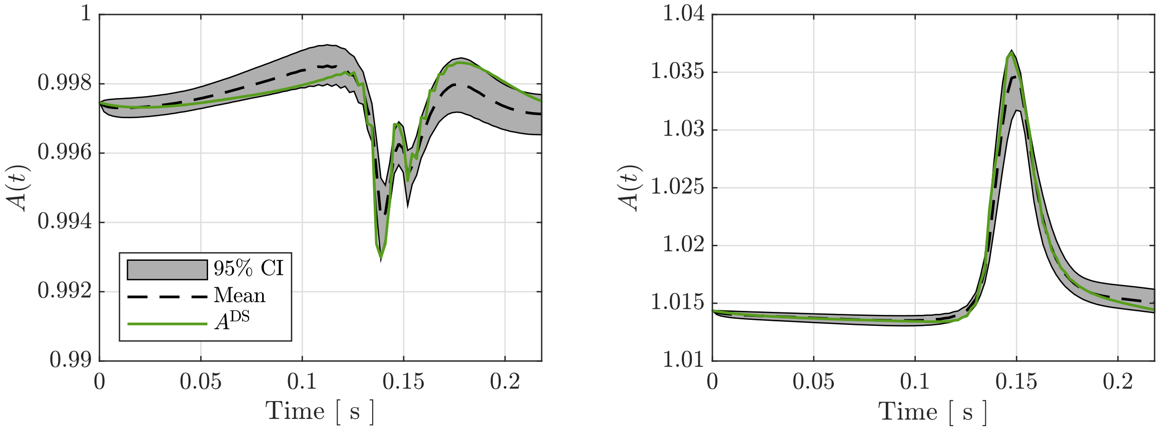
Branch 4, Group 1: $\hat{E} = 1.0116$. Group : $\hat{E} = 1.6672$.

Propagation of uncertainties: Branch 1



Branch 1, Group 1: Confidence interval (CI). The best case estimation scenario.

Propagation of uncertainties: Branch 4



Branch 4, Group 1: Confidence interval (CI). The worst case estimation scenario.

Concluding remarks

- An advantage of the reduced-order model is that it can be computed by numerically solving a nonlinear, first-order differential equations system rather than a finite element model;
- Following the adopted methodology, the reduced-order model is adequate for simulating viscoelastic internal variables;
- The responses of the reduced-order model are in conformity with the data;
- The main efforts on the calibration strategy are related to the Bayesian inference;
- Obtaining a robust and less computational expensive model may facilitate research and simulations to obtain a more robust tire.

Performed activities

Publications

- The manuscript **”On the use of stochastic Bouc-Wen model for simulating viscoelastic internal variables from a finite element approximation of steady-rolling tire”** was submitted to the Journal of Vibration and Control.

Other activities

- **PAADES B** in Dynamics course:
  - 48h of teaching support activities;
  - 12h of teaching activities.

Following steps

International Dual Degree PhD: UBFC (France) and UdeS (Canada)

- I started my PhD studies at Besançon, France.
- Thesis subject: **"Self-adaptive thermally driven multilayered composite for vibration control"**.

Availability of materials

- The MATLAB scripts used in numerical simulations are available in the following GitHub repository: `https://github.com/rafaelraqueti/UQ_Bouc-Wen_calibration.git`;
- The dataset used in this work belongs to Michelin and is not available in this repository.

Acknowledgements

Dataset:

- Manufacture Française des Pneumatiques **Michelin**.

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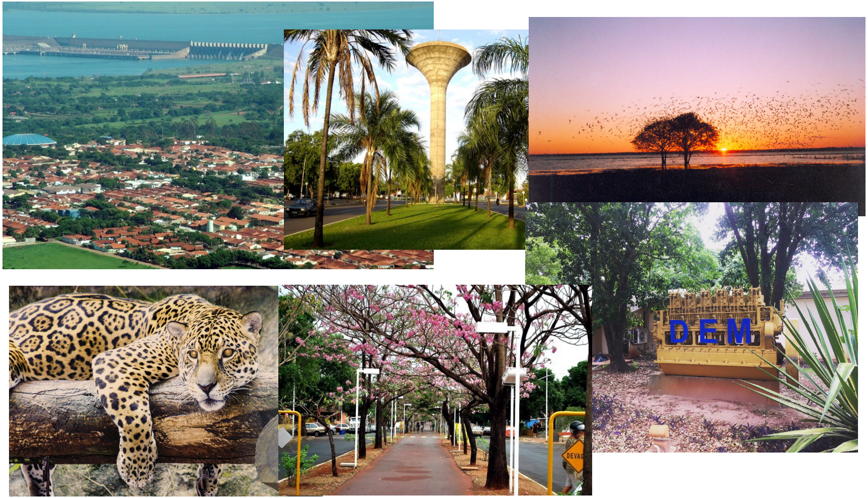
PROGRAMA DE PÓS-GRADUAÇÃO
EM ENGENHARIA MECÂNICA

Thank you for your attention!



Ilha Solteira - SP, Brazil.

Appendix - Estimated values

Branch	Group	Estimator	c	k	α	ν	σ_{ϵ}^{2}
Branch 1	Group 1	CE	0.0018	1.0000	0.1152	1.0220	n/a
		MAP	0.0016	1.0000	0.0227	2.0326	0.0062
	Group 2	CE	0.0035	1.0000	0.0418	1.8039	n/a
		MAP	0.0034	1.0000	0.0551	2.4126	0.0069

Elapsed time for Branch 1, Group 1: 63 min. (CE method) and 311 min. (Bayesian inference). Group 2: 39 min. (CE method) and 172 min. (Bayesian inference).

Appendix - Estimated values

Branch	Group	Estimator	c	k	α	ν	σ_ϵ^2
Branch 2	Group 1	CE	0.0020	1.0000	0.0194	1.5031	n/a
		MAP	0.0018	1.0001	0.1108	1.1788	0.0052
	Group 2	CE	0.0043	1.0000	0.0511	2.3418	n/a
		MAP	0.0042	1.0000	0.0433	2.1276	0.0040

Elapsed time for Branch 2, Group 1: 27 min. (CE method) and 241 min. (Bayesian inference). Group 2: 22 min. (CE method) and 103 min. (Bayesian inference).

Appendix - Estimated values

Branch	Group	Estimator	c	k	α	ν	σ^2_ϵ
Branch 3	Group 1	CE	0.0027	1.0000	0.1819	1.1381	n/a
		MAP	0.0027	1.0000	0.2068	1.1640	0.0016
	Group 2	CE	0.0072	1.0000	0.0738	1.9416	n/a
		MAP	0.0072	1.0000	0.0738	1.9416	0.0100

Elapsed time for Branch 3, Group 1: 35 min. (CE method) and 149 min. (Bayesian inference). Group 2: 26 min. (CE method) and 83 min. (Bayesian inference).

Appendix - Estimated values

Branch	Group	Estimator	c	k	α	ν	σ_ϵ^2
Branch 4	Group 1	CE	0.0049	1.0001	1.0411	1.0418	n/a
		MAP	0.0057	1.0001	0.6845	1.0911	0.0028
	Group 2	CE	0.0154	1.0000	0.1077	1.8369	n/a
		MAP	0.0154	1.0000	0.1077	1.8369	0.0100

Elapsed time for Branch 4, Group 1: 50 min. (CE method) and 173 min. (Bayesian inference). Group 2: 12 min. (CE method) and 69 min. (Bayesian inference).