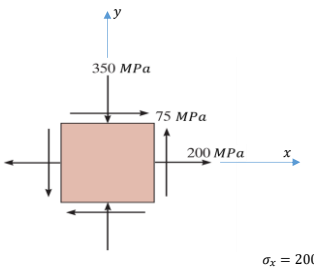
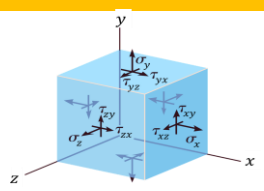
Resistência dos Materiais - Aula 10

Exercício 1:



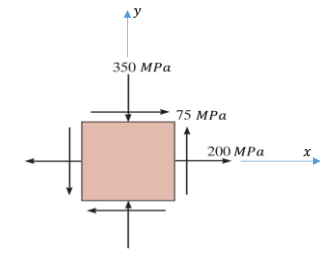


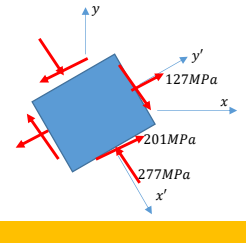
$\sigma_x = 200MPa \qquad \sigma_y = -350MPa \qquad \tau_{xy} = 75MPa \qquad \theta = -60°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} * \cos(2 * \theta) + \tau_{xy} * sen(2 * \theta) = \frac{200 + (-350)}{2} + \frac{200 - (-350)}{2} * \cos(2 * (-60)) + 75 * sen(2 * (-60)) = -277MPa$$

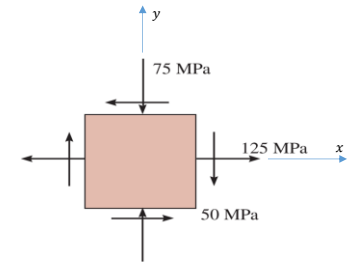
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} * \cos(2 * \theta) - \tau_{xy} * sen(2 * \theta) = \frac{200 + (-350)}{2} - \frac{200 - (-350)}{2} * \cos(2 * (-60)) - 75 * sen(2 * (-60)) = 127MPa$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} * sen(2 * \theta) + \tau_{xy} * cos(2 * \theta) = -\frac{200 - (-350)}{2} * sen(2 * (-60)) + 75 * \cos(2 * (-60)) = 201MPa$$





Exercício 2:



$\sigma_x = 125MPa \qquad \sigma_y = -75MPa \qquad \tau_{xy} = -50MPa$

Tensões Principais:

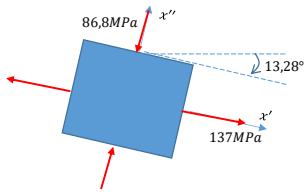
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{125 + (-75)}{2} + \sqrt{\left(\frac{125 - (-75)}{2}\right)^2 + (-50)^2} = 137MPa$$

$$\sigma_2 = \frac{125 + (-75)}{2} - \sqrt{\left(\frac{125 - (-75)}{2}\right)^2 + (-50)^2} = -86{,}8MPa$$

$$\tan(2 * \theta_p) = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{-50}{\frac{125 - (-75)}{2}} = -0{,}5$$

$$\theta_p = -13{,}28°$$



Tensões de Cisalhamento Máximas no Plano:

$$\tan(2 * \theta_s) = -\frac{\frac{\sigma_x - \sigma_y}{2}}{\tau_{xy}} = -\frac{\frac{125 - (-75)}{2}}{-50} = 2$$

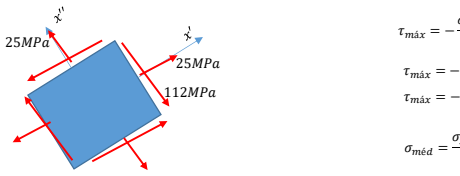
$$\theta_s = 31{,}7°$$

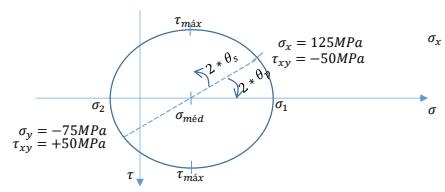
$$\tau_{máx} = -\frac{\sigma_x - \sigma_y}{2} * sen(2 * \theta_s) + \tau_{xy} * \cos(2 * \theta_s)$$

$$\tau_{máx} = -\frac{125 - (-75)}{2} * sen(2 * 31{,}7°) + (-50) * \cos(2 * 31{,}7°)$$

$$\tau_{máx} = -112MPa$$

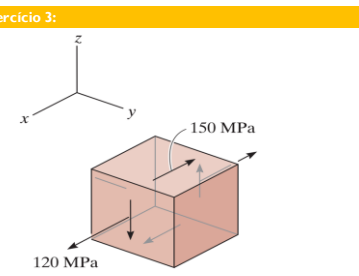
$$\sigma_{méd} = \frac{\sigma_x + \sigma_y}{2} = \frac{125 - 75}{2} = 25MPa$$

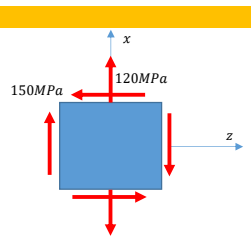




$\sigma_x = 125MPa \qquad \sigma_y = -75MPa \qquad \tau_{xy} = -50MPa$

Exercício 3:





Teoria:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Para este exercício:

$$\sigma_{1,2} = \frac{\sigma_z + \sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2}$$

Tensões Principais:

$$\sigma_1 = \frac{0 + 120}{2} + \sqrt{\left(\frac{0 - 120}{2}\right)^2 + (-150)^2} = 221{,}55MPa$$

$$\sigma_2 = \frac{0 + 120}{2} - \sqrt{\left(\frac{0 - 120}{2}\right)^2 + (-150)^2} = -101{,}56MPa$$

$$\sigma_3 = 0Pa$$

Reordenar Tensões Principais por: $\sigma_1 \geq \sigma_2 \geq \sigma_3$

$$\sigma_1 = 221{,}55MPa \qquad \sigma_2 = 0Pa \qquad \sigma_3 = -101{,}56MPa$$

Tensão de Cisalhamento Máxima no Plano (Extra):

Teoria:

$$\tau_{máx} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

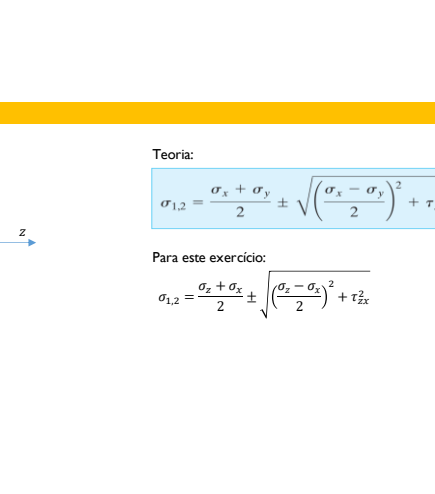
Para este Exercício:

$$\tau_{máx} = \pm \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2}$$

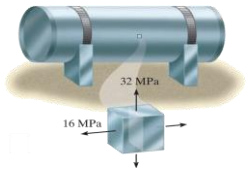
$$\tau_{máx} = \pm \sqrt{\left(\frac{0 - 120}{2}\right)^2 + (-150)^2} = \pm 161{,}55MPa$$

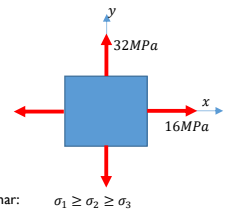
Tensão de Cisalhamento Máxima Absoluta:

$$\tau_{máx_{abs}} = \frac{\sigma_1 - \sigma_3}{2} = \frac{221{,}55 - (-101{,}56)}{2} = 161{,}55MPa$$



Exercício 4:





$\sigma_x = 16MPa$

$\sigma_y = 32MPa$

$\sigma_z = 0Pa$

Tensões principais já recebidas do enunciado, basta reordenar: $\sigma_1 \geq \sigma_2 \geq \sigma_3$

$\sigma_1 = 32MPa \qquad \sigma_2 = 16MPa \qquad \sigma_3 = 0Pa$

(Extra): Tensões de Cisalhamento Máxima no Plano xy:

$$\tau_{máx} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = \pm \sqrt{\left(\frac{16 - 32}{2}\right)^2 + (0)^2} = \pm 8MPa$$

Tensão de Cisalhamento Máxima Absoluta:

$$\tau_{máx_{abs}} = \frac{\sigma_1 - \sigma_3}{2} = 16MPa$$