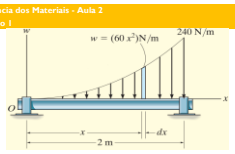
Resistência dos Materiais - Aula 2

Exercício 1



Força Resultante:

$$Fr = \int_L^{\square} w\,dx = \int_0^2 60x^2\,dx = 60\int_0^2 x^2\,dx = \frac{60}{3} * x^3\Big|_0^2 = 20 * (2^3 - 0^3) = 160N$$

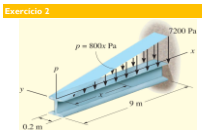
Ponto de aplicação da Força Resultante:

$$\bar{x} = \frac{\int_L^{\square} x * w\,dx}{\int_L^{\square} w\,dx}$$

$$\int_L^{\square} x * w\,dx = \int_0^2 x * 60 * x^2\,dx = 60 * \int_0^2 x^3\,dx = \frac{60}{4} * x^4\Big|_0^2 = 15 * (2^4 - 0^4) = 240Nm$$

$$\bar{x} = \frac{\int_L^{\square} x * w\,dx}{\int_L^{\square} w\,dx} = \frac{240}{160} = 1{,}5m$$

Exercício 2



Simplificar o carregamento em pressão para um carregamento sobre uma linha:

$$w(x) = p(x) * 0{,}2 = 800 * x * 0{,}2 = 160 * x\left[\frac{N}{m}\right]$$

Força Resultante do carregamento sobre uma linha:

$$Fr = \int_L^{\square} w\,dx = \int_0^9 160x\,dx = 160\int_0^9 x\,dx = \frac{160}{2} * x^2\Big|_0^9 = 80 * (9^2 - 0^2) = 6480N$$

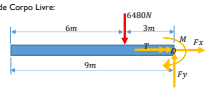
Localização da Força Resultante:

$$\bar{x} = \frac{\int_L^{\square} x * w\,dx}{\int_L^{\square} w\,dx}$$

$$\int_L^{\square} x * w\,dx = \int_0^9 x * 160 * x\,dx = 160\int_0^9 x^2\,dx = \frac{160}{3} * x^3\Big|_0^9 = \frac{160}{3} * (9^3 - 0^3) = 38880Nm$$

$$\bar{x} = \frac{\int_L^{\square} x * w\,dx}{\int_L^{\square} w\,dx} = \frac{38880}{6480} = 6m$$

Diagrama de Corpo Livre:



$T = 0Nm$
$Fx = 0N$

Equilíbrio de Forças na Vertical:

$\sum Fv = 0 \quad +\uparrow$

$-6480 + Fy = 0$

$Fy = 6480N$

Equilíbrio de Momentos:

$\sum M_O = 0 \quad +\circlearrowleft$

$-6480 * 3 + M = 0$

$M = 19440Nm$

Exercício 3

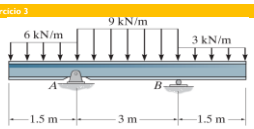
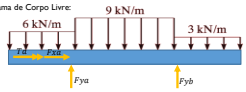


Diagrama de Corpo Livre:



$Fxa = 0N$
$Ta = 0Nm$

Equações dos Carregamentos Distribuídos:

$w_1 = 6000N/m$
$w_2 = 9000N/m$
$w_3 = 3000N/m$

Força resultante do primeiro carregamento distribuído:

$$Fr1 = \int_L^{\square} w_1\,dx = \int_0^{1{,}5} 6000\,dx = 6000\int_0^{1{,}5} dx = 6000 * x\Big|_0^{1{,}5} = 6000 * (1{,}5 - 0) = 9000N$$

$$\bar{x}_1 = \frac{\int_L^{\square} x * w_1\,dx}{\int_L^{\square} w_1\,dx}$$

$$\int_L^{\square} x * w_1\,dx = \int_0^{1{,}5} 6000 * x\,dx = 6000\int_0^{1{,}5} x\,dx = 6000 * \frac{1}{2} * x^2\Big|_0^{1{,}5} = 3000 * (1{,}5^2 - 0^2) = 6750Nm$$

$$\bar{x}_1 = \frac{6750}{9000} = 0{,}75m$$

Força resultante do segundo carregamento distribuído:

$$Fr2 = \int_L^{\square} w_2\,dx = \int_{1{,}5}^{4{,}5} 9000\,dx = 9000 * x\Big|_{1{,}5}^{4{,}5} = 9000(4{,}5 - 1{,}5) = 27000N$$

$$\bar{x}_2 = \frac{\int_L^{\square} x * w_2\,dx}{\int_L^{\square} w_2\,dx}$$

$$\int_L^{\square} x * w_2\,dx = \int_{1{,}5}^{4{,}5} x * 9000\,dx = \frac{9000}{2} * x^2\Big|_{1{,}5}^{4{,}5} = 4500 * (4{,}5^2 - 1{,}5^2) = 81000Nm$$

$$\bar{x}_2 = \frac{81000}{27000} = 3m$$

Força resultante do terceiro carregamento distribuído:

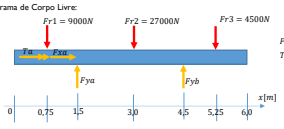
$$Fr3 = \int_L^{\square} w_3\,dx = \int_{4{,}5}^{6} 3000\,dx = 3000 * x\Big|_{4{,}5}^{6} = 3000 * (6 - 4{,}5) = 4500N$$

$$\bar{x}_3 = \frac{\int_L^{\square} x * w_3\,dx}{\int_L^{\square} w_3\,dx}$$

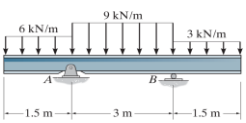
$$\int_L^{\square} x * w_3\,dx = \int_{4{,}5}^{6} x * 3000\,dx = \frac{3000}{2} * x^2\Big|_{4{,}5}^{6} = 1500 * (6^2 - 4{,}5^2) = 23625Nm$$

$$\bar{x}_3 = \frac{23625}{4500} = 5{,}25m$$

Diagrama de Corpo Livre:



$Fxa = 0N$
$Ta = 0Nm$



Equilíbrio de Forças:

$\sum Fv = 0 \quad +\uparrow$

$-9000 + Fya - 27000 + Fyb - 4500 = 0$

$Fya + Fyb = 40500N$ *Equação 1*

$$\begin{cases} Fya + Fyb = 40500N \\ Fya + 3 * Fyb = 74250N \end{cases}$$

$0 - 2 * Fyb = -33750$

$Fyb = 16875N$

Equilíbrio de Momentos:

$\sum M_o = 0 \quad +\circlearrowleft$

$+9000 * 0{,}75 - Fya * 1{,}5 + 27000 * 3 - Fyb * 4{,}5 + 4500 * 5{,}25 = 0$

$6750 - Fya * 1{,}5 + 81000 - Fyb * 4{,}5 + 23625 = 0$

$Fya * 1{,}5 + Fyb * 4{,}5 = 111375$

$Fya + 3 * Fyb = 74250N$ *Equação 2*

Substituir Fyb na Equação 1

$Fya + 16875 = 40500N$

$Fya = 23625N$