EM423 – RESISTÊNCIA DOS MATERIAIS
AULA 8

Universidade Estadual de Campinas
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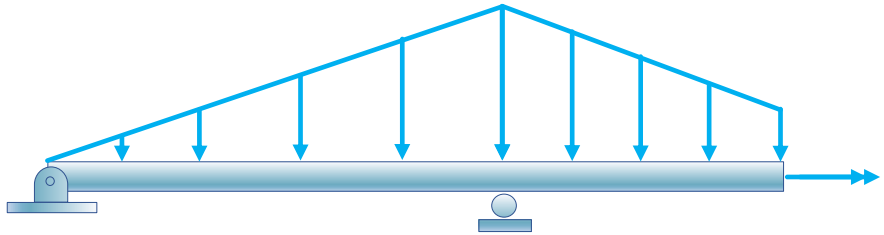
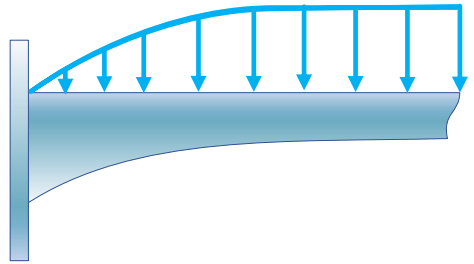
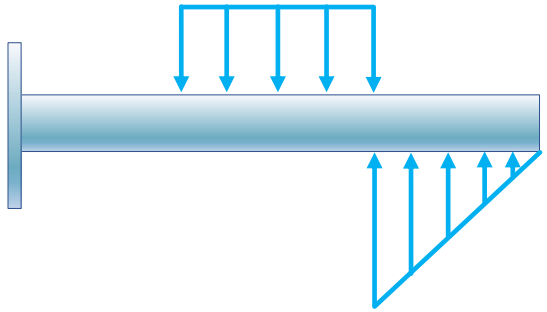
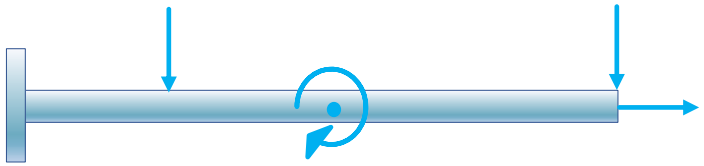
ESFORÇOS EXTERNOS

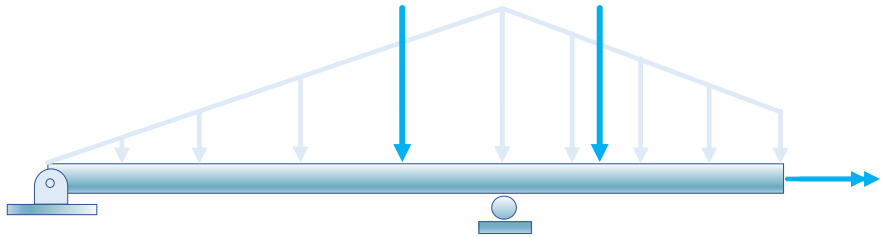


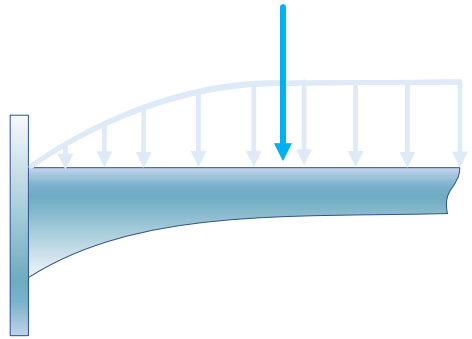


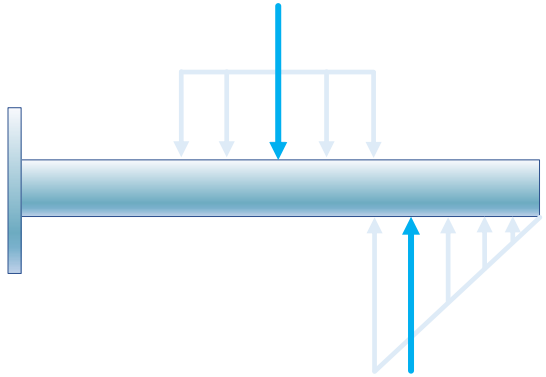


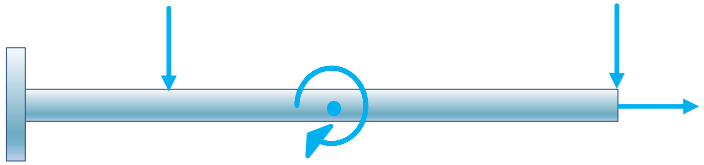


ESFORÇOS EXTERNOS

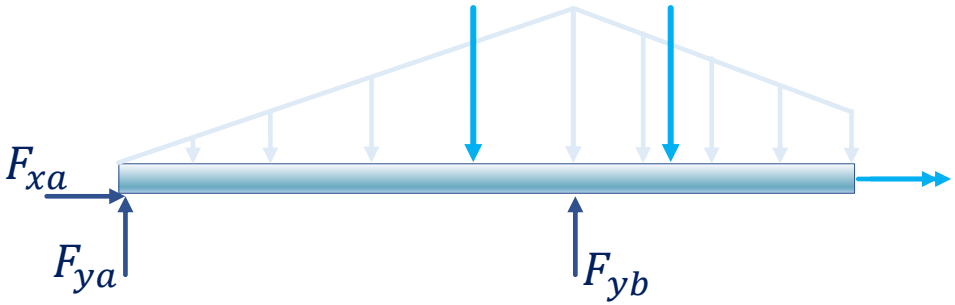


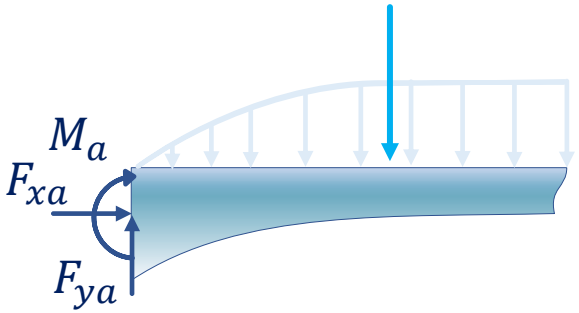


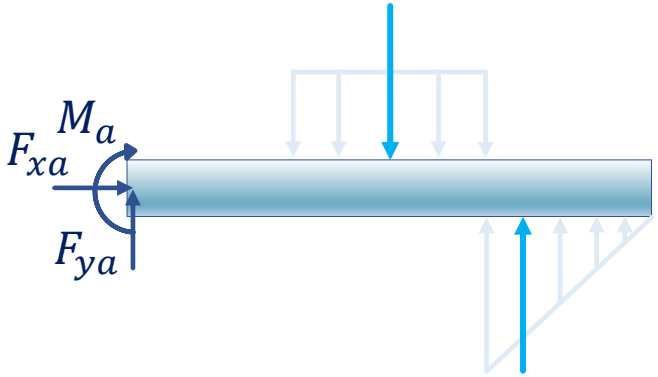


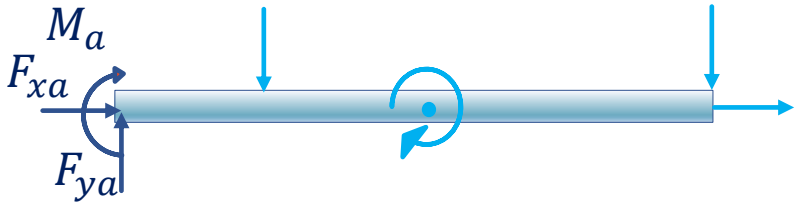


ESFORÇOS EXTERNOS

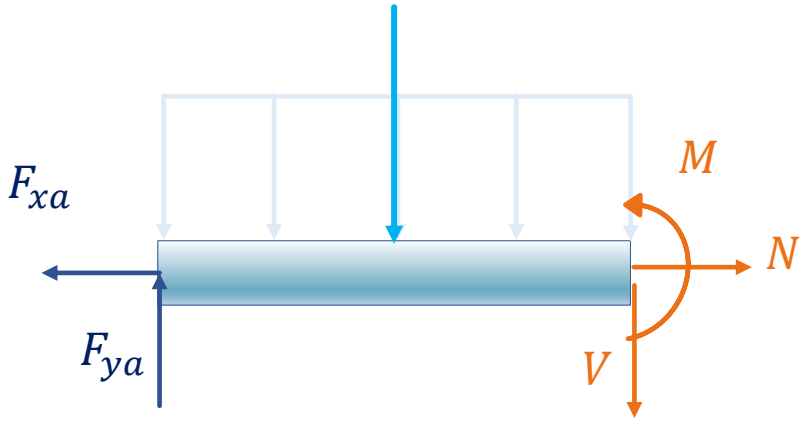




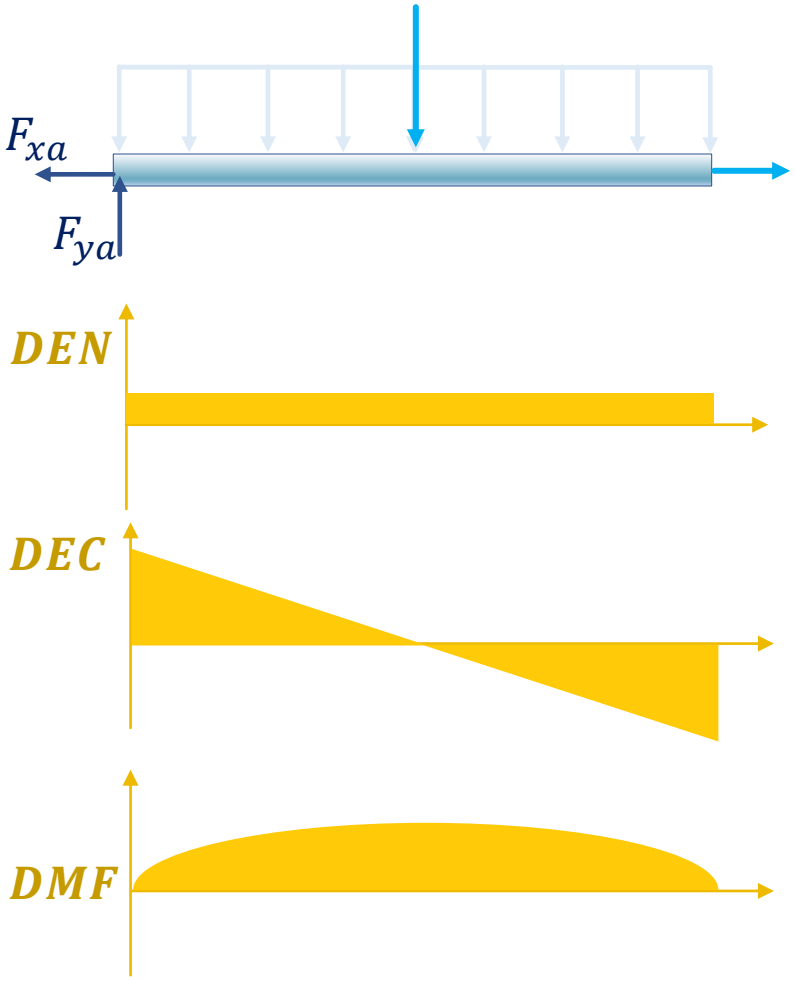




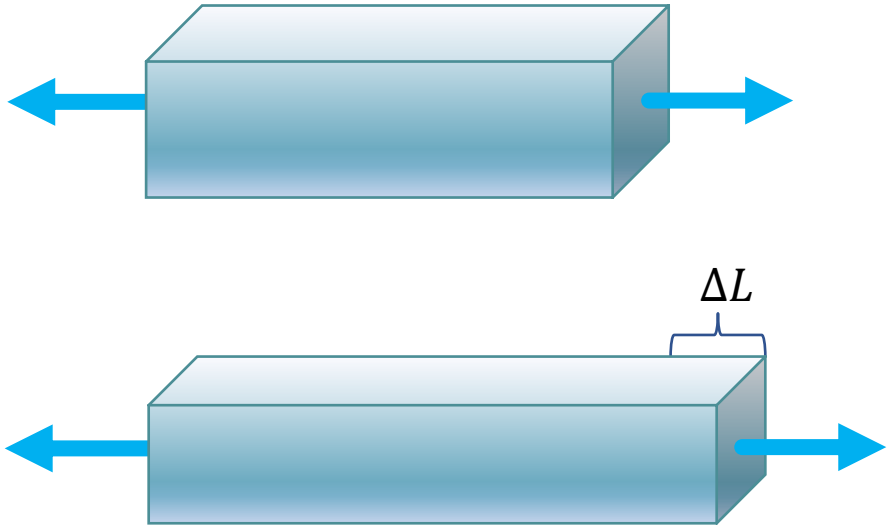
ESFORÇOS INTERNOS

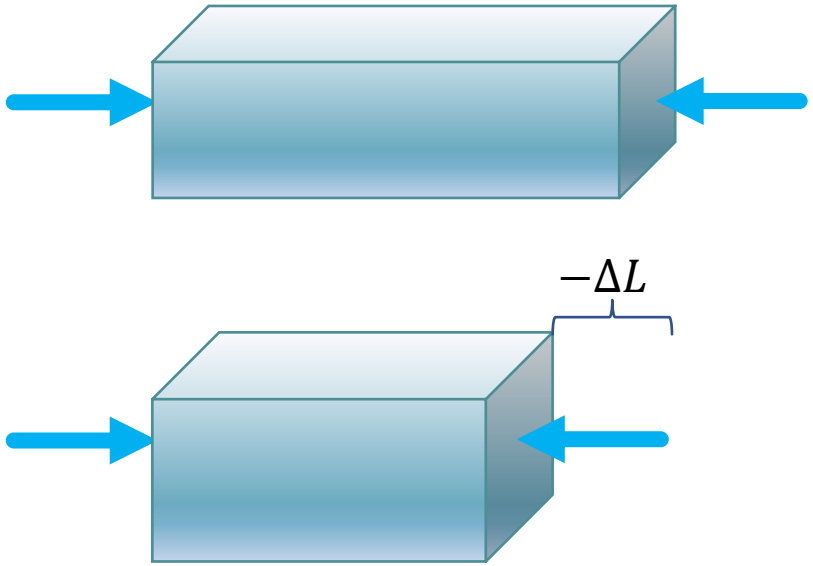


Convenção

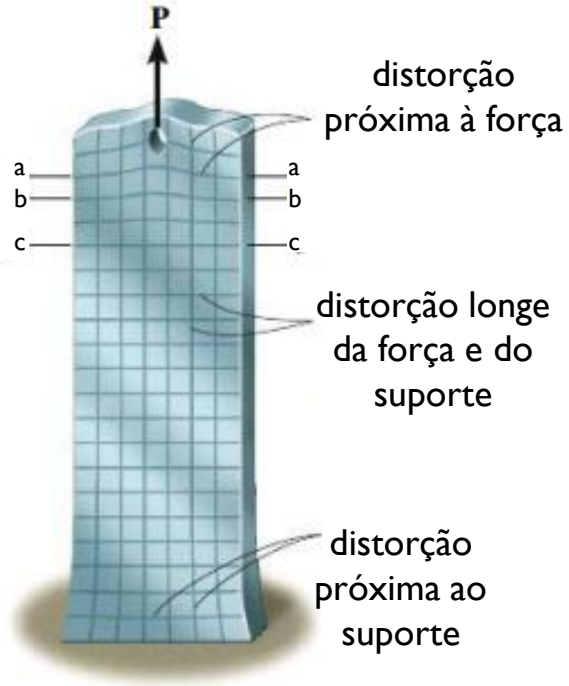


ALONGAMENTO





TENSÃO NORMAL



Seção a — a

Seção b — b

Seção c — c

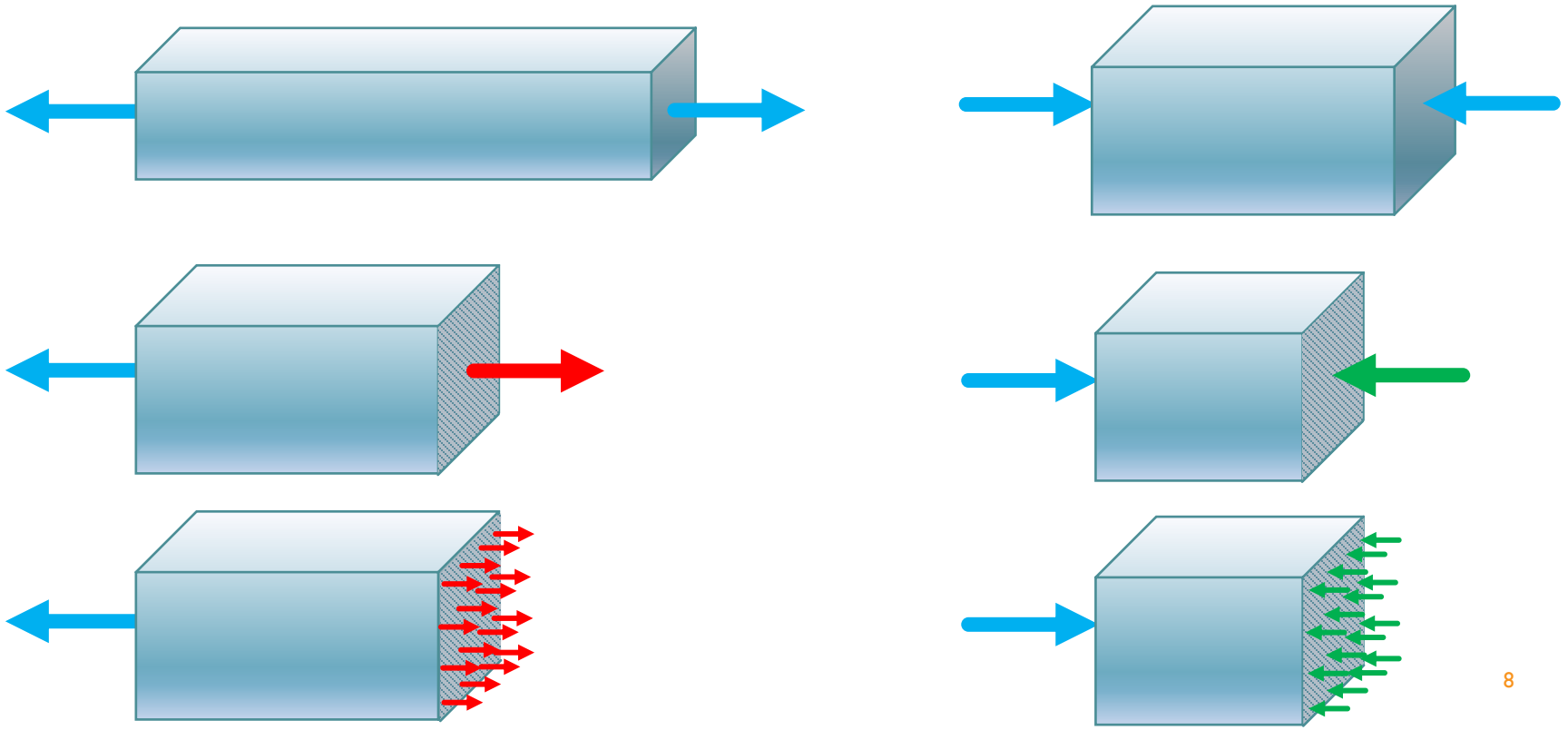
TENSÃO NORMAL

Tensão Normal

$$\sigma = \frac{N(x)}{A}$$

Unidade

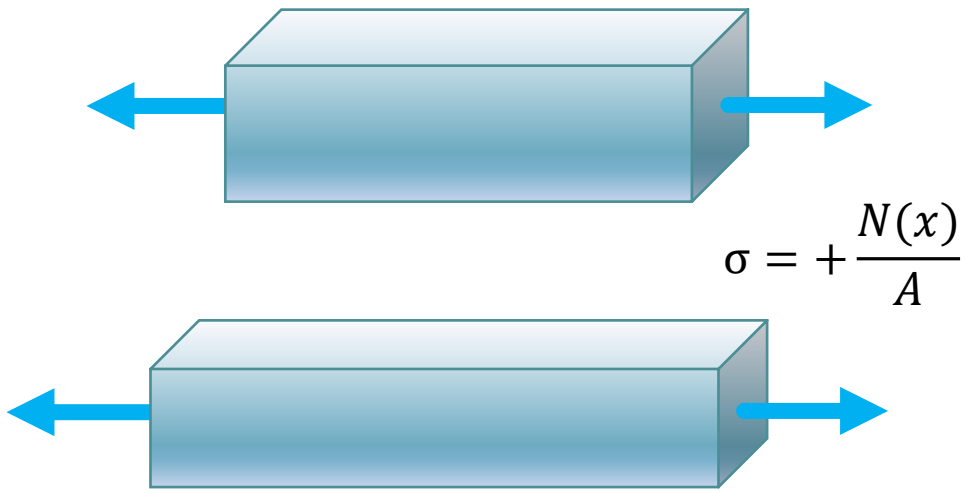
$$\frac{N}{m^2} = Pa$$



TENSÃO NORMAL

Define-se que um corpo está em TRAÇÃO quando as forças e tensões agem para alongá-lo.

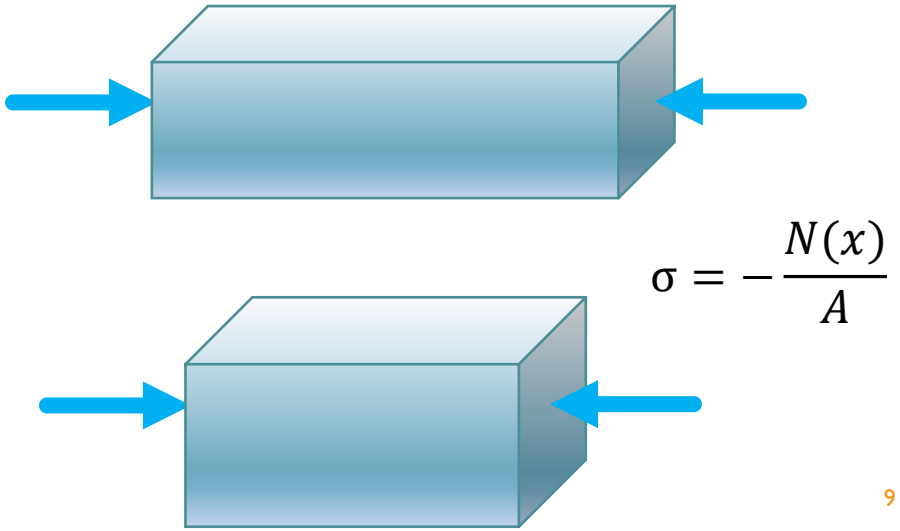
A convenção de sinais que utilizamos para obtenção dos esforços internos trata a TRAÇÃO como um PADRÃO POSITIVO



$$\sigma = +\frac{N(x)}{A}$$

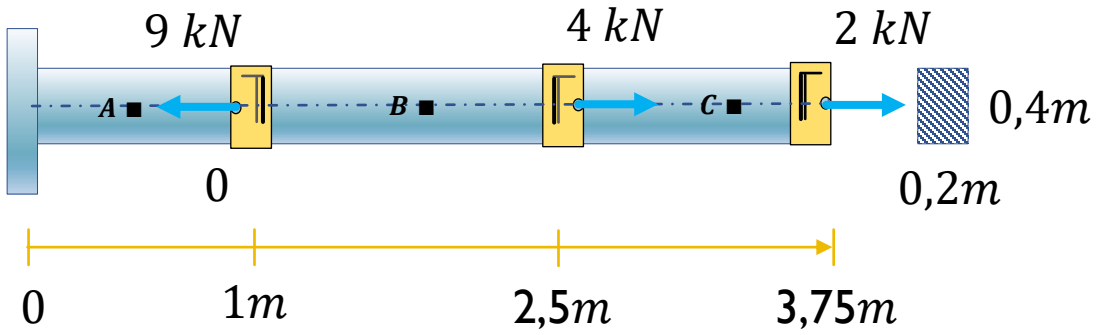
Define-se que um corpo está em COMPRESSÃO quando as forças e tensões agem para encurtá-lo.

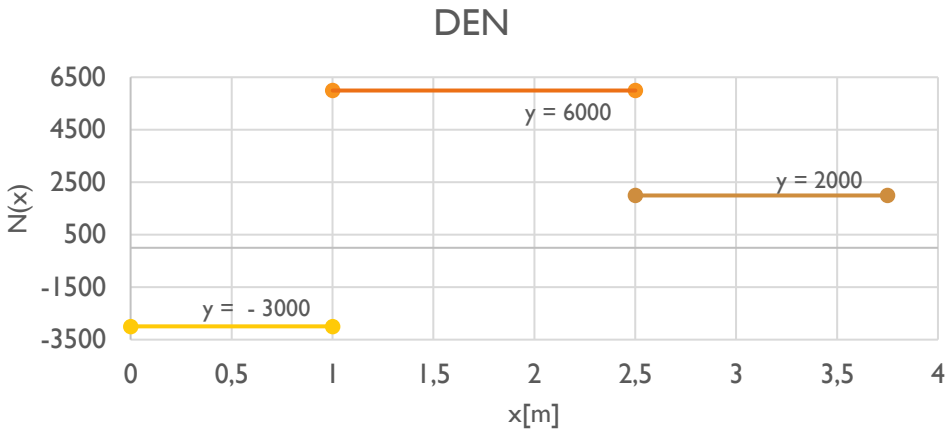
A convenção de sinais que utilizamos para obtenção dos esforços internos trata a COMPRESSÃO como um PADRÃO NEGATIVO



$$\sigma = -\frac{N(x)}{A}$$

EXERCÍCIO 1 – CÁLCULO DAS TENSÕES NORMAIS DEVIDO A FORÇAS AXIAIS





1 - Cálculo da Área

$$A = 0{,}2 * 0{,}4 = 0{,}08\ m^2$$

1 - Cálculo das Tensões

$$\sigma_A = -\frac{3000}{0{,}08} = -37{,}5\ kPa \rightarrow Compressão$$

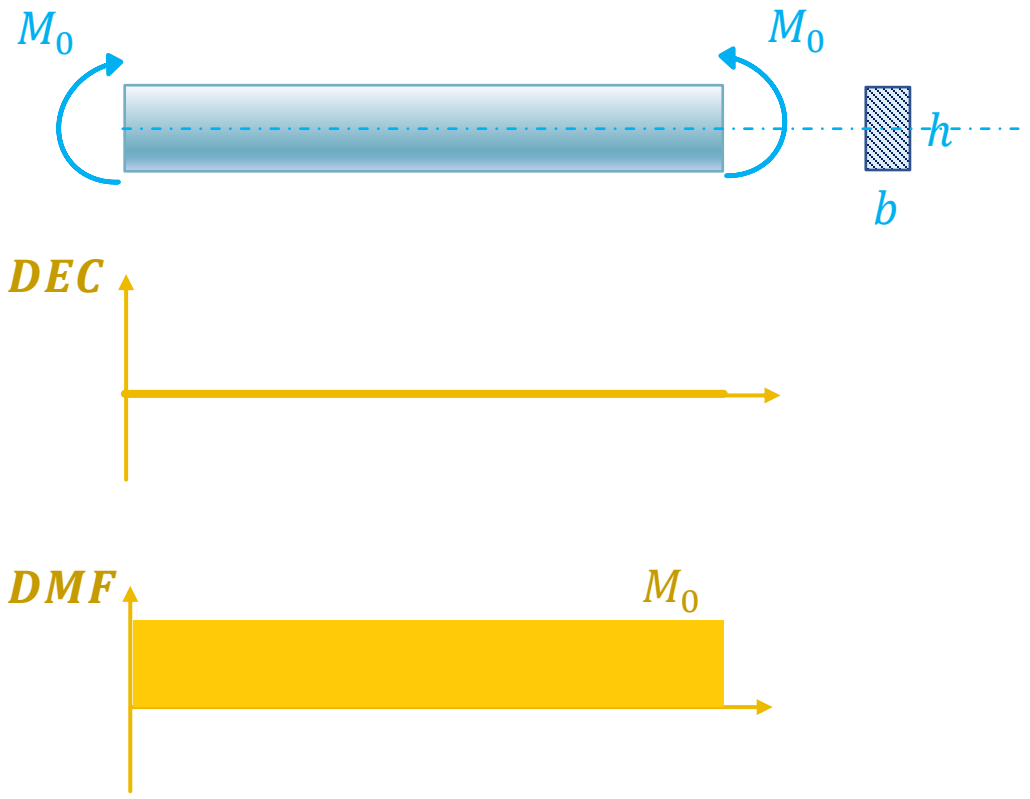
$$\sigma_B = -\frac{6000}{0{,}08} = +75\ kPa \rightarrow Tração$$

$$\sigma_C = -\frac{2000}{0{,}08} = +25\ kPa \rightarrow Tração$$

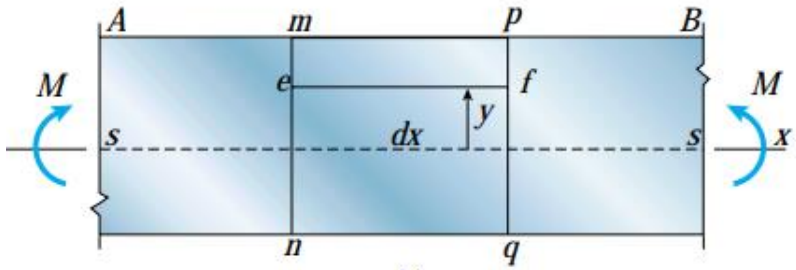
TENSÃO NORMAL NA FLEXÃO

- Esta viga está em flexão pura

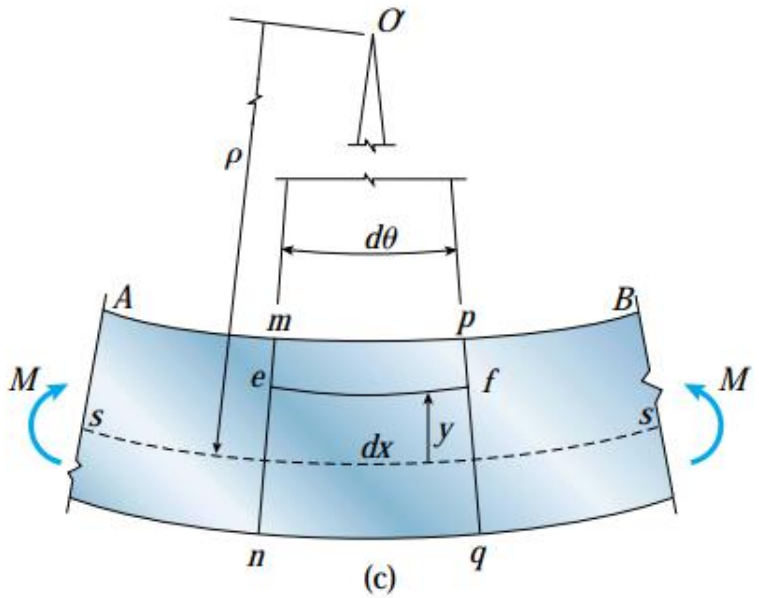
- Exclusivamente sob ação de momentos fletores.



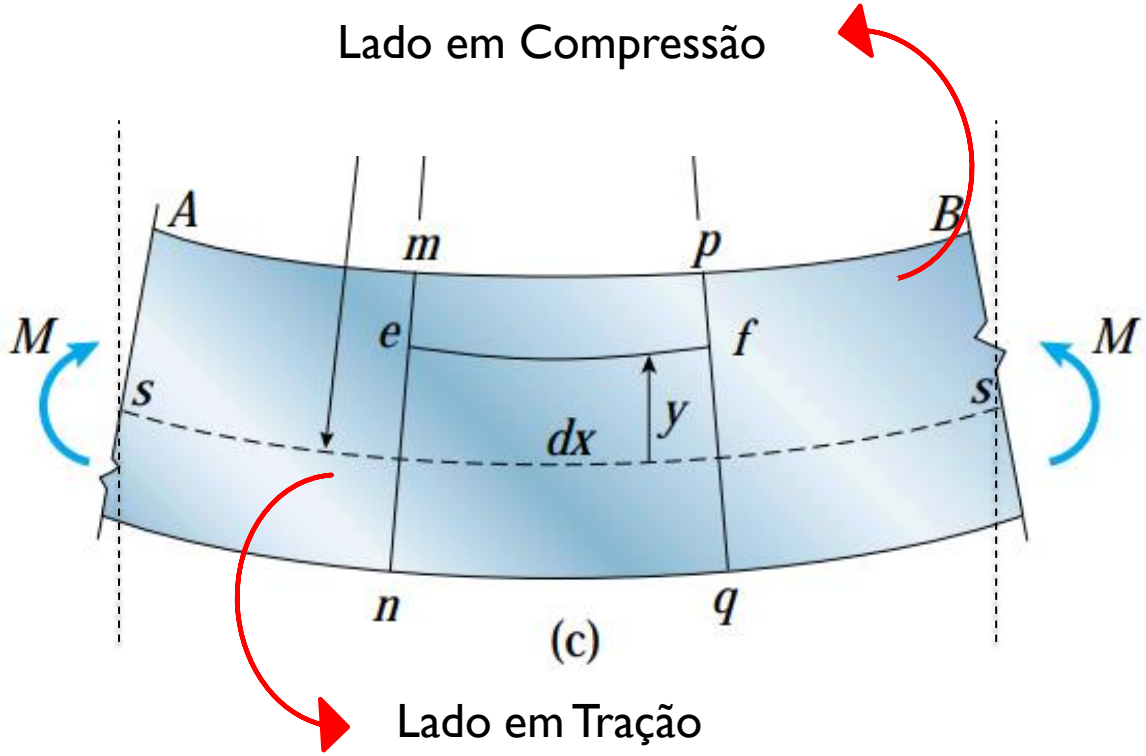
TENSÃO NORMAL NA FLEXÃO



(a)



(c)

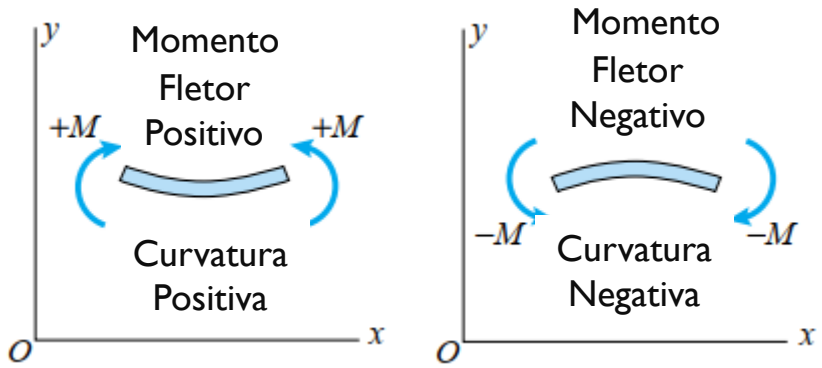


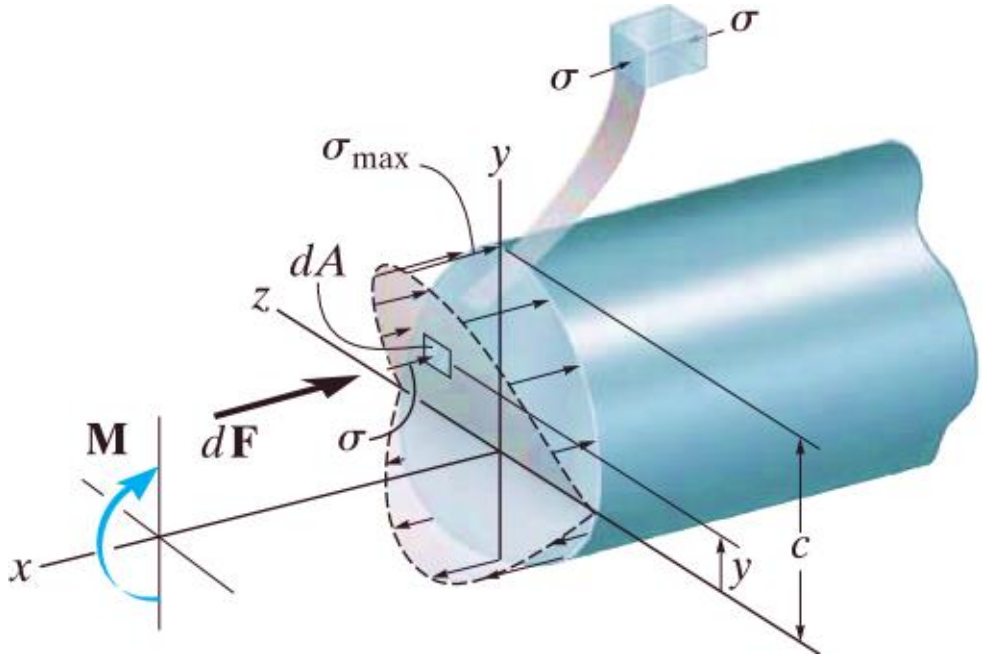
(c)

TENSÃO NORMAL NA FLEXÃO

- A Tensão Normal provocada por flexão é :

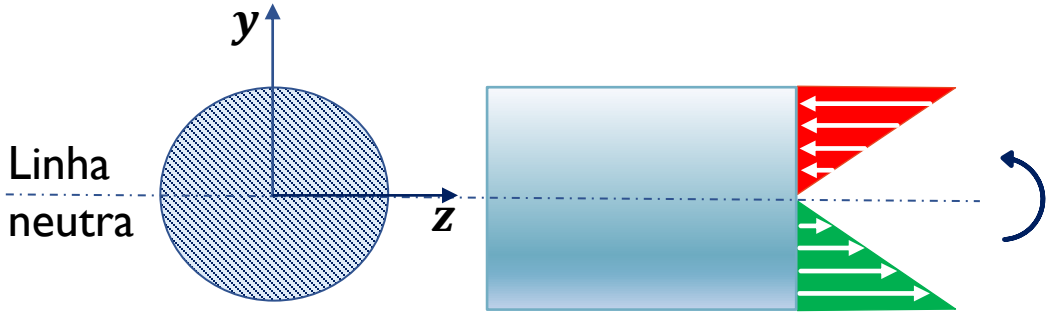
$$\sigma = -\frac{M_z * y}{I}$$

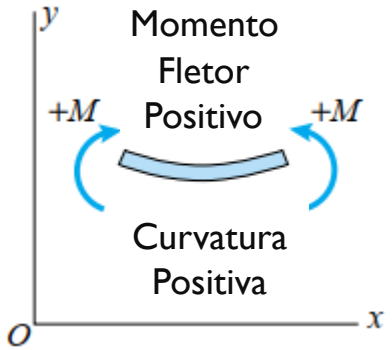


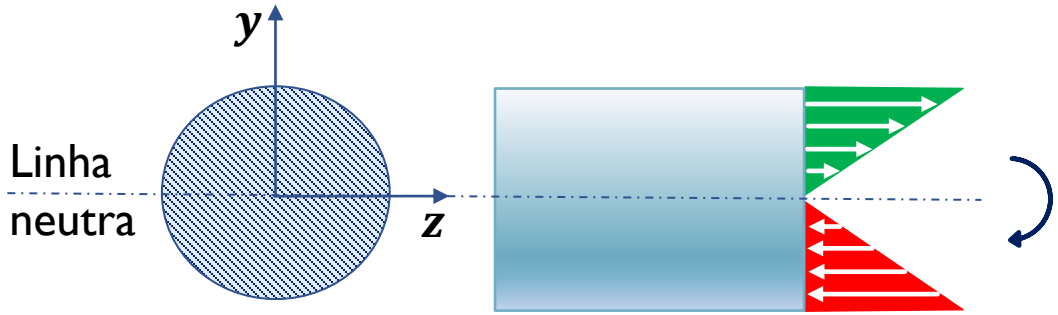


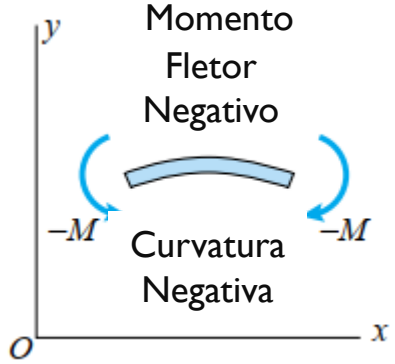
Para Flexão Pura, a Tensão Normal é nula na linha horizontal de simetria e mais intensa nos pontos máximos superiores e inferiores.

TENSÃO NORMAL NA FLEXÃO



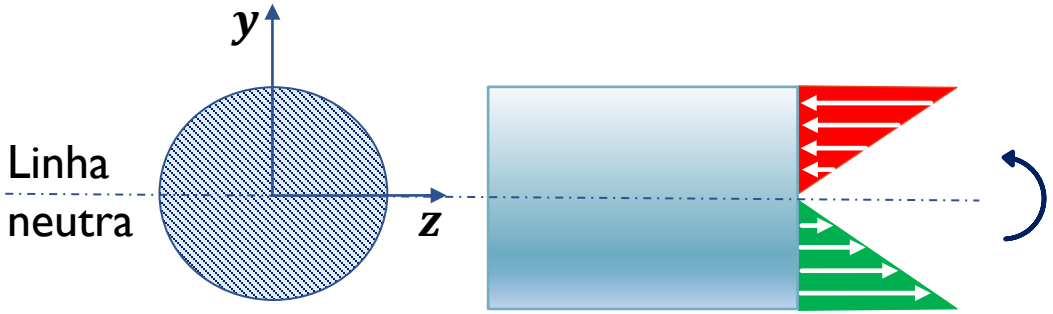


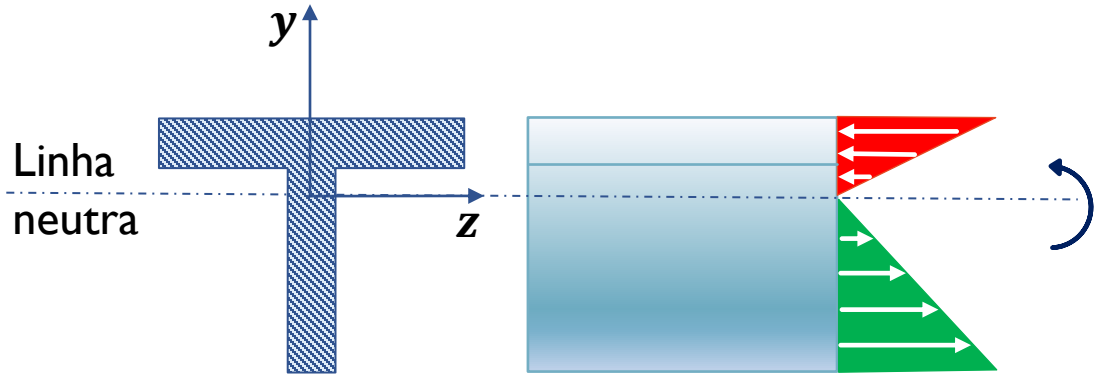


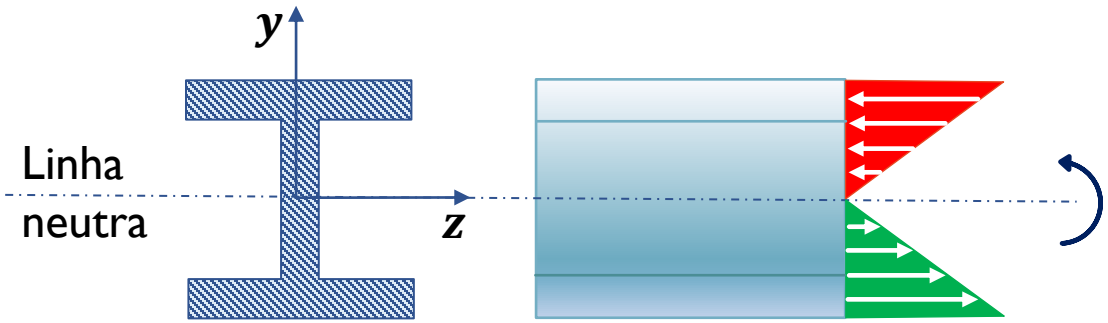


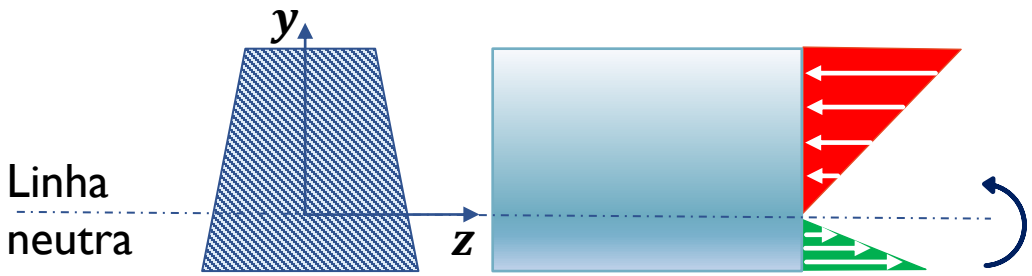
$$\sigma = -\frac{M_z * y}{I}$$

TENSÃO NORMAL NA FLEXÃO



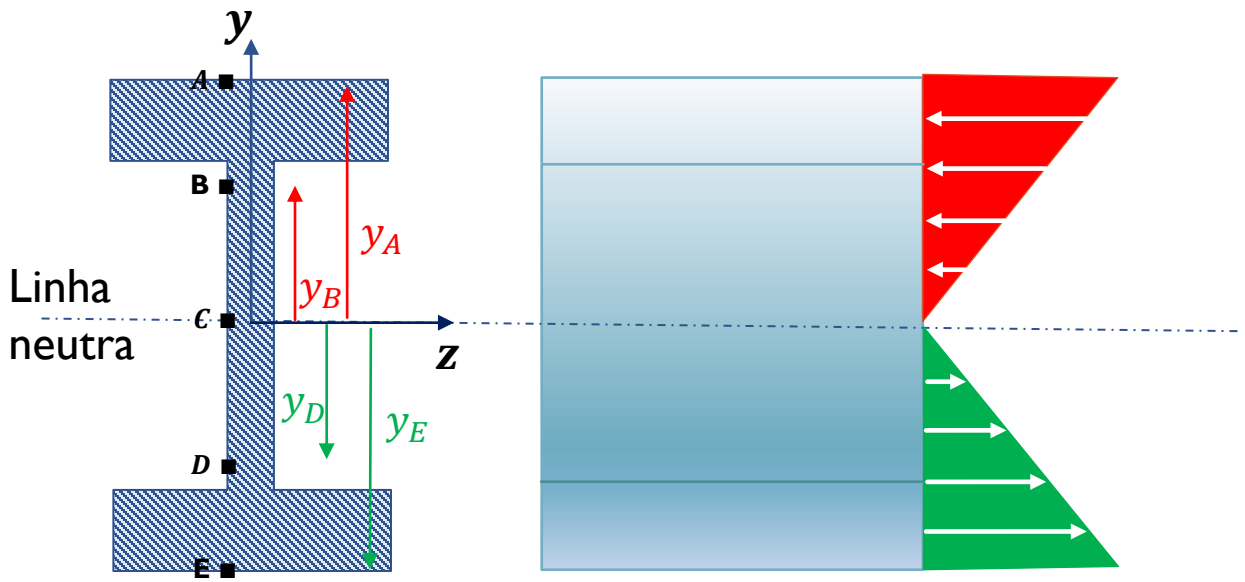




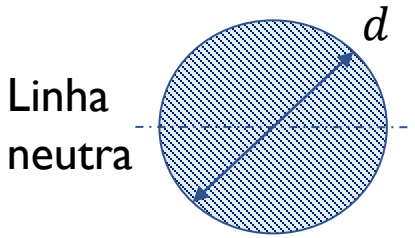


TENSÃO NORMAL NA FLEXÃO

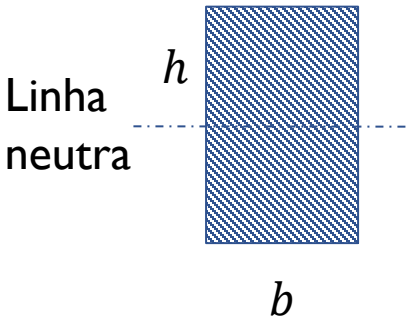
- Ponto A : $\sigma_A = \sigma_{máx}[compressão]$
- Ponto B : $\sigma_A < \sigma_B < \sigma_c$
- Ponto C : $\sigma_c = \sigma = 0$
- Ponto D : $\sigma_C < \sigma_D < \sigma_E$
- Ponto E : $\sigma_E = \sigma_{máx}[tração]$



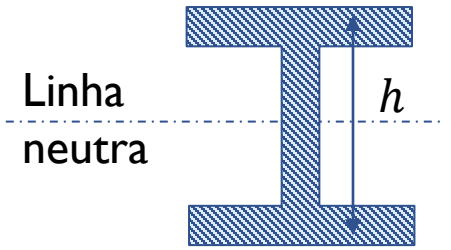
LINHA NEUTRA



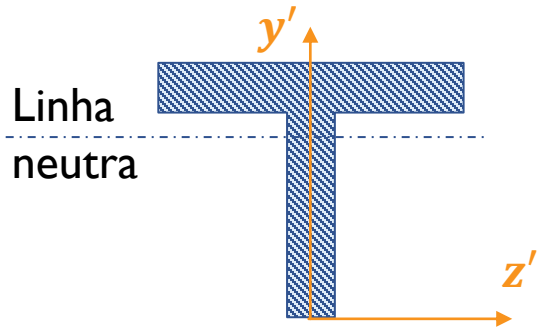
$$LN = \frac{d}{2}$$



$$LN = \frac{h}{2}$$

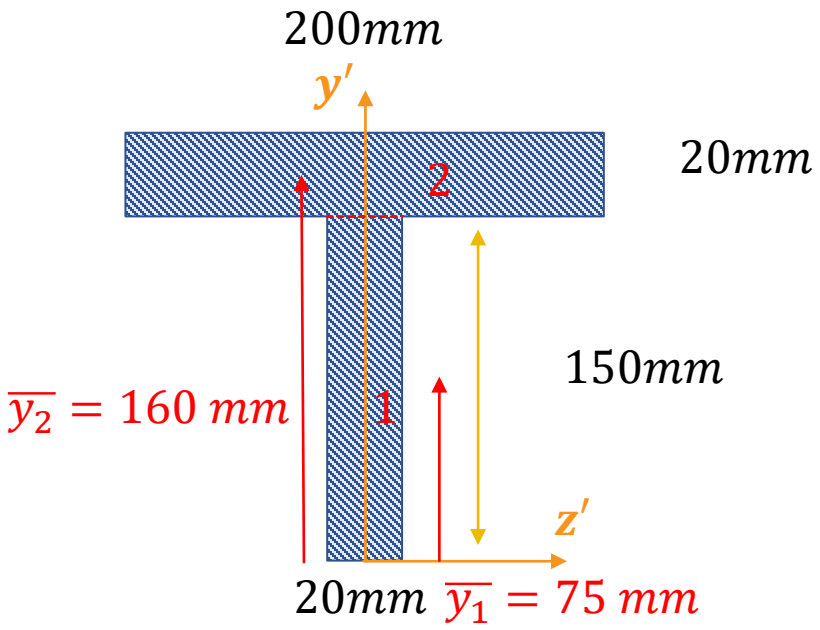


$$LN = \frac{h}{2}$$



$$LN = \bar{y} = \frac{\sum \bar{y}A}{\sum A}$$

EXERCÍCIO 2 - CÁLCULO DA LINHA NEUTRA



$$LN = \bar{y} = \frac{\sum \bar{y}A}{\sum A}$$

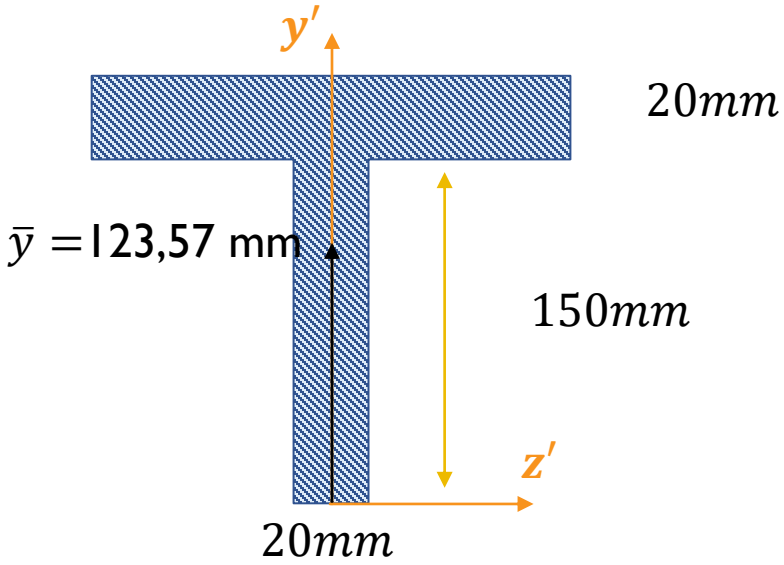
$$A_1 = 150 * 20 = 3000\ m^2$$

$$A_2 = 200 * 20 = 4000\ m^2$$

$$\bar{y} = \frac{75 * 3000 + 160 * 4000}{7000}$$

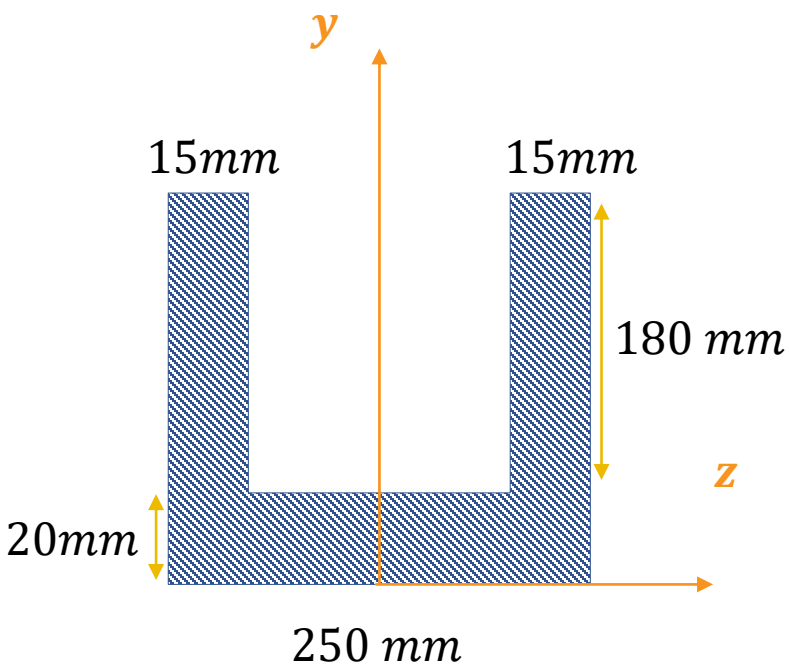
$$\bar{y} = 123{,}57 \text{ mm}$$

Resposta:

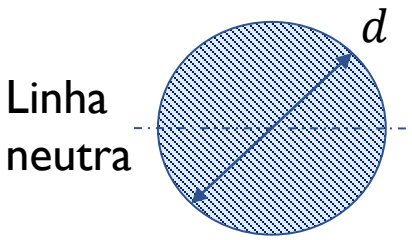


EXERCÍCIO PROPOSTO I - CÁLCULO DA LINHA NEUTRA

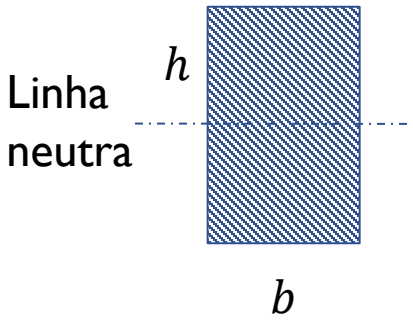
$$LN = \bar{y} = \frac{\sum \bar{y} A}{\sum A}$$



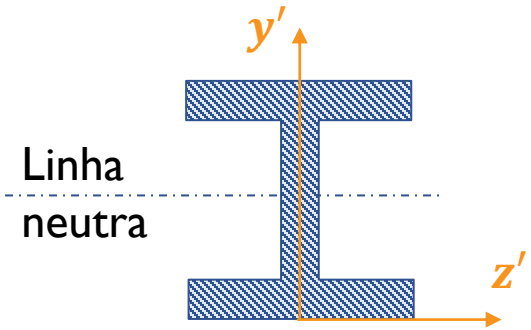
MOMENTO DE INÉRCIA INERCIA

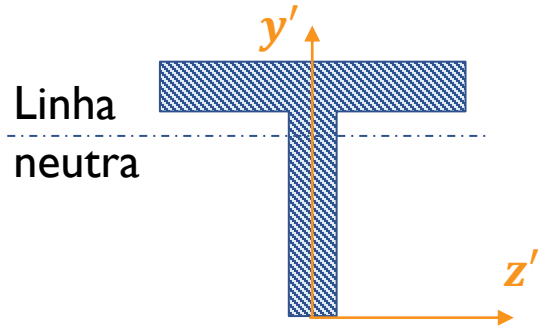


$$I = \frac{\pi d^4}{64}$$



$$I = \frac{bh^3}{12}$$

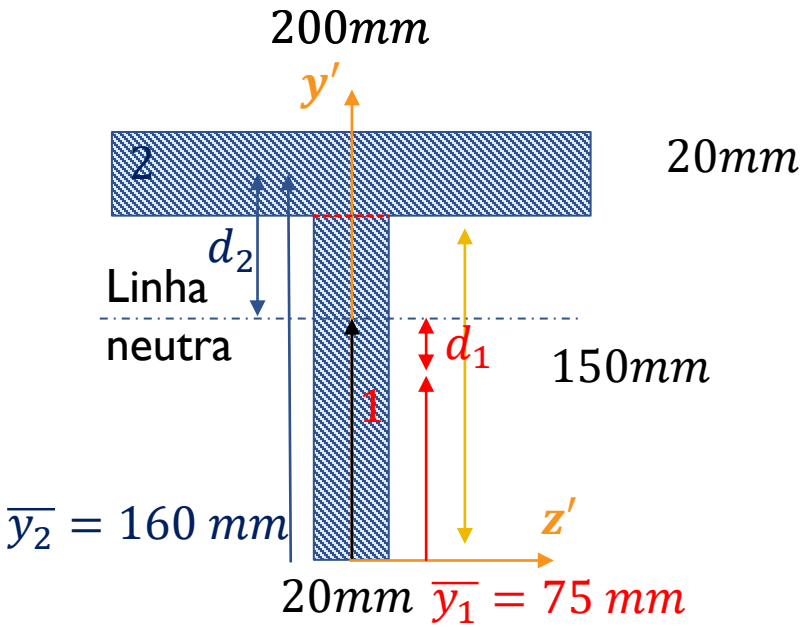




Teorema dos Eixos Paralelos

$$I = \sum (I + Ad^2)$$

EXERCÍCIO 3 - CÁLCULO INÉRCIA
TEOREMA DOS EIXOS PARALELOS



$LN = \bar{y} =$123,57 mm

$$I = \sum (I + Ad^2)$$

$$A_1 = 20 * 150 = 3000\ mm^2 \qquad A_2 = 20 * 200 = 4000\ mm^2$$

$$I_1 = \frac{20 * 150^3}{12} \qquad I_2 = \frac{200 * 20^3}{12}$$

$$I = \left[\frac{20*150^3}{12} + 20 * 150 * (123{,}57 - 75)^2\right] + \left[\frac{200*20^3}{12} + 20 * 200 * (160 - 123{,}57)^2\right]$$

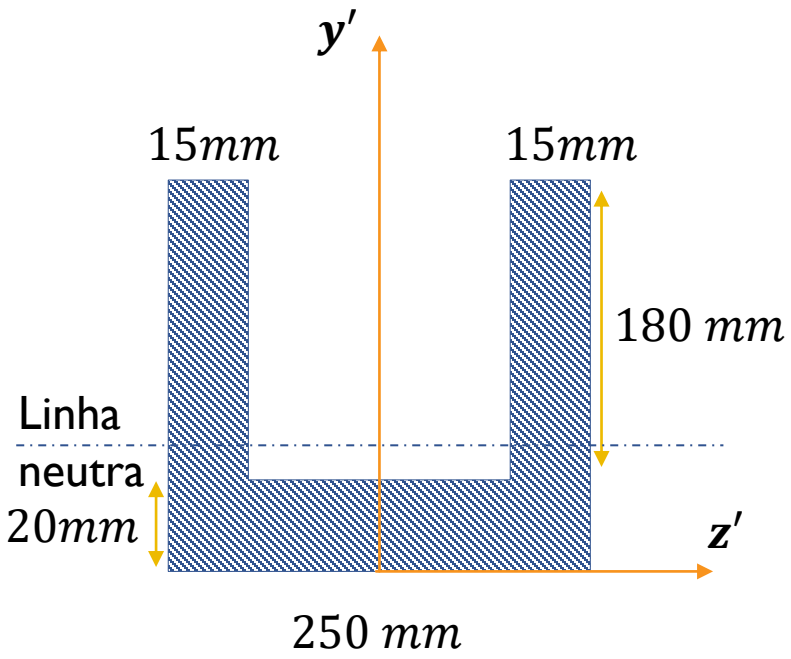
$$I = 1{,}814 * 10^7 mm^4$$

$d_1 = (123{,}57 - 75)$

$d_2 = (160 - 123{,}57)$

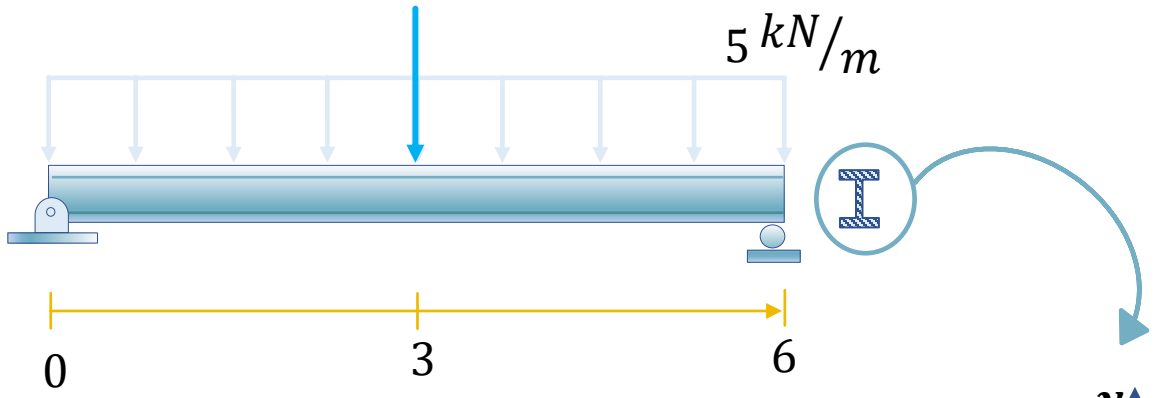
EXERCÍCIO PROPOSTO 2 - CÁLCULO INÉRCIA TEOREMA DOS EIXOS PARALELOS

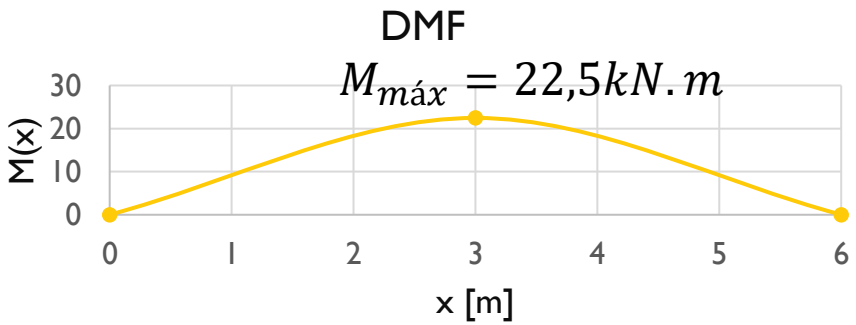
$$I = \sum (I + Ad^2)$$

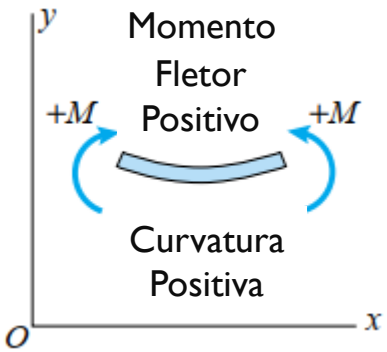


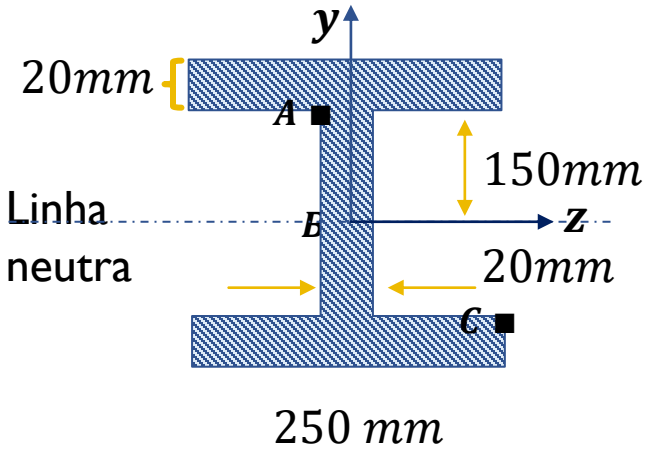
EXERCÍCIO 3 – CÁLCULO DE TENSÃO NORMAL NA FLEXÃO

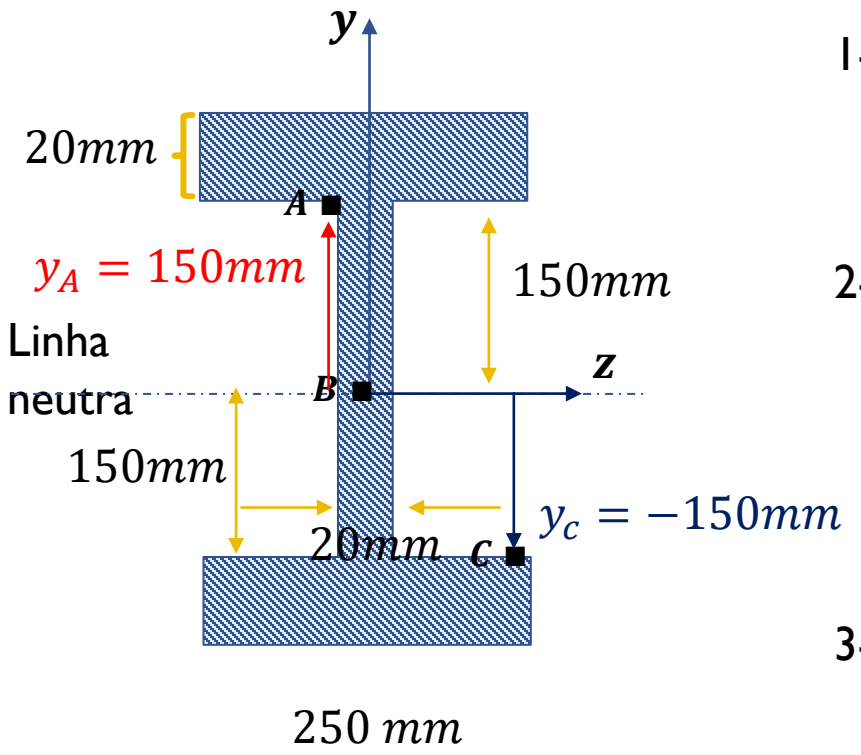
Determinar as tensões máximas











1- Inércia

$$I = 301{,}3 * 10^6 m^4$$

2- Tensão no ponto A

$$\sigma_A = -\frac{22500 * 0{,}15}{301{,}3 * 10^6}$$

$$\sigma_A = -11{,}2\ Mpa \rightarrow Compressão$$

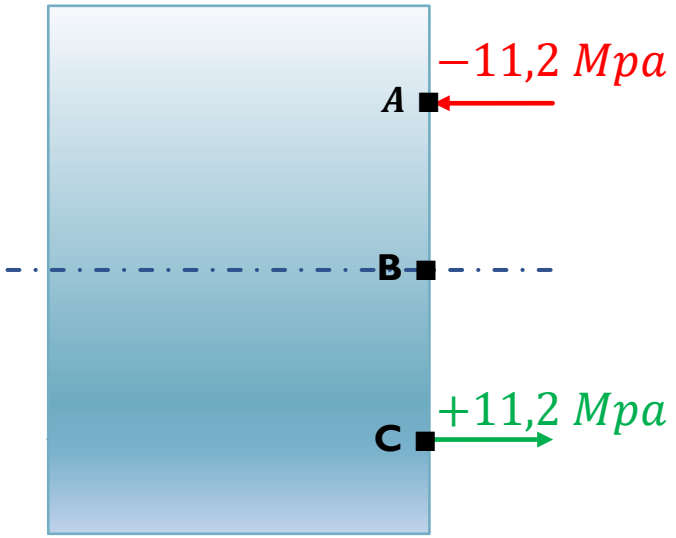
3- Tensão no ponto B

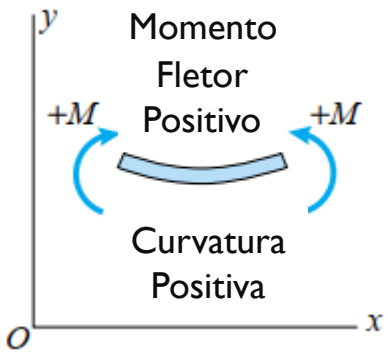
$$\sigma_b = 0\ Mpa$$

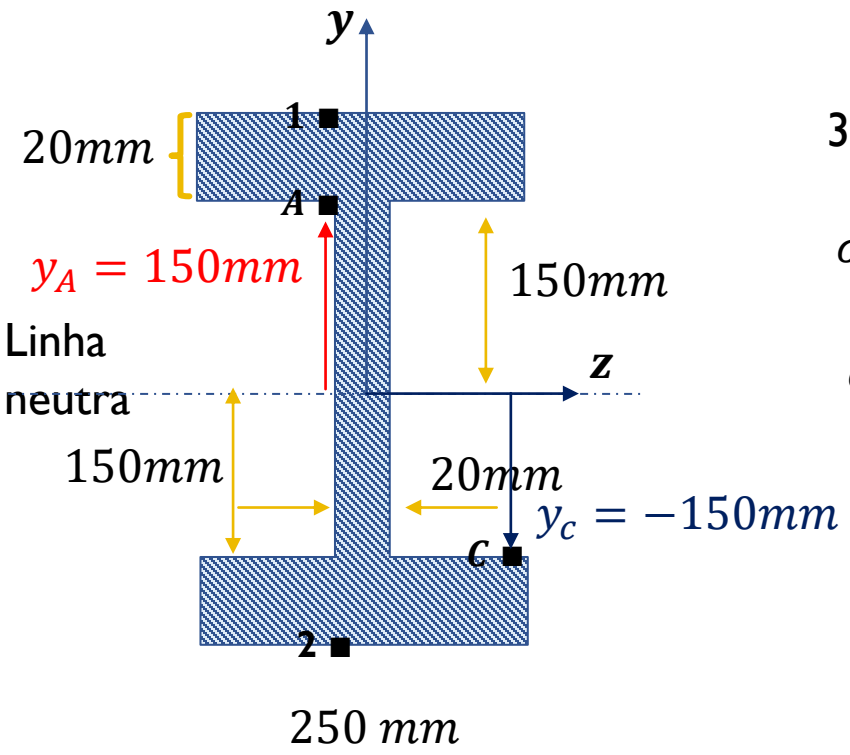
4- Tensão no ponto C

$$\sigma_C = -\frac{22500 * (-0{,}15)}{301{,}3 * 10^6}$$

$$\sigma_C = +11{,}2\ Mpa \rightarrow Tração$$







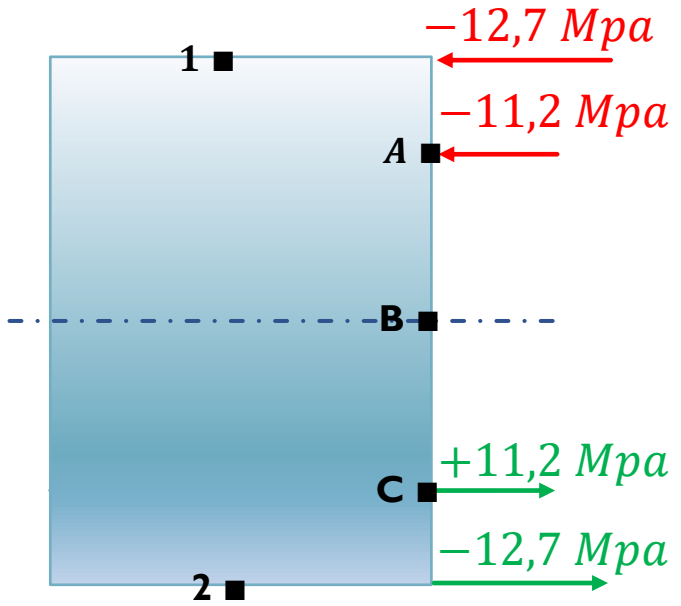
3- Tensão nas extremidades

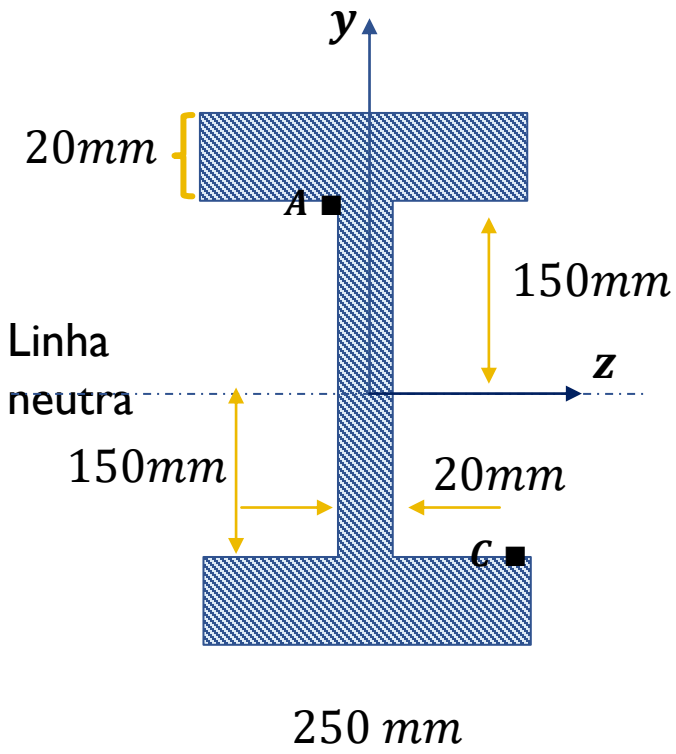
$$\sigma_{1Máx} = -\frac{22500 * 0{,}17}{301{,}3 * 10^6}$$

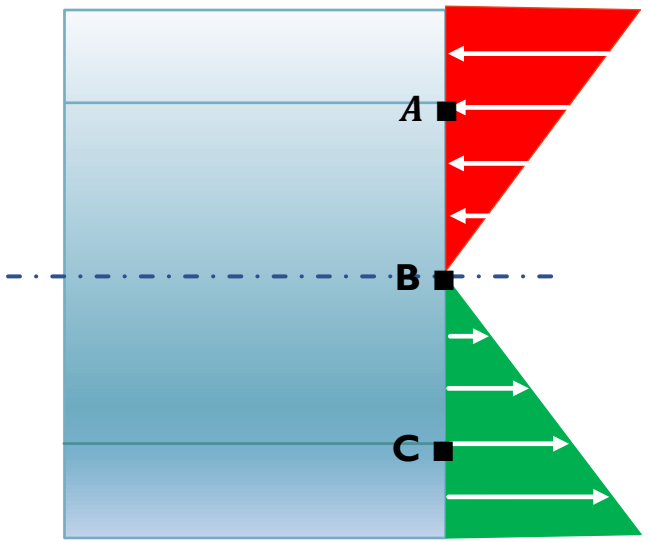
$$\sigma_{1Máx} = -12{,}7\ Mpa \rightarrow Compressão$$

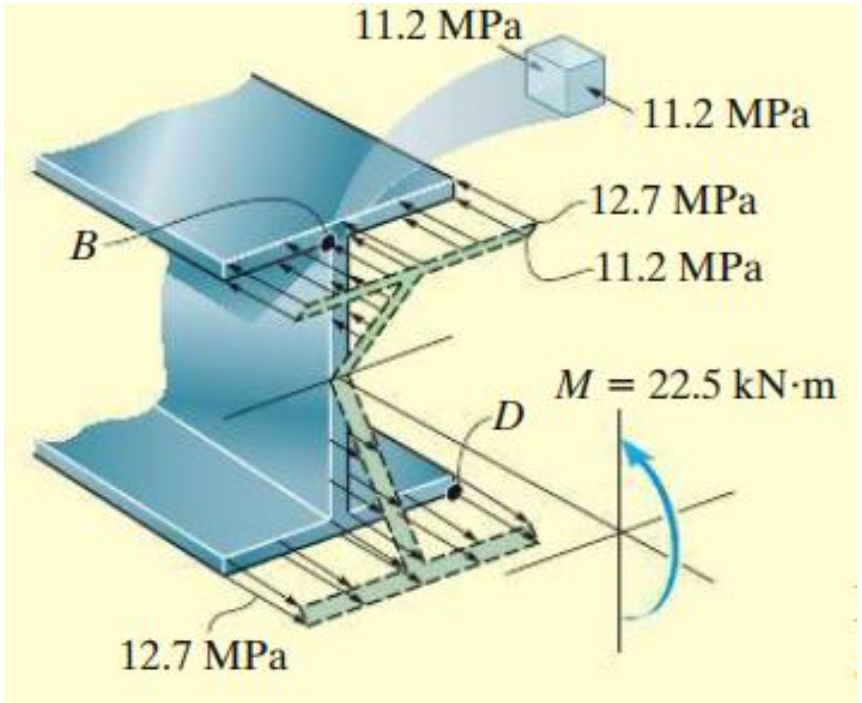
$$\sigma_{2\ Máx} = -\frac{22500 * (-0{,}17)}{301{,}3 * 10^6}$$

$$\sigma_{2\ Máx} = +12{,}7\ Mpa \rightarrow Tração$$




y
20mm
A
150mm
Linha
neutra
z
150mm
20mm
C
250 mm

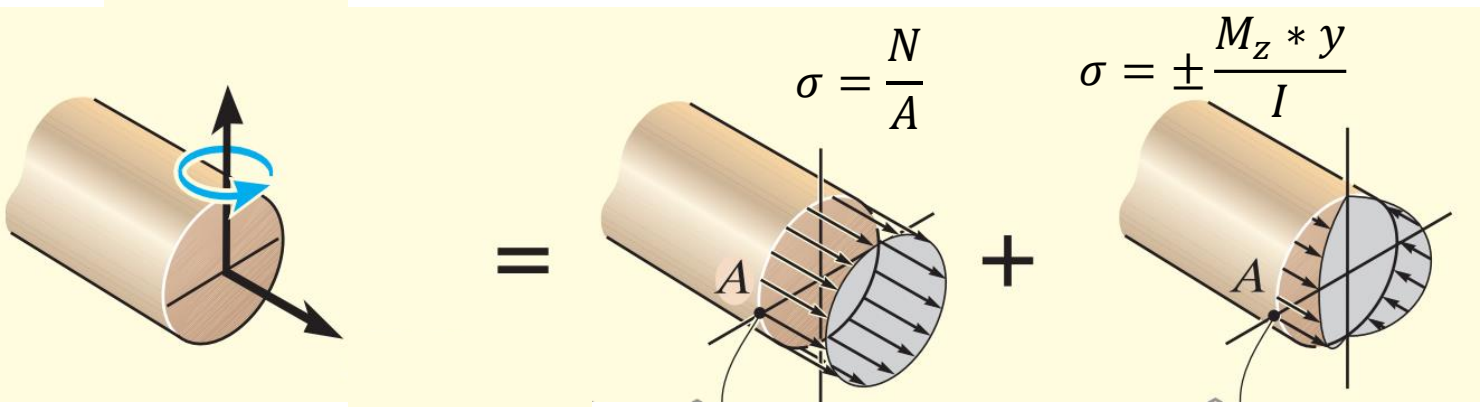

A
B
C


11.2 MPa
11.2 MPa
12.7 MPa
B
11.2 MPa
M = 22.5 kN·m
D
12.7 MPa

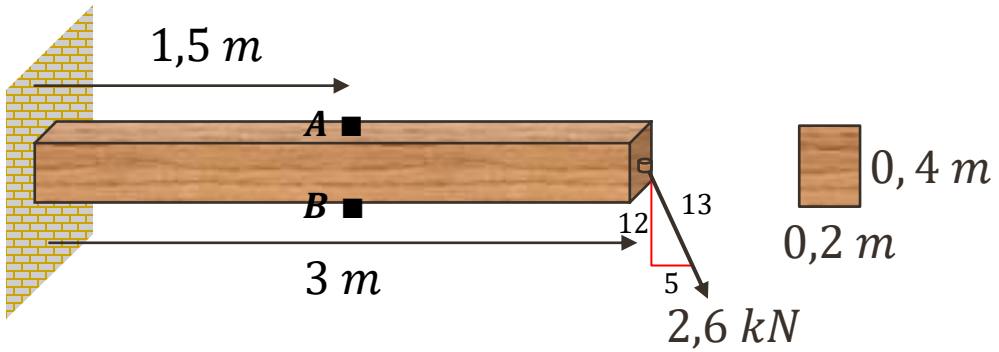
SUPERPOSIÇÃO DA TENSÃO NORMAL

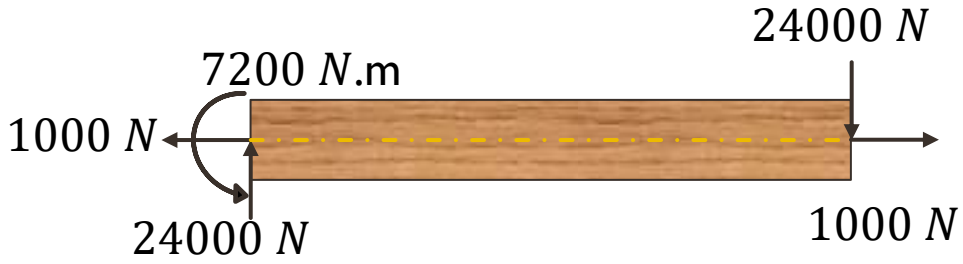
- É possível associar as tensões normais causadas por momentos fletores e por forças normais por meio de uma simples soma:

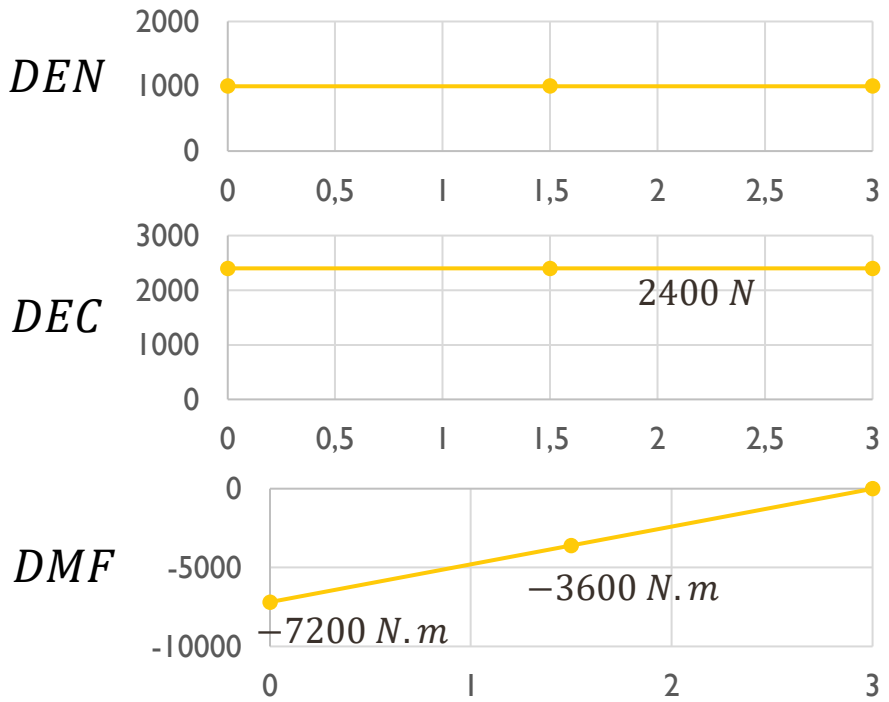
$$\sigma = -\frac{M_z * y}{I} + \frac{N}{A}$$

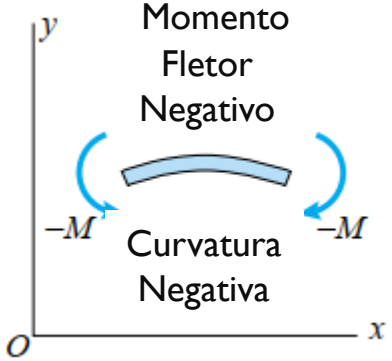


EXERCÍCIO 4 – CÁLCULO DE TENSÃO NORMAL COMBINADA







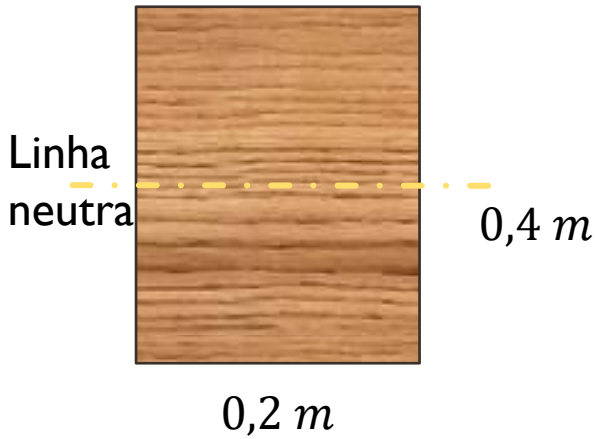


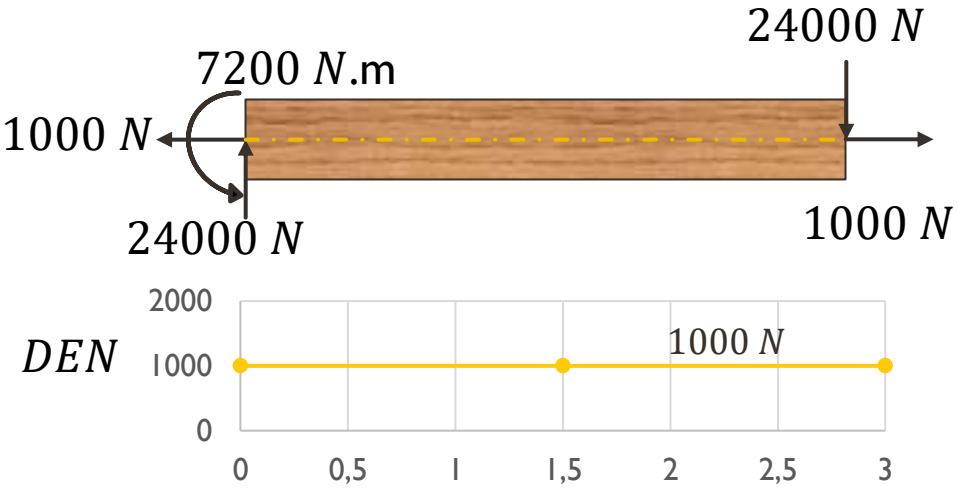
I- Área e Inércia

$$A = 0{,}2 * 0{,}4 = 0{,}08\ m^2$$

$$I = \frac{0{,}2 * 0{,}4^3}{12}$$

$$I = 1{,}066 * 10^{-3}\ m^2$$



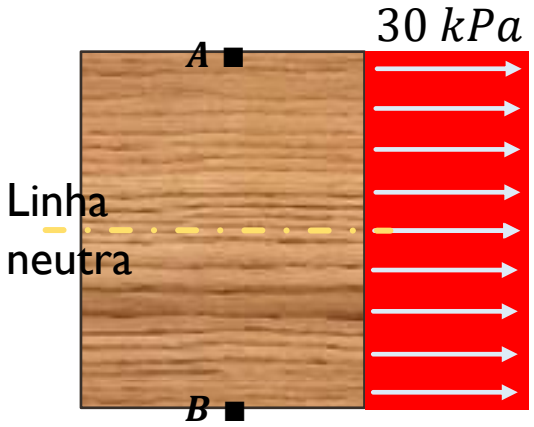


I- Tensão Normal relativa a $N(1{,}5)$

$$\sigma = \frac{N(x)}{A}$$

$$\sigma_A = \frac{1000}{0{,}08} = +30\ kPa \rightarrow Tração$$

$$\sigma_B = \frac{1000}{0{,}08} = +30\ kPa \rightarrow Tração$$



3 – Tensão Normal da Flexão

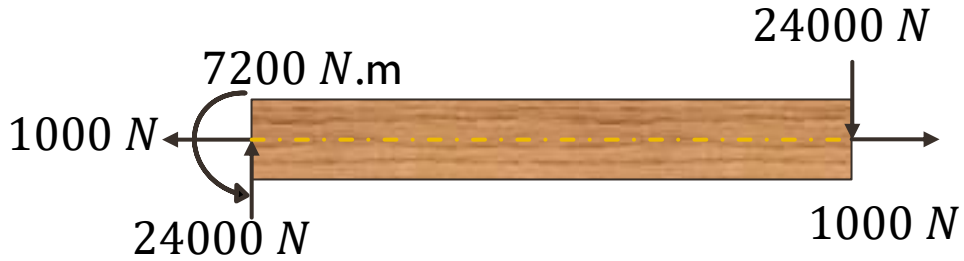
$$\sigma = -\frac{M_z y}{I}$$

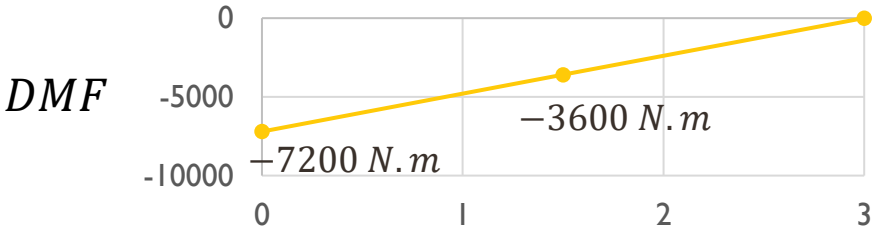
$$\sigma_A = -\frac{(-3600) * 0{,}2}{1{,}066 * 10^{-3}}$$

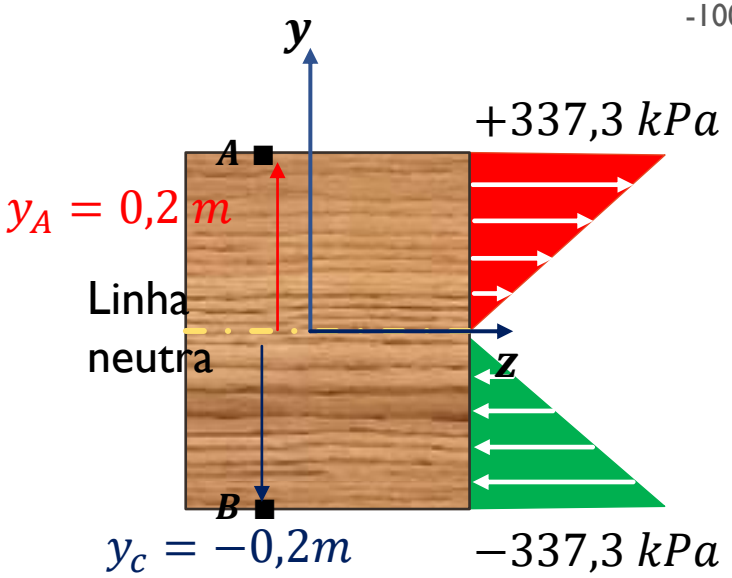
$$\sigma_A = +337{,}3\ kPa \rightarrow Tração$$

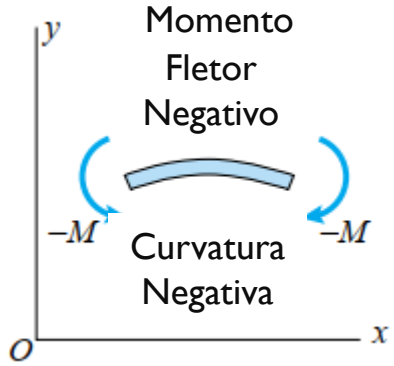
$$\sigma_B = -\frac{(-3600) * (-0{,}2)}{1{,}066 * 10^{-3}}$$

$$\sigma_B = -337{,}3\ kPa \rightarrow Compressão$$

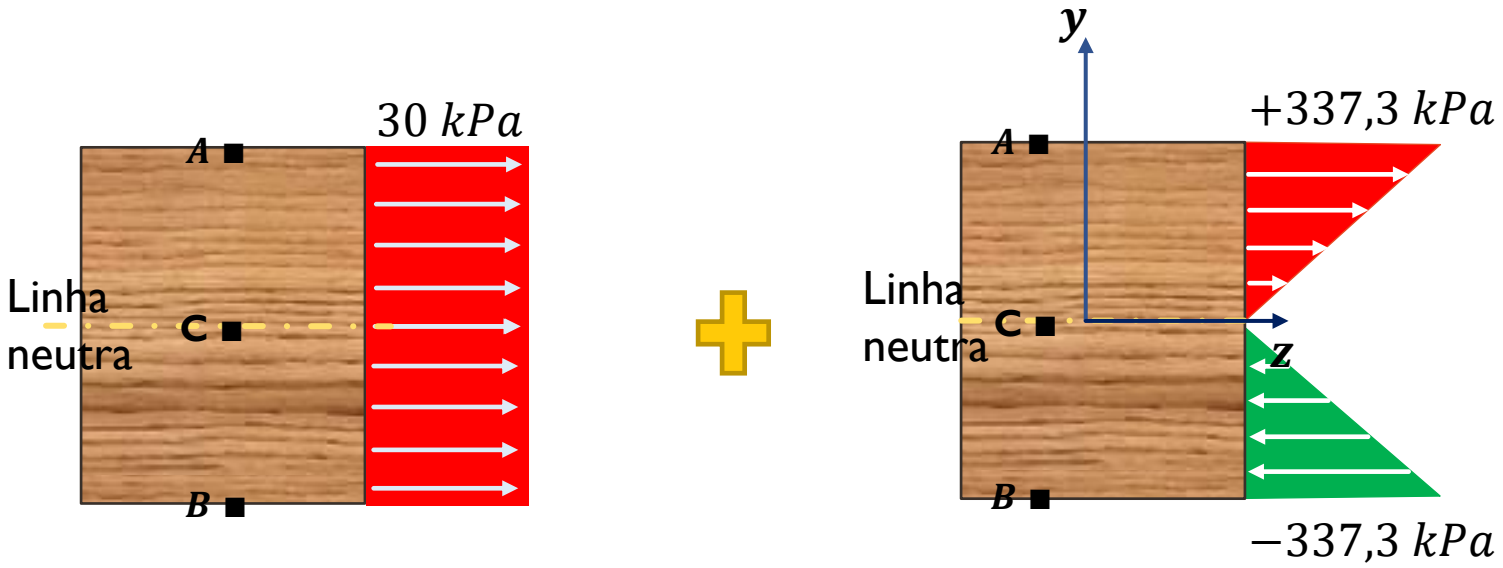








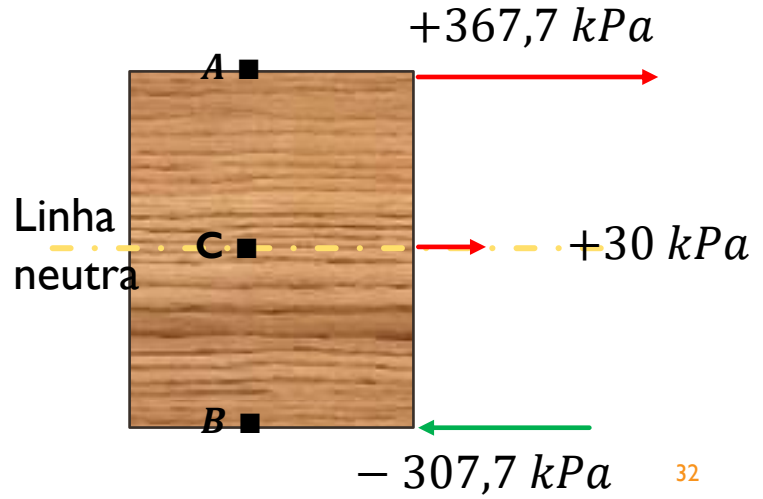
4 - Superposição



$$\sigma_A = 30 + 337{,}3 = +367{,}7\ kPa \rightarrow Tração$$

$$\sigma_B = 30 - 337{,}3 = -307{,}7\ kPa \rightarrow Compressão$$

$$\sigma_C = 30 + 0 = +30\ kPa \rightarrow Tração$$



REFERÊNCIAS

- GERE, J. M. Mecânica dos materiais. Tradução da: 7. edição americana São Paulo, SP: Cengage Learning, 2011. E-BOOK.
- HIBBELER, R. C., Resistência de materiais. Prentice Hall, 2010.
- SCHIEL, F. - Introdução à resistência dos materiais, apostila, vol.1, Escola de Engenharia de São Carlos, depto de publicações.
- COELHO, E.; MORI, D. e outros - Exercícios propostos de resistência dos materiais - Escola de Engenharia de São Carlos, depto de publicações.
- NASH, W. - Resistência dos materiais, coleção SCHAUM, Ed. Mc Graw Hill.
- BEER, Ferdinand - Resistência dos materiais, Ed. Mc Graw Hill.
- TIMOSHENKO, S. - Resistência dos Materiais, Ed. livros técnicos e científicos, vol.1.
- WILLEM, N.; EASLEY, J.; ROLFE, S. - Resistência dos materiais, Ed. Mc Graw Hill.