# A Simple Model of Radiation Transfer

We have seen that several gases in the atmosphere absorb strongly in the infra-red.

Each gas has a complex absorption pattern made up of many different individual vibrational and rotational transitions.  The way these different absorptions interact is not straightforward and should be accounted for in a detailed description of radiative transfer through the atmosphere.

However, we can provide a general picture of the processes taking place in the atmosphere by deriving a simple model of radiative transfer based on an atmosphere that is transparent to incoming shortwave radiation and includes only one trace gas that absorbs uniformily at all infra-red wavelengths.

This model atmosphere is known as a *Grey atmosphere*.

The model is further simplified by neglecting scattering and assuming that radiation is either emitted or absorbed only in the vertical direction.

We also assume that each level of the atmosphere is in local thermodynamic equilibrium.

Firstly, we need to describe the absorption of light by an absorbing species in the atmosphere. The intensity of light of wavelength $\lambda$, $I(\lambda)$, which passes through a depth dz of an absorber with number concentration, n, is reduced by an amount $dI(\lambda)$ given by:

$$\mathrm{dI}(\lambda) = -\mathrm{I}(\lambda)\mathrm{n}\sigma(\lambda)\mathrm{dz} = I(\lambda)d\chi$$

where $\sigma(\lambda)$ is the absorption cross section at wavelength $\lambda$ and is constant for any given species, and $\chi$ is the optical depth. We can obtain the intensity of light transmitted a distance z through the absorber, $I_z(\lambda)$, by integrating the above:

$$\mathrm{I_z}(\lambda) = \mathrm{I_0}(\lambda)\exp\left\{-\int_0^z \mathrm{n}\sigma(\lambda)\mathrm{dz}\right\}$$

where $I_0(\lambda)$ is the initial intensity of light of wavelength $\lambda$. In cases where the concentration of the absorber is independent of the depth of the absorbing slab the above relation becomes the *Beer-Lambert Law*:

$$\mathrm{I_z}(\lambda) = \mathrm{I_0}(\lambda)\exp\{-\mathrm{n}\sigma(\lambda)\mathrm{z}\}$$

This is not the case for a vertical slice through the atmosphere.

In our simplified model the single species absorbs uniformly over all wavelengths so we can simplify the above to give:

$$I_z = I_0 \exp\left\{ -\int_0^z n\sigma dz \right\} = I_0 \exp\left\{ \int_{\chi_0}^{0} d\chi \right\}$$

where $\chi_0$ is by convention the optical depth at the base of the atmosphere.

So far we have only considered the absorption of light. However, we know that the layer will re-emit radiation as a blackbody in a similar way so we must also include the intensity of emitted radiation, B, and assuming Kirchoff's law:

$$dI = -In\sigma dz + Bn\sigma dz = (I - B)d\chi$$

Furthermore, in any slice of the atmosphere there may be some downwelling longwave radiation arising from blackbody emission of the layers above so we should treat both the upwelling and downwelling radiative fluxes, $F^{\uparrow}$ and $F^{\downarrow}$, separately:

$$\frac{dF^{\uparrow}}{d\chi} = F^{\uparrow} - \pi B \qquad \text{and} \qquad -\frac{dF^{\downarrow}}{d\chi} = F^{\downarrow} - \pi B$$

The net flux through a layer is given by $F = F^{\uparrow} - F^{\downarrow}$, the difference between the upwelling and downwelling radiation.

As we have assumed that the atmosphere is in local thermodynamic equilibrium the flux must not change with height and is therefore constant throughout the depth of the atmosphere.

By summation and subtraction of the upward and downward fluxes we obtain:

$$\frac{dF}{d\chi} = \overline{F} - 2\pi B \qquad \text{and} \qquad \frac{d\overline{F}}{d\chi} = F$$

where $\overline{F}$ is the total flux leaving one layer. $\overline{F} = F^{\uparrow} + F^{\downarrow}$

As F is constant these expressions are easily integrated to give:

$$\overline{F} = 2\pi B \quad \text{and .} \quad \overline{F} = F\chi + c$$

The blackbody emission flux of the outermost layer of the Earth's atmosphere is given by $\pi B_0$, and this must be equal to half of the total flux from this layer, $\overline{F}$.

As there are no overlying layers to supply a contribution to F↓, $\overline{F}$ = F and so:

$$B = \frac{F}{2\pi}\chi + B_0 = \frac{F}{2\pi}(\chi + 1)$$

The blackbody emission decreases linearly with height from the surface to the top of the atmosphere and there is a constant difference between the up and downwelling fluxes, i.e. F= F↑ - F↓ is constant.

Furthermore, as there is no heat gained or lost by the atmosphere, the upwelling longwave radiation leaving the top of the atmosphere must be equal to the solar radiation absorbed at the Earth's surface, $F_s$ so at the top of the atmosphere

F↑ =$F_s$. So $F_s$=F as F↓=0 at the top of the atmosphere

Consider the boundary conditions at the surface. We must balance the upward flux of radiation emitted by the Earth at a temperature $T_s$, $B(T_s)$, with the downwelling short and longwave radiation.

$$\pi B(T_s) = F_s + F^{\downarrow}(\chi_s)$$

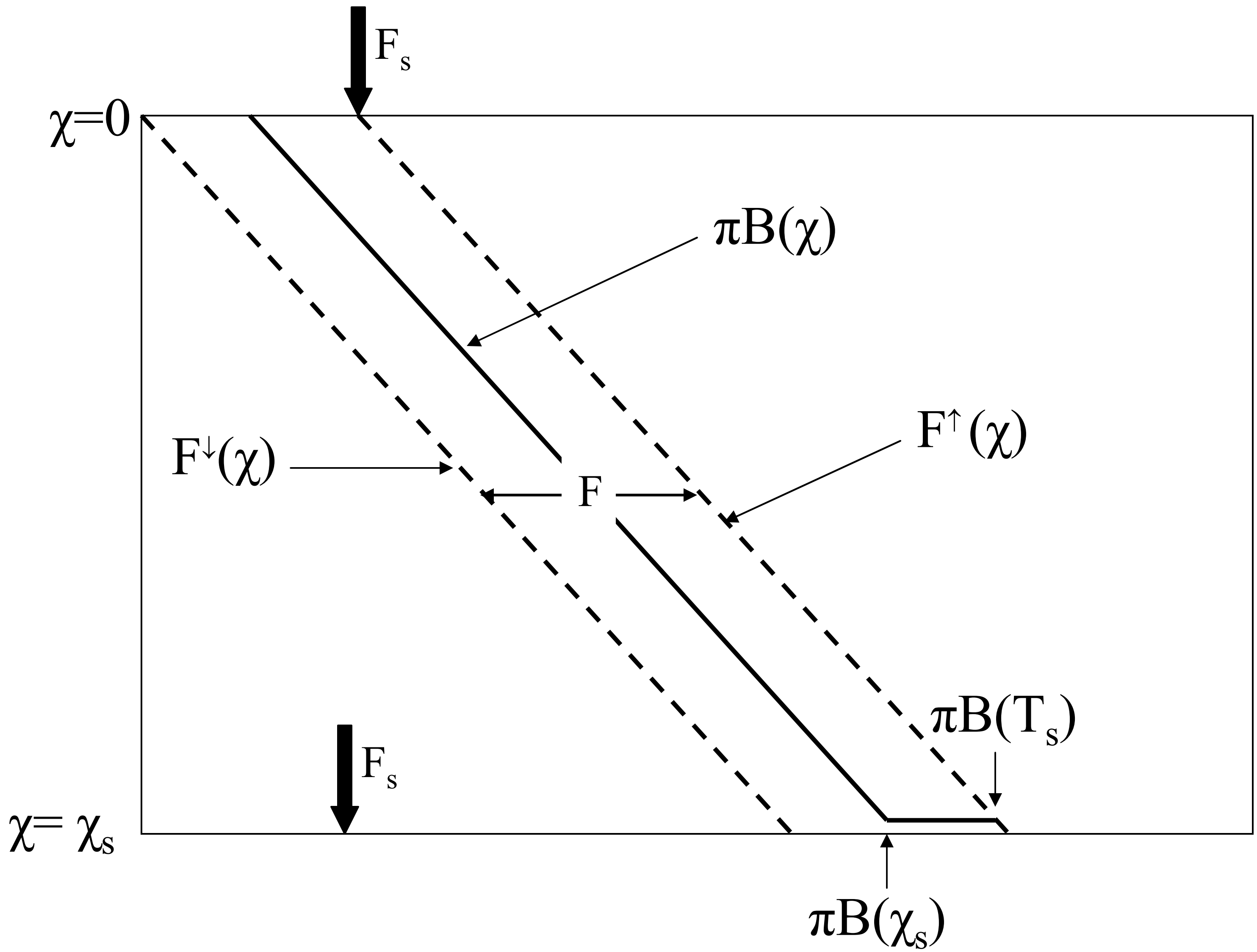
where $\chi_s$ is the optical depth of the lowest layer of the atmosphere. Howeve~~r~~,

$$\overline{F} - F = 2F^{\downarrow} = 2\pi B - F_s$$

which when evaluated at the surface gives

$$\pi B(T_s) = \pi B(\chi_s) + F_s/2.$$

This expression implies that there is a temperature discontinuity between the surface and the cooler lowest layer of the atmosphere.



$F_s$
$\chi=0$
$\pi B(\chi)$
$F^{\uparrow}(\chi)$
$F^{\downarrow}(\chi)$
F
$\pi B(T_s)$
$F_s$
$\chi = \chi_s$
$\pi B(\chi_s)$

If we assume that our absorber varies in concentration solely as a function of pressure then we can express its optical density in the atmosphere as

$$\chi = \chi_s \exp\{-z/H_s\}$$

where z is the height above the surface and $H_s$ is the scale height in the surface layers, approximately 7 km.

When the atmosphere is optically thin, <1, radiation traverses the level with little interaction.

When >1 radiation is absorbed efficiently within the layer and successive layers do not strongly interact.

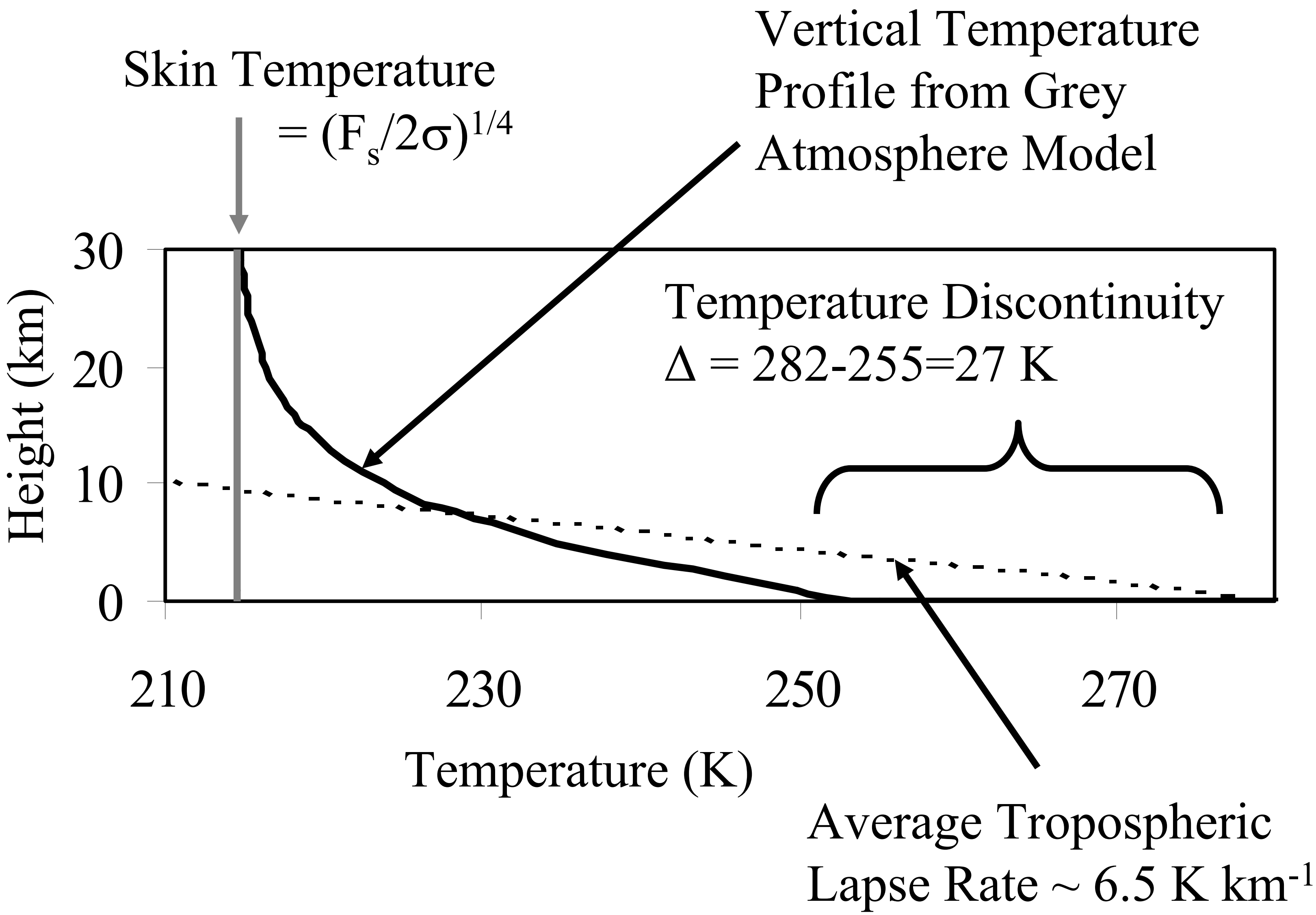
We will therefore choose $\chi_s$=1 to crudely illustrate how our model works.

We can evaluate $\chi$ and so derive B as a function height.

However:

$$F_B = \pi B = \pi \int_0^\infty B(\lambda, T) d\lambda = \sigma T^4$$

and so we can calculate the temperature at any level in the atmosphere.


Skin Temperature
$= (F_s/2\sigma)^{1/4}$
Vertical Temperature
Profile from Grey
Atmosphere Model
Temperature Discontinuity
$\Delta$ = 282-255=27 K
30
20
10
0
Height (km)
210
230
250
270
Temperature (K)
Average Tropospheric
Lapse Rate ~ 6.5 K $km^{-1}$

If the atmosphere did not interact with outgoing longwave radiation ($\chi$ =0) the atmospheric temperature would be constant and be equal to the skin temperature.

The lower stratospheric temperature profile behaves in a similar way to the model.

In the troposphere convection is the dominant transfer mechanism and the rate of decrease of temperature with height, or *lapse rate* is determined by this process.

The average lapse rate in the troposphere is around 6.5 K $km^{-1}$, somewhere between the lapse rate of dry air and that of cloudy air.

If we include this lapse rate in the model, the air in contact with the ground is heated by the surface, becomes buoyant and initiates convection. and the temperature of the overlying layers decreases with height at the average tropospheric lapse rate.

The temperature determined from convection is greater than that predicted by the radiative scheme to a height of 8 km, above which the radiative scheme predicts warmer temperatures than the lapse rate.

The model predicts a convective turbulent lower atmosphere that is well mixed and a transition at around 8 km to a stable atmosphere whose T structure is controlled by radiative processes.  This is a little lower than the observed tropopause but predicts the broad T characteristics of the atmosphere below 20 km.

# More Complex Radiative Transfer

The absorption spectrum of each radiatively active gas is composed of many individual lines. At some wavelengths much of the light is transmitted, while at other wavelengths the absorption by the atmosphere is total.

Although the individual absorption lines arise from discrete transitions between particular vibrational and rotational energy levels within an absorbing molecule the individual lines are not infinitely narrow. The lifetime of the state to which the molecule has been excited can never be predicted exactly as there is always some small but finite uncertainty in the energy of the excited state.

This uncertainty in the decay time leads to a broadening of the transition line over a range of frequencies. However for vibrational and rotational transitions in the infra-red part of the spectrum other line broadening mechanisms are more important.

**Pressure Broadening:** The absorbing molecules can collide with other molecules whilst in the excited state and this affects the re-radiation of light from the molecule.

**Doppler broadening:** occurs as a result of the absorbing or emitting molecule moving in either the same or opposite direction as the photon of light it emits. This leads to a small frequency shift, observed as line broadening. In the troposphere the frequency of collisions is high and pressure broadening is the dominant mechanism.

# More Complex Radiative Transfer 2

**Saturation:** Several gases in the atmosphere absorb very strongly and certain individual transitions effectively absorb all light of that wavelength through the atmospheric column and the absorptance approaches unity. Radiation at these wavelengths has effectively been removed and the only further change in the absorption spectrum of the atmosphere after further passage through the absorbing atmosphere is at the edges of the line where the gas is more weakly absorbing. Saturation will therefore change the shape of the absorbing band through the depth of the absorbing column. In the case where the spectrum of an absorbing gas contains many strong absorption lines close together, saturation may lead to the merging of these lines and the absorption spectrum may become continuous.

Furthermore, unlike the simple model we discussed above, there are many different absorbing species in the real atmosphere, each with their own spectral characteristics. This can lead to some of the absorbing features of different gases overlapping and saturation at some wavelengths may occur even though the contribution from each of the individual absorbing molecules may be weak.

Part of the 14 μm band of $CO_2$, and the complex structure in the band is immediately evident. At some wavelengths much of the light is transmitted, while at other wavelengths the absorption by the atmosphere is total.