**Energy and Climate**

Outline answers to coursework problems

1. Application of inverse square law.

Ratios are $\left(\frac{149.6}{147}\right)^2$ and $\left(\frac{149.6}{152}\right)^2$ giving 3.6% increase at perihelion and 3.1% decrease at aphelion.

Use $T_e^4 = (1-A)\frac{F}{4R^2\sigma} = (1-A)\frac{S}{4\sigma}$ from notes to give $\frac{4\Delta T_e}{T_e} = \frac{\Delta S}{S}$

With $T_e \approx 256\text{K}$ this gives a 2.3 K increase at perihelion and 2.0 K decrease at aphelion.

2. Consider radiation balance of latitude ring between latitudes $\theta$ and $\theta + \delta\theta$.

Area of surface $= 2\pi R\, R\cos\theta\delta\theta$, area intercepted normal to solar beam $= 2R\cos\theta\, R\delta\theta\cos\theta$.

Balance of incoming and emitted fluxes then gives required result, making use of the formula for the global equilibrium radiative temperature $\overline{T_e}$ from notes.
Noting that, at present, with $\overline{T_e} = 256$ K the formula gives a temperature at the equator of 272 K and remembering this is an annual mean, the surface is unlikely to be ice covered.

However, if for some reason the solar flux is first reduced so that the surface becomes ice covered and the albedo is increased to 0.8, and then returns to it's original value, the value of $\overline{T_e}$ will be multiplied by a factor $((1\text{-}0.8)/(1\text{-}0.3))^{0.25}$ giving a mean temperature of 187 K and an equatorial value of 199 K. It is unlikely that with this annual mean value that the ice would melt even at the equator.

3. Simple approximate integration of Planck's law using standard notation

$$B_\lambda(T)d\lambda = \frac{2hc^2}{\lambda^5\left[\exp\left(\frac{ch}{k\lambda T}\right)-1\right]}d\lambda$$

Using $T = 6000$ K, and choosing $d\lambda = \lambda/\sqrt{2}$

| $\lambda$ ($10^{-7}$ m) | $B_\lambda$ ($10^{10}$ mks) | $B_\lambda d\lambda$ ($10^{7}$ mks) |
|---|---|---|
| 1 | 0.045 | 0.00 |
| 2 | 228.5 | 0.05 |
| 4 | 2888 | 1.16 |
| 8 | 1903 | 1.52 |
| 16 | 325.9 | 0.52 |
| 32 | 31.75 | 0.10 |
| 64 | 2.435 | 0.02 |
| 128 | 0.168 | 0.00 |
| TOTAL | | 3.37 |

Using the wavelength bands 0.4 -0.5656 and 0.5656-0.7 µm gives a total flux of $0.73\times10^7$ mks, or 21.7% of the total.

4. Radiation balance at surface.

Let surface emit flux $F$, the glass absorbs $0.8F$. Hence glass must also emit $0.8F$, half upwards and half downwards.

Hence total radiative flux downwards at surface is $S + 0.4F$. But radiation balance at surface implies this must be balanced by upward flux at surface, $F$.

Hence $F = S / 0.6$, and the downward flux is $S$ (1+0.4/0.6) = $5S / 3$.

5. Radiation balance in each layer.

Denote lowest layer by subscript 1, etc and temperature of layer by $T$.

Apply Stefan Boltzmann law to each layer and remember that each layer must emit the same amount of radiation as it receives. Assume that equal amounts of energy are radiated upwards and downwards from each layer and incident short wave flux is $S$.

Surface receives $S + \sigma T_1^4 = \sigma T_s^4$
Layer 1 receives $\sigma T_s^4 + \sigma T_2^4 = 2\sigma T_1^4$
Layer 2 receives $\sigma T_1^4 + \sigma T_3^4 = 2\sigma T_2^4$
Layer 3 receives $\sigma T_2^4 = 2\sigma T_3^4$
Radiation emitted upwards from layer 3 must balance the incoming solar radiation since no other radiation escapes, hence
$\sigma T_3^4 = S$

These can be manipulated to give:
$T_1 = \left(\frac{3}{4}\right)^{1/4} T_s$, $T_2 = \left(\frac{2}{4}\right)^{1/4} T_s$, $T_3 = \left(\frac{1}{4}\right)^{1/4} T_s$

6. Application of $B_g = \frac{\phi}{2\pi}(\chi_0 + 2)$ from notes.

$\phi$ is the difference between the upward and downward long wave fluxes, which is independent of height in steady state. The net long wave flux, $\phi$, and incoming short wave flux at the surface must also balance.

Hence $\phi = \pi B_m$ where $B_m$ is the black body radiation emitted at the measured radiative equilibrium temperature and the factor of $\pi$ is introduced because the radiation is emitted in all directions.

Hence at surface if the measured effective temperature is $T_m$, $B_m = \sigma T_m^4$, so that $\phi = \pi\sigma T_m^4$. But $B_g = \sigma T_s^4$, the black body flux from the surface where $T_s$, is the measured surface temperature.

Substituting in $B_g = \frac{\phi}{2\pi}(\chi_0 + 2)$ gives $\chi_0 = 2\left(\frac{T_s}{T_m}\right)^4 - 2$

Using the values given for Earth $\chi_0 = 1.14$, for Venus $\chi_0 = 224$.

7. ANSWER TO BE ASSESSED

8. Conservation of absolute angular momentum.

Let $\bar{u}$ be the relative zonal wind speed at latitude $\phi$. The absolute angular momentum is then $\Omega a^2 \cos^2\phi + \bar{u} a \cos\phi$

For a ring of air at rest at the equator, the angular momentum is $\Omega a^2$, so that conservation of angular momentum as the ring moves northwards implies
$\Omega a^2 \cos^2\phi + \bar{u} a \cos\phi = \Omega a^2$

and hence

$$\bar{u} = \frac{\Omega a \sin^2\phi}{\cos\phi}$$

Using $a = 6.4\times10^6$ m, and $\Omega = 7.3\times10^{-5}$ rad s$^{-1}$ gives at 45°N

$\bar{u} = 330$ m s$^{-1}$

Note that this is unrealistically large value. Momentum is removed by eddy fluxes.

9. Consider a two-layer model in which air is transported polewards.

Heat transport in a layer is $\rho U c_p \Delta T$ where $\Delta T$ is the difference between the start and end temperatures.

$$\Delta T = \frac{\partial \bar{T}}{\partial y}\Delta y$$

Consider the total transport by the two layers $c_p \Delta y\left(\rho_1 U_1\left(\frac{\partial T}{\partial y}\right)_1 A_1 + \rho_2 U_2\left(\frac{\partial T}{\partial y}\right)_2 A_2\right)$

where $A$ is the vertical cross section of the layer

But since the net poleward mass transport must be zero, in the long term mean, $\rho_1 U_1 A_1 = -\rho_2 U_2 A_2$ and the total heat transport is then given by

$$c_p \Delta y \rho_1 U_1 A_1\left(\left(\frac{\partial T}{\partial y}\right)_1 - \left(\frac{\partial T}{\partial y}\right)_2\right)$$

Assuming a lower layer 3 km deep and extending around the globe at 50°N gives $A_1 = 7.75\times10^{10}$ m$^2$

Using $\rho_1 = 1$ kg m$^{-3}$ and a total heat flux of $3\times10^{15}$ W gives $U \approx 20$ m s$^{-1}$.

The observed mean meridional velocity is very much weaker. This is because the circulation at this latitude is not a direct circulation, but consists of waves which are more effective at transporting heat.

10. ANSWER TO BE ASSESSED

Answers to assessed questions

7. Latent heat flux determines the rate of evaporation. In the global average, precipitation must balance evaporation. Also, the mean residence time is the ratio of the total amount of water in the troposphere to the flux into, or out of, the troposphere.

Latent heat flux = $0.7 \times (0.48 - 0.16) \times 340 = 76.16$ W m$^{-2}$

Now latent heat of vaporisation = $2.5 \times 10^6$ J kg$^{-1}$ or $2.5 \times 10^9$ J m$^{-3}$

Hence water flux at surface is $3 \times 10^{-8}$ m s$^{-1}$ and the mean annual evaporation or precipitation rate is 0.94 m. Most of the precipitation falls in the tropical regions.

The total water flux into the atmosphere is $3 \times 10^{-8} A$ m$^3$ s$^{-1}$ where $A$ is the surface area of the earth, or $3\times10^{-8} A\rho_w$ kg s$^{-1}$ where $\rho_w$ is the density of water.

Total mass of water in the troposphere is $Ah\bar{\rho}$

Hence the average lifetime of a molecule in the troposphere is $\dfrac{Ah\bar{\rho}}{3\times10^{-8} A\rho_w}$

This gives $10^6$ s or about 11-12 days.

10. Angular momentum = $I\omega$ where $I$ is moment of inertia about diameter and $\omega$ is angular velocity.

For atmosphere (spherical shell of radius $a$) $I_a = \dfrac{2}{3} M_a a^2$ where $M$ is mass, and for the Earth

$$I_e = \frac{3}{5} M_e a^2$$

If atmosphere and Earth have same angular velocity, ratio of angular momentum of atmosphere to that of Earth is $I_a / I_e$.

Thus ratio is $\dfrac{\rho_a}{\rho_e} \dfrac{10d}{3a}$ where $\rho$ is the mean density, $a$ the radius of the Earth and $d$ the depth of the atmosphere.

From the hydrostatic equation, $\rho_a d = p/g$ where $p$ is the mean surface pressure.

Taking $p = 1013$ mb $= 1.013\times10^5$ Pa
$g = 9.8$ m s$^{-1}$
$a = 6.4\times10^6$ m
$\rho_e = 5.5\times10^3$ kg m$^{-3}$

Gives ratio $10^{-6}$

To conserve angular momentum of the earth-atmosphere system, an increase in the angular velocity of the atmosphere by an amount $\Delta\omega_a$ must be accompanied by a decrease in that of the solid earth by $\Delta\omega_e = -\frac{I_a}{I_e}\Delta\omega_a$.

If the length of the day is $t$, then

$$\frac{\Delta t}{t} = -\frac{\Delta\omega_e}{\omega_e}$$

but $\omega_e = 2\pi/t$, so $\Delta t = -\frac{t^2\Delta\omega_e}{2\pi}$

An increase in velocity of 5 m $s^{-1}$ at the equator implies $\Delta\omega_a = 5/a$, and

an *increase* in the length of the day of $9\times10^{-4}$ s.