1   The US Geological Survey estimate of cumulative world production of natural gas was 1.72 Q up to the end of 1992. They also estimated the total resource to be 11.4 Q. In a previous estimate in 1985 the USGS gave the cumulative production to be 1.15. If gas, like oil, is explored for and priced such that the known reserves are always around 10 years ahead of current levels of production, estimate when production is likely to reach it's peak.

[The Verhulst equation is $\dfrac{dr}{dt} = cr(R - r)$, where r is the cumulative reserve and R the total resource]

$$\dfrac{dr}{dt} = cr(R - r) \text{ so } \dfrac{1}{R}\dfrac{dr}{dt} = cR\,\dfrac{r}{R}\left(1 - \dfrac{r}{R}\right), \text{ if we let } f = \dfrac{r}{R}$$

$$\dfrac{df}{dt} = cRf(1 - f) \text{ and so } \dfrac{df}{f(1 - f)} = cRdt$$

$$\int_{f_0}^{f} \dfrac{df}{f(1-f)} = \int_{t_0}^{t} cRdt \quad \text{now use} \quad \dfrac{df}{f(1-f)} = \dfrac{df}{f} + \dfrac{df}{1-f}$$

$$\int_{f_0}^{f} \dfrac{df}{f} + \int_{f_0}^{f} \dfrac{df}{1-f} = \int_{t_0}^{t} cRdt \quad \text{this gives} \quad \ln\left[\dfrac{f(t)}{f(t_0)}\right] - \ln\left[\dfrac{1-f(t)}{1-f(t_0)}\right] = cR(t - t_0)$$

re-arrange in terms of f(t) $\quad \dfrac{f(t)}{f(t_0)}\dfrac{1-f(t_0)}{1-f(t)} = e^{cR(t-t_0)}$

$$\dfrac{f(t)}{1-f(t)} = \dfrac{f(t_0)}{1-f(t_0)}e^{cR(t-t_0)} \quad \text{and then} \quad \dfrac{1-f(t)}{f(t)} = \left(\dfrac{1}{f(t_0)} - 1\right)e^{-cR(t-t_0)}$$

$$f(t) = \dfrac{f(t_0)}{f(t_0) + (1 - f(t_0))e^{-cR(t-t_0)}} \quad \text{re-substitute for r and R}$$

$$\dfrac{r(t)}{R} = \dfrac{r_0/R}{r_0/R + \left(1 - r_0/R\right)e^{-cR(t-t_0)}}$$

If 10 years lag between cumulative reserve and cumulative production then
r(1982) = p(1992) = 1.72 Q   and   r(1975) = p(1985) = 1.15 Q

R = 11.4 Q
find cR from above equation, cR = 0.0657
need to maximise dr/dt i.e. maximum discovered reserve= $d^2r/dt^2 = c(R-2r)=0$
maximum discovered reserve when r=R/2
so solve for t in solution to Verhulst equation. T = 1970 + 33 years = 2003.
If production lags behind reserve by 15 years then production peaks in 2018.