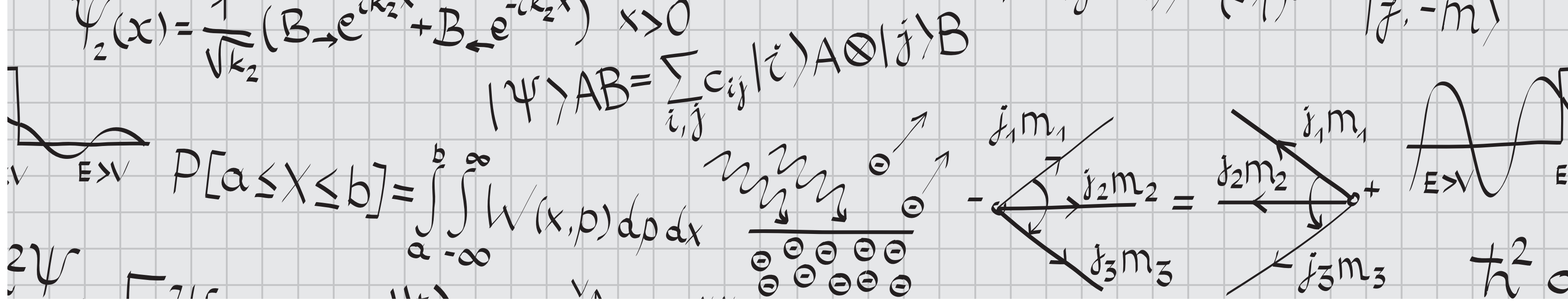

9 Special Tips and Tricks for the Physics GRE

One of the main reasons we wanted to write a Physics GRE review book is that none of the existing review materials address *both* general test-taking strategies and strategies specific to physics problems. We'll make some general suggestions applicable for multiple-choice tests at the end of this chapter, but we'll start with several important and physics-specific tips and tricks.

9.1 Derive, Don't Memorize

If you're just beginning your GRE preparation, and you've started looking through your freshman year textbook, you're probably overwhelmed by the sheer number of formulas. If you're like most physics students, you probably don't even remember learning many of them! But for better or for worse, the Physics GRE is a test of outside knowledge, and you need to know certain formulas to answer many of the questions. And the formula sheet provided at the beginning of the test is worse than useless: numerical values of constants you'll never need, a couple random definitions, and three moments of inertia. Obviously we're going to need an efficient way to remember all the missing formulas.

Richard Feynman (famous twentieth-century physicist and co-inventor of quantum electrodynamics) has a wonderful piece of advice on this sort of thing: "knowledge triangulation." No one can possibly remember all the formulas, but if you can remember a few key facts, you can reconstruct most of the rest of your knowledge, and "triangulate" unknown facts from known ones. The key to this is remembering the basic steps in the important derivations in all the key areas tested on the Physics GRE.

Try this: divide up your formulas into categories based on how involved the derivations are. Class 1 would be the absolute basics, such as $\mathbf{F} = m\mathbf{a}$, expressions for kinetic energy ($\frac{1}{2}mv^2$ for translational, $\frac{1}{2}I\omega^2$ for rotational), the rest energy $E_0 = mc^2$ of a particle, and so on. These are essentially definitions of important physical quantities, rather than actual formulas. Class 2 would be formulas that you could quickly derive in a couple steps from the Class 1 formulas. This might include formulas for recoil velocities in one-dimensional elastic collisions where one mass is at rest (apply conservation of momentum and energy) and the cyclotron frequency of a charged particle in a magnetic field (use the fact that the magnetic field provides the centripetal force required for uniform circular motion). Class 3 would be any formula or equation that you expect will take more than two or three lines of algebra to derive, such as normal mode frequencies for a pair of coupled springs or second-order energy shifts in quantum-mechanical perturbation theory.

Now, focus your attention on memorizing the Class 1 formulas, and the steps in the derivations that lead to the Class 2 formulas. Start a formula sheet containing the Class 3 formulas, adding them as you come across them in your studying, and memorize them as you go. Also, include a sketch of the derivations of the Class 2 formulas, but *don't* include the formula itself. Your notes might look like this:

EM boundary conditions at a conductor: apply Maxwell's equations using infinitesimally thin pillboxes and loops

That way, every time you review your formula sheet, you'll force yourself to rederive these formulas. If you find you can't

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do this after several tries, promote it to a Class 3 formula and write it down.

Of course, this classification is a very individual process, and will depend strongly on which subjects you consider your strengths or weaknesses. But a good target is to have no more than ten Class 3 formulas for the major subjects (classical mechanics, electricity and magnetism), and no more than five Class 3 formulas for each of the smaller subject areas. Anything else is probably overkill, assuming you're familiar enough with the basics to know the Class 1 formulas by heart. And despite what the GRE formula sheet may suggest, moments of inertia are *not* worth memorizing. We would consider the formula $I = mr^2$ for a point mass a Class 1 formula, and everything else Class 2 (just integrate, or use the parallel axis theorem).

You can go even further and develop mnemonics for memorizing Class 3 formulas by treating them as Class 2 formulas, and doing a quick-and-dirty "derivation." Here are a couple of examples. The formula for the Bohr radius of the hydrogen atom, $a_0 = 4\pi\epsilon_0\hbar^2/m_e e^2$, is both completely ubiquitous in quantum mechanics and a huge mess. But instead of memorizing the expression, you can cheat slightly and derive it using mostly classical mechanics and a little quantum mechanics. Apply the uncertainty principle in the form $\Delta r \Delta p \sim \hbar$ to the Bohr model of the hydrogen atom, where we assume the electron executes uniform circular motion in the Coulomb field of the proton. Putting $\Delta r = r$ and $\Delta p = p$, and turning the $\sim$ sign into an $=$ sign, we obtain precisely the Bohr radius. (Try it yourself!) Strictly speaking, of course, this derivation is completely bogus: the p appearing in the uncertainty relation should really be the *radial* momentum, the right-hand side should be $\hbar/2$, and setting $\Delta r = r$ is dubious at best. However, if you just treat this derivation as a mnemonic, you have a two-line derivation of a Class 3 formula, which takes it off your list of formulas to memorize. A simpler example, but one that may be a little too advanced for the Physics GRE, is the Schwarzschild radius of a black hole. Treat light like a "particle" of mass m and kinetic energy $\frac{1}{2}mv^2$, and find the starting radius R for which the escape velocity from a body of mass M is the speed of light $v = c$. You'll find the mass m cancels out, and that light can only escape to infinity for $R > 2GM/c^2$, the Schwarzschild radius. Again, the right answer for the wrong reasons, but it's quick and it works.

Keep an eye out for mnemonics like this, and you should be able to keep your formula sheet to a manageable size. That way you can devote more of your study time to reviewing and doing practice problems, rather than cramming your brain full of formulas.

9.2 Dimensional Analysis

Physical quantities have units. This may not seem like a profound statement, but it is an extraordinarily powerful tool for getting order-of-magnitude answers to physical questions, without ever doing involved computations. On the GRE, it offers an interesting alternative problem-solving method thanks to the multiple-choice format. The *very first* thing you should do when you see a tough-looking question is to scan the answer choices to see if they all have the same units. If *not*, there's a decent chance that only one of the answer choices has the correct units, and by identifying the units you want for the problem in question, you can get to the correct answer by dimensional analysis alone.

Here are some answer choices similar to those that appeared on a 2008 ETS-released test:

(A) h/f
(B) hf
(C) h/λ
(D) λf
(E) $h\lambda$

Without even knowing the question, only one of these choices can possibly be correct, because they all have different units. A question this easy is relatively rare, but you might expect to see a few problems on each test that can be solved with dimensional analysis.

A somewhat more common example is:

(A) $R\sqrt{l/g}$
(B) $R\sqrt{g/l}$
(C) $R\sqrt{2l/g}$
(D) Rg/l
(E) $R^2 l/2g$

Assuming R and l stand for lengths (g always has its usual meaning of gravitational acceleration), a quick scan shows that A and C have the same units, while all the others are different. So once we know which units we're looking for, at best we've solved the problem, and at worst we're down to two choices, A and C.

Because dimensional analysis can at least be used as a check on the answers of many problems on the GRE, it's an excellent fallback tool in case you forget exactly how to approach a problem or draw a complete blank. It pays to get *very* comfortable with computing units for quantities, so here's Example 9.1 to practice with.

Since this kind of dimensional analysis comes up so often, we *strongly* recommend coming up with your own method for solving these dimensional equations. Some combination

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EXAMPLE 9.1

Which of the following gives the uncertainty Δx^2 for the ground state of the harmonic oscillator?

(A) $\dfrac{\hbar}{2m\omega}$

(B) $\dfrac{\hbar^2}{m\omega}$

(C) $\dfrac{\hbar\omega}{m}$

(D) $\dfrac{\omega}{2\hbar m}$

(E) $\dfrac{\hbar\omega}{m^2}$

We're looking for a quantity with units of (length)^2. First, let's do the dimensional analysis the straightforward way, listing the dimensions of all the variables as powers of mass M, length L, and time T, the three fundamental units in the SI system:

- $\hbar$: ML^2T^{-1}
- m: M
- ω: T^{-1}

The most general combination we can form is $\hbar^a m^b \omega^c$, and we want this to have units of L^2, so we get a system of linear equations in a, b, and c that we can solve:

$$\begin{aligned} a + b &= 0 \\ 2a &= 2 \\ -a - c &= 0. \end{aligned}$$

It's straightforward to see that $a = 1$, $b = -1$, and $c = -1$; in other words, $\hbar/m\omega$, choice A. We're off by a factor of 2, but who cares: only choice A has the correct units. In fact, writing down the linear equations was probably a waste of time, since we could have just as easily stared at the list of units for $\hbar$, m, and ω and determined that the quantity we were looking for was $\hbar/m\omega$ right away.

For an alternate method, we could have avoided the ugly units of $\hbar$ by remembering that $\hbar\omega$ has nice units of energy. One form of energy is kinetic energy, $\frac{1}{2}mv^2$, so to get units of L^2 we need to divide energy by one power of M and multiply by two powers of T. This gives

$$\hbar\omega \times \frac{1}{m} \times \frac{1}{\omega^2} = \frac{\hbar}{m\omega},$$

as before. Note how much faster this was than actually computing the uncertainty for the harmonic oscillator, either by using operator methods or the position-space wavefunction!

of memorizing the MLT units for common constants, remembering useful combinations of constants with nice units like q^2/ϵ_0, and mnemonic methods would be an excellent start.

9.3 Limiting Cases

A careful analysis of limiting cases is one of the most efficient ways to check your work on physics problems. This is especially true for the GRE, where you'll often be able to hone in on the correct answer choice by considering limiting cases, even when dimensional analysis fails.

What exactly constitutes a "limiting case," of course, depends on the problem. Some of the more common ones include letting a quantity such as a mass, velocity, or energy go to zero or infinity, and seeing if the result makes sense in this limit. Here's a simple example: say you have a block of mass m on an inclined plane at an angle θ from the horizontal,

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EXAMPLE 9.2

Consider the classic problem of a wheel of mass M and radius R up against a ledge of height h, shown in Fig. 9.1. What horizontal force F do you have to apply at the axle to roll the wheel up over the ledge? (Try this problem yourself before reading the rest of the discussion.)

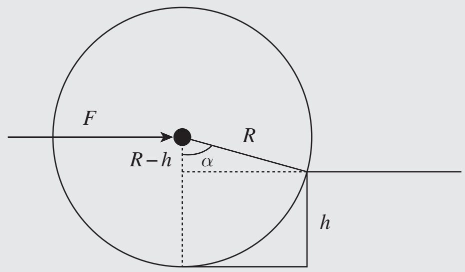

Figure 9.1 A wheel being pushed up a ledge.

This problem is solved most simply by considering the torques about the contact point with the ledge; the ledge exerts some complicated force on the wheel, but we can ignore this entirely because it exerts no torque about the contact point. The wheel will roll up if the torque due to the horizontal force exceeds the torque due to gravity:

$$\tau_g = RMg\sin\alpha = Mg\sqrt{R^2-(R-h)^2},$$
$$\tau_F = RF\sin(\pi/2-\alpha) = RF\cos\alpha = F(R-h),$$
$$\tau_F > \tau_g \implies F > Mg\frac{\sqrt{2Rh-h^2}}{R-h}.$$

Now, let's say we made a mistake calculating $\sin\alpha$, and wrote $\sin\alpha = \frac{\sqrt{R^2-h^2}}{R}$. This gives $F > Mg\sqrt{\frac{R+h}{R-h}}$. This sort of looks right: it has the right dimensions, and it goes to infinity as $h \to R$, which makes sense (you're never going to be able to push the wheel over the ledge using just a sideways force if the ledge is as high as the radius). However, taking the limiting case of $h \to 0$, we find $F > Mg$. This certainly doesn't make sense: if the ledge disappears, then any force, however small, will allow the wheel to keep rolling. So we know we've made a mistake somewhere.

and you can't remember whether the component of the gravitational force along the ramp is $mg\cos\theta$ or $mg\sin\theta$. Instead of fussing around with similar triangles, just consider what happens when θ is either 0 or $\pi/2$. In the first case, the ramp is horizontal, so the block doesn't slide; in other words, gravity doesn't act at all along the direction of the ramp. In the second case, the ramp is vertical, so the entire force of gravity mg acts downwards and the block just falls straight down. Either of these tell you immediately that the force we're looking for is $mg\sin\theta$.

Checking limiting cases is an extremely powerful strategy if you're running out of time at the end of the test. For sets of answer choices that contain algebraic expressions differing by more than just numerical factors, checking the limiting behavior of the answers usually will let you eliminate some choices. Remember that eliminating even *one* answer choice gives you a positive expected value for that question. If you can quickly identify the relevant limiting cases, and check them against the answer choices, you can often eliminate up to three wrong answers in under a minute. See Example 9.2.

9.4 Numbers and Estimation

Broadly speaking, there are two kinds of physicists: theorists and experimentalists. If you're a theorist, you're probably more comfortable with formulas than numbers, and you might not remember the last time you had to calculate an explicit temperature, energy, or pressure. But a large part of the Physics GRE requires you to think like an experimentalist, estimating rough orders of magnitudes for various

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physical quantities. Here we'll talk about some strategies for doing so.

First of all, there are some numbers you should just know *cold*. These are the numbers that show up so often in real physics problems that if you haven't already memorized them, you will have after less than a few months of graduate research in the relevant field. Perversely, many of these are *not* the numbers that show up on the Table of Information on the first page of the GRE. Here's the most important example: the binding energy of hydrogen is 13.6 eV. You could memorize the formula for the Bohr energies, $E_n = -\frac{1}{n^2}\frac{m_e e^4}{2(4\pi\epsilon_0)^2\hbar^2}$, plug in all the constants given in the table, and find E_1 after a ton of arithmetic ... or you can memorize this one number.

Actually, this number tells you quite a lot: if you remember that the mass of the electron is about 0.5 MeV$/c^2$ (another number to memorize – see below), this means that you can treat the hydrogen atom nonrelativistically, because the electron's binding energy is much less than its mass. If you know that X-rays have energies of the order of keV, you know that hydrogen atom transition energies are safely below this range, in the ultraviolet. And you know that atoms close to hydrogen in the periodic table will have roughly similar ionization energies: more specifically, the binding energy of each electron increases as the square of the nuclear charge Z, so the ground state energy of helium is about $(13.6)(2^2)(2) \approx 110$ eV, and the binding energy of lithium is about $(13.6)(3^2)(3) \approx 370$ eV. To be clear, these numbers are just approximations – you've probably treated the helium atom using the variational principle in your quantum mechanics class, and you've seen the ground state energy is somewhat less than 110 eV. But these rough estimates are plenty for the GRE – in fact, estimating the binding energy of lithium is a practice question on the Sample Question set released by ETS.

Other important numbers show up as commonly used combinations of fundamental constants. If you're like us, you probably had to memorize the value for h in high school chemistry – but when's the last time you actually had to use the value for *h by itself* in a calculation? If you're calculating anything in quantum mechanics, you use $\hbar$, and if you're doing anything relativistic, you use $\hbar c$. For speed, these combinations are worth memorizing, because they're the ones that you'll actually need. But note that they are currently listed in the GRE formula sheet in case you forget. Similarly, Boltzmann's constant k_B is almost *never* used by itself, but always in combination with temperature. But if you remember that room temperature is about $300\,\text{K} \approx \frac{1}{40}$ eV, you can get the value if you need it. When dealing with combinations of constants, equally important is remembering the units: $\hbar c \approx 200\,\text{MeV}\cdot\text{fm}$ has units of energy $\times$ distance, which tells you that the characteristic distance associated with an object with energy 0.5 MeV is $(200/0.5) \times 10^{-15}$ m, or about 4×10^{-13} m: this is a rough estimate of the Compton wavelength of the electron. (Actually, this is off by a factor of 2π, but who cares? It's good enough as an order of magnitude.)

Based on our experience reviewing past GREs, here is a list of the top five numbers to memorize (in order of importance):

- 13.6 eV – energy of the ground state of hydrogen
- 511 keV – mass of the electron in units of c^2
- 1.22 – coefficient appearing in the Rayleigh criterion, $D\sin\theta = 1.22\lambda$
- 2.9×10^{-3} m·K – Wien's displacement law constant
- 2.7 K – temperature of the cosmic microwave background

You can almost certainly get by with just these numbers. Not included in this list are other numbers you can derive in one or two short steps from numbers given in the Table of Information, such as $\hbar c$ as discussed above.[1]

9.5 Answer Types (What to Remember in a Formula)

The Physics GRE is tricky. Compared to other tests of similar subject matter, such as the AP Physics test or the Physics section of the MCAT, the testmakers throw in answer choices that are deliberately designed to mislead you. Being aware of the common patterns of answer choices can help you avoid these traps, and can often suggest the most efficient approach to a problem. In order of increasing difficulty, here are some patterns you should be aware of.

- **Answer choices with different dimensions.** This was covered in Section 9.2 above, and these problems are some of the easiest because of the possibility of eliminating many answer choices without actually doing any calculations.
- **Order of magnitude.** This was touched on in Section 9.4, and similar to dimensional analysis questions, one can get pretty far just by knowing rough orders of magnitude for common physical situations. See Example 9.3.
- **"Which power of two?"** This pattern is best illustrated by a couple of examples:

[1] Be careful! The most recent GRE included hc on the formula sheet, but often $\hbar c$ is the more useful quantity.

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EXAMPLE 9.3

The average intermolecular spacing of air molecules in a room at standard temperature and pressure is closest to

(A) 10^{-12} cm
(B) 10^{-9} cm
(C) 10^{-6} cm
(D) 10^{-3} cm
(E) 1 cm

While you could try to calculate this quantity exactly, using the fact that one mole of gas occupies 22.4 L at STP and so on, it's best just to recognize that A is the scale of nuclear diameters, B is the scale of atomic diameters, and E is macroscopic, which just seems incorrect. So, by common sense, we've narrowed it down to C and D.

1. (A) 2
   (B) 4
   (C) 8
   (D) 16
   (E) 32

2. (A) 0
   (B) $a/3$
   (C) $a/\sqrt{3}$
   (D) a
   (E) $3a$

While the first set is numeric and the second set is symbolic, they're both testing the same thing: do you know the correct power law for a given variable in a certain formula? Often these answer choices will all have the same dimensions, so dimensional analysis won't help you. But the fact that the choices almost always involve nice numbers suggests that memorizing the various constants that accompany formulas is mostly useless: all that matters is the dependence on the various parameters in the problem. As we've emphasized many times, this is especially apparent in the formula for the Bohr energies, where the dependence on reduced mass, nuclear charge, and principal quantum number are all important. On a similar note, if a formula has a simple power-law dependence, such as the Rayleigh formula for small-particle scattering, it's worth simply committing it to memory without asking too many questions about where it came from.

- **Same units, different limiting cases.** This pattern might come from a problem with an angle that can range from 0 to 90°, two unequal masses m and M, or two springs with different spring constants k_1 and k_2. But in any case, while dimensional analysis isn't helpful, taking limiting cases as discussed in Section 9.3 can often help narrow down the answer choices. This pattern lies right on the border between trying to do the problem from the beginning, and forgoing any calculations and just using limiting cases instead. Use your best judgment based on which method you think will be the fastest based on your own strengths and weaknesses.
- **Same units, different numerical factors.** This pattern, which looks like

  (A) $\cos(l/d)$
  (B) $\cos(2l/d)$
  (C) $\cos(l/2d)$
  (D) $\cos(l^2/d^2)$
  (E) $\cos(l^2/2d^2)$

  is tricky, because dimensional analysis is useless, and limiting cases are almost useless. Worse, many of the answer choices only differ by dividing instead of multiplying, increasing the possibility that you land on a trap answer choice by an arithmetic mistake. This pattern is a clue to *slow down*, work through the problem carefully, and try not to refer to the answer choices at any point during your calculation.
- **Random numbers.** Sometimes, you'll have to work out a problem numerically, and all the answer choices will be numbers with no obvious relation to one another. This arises most often in basic kinematics and mechanics problems, where luckily the physics is not an issue – the strategy is just to work slowly and make sure you don't make an arithmetic mistake. Equally as important, many of the wrong answer choices are likely correct answers to an *intermediate* step in the calculation, so, just as mentioned above, try *not* to refer to the answer choices until you're absolutely

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through calculating. This reduces the chance you'll get distracted by a trap answer.

9.6 General Test-Taking Strategies

- **Don't practice with a calculator!** You won't be allowed a calculator for the GRE, and the arithmetic required for each problem has been deliberately simplified to avoid messy long division. Resist any temptation to practice with a calculator. We have tried to make the problems in this book as similar as possible to GRE problems, so you should get used to the kinds of arithmetic simplifications you might see on the real test: square roots and factors of π will either be given in the answer choices or be close to whole numbers, and so on.
- **Don't study the night before the exam.** In particular, don't try to cram formulas. Remember, you can derive most of the ones you need from simpler formulas, which should be intimately familiar to you since you've been studying hard for the past several months.
- **Consider working problems symbolically first, only plugging numbers in at the end.** Many GRE problems have extraneous information. If you feel comfortable with algebraic manipulation, it may be a good idea to assign variables to given numbers (if a radius is given as 5 cm, just call it R), find the solution algebraically, and plug in numbers only at the end. Not only will this reduce the amount of arithmetic you have to do, but it may eliminate mistakes because you can check the dimensions of your answer easily. You may also find that you didn't need to use all the information given, or that some factors canceled out.
- **Always guess.** Unlike some other multiple-choice tests, the current version of the GRE does *not* penalize wrong answers. (You may see different instructions in older versions of the test released by ETS, but the most recently released test from 2017 has the new instructions.) Therefore, it is always to your advantage to guess. However, random guessing should be a last resort, and the tips and tricks detailed above will sometimes make it possible to narrow down the answer choices *completely* without ever actually sloving the problem from first principles!
- **Avoid time sinks.** Feel free to take a first pass through the test doing only the problems you feel you know how to answer immediately. On your second pass through the test, you can tackle the more calculation-heavy problems, but try not to spend more than 5 minutes on any particular problem. You should be averaging 1.7 minutes per problem, so it may be worth making a note of the time when you start your second pass to make sure you have enough time to finish the problems that are left. If it looks like you're getting stuck in a time sink, look over the answer choices and see if limiting cases or dimensional analysis can help you narrow down the answer choices. You can always come back to the really tough problems in the last half hour before the test ends.

9.7 Problems: Tips and Tricks

1. Optical phonons in a solid can be excited by infrared light. The typical energy of optical phonons is

   (A) 10^{-4} eV
   (B) 0.1 eV
   (C) 100 eV
   (D) 100 KeV
   (E) 100 MeV

2. A capacitor filled with a dielectric of dielectric constant ϵ is connected to a battery of fixed voltage. If ϵ is doubled, the energy stored in the capacitor is multiplied by a factor of

   (A) 1/4
   (B) 1/2
   (C) 1
   (D) 2
   (E) 4

3. In nuclear magnetic resonance experiments, a nucleus with magnetic moment μ in an external magnetic field B will resonantly absorb radiation of frequency

   (A) $\mu B/\hbar$
   (B) $\mu\hbar/B$
   (C) $\mu/(B\hbar)$
   (D) $\mu\hbar B$
   (E) $B/(\mu\hbar)$

4. Metal A has a Fermi energy of 5 eV and a density of 3 g/cm^3. Metal B has the same number of valence electrons as metal A, but a density of 24 g/cm^3. The Fermi energy of metal B is approximately

   (A) 2.5 eV
   (B) 5 eV
   (C) 10 eV
   (D) 20 eV
   (E) 40 eV

5. A particle of mass m is attached to a spring with spring constant k and feels a frictional force $F = bv$ proportional to its velocity. The particle starts at position x_0 at time $t = 0$ and is observed to undergo oscillatory motion.

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Which of the following could describe its position x as a function of time?

(A) $x_0\, e^{-\frac{m}{2b}t}$

(B) $x_0 e^{-\sqrt{\frac{k}{m}}t} \cos\left(\sqrt{\frac{k}{m}}t\right)$

(C) $x_0\, e^{-\frac{b}{2m}t} \sin\left(\sqrt{\frac{m}{k} - \frac{m}{b}}t\right)$

(D) $x_0\, e^{-\frac{b}{2k}t} \cos\left(\sqrt{\frac{b}{m} - \frac{k^2}{2m^2}}t\right)$

(E) $x_0\, e^{-\frac{b}{2m}t} \cos\left(\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}t\right)$

9.8 Solutions: Tips and Tricks

1. B – Knowing that the binding energy of hydrogen is 13.6 eV (one of your numbers to memorize) tells you that energy transitions in hydrogen are at the eV scale. Since some of the hydrogen transition lines are in the infrared spectrum, this means that infrared photons have energies close to an eV.
2. D – Inserting a dielectric into a capacitor multiplies its capacitance by ϵ. The energy stored in the capacitor can be written as $U = \frac{1}{2}CV^2$, which is useful because we are told the capacitor is at constant voltage. Therefore, the energy scales linearly with C, and hence linearly with ϵ, choice D. This is a classic "which power of 2" problem.
3. A – All the answer choices have different units, so dimensional analysis is the way to go here. Rather than deal with the messy units of electromagnetism, we can remember that the energy of a dipole (anti)aligned with a magnetic field is $U = \mu B$. So μB has units of energy, but so does $\hbar\omega$, which is the energy of a photon of angular frequency ω. This tells us that $\mu B/\hbar$ has units of frequency.
4. D – Since the answer choices resemble a "which power of 2" problem, we know we are not interested in the constants that appear in the formula for the Fermi energy, so this is a good candidate for "derive, don't memorize." Electrons in a metal fill up a Fermi sphere with radius p_F and volume proportional to p_F^3. The total number of electrons is proportional to the density ρ, so $p_F \propto \rho^{1/3}$. The Fermi energy is $E_F = p_F^2/2m_e$, so $E_F \propto \rho^{2/3}$, which is the relation we need to solve this problem. If the density is increased by a factor of 8, the Fermi energy is increased by a factor of $8^{2/3} = 4$, which gives choice D.
5. E – A direct solution of this problem by solving the second-order differential equation for a damped harmonic oscillator would be a time sink. Instead, we can use limiting cases and dimensional analysis. A useful limit to take is $b \to 0$, the case of an undamped oscillator. In that case, we know that the amplitude should be constant and the angular frequency should be $\omega = \sqrt{k/m}$. Only choice E satisfies those criteria. Even if you didn't remember that formula for ω, choices A, C, and D have incorrect units in the exponentials and trig functions. B has correct units, but the fact that it is independent of b is suspicious, since as $b \to 0$ the amplitude should stay constant.