Sample Exams and Solutions

Sample Exam 1

TABLE OF INFORMATION

Rest mass of the electron	m_e	$=$	9.11×10^{-31} kg
Magnitude of the electron charge	e	$=$	1.60×10^{-19} C
Avogadro's number	N_A	$=$	6.02×10^{23}
Universal gas constant	R	$=$	8.31 J/(mol · K)
Boltzmann's constant	k	$=$	1.38×10^{-23} J/K
Speed of light	c	$=$	3.00×10^{8} m/s
Planck's constant	h	$=$	6.63×10^{-34}J · s $=$ 4.14×10^{-15} eV · s
	$\hbar$	$=$	$h/2\pi$
	hc	$=$	1240 eV · nm
Vacuum permittivity	ϵ_0	$=$	8.85×10^{-12} C^2/(N · m^2)
Vacuum permeability	μ_0	$=$	$4\pi \times 10^{-7}$ T · m/A
Universal gravitational constant	G	$=$	6.67×10^{-11} m^3/(kg · s^2)
Acceleration due to gravity	g	$=$	9.80 m/s^2
1 atmosphere pressure	1 atm	$=$	1.0×10^{5} N/m^2 $=$ 1.0×10^{5} Pa
1 angstrom	1 Å	$=$	1×10^{-10} m $=$ 0.1 nm

Prefixes for Powers of 10

10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	µ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P

Rotational inertia about center of mass

Rod	$\frac{1}{12}M\ell^2$
Disk	$\frac{1}{2}MR^2$
Sphere	$\frac{2}{5}MR^2$

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SAMPLE EXAM 1

Time — 170 minutes

100 questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then fill in the corresponding space on the answer sheet.

1. A centrifuge can be used to simulate large gravitational forces. Consider a centrifuge consisting of an arm of length 4 meters, rotating about a fixed pivot at constant speed. What must this speed be to simulate a gravitational acceleration of $9g$? ($\sqrt{|g|} = \sqrt{9.8}$)

   (A) $2\sqrt{|g|}$ m/s
   (B) $3\sqrt{|g|}$ m/s
   (C) $6\sqrt{|g|}$ m/s
   (D) $18\sqrt{|g|}$ m/s
   (E) $36\sqrt{|g|}$ m/s

2. A block of mass m moving with velocity v collides with a heavier block of mass $4m$, initially at rest. If the collision is perfectly elastic, what is the velocity of the heavier block after the collision?

   (A) $4v$
   (B) $(1/4)v$
   (C) v
   (D) $(5/2)v$
   (E) $(2/5)v$

3. An LC circuit, consisting of a solenoid and a parallel-plate capacitor, has resonant frequency ω. If the linear dimensions of all circuit elements are doubled, the new resonant frequency is

   (A) $\sqrt{2}\omega$
   (B) 2ω
   (C) ω
   (D) $\omega/2$
   (E) $\omega/\sqrt{2}$

4. A point dipole with dipole moment $\mathbf{p} = p\hat{\mathbf{z}}$ is placed at the center of a thin spherical conducting shell of radius R. What is the electric field outside the shell?

   (A) $\dfrac{1}{4\pi\epsilon_0}\dfrac{p}{r^2R}\hat{\mathbf{r}}$
   (B) 0
   (C) $\dfrac{1}{4\pi\epsilon_0}\dfrac{3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}}{r^3}$
   (D) $-\dfrac{1}{4\pi\epsilon_0}\dfrac{p}{r^2R}\hat{\mathbf{r}}$
   (E) $-\dfrac{1}{4\pi\epsilon_0}\dfrac{3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}}{r^3}$

5. The ground state energy of helium is 79 eV. If the ground state wavefunction of helium were a simple product of $1s$ wavefunctions, $\Psi_{100}(\mathbf{r}_1)\Psi_{100}(\mathbf{r}_2)$, the predicted ground state energy would be 108 eV. What is the MAIN factor that accounts for this discrepancy?

   (A) Electron–electron Coulomb repulsion
   (B) Nonzero orbital angular momentum in the ground state
   (C) Spin–spin coupling between the orbital electrons
   (D) Spin–spin coupling between the nucleons
   (E) None of these

6. The energy of gamma rays from a transition of a nucleus from the first excited state to its ground state is measured. Which of the following is true of the measurement?

   (A) Gamma rays from this transition are part of a continuum of gamma rays from the de-excitation of low-lying states.
   (B) The measured mean energy must correspond to the energy of a vibrational state of the nucleus.
   (C) The measured width of the spectral peak must be $\hbar/(2\tau)$, where τ is the lifetime of the excited state.
   (D) The measured mean energy is greater than the true transition energy.
   (E) The measured mean energy is less than the true transition energy.

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7. A system of electrons is in a box of fixed volume. If the number of electrons in the box is doubled, the Fermi energy is multiplied by a factor of

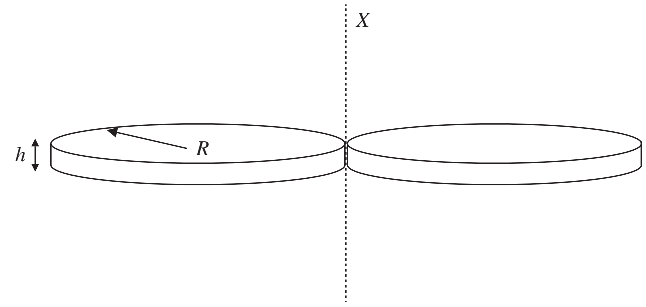
(A) $2^{-1/2}$
(B) $2^{1/2}$
(C) $2^{2/3}$
(D) 2
(E) $2^{3/2}$

8. A gas of electrons is confined to a two-dimensional surface at $z = 0$ but is otherwise free to move in the x- and y-directions. An external magnetic field is applied so that the electrons feel a harmonic oscillator potential, $U = \frac{1}{2}m\omega^2(x^2 + y^2)$. The temperature of the system is well above the Fermi temperature. What is the specific heat per particle of the electron gas?

(A) $\frac{1}{2}k$
(B) k
(C) $\frac{3}{2}k$
(D) $2k$
(E) $\frac{5}{2}k$

9. A particle with mass m and angular momentum l moves in a constant central potential $U(r) = -k/r$, with $k > 0$. What, if any, is the radius of its stable circular orbit?

(A) The particle has no allowed stable circular orbit.
(B) $\dfrac{l^2}{mk}$
(C) $\dfrac{l^2}{2mk}$
(D) $\dfrac{2l^2}{mk}$
(E) $\dfrac{2l^2}{3mk}$



10. Two identical disks shown in the figure above, each of thickness h, radius R, and mass M, are rigidly attached at a point on their edges. What is the moment of inertia of the pair of disks about an axis X, perpendicular to the plane of the disks, which passes through the point where the disks are connected?

(A) MR^2
(B) $\frac{3}{2}MR^2$
(C) $3MR^2$
(D) $6MR^2$
(E) $\frac{3}{2}MRh$

11. A distant galaxy is located at redshift 2. What is the observed wavelength of the 21 cm hyperfine transition line of hydrogen originating from the galaxy?

(A) 7 cm
(B) 10.5 cm
(C) 21 cm
(D) 42 cm
(E) 63 cm

12. A quantum system is prepared in a state $|\Psi\rangle$, which is a superposition of three energy eigenstates. The first two states in $|\Psi\rangle$ have energies E_0 and $2E_0$, and they are measured with probabilities $1/4$ and $1/2$, respectively. If the energy expectation value in the state $|\Psi\rangle$ is $\frac{9}{4}E_0$, what is the energy of the third eigenstate in $|\Psi\rangle$?
   (A) $\frac{3}{4}E_0$
   (B) E_0
   (C) $2E_0$
   (D) $4E_0$
   (E) $\frac{43}{16}E_0$

13. The Planck mass is given by which of the following expressions?
   (A) $\sqrt{\frac{\hbar G}{c^3}}$
   (B) $\sqrt{\frac{\hbar G}{c^5}}$
   (C) $\sqrt{\frac{\hbar c^3}{G}}$
   (D) $\sqrt{\frac{\hbar c^5}{Gk^2}}$
   (E) $\sqrt{\frac{\hbar c}{G}}$

14. A spaceship traveling at $0.6c$ towards a planet transmits a signal at 1 GHz to the planet's inhabitants. What frequency is the signal when it is received on the planet?
   (A) 1 GHz
   (B) 2 GHz
   (C) 2.5 GHz
   (D) 4 GHz
   (E) 8 GHz

15. How much work is required to move a point charge q from infinity to a distance d above an infinite conducting plane?
   (A) $-\frac{1}{4\pi\epsilon_0}\frac{q^2}{4d}$
   (B) $-\frac{1}{4\pi\epsilon_0}\frac{q^2}{2d}$
   (C) $\frac{1}{4\pi\epsilon_0}\frac{q^2}{4d}$
   (D) $\frac{1}{4\pi\epsilon_0}\frac{q^2}{2d}$
   (E) 0

16. An initially uncharged 10-μF parallel-plate capacitor is charged with a constant current of 1 mA. What is the potential difference between the plates after one second?
   (A) 0.01 V
   (B) 1 V
   (C) 10 V
   (D) 100 V
   (E) 1000 V

17. A particle in a one-dimensional infinite square well between $x = 0$ and $x = L$ is subject to the following perturbation:

$$\delta V(x) = \begin{cases} V_0, & x < L/2, \\ 0, & \text{otherwise.} \end{cases}$$

   What is the leading-order shift in the energy of the first excited state? Recall that the wavefunction for the first excited state is

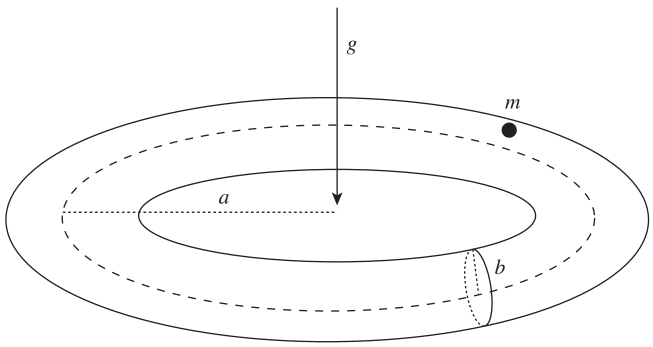
$$\psi(x) = \sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}.$$

   (A) $-V_0$
   (B) V_0
   (C) $V_0/4$
   (D) 0
   (E) $V_0/2$

18. Which of the following is NOT true about the isothermal expansion phase of a Carnot cycle?
   (A) The free energy of the gas increases.
   (B) The entropy of the gas increases.
   (C) The isothermal expansion phase is reversible.
   (D) The expansion takes place at the temperature of the "hot" reservoir.
   (E) The gas does work on its surroundings.

19. Monochromatic blue light of wavelength 450 nm is shined on a slit of width a. A diffraction pattern is observed on a screen 10 m away. What must a be such that the width of the central diffraction maximum is 100 times the width of the slit?

(A) 45 nm
(B) 450 nm
(C) 0.045 mm
(D) 0.21 mm
(E) 0.30 mm



Questions 20 and 21 refer to a particle of mass m, confined to the surface of a torus with central radius a and cross-sectional radius b, oriented such that the Earth's gravitational field points perpendicular to the plane of the circle of radius a. Letting ϕ and θ be the angular coordinates on the circles of radii a and b, respectively, a Lagrangian for this system is

$$L = \frac{1}{2}m(a + b\cos\theta)^2\dot{\phi}^2 + \frac{1}{2}mb^2\dot{\theta}^2 - mgb\sin\theta.$$

20. What is the conjugate momentum to ϕ?

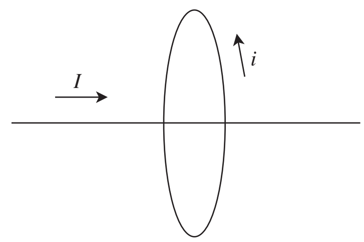
(A) $\frac{1}{2}m\dot{\phi}(a + b\cos\theta)^2$
(B) $m\dot{\phi}(a + b\cos\theta)^2$
(C) $\frac{1}{2}mb^2\dot{\theta}$
(D) $mb^2\dot{\theta}$
(E) $mgb\cos\theta$

21. Which of the following quantities represents the total energy?

(A) L
(B) $L + mgb\sin\theta$
(C) $L - mgb\sin\theta$
(D) $L + 2mgb\sin\theta$
(E) $L - 2mgb\sin\theta$

22. A resistor with resistance R and an inductor with inductance L are in series with a voltage source. For $t < 0$, the voltage is 0. For $t > 0$, the voltage source is V. What time t does it take for the voltage across the inductor to drop to half of its initial level?

(A) $\frac{L\ln 2}{R}$
(B) $\frac{L}{R}$
(C) $\frac{L}{R\ln 2}$
(D) $\frac{2L}{R}$
(E) 0



23. A straight wire carrying current I passes through the center of a circular wire carrying current i. If the circular loop of wire has radius R, what is the tension on the circular wire due to the field produced by the straight wire?

(A) $\frac{\mu_0 iI}{2\pi R^2}$
(B) $\frac{\mu_0 I^2}{2\pi R}$
(C) $\frac{\mu_0 i^2}{2\pi R}$
(D) $\frac{\mu_0 iI}{2\pi R}$
(E) 0

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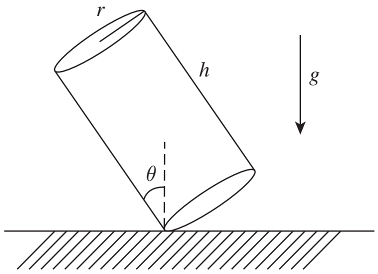


24. A uniform cylinder of height h and radius r is placed on a flat surface and tipped at an angle θ from the vertical. Find θ_0 such that, when the cylinder is released from $\theta > \theta_0$, it falls over.

(A) $\arctan(2r/h)$
(B) $\arctan(r/h)$
(C) $\arctan(r/2h)$
(D) $\arccos(2r/h)$
(E) $\arccos(r/h)$

25. Consider a beam of muons with energy 3 GeV. The muon's mass is approximately 100 MeV/c^2, and its lifetime at rest is 2×10^{-6} s. What is the muon lifetime measured by an experimenter in the lab?

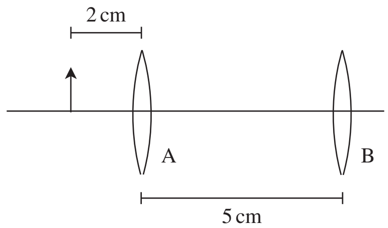
(A) 67 ns
(B) 20 μs
(C) 60 μs
(D) 20 ms
(E) 60 ms

26. Graphene, a two-dimensional allotrope of carbon, displays unusual electronic properties. In particular, the dispersion relation for conduction electrons in graphene is

(A) $\omega \propto \sqrt{|k|}$
(B) $\omega \propto |k|$
(C) $\omega \propto |k|^2$
(D) $\omega \propto |k|^3$
(E) $\omega \propto |k|^4$

27. An electron placed in a one-dimensional harmonic oscillator potential $V = \frac{1}{2}kx^2$ is subject to a uniform electric field $\mathbf{E} = E_0\hat{\mathbf{x}}$. For small E_0, the lowest-order, nonzero correction to the ground state energy is

(A) independent of E_0
(B) proportional to E_0
(C) proportional to E_0^2
(D) proportional to E_0^3
(E) proportional to E_0^4



28. In the optical arrangement shown above, converging lenses A and B both have focal length 5 cm. An object is placed 2 cm to the left of lens A. Where is the image of the object located?

(A) 5 cm to the right of B
(B) 6.25 cm to the right B
(C) 12.5 cm to the left of B
(D) 12.5 cm to the right of B
(E) No image is formed.

29. A star of mass m orbits a galaxy of mass M in a circular orbit. The MOND theory postulates that, at small accelerations, Newton's Second Law is replaced by the force law $F = ma^2/a_0$, where a_0 is a constant with dimensions of acceleration. Assuming the MOND force law and Newton's Law of Gravity, what is the relation between the velocity v of the star and the radius of its orbit r?

(A) v is independent of r
(B) v is proportional to $r^{-1/2}$
(C) v is proportional to $r^{1/4}$
(D) v is proportional to r^{-1}
(E) v is proportional to r^2

30. Which values of spin quantum numbers are NOT possible for a system consisting of a spin-1 particle and a spin-2 particle?

(A) $s = 3, m_s = 3$
(B) $s = 1, m_s = 0$
(C) $s = 2, m_s = 1$
(D) $s = 2, m_s = 0$
(E) $s = 0, m_s = 0$

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31. The radial wavefunction of the $2p$ state of hydrogen is

$$R_{21}(r) = \frac{1}{\sqrt{24}} a_0^{-5/2} r \exp(-r/2a_0),$$

where a_0 is the Bohr radius. What is the most probable value of r in this state?

(A) $a_0/2$
(B) a_0
(C) $2a_0$
(D) $4a_0$
(E) $6a_0$

32. Mass spectrometry uses which of the following physical properties of ions to determine the chemical makeup of a substance?

(A) Dipole moment
(B) Nuclear spin
(C) Charge-to-mass ratio
(D) Atomic number
(E) Electronegativity

33. A musician tuning a violin to a tuning fork at 440 Hz hears a beat frequency of 3 Hz. What is the frequency of the note produced by the violin?

(A) 428 Hz
(B) 434 Hz
(C) 437 Hz
(D) 443 Hz
(E) It is impossible to tell from the given information.

34. A spaceship is traveling directly towards a planet at speed $0.5c$. When the ship is a distance of 1 light-hour away from the planet (as measured in the frame of the planet), it fires a missile at the planet with speed $0.5c$. An observer on the planet sees the flash of light from the missile at time t_1, followed by the missile impact at time t_2. What is $t_2 - t_1$?

(A) 0 min
(B) 10 min
(C) 15 min
(D) 30 min
(E) 60 min

35. An ice skater is spinning with arms extended at an angular velocity of 5.0 radians/second. After drawing her arms in, her new angular velocity is 8.0 radians/second. If the skater's moment of inertia with arms extended was I, her moment of inertia with arms drawn in is

(A) I
(B) $3I$
(C) $\frac{8}{5}I$
(D) $\frac{5}{8}I$
(E) $\sqrt{\frac{5}{8}}I$

36. Suppose that a particle in a one-dimensional system has a Lagrangian L with a potential that is constant in time and such that

$$\frac{\partial L}{\partial t} = 0,$$
$$\frac{\partial L}{\partial x} = 0.$$

Which of the following must be true?

I. Energy is conserved.
II. Linear momentum is conserved.
III. The potential is nonzero.
IV. The Euler–Lagrange equations are not satisfied.

(A) I only
(B) II only
(C) I and II
(D) I, II, and III
(E) I, III, and IV

37. A beam of particles with luminosity $10^{22}\ \text{cm}^{-2}\,\text{s}^{-1}$ is incident upon a target with scattering cross section $10^{-20}\ \text{cm}^2$. Assuming a detector has an efficiency of 0.5 for detecting products of the scattering process, how many events will the detector see if the experiment runs for 1 day? Recall that 1 day = 8.64×10^4 seconds.

(A) 4.20×10^2 events
(B) 4.00×10^4 events
(C) 7.20×10^5 events
(D) 4.32×10^6 events
(E) 8.64×10^6 events

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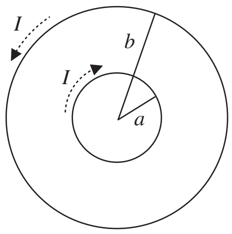
38. An electron in a cyclotron moves in a circular orbit at a fixed radius in the presence of a constant magnetic field **B**. If the strength of the magnetic field is tripled, by what factor must the electron's momentum change to keep it orbiting at the same radius?

(A) $\sqrt{3}$
(B) 3
(C) $1/\sqrt{3}$
(D) $1/3$
(E) $3/2$



39. Two circular loops of wire of radii a and b are oriented concentrically in the same plane, and they each carry a current I circulating in opposite directions, as shown in the figure above. What is the magnetic field at the center of the loops?

(A) $\frac{\mu_0 I}{2}\left(\frac{1}{a}-\frac{1}{b}\right)$, pointed out of the page
(B) $\frac{\mu_0 I}{2}\left(\frac{1}{a}-\frac{1}{b}\right)$, pointed into the page
(C) $\frac{\mu_0 I}{4}\left(\frac{1}{a}-\frac{1}{b}\right)$, pointed into the page
(D) $\frac{\mu_0 I}{2}\frac{1}{a}$, pointed out of the page
(E) 0

40. Which of the following is true about the total orbital angular momentum operator, L^2, of a particle subjected to an arbitrary force?

I. Always commutes with L_x, L_y, L_z
II. Always commutes with the total angular momentum J^2
III. Always commutes with the Hamiltonian

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I, II, and III

41. A quantum system has a Hamiltonian given by

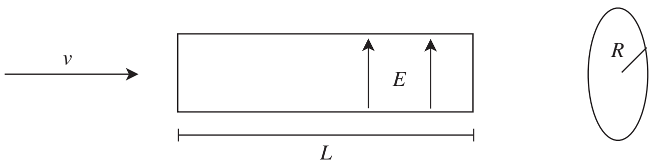
$$H = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix},$$

where a, b, c are real positive constants. What are the possible results of a measurement of the energy of the system?

(A) $b, \pm a$
(B) $a, \pm b$
(C) $a, b, a+b$
(D) $a, \pm\sqrt{ab}$
(E) $a, \pm b^2$

42. If magnetic monopoles existed, which of the following expressions would be proportional to the "magnetic charge" of the monopole? You may assume that there are no other sources of electric or magnetic fields present.

(A) $\int (\nabla \cdot \mathbf{E})\, dV$
(B) $\int (\nabla \cdot \mathbf{B})\, dV$
(C) $\int |\mathbf{E}|^2\, dV$
(D) $\int |\mathbf{B}|^2\, dV$
(E) $\int (\mathbf{E} \cdot \mathbf{B})\, dV$



43. A beam of nonrelativistic protons (mass m, charge q) of velocity v enters a region of length L with an electric field E perpendicular to the direction of the beam. At the end of the region of length L is a circular target of radius R. Assuming that the diameter of the beam is much smaller than R, what is the minimum electric field E needed to deflect all protons before they strike the target?

(A) $\dfrac{mLv^2}{2qR^2}$
(B) $\dfrac{2mLv^2}{qR^2}$
(C) $\dfrac{mRv^2}{q^2L^2}$
(D) $\dfrac{2mRv^2}{qL^2}$
(E) $\dfrac{4mLv^2}{qR^2}$

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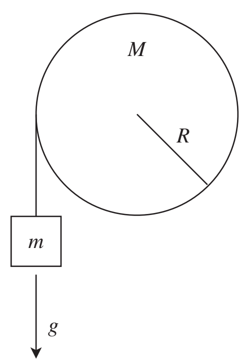
44. Put the following in chronological order, starting with the earliest.

I. Epoch of reionization
II. Nucleosynthesis
III. Inflation

(A) I, II, III
(B) I, III, II
(C) II, I, III
(D) III, I, II
(E) III, II, I

45. For a *monoatomic* ideal gas, which of the following is constant during adiabatic changes of state?

(A) $PV^{1/2}$
(B) PV
(C) $PV^{5/3}$
(D) $PV^{7/5}$
(E) $PV^{9/7}$



46. A string of length L and negligible mass is completely wound around a solid cylinder of uniform density, of mass M and radius R, and it has a small weight of mass m attached to its end. If the weight is released from rest under the influence of gravity, what is its velocity when the string is entirely unwound?

(A) $\sqrt{\dfrac{4mgL}{M+2m}}$
(B) $\sqrt{\dfrac{2mgL - MR^2}{2m}}$
(C) $\sqrt{2gL}$
(D) $\sqrt{\dfrac{2(m+M)gL}{m}}$
(E) $\sqrt{\dfrac{2mgL - 2MR^2}{m}}$

47. An object is placed at rest in a potential field $U(x, y, z) = x + y^2 - \cos z$. What is the force on the object?

(A) $\mathbf{F}(x, y, z) = -\hat{\mathbf{x}} - 2y\hat{\mathbf{y}} - \sin z\hat{\mathbf{z}}$
(B) $\mathbf{F}(x, y, z) = x\hat{\mathbf{x}} + 2y\hat{\mathbf{y}} - \cos z\hat{\mathbf{z}}$
(C) $\mathbf{F}(x, y, z) = -x\hat{\mathbf{x}} - 2y\hat{\mathbf{y}} + \cos z\hat{\mathbf{z}}$
(D) $\mathbf{F}(x, y, z) = -\hat{\mathbf{x}} - 2y\hat{\mathbf{y}} + \cos z\hat{\mathbf{z}}$
(E) $\mathbf{F}(x, y, z) = \hat{\mathbf{x}} + 2y\hat{\mathbf{y}} + \sin z\hat{\mathbf{z}}$

48. Consider a system with three energy levels $-\epsilon$, 0, ϵ, and degeneracies $d(-\epsilon) = 2$, $d(0) = 1$, $d(\epsilon) = 3$. What is the energy of the system as $T \to \infty$?

(A) $\epsilon/5$
(B) $\epsilon/6$
(C) $5\epsilon/6$
(D) 0
(E) ϵ

49. In process 1, a monoatomic ideal gas is heated from temperature T to temperature $2T$ reversibly and at constant volume. In process 2, a monoatomic ideal gas freely expands from V to $2V$. Which is the correct relationship between the change in entropy ΔS_1 in process 1 and the change in entropy ΔS_2 in process 2?

(A) $0 < \Delta S_1 < \Delta S_2$
(B) $0 < \Delta S_1 = \Delta S_2$
(C) $0 = \Delta S_1 < \Delta S_2$
(D) $0 < \Delta S_2 < \Delta S_1$
(E) $\Delta S_1 = \Delta S_2 < 0$

50. An electromagnetic wave propagates in vacuum with electric field $E_0 \cos(kx - \omega t)\hat{\mathbf{z}}$. What is the average magnitude of the Poynting vector in SI units, where the average is taken over one period of oscillation?

(A) $\dfrac{4E_0^2}{c\mu_0}$
(B) 0
(C) $\dfrac{E_0^2}{c\mu_0}$
(D) $\dfrac{E_0^2}{2c\mu_0}$
(E) $-\dfrac{E_0^2}{2c\mu_0}$

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51. An observation of the reaction $e^+e^- \rightarrow \gamma$ would necessarily violate which of the following conservation laws?

(A) Lepton number
(B) Baryon number
(C) Energy–momentum
(D) Angular momentum
(E) Charge conservation

52. The nucleus can be modeled as a degenerate Fermi gas. If the Fermi momentum of nucleons in the carbon nucleus is measured to be 40 MeV/c, which of the following is an approximate lower bound on the nuclear radius?

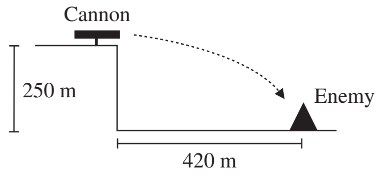
(A) 1 nm
(B) 10 pm
(C) 100 fm
(D) 1 fm
(E) 0.01 fm

53. Which of the following does NOT obey Bose–Einstein statistics?

(A) Neutrinos
(B) Photons
(C) ^{4}He nuclei
(D) ^{4}He atoms
(E) Pions

54. The observation of a sharp line of gamma rays of energy 511 keV from the center of our galaxy is most naturally explained by which of the following processes?

(A) Helium hyperfine transitions
(B) Hawking radiation
(C) Ammonia maser transitions
(D) Electron–positron annihilation
(E) Supernovae

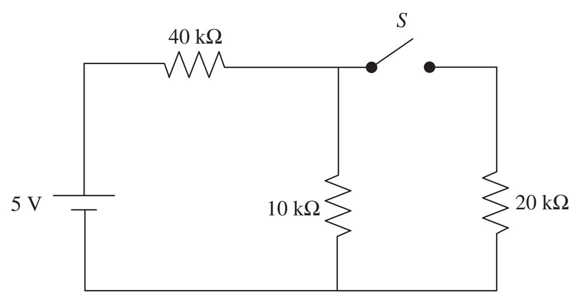


55. A soldier can fire a cannon horizontally from the top of a 250 m cliff. He wants to hit an enemy encampment at a 420 m horizontal distance from the cliff. What must the initial velocity of his cannonball be in order to strike the encampment, neglecting air resistance?

(A) 22.4 m/s
(B) 39.6 m/s
(C) 58.8 m/s
(D) 94.9 m/s
(E) 134.2 m/s

56. A rocket traveling at constant speed v in empty space instantaneously expels 10% of its mass in fuel, which is ejected at speed $v/2$ relative to the rocket. What is the final speed of the rocket? You may assume $v \ll c$.

(A) $v/18$
(B) $21v/20$
(C) $19v/18$
(D) $11v/10$
(E) $3v/2$



57. Consider the circuit shown in the diagram. When switch S is open, the current through the 10 kΩ resistor is I_1. After switch S is closed, the current through the same resistor is I_2. What is I_2/I_1?

(A) 1/4
(B) 5/7
(C) 1
(D) 15/14
(E) 2

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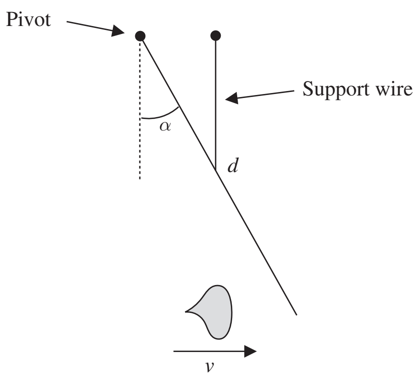


58. A rod of length d and mass M is attached to a pivot and suspended at an angle α from the vertical using a support wire, as shown in the diagram. A lump of clay of mass m is fired at the end of the rod with a velocity v. Just before the clay makes contact with the rod, the wire is cut. Assuming the clay and rod stick together after collision, what is the angular velocity in radians of the rod–clay system? (You may treat the lump of clay as a point mass.)

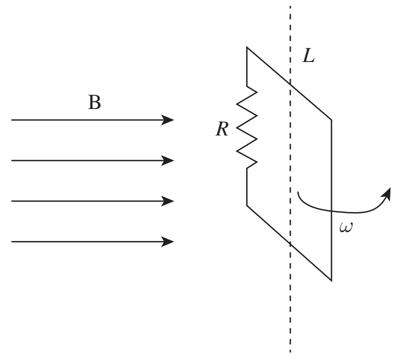
(A) $\dfrac{mv\cos\alpha}{(M+m)d}$

(B) $\dfrac{3mv\sin\alpha}{(M+m)d}$

(C) $\dfrac{3mv\cos\alpha}{(M+3m)d}$

(D) $\dfrac{3mv}{(M+3m)d}$

(E) $\dfrac{3mv}{Md}$



59. A square loop of wire of side length L, containing a load resistor R, is oriented perpendicular to the xy-plane and rotates about the z-axis at angular frequency ω in the presence of a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{x}}$, as shown in the diagram. If $L = 10$ cm, $B_0 = 2$ tesla, and $R = 100.0\ \Omega$, what must ω be so that the average power dissipated in the resistor is 0.5 W?

(A) 25 rad/s
(B) 50 rad/s
(C) 314 rad/s
(D) 354 rad/s
(E) 500 rad/s

60. In calculating the entropy of a microcanonical ensemble, the inverse temperature $\beta = 1/kT$ can be viewed as a Lagrange multiplier enforcing the constraint of fixed total energy. Similarly, the chemical potential μ is related to the Lagrange multiplier for

(A) fermion number
(B) particle number
(C) pressure
(D) volume
(E) magnetization

61. A spin-1/2 particle interacts with a magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ through a Hamiltonian $H = (-\mu_B g B/2\hbar)\sigma_z$, where μ_B is the Bohr magneton and g is the particle's gyromagnetic ratio. Consider a system of these spin-1/2 particles in equilibrium at temperature T. Let A be the ratio of the number of spin-up particles to spin-down particles. If the strength of the magnetic field is doubled, the new ratio of spin-up to spin-down particles is

(A) A^{-2}
(B) A
(C) A^2
(D) e^A
(E) $A\exp(\mu_B g B/\hbar kT)$

62. Which of the following is equivalent to $\nabla^2(1/r)$?

(A) $-4\pi\delta^3(\mathbf{r})$
(B) $4\pi\delta^3(\mathbf{r})$
(C) 0
(D) 4π
(E) -4π

63. The mass of the proton is 1.67×10^{-27} kg. Which of the following is closest to the Compton wavelength of the proton?

(A) 10^{-15} m
(B) 10^{-13} m
(C) 10^{-12} m
(D) 10^{-10} m
(E) 10^{-9} m

Questions 64 and 65 refer to the following scenario. A K^0 of mass m_K and energy E in the lab frame decays to a π^+ and a π^-, both of mass m_π. The π^+ is observed to be emitted parallel to the K^0 momentum.

64. What is the speed of the π^+ in the K^0 rest frame?

(A) $(1 - 4m_K^2/m_\pi^2)^{1/2}c$
(B) $(1 - 4m_\pi^2/m_K^2)^{1/2}c$
(C) $(1 - m_K^2/m_\pi^2)^{1/2}c$
(D) $(1 - m_\pi^2/m_K^2)^{1/2}c$
(E) $2(m_\pi^2/m_K^2)^{1/2}c$

65. What must be the initial K^0 energy such that the π^- is stationary in the lab frame?

(A) $\dfrac{m_\pi^2 c^2}{2m_K}$
(B) $\dfrac{m_K c^2}{2}$
(C) $\dfrac{m_\pi c^2}{2}$
(D) $\dfrac{(m_K^2 + m_\pi^2)c^2}{2m_\pi}$
(E) $\dfrac{m_K^2 c^2}{2m_\pi}$

66. A clarinet can be treated as a half-open pipe, where sounds are produced by standing pressure waves. For a clarinet of length 0.6 m, which of the following is a possible wavelength of a standing wave?

(A) 0.3 m
(B) 0.6 m
(C) 0.8 m
(D) 1.2 m
(E) 1.5 m

67. A sphere has a polarization of $\mathbf{P}(\mathbf{r}) = Cr^2\hat{\mathbf{r}}$. What is the electric field inside the sphere? (You may find the following fact useful: $\nabla \cdot (v(r)\hat{\mathbf{r}}) = \frac{1}{r^2}\frac{d}{dr}(r^2 v(r))$.)

(A) $-\dfrac{4Cr^2}{\epsilon_0}\hat{\mathbf{r}}$
(B) $\dfrac{2Cr^2}{\epsilon_0}\hat{\mathbf{r}}$
(C) $-\dfrac{Cr^2}{\epsilon_0}\hat{\mathbf{r}}$
(D) $\dfrac{Cr^2}{4\pi\epsilon_0}\hat{\mathbf{r}}$
(E) 0

68. Suppose an electromagnetic plane wave propagating in vacuum in the $+\hat{\mathbf{z}}$-direction has a polarization with the electric field in the $+\hat{\mathbf{x}}$-direction immediately before it strikes a perfect conductor at normal incidence. What are the directions of the **E** and **B** vectors of the transmitted wave?

(A) **E** in $+\hat{\mathbf{x}}$-direction & **B** in $+\hat{\mathbf{y}}$-direction
(B) **E** in $-\hat{\mathbf{x}}$-direction & **B** in $+\hat{\mathbf{y}}$-direction
(C) **E** in $+\hat{\mathbf{x}}$-direction & **B** in $-\hat{\mathbf{y}}$-direction
(D) **E** in $-\hat{\mathbf{x}}$-direction & **B** in $-\hat{\mathbf{y}}$-direction
(E) There is no transmitted wave in a perfect conductor.

69. What is the value of the following commutator?

$$[[[L_x, L_y], L_x], L_x]$$

(A) $-i\hbar^3 L_z$
(B) $i\hbar^3 L_z$
(C) $-i\hbar^3 L_y$
(D) $i\hbar^3 L_y$
(E) $-i\hbar^3 L_x$

70. The vibrational frequency of diatomic oxygen is approximately 5×10^{13} Hz. The temperature at which the vibrational modes of O_2 will begin to be excited is closest to

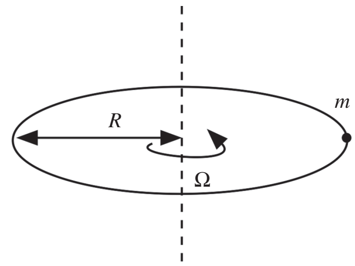
(A) 20 K
(B) 200 K
(C) 2,000 K
(D) 20,000 K
(E) 2×10^5 K

71. Which of the following does NOT represent a possible observable, written in the position basis, for a free particle in three dimensions?

(A) $-i\hbar\nabla$
(B) $x^2\partial/\partial y$
(C) $x\partial^2/\partial y^2$
(D) $x^2y^2z^2$
(E) xyz

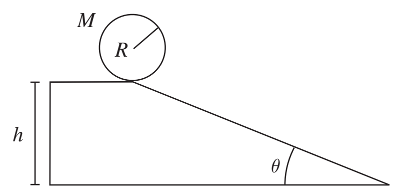
72. The BCS theory of superconductivity explains the superconducting properties of metals at low temperature by supposing that a macroscopic number of metallic electrons all lie in the same ground state. Why does this not violate the Pauli exclusion principle?

(A) BCS theory is incorrect.
(B) Electrons pair off into Cooper pairs, which behave as bosons.
(C) Spin–spin coupling prevents electrons from being in the same state.
(D) The Pauli exclusion principle does not apply to systems at low temperature.
(E) Electrons are not fermions.



73. A hoop of radius R rotates at constant angular velocity Ω. A small bead of mass m is attached to the hoop, with a frictional force on the bead proportional to the difference in velocity between the bead and edge of the hoop, $F = k(R\Omega - R\omega)$, where ω is the angular velocity of the bead. If the bead begins at angular velocity ω_0, which of the following describes its subsequent motion?

(A) $\omega(t) = \omega_0 e^{-kt/m}$
(B) $\omega(t) = \Omega - \omega_0 e^{-kt/m}$
(C) $\omega(t) = \Omega - \omega_0 e^{-mt/k}$
(D) $\omega(t) = \Omega(1 - e^{-kt/m})$
(E) $\omega(t) = \Omega - (\Omega - \omega_0)e^{-kt/m}$



74. Consider a cylinder of radius R, mass M, and density $\rho(r) = Ar^{\alpha}$ that starts at rest and rolls without slipping down an inclined plane of height h at an angle θ. Assuming no rolling friction, the final velocity of the cylinder at the bottom depends ONLY on

(A) θ and α
(B) M and R
(C) h and M
(D) h and θ
(E) h and α

75. The Δ is a spin-3/2 bound state of three spin-1/2 quarks. The spin part of the wavefunction of the state with $m = +3/2$ is $|\Psi\rangle = |\uparrow\uparrow\uparrow\rangle$. What is the spin part of the wavefunction with definite spin $m = -1/2$?

(A) $|\uparrow\downarrow\downarrow\rangle$
(B) $\frac{1}{\sqrt{3}}(|\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle)$
(C) $\frac{1}{\sqrt{3}}(-|\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle)$
(D) $\frac{1}{\sqrt{3}}(|\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle)$
(E) $|\downarrow\downarrow\downarrow\rangle$

76. What is true of the electromagnetic field at a p-n junction at equilibrium with zero bias voltage applied?

(A) The electric field points toward the p-type semiconductor.
(B) The electric field points toward the n-type semiconductor.
(C) The electric field is parallel to the interface between the p-type and n-type semiconductors.
(D) There is no electromagnetic field.
(E) There is no electric field, but there is a magnetic field pointing toward the n-type semiconductor.

77. In an inertial frame S, two events E_1 and E_2 occur at $(t, x, y, z) = (3, 4, 1, 1)$ and $(1, 3, 0, 1)$, respectively (in units where $c = 1$). In another inertial frame S', which of the following could an observer measure as the spacetime 4-vector between E_1 and E_2?

(A) $(1, 0.5, 1, 1)$
(B) $(2, 1, 0, 0)$
(C) $(3, 2, \sqrt{3}, 0)$
(D) $(2, 0, \sqrt{3}, 0)$
(E) None of these

78. A dark matter experiment takes data for a time T and observes no events. What is the 90% confidence level upper limit that one can place on the event rate in the detector?

(A) One cannot place a limit at the 90% confidence level for this experiment.
(B) $-(1/T)\ln 0.9$
(C) $-(1/T)\ln 0.1$
(D) $(1/T)\ln 0.9$
(E) 0

79. If the proton were a spin-0 particle, which of the following features of the hydrogen energy spectrum would be absent?

(A) Lyman series
(B) Balmer series
(C) 21 cm hyperfine transition
(D) Lamb shift
(E) fine-structure splitting of the $2p$ state

80. An electron neutrino emitted from the Sun may be detected as a tau neutrino on Earth because:

(A) Conservation of lepton number does not apply to tau neutrinos.
(B) Electron neutrinos from the Sun can annihilate and be reemitted as a pair of tau neutrinos.
(C) Electron neutrinos interact with the Earth's magnetic field.
(D) A freely propagating neutrino is a superposition of electron and tau neutrinos.
(E) Electron neutrinos decay faster than tau neutrinos.

81. A pair of electrons is trapped in a "quantum dot." A magnetic field is applied along the z-direction so that the singlet state has energy $-\epsilon$, and the triplet state has energies $-\epsilon/2$, $-\epsilon$, and $-3\epsilon/2$ for spins $+\hbar$, 0, and $-\hbar$ along the z-axis, respectively. What is the probability of finding the electrons in the triplet state, at temperature T?

(A) 0
(B) 1
(C) $\dfrac{2}{2 + e^{\epsilon/2kT} + e^{-\epsilon/2kT}}$
(D) $\dfrac{e^{\epsilon/2kT} + e^{-\epsilon/2kT}}{2 + e^{\epsilon/2kT} + e^{-\epsilon/2kT}}$
(E) $\dfrac{1 + e^{\epsilon/2kT} + e^{-\epsilon/2kT}}{2 + e^{\epsilon/2kT} + e^{-\epsilon/2kT}}$

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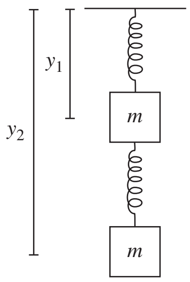


82. The diagram above illustrates a system consisting of a block of mass m hanging from a spring of spring constant k, with another block of mass m hanging from the first block by another spring of spring constant k. What is the total energy of this system?

(A) $\frac{1}{2}m(\dot{y_1}^2+\dot{y_2}^2)+\frac{1}{2}k(y_1^2+(y_2-y_1)^2)-mg(y_1+y_2)$
(B) $\frac{1}{2}m(\dot{y_1}^2+\dot{y_2}^2)+\frac{1}{2}k(y_1^2+(y_2-y_1)^2)+mg(y_1+y_2)$
(C) $\frac{1}{2}m(\dot{y_1}^2+\dot{y_2}^2)-\frac{1}{2}k(y_1^2+(y_2-y_1)^2)+mg(y_1+y_2)$
(D) $\frac{1}{2}m(\dot{y_1}^2+\dot{y_2}^2)-\frac{1}{2}k(y_1^2+(y_2-y_1)^2)-mg(y_1+y_2)$
(E) $-\frac{1}{2}m(\dot{y_1}^2+\dot{y_2}^2)-\frac{1}{2}k(y_1^2+(y_2-y_1)^2)-mg(y_1+y_2)$

83. A particle of mass m is in the ground state of an infinite square well of size a, with energy E. The well suddenly expands to size $2a$. What is E'/E, where E' is the expectation value of the energy of the particle after this sudden expansion?

(A) 0
(B) 1
(C) $1/\sqrt{2}$
(D) 1/2
(E) 1/4

84. A particle of mass m and energy E is incident from the left on a delta-function barrier, $V(x) = \alpha\delta(x)$ with $\alpha > 0$. Which of the following gives the coefficient of reflection for the system?

(A) α^2
(B) $\alpha^2 E$
(C) $\dfrac{\alpha}{\hbar}\sqrt{\dfrac{m}{2E}}$
(D) $\dfrac{1}{1+2\hbar^2 E/m\alpha^2}$
(E) $\dfrac{1}{1+m\alpha^2/2\hbar^2 E}$

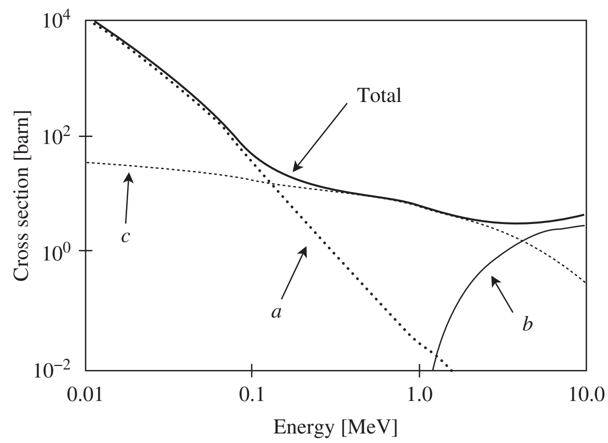
85. Which of the following is NOT true about the $2s \to 1s$ transition in the hydrogen atom?

(A) The dominant decay mode is two-photon emission.
(B) It violates $\Delta l = \pm 1$.
(C) It violates $\Delta m = \pm 1$ or 0.
(D) It cannot occur in the electric dipole approximation.
(E) None of these.

86. Measurements of the electric dipole moment of the neutron provide sensitive tests of fundamental physics. If the neutron were found to have a nonzero electric dipole moment, one could directly conclude that which of the following symmetries is violated?

I. Parity
II. Charge conjugation
III. Time reversal

(A) I
(B) II
(C) III
(D) I and II
(E) I and III



87. The figure above shows the total cross section for photon scattering on a Pb atom as well as the cross sections for several individual process. Why does curve b drop quickly near 1 MeV?

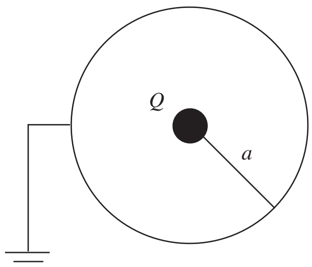
(A) Penetration depth of low-energy photons is small.
(B) Interactions with electrons become significant.
(C) 1.022 MeV threshold for pair production.
(D) Pb has no absorption lines below 1 MeV.
(E) Conservation of angular momentum.

88. Let $f(x) = x$ for $x \in [-\pi, \pi]$. What is the coefficient a_1 of the cos x term in the Fourier series for $f(x)$?

(A) 0
(B) π
(C) 1
(D) 2
(E) 4

89. Suppose that the magnetic field in a region of space is given by $\mathbf{B} = B_0(\hat{\mathbf{x}} + 2x\hat{\mathbf{z}})$. Which of the following could be the vector potential?

(A) $B_0(x\hat{\mathbf{y}} + x^2\hat{\mathbf{z}})$
(B) $-B_0(x\hat{\mathbf{y}} + x^2\hat{\mathbf{z}})$
(C) $-B_0(x^2\hat{\mathbf{y}} + y\hat{\mathbf{z}})$
(D) $B_0(y^2\hat{\mathbf{x}} + z\hat{\mathbf{y}})$
(E) $B_0(x^2\hat{\mathbf{y}} + y\hat{\mathbf{z}})$



90. Consider a charge configuration consisting of a ball of charge Q surrounded by a thin conducting shell of radius a. The conductor initially has no net charge, but is then connected to ground (the potential at infinity). What is the change in energy of the configuration?

(A) $-\dfrac{Q^2}{4\pi\epsilon_0 a}$
(B) $\dfrac{Q^2}{4\pi\epsilon_0 a}$
(C) $\dfrac{Q^2}{8\pi\epsilon_0^2 a^2}$
(D) $-\dfrac{Q^2}{8\pi\epsilon_0 a}$
(E) $\dfrac{Q^2}{8\pi\epsilon_0 a}$

91. Without the hypothesis of quark color, the quark model would be unable to explain the existence of which of the following spin-3/2 baryons? You may assume the quarks have zero relative orbital angular momentum.

(A) *udd*
(B) *uud*
(C) *uuu*
(D) *uds*
(E) All of the above baryons are allowed.

92. In tabletop atomic spectroscopy experiments using free nuclei, the difference between the frequencies of emitted and absorbed photons driven by the same electronic transition is due to

(A) measurement error
(B) nuclear recoil
(C) gravitational redshift
(D) time dilation
(E) none of these

93. A sequence of NAND gates can create which of the following effective logic gates?

(A) AND
(B) OR
(C) NOT
(D) NOR
(E) all of the above

94. A particle moving in one dimension has the following Lagrangian:

$$L = \frac{1}{2}A\dot{q}^2 - Bq^2.$$

What is the equation of motion of the particle?

(A) $\dot{q} = \dfrac{2B}{A}q$
(B) $\dot{q} = -\dfrac{2B}{A}q$
(C) $\ddot{q} = \dfrac{2B}{A}q$
(D) $\ddot{q} = \dfrac{2A}{B}q$
(E) $\ddot{q} = -\dfrac{2B}{A}q$

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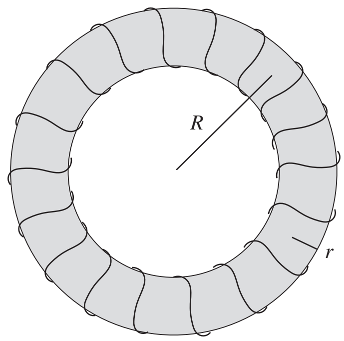

95. A toroidal solenoid of radius R and cross-sectional radius $r \ll R$ has N winds and carries current R. The volume enclosed by the torus is $2\pi^2 Rr^2$. What is the energy stored in the toroidal solenoid?

(A) 0
(B) $\dfrac{\mu_0 N I^2 r^2}{4\pi R^3}$
(C) $\dfrac{\mu_0 N^2 I^2 r^2}{4\pi R^3}$
(D) $\dfrac{\mu_0 N I^2 r^2}{4R}$
(E) $\dfrac{\mu_0 N^2 I^2 r^2}{4R}$

96. Which of the following is true about a longitudinally polarized wave in three dimensions?

I. There are two linearly independent polarization vectors.
II. The polarization vector(s) is/are perpendicular to the wavevector.
III. The polarization vector(s) is/are parallel to the wavevector.

(A) III only
(B) II only
(C) I only
(D) I and II
(E) I and III

97. Deep water waves obey the dispersion relation $\omega = A\sqrt{k}$, where A is a constant. What is the correct relationship between phase velocity and group velocity for deep water waves?

(A) $v_{\text{phase}} = \frac{1}{2} v_{\text{group}}$
(B) $v_{\text{phase}} = v_{\text{group}}$
(C) $v_{\text{phase}} = 2 v_{\text{group}}$
(D) $v_{\text{phase}} v_{\text{group}} = A^4 t^2$
(E) none of these

98. When light of 5000 Å is shined on a thin film of oil ($n = 1.5$) that sits on top of a medium with $n = 2.0$, the intensity of reflected light is minimized. What is the thickness of the oil?

(A) 4×10^{-8} m
(B) 8.33×10^{-8} m
(C) 1.67×10^{-7} m
(D) 1.25×10^{-7} m
(E) 5.0×10^{-7} m

99. Suppose a particle has a normalized wavefunction $\psi(x)$ given by

$$\psi(x) = \begin{cases} \sqrt{3}(1-x), & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

What is the expectation value of the position of this particle?

(A) 0
(B) 1
(C) 1/12
(D) 1/4
(E) 1/2

100. What are the energy levels of a quantized system consisting of a massless rigid rod of length a connecting two masses m, where n is a non-negative integer?

(A) $\dfrac{\hbar^2 n(n+1)}{ma^2}$
(B) $\dfrac{\hbar^2 n(n+1)}{2ma^2}$
(C) $\dfrac{\hbar^2 n}{2ma^2}$
(D) $\dfrac{\hbar^2 n}{ma^2}$
(E) $\dfrac{\hbar^2 (n+1)}{ma^2}$

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Sample Exam 2

TABLE OF INFORMATION

Rest mass of the electron	m_e	$=$	9.11×10^{-31} kg
Magnitude of the electron charge	e	$=$	1.60×10^{-19} C
Avogadro's number	N_A	$=$	6.02×10^{23}
Universal gas constant	R	$=$	8.31 J/(mol · K)
Boltzmann's constant	k	$=$	1.38×10^{-23} J/K
Speed of light	c	$=$	3.00×10^{8} m/s
Planck's constant	h	$=$	$6.63 \times 10^{-34}\mathrm{J \cdot s} = 4.14 \times 10^{-15}\ \mathrm{eV \cdot s}$
	$\hbar$	$=$	$h/2\pi$
	hc	$=$	$1240\ \mathrm{eV \cdot nm}$
Vacuum permittivity	ϵ_0	$=$	$8.85 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$
Vacuum permeability	μ_0	$=$	$4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$
Universal gravitational constant	G	$=$	$6.67 \times 10^{-11}\ \mathrm{m^3/(kg \cdot s^2)}$
Acceleration due to gravity	g	$=$	$9.80\ \mathrm{m/s^2}$
1 atmosphere pressure	1 atm	$=$	$1.0 \times 10^5\ \mathrm{N/m^2} = 1.0 \times 10^5$ Pa
1 angstrom	1 Å	$=$	1×10^{-10} m $= 0.1$ nm

Prefixes for Powers of 10

10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	µ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P

Rotational inertia about center of mass

Rod	$\frac{1}{12}M\ell^2$
Disk	$\frac{1}{2}MR^2$
Sphere	$\frac{2}{5}MR^2$

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SAMPLE EXAM 2

Time — 170 minutes

100 questions

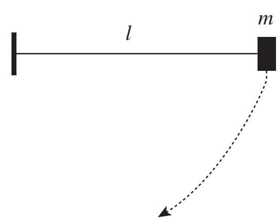
Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then fill in the corresponding space on the answer sheet.

1. A ball of mass m is dropped from a tall building, and experiences a velocity-dependent air resistance force $F = bv$. What is its terminal velocity?

   (A) $\frac{b}{mg}$
   (B) $\frac{mb}{g}$
   (C) $e^{b/m}$
   (D) $\frac{mg}{b}$
   (E) $\frac{mg}{b}(1 - e^{-b/m})$

2. A charged particle moving in the direction $\hat{\mathbf{n}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + \hat{\mathbf{y}})$ enters a region of uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{x}}$. The path of the particle after it enters the field is a

   (A) circle
   (B) cycloid
   (C) helix
   (D) straight line
   (E) logarithmic spiral



3. A massless rope of length l, attached to a fixed pivot at one end and with a mass m at the other end, is held horizontally and then released, as shown in the diagram. When the mass is at its lowest point, the tension in the rope is

   (A) 0
   (B) $gl/2$
   (C) mg
   (D) $2mg$
   (E) $3mg$

4. A particle of charge q and mass m is suspended from a massless string. A constant electric field of known magnitude is turned on, perpendicular to the direction of gravity, and the rope forms some angle α with the vertical. A measurement of α determines which of the following quantities?

   (A) m
   (B) q
   (C) q/m
   (D) qm
   (E) none of the above

5. A hydrogen atom transitions from the $n = 3$ to $n = 2$ states by emitting a photon. What is the wavelength of the photon?

   (A) 347 nm
   (B) 657 nm
   (C) 985 nm
   (D) 2.32 μm
   (E) 1.34 mm

6. For a quantum operator to represent a physical observable, it must be

   (A) Hermitian
   (B) positive-definite
   (C) finite-dimensional
   (D) symmetric
   (E) none of the above

7. If the net force on an object is zero, which of the following MUST be true?

   I. Its angular momentum is constant.
   II. Its velocity is zero.
   III. Its acceleration is zero.

   (A) I only
   (B) II only
   (C) III only
   (D) I and II
   (E) I and III

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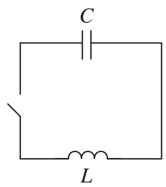
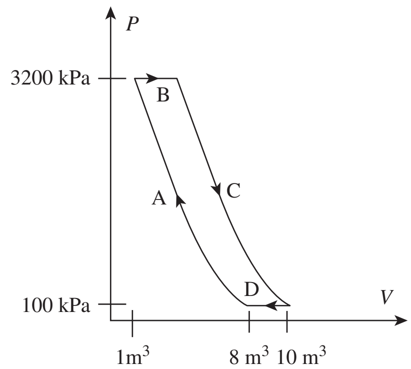
8. Fluorine is not naturally found as free atoms, but rather in compounds as the ion Fl^-. The electron configuration of a neutral fluorine atom is

(A) $1s^2\,2s^1$
(B) $1s^2\,2s^2$
(C) $1s^2\,2s^2\,2p^1$
(D) $1s^2\,2s^2\,2p^5$
(E) $1s^2\,2s^2\,2p^6$



9. In the circuit shown in the diagram, the capacitor is initially charged, and the switch is closed at $t = 0$. Assuming all circuit elements have negligible resistance, the peak magnitude of the current is achieved at

(A) $t = \frac{\pi}{4}\sqrt{\frac{L}{C}}$
(B) $t = \frac{\pi}{\sqrt{LC}}$
(C) $t = 2\pi\sqrt{LC}$
(D) $t = \frac{\pi}{2}\sqrt{LC}$
(E) $t = \frac{\pi}{2\sqrt{LC}}$



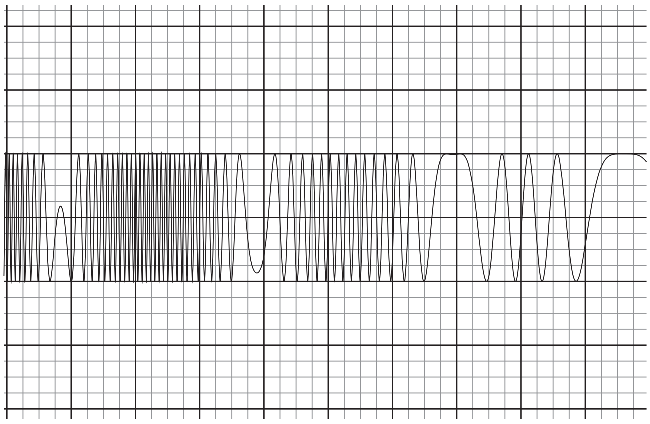
Questions 10 and 11 refer to the P–V diagram of an ideal gas undergoing the Brayton cycle. Steps A and C are isentropic, and steps B and D are isobaric.

10. What is the approximate work done by the gas over one cycle?

(A) −6,200 kJ
(B) −3,100 kJ
(C) 0
(D) 3,100 kJ
(E) 6,200 kJ

11. The gas used in the cycle is most likely

(A) monoatomic
(B) diatomic
(C) triatomic
(D) ionized
(E) heteronuclear



12. The oscilloscope trace shown in the diagram is an example of

(A) frequency modulation
(B) amplitude modulation
(C) pulse-code modulation
(D) single-sideband modulation
(E) clipping

13. Two exoplanets, A and B, are discovered orbiting a star much more massive than either planet. The semimajor axes of the orbits of A and B are found to be a and $a/2$, respectively. What is the ratio of the area enclosed by the orbit of planet A to the area enclosed by the orbit of planet B?

(A) $1/4$
(B) $1/(2\sqrt{2})$
(C) $1/2$
(D) $2\sqrt{2}$
(E) It cannot be determined from the information given.

14. A spin-1/2 particle has the angular wavefunction

$$\psi(\theta,\phi) = \frac{1}{\sqrt{2}}\left(Y_3^0(\theta,\phi) + Y_2^1(\theta,\phi)\right),$$

where $Y_l^m(\theta,\phi)$ are the normalized spherical harmonics. Which of the following is a possible result of measuring the particle's total spin quantum numbers j and m_j?

(A) $j = 3,\ m_j = 0$
(B) $j = 2,\ m_j = 1/2$
(C) $j = 7/2,\ m_j = -1/2$
(D) $j = 7/2,\ m_j = 3/2$
(E) $j = 9/2,\ m_j = -1/2$

15. The normalized energy eigenfunctions of the infinite square well of size L are $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(n\pi x/L)$. The expectation value of energy of the state

$$\Psi = \frac{1}{\sqrt{2}}\psi_2 + \frac{1}{\sqrt{3}}\psi_3 + \frac{1}{\sqrt{6}}\psi_4$$

for a particle of mass m is

(A) $\dfrac{4\pi^2\hbar^2}{3mL^2}$
(B) $\dfrac{8\pi^2\hbar^2}{3mL^2}$
(C) $\dfrac{17\pi^2\hbar^2}{4mL^2}$
(D) $\dfrac{14\pi^2\hbar^2}{3mL^2}$
(E) $\dfrac{23\pi^2\hbar^2}{6mL^2}$

16. An ideal gas is maintained at a temperature of 250 K through contact with a thermal reservoir and is free to expand against a piston. If 5000 J of heat is slowly added to the gas, what is the change in entropy of the gas?

(A) 10 J/K
(B) $10 \ln 2$ J/K
(C) 20 J/K
(D) 40 J/K
(E) 500 J/K

17. The photoelectric effect provides direct experimental evidence for which of the following properties of light?

I. It has two linearly independent polarization states.
II. It carries kinetic energy proportional to its frequency.
III. It travels at a constant speed c in vacuum.

(A) I only
(B) II only
(C) I and II
(D) II and III
(E) I, II, and III

18. The Hamiltonian $H = eE_0 z$, describing an atomic electron of charge $-e$ interacting with a uniform electric field in the z-direction, is responsible for

(A) the Zeeman effect
(B) the Lamb shift
(C) hyperfine splitting
(D) the Stark effect
(E) stimulated emission

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19. A block of mass m_1 moving with velocity v collides elastically with a block of mass m_2 at rest. If m_1 continues moving in the same direction as it did prior to the collision, one can conclude

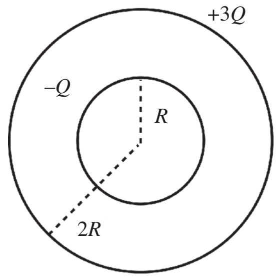
(A) $m_1 > m_2$
(B) $m_1 = m_2$
(C) $m_1 < m_2$
(D) momentum was not conserved in this collision
(E) none of the above

20. A 20 cm tall slice of a spherical mirror is oriented such that the image of a 2-meter tall person 1 meter away from the ATM will just fill the surface of the mirror. What must the radius of curvature R and convexity of the mirror be?

(A) $R = 22$ cm, convex
(B) $R = 40$ cm, concave
(C) $R = 80$ cm, concave
(D) $R = 4.5$ m, convex
(E) $R = 18$ m, convex

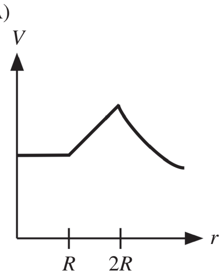
21. The quantized vibrations of a crystal lattice are called

(A) photons
(B) anyons
(C) phonons
(D) vibrons
(E) rotons

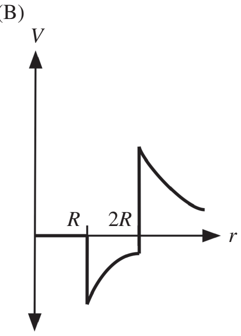


22. Shown in the diagram are two concentric thin spherical shells of radii R and $2R$, the outer one carrying charge $+3Q$ and the inner one carrying charge $-Q$. Setting the electric potential equal to zero at infinity, which of the following graphs best represents the electric potential as a function of r, the distance from the center of the shells?

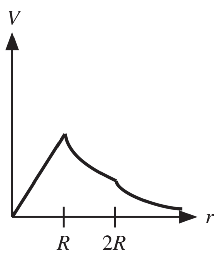
(A)



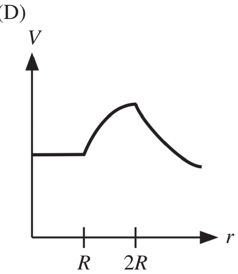
(B)



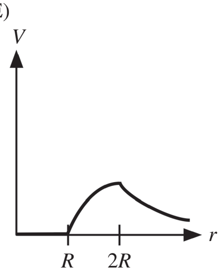
(C)



(D)



(E)



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23. The force-carrying particle responsible for binding the quarks in the proton is called the

(A) photon
(B) gluon
(C) W boson
(D) Z boson
(E) Higgs boson

24. A charge $-e$ at the origin is subject to a uniform electric field $\mathbf{E} = -E\hat{\mathbf{x}}$. After traveling to $(3, 4)$, what is the change in potential energy of the charge?

(A) $-3Ee$
(B) $3Ee$
(C) $-5Ee$
(D) $5Ee$
(E) $-7Ee$

25. Which of the following are true statements about Gauss's law for magnetism, $\nabla \cdot \mathbf{B} = 0$?

I. It implies that magnetic monopoles do not exist in nature.
II. It is incompatible with the continuity equation for $\mathbf{J}$.
III. It allows the magnetic field to be written in terms of a vector potential as $\mathbf{B} = \nabla \times \mathbf{A}$.

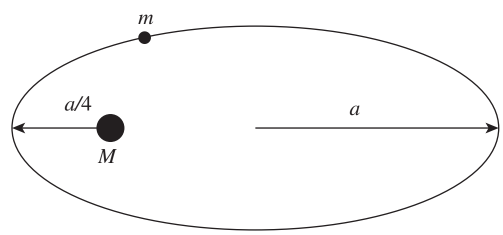
(A) I only
(B) II only
(C) III only
(D) I and II
(E) I and III

26. A relativistic particle of mass m has momentum $p = mc$. What is the particle's energy?

(A) mc^2
(B) $\sqrt{2}mc^2$
(C) $2mc^2$
(D) $4mc^2$
(E) none of the above

27. The hot, dense gas of electrons and positive ions known as a plasma is capable of supporting charge density waves known as plasma oscillations. Let n_e be the number density of electrons, e the charge of the electrons, and m^* an effective mass of the electrons in the plasma. The frequency of plasma oscillations is

(A) $\omega = \dfrac{\epsilon_0}{n_e e^2 m^*}$
(B) $\omega = \dfrac{m^* e^2}{n_e \epsilon_0}$
(C) $\omega = \dfrac{e^2}{n_e \epsilon_0}$
(D) $\omega = \sqrt{\dfrac{n_e e^2 m^*}{\epsilon_0}}$
(E) $\omega = \sqrt{\dfrac{n_e e^2}{m^* \epsilon_0}}$



28. A planet of mass m orbits a star of mass M in an elliptical orbit with semimajor axis a, as shown in the diagram. The distance of closest approach to the star is $a/4$. Assuming $m \ll M$, the ratio of the planet's speed at perigee (when the planet is closest to the star) to the planet's speed at apogee (when the planet is furthest away from the star) is

(A) 1/4
(B) 1/3
(C) 4
(D) 7
(E) 16

29. Two identical sailboats race across a lake, starting from rest. Boat 1 reaches the finish line first with velocity v_1, and boat 2 arrives later with velocity $v_2 > v_1$. Let F_{t1} and F_{t2} be the average force per unit *time* on boats 1 and 2, respectively, and let F_{d1} and F_{d2} be the average force per unit *distance* on boats 1 and 2. Which of the following MUST be true?

(A) $F_{t1} > F_{t2}$
(B) $F_{t1} < F_{t2}$
(C) $F_{d1} > F_{d2}$
(D) $F_{d1} < F_{d2}$
(E) none of the above

30. The first two normalized position-space energy eigenfunctions of the harmonic oscillator Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$ are

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar},$$
$$\psi_1(x) = \sqrt{2}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} x e^{-m\omega x^2/2\hbar}.$$

A delta-function perturbation $V(x) = \epsilon\delta(x)$ is added to the harmonic oscillator Hamiltonian. What are the new energies E_0 and E_1 to first order in perturbation theory?

(A) $E_0 = \hbar\omega/2, E_1 = 3\hbar\omega/2$
(B) $E_0 = \hbar\omega/2 + \epsilon\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}, E_1 = 3\hbar\omega/2$
(C) $E_0 = \hbar\omega/2 + \epsilon\sqrt{\frac{m\omega}{\pi\hbar}}, E_1 = 3\hbar\omega/2$
(D) $E_0 = \hbar\omega/2 + \epsilon\sqrt{\frac{m\omega}{\pi\hbar}}, E_1 = 3\hbar\omega/2 + 2\epsilon\sqrt{\frac{m\omega}{\pi\hbar}}$
(E) $E_0 = \hbar\omega + \epsilon\sqrt{\frac{m\omega}{\pi\hbar}}, E_1 = 2\hbar\omega$

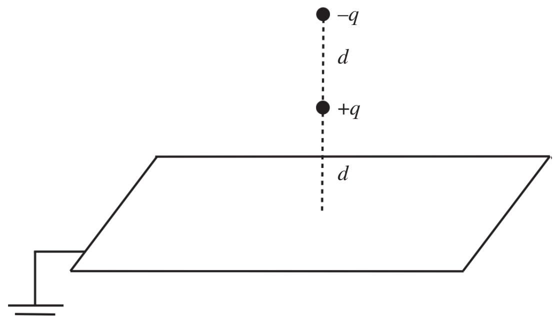
31. At sufficiently high temperature T, which of the following contributes to the total energy of a diatomic molecule?

I. Translational kinetic energy
II. Rotational kinetic energy
III. Vibrational potential energy

(A) I only
(B) II only
(C) I and II
(D) I and III
(E) I, II, and III

32. During the adiabatic expansion phase of a Carnot cycle, one mole of gas expands to twice its original size. The change in entropy of the gas during this process is

(A) $R \ln 2$
(B) $-R \ln 2$
(C) $2R$
(D) $-2R$
(E) 0



33. A particle of charge $+q$ is placed at the point $(0, 0, d)$, between an infinite grounded conducting plate at $z = 0$ and a stationary charge $-q$ at $(0, 0, 2d)$, as shown in the diagram. What is the force on the charge $+q$?

(A) $-\dfrac{q^2}{8\pi\epsilon_0 d^2}\hat{\mathbf{z}}$
(B) $-\dfrac{7q^2}{24\pi\epsilon_0 d^2}\hat{\mathbf{z}}$
(C) $\dfrac{11q^2}{72\pi\epsilon_0 d^2}\hat{\mathbf{z}}$
(D) $\dfrac{25q^2}{72\pi\epsilon_0 d^2}\hat{\mathbf{z}}$
(E) $\dfrac{31q^2}{144\pi\epsilon_0 d^2}\hat{\mathbf{z}}$

Questions 34 and 35 refer to the following scenario. A new star is discovered with an optical telescope, from which it is deduced that the star emits most of its power in the orange region of the visible spectrum, at wavelength approximately 600 nm.

34. Assuming the star behaves as a blackbody and neglecting possible redshift, what is its approximate temperature?

(A) 200 K
(B) 5000 K
(C) 6×10^4 K
(D) 3×10^6 K
(E) 2×10^9 K

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35. It is later discovered that the star is receding at a peculiar velocity of $0.2c$. The ratio between the true temperature T_{true} and the measured temperature T_{meas} is

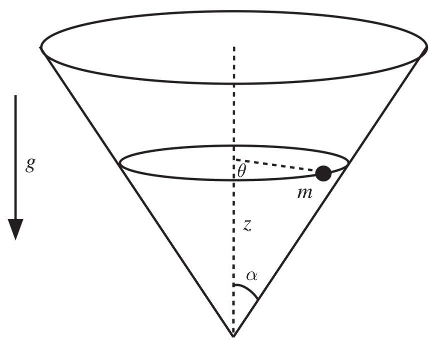
(A) $1/5$
(B) $2/3$
(C) $3/2$
(D) $\sqrt{2/3}$
(E) $\sqrt{3/2}$

36. A one-dimensional system has Lagrangian

$$L(q, \dot{q}, t) = A\dot{q}^2 + \sin(q/L - \omega t)$$

for constants A, L, and ω. What is the Euler–Lagrange equation of motion?

(A) $\dot{q} = \frac{\omega}{2AL} \sin(q/L - \omega t)$
(B) $\dot{q} = -\frac{\omega}{A} \cos(q/L - \omega t)$
(C) $\dot{q} = -\frac{\omega}{2A} \cos(q/L - \omega t)$
(D) $\ddot{q} = \frac{1}{2AL} \cos(q/L - \omega t)$
(E) $\ddot{q} = -\frac{1}{AL} \cos(q/L - \omega t)$



37. A particle of mass m is constrained to move on a cone of opening angle α, oriented as shown in the diagram. The Hamiltonian for this system is given by

$$H = \frac{p_z^2}{2m} \cos^2 \alpha + \frac{p_\theta^2}{2mz^2} \cot^2 \alpha + mgz.$$

What are Hamilton's equations for the coordinate z?

(A) $\dot{p}_z = \frac{p_z}{m} \cos^2 \alpha - mg; \dot{z} = \frac{p_z^2}{2mz^2} \cot^2 \alpha$
(B) $\dot{p}_z = \frac{p_\theta^2}{2mz} \cot^2 \alpha - mg; \dot{z} = \frac{p_z}{m} \cos^2 \alpha$
(C) $\dot{p}_z = \frac{p_z}{m} \cos^2 \alpha; \dot{z} = \frac{p_\theta^2}{mz} \cot^2 \alpha - mg$
(D) $\dot{p}_z = \frac{p_\theta^2}{mz^3} \cot^2 \alpha - mg; \dot{z} = \frac{p_z}{m} \cos^2 \alpha$
(E) $\dot{p}_z = -\frac{p_\theta^2}{mz} \cot^2 \alpha + mg; \dot{z} = -\frac{p_z}{m} \cos^2 \alpha$

38. The specific heat at constant volume, C_V, of a solid is observed at low temperatures T to follow the Debye law $C_V = AT^3$, with A a constant. What is the internal energy of the solid $U(T)$ as a function of temperature, assuming $U(0) = 0$, in the regime of validity of the Debye law?

(A) $3AT^2$
(B) AT^3
(C) $\frac{1}{3}AT^3$
(D) $\frac{1}{4}AT^4$
(E) AT^4

39. Let $|n\rangle$ denote a set of real, orthonormal energy eigenfunctions of a Hamiltonian $\hat{H}$ in one dimension, with energies E_n. Let $\hat{p}$ denote the momentum operator. Which of the following must be true?

I. $\langle m|n\rangle = m + n$
II. $|n\rangle$ is an eigenfunction of $\hat{p}$
III. $\langle m|\hat{H}|n\rangle = \delta_{mn}E_n$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

40. The Meissner effect refers to the tendency of superconductors to

(A) develop a surface charge density
(B) expel magnetic fields
(C) acquire a finite resistance at a critical temperature T_c
(D) spontaneously develop an internal electric field
(E) have persistent currents

41. The hydrogen isotope tritium, ^{3}H, contains one proton and two neutrons and has a half-life of approximately 12 years. The binding energy of tritium is closest to

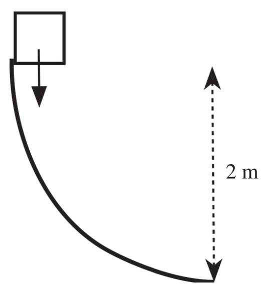
(A) 8.5 eV
(B) 8.5 keV
(C) 8.5 MeV
(D) 8.5 GeV
(E) 8.5 TeV

42. A positive muon stopped by matter can attract an electron to form an exotic bound state known as muonium, where the muon (which has the same charge as the proton) acts as the nucleus. Let m_μ be the mass of the muon, m_e the mass of the electron, and m_p the mass of the proton. What is the Bohr radius of muonium, in terms of the Bohr radius of ordinary hydrogen a_0 and the masses of the particles?

(A) $a_0 \dfrac{m_p}{m_\mu}$
(B) $a_0 \dfrac{m_\mu}{m_p}$
(C) $a_0 \left(\dfrac{m_\mu}{m_p}\right)^2$
(D) $a_0 \dfrac{m_p(m_e + m_\mu)}{m_\mu(m_e + m_p)}$
(E) $a_0 \dfrac{m_p(m_e + m_p)}{m_\mu(m_e + m_\mu)}$

43. Let $\hat{x}$ and $\hat{p}$ be the quantum-mechanical position and momentum operators, respectively. The commutator $[\hat{x}, \hat{p}^2]$ is equivalent to which of the following?

(A) 0
(B) $i\hbar$
(C) $-i\hbar\hat{x}$
(D) $2i\hbar\hat{p}$
(E) $2i\hbar$



44. A block of mass 2 kg slides down a ramp in the shape of a quarter-circle of radius 2 m, as shown in the diagram. If the block reaches the bottom of the ramp with velocity 4 m/s, then, ignoring air resistance, the work done by friction during the slide down the ramp is most nearly

(A) 0 J
(B) 8 J
(C) 12 J
(D) 24 J
(E) 40 J

45. Consider an infinite charge-carrying wire, of charge per unit length λ. Setting the zero of electric potential at distance a from the wire, what is the electric potential as a function of the distance r from the wire?

(A) $\dfrac{\lambda}{2\pi\epsilon_0} \ln(a/r)$
(B) $\dfrac{\lambda}{4\pi\epsilon_0} \ln(a/r)$
(C) $\dfrac{\lambda}{2\pi\epsilon_0} \ln(r/a)$
(D) $\dfrac{\lambda}{4\pi\epsilon_0 r}$
(E) $\dfrac{\lambda(r-a)}{4\pi\epsilon_0 r}$

46. An ideal beam-splitter is an optical device that lets part of an incident beam of light pass through and reflects the remainder, with no absorption taking place in the beam-splitter. Let the incident beam have complex amplitude E, the reflected beam have amplitude E_r, and the transmitted beam have amplitude E_t. Which of the following MUST be true?

(A) $E_r = E/\sqrt{2}$
(B) $E_r = E/2$
(C) $E = E_r + E_t$
(D) $|E|^2 = |E_r|^2 + |E_t|^2$
(E) $|E|^4 = |E_r|^4 + |E_t|^4$

47. The Hamiltonian operator for a free particle of mass m moving in three dimensions is

(A) $-\dfrac{\hbar^2\nabla^2}{m}$
(B) $-\dfrac{\hbar^2\nabla^2}{2m}$
(C) $-i\hbar\nabla$
(D) $\dfrac{\hbar^2\nabla^2}{2m}$
(E) 0

48. In proton therapy, medium-energy protons are directed at a cancer patient's tumor in order to irradiate it. Which of the following pieces of information would be MOST useful in determining the correct energy and angle with which to fire the protons?

(A) The charge-to-mass ratio of the proton
(B) The distance traveled in human tissue as a function of energy
(C) The cross section for proton scattering on carbon nuclei
(D) The mean lifetime of the proton
(E) The binding energy of the proton

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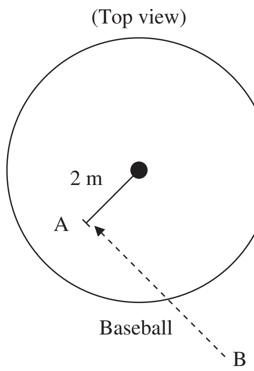


49. Student A, of mass 100 kg, stands 2 meters from the center of a circular platform which is free to rotate on frictionless bearings. Student B, not standing on the platform, tosses student A a baseball of mass 0.09 kg, which reaches student A with a velocity of 20 m/s directed perpendicular to the line joining student A and the center of the platform. If the platform has moment of inertia 200 kg$\cdot$m^2, what is its approximate angular velocity after student A catches the baseball?

(A) 0.006 rad/s
(B) 0.009 rad/s
(C) 0.018 rad/s
(D) 0.067 rad/s
(E) 0.1 rad/s

50. In Compton scattering, the change in wavelength of the scattered light is given in terms of the electron mass m_e and the scattering angle θ by which of the following?

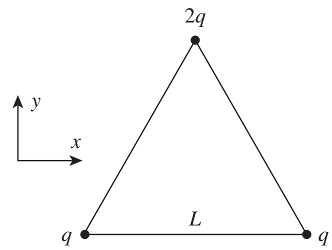
(A) $\frac{h}{m_e c}(1 - \cos\theta)$
(B) $\frac{m_e}{hc}(1 + \cos\theta)$
(C) $\frac{h}{m_e c}(1 + \sin\theta)$
(D) $\frac{m_e}{hc}\sin^2\theta$
(E) $\frac{1}{hcm_e}(1 - \sin\theta)$

51. The binding energy of the electron in the Li^{++} ion is approximately

(A) 1.51 eV
(B) 13.6 eV
(C) 40.8 eV
(D) 122.4 eV
(E) 1102 eV

52. Which of the following statements is NOT consistent with the three laws of thermodynamics?

(A) The entropy of a perfect crystal of a pure substance must approach zero at absolute zero.
(B) The entropy of an isolated system can sometimes decrease.
(C) The ground state degeneracy of a system determines its entropy.
(D) Absolute zero can never be reached in experiments.
(E) The entropy of a system can be nonzero at absolute zero.



53. Questions 53 and 54 refer to the diagram above, with charges q, q, and $2q$ placed at the corners of an equilateral triangle of side length L, and the $\hat{\mathbf{x}}$- and $\hat{\mathbf{y}}$-axes oriented as shown. What is the electric field at the center of the triangle?

(A) $\frac{q}{4\pi\epsilon_0 L^2}(\hat{\mathbf{x}} + \sqrt{3}\hat{\mathbf{y}})$
(B) $-\frac{3q}{4\pi\epsilon_0 L^2}\hat{\mathbf{y}}$
(C) $\frac{4q}{3\pi\epsilon_0 L^2}\hat{\mathbf{y}}$
(D) $-\frac{q}{\pi\epsilon_0 L\sqrt{3}}\hat{\mathbf{y}}$
(E) 0

54. What is the electric potential at the center of the triangle, relative to infinity?

(A) $\frac{q\sqrt{3}}{\pi\epsilon_0 L}$
(B) $\frac{q\sqrt{3}}{4\pi\epsilon_0 L}$
(C) $\frac{4q\sqrt{3}}{3\pi\epsilon_0 L}$
(D) $\frac{4q}{3\pi\epsilon_0 L^2}$
(E) 0

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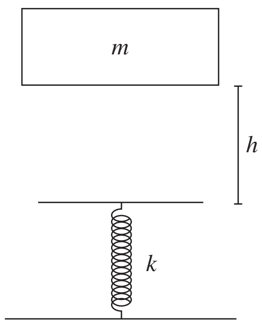
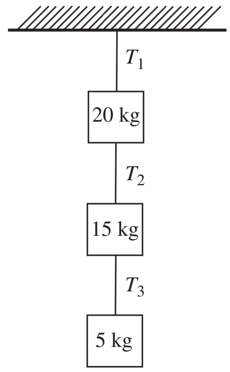
55. A car engine operates between a cold reservoir at temperature 27 °C and a hot reservoir at 127 °C. What is the minimum amount of heat the engine must absorb from the hot reservoir in a period of 1 minute to obtain a power output of 100 kW?

(A) 1,500 kJ
(B) 6,000 kJ
(C) 8,000 kJ
(D) 18,000 kJ
(E) 24,000 kJ



56. A brick of mass m falls onto a massless spring with spring constant k from a height h above it. What is the maximum distance the spring will be compressed from its equilibrium length?

(A) $\frac{mg}{k}$
(B) $\frac{mk}{2gh}$
(C) $\frac{k}{mg}\left(1+\sqrt{\frac{mg}{kh}}\right)$
(D) $\frac{mg}{k}\left(1+\sqrt{\frac{2kh}{mg}}\right)$
(E) $\frac{mg}{k}\left(1+\sqrt{1+\frac{2kh}{mg}}\right)$



57. Three weights are suspended from a ceiling using massless ropes, as shown in the diagram. The tensions in the ropes are T_1, T_2, and T_3. What is T_1/T_3?

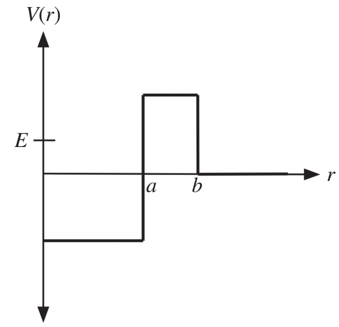
(A) 0.25
(B) 1
(C) 3
(D) 4
(E) 8

58. A particle's normalized spin wavefunction has the form

$$\psi(\theta,\phi)=\sqrt{\frac{3}{2\pi}}\sin\theta\cos 2\phi\sin\phi.$$

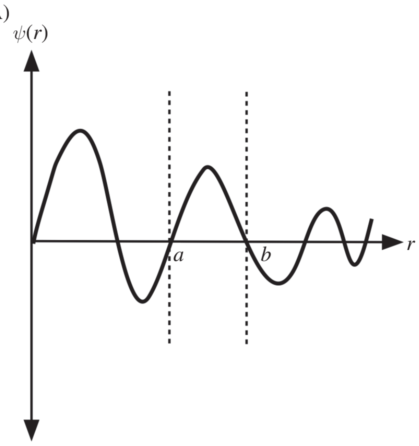
What is the expectation value of the particle's z-component of orbital angular momentum L_z?

(A) 0
(B) $-3\hbar/2\pi$
(C) $3\hbar/2\pi$
(D) $-3\hbar/\pi$
(E) $3\hbar/\pi$

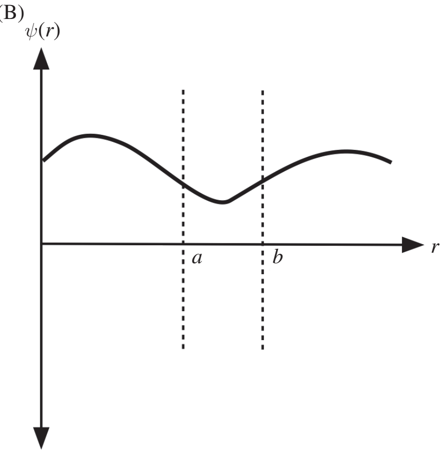


59. The strong nuclear force binding an alpha particle to a nucleus can be modeled by the potential shown in the diagram. Which of the following plots best illustrates the radial wavefunction of an alpha particle with energy E that tunnels out of the nucleus in alpha decay?

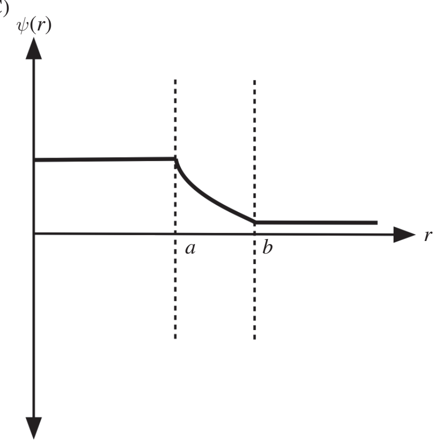
(A)



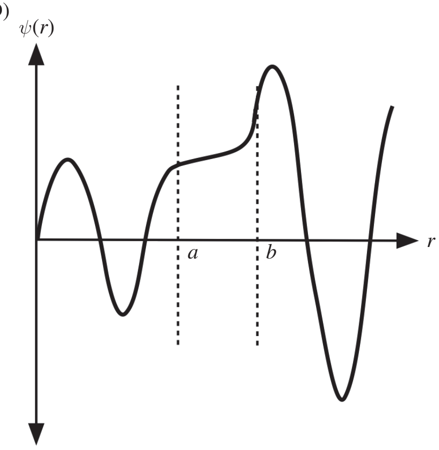
(B)



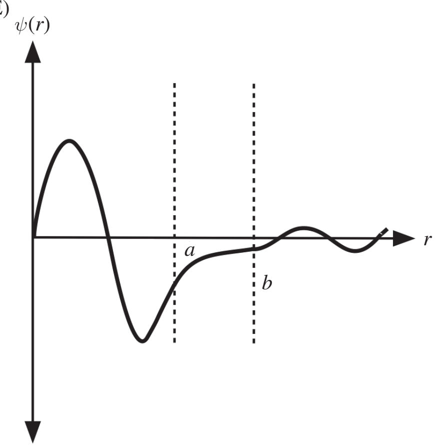
(C)



(D)



(E)



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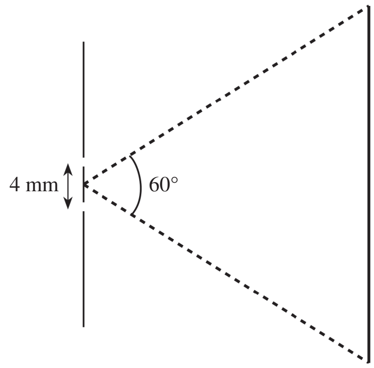


60. A coherent beam of monochromatic light of wavelength 500 nm is directed towards two very thin slits separated by a distance 4 mm. Far behind the slits is a screen covering an angular region of 60°, as shown in the diagram. Approximately how many bright interference bands are visible on the screen?

(A) 0
(B) 4,000
(C) 6,800
(D) 8,000
(E) 13,600

61. A stationary telescope monitoring a rocket ship observes the ship emitting flashes of light at 1-second intervals. If the ship begins moving toward the telescope at speed $0.6c$, with what period does the telescope observe the light flashes?

(A) 0.5 s
(B) 0.8 s
(C) 1 s
(D) 1.2 s
(E) 2 s

62. A particle moving at speed $0.8c$ enters a tube of length 30 m and hits a target at the end of the tube. How far away is the target when the particle enters the tube, in the reference frame of the particle?

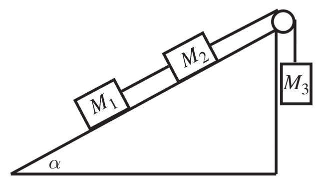
(A) 18 m
(B) 24 m
(C) 30 m
(D) 50 m
(E) 60 m

63. The volume of the first Brillouin zone of a simple cubic lattice of lattice spacing a is

(A) a^3
(B) $1/a^3$
(C) $(a/2\pi)^3$
(D) $(2\pi/a)^3$
(E) $a^3/2\pi$

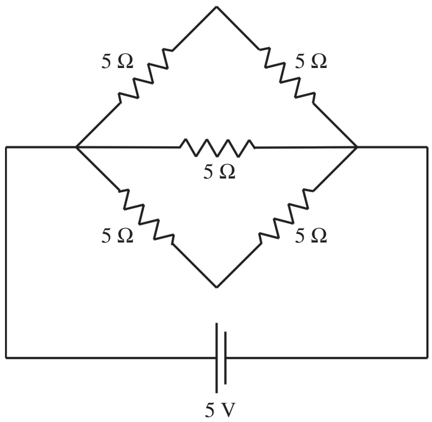
64. Early observations of beta decay of the neutron showed that the emitted electron had a broad energy spectrum, rather than a fixed energy. This was taken as evidence for the existence of the

(A) neutrino
(B) positron
(C) muon
(D) strange quark
(E) pion



65. Blocks of masses M_1, M_2, and M_3 are arranged on a frictionless inclined plane at angle α as shown in the diagram. The pulley at the top of the plane is frictionless and massless, and the system is in static equilibrium. What is α in terms of M_1, M_2, and M_3?

(A) $\tan^{-1}\left(\frac{M_3}{M_1}\right)$
(B) $\sin^{-1}\left(\frac{M_3}{M_1+M_2}\right)$
(C) $\sin^{-1}\left(\frac{M_1+M_2}{M_3}\right)$
(D) $\cos^{-1}\left(\frac{M_1+M_2}{M_3}\right)$
(E) $\cos^{-1}\left(\frac{M_3}{M_1+M_2}\right)$



66. A 5 V battery supplies the emf for the circuit shown in the diagram, where all resistors are 5 Ω. What current flows through the circuit? (You may assume the wires are resistanceless and the battery has negligible internal resistance.)

(A) 0.2 A
(B) 0.5 A
(C) 1 A
(D) 2 A
(E) 5 A

67. Questions 67 and 68 refer to the Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$
$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

What is the determinant of the matrix

$$M = \sigma_x^2 + \sigma_y^2 + \sigma_z^2?$$

(A) 0
(B) 1
(C) 3
(D) 4
(E) 9

68. The state of a spin-1/2 particle is described by the spinor

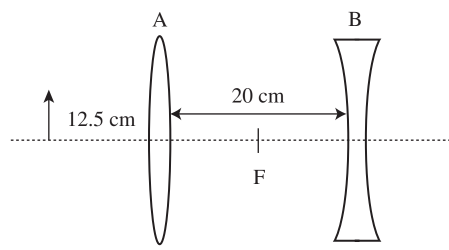
$$\eta = \mathcal{N}\begin{pmatrix} i \\ 2-i \end{pmatrix}$$

where $\mathcal{N}$ is a normalization constant. What is the expectation value of S_x, the spin projection onto the x-axis, in the state η?

(A) $-\hbar$
(B) $-\hbar/3$
(C) $-\hbar/6$
(D) 0
(E) $\hbar/2$

69. Which of the following could represent the displacement of a standing wave?

(A) $\cos(kx - \omega t)$
(B) $\sin(kx - \omega t)$
(C) $(x - vt)^2$
(D) $\sin kx \cos \omega t$
(E) $\omega t \sin^2 kx$



70. A converging lens A and a diverging lens B, both with focal length 10 cm, are arranged so that the midpoint between the lenses F coincides with both lenses' foci. An object is placed 12.5 cm to the left of A. Which of the following gives the correct position and orientation of the image?

(A) 5 cm to the right of A, inverted
(B) 5 cm to the right of A, upright
(C) 7.5 cm to the right of A, inverted
(D) 30 cm to the right of B, inverted
(E) 30 cm to the right of B, upright

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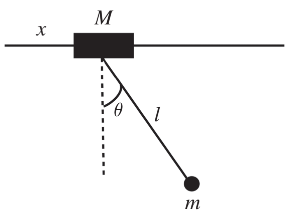
71. A parallel-plate capacitor with capacitance C is at rest in frame $\mathcal{S}$, with the plates of the capacitor parallel to the xy-plane. In frame $\overline{\mathcal{S}}$, moving in the positive $\hat{\mathbf{x}}$ direction at speed v, what is the new capacitance in terms of C?

(A) C
(B) $C\sqrt{1 - v^2/c^2}$
(C) $\dfrac{C}{\sqrt{1 - v^2/c^2}}$
(D) $C(1 - v^2/c^2)$
(E) $\dfrac{C}{1 - v^2/c^2}$

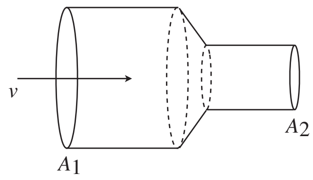
72. In underground particle detection experiments, the main source of naturally occurring ionizing radiation at energies above 200 MeV is

(A) thermal radiation
(B) cosmic ray muons
(C) solar neutrinos
(D) seismic noise
(E) solar flares



73. A point mass m is attached with a massless rod of length l to a pivot of mass M, which is free to slide along a frictionless bar. Letting x be the position of the pivot and θ the angle of the rod, what is a possible Lagrangian for this system?

(A) $L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta$
(B) $L = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta$
(C) $L = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 + ml\dot{x}\dot{\theta}\cos\theta + mgl\cos\theta$
(D) $L = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 + 2ml\dot{x}\dot{\theta}\sin\theta - mgl\cos\theta$
(E) $L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2\sin^2\theta + ml\dot{x}\dot{\theta}\cos^2\theta - mgl\cos\theta$



74. A pipe has cross-sectional area A_1 at one point, but subsequently narrows to a cross-sectional area A_2. If the pressure of an incompressible fluid of density ρ flowing toward the narrow end is p in the first region, and its velocity is v, what is the pressure in the second narrow region?

(A) p
(B) $\dfrac{1}{2}\dfrac{A_2^2}{A_1^2}\rho v^2$
(C) $p + \dfrac{1}{2}\left(\dfrac{A_2^2}{A_1^2} - 1\right)\rho v^2$
(D) $p + \dfrac{1}{2}\left(1 - \dfrac{A_1^2}{A_2^2}\right)\rho v^2$
(E) $p + \dfrac{1}{2}\left(\dfrac{A_2^2}{A_1^2} + 1\right)\rho v^2$

75. A particle in three dimensions has normalized radial wavefunction

$$\psi(r) = \begin{cases} 0, & 0 \leq r \leq a \\ \sqrt{a}/r^2, & r > a. \end{cases}$$

What is the probability the particle will be found between $r = a$ and $r = 2a$?

(A) $1/3$
(B) $1/2$
(C) $1/\sqrt{3}$
(D) $1/\sqrt{2}$
(E) 1

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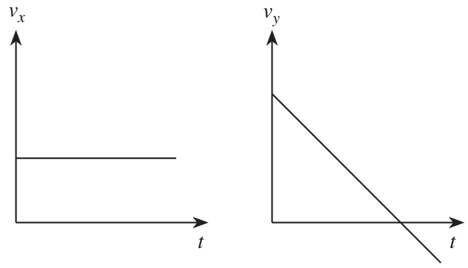

76. The diagram above shows plots of components of velocity v_x and v_y versus time t, with identical scales on both plots. Gravity acts in the $-\hat{\mathbf{y}}$-direction. Ignoring air resistance, these plots could represent which of the following scenarios?

(A) A ball dropped from the top of a high building
(B) A rock thrown from a high building at an angle of 45° below the horizontal
(C) A brick thrown from ground level at an angle of 60° above the horizontal
(D) A golf ball on an elevated tee struck at an angle of 30° above the horizontal
(E) A mass attached to a vertical spring which is compressed and then released

77. A boater wearing sunglasses with the polarization axis vertical observes that the intensity of sunlight reflected off the water and transmitted through her sunglasses gradually decreases once the Sun is overhead, and goes to zero when the Sun is 30° above the horizon. What is the index of refraction of the water? You may assume the index of refraction of air is 1.

(A) $1/2$
(B) 1
(C) $\sqrt{2}$
(D) $\sqrt{3}$
(E) 2

78. A body of mass m and charge q at the origin is subjected to an electric field $\mathbf{E}(t) = E_0 \sin(\omega t)\hat{\mathbf{x}}$ for a time $T \gg 2\pi/\omega$. Which of the following will cause the average power radiated by the charge to decrease?

(A) Increasing m
(B) Increasing q
(C) Increasing E_0
(D) Decreasing ω
(E) Decreasing T

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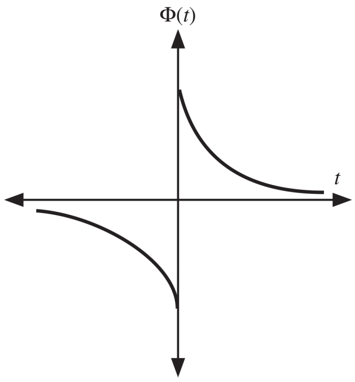
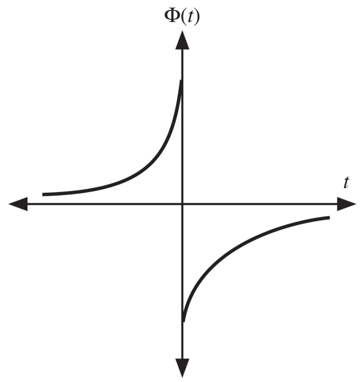
79. A positively charged particle q is traveling at constant (nonrelativistic) velocity in the $+\hat{\mathbf{z}}$-direction and passes through the center of a loop of wire lying in the xy-plane, at $t = 0$. Which of the following plots best illustrates $\Phi(t)$, the electric flux through the loop as a function of t? Assume that the normal to the loop is parallel to the velocity vector of the charge.

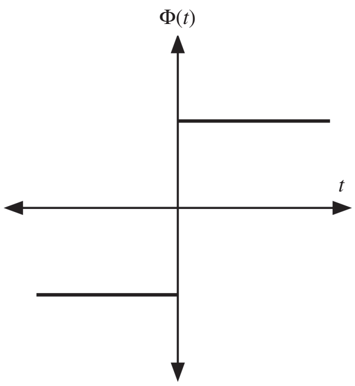
(A)



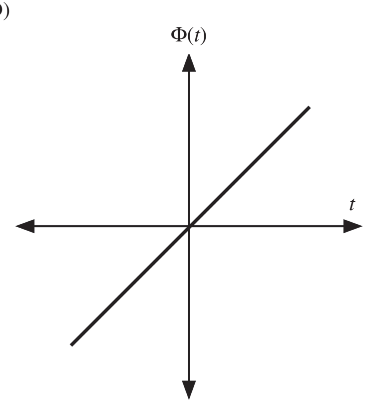
(B)



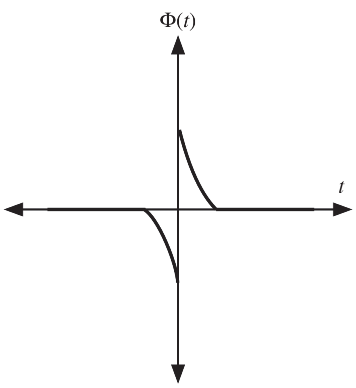
(C)



(D)



(E)



80. For an ideal gas in a container of fixed volume, the most probable speed of the gas molecules as a function of pressure P is proportional to

(A) $P^{-1/2}$
(B) $P^{1/2}$
(C) P
(D) $P^{5/3}$
(E) P^2

81. An ambulance with its siren blaring at constant frequency f drives in a straight line at constant speed directly toward an observer. Which of the following is true of f_{obs}, the siren frequency heard by the observer?

(A) $f_{\text{obs}} = f$.
(B) f_{obs} is constant and greater than f.
(C) f_{obs} is constant and less than f.
(D) f_{obs} continuously increases as the ambulance approaches the observer.
(E) f_{obs} continuously decreases as the ambulance approaches the observer.

82. A photon of energy 25 MeV collides head-on with an electron of energy 50 MeV in the laboratory frame. What is the velocity of the photon in the center-of-momentum frame of the electron and photon?

(A) $0.33c$
(B) $0.5c$
(C) $0.66c$
(D) $0.95c$
(E) c

83. In the quark model, mesons such as the pion are composed of

(A) two quarks
(B) two antiquarks
(C) a quark and an antiquark
(D) three quarks
(E) two baryons

84. Which of the following is the MAIN factor that prevents neutron stars from gravitationally collapsing?

(A) Pauli exclusion principle
(B) Angular momentum
(C) Tidal forces
(D) Spin-down
(E) Strong nuclear force

85. A capacitor C is in an RC circuit with an initial charge Q_0. When the circuit is closed, the energy dissipated in the resistor is used to heat a material of specific heat c_p and mass m, with an efficiency ϵ. Assuming that the material is thermally isolated from everything except the resistor and that the heat capacity of the resistor is negligible compared with the material, what is the change in temperature of the material a long time after the capacitor is discharged?

(A) $\dfrac{Q_0^2\epsilon}{2mc_pC}$
(B) $\dfrac{Q_0^2\epsilon}{mc_pC}$
(C) $\dfrac{2Q_0^2\epsilon}{mc_pC}$
(D) $\dfrac{Q_0^2}{2\epsilon mc_pC}$
(E) $\dfrac{Q_0^2}{\epsilon mc_pC}$

86. Two infinite wires a distance d apart carry equal current I in opposite directions. The force per unit length of one wire acting on the other

(A) has magnitude $\frac{\mu_0 I^2}{2\pi d}$ and is attractive
(B) has magnitude $\frac{\mu_0 I^2}{2\pi d}$ and is repulsive
(C) has magnitude $\frac{\mu_0 I^2}{4\pi d^2}$ and is attractive
(D) has magnitude $\frac{\mu_0 I^2}{4\pi d^2}$ and is repulsive
(E) is zero

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87. A system of two spin-1/2 particles is subject to the Hamiltonian $H = -A\mathbf{S}_1 \cdot \mathbf{S}_2$, with $A > 0$. What is the degeneracy of the ground state of this system?

(A) 1
(B) 2
(C) 3
(D) 4
(E) This system does not have a ground state.

88. When ultraviolet light of wavelength 350 nm is shined on a container of gas whose molecules have diameter 1 nm, the intensity of the scattered light is I. If the experiment were repeated with the same incident intensity of red light (wavelength 700 nm), the intensity of the scattered light would be

(A) $I/64$
(B) $I/16$
(C) I
(D) $2I$
(E) $16I$

89. The principal decay mode of the 3*s* state of hydrogen is to

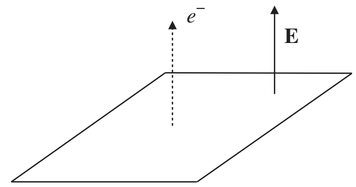
(A) 1*s*
(B) 2*s*
(C) 2*p*
(D) 3*p*
(E) nothing; the 3*s* state is stable

90. A new particle's mass is measured in a high-energy physics experiment, and the value reported as $5.43\ \text{GeV} \pm 0.08\ \text{GeV} \pm 0.06\ \text{GeV}$, where the first error is systematic and the second is statistical. The total error on the measurement is

(A) 0.0048 GeV
(B) 0.01 GeV
(C) 0.02 GeV
(D) 0.10 GeV
(E) 0.14 GeV

91. A Geiger counter monitoring a radioactive sample records 64 counts in a 1-minute window. The fractional uncertainty on the counting rate is

(A) 1/64
(B) 1/8
(C) 1/4
(D) 1/2
(E) not determinable from the information given



92. An electron is ejected from a metal sheet in the direction normal to the sheet with kinetic energy of 10 eV. A uniform electric field of 100 V/m is applied normal to the sheet, as shown in the diagram. What is the maximum height above the sheet achieved by the electron? You may ignore the effects of gravity.

(A) 1 mm
(B) 1 cm
(C) 10 cm
(D) 1 m
(E) The electron accelerates off to infinity.

93. A bullet of mass 5 g is fired at a block of wood of mass 1 kg, which is hanging from a massless rigid rod of length 0.4 m. The block is thick enough to stop the bullet entirely inside. Which of the following is closest to the minimum velocity of the bullet such that the block makes a complete vertical revolution?

(A) 200 m/s
(B) 400 m/s
(C) 800 m/s
(D) 1000 m/s
(E) 1600 m/s

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94. A radio telescope is trained on a binary star system whose angular separation on the sky is 0.061 radians. What is the minimum diameter of the telescope in order to resolve both stars in the binary by observing radio frequency radiation at 200 MHz? (Ignore any atmospheric effects.)

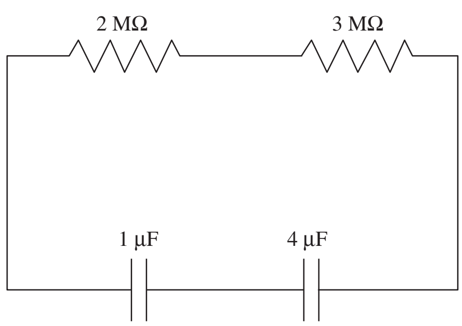
(A) 0.11 m
(B) 1.5 m
(C) 13.3 m
(D) 20 m
(E) 30 m

95. An electron–positron collider creates muons through the reaction

$$e^+ + e^- \to \mu^+ + \mu^-.$$

In the center-of-momentum frame, what is the minimum speed of the electron for this reaction to occur, in terms of the masses m_e and m_μ of the electron and muon?

(A) $(1 - m_e/m_\mu)c$
(B) $\left(\sqrt{1 - m_e/m_\mu}\right) c$
(C) $\left(\sqrt{1 - m_e^2/(2m_\mu^2)}\right) c$
(D) $\left(\sqrt{1 - m_e^2/m_\mu^2}\right) c$
(E) This process can occur at any speed.



96. What is the time constant of the circuit shown in the diagram?

(A) 2 s
(B) 4 s
(C) 6 s
(D) 10 s
(E) 25 s

97. The self-inductance of an ideal solenoid is L. If the number of coils per unit length is tripled while all other parameters remain the same, the new self-inductance is

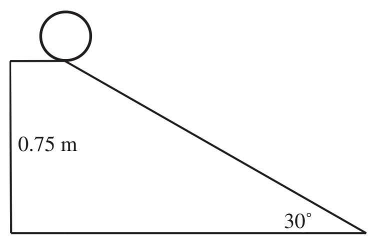
(A) $L/9$
(B) $L/3$
(C) $L\sqrt{3}$
(D) $3L$
(E) $9L$

98. Free electron lasers produce coherent light through which of the following mechanisms?

(A) Spontaneous emission
(B) Synchrotron radiation
(C) Population inversion
(D) Optical pumping
(E) Electric dipole transitions

99. What is the magnetic field due to an infinite surface current $\mathbf{K} = K_0\hat{\mathbf{y}}$ flowing along the xy-plane?

(A) $-\frac{\mu_0 K}{2}\hat{\mathbf{z}}$ for $z < 0$, $\frac{\mu_0 K}{2}\hat{\mathbf{z}}$ for $z > 0$
(B) $\frac{\mu_0 K}{2}\hat{\mathbf{z}}$ for $z < 0$, $-\frac{\mu_0 K}{2}\hat{\mathbf{z}}$ for $z > 0$
(C) $-\frac{\mu_0 K}{2}\hat{\mathbf{x}}$ for $z < 0$, $\frac{\mu_0 K}{2}\hat{\mathbf{x}}$ for $z > 0$
(D) $\frac{\mu_0 K}{2}\hat{\mathbf{x}}$ for $z < 0$, $-\frac{\mu_0 K}{2}\hat{\mathbf{x}}$ for $z > 0$
(E) $-\frac{\mu_0 K}{2}\hat{\mathbf{y}}$ for $z < 0$, $\frac{\mu_0 K}{2}\hat{\mathbf{y}}$ for $z > 0$



100. A sphere of radius 20 cm and mass 45 g is placed atop a ramp of height 0.75 m and inclination angle 30°. If the ramp were frictionless, the sphere would slide down the ramp in a time t. With friction, the sphere would roll without slipping down the ramp, and reach the bottom in a time t'. What is t'/t?

(A) $\sqrt{2/5}$
(B) $\sqrt{7/10}$
(C) 1
(D) $\sqrt{7/5}$
(E) $\sqrt{3}$

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Sample Exam 3

TABLE OF INFORMATION

Rest mass of the electron	m_e	$=$	9.11×10^{-31} kg
Magnitude of the electron charge	e	$=$	1.60×10^{-19} C
Avogadro's number	N_A	$=$	6.02×10^{23}
Universal gas constant	R	$=$	8.31 J/(mol · K)
Boltzmann's constant	k	$=$	1.38×10^{-23} J/K
Speed of light	c	$=$	3.00×10^{8} m/s
Planck's constant	h	$=$	6.63×10^{-34}J · s $=$ 4.14×10^{-15} eV · s
	$\hbar$	$=$	$h/2\pi$
	hc	$=$	1240 eV · nm
Vacuum permittivity	ϵ_0	$=$	8.85×10^{-12} $C^2/(N \cdot m^2)$
Vacuum permeability	μ_0	$=$	$4\pi \times 10^{-7}$ T · m/A
Universal gravitational constant	G	$=$	6.67×10^{-11} $m^3/(kg \cdot s^2)$
Acceleration due to gravity	g	$=$	9.80 m/s^2
1 atmosphere pressure	1 atm	$=$	1.0×10^{5} N/m^2 $=$ 1.0×10^{5} Pa
1 angstrom	1 Å	$=$	1×10^{-10} m $=$ 0.1 nm

Prefixes for Powers of 10

10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	µ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P

Rotational inertia about center of mass

Rod	$\frac{1}{12}M\ell^2$
Disk	$\frac{1}{2}MR^2$
Sphere	$\frac{2}{5}MR^2$

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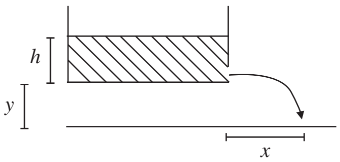
SAMPLE EXAM 3

Time — 170 minutes

100 questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then fill in the corresponding space on the answer sheet.



1. A bath of water has a hole in the bottom of one side, as shown in the figure. At what horizontal distance x from the edge of the bath does the draining water land? Neglect effects due to viscosity and surface tension.

   (A) $\sqrt{2hy}$
   (B) $\frac{\sqrt{hy}}{2}$
   (C) $2\sqrt{hy}$
   (D) $\frac{h}{2}$
   (E) $\frac{2h^2}{y}$

2. When monochromatic blue light of wavelength 450 nm is shined on a sample of hydrogen *atoms*, the intensity of the scattered light is I_0. If the Bohr radius of hydrogen were doubled, what would be the approximate intensity of scattered light?

   (A) $I_0/16$
   (B) $I_0/4$
   (C) $4I_0$
   (D) $16I_0$
   (E) $64I_0$

3. Two objects in the sky have angular separation 1 arcminute, and emit a broad spectrum of radiation. A telescope with aperture diameter 1 cm could resolve the objects by observing which of the following kinds of radiation? Note that 1 arcminute is approximately 3×10^{-4} rad.

   I. Radio
   II. Visible
   III. X-ray

   (A) I only
   (B) II only
   (C) III only
   (D) I and II
   (E) II and III

4. A block slides frictionlessly on ice at a constant velocity of 10 m/s. The block suddenly encounters a rough patch where its coefficient of kinetic friction suddenly increases from 0 to 0.5. How far does the block slide before stopping?

   (A) 5 m
   (B) 10 m
   (C) 15 m
   (D) 20 m
   (E) 100 m

5. The Euler–Lagrange equations are valid for systems with which of the following properties?

   I. Systems with time-dependent potentials
   II. Systems without rotational symmetry
   III. Systems acted on by only conservative forces

   (A) II
   (B) III
   (C) I and II
   (D) I and III
   (E) I, II, and III

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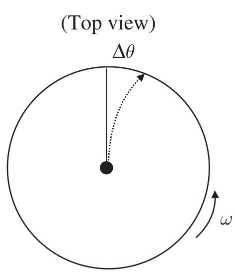
6. Order the following corrections to the Bohr energies of hydrogen from smallest to largest.

   I. Fine structure
   II. Hyperfine splitting
   III. Lamb shift

   (A) I, II, III
   (B) I, III, II
   (C) II, III, I
   (D) III, II, I
   (E) III, I, II

7. What is the difference in energy between the $n = 5$ and the $n = 1$ states of the one-dimensional quantized harmonic oscillator?
   (A) $\hbar\omega$
   (B) $2\hbar\omega$
   (C) $4\hbar\omega$
   (D) $8\hbar\omega$
   (E) $16\hbar\omega$



8. A person stands at the center of a frictionless disk of radius R rotating at angular velocity ω, and pushes a puck radially outwards at velocity v. What is the angle $\Delta\theta$ between the point on the edge of the disk where the puck was aimed and the point on the edge of the disk where the puck actually arrives?

   (A) $v/(\omega R)$
   (B) $\omega R/v$
   (C) $2\omega R/v$
   (D) $\omega R/(2v)$
   (E) 0

9. Two parameters x and y were measured with uncorrelated uncertainties Δx and Δy. What is the uncertainty of the quantity $z = x/y$?

   (A) $\sqrt{(\Delta x/x)^2 + (\Delta y/y)^2}$
   (B) $\sqrt{(\Delta x/x)^2 - (\Delta y/y)^2}$
   (C) $z\sqrt{(\Delta x/x)^2 + (\Delta y/y)^2}$
   (D) $z\sqrt{(\Delta x/x)^2 - (\Delta y/y)^2}$
   (E) $z\sqrt{(\Delta x/x)^2 + 2(\Delta y/y)^2}$

10. A circuit made only of which of the following circuit elements may function as a bandpass filter?

    (A) One resistor, one inductor, and one capacitor
    (B) One resistor and one inductor
    (C) One resistor and one capacitor
    (D) Two resistors
    (E) Two capacitors

11. Consider a planet of mass m that orbits a star of mass $M \gg m$. For a fixed orbital angular momentum L, what is the relationship between the energies of the three possible orbit shapes: circular ($E_{\rm cir}$), elliptical ($E_{\rm ell}$), or hyperbolic ($E_{\rm hyp}$)?

    (A) $E_{\rm cir} < E_{\rm ell} < E_{\rm hyp}$
    (B) $E_{\rm ell} < E_{\rm hyp} < E_{\rm cir}$
    (C) $E_{\rm hyp} < E_{\rm cir} < E_{\rm ell}$
    (D) $E_{\rm cir} < E_{\rm hyp} < E_{\rm ell}$
    (E) $E_{\rm ell} < E_{\rm cir} < E_{\rm hyp}$

12. For a system of electrons at zero temperature, the energy of the highest occupied quantum state is called the

    (A) zero-point energy
    (B) Einstein energy
    (C) Fermi energy
    (D) Bose energy
    (E) binding energy

13. A beam is made up of particles that have a lifetime of 10^{-8} s at rest. If the beam travels at $0.8c$, at what location down the beamline is there only a fraction $1/e$ of the particles remaining?

    (A) 6.2 m
    (B) 4.0 m
    (C) 3.6 m
    (D) 2.7 m
    (E) 2.2 m

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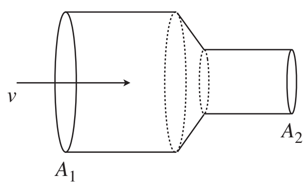
14. An atom has electron configuration $1s^2 2s^2 2p^3$. A measurement of the total orbital angular momentum of the outermost electron in the ground state could return which of the following?

(A) $\hbar$
(B) $\hbar\sqrt{2}$
(C) $2\hbar$
(D) $\hbar\sqrt{6}$
(E) $3\hbar$

15. Which of the following nuclei were produced during Big Bang nucleosynthesis?

I. ^{3}He
II. ^{7}Li
III. ^{55}Fe

(A) I
(B) III
(C) I and II
(D) I and III
(E) I, II, and III



16. A horizontal tube has a wide section of cross-sectional area A_1 and a narrow section with cross-sectional area A_2, as shown in the diagram. If an incompressible fluid of density ρ moves with velocity v through the wide part of the horizontal tube, what is the velocity of the fluid in the narrow section of the tube?

(A) v
(B) $v\frac{A_1}{A_2}$
(C) $v\frac{A_2}{A_1}$
(D) $v\frac{A_1^2}{A_2^2}$
(E) $v\frac{A_2^2}{A_1^2}$

17. A car accelerates from rest at 5 m/s^2. What is its speed after traveling 40 m?

(A) 9.7 m/s
(B) 15 m/s
(C) 20 m/s
(D) 30 m/s
(E) 54.2 m/s

18. A bullet of mass m is fired at velocity v into a block of mass M at rest on a table, where it stops and is embedded. If there is a coefficient of friction μ between the block and the table, how much time does it take for the block to come to rest?

(A) $\dfrac{Mv}{\mu g(m+M)}$
(B) $\dfrac{\mu mv}{g(m+M)}$
(C) $\dfrac{v}{\mu g}$
(D) $\dfrac{mv}{\mu gM}$
(E) $\dfrac{mv}{\mu g(m+M)}$

19. Thermal fluctuations produce voltage fluctuations in all resistors. Which of the following is the spectral density (units of V Hz$^{-1/2}$) of voltage fluctuations in a resistor at temperature T and of resistance R?

(A) $\sqrt{4kT/R}$
(B) $\sqrt{4kR/T}$
(C) $\sqrt{4kTR}$
(D) $\sqrt{kT}$
(E) $\sqrt{4R}$

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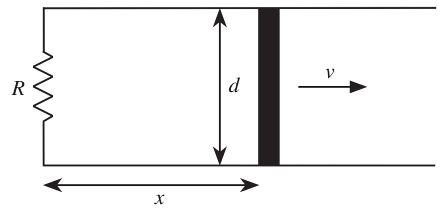


20. A metal bar is pulled at constant velocity $v\hat{\mathbf{x}}$ along two metal rails a distance d apart connected by a resistor of resistance R, as shown in the diagram. There is a constant magnetic field, pointing into the page, of magnitude B. At time T, how much energy has been dissipated in the resistor thus far, as a function of T?

(A) $\dfrac{2BvdT}{R}$
(B) $\dfrac{2(Bvd)^2T}{R}$
(C) $\dfrac{BvdT}{R}$
(D) $\dfrac{(Bvd)^2T}{R}$
(E) $(Bvd)^2TR$

21. If the rms velocity of H_2 gas at 300 K is v, which of the following is closest to the rms velocity of helium gas at the same temperature?

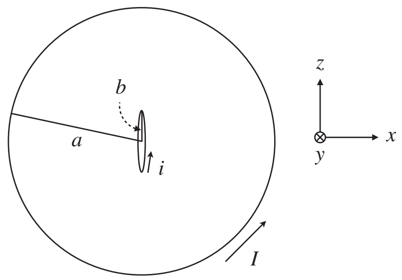
(A) $v/8$
(B) $v/4$
(C) $v/2$
(D) $v/\sqrt{2}$
(E) $2v$

22. A circuit consists of a capacitor C, a resistor R, and an inductor L all in series. The circuit is driven by an AC generator at frequency ω. What is the driving frequency at which the current through the circuit is maximized?

(A) $\sqrt{\dfrac{1}{LC}-\left(\dfrac{R}{2L}\right)^2}$
(B) $\sqrt{\dfrac{1}{LC}-\left(\dfrac{1}{2RC}\right)^2}$
(C) $\sqrt{\dfrac{1}{LC}-\left(\dfrac{R}{L}\right)^2}$
(D) $\sqrt{\dfrac{1}{LC}}$
(E) $\sqrt{\dfrac{1}{LC}-\left(\dfrac{1}{RC}\right)^2}$

23. A pipe with two open ends is 20 cm long. What is the fundamental frequency of the pipe? (You may assume the speed of sound is 343 m/s.)

(A) 1715 Hz
(B) 858 Hz
(C) 563 Hz
(D) 429 Hz
(E) 205 Hz



24. A current loop of radius a carrying current I is centered at the origin and lies in the xy-plane. Another loop, carrying current $i \ll I$, and of radius $b \ll a$, is centered at the origin and lies in the xz-plane. What is the torque on the smaller loop about its center?

(A) $\dfrac{\mu_0 \pi iIb^2}{a}$

(B) $\dfrac{\mu_0 \pi iIb^2}{2a}$

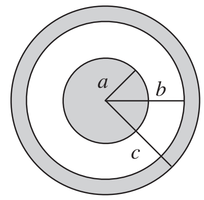
(C) $\dfrac{\mu_0 \pi iIb^3}{2a^2}$

(D) $\dfrac{\mu_0 \pi iIb^2}{4a^2}$

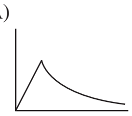
(E) $\dfrac{3\mu_0 \pi iIb^2}{2a^2}$

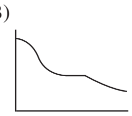
25. A merry-go-round of radius R rotates at an angular velocity of Ω. A ball A is released at radius $R/2$, initially at rest, by a person standing on the merry-go-round. An identical ball B is released at radius R, also at rest. In the noninertial reference frame of the rotating merry-go-round, what is the ratio of the acceleration experienced by ball A to ball B immediately after they are released?

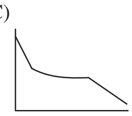
(A) 0
(B) 1/2
(C) 1
(D) 2
(E) 4

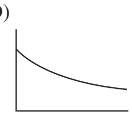


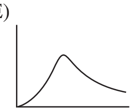
26. A ball of uniform charge density and radius a is surrounded by a conducting shell of inner radius b and outer radius c. Which could be the potential as a function of radius?

(A)


(B)


(C)


(D)


(E)


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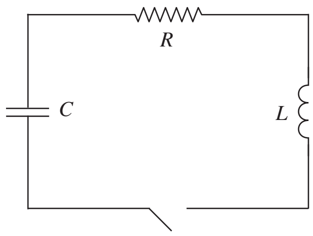
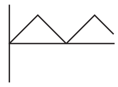
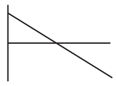


27. A charged capacitor is in series with a resistor and inductor, as in the diagram. Which of the following could be a graph of the current when the switch is closed?

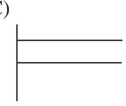
(A)



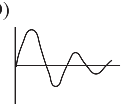
(B)



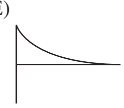
(C)



(D)



(E)

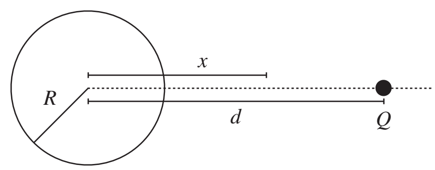


28. A beam of electrons (mass m and charge q) with uniform velocity enters a region of constant magnetic field B perpendicular to the beam direction. Assuming that the electrons are able to follow a circular path completely within the field, how long does it take for the beam to make one complete revolution?

(A) $2\pi m/(qB)$
(B) $\pi m/(qB)$
(C) $m/(2\pi qB)$
(D) $m/(\pi qB)$
(E) $m/(qB)$

29. Photons of wavelength 10 nm are incident on a crystal with interatomic space 80 nm. At what angle is the first-order maximum in the diffraction pattern?

(A) 7.2°
(B) 3.6°
(C) 2.4°
(D) 1.8°
(E) 0.9°



30. A charge Q is brought to a distance d from the center of a grounded conducting sphere of radius R. What is the electric potential at a distance $x > R$ from the center of the sphere along the axis between the charge and the sphere?

(A) $\dfrac{Q}{2\pi\epsilon_0}\left(\dfrac{1}{|x-d|} - \dfrac{d}{|xd-R^2|}\right)$

(B) $\dfrac{Q}{4\pi\epsilon_0}\left(\dfrac{1}{|x-d|} - \dfrac{R}{|xd-R^2|}\right)$

(C) $\dfrac{Q}{4\pi\epsilon_0}\left(\dfrac{1}{|x-d|} - \dfrac{1}{|2x-R|}\right)$

(D) $\dfrac{Q}{4\pi\epsilon_0}\left(\dfrac{1}{|x-d|} - \dfrac{1}{|x|}\right)$

(E) 0

31. What type of lattice is the reciprocal lattice to a simple cubic lattice in three dimensions?

(A) Face-centered cubic
(B) Simple cubic
(C) Body-centered cubic
(D) Simple hexagonal
(E) None of the above

32. The magnetic vector potential in a region of space is given by

$$\mathbf{A}(x, y) = Ay^2\hat{\mathbf{x}} + Cx^2\hat{\mathbf{z}},$$

where A and C are constants. What is the magnetic field in this region?

(A) $-2Cx\hat{\mathbf{y}} - 2Ay\hat{\mathbf{z}}$
(B) $2Cx\hat{\mathbf{y}} + 2Ay\hat{\mathbf{z}}$
(C) 0
(D) $-Cx\hat{\mathbf{y}} - Ay\hat{\mathbf{z}}$
(E) $Cx\hat{\mathbf{y}} + Ay\hat{\mathbf{z}}$

33. The CMB has a temperature of 2.7 K and has a peak intensity at a wavelength of approximately 1 mm. If the CMB were at 5 K, what would be the wavelength with the maximal intensity?

(A) 1 mm
(B) 0.54 mm
(C) 0.32 mm
(D) 5.3 mm
(E) 57 mm

34. Which of the following decays is allowed in the Standard Model?

(A) $\mu^- \to e^- + \nu_e$
(B) $\pi^- \to \gamma$
(C) $\Delta^+ \to p + n$
(D) $\pi^+ \to 2\gamma$
(E) $K^+ \to \mu^+ + \nu_\mu$

35. A nearby star is moving away from the Earth with peculiar velocity $0.1c$. It appears to have an effective blackbody temperature of 10^4 K. What is its true effective blackbody temperature? (Assume a negligible cosmological redshift.)

(A) 0.6×10^4 K
(B) 0.9×10^4 K
(C) 10^4 K
(D) 1.1×10^4 K
(E) 1.4×10^4 K

36. A two-level system has energies $\pm\epsilon$. What is the average energy of the system as the temperature $T \to \infty$?

(A) -2ϵ
(B) $-\epsilon$
(C) 0
(D) ϵ
(E) 2ϵ

37. An ideal gas is confined to half of a rigid box with volume $2V$. If a valve is opened suddenly, letting the gas suddenly fill the full volume of the box, which of the following is unchanged?

I. Internal energy U
II. Temperature T
III. Entropy S

(A) I
(B) III
(C) I and II
(D) I and III
(E) I, II, and III

38. Suppose a heat engine that transfers heat from a warm bath at temperature T_H to a cold bath at T_C has an efficiency

$$e = \frac{T_H + T_C}{T_H + 2T_C}.$$

Which of the following must be violated?

(A) Conservation of energy
(B) First Law of Thermodynamics
(C) Second Law of Thermodynamics
(D) Third Law of Thermodynamics
(E) Postulate of equal *a priori* probabilities

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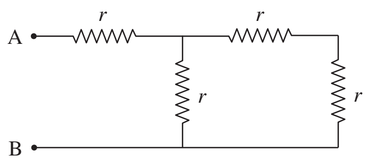
39. Two spaceships pass each other. Spaceship A moves relative to a nearby planet at velocity v_1, while spaceship B moves at velocity v_2 relative to the planet. How fast does spaceship A move relative to spaceship B?

(A) $\dfrac{v_1 - v_2}{1 + v_1 v_2/c^2}$
(B) $\dfrac{|v_1 + v_2|}{1 - v_1 v_2/c^2}$
(C) $|v_1 - v_2|$
(D) $\dfrac{v_1 + v_2}{1 + v_1 v_2/c^2}$
(E) $\dfrac{|v_1 - v_2|}{1 - v_1 v_2/c^2}$



40. Consider the network of resistors with resistance r, shown in the figure. What is the equivalent resistance between terminals A and B?

(A) $r/3$
(B) $2r/3$
(C) r
(D) $4r/3$
(E) $5r/3$

41. An ideal monoatomic gas initially at pressure P undergoes adiabatic expansion from a volume V to a volume $2V$. What is the final pressure of the gas?

(A) P
(B) $P/2$
(C) $P/4$
(D) $2^{-5/3}P$
(E) $2^{-7/3}P$

42. What is the expectation value of L_z for the following wavefunction:

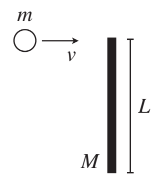
$$\psi(\theta,\phi) = \frac{1}{\sqrt{2}}\left(Y_1^{-1}(\theta,\phi) + Y_1^0(\theta,\phi)\right),$$

where $Y_l^m(\theta,\phi)$ are the spherical harmonics?

(A) $-\hbar$
(B) $-\hbar/2$
(C) 0
(D) $\hbar/2$
(E) $\hbar$

43. Which of the following statements is true in general for one-dimensional spin-0 quantum mechanical systems?

(A) All states are energy eigenstates.
(B) Energies are always quantized.
(C) Eigenvalues of Hermitian operators are real.
(D) All states have real-valued wavefunctions in the x basis.
(E) None of the above.



44. A mass m moves at speed v perpendicular to a rod of uniform density, mass M, and length L on a frictionless table. Suppose $m \ll M$. If the mass collides with the end of the rod and sticks to it, at what angular speed does the rod begin to rotate? (You may treat the mass m as a point particle.)

(A) $\dfrac{3mv}{2ML}$
(B) $\dfrac{3mv}{ML}$
(C) $\dfrac{6mv}{ML}$
(D) $\dfrac{12mv}{ML}$
(E) $\dfrac{Mv}{2mL}$

45. An electron is in a magnetic field and has a Hamiltonian $H = \alpha \mathbf{S} \cdot \mathbf{B}$. If the electron is aligned with the magnetic field at $t = 0$, what is its time-dependent wavefunction? ($|+\rangle$ represents a spinor aligned with the magnetic field.)

(A) $\exp(-i\alpha Bt/2)\,|+\rangle$
(B) $\exp(+i\alpha Bt/2)\,|+\rangle$
(C) $\exp(-i\alpha Bt)\,|+\rangle$
(D) $\exp(+i\alpha Bt)\,|+\rangle$
(E) $\exp(-2\pi i\alpha Bt/\hbar)\,|+\rangle$

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46. Which of the following is true of observables in quantum mechanics?

    I. They are represented by Hermitian operators.
    II. Multiple observables can never be simultaneously measured.
    III. The operators representing observables must have real eigenvalues.

    (A) I only
    (B) II only
    (C) I and II
    (D) I and III
    (E) I, II, and III

47. A mass attached to a spring oscillates at frequency f. If the mass attached to the spring triples and the spring constant doubles, what is the new frequency of oscillation of the system?

    (A) $(\sqrt{2/3})f$
    (B) $(2/3)f$
    (C) $(4/9)f$
    (D) $(\sqrt{1/2})f$
    (E) $(1/2)f$

48. A quantum particle has normalized wavefunction $\psi(x) = \sqrt{5}x^2$ on the interval $[0, 1]$. What is the probability that the particle is found at $0 \leq x \leq 1/2$?

    (A) 1/2
    (B) 1/4
    (C) 1/8
    (D) 1/16
    (E) 1/32

49. A spin-1/2 particle is in a state

$$|\psi\rangle = \sqrt{\frac{2}{3}}\,|m = 1/2\rangle + \sqrt{\frac{1}{3}}\,|m = -1/2\rangle\,.$$

    What is the expectation value of the z-component of its spin?

    (A) $3\hbar$
    (B) $\hbar$
    (C) $\hbar/2$
    (D) $\hbar/3$
    (E) $\hbar/6$

50. Which of the following physical processes is responsible for producing the photons from a carbon dioxide laser?

    (A) Pair annihilation
    (B) Bremsstrahlung
    (C) Transitions of nuclear energy levels
    (D) Transitions between vibrational molecular energy levels
    (E) Photoelectric effect

51. A car of mass M pulls a trailer of mass m by a cord. The car's engine exerts a force F on the car. Suppose that the trailer has a coefficient of rolling friction μ, but neglect the coefficient of friction of the car. What is the tension in the cord between the car and trailer?

    (A) $\dfrac{m(F - \mu Mg)}{M - m}$
    (B) $\dfrac{m(F + \mu Mg)}{M + m}$
    (C) $\dfrac{m(F - \mu Mg)}{M + m}$
    (D) $\dfrac{\mu m(F - Mg)}{M + m}$
    (E) $\dfrac{m(F + \mu Mg)}{\mu(M + m)}$

52. A spin-2 particle has orbital angular momentum $l = 4$. What is the smallest possible value of its total angular momentum quantum number j?

    (A) 6
    (B) 5
    (C) 4
    (D) 3
    (E) 2

53. An element with a ground state electron configuration of $1s^2 2s^2 2p^6$ is best characterized as a(n)

    (A) alkali metal
    (B) rare earth metal
    (C) semiconductor
    (D) halogen
    (E) noble gas

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54. The ground state wavefunction for the harmonic oscillator is

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/(2\hbar)}.$$

Consider a perturbation to the Hamiltonian given by

$$\delta V(x) = \alpha e^{-\beta x^2}.$$

What is the first-order correction to the ground state energy? Note that

$$\int_{-\infty}^{\infty} e^{-x^2/c^2} = c\sqrt{\pi}.$$

(A) $\alpha\sqrt{\dfrac{\beta\hbar}{m\omega}}$
(B) $\alpha\sqrt{\dfrac{m\omega+\beta\hbar}{m\omega}}$
(C) $\alpha\sqrt{\dfrac{m\omega}{m\omega+\beta\hbar}}$
(D) $\alpha\sqrt{\dfrac{2m\omega+\beta\hbar}{m\omega}}$
(E) $\alpha\sqrt{\dfrac{m\omega}{2m\omega+\beta\hbar}}$

55. Let γ be the path in the xy-plane that traverses the square with vertices $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$, in that order. What is the line integral $\oint_\gamma \mathbf{f}\cdot d\mathbf{l}$ of the function $\mathbf{f}(x,y) = y\hat{\mathbf{x}} + x\hat{\mathbf{y}}$?

(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

56. Magnetic flux is quantized in type II superconductors. What is the unit of magnetic flux quanta?

(A) $2eh$
(B) $\dfrac{e}{2h^2}$
(C) $\dfrac{h}{2e}$
(D) $\dfrac{h}{2e^2}$
(E) $\dfrac{e^2}{2h}$

57. Positronium is a bound state of an electron and a positron. Which of the following are true facts about positronium?

I. It obeys Bose–Einstein statistics.
II. Its binding energy is -6.8 eV.
III. It can decay into a single photon.

(A) I only
(B) III only
(C) I and II
(D) I and III
(E) I, II, and III

58. A billiard ball m rolls without slipping with velocity $\mathbf{v}$ on a pool table. After striking the wall of the table in an elastic collision, the ball has velocity $\mathbf{v}'$. The change in the ball's momentum is $\Delta\mathbf{p}$. Which of the following must be true?

I. $|\mathbf{v}'| = |\mathbf{v}|$
II. $\mathbf{v}' = -\mathbf{v}$
III. $|\Delta\mathbf{p}| = m|\mathbf{v}' - \mathbf{v}|$

(A) I only
(B) I and II
(C) I and III
(D) II and III
(E) I, II, and III

59. Which of the following transitions of the hydrogen atom is allowed in the electric dipole approximation? (The entries in parentheses are (n, l, m).)

(A) $(2,1,0) \to (1,0,0)$
(B) $(2,0,0) \to (1,0,0)$
(C) $(3,2,0) \to (1,0,0)$
(D) $(3,2,2) \to (2,1,0)$
(E) $(3,0,0) \to (2,0,0)$

60. Suppose that a satellite orbiting the Sun can be approximated as a perfect blackbody. Assuming the body is in equilibrium with its surroundings, what is the ratio of its blackbody temperature at radius $2R$ from the Sun to the blackbody temperature at radius R from the Sun?

(A) $2^{-3/2}$
(B) 2^{-1}
(C) $2^{-1/2}$
(D) $2^{-1/3}$
(E) $2^{-1/4}$

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61. Gas A consists of atoms of mass m, and gas B consists of diatomic molecules where each atom has mass $m/2$. Which of the following best describes the rms speeds v_A and v_B of the particles of gases A and B at temperature T? You may assume T is less than the temperature at which gas B dissociates.

(A) $v_A < v_B$ at low T, $v_A > v_B$ at high T
(B) $v_A > v_B$ at low T, $v_A < v_B$ at high T
(C) $v_A = v_B$ at low T, $v_A > v_B$ at high T
(D) $v_A < v_B$ for all T
(E) $v_A = v_B$ for all T

62. A merry-go-round can be approximated as a disk of uniform density with radius r, and mass M. A merry-go-round is spinning at angular velocity ω before a person steps onto it. What is the change in angular velocity after a person of mass m steps onto the edge of the merry-go-round?

(A) $\frac{2\omega m}{M+2m}$
(B) $\frac{\omega M}{M+2m}$
(C) $\frac{2\omega(m+M)}{M+2m}$
(D) $\frac{2\omega m}{M+m}$
(E) $\frac{\omega m}{M+m}$

63. The He^+ ion experiences an atomic transition from the $n = 2$ state to the $n = 1$ state. What is the energy of the emitted photon?

(A) 10.2 eV
(B) 13.6 eV
(C) 27.2 eV
(D) 31.4 eV
(E) 40.8 eV

64. A string of mass density $\mu = 1$ g/cm and tension $T = 4 \times 10^3$ N is fixed at both ends. What is the speed of waves on the string?

(A) 2 m/s
(B) 20 m/s
(C) 200 m/s
(D) 2,000 m/s
(E) 2×10^5 m/s

65. Six charges $+q$ are fixed at the corners of a cube of side length a. A test charge $-q$ is placed at the center of the cube and released. Which of the following MUST be true of this configuration of charges?

I. The force on the test charge due to the six charges $+q$ is zero.
II. The electric potential due to the six charges $+q$ is zero.
III. The test charge is in stable equilibrium.

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III

66. A particle of mass m and charge q in a region of homogeneous magnetic field undergoes cyclotron motion at speed $v = 0.6c$ with cyclotron radius R. If the particle's speed is increased to $v = 0.8c$, what is the new cyclotron radius?

(A) $3R/4$
(B) R
(C) $5R/4$
(D) $4R/3$
(E) $16R/9$

67. Unpolarized light is incident on two polarizing filters oriented at 30° to one another. What is the intensity of transmitted light as a fraction of the incident light intensity?

(A) 1/4
(B) 3/8
(C) 1/2
(D) 3/4
(E) 1/8

68. What is the expectation value of the operator

$$\mathcal{O} = |[\hat{x}, \hat{p}]|^2$$

in the ground state of the infinite square well of size L, centered on $x = 0$?

(A) 0
(B) $\hbar^2$
(C) L^2
(D) $\hbar^2 L^2$
(E) $\hbar^2/L^2$

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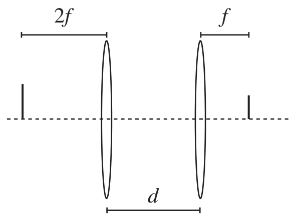
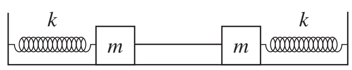
69. What is the inductance of a cylindrical solenoid with N turns, length ℓ, and radius $R \ll \ell$?

(A) $2\mu_0 N^2 R^2/\ell$
(B) $2\mu_0 N^2 \pi R^2/\ell$
(C) $\mu_0 N \pi R^2/\ell$
(D) $\mu_0 N^2 \pi R^2/\ell$
(E) $\mu_0 N^2 R^2/\ell$



70. Two converging lenses of focal length $f/2$ are placed in series, separated by a distance d. The object is placed a distance $2f$ to the left of the left lens, and the image is upright and located a distance f to the right of the right lens. What is d?

(A) $(2/3)f$
(B) $(5/3)f$
(C) $(7/3)f$
(D) $3f$
(E) $9f$



71. Two masses m are connected by springs with spring constants k and a massless rigid rod, as shown in the diagram. What is the frequency of oscillation of the system?

(A) $\sqrt{3k/m}$
(B) $\sqrt{k/m}$
(C) $\sqrt{2k/m}$
(D) $\sqrt{k/2m}$
(E) $2\sqrt{k/m}$

72. Suppose an atomic transition has a lifetime of 3×10^{-10} s. The natural line width of this transition is closest to

(A) 10^{-2} eV
(B) 10^{-4} eV
(C) 10^{-6} eV
(D) 10^{-8} eV
(E) 10^{-10} eV

73. Which of the following quantities change under a general gauge transformation in electromagnetism?

I. Electric potential
II. Electric field
III. Magnetic field

(A) I only
(B) II only
(C) I and II
(D) II and III
(E) I, II, and III

74. The degeneracy of the second excited state of the three-dimensional infinite square well is

(A) 1
(B) 2
(C) 3
(D) 8
(E) 9

75. What is the capacitance of two concentric thin conducting spheres of radii a and $b > a$?

(A) $4\pi\epsilon_0 ab/(b-a)$
(B) $4\pi\epsilon_0 a^2/(b-a)$
(C) $4\pi\epsilon_0 b^2/(b-a)$
(D) $4\pi\epsilon_0 \ln(b/a)$
(E) $4\pi\epsilon_0$

76. How many distinct spin states can be formed by three distinguishable spin-1/2 particles?

(A) 1
(B) 2
(C) 4
(D) 7
(E) 8

77. A thin lens is made of a material with an index of refraction of 1.5. If the radius of curvature of the left side of the lens is 10 cm, and the focal length is 1 m, what is the radius of curvature of the right side of the lens?

(A) 12.5 cm
(B) 6.25 cm
(C) 25 cm
(D) 3.125 cm
(E) 25 cm

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78. A car drives at velocity v through a garage of length l, with a front and rear door. The front door is initially open and the rear door is initially closed. In the frame of the garage, the rear door opens when the car is just about to collide with it, and the front door closes at the same time. In the frame of the car, how much time separates the opening of the rear door from the closing of the front door? ($\gamma = 1/\sqrt{1 - v^2/c^2}$.)

(A) 0
(B) $\dfrac{\gamma v l}{c^2}$
(C) $\dfrac{v l}{c^2}$
(D) $\dfrac{\gamma c l}{v^2}$
(E) $\dfrac{v l}{\gamma c^2}$

79. Consider the change in entropy of an ideal gas in the following situations:

- ΔS_1: temperature doubles, pressure constant
- ΔS_2: temperature doubles, pressure doubles
- ΔS_3: temperature constant, volume doubles

Which of the following is true?

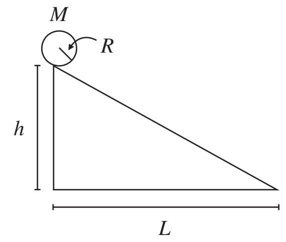
(A) $\Delta S_1 < \Delta S_2 < \Delta S_3$
(B) $\Delta S_1 < \Delta S_3 < \Delta S_2$
(C) $\Delta S_2 < \Delta S_3 < \Delta S_1$
(D) $\Delta S_3 < \Delta S_1 < \Delta S_2$
(E) $\Delta S_3 < \Delta S_2 < \Delta S_1$

80. A mass m attached to a spring of constant k is driven by a force $F(t) = F_0 \sin(\omega t)$. What is the late-time amplitude of the spring oscillations, assuming friction is small but sufficient to damp out transient oscillations?

(A) $\dfrac{F_0}{k}$
(B) $\dfrac{F_0}{m\omega^2}$
(C) $\dfrac{F_0}{|k - 2m\omega^2|}$
(D) $\dfrac{F_0}{|k - m\omega^2|}$
(E) $\dfrac{F_0}{|4k - m\omega^2|}$

81. An event occurs at $(t, x, y, z) = (0\text{ s}, 5\text{ m}, 10\text{ m}, 0\text{ m})$ in a reference frame S. At what position x' does the event occur in a reference frame S' with coordinates (t', x', y', z') that is moving at velocity $0.8c$ along the x-axis, relative to S? You may assume that the origin of coordinates coincides in both reference frames.

(A) 15.67 m
(B) 11.33 m
(C) 8.33 m
(D) 6.67 m
(E) 3.00 m



82. A dowel of radius R, mass M, and uniform mass density rolls without slipping down a ramp of length L and height h. What is its speed at the bottom of the ramp?

(A) $\sqrt{2gh}$
(B) $\sqrt{gh}$
(C) $\sqrt{gL}$
(D) $\sqrt{\dfrac{Rgh}{3L}}$
(E) $2\sqrt{\dfrac{gh}{3}}$

83. A positively charged particle, initially moving in the $\hat{\mathbf{x}}$-direction, enters a region containing uniform electric and magnetic fields, $\mathbf{E} = E_0(\hat{\mathbf{x}}+2\hat{\mathbf{y}})$ and $\mathbf{B} = B_0\hat{\mathbf{z}}$. Which of the following is a true statement about the motion of the particle?

(A) The particle moves in a circle.
(B) The particle is confined to the xz-plane.
(C) The particle is confined to the xy-plane.
(D) No work is done on the particle.
(E) The particle moves in a straight line.

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84. Dark matter, a hypothetical component of matter in the universe, which feels the gravitational force but does not interact with electromagnetism, can potentially explain all of the following observations EXCEPT:

(A) Stars at the outskirts of galaxies rotate faster than expected based on the mass inferred from luminous matter.
(B) Galaxies appear to be receding from us at a rate proportional to their distance.
(C) Fits to cosmic microwave background anisotropy data require a nonzero energy density which does not couple to photons.
(D) Gravitational lensing observations of colliding galaxies show that the luminous mass and the gravitational mass are displaced from one another.
(E) The gravitational potential of baryons alone would not be sufficient to counteract cosmic expansion and form structure such as galaxies.

85. A Geiger counter measures 1,250 events near a radioactive source during 10 seconds, and 350 events during 10 seconds when the source is removed. What is the uncertainty of the rate of events due to the source?

(A) 3.0 Hz
(B) 4.0 Hz
(C) 18.7 Hz
(D) 35.4 Hz
(E) 40.0 Hz

86. A signal pulse contains a current that has an exponential risetime of 10 ms and an exponential falltime of 100 ms. Approximately what minimum bandwidth should be used to view the pulse in frequency space on a spectrum analyzer?

(A) 0.01 Hz
(B) 1 Hz
(C) 1 kHz
(D) 1 MHz
(E) 1 GHz

87. A fit of a histogram to a model has a χ^2 statistic of 45.6. Which of the following should be used with the χ^2 value to determine whether the model represents the data well?

I. The number of entries in the histogram
II. The number of parameters in the model
III. The number of bins in the histogram

(A) II only
(B) III only
(C) I and III only
(D) II and III only
(E) I, II, and III

88. Which of the following is NOT produced in the *pp* cycle of the Sun?

(A) ^{2}H
(B) ^{3}He
(C) ^{4}He
(D) ^{8}B
(E) ^{11}C

89. What is the mean energy at temperature T of a two-state system with energy states 0 and ϵ?

(A) $\epsilon/2$
(B) $\epsilon/(1+e^{-\epsilon/kT})$
(C) $\epsilon/(1-e^{\epsilon/kT})$
(D) $\epsilon/(1+e^{\epsilon/kT})$
(E) $\epsilon(1+e^{\epsilon/kT})$

90. If the Hubble parameter suddenly were increased to twice its present value, which of the following would change?

(A) Fundamental particle masses
(B) Distance between Earth and distant galaxies
(C) Redshift of distant galaxies
(D) Cosmological constant
(E) Gravitational constant G

91. A satellite in orbit around the Earth monitors seismic activity through interferometry, by comparing the phase of a reflected wave before and after an earthquake. If the typical displacement of a point on Earth is 5 cm, which of the following radiation frequencies would be most appropriate to use for this measurement?

(A) 1 Hz
(B) 1 kHz
(C) 1 MHz
(D) 1 GHz
(E) 1 THz

92. Doping a semiconductor does which of the following to the band structure?

(A) No effect on band structure
(B) Eliminates the valence band
(C) Increases the gap energy
(D) Adds additional states between valence and conduction bands
(E) Eliminates the energy gap

93. An infinite conducting cylinder of radius a carries a surface charge density of σ. Assuming that the potential on the surface is 0, what is the potential at a distance $r > a$?

(A) $\dfrac{1}{\epsilon_0} a\sigma \ln \dfrac{r}{a}$
(B) $\dfrac{1}{\epsilon_0} a\sigma \ln \dfrac{a}{r}$
(C) $\dfrac{1}{2\epsilon_0} a\sigma \ln \dfrac{r}{a}$
(D) $\dfrac{1}{2\epsilon_0} a\sigma \ln \dfrac{a}{r}$
(E) 0

94. A toy car travels through a vertical loop on a track. If the radius of the loop is 50 cm, what is the minimum initial speed needed by the car at the bottom of the loop to successfully complete the loop without losing contact with the track?

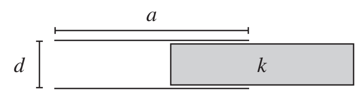
(A) 1.4 m/s
(B) 4.5 m/s
(C) 5.0 m/s
(D) 7.8 m/s
(E) 9.6 m/s

95. Which of the following is equivalent to the commutator $[S_x S_y, S_y]$, where S_x and S_y are quantum-mechanical spin operators?

(A) 0
(B) $i\hbar S_z S_x$
(C) $-i\hbar S_x S_z$
(D) $i\hbar S_y S_x$
(E) $i\hbar S_z S_y$

96. A particle is under the influence of a central potential $U(r)$ with the property that all bound orbits are closed. The particle has an energy E such that $V_{\min} < E < 0$, where $V_{\min}$ is the minimum of the effective potential derived from $U(r)$. Which of the following characterizes the shape of the orbit?

(A) Circular
(B) Elliptical
(C) Parabolic
(D) Hyperbolic
(E) The answer cannot be determined from the information given.



97. A square parallel-plate capacitor has side lengths a and separation d between the plates. A constant voltage V_0 is applied between the plates. A block of dielectric material of dielectric constant κ and the same area and thickness as the capacitor is slowly inserted into the capacitor. What is the change in energy stored in the capacitor by the time the dielectric is fully inserted?

(A) 0
(B) $\dfrac{(\kappa - 1)\epsilon_0 a^2 V_0^2}{4d}$
(C) $\dfrac{(\kappa - 1)\epsilon_0 a^2 V_0^2}{d}$
(D) $\dfrac{(\kappa - 1)\epsilon_0 a^2 V_0^2}{2d}$
(E) $\dfrac{\kappa \epsilon_0 a^2 V_0^2}{2d}$

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98. An object is placed at rest in a potential field $U(x, y, z) = Ax + By^2 - C\cos z$, where A, B, C are constants. What is the force on the object?

(A) $\mathbf{F}(x, y, z) = -A\hat{\mathbf{x}} - 2By\hat{\mathbf{y}} - C\sin z\hat{\mathbf{z}}$
(B) $\mathbf{F}(x, y, z) = Ax\hat{\mathbf{x}} + 2By\hat{\mathbf{y}} - C\cos z\hat{\mathbf{z}}$
(C) $\mathbf{F}(x, y, z) = -Ax\hat{\mathbf{x}} - 2By\hat{\mathbf{y}} + C\cos z\hat{\mathbf{z}}$
(D) $\mathbf{F}(x, y, z) = -A\hat{\mathbf{x}} - 2By\hat{\mathbf{y}} + C\cos z\hat{\mathbf{z}}$
(E) $\mathbf{F}(x, y, z) = A\hat{\mathbf{x}} + 2By\hat{\mathbf{y}} + C\sin z\hat{\mathbf{z}}$

99. Suppose a hydrogen atom is in a uniform external electric field of magnitude E. What is the correction to the ground state energy calculated from first-order perturbation theory? The normalized ground state wavefunction of hydrogen is $\psi(r, \theta, \phi) = \frac{1}{\sqrt{\pi}a_0^{3/2}}e^{-r/a_0}$.

(A) eE
(B) $\frac{1}{2}eE$
(C) $\frac{3}{2}eE$
(D) $\frac{16}{5}eE$
(E) 0

100. In Mössbauer spectroscopy, a source of photons of energy E is moved with velocity $v \ll c$ relative to a target material. The absorption of photons by the target material is then measured, with the Doppler shift from the source velocity producing a small variation in the photon energy. If the absorption peaks of two lines correspond to source velocities of 0 and v, what is the energy splitting between the lines to lowest order in v?

(A) Ev^4/c^4
(B) Ev^3/c^3
(C) Ev^2/c^2
(D) Ev/c
(E) $E\sqrt{v/c}$

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Answers to Sample Exam 1

1. C
2. E
3. D
4. B
5. A
6. E
7. C
8. D
9. B
10. C
11. E
12. D
13. E
14. B
15. A
16. D
17. E
18. A
19. E
20. B
21. D
22. A
23. E
24. A
25. C
26. B
27. C
28. D
29. A
30. E
31. D
32. C
33. E
34. C
35. D
36. C
37. D
38. B
39. B
40. D
41. B
42. B
43. D
44. E
45. C
46. A
47. A
48. B
49. D
50. D
51. C
52. D
53. A
54. D
55. C
56. C
57. B
58. C
59. E
60. B
61. C
62. A
63. A
64. B
65. E
66. C
67. C
68. E
69. B
70. C
71. B
72. B
73. E
74. E
75. D
76. A
77. C
78. C
79. C
80. D
81. E
82. A
83. B
84. D
85. C
86. E
87. C
88. A
89. E
90. D
91. C
92. B
93. E
94. E
95. E
96. A
97. C
98. B
99. D
100. A

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Answers to Sample Exam 2

1. D
2. C
3. E
4. C
5. B
6. A
7. C
8. D
9. D
10. E
11. A
12. A
13. E
14. C
15. E
16. C
17. B
18. D
19. A
20. A
21. C
22. D
23. B
24. A
25. E
26. B
27. E
28. D
29. D
30. C
31. E
32. E
33. E
34. B
35. E
36. D
37. D
38. D
39. C
40. B
41. C
42. D
43. D
44. D
45. A
46. D
47. B
48. B
49. A
50. A
51. D
52. B
53. B
54. A
55. E
56. E
57. E
58. A
59. E
60. D
61. A
62. A
63. D
64. A
65. B
66. D
67. E
68. C
69. D
70. B
71. B
72. B
73. C
74. D
75. B
76. C
77. D
78. A
79. B
80. B
81. B
82. E
83. C
84. A
85. A
86. B
87. C
88. B
89. C
90. D
91. B
92. C
93. C
94. E
95. D
96. B
97. E
98. B
99. C
100. D

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Answers to Sample Exam 3

1. C
2. E
3. E
4. B
5. E
6. C
7. C
8. B
9. C
10. A
11. A
12. C
13. B
14. B
15. C
16. B
17. C
18. E
19. C
20. D
21. D
22. D
23. B
24. B
25. B
26. B
27. D
28. A
29. B
30. B
31. B
32. A
33. B
34. E
35. D
36. C
37. C
38. C
39. E
40. E
41. D
42. B
43. C
44. C
45. A
46. D
47. A
48. E
49. E
50. D
51. B
52. E
53. E
54. C
55. C
56. C
57. C
58. C
59. A
60. C
61. E
62. A
63. E
64. C
65. A
66. E
67. B
68. B
69. D
70. B
71. B
72. C
73. A
74. C
75. A
76. E
77. A
78. B
79. E
80. D
81. C
82. E
83. C
84. B
85. B
86. C
87. D
88. E
89. D
90. C
91. D
92. D
93. B
94. C
95. E
96. B
97. D
98. A
99. E
100. D

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Solutions to Sample Exam 1

1. C – This is a simple application of the centripetal acceleration formula:
$$v = \sqrt{ar} = \sqrt{9 \cdot 4g} = 6\sqrt{|g|} \text{ m/s}.$$
2. E – Although we have to just calculate this one, it is good practice for working through basic kinematics problems quickly. If you find yourself getting bogged down in numerical factors, this is a good example of a problem to skip and save for later, as there shouldn't be too many problems of this type on the exam. Recall that elastic collisions conserve both linear momentum and energy. If we call the final velocity of the heavy block v_1 and the final velocity of the light block v_2, and drop all the m's, then we just need to solve the following system of equations:
$$\begin{aligned} v &= 4v_1 + v_2, \\ \frac{1}{2}v^2 &= \frac{1}{2}4v_1^2 + \frac{1}{2}v_2^2. \end{aligned}$$
Plugging the first equation into the second for v, we get
$$\begin{aligned} (4v_1 + v_2)^2 &= 4v_1^2 + v_2^2, \\ 16v_1^2 + 8v_1v_2 + v_2^2 &= 4v_1^2 + v_2^2, \\ 12v_1^2 + 8v_1v_2 &= 0, \\ v_2 &= -\frac{3}{2}v_1. \end{aligned}$$
Then, plugging this into the conservation of momentum equation, we get
$$\begin{aligned} v &= 4v_1 - \frac{3}{2}v_1, \\ v_1 &= \frac{2}{5}v. \end{aligned}$$
3. D – The oscillation frequency of an LC circuit is a good quantity to memorize:
$$\omega = \frac{1}{\sqrt{LC}}.$$
If you (understandably) forget it, it is straightforward to reconstruct it from dimensional analysis by looking for combinations of L and C that give units of s^{-1}. The capacitance scales as $C \sim A/d$, so C increases by a factor of 2 under the doubling. Assuming that our inductor is a solenoid with a fixed number of loops, then the inductance scales just like the solenoid inductance (a 20 second derivation if you forget it) $L \sim A/\ell$. Overall, the frequency drops in half.
4. B – A dipole contains no net charge, and the charge on a conductor will rearrange itself to completely cancel out the dipole field.
5. A – The approximation of a product wavefunction comes from solving the Schrödinger equation by separation of variables, which is only possible when the Coulomb repulsion term is ignored.
6. E – Since the emitted photon carries momentum, the entire atom must recoil slightly to conserve linear momentum. This means that the total energy released in the transition is divided between the gamma and the recoil of the atom. As a result, the photon energy will be slightly less than the true transition energy of the atomic level. Choices A–C simply do not make sense in the context of the problem, and D appears to violate the conservation of energy, so E is the correct choice.
7. C – The Fermi energy is $E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$, where n is the density. Since we are at fixed volume, doubling the number of particles doubles the density, which multiplies E_F by a factor of $2^{2/3}$, choice C.
8. D – Since the gas is well above the Fermi temperature, it is essentially classical. By the equipartition theorem, each quadratic degree of freedom contributes specific heat per particle $k/2$. There are two quadratic degrees of freedom for the kinetic part of the Hamiltonian and two quadratic degrees of freedom for the potential part, so the specific heat is $2k$.
9. B – Recall that the effective potential for radial motion is $V_{\text{eff}}(r) = \frac{l^2}{2mr^2} + U(r)$. The radii of circular orbits are found by solving $\frac{dV_{\text{eff}}}{dr} = 0$, and stability is determined by the sign of $\frac{d^2V_{\text{eff}}}{dr^2}$. Here,
$$\frac{dV_{\text{eff}}}{dr} = -\frac{l^2}{mr^3} + \frac{k}{r^2},$$
and setting this equal to zero and solving for r gives $r = \frac{l^2}{mk}$, choice B. We could check that this is a minimum of V_{eff} by computing the second derivative, but it's easier to just think about the behavior of the potential at $r = 0$ and $r = \infty$. At $r = 0$, the repulsive centrifugal barrier dominates and $V_{\text{eff}} \to +\infty$. As r approaches infinity, the central potential $U = -k/r$ decays more slowly than the centrifugal term, so V_{eff} approaches zero from *below*. Since V_{eff} has only a single critical point, sketching the graph of V shows that this radius is indeed a minimum, and hence an allowed stable circular orbit.
10. C – This is a straightforward application of the parallel axis theorem. Recall from the formula sheet that the

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moment of inertia for a single disk about its central axis is $I_{CM} = \frac{1}{2}MR^2$. From the parallel axis theorem, we can read off the moment of inertia for a single disk about the axis going through the edge:

$$I_{\text{disk}} = I_{CM} + \frac{1}{2}MR^2 = MR^2 + \frac{1}{2}MR^2 = \frac{3}{2}MR^2.$$

And finally, the moment of inertia of two bodies that are rigidly attached to each other is equal to the sum of the moments of inertia of the individual bodies, so we have the final result

$$I_{\text{total}} = 2\frac{3}{2}MR^2 = 3MR^2.$$

11. E – Redshift z is defined by $1 + z = \lambda_{\text{obs}}/\lambda_{\text{emit}}$. So a galaxy at redshift 2 has wavelengths expanded by a factor of 3, and the 21 cm hydrogen line gets redshifted to 63 cm, choice E. Beware the trap answer D: redshift 2 does *not* mean wavelengths are expanded by a factor of 2!
12. D – The probabilities of states 1 and 2 are $\frac{1}{4}$ and $\frac{1}{2}$, so the probability of measuring state 3 must be $\frac{1}{4}$. We can write the wavefunction as

$$|\Psi\rangle = c_1|1\rangle + c_2|2\rangle + c_3|3\rangle,$$

and the energy expectation value is

$$\langle E\rangle = c_1^2(E_0) + c_2^2(2E_0) + c_3^2X.$$

The squares of the coefficients are the probabilities of measuring each energy, so we can just plug in values and solve for X:

$$\begin{aligned}\frac{9}{4}E_0 &= \frac{1}{4}E_0 + \frac{1}{2}2E_0 + \frac{1}{4}X,\\ \frac{1}{4}X &= \frac{9-5}{4}E_0,\\ X &= 4E_0.\end{aligned}$$

13. E – The Planck mass is the unique combination of fundamental constants $\hbar$, c, and G that has units of mass. Expressing all the constants in terms of mass, length, and time:

$$\begin{aligned}\hbar &= [M][L]^2[T]^{-1},\\ c &= [L][T]^{-1},\\ G &= [M]^{-1}[L]^3[T]^{-2}.\end{aligned}$$

For a combination $(\hbar)^p(c)^q(G)^r$ to have units of mass, or $[M]^1$, we need:

$$\begin{aligned}p - r &= 1,\\ 2p + q + 3r &= 0,\\ -p - q - 2r &= 0.\end{aligned}$$

Using your favorite method to solve systems of linear equations, we find $p = 1/2$, $q = 1/2$, $r = -1/2$, so $M_P = \sqrt{\frac{\hbar c}{G}}$, which is E.

14. B – This is a straightforward application of the relativistic Doppler shift formula:

$$\frac{\nu'}{\nu} = \sqrt{\frac{1+\beta}{1-\beta}},$$

where $\beta = v/c$ is the relative velocity of the source with respect to the observer. The easy way to keep track of the signs in the numerator and the denominator is to remember that the observed frequency *increases* when the source is moving towards the observer. Or, if you're partial to astrophysics, when the observer is receding, the signal is *redshifted* (in other words, the frequency decreases). Here the source is approaching, so $\beta = 0.6$ is positive and

$$\frac{\nu'}{\nu} = \sqrt{\frac{1.6}{0.4}} = \sqrt{4} = 2,$$

so the frequency doubles: $\nu' = 2$ GHz, choice B.

15. A – This is a classic method of images problem, with a small twist. To calculate the work, we first need to know the potential energy of the initial charge configuration with the point at infinity. This is just $U_0 = 0$, by definition. The final potential energy can be determined from the image charge configuration, which consists of our charge $+q$ at a distance d above the plane, and an image charge $-q$ at a distance d below the plane. Be sure to remember your signs! One of the charges must be negative because the potential on the conducting plane must, by definition, always be zero (this is the entire point of the method of images). The potential energy of two opposite point charges separated by a distance $2d$ is given by

$$U_{\text{image}} = -\frac{1}{4\pi\epsilon_0}\frac{q^2}{2d}.$$

Now the trick: the total potential energy of the point charge plus plane is half the potential energy of the two point charges. This is because we can think of the total energy of the configuration as

$$U = \int_{\text{all space}} \frac{\epsilon_0}{2}E^2\, dV,$$

but since the plane is a conductor, the electric field $E = 0$ below the plane. By symmetry about the plane, the potential energy of the charge with conductor is half the energy of the two charges. Thus we find

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$$W = \Delta U = \frac{U_{\text{image}}}{2} - 0 = -\frac{1}{4\pi\epsilon_0}\frac{q^2}{4d},$$

which is choice A. You could also compute this by integrating the force on the point charge, as discussed in Section 2.1.7.

16. D – The charge after one second is $Q = (10^{-3}\ \text{C} \cdot \text{s}^{-1})(1\ \text{s}) = 10^{-3}$ C. From $V = Q/C$, we get $V = 10^{-3}/10^{-5} = 100$ V.
17. E – The leading-order perturbation is simply

$$\begin{aligned}\langle 2|\,\delta V\,|2\rangle &= \frac{2}{L}\int_0^{L/2} V_0 \sin^2\left(\frac{2\pi x}{L}\right) dx \\ &= V_0/2.\end{aligned}$$

18. A – From $dE = T\,dS - P\,dV$, we do a Legendre transform to get $dF = -S\,dT - P\,dV$. Since the expansion is isothermal, it takes place at constant temperature, so the first term vanishes; the second term is positive because the volume increases, so dF is negative and the free energy decreases, rather than increases. This is consistent with the definition of "free" energy, which is essentially the available energy the gas has to do work on its surroundings. Since it does work in the isothermal expansion phase, at the end of the expansion it has less free energy available to do work. All the other statements are true; see the discussion in Section 4.2.6.
19. E – The angle of the first diffraction minimum is $\sin\theta = \lambda/a$, where λ is the wavelength of the incident light. Since the screen is far away, we approximate $\sin\theta \approx \theta$, so the angular width of the central maximum is 2θ. For a screen a distance L away, the width of the maximum as seen on the screen is $L\tan\theta \approx L\theta$. So we want

$$\begin{aligned}2L\frac{\lambda}{a} = 100a &\implies a^2 = 9\times 10^{-8}\ \text{m}^2 \\ &\implies a = 3\times 10^{-4}\ \text{m} = 0.3\ \text{mm}.\end{aligned}$$

20. B – From the definition of conjugate momentum,

$$p_\phi = \frac{\partial L}{\partial\dot\phi} = m(a + b\cos\theta)^2\dot\phi,$$

choice B.

21. D – The Lagrangian isn't explicitly dependent on time, so the first two terms in L represent the kinetic energy T and the third represents the potential energy U. Since $L = T - U$, to get the total energy $T + U$ we have to take $L + 2U$, which is choice D.
22. A – The decay time constant for an RL circuit is $\tau = L/R$. This represents the amount of time required for the voltage across the inductor to drop to $1/e$ of its initial level, or, in other words, $V(t) = V_0 e^{-t/\tau}$. Setting $V(t) = V_0/2$, we find that $t = \tau\ln 2 = \frac{L}{R}\ln 2$.
23. E – The straight wire produces a magnetic field that circles azimuthally around in the $\hat{\boldsymbol\phi}$-direction. The tension on the circular wire, if any, will be the result of the Lorentz force that the moving charges in the circular wire feel due to the magnetic field established by the straight wire. But the current in the circular wire is also flowing in the $\hat{\boldsymbol\phi}$-direction, parallel to the magnetic field. The electrons flowing in the circular wire therefore feel no force and there is no tension. So the correct answer is E.
24. A – Considering the limit of $r \gg h$ eliminates C and D immediately, but we need to calculate to get the exact numerical factors. The cylinder will fall over when it is tipped just past the point where the center of mass is directly above the point of contact of the cylinder with the ground. At that point, the angle θ forms a right triangle with side lengths $h/2$ and r, so we have $\tan\theta = r/(h/2) = 2r/h$, and the angle of the cylinder with the horizontal is A.
25. C – The γ factor is $\gamma = 3000\ \text{MeV}/100\ \text{MeV} = 30$. In the (stationary) lab frame, the lifetime of the muon is $\tau = \gamma\tau_{\text{rest}} = 30\tau_{\text{rest}} = 60\ \mu\text{s}$.
26. B – This problem is a bit of trivia that is difficult to guess. The only way to guess it is to notice that the dispersion relation in B gives energy as proportional to momentum, which is ordinarily true for particles in the extreme relativistic limit (i.e. $E = pc$). Given that the problem refers to unusual electronic properties, it might seem reasonable that this unusual dispersion relation would produce unusual properties. This is indeed what happens in graphene, where electrons and holes behave like massless Dirac fermions, a distinctive feature of a semiconductor with no gap energy and linear dispersion.
27. C – The perturbing potential $V(x) = -qE_0x$ is an odd function, while the ground state wavefunction for the harmonic oscillator is even. Therefore, the first-order perturbation will vanish, and the leading-order correction will come from second-order perturbation theory and be proportional to E_0^2.
28. D – We can use the lens equation twice. For the first lens, we have

$$\frac{1}{5\ \text{cm}} = \frac{1}{2\ \text{cm}} + \frac{1}{s_1'},$$

which implies that $s_1' = -10/3$ cm. The negative sign implies that the image is located to the left of lens A,

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or 25/3 cm to the left of lens B. Using the lens equation again for the second lens, we find that

$$\frac{1}{5\text{ cm}} = \frac{3}{25\text{ cm}} + \frac{1}{s_2'},$$

which implies that $s_2' = 12.5$ cm. Since the answer is positive, it is located to the right of lens B.

29. A – As is typical for uniform circular motion, we want to set the gravitational force equal to the centripetal force. The acceleration in uniform circular motion is still given by $a = v^2/r$, and the gravitational force is still $F_{\text{grav}} = GMm/r^2$. The MOND force law requires that we set $F_{\text{grav}} = ma^2/a_0$, giving

$$\frac{GMm}{r^2} = m\frac{(v^2/r)^2}{a_0}$$
$$\implies v = (GMa_0)^{1/4}.$$

The factors of r cancel out, and v ends up independent of r.

30. E – When combining spins l_1 and l_2 (where without loss of generality we can take $l_1 < l_2$), we can get all values of l between l_2+l_1 and l_2-l_1 in integer steps. In the present case, with $l_1 = 1$ and $l_2 = 2$, we can get $s = 3, 2, 1$. The only condition on m_s is $|m_s| \leq s$, so choices A–D are all fine. Since $s = 0$ is impossible, the answer is E.

31. D – The radial probability density for the $2p$ state is given by

$$r^2|R_{21}(r)|^2 = \frac{1}{24}a_0^{-5}r^4\exp(-r/a_0),$$

where the factor of r^2 is from the volume element in spherical coordinates, $dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$. To find the most probable value, we take the derivative and set to zero (and in doing so, we can ignore all the annoying constants out front):

$$4r^3e^{-r/a_0} - \frac{r^4}{a_0}e^{-r/a_0} = 0$$
$$\implies r = 4a_0,$$

choice D. Forgetting the factor of r^2 from the volume element leads to trap answer C, forgetting to square the wavefunction leads to E, and forgetting both also leads to C. These are all very common mistakes – don't make them!

32. C – This is almost a giveaway, since the "mass" in "mass spectrometry" does indeed refer to the mass of the particle involved. However, it is important to note that only the charge-to-mass ratio can be measured by electromagnetic fields, so the answer can't be D, which can change the mass without affecting the charge.

33. E – Beat frequencies are caused by destructive interference between two closely spaced frequencies, resulting in a modulation with a long enough period that the minimum of each cycle is heard independently. For this problem, all that is relevant is the sum-to-product identity

$$\cos 2\pi at + \cos 2\pi bt = 2\cos\left(2\pi\frac{a+b}{2}t\right)\cos\left(2\pi\frac{a-b}{2}t\right).$$

The second term in the product modulates the wave; we hear beats when its amplitude is zero, which occurs at frequency $a - b$. Note that this is *twice* the apparent frequency of the cosine! If all we know is that this frequency is 3 Hz, and that one of a or b is 440 Hz, it is impossible to determine whether the other frequency is 443 Hz or 437 Hz, since both would give the same beat frequency (the sign of $a - b$ is irrelevant because cosine is even). Hence the correct answer is E.

34. C – This is a relativistic velocity addition problem. In the frame of the planet, the speed of the missile is given by the velocity addition formula:

$$v_{\text{missile}} = \frac{0.5c + 0.5c}{1 + (0.5c)(0.5c)/c^2} = \frac{c}{1.25} = 0.8c.$$

Light travels at c in all frames, so the information that the missile has been fired reaches the planet at $1c$. Since the spaceship is 1 light-hour away ($c \times 1$ h), the observer sees the flash at $t_1 = (c \times 1\text{ h})/1c = 1$ h, and the missile hits at $t_2 = (c \times 1\text{ h})/0.8c = 1.25$ h, so the difference is 0.25 h or 15 min, choice C.

35. D – There are no external torques here, so angular momentum is conserved. Thus $I_0\omega_0 = I_1\omega_1$, and plugging in numbers we arrive at D.

36. C – IV is obviously false since we use the Euler–Lagrange equations to construct the equations of motion for any system with a Lagrangian. III is not always true because we are always free to add a constant to the potential – the fact that L is independent of x only means that the potential must be constant in space, and we can set that constant to zero if we wish.

On the other hand, homogeneity of time *does* imply conservation of energy, and homogeneity of space implies linear momentum conservation. If these statements are unfamiliar to you, then let's prove them. Since L depends on x and $\dot{x}$ only, the total time derivative of the Lagrangian is

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$$\frac{dL}{dt} = \frac{\partial L}{\partial x}\dot{x} + \frac{\partial L}{\partial \dot{x}}\ddot{x} + \frac{\partial L}{\partial t}.$$

Using the Euler–Lagrange equations and the fact that $\frac{\partial L}{\partial t} = 0$, we have

$$\begin{aligned}\frac{dL}{dt} &= \dot{x}\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} + \frac{\partial L}{\partial \dot{x}}\ddot{x}\\ &= \frac{d}{dt}\left(\dot{x}\frac{\partial L}{\partial \dot{x}}\right),\\ 0 &= \frac{d}{dt}\left(\dot{x}\frac{\partial L}{\partial \dot{x}} - L\right).\end{aligned}$$

But the quantity in parentheses is precisely the energy of the system that we obtain when we construct the Hamiltonian of the system from the Lagrangian via the Legendre transform.

The situation for momentum is more simple. By Euler–Lagrange, we have

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = 0,$$

and therefore

$$\frac{\partial L}{\partial \dot{x}} = p,$$

where p is a constant of motion that is defined to be the linear momentum.

37. D – The formula we want is $N = \epsilon \mathcal{L} \sigma T$, where N is the number of events seen, ϵ is the detector efficiency, $\mathcal{L}$ is the luminosity, σ is the cross section, and T is the running time. Note that this just comes straight from dimensional analysis – any possible numerical factors are all absorbed in the definitions of luminosity, cross section, and efficiency. There are 8.64×10^4 seconds in a day, so $N = (0.5)(10^{22})(10^{-20})(8.64\times 10^4) = 4.32\times 10^6$, choice D.
38. B – Cyclotron motion is simple enough that it can be derived in a matter of seconds from the Lorentz force law and centripetal force. Rewriting a bit, we find that

$$r = \frac{p}{qB}.$$

Triple the field, and the momentum must also be tripled to maintain constant radius.
39. B – Without calculating anything, we can reason that only choices B and C are allowed. The current is the same in both loops of wire and the field from the smaller loop (pointed into the page, by the right-hand rule) will be stronger than the field from the larger loop (pointed out of the page). So the total field must also point into the page. If you remember the formula for the magnetic field at the center of a single circular loop of wire of radius r,

$$B = \frac{\mu_0 I}{2r},$$

then you can immediately find that the answer is

$$B_{\text{tot}} = \frac{\mu_0 I}{2}\left(\frac{1}{a} - \frac{1}{b}\right),$$

which is choice B.

If you are in a bind and don't remember the formula for the field at the center of a circular loop of wire, it is easy to derive from the Biot–Savart law:

$$\begin{aligned}\mathbf{B} &= \frac{\mu_0}{4\pi}\int \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}\\ &= \frac{\mu_0 I}{4\pi r}\int_0^{2\pi} d\phi\, \hat{\mathbf{z}}\\ &= \frac{\mu_0 I}{2r}\hat{\mathbf{z}}.\end{aligned}$$

40. D – Recall that L^2 commutes with each component of L because it commutes with L_z. For the same reason, L^2 commutes with $J^2 = L^2 + S^2 + 2\mathbf{L}\cdot\mathbf{S}$: $\mathbf{L}$ and $\mathbf{S}$ act on different parts of the wavefunction, so all components of $\mathbf{L}$ commute with all components of $\mathbf{S}$, and L^2 commutes with the last term because it commutes with $\mathbf{L}$ as well. However, L^2 only commutes with rotationally symmetric Hamiltonians, and a Hamiltonian need not have rotational symmetry: for example, an atom in a strong magnetic field which picks out a particular direction in space.
41. B – The possible energies are just the eigenvalues of the Hamiltonian matrix, which we can obtain by solving the equation

$$\begin{aligned}\det(H - \lambda I) &= 0,\\ (a-\lambda)(\lambda^2 - b^2) &= 0.\end{aligned}$$

The solutions are clearly $\lambda = a$ and $\lambda = \pm b$, choice B.
42. B – If there were magnetic charges, we would have a Maxwell equation $\nabla\cdot\mathbf{B} \propto \rho_m$ just like we have a Maxwell equation for electric (monopole) charges $\nabla\cdot\mathbf{E} \propto \rho_e$, and Gauss's law for magnetism would give the total charge by integrating over space. This is choice B.
43. D – We could solve this one exactly, but it is clear from the answer choices that we can use dimensional analysis and scaling arguments to rule out answers much more quickly. Choice C does not have the correct units for an electric field (N/C). And choices A, B, and E all have the wrong scaling with the radius of the target.

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As the radius of the target increases, the electric field required to deflect the beam should increase. In all of these answers, R appears in the denominator, implying the opposite behavior.

On the other hand, it is not too difficult to solve the problem the normal way, which would be necessary on the exam if a different set of answer choices were given. The perpendicular electric field only provides an acceleration perpendicular to the direction of motion, so we can solve this just like an analogous kinematics problem where gravity provides the perpendicular force. The velocity in the direction of the beam stays v, so the time it takes to strike the target is $t = L/v$. On the other hand, the perpendicular acceleration is $a_\perp = qE/m$, so in time t the protons are deflected $\frac{1}{2}a_\perp t^2 = \frac{qE}{2m}t^2$ in the perpendicular direction. Setting this equal to R, plugging in $t = L/v$, and solving for E, we obtain $E = \dfrac{2mRv^2}{qL^2}$, choice D.

44. E – In the standard cosmology, the Big Bang is immediately followed by inflation, at the end of which elementary particles are produced in an era known as "reheating." At this stage, quarks and gluons are free particles. Once the universe cools enough to allow quarks to form baryons, we get nucleosynthesis. Even without knowing the exact timeline governing I and II, we can deduce immediately that II precedes I, since (re-)ionization refers to the stripping of an electron from a neutral atom, and to have atoms we must first have nuclei, which are formed during nucleosynthesis. So the correct order is III, II, I, choice E.

45. C – For an adiabatic process, we have $PV^\gamma = \text{const}$, where $\gamma \equiv c_P/c_V = (\alpha+1)/\alpha$ and 2α is the number of degrees of freedom for the system. For an ideal gas $\alpha = 3/2$ and $\gamma = 5/3$.

46. A – A solid cylinder of uniform mass density has moment of inertia $I = \frac{1}{2}MR^2$. The potential energy mgL of the weight when the rope is wound is entirely converted to kinetic energy once the rope is unwound. Note that the kinetic energy has contributions from both the rotational energy of the cylinder and the velocity of the weight. So by conservation of energy,

$$mgL = \frac{1}{2}mv^2 + \frac{1}{4}MR^2\frac{v^2}{R^2} \implies v = \sqrt{\frac{4mgL}{M+2m}},$$

choice A.

47. A – The force on an object is related to the potential energy by

$$\mathbf{F} = -\nabla U.$$

All we need is to take the gradient of the potential in the question:

$$\begin{aligned}\mathbf{F} &= -\hat{\mathbf{x}}\frac{\partial}{\partial x}(x) - \hat{\mathbf{y}}\frac{\partial}{\partial y}\left(y^2\right) + \hat{\mathbf{z}}\frac{\partial}{\partial z}(\cos z)\\ &= -\hat{\mathbf{x}} - 2y\hat{\mathbf{y}} - \sin z\hat{\mathbf{z}}.\end{aligned}$$

48. B – The formula for the energy is

$$E = \frac{\sum_\epsilon \epsilon d(\epsilon)e^{-\epsilon/kT}}{\sum_\epsilon d(\epsilon)e^{-\epsilon/kT}}.$$

Plugging in the given energies and degeneracies,

$$E = \frac{(-2\epsilon)e^{\epsilon/kT} + (3\epsilon)e^{-\epsilon/kT}}{2e^{\epsilon/kT} + 1 + 3e^{-\epsilon/kT}}.$$

As $T \to \infty$, the exponent goes to zero, so each of the exponential factors collapses to 1. Thus the $T \to \infty$ limit of the energy is

$$E = \frac{-2\epsilon + 3\epsilon}{2+1+3} = \frac{\epsilon}{6},$$

choice B. This last step is equivalent to saying that, at infinite temperature, each state becomes equally probable, so we can forget about the Boltzmann statistics and just calculate a weighted mean.

49. D – In process 1 we have $\Delta S_1 = \int dQ/T = C\ln 2 = (3/2)Nk\ln 2$. In process 2, we have have $\Delta S_2 = Nk\ln 2$ from the equation for the entropy of an ideal gas. So $\Delta S_1 > \Delta S_2$.

50. D – The Poynting vector is

$$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}),$$

and if the electric field is $\mathbf{E} = E_0\cos(kx-\omega t)\hat{\mathbf{z}}$, then the magnetic field is $\mathbf{B} = -(1/c)E_0\cos(kx-\omega t)\hat{\mathbf{y}}$ for propagation in the $\hat{\mathbf{x}}$-direction. The average magnitude of the Poynting vector is simply

$$\langle \mathbf{S}\rangle = \frac{1}{\mu_0 c}\left\langle E_0^2\cos^2(kx-\omega t)\right\rangle = \frac{E_0^2}{2\mu_0 c}.$$

(A very useful fact to remember is that the average of $\sin^2$ or $\cos^2$ over one period is $1/2$.)

51. C – If we consider the reaction in the CM frame of the e^+e^- system, then the γ must be at rest to conserve energy–momentum. But γ always travels at the speed of light, so conservation of energy–momentum is violated.

52. D – This is a straightforward application of the uncertainty principle. We know that

$$\Delta p \Delta x \gtrsim \hbar.$$

So if r is the nuclear radius,

$$r \sim \Delta x \gtrsim \frac{\hbar}{\Delta p}.$$

Using our numerical values, we have

$$r \gtrsim \frac{\hbar c}{40 \text{ MeV}} = \frac{197}{40} \text{ fm} \sim 5 \text{ fm},$$

closest to choice D. Note that we have dropped lots of factors of 2 along the way, which is fine for these "powers of 10" type of problems.

53. A – Choices B–E all are spin-1 objects and therefore are bosons, which obey Bose–Einstein statistics. You should know that the photon has spin-1 from quantum mechanics. The statistics of helium nuclei and atoms follow from the rules of addition of angular momentum: the nucleus has four fermions (two protons and two neutrons), and the atom has two more fermions (the two atomic electrons), and an even number of spin-1/2 particles behaves as a boson with integer spin. Pions are a little tricky, but you might remember that in the quark model they are bound states of a quark and an antiquark, both fermions, so again an even number of fermions gives a boson. Only A, the neutrino, is a spin-1/2 particle and fermion that obeys Fermi statistics.

54. D – The mass of the electron is 511 keV/c^2, which is one of those ubiquitous numbers that should be memorized. So any time the energy scale 511 keV shows up, it must have something to do with electrons. In this case, electrons and positrons at rest in the galactic center can annihilate to two photons, each of energy 511 keV. (Incidentally, the fact that the line is "sharp" means that most of the electrons and positrons in the galactic center are moving slowly, since if annihilation took place between two highly energetic particles, the photons would be boosted in the galactic rest frame. The spectrum observed on Earth would then have a sharp "edge" at 511 keV and a long tail extending to higher energies.)

55. C – From basic kinematics, the cannonball falls a vertical distance of 250 m in time $\frac{1}{2}gt^2$; solving gives $t = \sqrt{500/g}$ seconds. In this time, the cannonball travels a horizontal distance of 420 m, so from $vt = 420$ m, we plug in our previous result for t to get

$$v = 420\sqrt{g/500} \approx 420\sqrt{1/50} \approx 420(1/7) \approx 60 \text{ m/s}.$$

This is closest to choice C, so we choose it and move on. This pattern of answer choices is typical of Physics GRE questions – since they're so widely spaced, it's not at all necessary to do arithmetic to three decimal places. The approximations $g \approx 10$ m/s and $\sqrt{50} \approx 7$ were just fine here.

56. C – Let the mass of the rocket be m. In the frame of the rocket, we balance the momentum of the exhaust with the momentum of the rocket at the moment of expulsion:

$$|\Delta p_{\text{fuel}}| = |\Delta p_{\text{rocket}}|,$$
$$\frac{v}{2}(0.1m) = (0.9m)|\Delta v_{\text{rocket}}|,$$
$$|\Delta v_{\text{rocket}}| = \frac{v}{18}.$$

Transforming to the given frame where the rocket is moving at speed v, the final velocity is $v + v/18 = 19v/18$. The signs are fixed by physical reasoning: if the exhaust is expelled backwards, the rocket's velocity increases.

57. B – Since all we care about is ratios, we can scale all elements of the circuit by some convenient numerical factor: let's divide all the resistances by 10 kΩ to make the numbers easier (so we're working in units of 10^{-4} A whenever we calculate currents). When S is open, the total resistance of the circuit is $4 + 1 = 5$, so the current is $I_1 = 1$. When S is closed, the two resistors in parallel have an effective resistance of $(1 + 1/2)^{-1} = 2/3$. So the total resistance is $4+2/3 = 14/3$, and the total current is $I = 15/14$. The voltage across the 40 kΩ resistor is $IR = 30/7$, so the voltage drop across the 10 kΩ resistor is $5 - 30/7 = 5/7$. Hence the current I_2 is 5/7 in our units; since $I_1 = 1$ in these units, this is also the desired ratio.

58. C – This is a ballistic pendulum problem with two twists: the rod is not massless, and it starts out an angle with respect to the projectile, so we have to be a little careful calculating the angular momentum. The initial angular momentum comes just from the clay and is $L = m\mathbf{v} \times \mathbf{r} = mvd\sin(90° - \alpha) = mvd\cos\alpha$, so by conservation of angular momentum, this is the angular momentum after the collision as well. We get the angular velocity from $L = I\omega$, where the moment of inertia of the clay–rod system is $I = \frac{1}{3}Md^2 + md^2$ (the first term from the moment of inertia of a rod about one end, and the second from the moment of inertia of a point mass). So

$$\omega = \frac{L}{I} = \frac{3mv\cos\alpha}{(M + 3m)d},$$

choice C.

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59. E – Because the loop rotates, the angle the normal to the loop makes with the magnetic field oscillates sinusoidally, and so does the flux: $\Phi = B_0 A \sin \omega t$. Thus the emf in the loop is $V = d\Phi/dt = B_0 A\omega \cos \omega t$, and the power dissipated in the resistor is $P = V^2/R = B_0^2 A^2 \omega^2 \cos^2 \omega t/R$. Solving for ω, and using the fact that the average of $\cos^2$ is $1/2$, we find

$$\omega = \frac{\sqrt{2PR}}{B_0 A} = 500 \text{ rad/s}.$$

60. B – If you do not recall the mathematical definition of chemical potential, you should at least remember that the chemical potential has something to do with the energy associated with changing the number of particles in a system. This reduces the options to A or B, but there is chemical potential for systems of bosons and fermions, so B is the correct choice.

To be more rigorous, recall that the entropy is $S = \sum_i p_i \ln p_i$. We wish to maximize the entropy with respect to the constraints:

$$0 = \sum_i \epsilon_i p_i - E,$$
$$0 = \sum_i n_i p_i - N,$$
$$0 = \sum_i p_i - 1.$$

To maximize S we introduce the Lagrange multipliers β, μ, and λ, we write

$$S = \sum_i p_i \ln p_i - \beta \left(\sum_i \epsilon_i p_i - E \right) - \mu \left(\sum_i n_i p_i - N \right) - \lambda \left(\sum_i p_i - 1 \right),$$

and solve for

$$\frac{\partial S}{\partial x} = 0.$$

This μ turns out to be the chemical potential and is clearly the Lagrange multiplier that enforces the particle number constraint.

61. C – At first glance, choices D and E do not seem to make much sense. Choice B is a bit suspicious too because it does not contain a factor of 2, implying that the spin distributions are independent of the external magnetic field. Choice A does not make much sense either because the number of spin-up particles should increase, not decrease, with an increase in magnetic field: alignment with the magnetic field is energetically favorable. So C seems to be the best choice.

To decide for sure, we can calculate. Since the eigenvalues of σ_z are ± 1, the possible energies are $E = \pm |H|$. The partition function is

$$Z = e^{E/kT} + e^{-E/kT}.$$

The ratio of spin-up to spin-down particles is just

$$A = \frac{e^{E/kT}}{e^{-E/kT}} = e^{2E/kT}.$$

If we double the magnetic field, we have $E \to 2E$, which implies that $A \to e^{4E/kT} = \left(e^{2E/kT}\right)^2 = A^2$.

62. A – While this is a good fact to memorize, we can get it quickly by recalling Poisson's equation from electromagnetism, in SI units:

$$\nabla^2 V = -\rho/\epsilon_0.$$

Since the potential of a point charge q at the origin is $V = \frac{q}{4\pi\epsilon_0}\frac{1}{r}$, and its charge density is $\rho = q\delta^3(\mathbf{r})$, we can read off $\nabla^2 V = -4\pi\delta^3(\mathbf{r})$.

63. A – The formula for Compton wavelength is

$$\lambda = \frac{h}{mc},$$

and plugging in numbers gives 1.32×10^{-15} m, which is closest to choice A. If you didn't happen to remember the formula for Compton wavelength, you could get it by dimensional analysis. We know the Compton wavelength has something to do with quantum mechanics, so h or $\hbar$ must make an appearance, and the mass of a quantum particle is its only distinguishing characteristic (besides its spin of course, but that has the same units as $\hbar$). To get units of length, we need another dimensionful constant, and c fits the bill. So at worst we would get $\lambda = \frac{\hbar}{mc}$ and be off by a factor of 2π, but happily the answer choices are widely spaced enough that this isn't an issue. Alternatively, we could remember that the defining length scale for nuclear interactions is the fermi, or femtometer, 10^{-15} m, which is approximately the range of the strong force. So it makes sense that the quantum "size" of the proton is close to this value, but certainly not much larger.

64. B – We will work in units where $c = 1$ until the very end of the problem. In its rest frame, the K^0 has energy m_K, and since the decay products have equal mass, the π^+ and π^- each get energy $m_K/2$. The boost factor of

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the pion is then $\gamma_{\pi^+} = (m_K/2)/m_\pi$. We now solve for v using $\gamma = 1/\sqrt{1-v^2}$:

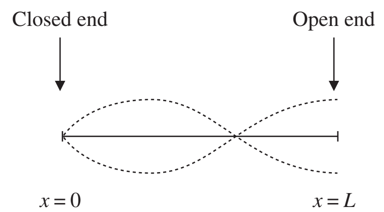
$$\frac{1}{\sqrt{1-v^2}} = \frac{m_K}{2m_\pi} \implies v = \sqrt{1 - 4m_\pi^2/m_K^2}.$$

Tacking on a factor of c to give dimensions of velocity gives choice B.

65. E – Again, we will use units where $c = 1$ until the end. The boost factor to go from the rest frame of the K^0 to the lab frame is $\gamma = E/m_K$. In the center-of-momentum frame (the rest frame of the K^0, after it decays), the π^- has an energy $m_K/2$, as found above. To find the energy of the π^- back in the lab frame, we use the γ that we found in the equation above to boost back. Since we want to know when the π^- is at rest, we want to set the energy after the boost equal to the rest energy of the π^-:

$$\gamma m_\pi = \frac{m_K}{2} \implies E = \frac{m_K^2}{2m_\pi}.$$

Since $m_K > 2m_\pi$ for the decay to be kinematically allowed, we have $E > m_K$, which passes a useful sanity check. Restoring a factor of c^2, we conclude that E is correct.



66. C – For a half-open pipe, the open end must be a pressure node, because the air inside and outside the pipe is at atmospheric pressure, hence it is a displacement antinode. On the other hand, the air at the closed end cannot go anywhere, so it is a displacement node. The allowable wavelengths λ in a half-open pipe of length L are then given by the constraint that $L = \lambda\,(1/4 + n/2)$, for non-negative integers n. This implies that

$$\lambda = \frac{4L}{2n+1}.$$

For $n = 1$, we have $\lambda = (4/3)(0.6\text{ m}) = 0.8$ m. The cartoon above shows the envelope of displacements as a function of distance along the pipe for this λ.

67. C – The field inside the sphere is as if there was a bound charge density of

$$\rho_b = -\nabla \cdot \mathbf{P}.$$

Plugging in the expression for the polarization, we find that

$$\rho_b = -C\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 r^2\right) = -4Cr.$$

The electric field can easily be solved using Gauss's law and exploiting the spherical symmetry of the problem:

$$\begin{aligned} E(r) &= \frac{1}{4\pi r^2 \epsilon_0}\int_0^r -4Cr' 4\pi r'^2 dr' \\ &= \frac{1}{r^2\epsilon_0}\int_0^r -4Cr'^3\, dr' = \frac{-Cr^2}{\epsilon_0}. \end{aligned}$$

68. E – The electric field inside a perfect conductor is zero, so the magnetic field must vanish as well, and there is no transmitted wave.

69. B – This just involves repeated application of the angular momentum identity

$$\left[L_i, L_j\right] = i\hbar\epsilon_{ijk}L_k.$$

Proceeding step-by-step, we have

$$\begin{aligned} \left[\left[\left[L_x, L_y\right], L_x\right], L_x\right] &= \left[\left[i\hbar L_z, L_x\right], L_x\right] \\ &= \left[(i\hbar)^2 L_y, L_x\right] \\ &= -(i\hbar)^3 L_z \\ &= i\hbar^3 L_z. \end{aligned}$$

70. C – The vibrational energies of diatomic molecules are approximately those of a harmonic oscillator, so we solve for T in $kT \simeq \hbar\omega = hf$. To do this quickly, it helps to take advantage of the fact that h is given in the formula sheet in units of eV, and use the mnemonic that kT at room temperature (300 K) is about 1/40 of an eV. We get about 2,400 K, which is closest to choice C. This problem illustrates a fact worth remembering – the vibrational degrees of freedom of light diatomic molecules are "frozen out" at room temperature, and are only unfrozen at temperatures an order of magnitude larger.

71. B – Observables must be Hermitian. The reason for the factor of i in the momentum operator $-i\hbar\nabla$ is precisely to make this single-derivative operator Hermitian – otherwise, we'd pick up an extraneous minus sign during integration by parts. So any operator involving only one derivative that does *not* have a factor of i can't be Hermitian.

72. B – Choices D and E are obviously incorrect. A is incorrect because the chief virtue of the BCS theory is that it *does* give the correct microscopic description of many superconductors. C is interesting, but B is a better answer because it not only escapes the violation of

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Pauli exclusion but also gives us a mechanism for superconductivity (bosons can occupy the same state, which produces superconductivity).

73. E – Analyze this system in a frame that rotates with the hoop, and consider only the equation of motion for the tangential component, since the bead is constrained to move on the hoop. There are no Coriolis or centrifugal force terms, so Newton's second law just reads

$$F = -kR\omega = mR\dot{\omega},$$

where ω here means the angular velocity *relative to the hoop*. This has solution $\omega(t) = Ce^{-kt/m}$. Moving back to the stationary reference frame with the replacement $\omega \to \omega + \Omega$, we have $\omega(t) = \Omega + Ce^{-kt/m}$. Imposing the initial condition $\omega(0) = \omega_0$ gives $C = \omega_0 - \Omega$, so

$$\omega(t) = \Omega - (\Omega - \omega_0)e^{-kt/m},$$

which is choice E.

We could also have proceeded by limiting cases. If $\omega_0 = \Omega$, the bead will experience no frictional force and will continue to rotate along with the hoop, $\omega = \Omega$, for all t. This eliminates all but choice E.

74. E – At the top of the hill, the cylinder has only potential energy. At the bottom, it will have purely kinetic energy, which is composed of both translational and rotational kinetic energy. Given the moment of inertia of the cylinder, we can solve the energy conservation equations to find the velocity; since the rotational kinetic energy depends on the moment of inertia, which depends on α, the final velocity must depend on α. Since there is no rolling friction, the difference in potential energy simply depends on the initial and final heights and not on the path taken, so the final velocity depends on h and not θ. This leaves choice E. All energies, kinetic and potential, are proportional to M, so this cancels when using conservation of energy to find the velocity. Similarly, the rolling without slipping condition $v = R\omega$ ensures that both translational and rotational kinetic energy are proportional to v and independent of R.

75. D – The spin-3/2 states are the states with the highest total spin that can be formed from three spin-1/2 particles. The maximal-spin states are always totally symmetric under exchange of the particles. D is the only choice that is totally symmetric. We could also derive this quickly by starting with $|\Psi\rangle$ and applying the lowering operator twice; or even more quickly, by starting with the $m = -3/2$ state $|\downarrow\downarrow\downarrow\rangle$ and applying the *raising* operator *once*.

76. A – A p-type semiconductor has an excess of positive charge carriers or holes, which are empty states in the valence band that electrons can fill. An n-type semiconductor has an excess of electrons. When p- and n-type materials are brought together, the electrons from the n-type material diffuse into the p-type material. This leaves a slight negative charge on the edge of the p-type material and a slight positive charge on the edge of the n-type material. The electric field thus points from n-type to p-type material.

77. C – The two reference frames S and S' must be related by a Lorentz transformation, so with the $(+,-,-,-)$ metric signature, the invariant interval of the position 4-vector in both reference frames must be equal. The separation of E_1 and E_2 in the original frame is (2,1,1,0), which has invariant interval 2. Checking each choice, the invariant interval for A is -1.25, B is 3, C is 2, and D is 1. Choice C is the same as in the original frame, and so is the correct answer.

78. C – With these answer choices, one can make considerable progress using pure dimensional analysis: the answer must have units of $(\text{time})^{-1}$, and must be non-negative. This leaves only B, C, and E, and E seems rather unreasonable. More formally, assuming that dark matter detection follows a Poisson process with mean rate λ, the probability of seeing zero events is $e^{-\lambda T}$. The 90% confidence level upper limit is the mean rate such that 90% of the time we would see in our experiment a number of events that is inconsistent with our measurement of zero events. Practically, this means that we want to find the mean rate that would produce more than 0 events 90% of the time. In other words, we want to find the rate that gives 0 events only 10% of the time. This is just $0.1 = \exp(-\lambda T)$, or $\lambda = -(1/T)\ln 0.1$.

A limiting-cases analysis also works here. As the confidence level grows and approaches 100%, the upper limit must get weaker (that is, the rate must be larger), since we can never be 100% confident that any finite rate will give zero events in every experiment. The only answer choice that goes to infinity as the confidence level goes to 100% is choice C; indeed, choices B and D both go to zero.

79. C – The 21 cm splitting comes from the hyperfine interaction, which is a spin–spin coupling between the electron and proton spins. Hence, this splitting (between the spin singlet and triplet configurations) is evidence for the proton having spin-1/2.

80. D – Choices A and E are obviously incorrect. B seems initially plausible, but this process would happen just as often with tau neutrinos, so it is far from clear that it would make up the discrepancy. C also seems vaguely plausible, but it does not give an obvious mechanism for production of tau neutrinos. D is the correct answer, with the difference between neutrino mass and flavor eigenstates being the basis of the famous neutrino oscillations. This is the same oscillation effect that takes place in two- and three-state quantum systems.
81. E – By rescaling energies, the partition function can be written as
$$Z = e^{\epsilon/2kT} + e^{-\epsilon/2kT} + 2.$$
The probability of being the triplet state is just
$$P = \frac{e^{\epsilon/2kT} + e^{-\epsilon/2kT} + 1}{e^{\epsilon/2kT} + e^{-\epsilon/2kT} + 2}.$$
82. A – Recalling that the total energy is $H = T + U$, we can solve this problem by a careful consideration of signs in the answer choices. The spring potential energy $U_s = \frac{1}{2}kx^2$, where x is some relative displacement, is always non-negative, so we have only choices A and B. With coordinates y_1 and y_2 as defined in the problem, positive displacements correspond to downward motion, so gravitational potential energy is actually *negative*. Thus we are left with choice A.
83. B – The expectation value of energy is
$$E = \int_0^a \psi^* H \psi \, dx.$$
After a sudden expansion, ψ stays constant. The potential V also stays constant on the interval $[0, a]$, but changes from ∞ to 0 on the interval $[a, 2a]$. Since ψ vanishes on $[a, 2a]$, the kinetic energy operator $T = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$ gives zero when acting on ψ. (If you're worried about the fact that the derivative of ψ is discontinuous at $x = a$, meaning that the second derivative is a delta function, note that $\psi(a) = 0$, and zero times a delta function is still zero.) The expectation value of energy after the expansion is therefore
$$\begin{aligned} E' &= \int_0^{2a} \psi^* H' \psi \, dx \\ &= \int_0^a \psi^* H \psi \, dx + \int_a^{2a} \psi^* H' \psi \, dx \\ &= \int_0^a \psi^* H \psi \, dx + 0 \\ &= E. \end{aligned}$$
By the way, this is an application of conservation of energy within the formalism of quantum mechanics – just as the temperature of an ideal gas remains constant during free expansion, the energy of a quantum system remains constant after a sudden change of potential.
84. D – This looks long and complicated, but it's really just a matter of limiting cases. A and B are eliminated by dimensional analysis, since the reflection coefficient must be dimensionless. To eliminate E, note that, as $\alpha \to 0$, the coefficient of reflection must go to zero because the barrier disappears, and the particle continues to propagate to $x > 0$ with probability 1. Choice C looks reasonable at first, but the reflection coefficient must always take a value between 0 and 1, by definition, for all values of parameters in the problem. If α is chosen sufficiently large, then the reflection coefficient of choice C is greater than 1, which is unphysical. This leaves only D.
85. C – The transition $2s \to 1s$ has $\Delta m = 0$ so it does not violate the dipole selection rule.
86. E – The electric dipole moment is a vector quantity, which changes sign under parity transformations, so a nonzero electric dipole moment violates parity. Interestingly, it *also* violates time-reversal invariance. To see this, recall that the neutron *does* have a nonzero magnetic dipole moment. Suppose the magnetic and electric dipole moments were parallel; then, under time-reversal, the magnetic one would change sign but the electric one would remain the same, and the system would not be invariant under time-reversal. So the *relative* orientations of the magnetic and electric dipole moments lead to a violation of time-reversal invariance.
87. C – The sharp drop of curve b is the signature of a process with an energy threshold around 1 MeV. Recalling that the electron mass is about 0.5 MeV, this must be the threshold for pair production.
88. A – $f(x)$ is an odd function on $[-\pi, \pi]$, and cosine is an even function, so all the cosine coefficients in the Fourier series vanish identically.
89. E – We are looking for an $\mathbf{A}$ satisfying $\nabla \times \mathbf{A} = \mathbf{B}$. It's simplest to consider the components of $\mathbf{B}$ one by one. Since $B_x = (\nabla \times \mathbf{A})_x = \partial A_z/\partial y - \partial A_y/\partial z$, with an eye on the answer choices we see that we can only satisfy this by taking $A_z = B_0 y$, since A_y is independent of z in all answer choices. This leaves only C and E. To get $B_z = 2xB_0$, we must have $2xB_0 = (\nabla \times \mathbf{A})_z = \partial A_y/\partial x - \partial A_x/\partial y$. Choice E has the correct sign, and we can check that it also satisfies $(\nabla \times \mathbf{A})_y = 0$.

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90. D – The change in energy can be obtained from the change in the electric field energy density outside the sphere. There is no change in energy density inside the sphere, so we will neglect this contribution. Before the grounding, the field outside the sphere has energy

$$\begin{aligned} U_{\text{before}} &= \frac{\epsilon_0}{2}\int_{\text{outside}} \mathbf{E}^2 dV \\ &= \frac{\epsilon_0}{2}\frac{1}{16\pi^2\epsilon_0^2}\int_a^\infty \frac{Q^2}{r^4}4\pi r^2 dr \\ &= \frac{1}{8\pi\epsilon_0}\int_a^\infty \frac{Q^2}{r^2}dr \\ &= \frac{Q^2}{8\pi\epsilon_0 a}. \end{aligned}$$

Afterwards, charge is induced on the conducting sphere to exactly cancel the electric field everywhere outside of the sphere of radius a, so $U_{\text{after}} = 0$. The change is therefore $\Delta U = -\frac{Q^2}{8\pi\epsilon_0 a}$, which is choice D.

91. C – Without color, spin-3/2 baryons with three *identical* quarks and zero orbital angular momentum would have symmetric wavefunctions under the interchange of any two quarks, violating the Pauli exclusion principle.

92. B – By momentum conservation, when a free nucleus emits a photon it must also recoil in the opposite direction, which causes the emitted photon to have a different energy (hence frequency) than it would if the nucleus were held stationary. You may have been thrown by choice C, since the Pound–Rebka experiment used a carefully contrived arrangement of absorbers and emitters in a vertical shaft so that the gravitational redshift was dominant. However, the adjective "tabletop" implies that the experiment takes place at (almost) constant gravitational potential, so gravitational redshift barely contributes.

93. E – It is a useful bit of trivia that a sequence of NAND gates can be combined to create any sequence of basic logical gates. (This is also true of NOR gates.) Even without knowing this, though, we can see fairly easily that A and C must be possible: since NAND is AND followed by NOT, to get an AND we just put two NAND gates in sequence and tie the output of the first to both inputs of the second, and to get NOT we can tie both inputs of a single AND gate together. So if at least two of the answer choices are possible, the answer must be E.

94. E – Applying the Euler–Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$$

to the given Lagrangian, we get $A\ddot{q} - (-2Bq) = 0$. Rearranging gives $\ddot{q} = -\frac{2B}{A}q$, choice E.

95. E – The field of a straight solenoid has uniform magnitude $\mu_0 nI$, where n is the number of turns per unit length; bending this solenoid around into a toroid of radius R sets $n = N/2\pi R$. (This is a hand-wavy argument, but it gives the correct answer, and it's an excellent way to remember the formula without having to rederive it from scratch.) The energy stored in the magnetic field is $U = \dfrac{1}{2\mu_0}\displaystyle\int \mathbf{B}^2\, dV$, where the integral is taken over all of space. Here the field is only nonzero inside the volume V, and inside this volume the field is approximately uniform (this follows from the statement that $r \ll R$), so

$$U = \frac{V}{2\mu_0}|\mathbf{B}|^2 = \frac{2\pi^2 Rr^2}{2\mu_0}\cdot\left(\frac{\mu_0 NI}{2\pi R}\right)^2 = \frac{\mu_0 N^2 I^2 r^2}{4R},$$

choice E.

96. A – Longitudinally polarized waves propagate in the same direction as the displacement of the wave medium. This means that the polarization vector is forced to be along the direction of propagation, excluding choices I and II. This narrows the answer to A.

97. C – The phase velocity is

$$v_{\text{phase}} = \frac{\omega}{k} = Ak^{-1/2}.$$

The group velocity is

$$v_{\text{group}} = \frac{d\omega}{dk} = \frac{1}{2}Ak^{-1/2},$$

so $v_{\text{phase}} = 2v_{\text{group}}$, choice C.

98. B – The setup $n_1 < n_2 < n_3$ occurs so often that it is probably useful to memorize the result. This is the configuration of an antireflective coating, and the 180° phase shift that occurs at *both* boundaries leads to the condition for destructive interference:

$$2n_{\text{film}}t = \left(m - \frac{1}{2}\right)\lambda,$$

where t is the thickness of the film. (Try to derive this formula if you forgot it.) We conclude that $t_{\min} = \lambda/4n$, which can be memorized with the mnemonic that tn for an antireflective coating is a quarter-wavelength. For the numbers given in this problem, we get $t = 8.33 \times 10^{-8}$ m. So B is correct.

99. D – The wavefunction is already normalized, so using $\langle x\rangle = \int |\Psi(x)|^2 x\, dx$, we have

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$$\langle x \rangle = 3 \int_0^1 x(1-x)^2 \, dx$$
$$= 3\left(\frac{1}{12}\right)$$
$$= \frac{1}{4}.$$

This matches choice D.

100. A – A rigid rod has only rotational degrees of freedom, so its energy is determined by its rotational quantum numbers. The classical formula is $T = L^2/2I$ for the kinetic energy of a rotating body, where I is the moment of inertia about the center of mass, so in the quantum mechanics setting we get $E = \hbar^2 n(n+1)/2I$. The center of mass of this rod is at the center of the rod, so the moment of inertia is $I = 2 \cdot m\left(\frac{a}{2}\right)^2 = \frac{1}{2}ma^2$. Thus,

$$E = \frac{\hbar^2 n(n+1)}{ma^2},$$

choice A.

Note that we could also have done the last step of the problem by calculating the reduced mass of the system, $\mu = \dfrac{m \cdot m}{m+m} = \dfrac{m}{2}$, and using the formula $I = \mu r^2$ for a single particle of mass μ.

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Solutions to Sample Exam 2

1. D – Rather than solving the equation of motion to find the whole trajectory of the particle, then taking the limit to find the velocity at $t = \infty$, we can simply solve for the velocity v_t at which the force due to gravity balances the air resistance. By Newton's Second Law, there will then be no net force, and the ball will continue to fall at this velocity – this is the terminal velocity. Solving $mg = bv$ for v, we get $v_t = mg/b$. For an alternate solution, note that only choice D has the correct units. Choice E looks like it has the correct units, but you can check that there is no dimensionless combination of the quantities m, g, and b (which are the only ones appearing in the problem), so nothing can appear in an exponential and we are back to choice D.
2. C – The particle has a component of velocity along the direction of the magnetic field. Since the Lorentz force is zero for this component, the particle will continue with the same velocity in the x-direction. However, the perpendicular component of $\mathbf{B}$ will cause the particle to execute cyclotron motion in the yz-plane. Superimposing these two components of velocity gives a helical path, choice C.
3. E – We first find the velocity at the lowest point, then find the tension needed to supply the required centripetal acceleration. The initial potential energy mgl is converted into kinetic energy $\frac{1}{2}mv^2$ at the bottom, so $v^2 = 2gl$. The centripetal force must be mv^2/l, so $F = m(2gl)/l = 2mg$. However, this is the *net* force: since there is already a force mg due to gravity acting downwards, the tension must provide a force $3mg$ acting upwards for a net force of $2mg$ upwards. Hence choice E.
4. C – Without actually setting up any equations, we can see that C must be correct. The electric field couples to charge (that is, the electric force is qE), but gravity couples to mass (with force mg), so balancing forces gives something like $mg = qE$ with some terms involving α thrown in as well. The only quantity we can get out of this equation, knowing both g and E, is q/m, and a single measurement of α is enough to determine this quantity.
5. B – This is an easy problem, but without a few tricks it is easy to get hung up on the computational details. The Bohr formula gives $E = -E_0/n^2$ for the nth excited state with $E_0 = 13.6$ eV. It's easy to eliminate E by remembering that most of the hydrogen spectrum falls near the visible range, and mm wavelengths correspond to radio waves. Deciding between the rest takes a little bit of computation. Using $hc = 1240 \text{ eV} \cdot \text{nm}$ (which is given in the Table of Information on recent tests), we can compute using a couple of numerical approximations to avoid nasty arithmetic:

$$\Delta E = E_0 \left(\frac{1}{4} - \frac{1}{9} \right) = 13.6 \text{ eV} \left(\frac{5}{36} \right) \approx 2 \text{ eV},$$
$$\lambda = \frac{hc}{\Delta E} \approx \frac{1240 \text{ eV} \cdot \text{nm}}{2 \text{ eV}} \approx 620 \text{ nm}.$$

   This is closest to B.
6. A – This follows from the axioms of quantum mechanics: Hermitian observables are guaranteed to have real eigenvalues, and since eigenvalues of operators are results of measurements which must be real numbers, operators must be Hermitian. Some counterexamples for the other choices: the Hamiltonian of a bound state has negative eigenvalues, which violates B; the momentum operator is an operator on infinite-dimensional function spaces, so violates C; the Pauli matrix $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ is Hermitian but not symmetric, so violates D.
7. C – III is the statement of Newton's Second Law. Choice I fails if an extended object is subject to two equal and opposite forces at different locations, creating a net torque, and choice II fails (for example) for an object in empty space moving with some initial velocity.
8. D – You might be tempted to think that this question requires rote memorization of the atomic number of fluorine, but there's an easy way to deduce the correct answer using the information provided and the particular selection of answer choices. The fact that the common form is Fl^- means that fluorine wants to attract an extra electron. This results from an electron shell that is one short of being full, which matches choice D. In fact, E is the electron configuration for both Fl^- and the noble gas neon, with a full $n = 2$ shell. By contrast, A is the electron configuration for lithium, which wants to *lose* an electron to get a full $n = 1$ shell.
9. D – The resonant frequency of an LC circuit is given by $\omega = 1/\sqrt{LC}$. If the capacitor is initially charged, then the charge in the circuit oscillates as $Q(t) = Q_0 \cos \omega t$, and the current is $I = dQ/dt = -Q_0\omega \sin \omega t$. Thus $|I|$ is maximized at $\omega t = \pi/2$, or $t = \frac{\pi}{2}\sqrt{LC}$, choice D.
10. E – The magnitude of the work done by the gas is $\int P\, dV$, which is just the area in the P–V-plane. In this case, the sum of the integrals under each curve is positive, so the total work done by the cycle is also positive. The shape

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of the P–V curve is pretty close to a parallelogram, with base $2\,\mathrm{m}^3$ and height 3,100 kPa, so the area is about 6,200 kJ, choice E.

11. A – During an adiabatic process the quantity PV^γ is constant, where $\gamma = C_p/C_v$. Looking at step A, at the endpoint we have $V = 1\,\mathrm{m}^3$, so independent of γ, $PV^\gamma = 3,200$ in the units given. Looking at the start of step A, we solve for γ:

$$(100)(8)^\gamma = 3200 \implies \gamma = 5/3.$$

A monoatomic gas has $\gamma = 5/3$, so choice A is correct.

12. A – The scope trace looks like a carrier wave whose frequency changes as a function of time, which is the defining characteristic of frequency modulation.

13. E – Planetary orbits have two independent parameters corresponding to the two conserved quantities in central-force motion: the energy of the orbit and the angular momentum. The semimajor axis is determined solely by the energy, but the semiminor axis (which is needed to find the area of the orbit) also depends on the angular momentum. Without this additional information, the area of the orbits can't be determined.

14. C – The choices for the orbital angular momentum of the particle are given by the angular wavefunction: we can have either $l = 3$ and $m_l = 0$, or $l = 2$ and $m_l = 1$. Adding these choices to the particle's spin ($s = 1/2$ and $m_s = \pm 1/2$) gives the following possibilities:

$$\begin{aligned} j &= 7/2, m_j = \pm 1/2 \\ j &= 5/2, m_j = \pm 1/2 \\ j &= 5/2, m_j = 1/2, 3/2 \\ j &= 3/2, m_j = 1/2, 3/2 \end{aligned}$$

Of these, only $j = 7/2$, $m_j = -1/2$ appears in the answer choices, and it is choice C.

15. E – You can either remember the energies of the infinite square well, $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$, or derive them from the given wavefunctions using the Hamiltonian operator $-\frac{\hbar^2}{2m^2}\frac{d^2}{dx^2}$. Fortunately the wavefunction is already given to us normalized, so we compute

$$\begin{aligned} \langle E \rangle &= \frac{1}{2}E_2 + \frac{1}{3}E_3 + \frac{1}{6}E_4 = \frac{\pi^2\hbar^2}{2mL^2}\left(\frac{2^2}{2} + \frac{3^2}{3} + \frac{4^2}{6}\right) \\ &= \frac{23\pi^2\hbar^2}{6mL^2}, \end{aligned}$$

choice E.

16. C – From the definition of entropy for a reversible process, $\Delta Q = T\Delta S$, so at constant temperature $\Delta S = \Delta Q/T = 20$ J/K, choice C.

17. B – In the photoelectric effect, electrons are ejected from a metal after bombardment of the metal with light of a suitably high energy. The fact that no electrons are ejected for light of low enough frequency, independent of the intensity of the light, implies that the energy of the light depends on the frequency rather than the intensity, and higher frequency means higher energy. Both I and III are true, but are not directly supported by this experiment.

18. D – This is pure fact recall. The splitting of spectral lines by electric fields is called the Stark effect, choice D.

19. A – This is fairly intuitive: we know that if $m_1 = m_2$, then m_1 will stop dead after the collision, and if $m_1 \ll m_2$, then m_1 will just bounce back in the opposite direction, so it's reasonable that if $m_1 > m_2$ the first mass will continue moving in the same direction. More formally, letting v_1 and v_2 be the final velocities of m_1 and m_2 respectively, we solve the conservation of momentum and energy equations $m_1 v = m_1 v_1 + m_2 v_2$ and $\frac{1}{2}m_1 v^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$ for v_1 to get

$$v_1 = v\frac{m_1 - m_2}{m_1 + m_2},$$

so indeed if $m_1 > m_2$, the numerator is positive and v_1 has the same sign as v.

20. A – We want an upright virtual image with magnification $m = 20\,\mathrm{cm}/2\,\mathrm{m} = 1/10$. This means we need $-s'/s = 1/10$, or $s' = -s/10$. Using the optics equation with $s = 1$ m,

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = -\frac{9}{s} = -9\,\mathrm{m}^{-1},$$

so $f \approx -0.11$ m and $R = 2f = -22$ cm. In particular, R is negative, so the mirror must be convex.

21. C – You could probably get this from some etymology: vibrations of a lattice are also known as sound, which has the Greek root *phon*, hence "phonons" (choice C).

22. D – The potential must be continuous, which eliminates choice B (which, incidentally, is a graph of the electric *field*). Furthermore, the potential should be constant inside the inner shell since there is no field there, which eliminates C. In the region between the two shells, the potential should look like $-1/r$, since the only field is due to the inner shell, and this eliminates A.

To decide between D and E, we need to do a quick calculation. Depending on the amount of charge on each shell, either D or E could be valid. Call the potential inside the inner shell $V_1(r)$, the potential in between the two shells $V_2(r)$, and the potential outside the outer shell $V_3(r)$. Since the electric field is the *derivative* of the potential, we can add arbitrary constant offsets C_1 and C_2 to the standard Coulomb potentials for the enclosed charge in these regions, so that they match on the boundaries according to the conditions $V_1(R) = V_2(R)$ and $V_2(2R) = V_3(2R)$. When we rewrite the boundary conditions in terms of the Coulomb potentials with the constant offsets, we get the following system of equations:

$$V_1(R) = V_2(R) \Longleftrightarrow C_1 = -\frac{q}{4\pi\epsilon_0 R} + C_2,$$
$$V_2(2R) = V_3(2R) \Longleftrightarrow -\frac{q}{8\pi\epsilon_0 R} + C_2 = +\frac{2q}{8\pi\epsilon_0 R}.$$

The offset C_1 is clearly positive, so $V_1(r) = C_1 > 0$ and D is the correct answer.

23. B – The quarks in the proton are held together by the strong nuclear force, which is mediated by gluons, choice B. The quarks are also electrically charged, but the electromagnetic force is orders of magnitude weaker than the strong force so is not responsible for binding.
24. A – Potential energy is related to electric potential by $U = qV$. The change in potential is $\Delta V = -\int \mathbf{E}\cdot d\mathbf{l}$, and since $\mathbf{E} \perp \hat{\mathbf{y}}$, only the x-component of the path contributes. This gives $\Delta V = 3E$ since $\mathbf{E}$ is constant, and $U = -3Ee$. To make sure we have the signs right, note that if e is positive, then our particle is negatively charged and moving in the opposite direction from the field lines. This is the direction it would move under the influence of the Lorentz force, so its energy should decrease, which is consistent with the minus sign.
25. E – I is true, as a magnetic monopole would provide a "source term" on the right-hand side of Gauss's law, just as it does for Gauss's law for electricity. In general, by Helmholtz's theorem, we can decompose a vector field such as the magnetic field into the gradient and curl of a scalar and vector potential as $\mathbf{X} = -\nabla\varphi + \nabla\times\mathbf{A}$. Thus III is true as well, since a divergence-free $\mathbf{B}$ implies that the scalar potential can always just be chosen to be zero without changing the gradient or curl of $\mathbf{B}$ in Maxwell's equations. Gauss's law is perfectly compatible with the continuity equation – Ampère's law is the one that needs adjusting, through the addition of Maxwell's displacement current.
26. B – Using $E^2 = p^2c^2 + m^2c^4$, we have $E^2 = 2m^2c^4$, so $E = \sqrt{2}mc^2$, choice B.
27. E – This is pure dimensional analysis. Only E has the correct units, so it is the correct answer. A quick way to see this is to recall the expression for the potential energy of two equal point charges, $W = \frac{e^2}{4\pi\epsilon_0}\frac{1}{r}$, which says that $e^2/\epsilon_0 r$ has units of energy. The only remaining units are nice and familiar powers of mass and length, so we see that E works out to have units of inverse time, or frequency.
28. D – It's easy to get lost in calculations in this problem, computing the total energy of the orbit and so on, but things are drastically simplified because at perigee and apogee the planet's path is perpendicular to the line connecting the planet and the star. Thus, its angular momentum at perigee and apogee is just $mv_p(a/4)$ and $mv_a(7a/4)$, respectively. By conservation of angular momentum in a central force field, we get $v_p/v_a = 7$, choice D.
29. D – The time-averaged force is defined as $\frac{1}{T}\int_{t_i}^{t_f} F\,dt$, where T is the total time. The distance-averaged force is defined similarly, by $\frac{1}{D}\int_{x_i}^{x_f} F\,dx$, where D is the total distance. The quantities inside the integral are also known as total impulse and total work, respectively. But total impulse is change in momentum, and total work is change in kinetic energy. Since boat 2 arrives with a higher speed, we know the impulse must be greater, but the denominator T is greater as well, so we can't definitively say whether F_t increases or decreases with respect to boat 1. However, the *distance* traveled by the two boats is identical, and $v_2 > v_1$ means the kinetic energy of boat 2 is greater than that of boat 1. This implies $F_{d1} < F_{d2}$, choice D.
30. C – The unperturbed energies are $E_0 = \hbar\omega/2$ and $E_1 = 3\hbar\omega/2$. The first-order perturbation theory formula gives

$$\delta E_n = \langle n|V(x)|n\rangle = \epsilon\int_{-\infty}^{\infty}|\psi_n(x)|^2\delta(x)\,dx = |\psi_n(0)|^2.$$

Since $\psi_1(x)$ vanishes at $x = 0$, it has no first-order shift. $|\psi_0(0)|^2 = \sqrt{m\omega/\pi\hbar}$, so $E_0 = \hbar\omega/2 + \epsilon\sqrt{m\omega/\pi\hbar}$ and $E_1 = 3\hbar\omega/2$, which is C.

31. E – The degrees of freedom associated to choices II and III freeze out at low temperatures, but at sufficiently high temperatures all three are active. Note that if we were dealing with a single atom rather than a diatomic molecule, choices II and III would *not* contribute.
32. E – All steps of the Carnot cycle are reversible, and reversible adiabatic processes take place at constant entropy. This problem illustrates a questionable, but standard, use of GRE terminology: "adiabatic" in the question statement was understood to mean "reversible

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adiabatic," not just a process where $\Delta Q = 0$. Indeed, free expansion is also adiabatic, but not reversible, and if the process in question were free expansion, choice A would be the correct answer.

33. E – This is the setup for the method of images, but we must remember there are *two* image charges: $-q$ at distance d below the plate and $+q$ at distance $2d$ below the plate. So the real charge $+q$ feels a force from the real charge $-q$ and both the image charges. The correct prescription is at this point to pretend there is no conductor and just calculate the force directly from all these charges. The force from the closest charge will dominate, so we know that q will be attracted upwards, and thus the force must be in the $+\hat{\mathbf{z}}$-direction, eliminating choices A and B. More precisely,

$$\mathbf{F} = q\mathbf{E} = \frac{q^2}{4\pi\epsilon_0}\left(\frac{1}{d^2} - \frac{1}{(2d)^2} + \frac{1}{(3d)^2}\right)\hat{\mathbf{z}}.$$

At this stage we can make sure all our minus signs are correct: the real charge $-q$ should attract, so the force from the first term should be upwards (positive), whereas the attraction of the image charge $-q$ should be a downwards force (negative), and the repulsion of the image $+q$ should be upwards (positive). Rearranging a bit, we get

$$\mathbf{F} = \frac{q^2}{4\pi\epsilon_0 d^2}\left(1 - \frac{1}{4} + \frac{1}{9}\right) = \frac{31q^2}{144\pi\epsilon_0 d^2},$$

which is E. This problem is rather tricky, because if instead you had tried to get the field by taking the derivative of the potential, you would have been off by a factor of 2 in the middle term (from the image charge associated to our test charge $+q$) and ended up with choice C. This subtlety is discussed in Griffiths *Introduction to Electrodynamics* Sections 3.2.1 and 3.2.3, and is due to the fact that there is no energy cost from moving the image charge along with the test charge. In our opinion, it's better to forget all the complicated reasoning and just memorize the fact that you can pretend that you get forces, rather than potentials, from the method of images.

34. B – Using Wien's displacement law, $T = 3\times10^{-3}\,\mathrm{m\cdot K}/\lambda$ (since the answer choices are widely separated we can approximate the constant to one significant figure), we find $T \approx 5000$ K.

35. E – A receding star will have its wavelengths redshifted, meaning that $\lambda_{\text{true}} < \lambda_{\text{meas}}$, and hence $T_{\text{true}} > T_{\text{meas}}$. This immediately knocks out choices A, B, and D. To find the exact ratio we use the relativistic Doppler shift formula, which can be applied directly to T rather than λ because of the inverse proportionality between T and λ and the fact that we know which direction the ratio must go.

$$\frac{T_{\text{true}}}{T_{\text{meas}}} = \sqrt{\frac{1+\beta}{1-\beta}} = \sqrt{\frac{1.2}{0.8}} = \sqrt{\frac{3}{2}},$$

so choice E is correct.

36. D – The Euler–Lagrange equation is

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q}.$$

Computing the derivatives, $\partial L/\partial q = \frac{1}{L}\cos(q/L - \omega t)$ and $\partial L/\partial \dot{q} = 2A\dot{q}$, so the Euler–Lagrange equation is

$$2A\ddot{q} = \frac{1}{L}\cos(q/L - \omega t),$$

which becomes choice D after dividing through by $2A$.

37. D – Hamilton's equations for the coordinate z and the conjugate momentum p_z are given by

$$\dot{p}_z = -\frac{\partial H}{\partial z}, \qquad \dot{z} = \frac{\partial H}{\partial p_z}.$$

Plugging in the Hamiltonian to the first equation, we find

$$\begin{aligned}\dot{p}_z &= -\frac{\partial}{\partial z}\left(\frac{p_\theta}{2mz^2}\cot^2\alpha + mgz\right)\\ &= \frac{p_\theta^2}{mz^3}\cot^2\alpha - mg.\end{aligned}$$

Plugging into the second equation, we find

$$\begin{aligned}\dot{z} &= \frac{\partial}{\partial p_z}\left(\frac{p_z^2}{2m}\cos^2\alpha\right)\\ &= \frac{p_z}{m}\cos^2\alpha.\end{aligned}$$

Note that it is easy to confuse the minus signs and coordinates in Hamilton's equations, which make for classic trap answer choices, so it pays to memorize these, or rederive them quickly by relating $H = p^2/2m + U(x)$ to Newton's law $F = -U'(x)$.

38. D – C_V is defined as $\partial U/\partial T$, so all we have to do is integrate C_V with respect to T. $\int AT^3\,dT = \frac{1}{4}AT^4$, so choice D is correct. We could have narrowed down the answer choices first, by dimensional analysis: since C_V has units of energy/temperature, $U(T)$ could only have been D or E.

39. C – Since the states $|n\rangle$ are orthonormal, $\langle m|n\rangle = \delta_{mn}$, so I is false. For II, recall that two operators commute if and only if they share the same eigenfunctions. Since $\hat{x}$

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does not commute with $\hat{p}$, then $[\hat{H}, \hat{p}] \neq 0$ for Hamiltonians whose potential term depends on x, thus the eigenfunctions of the Hamiltonian and the momentum operator are not shared in general. III is true, and follows from both orthonormality and the fact that the states $|n\rangle$ are energy eigenfunctions with eigenvalues E_n. So C is the correct answer.

40. B – As is typical for specialized topics problems, this is pure fact recall. Expulsion of magnetic fields is an interesting characteristic of superconductors. For the record, A and D are false, and C and E are true but irrelevant.
41. C – Energy scales of MeV are typical of nuclear binding energies and reactions, so C is the only choice that has roughly the correct order of magnitude. One way of remembering this is that $\hbar c \approx 200\,\text{MeV} \cdot \text{fm}$, and nuclear dimensions are on the order of several femtometers. But we could get pretty far just by a process of elimination. The binding energy certainly can't be more than three times the mass of a nucleon (in units of c^2), or else tritium would have negative mass. Since nucleons have masses of about 1 GeV/c^2, choices D and E are impossible. Furthermore, energies of eV are typical of chemical reactions, rather than nuclear reactions, so the only remaining reasonable choices are B and C.
42. D – If we keep the nucleus with charge +1 and the orbiting particle with charge -1, the Bohr radius goes as $a \sim 1/\mu$, where μ is the reduced mass of the system. The reduced mass of the hydrogen atom is $\frac{m_e m_p}{m_e + m_p}$, and the reduced mass of muonium is $\frac{m_e m_\mu}{m_e + m_\mu}$, so taking the correct ratio gives

$$a = a_0 \frac{m_e m_p / m_e + m_p}{m_e m_\mu / m_e + m_\mu} = a_0 \frac{m_p(m_e + m_\mu)}{m_\mu(m_e + m_p)},$$

which is D. For an alternate solution, we can use limiting cases: in the limit $m_e \to 0$, we must have $a = a_0$ since the nucleus becomes infinitely heavy compared to the electron, and the reduced mass is then independent of the nucleus mass. Choice D is the only one that satisfies this property.
43. D – Just apply the canonical commutation relation $[\hat{x}, \hat{p}] = i\hbar$ repeatedly. Using the commutator identity $[A, BC] = [A, B]C + B[A, C]$, we have

$$[\hat{x}, \hat{p}^2] = [\hat{x}, \hat{p}]\hat{p} + \hat{p}[\hat{x}, \hat{p}] = (i\hbar)\hat{p} + \hat{p}(i\hbar) = 2i\hbar\hat{p},$$

which is D.
44. D – The work–energy theorem tells us that the difference between the initial and final energies must be the work done by friction; in the absence of friction, energy would be conserved. In symbols, let h be the radius of the quarter-circle ramp, v the block's velocity at the bottom, and m its mass.

$$\begin{aligned} W &= mgh - \frac{1}{2}mv^2 \\ &\approx (2\text{ kg})(10\text{ m/s}^2)(2\text{ m}) - (0.5)(2\text{ kg})(4\text{ m/s})^2 \\ &= (40 - 16)\text{ J} \\ &= 24\text{ J}, \end{aligned}$$

where as usual we have approximated $g \approx 10\text{ m/s}^2$.
45. A – Potentials of line sources are logarithmic, which eliminates D and E. Furthermore, the potential should blow up to $+\infty$ at $r = 0$, which eliminates C. To decide between A and B, we have to go back to Gauss's law to find the field. Taking a Gaussian cylinder of height h and radius r centered on the wire, the charge enclosed is $Q = \lambda h$. The field should be radial, so there is no field parallel to the endcaps of the cylinder, and the fact that the field is constant at constant radius r gives $E(2\pi rh) = \lambda h/\epsilon_0$, so $E = \lambda/2\pi r\epsilon_0$. Computing V we find

$$V = -\int_a^r E(r)\,dr = -\frac{\lambda}{2\pi\epsilon_0}(\ln r - \ln a) = \frac{\lambda}{2\pi\epsilon_0}\ln(a/r),$$

matching choice A.
46. D – If there is no absorption, the sum of the intensities of the outgoing beams is equal to the incident intensity. Since the intensity is proportional to the square of the amplitude, choice D is correct. Choice A is true for a 50/50 beam-splitter, which divides an incoming beam into two equal-intensity beams, but this is a special case and not true in general.
47. B – The classical free-particle Hamiltonian is $p^2/2m$, so to get the quantum operator we make the replacement $p \to -i\hbar\nabla$ to get

$$H = (-i\hbar\nabla)^2/2m = -\hbar^2\nabla^2/2m,$$

matching choice B.
48. B – This is mostly common sense. The penetration depth of the protons as a function of energy would be the most useful piece of information, since this would allow the proton depth to be matched with the tumor position. Note that protons of a fixed energy traveling through matter tend to penetrate to a relatively constant depth, losing the majority of their energy near the end of their track (a phenomenon usually referred to as the "Bragg peak" of their dE/dx curve). This makes them particularly suitable for cancer treatment because the physical location of a dose from a proton beam is well localized.

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This is in contrast to neutrons, for example, which tend to diffuse much more broadly as they lose their energy.

49. A – First using conservation of *angular* momentum, the angular momentum of the baseball is $L = (0.09\text{ kg})(20\text{ m/s})(2\text{ m}) = 3.6\text{ kg}\cdot\text{m}^2/\text{s}$, so this must be the angular momentum of the student–ball–platform system after it starts rotating. The total moment of inertia of the system (ignoring the ball, whose mass is negligible compared to the student) is $I = m_{\text{student}}r^2 + I_{\text{platform}} = (100\text{ kg})(2\text{ m}^2) + 200\text{ kg}\cdot\text{m}^2 = 600\text{ kg}\cdot\text{m}^2$. Using $L = I\omega$, we find $\omega = 0.006$ rad/s. Note that forgetting the contribution of the platform's moment of inertia would lead to choice B, and forgetting the student would lead to choice C.
50. A – You might think you needed to memorize the formula for Compton scattering, but really all you need here is to take the correct limits. As $\theta \to 0$, there should be no change in wavelength since no scattering takes place: this eliminates B, C, and E. However, D has the wrong units, which leaves only choice A. You could also observe that there should be a nonzero wavelength shift at $\theta = \pi$, where the incident light is reflected directly back off the electron accompanied by a suitable change in kinetic energy; this also eliminates D.
51. D – Li^{++} is a hydrogen-like ion with one electron orbiting a nucleus of charge $+3$. Recalling that the binding energy of hydrogen-like systems is proportional to the square of the nuclear charge Z, the binding energy of Li^{++} is $3^2 = 9$ times the binding energy of hydrogen. 9×13.6 eV = 122.4 eV, so D is correct.
52. B – Choice B is false because it is forbidden by the Second Law of Thermodynamics. The Third Law comes in various forms, but all of them require that the entropy approaches a constant at absolute zero. A, C, and E are all true by the Boltzmann definition of entropy $S = k\ln\Omega$, where Ω is the degeneracy of the system. A perfect crystal of a pure substance has a nondegenerate ground state, so $\Omega = 1$ and $S = 0$. But if the ground state were degenerate, then S could conceivably be nonzero at absolute zero when the system is in the ground state. Since a system at absolute zero is in its ground state, the Boltzmann definition of entropy implies that the entropy of a system approaches a constant at absolute zero. The fact that D follows from the Third Law is somewhat less obvious, but also true and proven in many textbooks.
53. B – By symmetry, if the triangle consisted of equal charges $+q$ at each vertex, the field at the center would be zero. So the field at the center is entirely due to the surplus of charge $+q$ at the top vertex compared to the other two vertices. The distance to the center of the triangle from a vertex is $L/\sqrt{3}$, so
$$\mathbf{E} = -\frac{q}{4\pi\epsilon_0}\frac{1}{(L/\sqrt{3})^2}\hat{\mathbf{y}} = -\frac{3q}{4\pi\epsilon_0 L^2}\hat{\mathbf{y}},$$
choice B.
54. A – We just use $V = \frac{q}{4\pi\epsilon_0}\frac{1}{r}$ with $r = L/\sqrt{3}$, and sum over all charges. In fact, since all charges are equidistant from the center, we can do the sum immediately:
$$V = \frac{2q+q+q}{4\pi\epsilon_0}\frac{1}{L/\sqrt{3}} = \frac{q\sqrt{3}}{\pi\epsilon_0 L},$$
which is choice A.
55. E – The thermodynamic limit for heat engine efficiency is $W/Q = 1 - T_C/T_H$, where W is the work output of the engine in one cycle, Q is the heat provided by the hot reservoir in one cycle, and T_C and T_H are the cold and hot reservoir temperatures, respectively. In the given problem $T_C = 300$ K and $T_H = 400$ K, so the maximum efficiency is $W/Q = 0.25$, and $Q_{\min} = 4W$. A power output of 100 kW corresponds to 100 kJ/s, so in 1 minute the work done is 6,000 kJ, and $Q_{\min} = 24,000$ kJ, choice E.
56. E – Doing the usual conservation of energy routine, we set the zero of potential energy at the top of the equilibrium position of the spring. The brick starts with potential energy mgh, and at maximum compression x the spring has potential energy $\frac{1}{2}kx^2$. However, there is an additional contribution $-mgx$ to potential energy from the compression of the spring below its equilibrium position. We have the following quadratic equation for x:
$$\begin{aligned}\frac{1}{2}kx^2 - mgx &= mgh\\ \Longrightarrow\ x &= \frac{mg \pm \sqrt{m^2g^2 + 2mghk}}{k}\\ &= \frac{mg}{k}\left(1 \pm \sqrt{1 + \frac{2kh}{mg}}\right).\end{aligned}$$
Discarding the spurious negative solution, we get choice E. We could also have done this by dimensional analysis and limiting cases. The only answers with correct units are A, D, and E, and in the limit $h \to 0$ (where the block is just sitting on top of the spring and then released), only E gives the correct compression $2mg/k$.
57. E – Solving this by free-body diagrams is straightforward but rather time consuming. Just using a little physics

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intuition, T_1 must support the mass of all the blocks, a total of 40 kg, while T_3 only has to support the lightest mass of 5 kg. So $T_1/T_3 = 8$, choice E.

58. A – Recall that the z-component of orbital angular momentum has the operator $\hat{L}_z = -i\hbar\partial/\partial\phi$. The hard way is to compute the expectation value from the definition,

$$\langle L_z \rangle = \int_0^{\pi}\int_0^{2\pi} d\phi\, d\theta\, \psi^*(\theta,\phi)\left(-i\hbar\frac{\partial}{\partial\phi}\right)\psi(\theta,\phi) = 0,$$

but the easy way is to note that this wavefunction is purely real, and so acting on it by $\hat{L}_z$ will give something imaginary. Expectation values of observables must be real, so the expectation value must be zero. Yet another way to see this is to remember that the ϕ dependence in spherical harmonics is carried by $e^{im\phi}$, and that sines and cosines are odd and even linear combinations of these exponentials with m values of opposite signs. So positive and negative L_z contribute equally and cancel in the end for any real orbital wavefunction.

59. E – The potential given is a finite square barrier, and at the given energy E, the barrier is a classically forbidden region for the alpha particle. So its wavefunction in that region must be an exponential, rather than a sinusoid. Furthermore, the tunneling probability is less than 1, so the amplitude of the wave outside the barrier must be less than the initial amplitude. Only choice E matches this description. Note that choice C is incorrect both because the wavefunction is not sinusoidal outside the barrier, and because the derivative of the wavefunction is discontinuous at both the walls; discontinuities in the derivative only occur in *infinite* potential wells, not finite ones.

60. D – Apply the double-slit relation $d\sin\theta = m\lambda$ to find m, with $d = 4\,\text{mm}$, $\lambda = 500\,\text{nm}$, and $\theta = 30°$, since θ measures the angle from the *center* of the arrangement. This gives $m = 4{,}000$, and we must multiply by 2 to cover both halves of the screen. So approximately 8,000 fringes are visible, choice D. (We say “approximately” because there is also a fringe at $\theta = 0$, but also the very last fringes at $\theta = 30°$ may or may not be visible at the edges of the screen.)

61. A – This is the setup for the relativistic Doppler shift. Since the ship is approaching the telescope, the period should decrease relative to the emitted period, since the light pulses are being received more often. This fixes the signs in the numerator and denominator:

$$\frac{T_{\text{obs}}}{T_{\text{emit}}} = \sqrt{\frac{1-\beta}{1+\beta}} = \sqrt{\frac{0.4}{1.6}} = \sqrt{\frac{1}{4}} = \frac{1}{2},$$

so $T_{\text{obs}} = 0.5$ s, choice A.

62. A – This is an example of length contraction: in the rest frame of the particle, the tube is approaching at speed $0.8c$, and so has its length contracted by a factor

$$\frac{1}{\gamma} = \sqrt{1 - v^2/c^2} = \sqrt{1 - 0.64} = 0.6.$$

So, in the particle's rest frame, the tube is $(30\,\text{m})(0.6) = 18\,\text{m}$ long, which is choice A.

63. D – The reciprocal lattice to a simple cubic lattice is also a simple cubic lattice, but with side length $2\pi/a$. The first Brillouin zone is again a cube, centered at one of the points on the reciprocal lattice, which bisects the reciprocal lattice vectors and hence also has side length $2\pi/a$. Thus its volume is $(2\pi/a)^3$, which is D.

64. A – Unfortunately this is just memorization. Beta decay is $n \to p + e + \nu$ (actually, the ν should be an antineutrino, but this distinction is irrelevant for the problem), and the third particle present in the final state is responsible for the broad energy spectrum of the remaining two. It is possible to narrow down the answer choices slightly using conservation laws: the positron, muon, and strange quark are all charged, so their presence in beta decay would violate charge conservation.

65. B – This one is simplest by limiting cases. Because we can always replace M_1 and M_2 along with the rope connecting them by a single block of mass $M_1 + M_2$, the answer must involve only $M_1 + M_2$ and not M_1 or M_2 individually. Furthermore, as $M_3 \to 0$, α must approach $0°$ or else M_1 and M_2 would slide down the ramp. Finally, if $M_1 + M_2 = M_3$, we must have $\alpha = 90°$. The only choice matching these limits is B.

66. D – The equivalent resistance of the top two resistors is $10\,\Omega$, as well as for the bottom two. Thus we have equivalent resistances of $10\,\Omega$, $5\,\Omega$, and $10\,\Omega$ in parallel, with total equivalent resistance of $(1/10 + 1/5 + 1/10)^{-1} = 2.5\,\Omega$. From $V = IR$, the current is $I = 2$ A, choice D.

67. E – There are several ways to do this, but the quickest is to remember that each of the Pauli matrices squares to the 2×2 identity matrix. You can of course check this by direct matrix multiplication, but it's a very useful fact to remember. Then $M = 3I_{2\times 2}$, which has determinant 9, choice E.

68. C – The expectation value of S_x is given by

$$\langle S_x \rangle = \eta^\dagger \hat{S}_x \eta = \frac{\hbar}{2}\eta^\dagger \sigma_x \eta.$$

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The normalization constant works out to be $\mathcal{N} = 1/\sqrt{6}$, so

$$\langle S_x \rangle = \frac{1}{6} \cdot \frac{\hbar}{2} \left(\begin{pmatrix} -i & 2+i \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} i \\ 2-i \end{pmatrix} \right) = -\hbar/6,$$

choice C. As it must, the expectation value comes out to be real: if it doesn't, you know you've made an arithmetic mistake.

69. D – A standing wave should be of the form $A(x)B(t)$ with B periodic in time. Only choice D matches this form.
70. B – First find the image formed by the lens A. Here the focal length is positive, so

$$\frac{1}{12.5\,\text{cm}} + \frac{1}{s_1'} = \frac{1}{10\,\text{cm}} \implies s_1' = 50\,\text{cm},$$

and the image is real and 50 cm to the right of A, so 30 cm to the right of B. The diverging lens acts next, and because the image is on the opposite side from the incoming light rays, it acts as a virtual object for B:

$$-\frac{1}{30\,\text{cm}} + \frac{1}{s_2'} = -\frac{1}{10\,\text{cm}} \implies s_2' = -15\,\text{cm},$$

so the image is virtual and 15 cm to the left of B, or 5 cm to the right of A. Since $s_1' > 0$, the first magnification is $m_1 = -s_1'/s_1 < 0$, so the first image is inverted. But both the second object and image distances s_2 and s_2' are negative, so $m_2 < 0$, and the final image is upright.
71. B – You could solve this by simply remembering the field transformations under a Lorentz transformation, but it's also instructive to reason geometrically. In $\overline{\mathcal{S}}$, there will be a length contraction along the direction of motion, reducing the area of the parallel plates by a factor of $1/\gamma = \sqrt{1 - v^2/c^2}$, but no change in the plate separation. The capacitance $C = \epsilon_0 A/d$ will then become $C/\gamma = C\sqrt{1 - v^2/c^2}$, choice B.
72. B – Muons have a mass of about 100 MeV, and cosmic ray muons are usually produced with energies substantially higher than this, so B is at least plausible. E might have high enough energy, but charged particles created by solar flares are usually captured by the Earth's magnetic field. Thermal radiation, seismic noise, and solar neutrinos have energies orders of magnitude too small, so B is the best choice.
73. C – The kinetic energy of the pivot is $\frac{1}{2}M\dot{x}^2$, and the pivot has no potential energy. Setting the zero of potential at the height of the bar, the potential energy of the mass m is $U = -mgl\cos\theta$. The position of m is $(X, Y) = (x + l\sin\theta, -l\cos\theta)$, so taking time derivatives,

$$(\dot{X}, \dot{Y}) = (\dot{x} + l\cos\theta\dot{\theta}, l\sin\theta\dot{\theta}).$$

The kinetic energy is

$$\begin{aligned} T &= \frac{1}{2}m(\dot{X}^2 + \dot{Y}^2) \\ &= \frac{1}{2}m(\dot{x}^2 + 2l\cos\theta\dot{x}\dot{\theta} + l^2\cos^2\theta\dot{\theta}^2 + l^2\sin^2\theta\dot{\theta}^2) \\ &= \frac{1}{2}m(\dot{x}^2 + l^2\dot{\theta}^2 + 2l\cos\theta\dot{x}\dot{\theta}), \end{aligned}$$

and taking $L = T - U$ gives choice C. Note that we had to start with the expression for kinetic energy in terms of Cartesian coordinates, rather than immediately jumping to polar coordinates, which would have neglected the contribution to the kinetic energy of the hanging mass coming from the motion of the pivot.
74. D – The relevant equations are the Bernoulli equation for an incompressible fluid,

$$\frac{1}{2}\rho v_1^2 + p_1 = \frac{1}{2}\rho v_2^2 + p_2,$$

and the continuity equation, which implies

$$v_1 A_1 = v_2 A_2.$$

Setting $p_1 = p$ and $v_1 = v$, we solve for the relevant variables and find

$$p_2 = p + \frac{1}{2}\rho v^2 \left(1 - \frac{A_1^2}{A_2^2}\right),$$

choice D. Note that this is consistent with Bernoulli's principle (in this context also known as the Venturi effect), where a decrease in cross-sectional area is accompanied by a decrease in pressure. For a partial alternate solution, we could have examined limiting cases: for $A_1 = A_2$, we must have $p_2 = p$, which eliminates B and E.
75. B – Calculating the probability (remembering the r^2 factor in the volume element),

$$\begin{aligned} P &= \int_a^{2a} |\psi(r)|^2\, r^2\, dr = a\int_a^{2a} \frac{1}{r^4}\, r^2\, dr \\ &= a\left(\frac{1}{a} - \frac{1}{2a}\right) = \frac{1}{2}, \end{aligned}$$

which is B. Note that we don't have to worry about the angular part of the wavefunction, which is assumed to be normalized on its own, and so we only have to do the radial part of the volume integral.
76. C – The v_x graph tells us that v_x is constant and positive, and v_y tells us that v_y is initially positive but decreases linearly, which means constant negative acceleration. In other words, something is falling, but started with positive velocity in both the x- and y-directions. And $v_y(0) \approx 2v_x(0)$, which means that the initial angle of the

velocity vector was about 60°, which is consistent with choice C. In particular, $v_y(0) > v_x(0)$, so choice D is not a possibility.

77. D – This is the setup for Brewster's angle: when the Sun is at Brewster's angle with respect to the normal to the ocean, the reflected wave is completely polarized parallel to the ocean. Since the polarization axis of the sunglasses is perpendicular to the ocean, no transmitted light passes through the polarizer. The information given in the problem tells us that Brewster's angle is 60°, so $n_2 = \tan 60° = \sqrt{3}$, choice D. Incidentally, choice A can be eliminated immediately since indices of refraction can't be less than 1.
78. A – A charge in a sinusoidally oscillating field is just a dipole oscillator, which radiates power according to the Larmor formula
$$P = \frac{q^2 a^2}{6\pi \epsilon_0 c^3},$$
where a is the acceleration of the charge. For the purposes of this problem, you don't need to remember all the constants and numerical factors: the crucial part is the dependence on q and a. The acceleration is obtained from $F = ma = qE$, giving $a = qE/m$, and (dropping the irrelevant constants)
$$P \propto q^2 (qE/m)^2 = \frac{q^4 E_0^2}{m^2} \cos^2 \omega t.$$
Taking the time average, $\langle \cos^2 \omega t \rangle = 1/2$, which is just a numerical factor (in particular, it's independent of ω). So we have
$$\langle P \rangle \propto \frac{q^4 E_0^2}{m^2},$$
which shows that if m is increased, $\langle P \rangle$ decreases. Increasing q and E_0 will increase $\langle P \rangle$. Changing T won't change the average power: since power is instantaneous energy per unit time, changing the measurement time can never change the power. Curiously, changing ω won't change the average power either, in contrast to the case of a dipole antenna where the emitted power scales as ω^4.
79. B – While actually calculating the flux as a function of time would require doing some tricky integrals, we can see that, because $q > 0$, the flux is positive from $t = -\infty$ to $t = 0$, and negative from $t = 0$ to $t = +\infty$. In particular, the flux must go to zero as $t \to \pm\infty$ because the charge is infinitely far away and the field dies off as $1/r^2$. Furthermore, because the strength of the electric field at $z = 0$ increases as the charge approaches, the flux does not simply increase at a constant rate, but accelerates. Finally, there is a discontinuity at $t = 0$ where the flux jumps from positive to negative, because at t slightly negative, all the field lines have components parallel to the normal to the loop, but at t slightly positive, all the field lines have components antiparallel. B is the only plot that satisfies all of these conditions.
80. B – The most probable speed of particles following the Maxwell distribution is $v = \sqrt{2kT/m}$. From the ideal gas law, P is directly proportional to T at fixed volume, so the most probable speed goes as $v \propto \sqrt{P}$, choice B.
81. B – The Doppler shift formula for a source approaching an observer in a straight line gives a constant observed frequency as the source approaches, and a different constant observed frequency as the source recedes because the sign of the relative velocity changes. The wavefronts are being compressed as the ambulance travels in the direction of the emitted signal, so the frequency is higher, hence choice B. This is a bit counterintuitive because the usual situation observed in real life is choice E, which corresponds to an ambulance passing by the observer with a nonzero distance of closest approach b. There, the reason for the sliding frequency is that the direction between the source and observer is constantly changing, and the Doppler shift only occurs in the direction of the source's motion, so there is an angle-dependent factor which changes continuously as the source moves past the observer.
82. E – Without doing any calculations, we know the answer must be E because the speed of a photon in *any* inertial reference frame is c. This is one of the axioms of special relativity: the speed of light is constant in any inertial reference frame.
83. C – Mesons are defined as bound states of a quark and an antiquark. Choice D describes baryons, not mesons, and none of the other choices are found in nature as bound states (except choice E, but that describes a nucleus).
84. A – Neutron stars are essentially giant spheres of neutrons. Since neutrons are fermions, they cannot all collapse to be in the same position state by the Pauli exclusion principle. All of the other choices could possibly be argued in vaguely plausible ways, but A is clearly the correct answer.
85. A – This can be quickly determined from energy conservation. The total energy stored in the capacitor is
$$E = \frac{Q_0^2}{2C}.$$

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Assuming that all of this energy is dissipated by the resistor, the heat transferred to the material is $\Delta Q = \frac{Q_0^2\epsilon}{2C}$, and the change in temperature can be found from $\Delta Q = mc_p\Delta T$:

$$\Delta T = \frac{\Delta Q}{mc_p} = \frac{Q_0^2\epsilon}{2mc_pC}.$$

86. B – We can get the field of an infinite wire from Ampère's law. Taking an Amperian loop around the wire at radius r, we get $2\pi rB = \mu_0 I$, so $B = \mu_0 I/2\pi r$, where we have assumed the field is circumferential. Setting $r = d$ and using $\mathbf{F} = I\,d\mathbf{l} \times \mathbf{B}$ gives a force per unit length $|\mathbf{F}|/dl = \mu_0 I^2/2\pi d$. The direction comes from the right-hand rule, or by remembering that wires behave oppositely from charges: like currents attract while opposite currents repel. Hence choice B.
87. C – Writing

$$\mathbf{S}_1\cdot\mathbf{S}_2 = \frac{1}{2}((\mathbf{S}_1+\mathbf{S}_2)^2 - \mathbf{S}_1^2 - \mathbf{S}_2^2) = \frac{1}{2}\left((\mathbf{S}_1+\mathbf{S}_2)^2 - \frac{3}{2}\hbar^2\right),$$

we see the possible values of H depend on the total spin $S_{\text{tot}}^2 = (\mathbf{S}_1 + \mathbf{S}_2)^2$; in other words, whether the two particles are in the singlet or triplet state. In the singlet state with spin 0, $H = -A(-\frac{3}{4}\hbar^2) = 3A\hbar^2/4$, and in the triplet state where $S_{\text{tot}}^2 = (1)(2)\hbar^2$, $H = -A\hbar^2/4$. However, there are three linearly independent total spin states in the triplet, so the triplet has a degeneracy of 3. Since $A > 0$, the triplet is the lowest-energy state, hence choice C.
88. B – The diameter of the molecules is only relevant in that they are much smaller than the wavelength of incident light. This puts us in the regime of Rayleigh scattering, where intensity is proportional to λ^{-4}. So doubling the wavelength multiplies the scattered intensity by 1/16, which is choice B. Incidentally, this is the same phenomenon that is responsible for the blue color of the sky, since solar light scattering off air and water molecules has a much higher intensity toward the blue end of the spectrum.
89. C – By the selection rules, we need $\Delta l = \pm 1$, so $3s$ must decay to $2p$ in the electric dipole approximation. All other decay modes will be suppressed relative to this "allowed" transition, hence choice C. (The selection rules for m will be satisfied if the $2p$ state also has $m = 0$.) Note that, depending on the total spin, the $3p$ state is at best degenerate with $3s$, up to hyperfine-structure corrections. This is a tiny energy splitting, so the transition would still prefer to go to $2p$.
90. D – Statistical and systematic errors are independent so they add in quadrature: $\sigma_{\text{tot}} = \sqrt{(0.08\text{ GeV})^2 + (0.06\text{ GeV})^2} = 0.10$ GeV.
91. B – Radioactive decays follow a Poisson probability distribution, whose variance is equal to N, the number of counts. Hence the fractional error is $\sigma/N = \sqrt{N}/N = 1/\sqrt{N}$, which for $N = 64$ gives 1/8, choice B.
92. C – This is a simple kinematics problem dressed up with some unfamiliar units. While we could convert the electron kinetic energy to joules and use $F = qE$ with the electron charge q in coulombs, it's much simpler to recall the definition of eV ("electron-volt") as the potential energy gained by an electron traveling through a potential difference of 1 V. The electron reaches its maximum height when its kinetic energy is converted entirely to potential energy, which occurs when the potential relative to the sheet is 10 V. 100 V/m = 1 V/cm, so the 10 V potential occurs at a height of 10 cm, choice C. (The electron is negatively charged, so an electric field directed along its direction of motion will cause it to decelerate – trap answer E would be correct for a positively charged particle.)
93. C – Let ℓ be the length of the rod, v be the initial velocity of the bullet, V be the velocity of the block immediately after the collision, and m and M be the masses of the bullet and block, respectively. Momentum is conserved in the collision with the block, so we can solve for V:

$$mv = (M+m)V$$
$$\implies V = \frac{m}{M+m}v \approx \frac{m}{M}v,$$

where the last approximation comes from the fact that the bullet is much lighter than the block. For the block to just barely make a complete revolution, the block must reach the very top of its circular path with zero velocity. There, it will have potential energy $(M+m)g(2\ell)$ with respect to its initial position, so we solve for V by conservation of energy:

$$\frac{1}{2}(M+m)V^2 = 2(M+m)g\ell$$
$$\implies V = 2\sqrt{g\ell}.$$

Plugging in for V and inserting the numbers in the problem gives

$$v = 2\frac{M}{m}\sqrt{g\ell} \approx 800\text{ m/s},$$

choice C.

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94. E – This is an application of the Rayleigh criterion, $\sin\theta = 1.22\lambda/D$. Solving for D gives $D = 1.22\lambda/\sin\theta$. Since $\theta = 0.061$ rad is very small, we can approximate $\sin\theta \approx \theta$. From $\lambda f = c$, we have $\lambda = 1.5$ m, and $D = 30$ m, choice E.
95. D – In the center-of-momentum frame, the electron and the positron each have energy γm_e where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the Lorentz factor. The total energy in this frame is $2\gamma m_e$, which must be at least the energy of the two muons at rest, $2m_\mu$. Solving $2\gamma m_e = 2m_\mu$ for v, we find choice D.
96. B – The time constant of an RC circuit is simply RC. We have two resistors in series, so the effective resistance is 2 MΩ + 3 MΩ = 5 MΩ, and two capacitors in series whose effective capacitance is $(1/1\,\mu\text{F} + 1/4\,\mu\text{F})^{-1} = 0.8\,\mu\text{F}$. Multiplying these gives a time constant of 4 s, which is B.
97. E – The field in the solenoid is proportional to the number of coils per unit length, n. But the flux through the solenoid is proportional to n^2, since we have to sum the fluxes through each of the n coils. From the definition of inductance, $L = \Phi/I$, L is also proportional to n^2, so tripling n increases L by a factor of 9, as in E.
98. B – In a free electron laser, electrons are forced in a sinusoidal path, and the resulting acceleration produces coherent synchrotron radiation, choice B. All the other choices are steps in the operation of normal lasers, but they all require electronic transitions in atoms – the key description "*free* electron" should at least eliminate choices A and E.
99. C – If we imagine the surface current as being made up of a bunch of parallel wires all pointing in the $\hat{\mathbf{y}}$-direction, we see that there are equal and opposite contributions to a field in the $\hat{\mathbf{z}}$-direction, from wires at $+x$ and $-x$ for any value of x. So there is no field in the $\hat{\mathbf{z}}$-direction. Moreover, there can't be any field parallel to the wires, so $\mathbf{B}$ must point in the $\pm\hat{\mathbf{x}}$-direction. This leaves only C and D. To get the sign right, apply the right-hand rule to all the wires, which shows that above the plane, the field from every wire is in the $+\hat{\mathbf{x}}$-direction. Since C and D only differ by a sign, we're done: we don't actually have to use Ampère's law to calculate the magnitude. Always be on the lookout for shortcuts like this on the GRE!
100. D – It turns out that *all* of the numerical values in the question are irrelevant. This is not an uncommon occurrence on the GRE, so it pays to work out everything in terms of variables until the end of the problem. Let m be the mass of the sphere, R its radius, h the height of the ramp, and v its velocity at the bottom. Two observations make this problem quite simple. First, while the forces on the sphere are different whether or not friction is present, they must be *constant* throughout the fall down the ramp. So the acceleration down the ramp is constant as well. Second, from kinematics we know $v^2 = 2a\Delta x$ for constant acceleration a, so combining this with $v = at$ gives $t = 2\Delta x/v$; the time is inversely proportional to the velocity at the bottom of the ramp. The distance traveled Δx will cancel out in the ratio t'/t.

    To find the velocity at the bottom, use conservation of energy. Without friction, $mgh = \frac{1}{2}mv^2$, so $v = \sqrt{2gh}$. With friction, there is an additional rotational kinetic energy $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{2}{5}mR^2)(\frac{v'^2}{R^2}) = \frac{1}{5}mv'^2$. Now we have

$$mgh = \left(\frac{1}{2} + \frac{1}{5}\right)mv'^2 \implies v' = \sqrt{\frac{10}{7}gh},$$

so $t'/t = v/v' = \sqrt{7/5}$, choice D. We could have narrowed the choices down to D or E from the beginning, since the additional rotational kinetic energy implies the velocity at the bottom will be less than in the frictionless case. From the constant acceleration argument, this means that $t' > t$, which eliminates choices A–C.

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Solutions to Sample Exam 3

1. C – Whenever a small amount of liquid with mass m is emitted from the bath, the potential energy released by the level of the water dropping is $\Delta U = mgh$. Neglecting small effects such as surface tension and viscosity, all of this energy is converted into kinetic energy with a velocity directed parallel to the ground. The velocity parallel to the ground is therefore

$$\frac{1}{2}mv^2 = mgh,$$
$$v = \sqrt{2gh}.$$

The same result can also be derived from Bernoulli's principle applied to the top and bottom of the bath, noting that atmospheric pressure is the same at both locations and cancels. In fact, it's common enough that it has its own name, "Torricelli's law," and may be worth memorizing so you don't have to repeat the derivation.

The time required for free fall in the vertical direction follows from basic kinematics:

$$t = \sqrt{2y/g}.$$

Substituting this into $x = vt$, we find that

$$x = 2\sqrt{hy}.$$

2. E – Hydrogen atoms are much smaller than 450 nm so the small-particle Rayleigh formula applies. If the Bohr radius were doubled by the a^6 dependence in the Rayleigh formula we get intensity $2^6 I_0 = 64 I_0$, choice E.
3. E – This is a straightforward application of the Rayleigh criterion. The minimum aperture diameter D needed to resolve objects with separation $\Delta\theta$ emitting light of wavelength λ is

$$D = \frac{1.22\lambda}{\Delta\theta},$$

where we have assumed $\Delta\theta$ is small so we can use the small-angle approximation. For the purposes of this problem, which is really an order-of-magnitude question, the coefficient 1.22 isn't even necessary. Radio waves have wavelengths on the mm to km scale, visible light has wavelengths between 400 and 700 nm, and x-rays have wavelengths of approximately 0.01 to 10 nm. Applying the criterion, we can estimate

$$D_{\text{radio}} \geq \frac{1.22(10^{-3}\text{ m})}{3 \times 10^{-4}\text{ rad}} \approx 5\text{ m},$$

$$D_{\text{vis}} \geq \frac{1.22(500 \times 10^{-9}\text{ m})}{3 \times 10^{-4}\text{ rad}} \approx 2\text{ mm}.$$

At this point we don't even have to do the calculation for x-rays (nor do we really have to know exactly what their wavelengths are), since we know that a 1 cm aperture is large enough to resolve visible light, so it must be large enough to resolve x-ray light, which has an even smaller wavelength. So both II and III are possible, giving choice E.
4. B – The force due to friction is $F = \mu mg$, so the acceleration due to friction is given by $ma = \mu mg$, or

$$a = \mu g,$$

opposite to the direction of motion. We can obtain the distance traveled by the useful kinematic identity

$$v_f^2 - v_i^2 = 2a\Delta x.$$

Plugging in numbers, we find that

$$\Delta x = \frac{v_i^2}{2a} = \frac{v_i^2}{2\mu g} = \frac{(10\text{ m/s})^2}{10\text{ m/s}^2} = 10\text{ m}.$$

Just as easily, we can use the work–energy theorem: the work done by friction to bring the block to rest must be equal to its initial kinetic energy, so

$$\mu mg\Delta x = \frac{1}{2}mv_i^2 \implies \Delta x = 10\text{ m}$$

as before. Note that the mass cancels out in both solutions.
5. E – The fastest way to solve this problem is to just think of some examples where you have used Lagrangians to solve a problem. Clearly, we use Lagrangians all the time to solve problems with conservative forces; for example, a particle in a gravitational field. There's also nothing special about rotational symmetry; for example, again, we can use Lagrangians to solve for motion in a uniform gravitational field at the surface of the Earth, which has cylindrical but not full rotational symmetry. Finally, one rarely deals with time-dependent potentials, but the fact that the Euler–Lagrange equations include time derivatives suggests that they do not assume anything to be constant with time.
6. C – The hyperfine structure is the smallest correction, orders of magnitude smaller than the fine-structure effects. It is produced by the interaction of the nuclear dipole moment with the magnetic field produced by the orbit of the electrons. The next smallest is the Lamb

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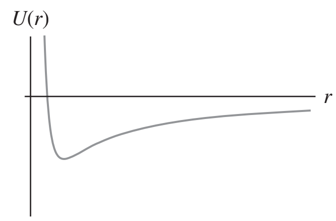
shift, which arises due to interactions between the electron and the vacuum and causes a shift in the relative magnitude of the s and p orbital energies for the $n = 2$ states. The fine-structure corrections arise due to spin–orbit coupling, relativistic effects, and the so-called Darwin term. The combined fine-structure effects are the largest contribution. The easiest way to remember the hierarchy is to remember the factors of the fine-structure constant α: fine-structure corrections are proportional to α^4, the Lamb shift is proportional to α^5, and hyperfine-structure corrections are proportional to $\alpha^4(m_e/m_p) \ll \alpha^5$.

7. C – The energies of the harmonic oscillator are just $E_n = \hbar\omega(n + 1/2)$. The difference between the $n = 5$ and $n = 1$ states is therefore $4\hbar\omega$.
8. B – While it is tempting to think of this problem as an application of the Coriolis force, it is much easier to solve in the nonrotating frame. It takes the puck a time $t = R/v$ to reach the edge of the disk. By this time, the disk has rotated through an angle $\Delta\theta = \omega t = \omega R/v$.
9. C – The answer is a common enough formula that you may have it memorized. If you do not, you can derive it from the usual rules for propagation of uncertainties. The uncertainty of the function $z = x/y$ is

$$\begin{aligned}(\Delta z)^2 &= \left(\frac{\partial z}{\partial x}\right)^2 (\Delta x)^2 + \left(\frac{\partial z}{\partial y}\right)^2 (\Delta y)^2 \\ &= \frac{1}{y^2}(\Delta x)^2 + \frac{x^2}{y^4}(\Delta y)^2 \\ &= z^2\left(\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2\right).\end{aligned}$$

Taking square roots to get Δz gives choice C.

10. A – A bandpass filter only allows signals to propagate that are between two frequencies (not to be confused with high-pass or low-pass filter, which are only one-sided). Intuitively, inductors suppress high-frequency signals because voltages are high for fast oscillations and capacitors suppress low-frequency ones because of excessive charge buildup. It therefore is most reasonable that a bandpass filter would require both an inductor and a capacitor. Choice A is the only option that has both of these circuit elements. (Note however that both high-pass and low-pass filters can be made with either RL or RC circuits, and stringing two of these circuits together would give a bandpass filter, but this requires more circuit elements than allowed by the answer choices.)



11. A – The effective potential is shown in the figure above. Since a circular orbit is at a fixed radius, E_{cir} corresponds to an energy at the minimum of the effective potential. An elliptical orbit is at a variable radius but is still bound, so E_{ell} corresponds to an energy above the minimum of the effective potential but less than zero. Finally, a hyperbolic orbit is unbound at all radii (even at $r \to \infty$), so $E_{\text{hyp}} > 0$. Putting it all together, we find that $E_{\text{cir}} < E_{\text{ell}} < E_{\text{hyp}}$.
12. C – Electrons are fermions, and so at zero temperature they cannot all collect in the ground state. Instead, they fill out the so-called Fermi sphere, and the energy of electrons at the boundary is called the Fermi energy, choice C. In a pinch, if you only remembered that electrons are fermions, you might be able to guess that the answer had something to do with Fermi.
13. B – The time dilation changes the apparent lifetime in the rest frame by a factor of $\gamma = 1/\sqrt{1-0.64} = 1/0.6 = 1.67$. The distance traveled during this time is $0.8c \times (1.67 \times 10^{-8}$ s), or about 4 m.
14. B – The outermost electron is in the p orbital, and so has $l = 1$. This means $L^2 = l(l+1)\hbar^2 = 2\hbar^2$, so $|\mathbf{L}| = \hbar\sqrt{2}$.
15. C – Superficially this may seem like a rather technical question from early universe cosmology. On the other hand, it is clear that I and II are light nuclei, while III is fairly heavy. Your intuition should tell you that light nuclei were probably the first elements produced after the Big Bang, leading you to guess C. Indeed this is correct. No elements heavier than beryllium were produced in the early universe before it cooled below the temperature needed for nucleosynthesis. Heavier elements were not produced until the first stars formed and combined the lighter elements into heavier ones via nuclear fusion.
16. B – The fluid conservation equation implies that

$$vA_1 = v_2 A_2,$$

so B is the correct choice. This follows immediately from conservation of mass and the fact that the fluid is incompressible.

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17. C – We can use the equation from basic kinematics:

$$v^2 = 2a\Delta x.$$

Plugging in numbers, we find that the velocity is 20 m/s.

18. E – While this problem can be analyzed to a large extent using limiting cases, it is also simple to work out explicitly. The velocity of the block–bullet system after the collision is obtained by conservation of momentum:

$$v' = \frac{mv}{m+M}.$$

The force due to friction is simply constant and given by

$$F = (m+M)a = -\mu(m+M)g.$$

From basic kinematics at constant acceleration, we know that

$$\Delta t = \frac{\Delta v}{a} = \frac{mv}{\mu g(m+M)}.$$

Alternatively, you could eliminate all choices except E by considering the limiting behavior of the masses and coefficient of friction.

19. C – This question refers to Johnson noise, the voltage fluctuations that arise in resistors due to thermal fluctuations that induce resistance fluctuations. The derivation of this formula is not difficult, but it may be unfamiliar to you. Luckily, we can solve this one by pure dimensional analysis. A spectral density of voltage fluctuations has units of V $\mathrm{Hz}^{-1/2}$. The funny units are chosen because rms values of noise sum in quadrature. It therefore is most natural to write a spectrum of square power over frequency because then noise simply adds. Choice C is the only option that has the correct units. Note that *almost always*, k and T show up in the combination kT which has units of energy. Here, it's most convenient to write energy as $[V][Q]$, or voltage times charge. From $V = IR$, resistance R has units of $V/I = [V][Q]^{-1}[T]$. To get $\mathrm{Hz}^{-1/2}$ or $[T]^{1/2}$, we want $\sqrt{R}$ somewhere, so computing the units,

$$\sqrt{kTR} \sim \sqrt{[V][Q][V][Q]^{-1}[T]} \sim [V][T]^{1/2},$$

and C has the right units as promised.

20. D – At time t the bar has traveled a distance $x = vt$. Since the magnetic field is perpendicular to the loop, the flux through the loop is

$$\Phi = \int \mathbf{B}\cdot d\mathbf{A} = Bxd.$$

By Faraday's law, the emf in the circuit is $\mathcal{E} = d\Phi/dt = Bvd$, so the power is $P = \mathcal{E}^2/R = (Bvd)^2/R$, and integrating this from $t = 0$ to $t = T$ gives the total energy $(Bvd)^2T/R$, choice D.

21. D – Recall that the rms velocity of an ideal gas is

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}.$$

Helium is a monoatomic gas since noble gases don't form chemical bonds under standard conditions, so helium gas, with a mass of approximately 4 amu, is approximately two times heavier than that of molecular hydrogen, with a mass of approximately 2 amu. Thus, its rms velocity is $v/\sqrt{2}$, choice D.

22. D – The resonant frequency of an LC circuit is

$$\omega_0 = \sqrt{\frac{1}{LC}},$$

which is also the frequency at which the current through an RLC circuit is maximized, so the answer is D. If you don't remember this fact, just remember that the resonant frequency is defined as the frequency where the imaginary part of the total impedance vanishes. Adding a series resistance only changes the *real* part of the impedance, so the resonant frequency is unchanged.

23. B – An open pipe has pressure nodes at both ends, so the wavelength of the fundamental vibration is $2L$. The frequency is

$$f = \frac{c}{\lambda} = \frac{343\ \mathrm{m\ s^{-1}}}{0.4\ \mathrm{m}} = 858\ \mathrm{Hz}.$$

24. B – Since the two loops are perpendicular to each other, the torque on the smaller loop is given by

$$N = mB,$$

where $m = \pi ib^2$ is the magnetic moment of the smaller loop, and B is the magnitude of the magnetic field at the center of the larger loop. This can be readily obtained from the Biot–Savart law, which reduces to

$$B = \frac{\mu_0}{4\pi}\frac{2\pi aI}{a^2} = \frac{\mu_0 I}{2a}.$$

Putting the pieces together, we find that the torque is

$$N = \frac{\mu_0\pi Iib^2}{2a}.$$

25. B – When ball A is released, it experiences a centrifugal acceleration in its reference frame:

$$a = \frac{v^2}{r} = \frac{\Omega^2 R}{2}.$$

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Ball B experiences an acceleration given by

$$a = \frac{v^2}{r} = \Omega^2 R,$$

so the ratio is 1/2.

26. B – The electric field inside the sphere can be quickly obtained from Gauss's law, and is found to be $\propto r$. By integration, we conclude that the potential is $\propto -r^2$. In the region between the charged sphere and the conductor, there is the usual $\propto r^{-1}$ potential. Inside the conductor, the potential is constant. Finally, outside the conductor, the potential is again $\propto r^{-1}$. Only choice B satisfies these requirements.
27. D – Only choices D and E are behaviors that are even remotely possible for an RLC circuit. E is almost a correct plot of an overdamped RLC circuit (the case with no visible oscillation from the inductor), but the current starts out at a nonzero value. This should not happen because the inductor should oppose the sudden increase in current, producing a more gradual increase from zero current to some maximum, as seen in choice D.
28. A – The cyclotron radius is

$$r = \frac{mv}{qB}.$$

If the electrons travel at velocity v, then we have

$$v = \frac{2\pi r}{T},$$

and so

$$T = \frac{2\pi m}{qB}.$$

29. B – This is classic Bragg diffraction. We can simply use the well-known formula for first-order diffraction:

$$2d \sin\theta = \lambda.$$

Solving for the angle in the small-angle approximation, we have

$$\theta \approx \frac{\lambda}{2d}.$$

Plugging in numbers, and approximating π by 3 when converting from radians to degrees,

$$\theta \approx \frac{10}{2 \cdot 80} \times \frac{180}{\pi} \approx \frac{30}{8} \approx 3.75^\circ,$$

which is closest to 3.6°, choice B.

30. B – If this strikes you as an insanely difficult problem for the GRE, realize that it can be solved without any calculation by considering limiting cases. The most useful limit here is $x \to R$: since the sphere is grounded and conducting, we know that the potential must vanish when $x = R$. Choices B and E are the only options that satisfy this basic requirement. Choice E is obviously wrong, because as $R \to 0$ we should still see some dependence on x from the potential of the charge Q by itself.
31. B – It is a useful fact that the simple cubic lattice is self-dual; that is, the lattice is equal to its reciprocal lattice. In case this isn't obvious, it should be clear to you from the definition of the reciprocal lattice. If $\mathbf{r} = la\hat{\mathbf{x}} + ma\hat{\mathbf{y}} + na\hat{\mathbf{z}}$ with $l, m, n \in \mathbb{Z}$ are the lattice sites of the simple cubic lattice, then the reciprocal lattice vectors are those vectors $\mathbf{k}$ satisfying the relation

$$e^{i\mathbf{k}\cdot\mathbf{r}} = 1.$$

It is clear that this relation will hold as long as $\mathbf{k} = \frac{2\pi}{a}(p\hat{\mathbf{x}} + q\hat{\mathbf{y}} + r\hat{\mathbf{z}})$, where $p, q, r \in \mathbb{Z}$. But these $\mathbf{k}$ vectors are just the points of another simple cubic lattice. So the reciprocal lattice of a simple cubic lattice is itself cubic.

32. A – This is a straightforward computation using the definition of the magnetic field in terms of the vector potential. Recall that we have

$$\mathbf{B} = \nabla \times \mathbf{A}.$$

Direct substitution of the vector potential into this equation gives choice A.

33. B – This is a simple application of Wien's displacement law, which holds for blackbody radiation. The CMB is one of nature's most perfect blackbodies, so Wien's law is applicable. It states that the wavelength of maximum intensity emitted by a blackbody is

$$\lambda = \frac{b}{T},$$

where b is a constant. We just set

$$\lambda T = \lambda' T',$$

so that

$$\lambda' = \frac{\lambda T}{T'} = \frac{1\ \text{mm} \cdot 2.7\ \text{K}}{5\ \text{K}} \approx 0.5\ \text{mm},$$

which is closest to B.

34. E – The best way to solve this problem is by a process of elimination. B and D are trivially forbidden by charge conservation. A is forbidden by conservation of lepton number (you cannot get two leptons from one). C is a bit more subtle, but it is forbidden because the Δ^+ is

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a low-lying excitation of the proton, which is not sufficiently massive to decay to a proton and neutron. In addition, this decay would violate baryon number since Δ^+, p, and n each have baryon number equal to 1. This leaves E: note that lepton number is conserved because μ^+, the antimuon, has mu-lepton number -1.

35. D – This is another application of Wien's displacement law. Relating observed to true quantities, we have

$$\frac{b}{T_{\text{obs}}} = \lambda_{\text{obs}} = \sqrt{\frac{1+\beta}{1-\beta}}\lambda_{\text{true}} = \sqrt{\frac{1+\beta}{1-\beta}}\frac{b}{T_{\text{true}}}.$$

The true temperature is therefore

$$T_{\text{true}} = \sqrt{\frac{1+\beta}{1-\beta}}T_{\text{obs}}.$$

Plugging in numerical values, we obtain about 1.1×10^4 K.

36. C – Intuitively, one might expect the energy to go to the larger of the two states, but this does not turn out to be the answer. The partition function is given by

$$Z = e^{-\beta\epsilon} + e^{\beta\epsilon}.$$

The average energy is given by

$$\langle E\rangle = \frac{\epsilon e^{-\beta\epsilon} - \epsilon e^{\beta\epsilon}}{e^{-\beta\epsilon} + e^{\beta\epsilon}}.$$

As $T \to \infty$, we have $\beta \to 0$, so

$$\langle E\rangle \to 0.$$

We can understand this result because $T \to \infty$ means $kT \gg \epsilon$, so the thermal fluctuations overpower the small energy splitting due to ϵ, and each state is equally populated.

37. C – If the gas expands, then clearly energy is conserved as long as the box is reasonably thermally isolated from its surroundings. Somewhat less obviously, the temperature is also unchanged. During free expansion of an ideal gas, we have $PV = P'V'$, and thus temperature must remain constant. Alternatively, you can see that the temperature is unchanged since the temperature is related to the internal energy of the gas via $U = (3/2)NkT$. The entropy of an ideal gas, on the other hand is given by

$$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right],$$

which has a clear volume dependence. Another formula that is probably more familiar to you and easier to remember for the exam is that the entropy change during free expansion of an ideal gas is

$$\Delta S = Nk\ln\left(\frac{V'}{V}\right).$$

This follows directly from the full expression for the entropy above, but it is quite a bit more practical for GRE-style questions.

38. C – Energy conservation is not necessarily violated because $e < 1$ is still possible. The First Law of Thermodynamics is essentially a restatement of conservation of energy, so this should be a good clue that neither of the first two choices are correct. The Third Law of Thermodynamics is the statement that objects cannot be cooled to absolute zero, which has nothing to do with the situation at hand. Finally, the postulate of equal *a priori* probabilities is a fundamental assumption of statistical mechanics, which states that all of the microstates corresponding to each macrostate of a system are equally probable. This clearly has nothing to do with the heat engine at hand. This leaves the Second Law of Thermodynamics as the correct answer.

We could also have seen this right away since, as $T_H \to T_C$, $e \to 2/3$. This is a clear violation of the Second Law since it would imply that useful work could be done between two reservoirs at the same temperature; in particular, it violates the Carnot bound $e = 1 - \frac{T_C}{T_H}$.

39. E – The addition of velocities formula is generally written in the form

$$s = \frac{u+v}{1+\frac{uv}{c^2}}.$$

It is critical to understand the notation here. This equation holds for a body A that is traveling at velocity v with respect to another body B that is traveling at velocity u with respect to a reference frame S. In this notation, s is the velocity of A with respect to the reference frame S. In the problem, we can identify v as the velocity v_1 of spaceship A with respect to the planet. We can identify s as the velocity v_2 of spaceship B with respect to the planet. And we want to solve for the absolute value of u, the speed of spaceship A relative to spaceship B. Making these substitutions and solving, we find that the speed is as given in the solution.

Equivalently, we can think of everything in the reference frame of B. Then u is the velocity of the planet relative to B, v is the velocity of A relative to the planet. and s is the velocity of A relative to B. Making the

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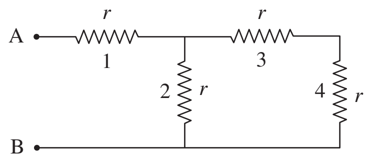

identification $u = -v_2$, $v = v_1$, we obtain the same result.

40. E – This is a straightforward equivalent resistance problem. Labeling the resistors as shown above, resistors 3 and 4 are in series, with equivalent resistance $2r$. This equivalent resistance is in parallel with resistor 2, giving

$$R_{234} = \left(\frac{1}{2r} + \frac{1}{r}\right)^{-1} = \frac{2r}{3}.$$

Adding resistor 1 in series, we get a total resistance of $5r/3$, or choice E.

41. D – The invariant of a gas undergoing a reversible adiabatic change is

$$PV^{\gamma} = \text{const},$$

where γ is the ratio of specific heat at constant pressure to specific heat at constant volume. For a monoatomic ideal gas, we have $\gamma = 5/3$. Thus we have

$$P = P_f \times 2^{5/3},$$

and

$$P_f = 2^{-5/3} P.$$

42. B – To compute this, we simply need to recognize that $L_z Y_l^m(\theta, \phi) = \hbar m Y_l^m(\theta, \phi)$. Using this fact, we have

$$\langle L_z \rangle = \frac{1}{2} \int d\Omega \left(Y_1^{*-1}(\theta, \phi) + Y_1^{*0}(\theta, \phi)\right)$$
$$L_z \left(Y_1^{-1}(\theta, \phi) + Y_1^0(\theta, \phi)\right) = -\frac{\hbar}{2}.$$

Even this setup is a little overkill here: since Y_1^{-1} has eigenvalue $-\hbar$ and Y_1^0 has eigenvalue 0, and they appear in the wavefunction with equal relative coefficients, the average must be $\frac{1}{2}(-\hbar + 0) = -\hbar/2$.

43. C – Choice A is clearly incorrect. Superpositions of energy states are permissible states, and they are certainly not energy eigenstates. Choice B is also incorrect. The simplest counterexample is the free particle (i.e. the solution of the Schrödinger equation with $V = 0$), which can have any positive energy. The wavefunction for this case $\psi(x) = Ae^{ikx}$ also is a simple counterexample for choice D. Choice C is, of course, true for quantum systems in any number of dimensions. You may remember that observables are represented by Hermitian operators, so it stands to reason that their eigenvalues should be real.

44. C – We cannot easily guess away the solutions, so we must solve this problem explicitly. We just need to use conservation of angular momentum. In order to do this, we need to know the moment of inertia of the mass–rod system. Since the mass is a point particle, the total moment of inertia is the sum of the point particle moment of inertia and the rod moment of inertia. And, since $m \ll M$, the center of mass of the new system is approximately the center of mass of the rod, so we compute both moments about the center of the rod:

$$I = \frac{1}{12} ML^2 + m\left(\frac{L}{2}\right)^2.$$

The angular momentum before the collision is equal to the angular momentum afterwards:

$$mv\left(\frac{L}{2}\right) = \left(\frac{1}{12} ML^2 + \frac{1}{4} mL^2\right)\omega.$$

So the solution is

$$\omega = \frac{6mv}{(M + 3m)L}.$$

Under the approximation $m \ll M$, we can drop the $3m$ term in the denominator, which just gives

$$\omega = \frac{6mv}{ML},$$

choice C.

45. A – The time dependence of the wavefunction is given by $e^{-iHt/\hbar}$. For an eigenstate we have $e^{-iEt/\hbar}$, where $E = \alpha\hbar B/2$ for the $|+\rangle$ state. The wavefunction for this state is thus $\exp(-i\alpha Bt/2)|+\rangle$.

46. D – Observables in quantum mechanics clearly must have real eigenvalues because we measure real numbers in physical experiments. We also require that observable operators be Hermitian, partially in order to ensure that the eigenvalues are real. So, I and III are correct. II is false because multiple operators can commute with the Hamiltonian and therefore can have simultaneous eigenstates.

47. A – The oscillation frequency of a mass on a spring is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

While you should definitely know this expression and how to derive it, you can easily derive it from dimensional analysis in a pinch. The equation of motion for

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this system only involves k and m, and there is only one combination of these constants that gives units of frequency. The prefactor is unimportant for this problem, since we're only interested in the fractional change of a quantity. That said, if we increase the mass by a factor of 3 and increase the spring constant by a factor of 2, then the new oscillation frequency is just

$$f = \frac{1}{2\pi}\sqrt{\frac{2k}{3m}} = \sqrt{\frac{2}{3}}f_0.$$

48. E – The probability of finding the particle on $[0, 1/2]$ is the integral over this interval of $|\psi|^2$:

$$P = 5\int_0^{1/2} x^4\,dx = \frac{1}{32}.$$

49. E – The spin-up and spin-down components of the wave function are orthogonal, so we simply have

$$\langle\psi|\,S_z\,|\psi\rangle = \frac{2}{3}\left(\frac{\hbar}{2}\right) + \frac{1}{3}\left(\frac{-\hbar}{2}\right) = \frac{\hbar}{6}.$$

50. D – Carbon dioxide lasers produce photons when electron impacts on nitrogen excite vibrational modes of the molecule. Collisions between nitrogen and CO_2 then excite vibration modes of the molecule, and the de-excitation produces the laser light.

51. B – This can be quickly solved by writing the equations of motion for the car and the trailer. For the car we have

$$Ma = F - T.$$

For the trailer, we have

$$ma = T - \mu mg.$$

Solving for T by substituting for a, we arrive at

$$T = \frac{m(F + \mu Mg)}{M + m}.$$

52. E – The total angular momentum obtainable when summing together angular momenta l and s are integral values between $|l+s|$ and $|l-s|$. The smallest total angular momentum is therefore $j = 2$.

53. E – Recall that noble gases such as helium, neon, and argon have full electron shells, which is the case here: each shell has $2n^2$ states, so there are two electrons in the $n = 1$ shell and eight electrons in the $n = 2$ shell. (This happens to be the electronic configuration for neon.) Alkali metals and rare earth metals have one or two additional electrons over a full shell, respectively, and halogens are one electron short of a full shell. Semiconductors are not easily identified from their electronic configurations.

54. C – This seems like a nasty problem, but considering limiting cases makes it easy. As $\beta \to 0$, the perturbation becomes constant, and thus the energy shift must also be the constant α: this eliminates A, D, and E. As $\beta \to \infty$, the perturbation disappears entirely, which leaves only choice C.

For completeness, here's the exact solution. The energy shift of the ground state is given by taking the expectation value of the perturbation with respect to the ground state wavefunction:

$$\begin{aligned}\langle 0|\,\delta V\,|0\rangle &= \alpha\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\int_{-\infty}^{\infty} e^{-m\omega x^2/\hbar}e^{-\beta x^2}\,dx\\ &= \alpha\left(\frac{m\omega}{m\omega + \beta\hbar}\right)^{1/2}.\end{aligned}$$

55. C – By Stokes's theorem, we can relate the line integral of $\mathbf{f}$ around a closed curve to the surface integral of $\nabla\times\mathbf{f}$ over a surface bounded by that curve. But $\nabla\times\mathbf{f} = 0$ so the line integral must vanish. We can also easily do the integral explicitly. The integral over the first segment is zero because only the $\hat{\mathbf{x}}$ part contributes, and $y = 0$; the same is true for the last segment. The second segment contributes 1, but the third segment contributes -1 since it is traversed in the $-\hat{\mathbf{x}}$-direction, so the total is zero as before.

56. C – All answers except C can be eliminated with dimensional analysis. The dimensions of magnetic flux are a bit tricky: it's magnetic field strength times area, but we can relate this to electrical charges by Faraday's law:

$$\begin{aligned}(B\text{-field})(\text{area}) &= (\text{voltage})(\text{time})\\ &= (\text{energy}\cdot\text{time})/\text{charge},\end{aligned}$$

which has units of $\text{J}\cdot\text{s}/\text{C}$.

57. C – I is true: electrons and positrons are both fermions, and a system consisting of an even number of fermions behaves as a boson and obeys Bose–Einstein statistics. This can be shown more rigorously by the rules for addition of angular momentum: two spin-1/2 particles can have a total spin of 0 or 1, both of which are integers rather than half-integers. II is also true, since the binding energy of a two-particle bound state is proportional to the reduced mass of the system. In the case of the hydrogen atom, the reduced mass is $m_p m_e/(m_e + m_p) \simeq m_e$ and the ground state energy is given by -13.6 eV. In the case of positronium, the reduced mass is $m_e^2/(2m_e) = m_e/2$, so the ground state energy must be about half that of hydrogen. III is false: going to the rest frame of positronium, a massive bound state cannot decay into

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a single massless photon without violating conservation of relativistic energy–momentum.

58. C – Energy is conserved in an elastic collision, and since the wall of the pool table doesn't move, the ball must have the same energy before and after the collision. Thus $\frac{1}{2}m|\mathbf{v}|^2 = \frac{1}{2}m|\mathbf{v}'|^2$, and $|\mathbf{v}| = |\mathbf{v}'|$, so I is true. II is only true if $\mathbf{v}$ is normal to the wall. III is true by the definition of momentum.
59. A – The only transition that is allowed is A. The others are forbidden by the selection rules $\Delta l = \pm 1$ and $\Delta m = 0, \pm 1$.
60. C – If the power of radiation emitted by the satellite is P, then the blackbody temperature is T such that $P = \sigma T^4$. If the radius of the orbit doubles, then the amount of radiation received from the Sun decreases by a factor of 4. In equilibrium, power emitted also decreases by a factor of 4. This means that the blackbody temperature must change by a factor of $4^{-1/4} = 2^{-1/2}$ in order to satisfy $P = \sigma T^4$.
61. E – The particles of gas A and gas B have the same total mass. By the equipartition theorem, the rms velocity *only* depends on the motion of the center of mass of the molecule, and thus should be the same for gases A and B. Gas B has rotational degrees of freedom at low temperatures and vibrational degrees of freedom at higher temperatures, but these only affect the specific heat and not the rms speed.
62. A – Since angular momentum is always conserved, we can use it to solve this problem quickly. The moment of inertia of the disk in the problem about the axis of rotation is

$$I_{\text{disk}} = \frac{1}{2}Mr^2.$$

The moment of inertia of the person at the edge of the disk is

$$I_{\text{person}} = mr^2.$$

The conservation equation is

$$I_{\text{disk}}\omega = (I_{\text{disk}} + I_{\text{person}})\omega',$$

so the change in angular velocity is

$$\Delta\omega = |\omega' - \omega| = \left|\frac{I_{\text{disk}}}{I_{\text{disk}} + I_{\text{person}}} - 1\right|\omega = \frac{2\omega m}{M + 2m}.$$

63. E – The equation for the energy levels of a hydrogen-like atom with Z protons is

$$E_n = \frac{Z^2(13.6 \text{ eV})}{n^2}.$$

Calculating the change in energy between the two states with $Z = 2$ for helium, we find that the emitted photon must have energy

$$\Delta E = 4(13.6 \text{ eV})\left(1 - \frac{1}{4}\right) = 40.8 \text{ eV}.$$

64. C – The given information suggests that the speed of a wave on a vibrating string only depends of μ and T, so by dimensional analysis, the quantity with the correct units is

$$c = \sqrt{\frac{T}{\mu}}.$$

In fact, this is the correct answer even up to dimensionless factors, but because the answer choices are of the "numbers and estimation" type, any such factors are irrelevant for getting the correct answer. Plugging in the given numbers,

$$c = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{4 \times 10^3 \text{ N}}{0.1 \text{ kg/m}}} = 200 \text{ m/s},$$

choice C.
65. A – By symmetry, the force on the test charge at the center is zero, so I is clearly true. II is false because the potential from each charge at the corners adds and is nonzero. III is false for a subtle, but important reason. Earnshaw's theorem states that any configuration of electrostatic charges cannot be in stable equilibrium. The charge at the center is in an unstable equilibrium: the force vanishes, but if the charge is perturbed in certain directions, it will move away from the center instead of returning to its original position.
66. E – The cyclotron radius is given by $R = \gamma mv/(qB)$, where B is the magnetic field strength and γ is the Lorentz factor. It's easy to remember this formula because it's identical to the nonrelativistic version, except with the momentum $p = mv$ replaced by the relativistic momentum γmv. For $v = 0.6c$, we have $\gamma = 5/4$ and $\gamma v = 3c/4$. For $v = 0.8c$, $\gamma = 5/3$ and $\gamma v = 4c/3$. All other factors cancel out in the ratio, so the new cyclotron radius is $\frac{4c/3}{3c/4}R = 16R/9$.
67. B – Let the initial intensity be I_0. The intensity of unpolarized light is attenuated through a single linear polarizer by a factor of 2. The emitted light leaving the first polarizer is now linearly polarized, with intensity $I' = I_0/2$. Malus's law gives the intensity I'' going through the second filter as

$$I'' = I' \cos^2\theta = \frac{I_0}{2}\cos^2\theta.$$

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For $\theta = 30°$, we have

$$I'' = \frac{I_0}{2}\left(\frac{3}{4}\right) = \frac{3}{8}I_0.$$

The total attenuation is therefore 3/8.

68. B – Recalling the fundamental commutation relation $[\hat{x}, \hat{p}] = i\hbar$, we see that

$$\mathcal{O} = |i\hbar|^2 = \hbar^2,$$

so $\mathcal{O}$ is just a constant operator. The expectation value of a constant in any state is just the value of that constant, so $\langle \mathcal{O} \rangle = \hbar^2$.

69. D – Recall that the inductance L is defined through

$$\Phi_B = LI.$$

We obtain the magnetic field through the solenoid by Ampère's law:

$$B\ell = \mu_0 NI.$$

The magnetic flux through the solenoid interior is

$$\Phi_B = \frac{\pi R^2 \mu_0 NI}{\ell},$$

so the inductance of *one* wind of the solenoid is

$$L = \frac{\pi R^2 \mu_0 N}{\ell}.$$

Since inductance adds in series, the inductance of N winds is

$$L = \frac{\pi R^2 \mu_0 N^2}{\ell}.$$

70. B – This is a simple application of the thin lens equation. The first lens satisfies

$$\frac{2}{f} = \frac{1}{2f} + \frac{1}{q},$$

where q is some undetermined position of the image from the first lens. The second lens satisfies

$$\frac{2}{f} = \frac{1}{d-q} + \frac{1}{f}.$$

Solving for d, we obtain

$$d = \frac{5}{3}f.$$

Note that we implicitly assumed that $d > q$, otherwise the sign conventions would be different; since the first equation gives $q = 2f/3$, this assumption is self-consistent.

71. B – We can replace the masses connected by the rod with an effective mass $2m$, coupled to a spring of effective spring constant $2k$ (one spring pulls while the other pushes, so the forces add, and are equivalent to a single spring with constant $2k$). This has an identical equation of motion to a single mass attached to a single spring, with the usual frequency $\omega = \sqrt{2k/2m} = \sqrt{k/m}$.

72. C – The natural line width is given by the energy–time uncertainty relation:

$$\Delta E \sim \frac{\hbar}{\Delta t}.$$

Plugging in numbers, we arrive at $\Delta E = 2.2 \times 10^{-6}$ eV, which is closest to C.

73. A – By definition, a general gauge transformation corresponds to changes of the scalar and vector potentials that leave the physical electric and magnetic fields unchanged. This information alone is sufficient to answer the question. In case you were wondering, however, a general gauge transformation corresponds to a change of the scalar potential by

$$V \to V - \frac{\partial f}{\partial t},$$

and a change of the vector potential by

$$\mathbf{A} \to \mathbf{A} + \nabla f,$$

where $f(\mathbf{r}, t)$ is some real-valued function.

74. C – The energies of the three-dimensional infinite square well are proportional to $n_x^2 + n_y^2 + n_z^2$, where n_x, n_y, and n_z are *positive* integers (if $n = 0$, the wavefunction is identically zero). The ground state has all of the n's equal to 1. The first excited state occurs when one of the n's is 2, and the others are 1. The second excited state has two of the n's equal to 2 and the third equal to 1. (Note that since $2^2 + 2^2 + 1^2 < 3^2 + 1^2 + 1^2$, this state has less energy than the state where one of the n's is 3.) There are three such combinations: (1, 2, 2), (2, 1, 2), and (2, 2, 1).

75. A – Suppose that there is a charge $+Q$ on the surface of the inner sphere, and a charge of $-Q$ on the surface of the outer sphere. The potential in the region between the two spheres is just the potential of a point charge:

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}.$$

The potential difference between the two spheres is therefore

$$\Delta V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right).$$

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Because $Q = CV$, we can read off the capacitance as

$$C = \frac{4\pi\epsilon_0 ab}{b-a}.$$

76. E – This can be solved just by dimension counting. The Hilbert space of a spin-1/2 particle has dimension 2, so the total Hilbert space of three such distinguishable particles has dimension $2^3 = 8$. This is true even if we rearrange the states into linear combinations with definite total spin, which for this problem happens to be completely unnecessary. Note that the adjective "distinguishable" is *crucial* here: without it, we would have to worry about symmetry or antisymmetry of the spatial and spin wavefunctions to satisfy Fermi–Dirac statistics.
77. A – This is a simple application of the lensmaker's equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

Solving for R_2, we obtain

$$R_2 = \left(\frac{1}{R_1} - \frac{1}{f(n-1)}\right)^{-1}.$$

Plugging in numbers, we find that $R_2 = 12.5$ cm. Incidentally, since R_2 is positive, the corresponding surface is concave, and we have an example of a convex–concave lens which happens to have positive curvature (as opposed to the more common case of a convex–convex converging lens, which always has positive curvature).
78. B – Let's define coordinates by setting the time at which the doors open and close in the garage's frame to be $t = 0$, and letting x be the distance from the front door. In the frame of the car, the first equation of the Lorentz transformation gives the time at which the doors open and close:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right).$$

The front door therefore closes at time $t'_1 = 0$. The rear door opens at time $t'_2 = \gamma(0 - vl/(c^2)) = -\gamma vl/c^2$. Notice, of course, that even though the doors move at the same instant in the frame of the garage, they do not move at the same time in the frame of the moving car. This is a prototypical example of the relativity of simultaneity.
79. E – This problem requires remembering the basic scaling law for the entropy of an ideal gas,

$$S = Nk \ln \frac{VT^{3/2}}{N} + \text{constants},$$

in light of the equation of state for an ideal gas $PV = NkT$. If the temperature doubles but pressure remains constant, then volume must also double. So $\Delta S_1 = (5/2)Nk\ln 2$. If temperature doubles but pressure also doubles, then volume must be constant. The entropy only changes by $\Delta S_2 = (3/2)Nk\ln 2$. Finally, if the temperature is constant, but the volume doubles, then the entropy only changes by $\Delta S_3 = Nk\ln 2$. The correct order is choice E.
80. D – The equation of motion for a forced oscillator is

$$m\ddot{x} = -kx + F_0\sin(\omega t).$$

This inhomogeneous ODE may look a little daunting at first, but it is clear that it is solved by a simple solution of the form

$$x(t) = A\sin(\omega t),$$

which allows us to cancel the sine in the driving force. This is the so-called "particular solution" to the ODE; there will be a general solution piece as well, but the assumptions in the problem allow us to ignore this "transient" term. Plugging in this *ansatz*, we obtain

$$-Am\omega^2 = -Ak + F_0,$$

and therefore

$$|A| = \frac{F_0}{|k - m\omega^2|}.$$

Alternatively, just use limiting cases and physical intuition: if the driving force is applied at the resonant frequency $\sqrt{k/m}$ of the system, the amplitude should blow up to infinity. Only choice D satisfies this condition.
81. C – There is no motion along the y- or z-directions, so these coordinates remain unchanged: $y' = y$ and $z' = z$. The x'-coordinate, on the other hand, can be found directly from the Lorentz transformation equations:

$$x' = \gamma\,(x - vt) = \frac{1}{\sqrt{1-0.8^2}}(5\text{ m}) = 8.33\text{ m}.$$

Choice E may be tempting since it is reminiscent of Lorentz contraction, but remember that lengths must be measured using simultaneous events in S', while the event in the problem does *not* occur at $t' = 0$.
82. E – This problem involves conservation of energy, which is pure potential at the top of the ramp but is a combination of translational and rotational energy at the bottom. To obtain the rotational energy, recall (from the formula sheet given at the beginning of the test) that the moment of inertia of a cylinder rotating about its axis is

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$$I = \frac{1}{2}MR^2.$$

The potential energy at the top of the ramp must be equal to the sum of the rotational and translational energy at the bottom of the ramp. Using the rolling-without-slipping condition $v = R\omega$, we have

$$\begin{aligned} Mgh &= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}Mv^2 + \frac{1}{4}MR^2\left(\frac{v}{R}\right)^2, \\ gh &= \frac{3}{4}v^2, \\ v &= 2\sqrt{\frac{gh}{3}}. \end{aligned}$$

83. C – The electric field is confined to the xy-plane, so since the particle starts out in the xy-plane, the electric force on the particle will certainly keep it in that plane. The magnetic force will be perpendicular to both **B** and the particle's path, but **B** is *already* perpendicular to the xy-plane, so the magnetic force will act along the vector perpendicular to the path, which also lies in the xy-plane. So C is correct. A would be true if there was only a magnetic field, D is false because the electric field does work, and E is false because of the magnetic contribution.
84. B – Without knowing too much about the details of dark matter, this question can be answered by recognizing that dark matter, like any other matter, is gravitationally *attractive*. Choice B is related to the expansion of the universe, which requires an energy density that is effectively *repulsive*: this is dark energy. Choices A, C, D, and E are all seen as compelling evidence in favor of the existence of dark matter.
85. B – Both the counting rates follow Poisson statistics, where the uncertainty on N counts is $\sqrt{N}$. The first count represents the number of signal + background events, while the second count with the source removed represents the number of background-only events. The uncertainty in the number of signal + background events is $\sqrt{1,250}$ events, and the uncertainty in the number of background events is $\sqrt{350}$. The number of signal events is obtained by subtracting the background, and the uncertainty on this count is given by

$$\Delta N = \sqrt{1,250 + 350} = \sqrt{1,600} = 40.$$

The rate uncertainty is therefore 4.0 Hz. Note that the errors *add* in quadrature, even though we are *subtracting* a background, because the two experiments are independent: forgetting this would lead to trap answer A.

86. C – While a precise answer to this question can be calculated, it is too laborious to do so during the exam. Just note that, since the fastest part of the pulse has a time constant of about 10 ms, it may produce some signal around $(0.01\text{ s})^{-1} = 100$ Hz. The slow part of the pulse may produce some signal around 10 Hz. We therefore need a bandwidth of at least 100 Hz, which is closest to 1 kHz, choice C.
87. D – In order to compute a p-value from a χ^2 statistic, we need to know the number of degrees of freedom. The χ^2 distribution looks different for each number of degrees of freedom. I is not relevant for a χ^2 test. II and III are ingredients that together determine the number of degrees of freedom. Neither alone is sufficient, however.
88. E – The *pp* cycle of the Sun consists of nine different nuclear reactions. It is rather fascinating and elegant, but chances are low that you would need to remember the details for an exam. Simple logic can lead to the right answer here, though. As the name suggests, the *pp* cycle starts with fusion of protons into heavier elements. The most natural guess for the nucleus not produced by the cycle would be the nucleus of highest atomic number, ^{11}C. This turns out to be the correct answer; the nucleus with the largest number of protons produced in the *pp* cycle of the Sun is ^{8}B.
89. D – The partition function of this two-state system is

$$Z = 1 + e^{-\beta\epsilon},$$

where $\beta = 1/kT$. It would be perfectly valid to compute the mean energy using $\langle E\rangle = -\partial \ln Z/\partial\beta$, but this problem is simple enough that, for GRE purposes, direct computation is faster:

$$\begin{aligned} \langle E\rangle &= \frac{1}{Z}\sum_i \epsilon_i e^{-\epsilon_i/kT} \\ &= \frac{\epsilon e^{-\epsilon/kT}}{1 + e^{-\epsilon/kT}} \\ &= \frac{\epsilon}{1 + e^{\epsilon/kT}}, \end{aligned}$$

where the last step follows from multiplying numerator and denominator by $e^{\epsilon/kT}$.

90. C – The Hubble parameter relates the velocity of receding objects in space to their distance from us through Hubble's law, $v = H_0 d$. So if H_0 were suddenly doubled, our distance to the objects would be unchanged and the velocity would have to double. Since redshift is determined by the velocity at which distant objects travel with

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respect to us, the redshift of these objects would have to change.

91. D – If the radiation used for interferometry has wavelength λ, the phase difference caused by a displacement Δx is $\Delta\phi = 2k\Delta x = 4\pi\,\Delta x/\lambda$ (the factor of 2 is from traversing the path twice, once from the satellite to the ground and again on the return, but since this is an order of magnitude problem, it won't really matter). This suggests that we want λ the same order of magnitude as Δx, but slightly larger so that the phase shift does not exceed 2π. If $\lambda = 5$ cm, the frequency is $\omega = c/\lambda = 6$ GHz, which is closest to choice D.
92. D – Doping a semiconductor adds additional unbound electrons or holes into the semiconductor by acting as donors or acceptors. This clearly influences the band structure somehow, so A is incorrect. B and E are incorrect because adding a small amount of dopant should not turn the semiconductor into a perfect insulator (B) or a perfect conductor with no band gap (E). C seems suspect because adding more free charge carriers should, if anything, decrease the energy needed for electrons to excite into the valence band.
93. B – We can get the radial electric field outside the cylinder from Gauss's law, using a Gaussian cylinder of radius r and length L. This just gives $E(2\pi rL) = \sigma(2\pi aL)/\epsilon_0$, or

$$E = \frac{a\sigma}{\epsilon_0 r}.$$

Integrating from a to some radius r, we obtain a potential

$$V(r) = -\int_a^r E(r')\,dr' = -\frac{a\sigma}{\epsilon_0}(\ln r - \ln a) = \frac{a\sigma}{\epsilon_0}\ln\frac{a}{r}.$$

Be careful with signs! Forgetting the minus sign in the potential is easy and leads to trap answer A.

94. C – This problem has two short steps: we need to find the speed of the car at the top of the loop in terms of the speed at the bottom, and then find what speed is needed in order for gravity to provide the required centripetal force. If the speed at the bottom is v_b, then the speed v_t at the top of the loop is given by

$$\frac{1}{2}mv_b^2 = 2mgR + \frac{1}{2}mv_t^2.$$

Canceling the mass and simplifying, we have

$$v_t^2 = v_b^2 - 4gR.$$

Setting the gravitational force equal to the centripetal force at the top of the loop, we have

$$\begin{aligned} mg &= \frac{mv_t^2}{R}, \\ gR &= v_b^2 - 4gR, \\ v_b &= \sqrt{5gR} \\ &\simeq \sqrt{25\ \mathrm{m^2/s^2}} \\ &\simeq 5.0\ \mathrm{m/s}. \end{aligned}$$

If v_b is smaller than this critical value, then gravity provides an extra radial force which causes the car to accelerate radially; in other words, to fall off the track. If v_b is larger than the critical value, the extra centripetal force must be provided by the normal force of the track.

95. E – Expanding out the commutator and factoring, we have

$$\begin{aligned} \left[S_xS_y, S_y\right] &= (S_xS_y)S_y - S_y(S_xS_y) \\ &= (S_xS_y)S_y - (S_yS_x)S_y = [S_x, S_y]S_y. \end{aligned}$$

(You could also have used the formula for commutators of products, $[AB, C] = A[B, C]+[A, C]B$.) Now applying the commutation algebra for spin angular momentum, we have $[S_x, S_y] = i\hbar S_z$, so the whole commutator is $i\hbar S_zS_y$.

96. B – Since we are not given the form of the potential, choice E may be tempting. However, Bertrand's theorem states that the only central potentials that produce closed noncircular orbits are the Kepler potential and the harmonic oscillator potential. Any central potential will have circular orbits at $E = V_{\min}$, but the problem tells us that $E > V_{\min}$ so the orbit cannot be circular. We already know that bound orbits of the Kepler potential with $E > V_{\min}$ are elliptical. For the harmonic oscillator, since $r^2 = x^2 + y^2 + z^2$, the coordinates decouple and the motion is a superposition of three harmonic oscillators in the three coordinate directions. The motion must lie in a plane, per the usual arguments for a central potential, so we can set the amplitude of the z oscillation to zero and just consider oscillators in the x- and y-directions. These are sinusoidal, but if they have different amplitudes and/or phases, they will produce ellipses. So the nonminimum energy orbits of a harmonic oscillator are also ellipses, and choice B is correct.
97. D – The energy stored in a capacitor of capacitance C at constant voltage is $E = \frac{1}{2}CV_0^2$. The capacitance with the dielectric inserted is $C = \kappa\epsilon_0 a^2/d$. The "bare"

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capacitance without the dielectric is $C = \epsilon_0 a^2/d$. The difference in energy is therefore

$$\Delta E = \frac{(\kappa - 1)\epsilon_0 a^2 V_0^2}{2d}.$$

98. A – To obtain the force from the potential energy, we just take the negative of the gradient:

$$\mathbf{F}(x, y, z) = -\nabla U = -\hat{\mathbf{x}}\frac{\partial}{\partial x}(Ax) - \hat{\mathbf{y}}\frac{\partial}{\partial y}\left(By^2\right)$$
$$+\,\hat{\mathbf{z}}\frac{\partial}{\partial z}(C\cos z) = -A\hat{\mathbf{x}} - 2By\hat{\mathbf{y}} - C\sin z\hat{\mathbf{z}}.$$

99. E – The ground state wavefunction of hydrogen is spherically symmetric, so we will denote it by $\psi(r)$ (as we will see, we don't actually need the functional form, symmetry is enough to solve this problem as stated). Defining the z-direction along the direction of the electric field, the perturbation Hamiltonian is $\Delta H = eE_0 z = eE_0 r\cos\theta$, and the first-order correction is given by

$$\int_0^\infty \int_0^{2\pi} \int_0^\pi \psi^*(r)\psi(r)(eE_0 r\cos\theta)r^2 \sin\theta\, d\theta\, d\phi\, dr = 0.$$

However, the θ integral vanishes:

$$\int_0^\pi \cos\theta\sin\theta\, d\theta = \frac{1}{2}\int_0^\pi \sin 2\theta\, d\theta$$
$$= -\frac{1}{4}(\cos 2\pi - \cos 0) = 0.$$

Thus the first-order correction vanishes.

100. D – We want the nonrelativistic limit of the relativistic Doppler shift, so we Taylor expand for $\beta \ll 1$:

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{1+\beta}{1-\beta}} = (1+\beta)^{1/2}(1-\beta)^{-1/2}$$
$$\approx (1+\beta/2)(1+\beta/2) \approx 1+\beta.$$

Thus the Doppler-shifted energy is

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda(1+v/c)} \approx \frac{hc}{\lambda}(1-v/c) = E(1-v/c),$$

so the energy shift is Ev/c. In fact, this is what we might have expected from using the nonrelativistic Doppler shift expression with wave speed c. Note that the sign of β or v doesn't matter here since all we care about is the magnitude of the energy difference.

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