



EQUATION INDEX

Classical Mechanics

$$(1.1) \quad x(t) = v_{0x}t + x_0, \quad y(t) = -\frac{1}{2}gt^2 + v_{0y}t + y_0 \quad (\text{p. 5})$$

$$(1.2) \quad v_f^2 - v_i^2 = 2a\Delta x \quad (\text{p. 5})$$

$$(1.3) \quad a = \frac{v^2}{r} \quad (\text{p. 5})$$

$$(1.4) \quad F = \frac{mv^2}{r} \quad (\text{p. 5})$$

$$(1.5) \quad \text{Translational kinetic energy:} \quad \frac{1}{2}mv^2 \quad (\text{p. 7})$$

$$(1.6) \quad \text{Rotational kinetic energy:} \quad \frac{1}{2}I\omega^2 \quad (\text{p. 7})$$

$$(1.7) \quad \text{Gravitational potential energy on Earth:} \quad mgh \quad (\text{p. 7})$$

$$(1.8) \quad \text{Spring potential energy:} \quad \frac{1}{2}kx^2 \quad (\text{p. 7})$$

$$(1.9) \quad \Delta U = -\int_a^b \mathbf{F} \cdot d\mathbf{l} \quad (\text{p. 8})$$

$$(1.10) \quad \mathbf{F}_{\text{grav}} = \frac{Gm_1m_2}{r^2} \hat{\mathbf{r}} \quad (\text{p. 8})$$

$$(1.11) \quad \mathbf{F} = -\nabla U \quad (\text{p. 8})$$

$$(1.12) \quad v = R\omega \quad (\text{p. 9})$$

$$(1.13) \quad E_{\text{initial}} + W_{\text{other}} = E_{\text{final}} \quad (\text{p. 11})$$

$$(1.14) \quad W = \Delta \text{KE} \quad (\text{p. 11})$$

$$(1.15) \quad W = \int \mathbf{F} \cdot d\mathbf{l} \quad (\text{p. 11})$$

$$(1.16) \quad \mathbf{L} = \mathbf{r} \times \mathbf{p} \quad (\text{p. 13})$$

$$(1.17) \quad \mathbf{L} = I\boldsymbol{\omega} \quad (\text{p. 13})$$

$$(1.18) \quad \boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} \quad (\text{p. 13})$$

$$(1.19) \quad L = I\omega \quad (\text{p. 13})$$

$$(1.20) \quad \tau = \frac{dL}{dt} \quad (\text{p. 13})$$

$$(1.21) \quad F_{\text{centrifugal}} = -m\Omega^2 r \quad (\text{p. 14})$$

$$(1.22) \quad F_{\text{Coriolis}} = -2m\boldsymbol{\Omega} \times \mathbf{v} \quad (\text{p. 14})$$

$$(1.23) \quad I = mr^2 \quad (\text{p. 14})$$

$$(1.24) \quad I = \int r^2 dm \quad (\text{p. 14})$$

$$(1.25) \quad I = I_{\text{CM}} + Mr^2 \quad (\text{p. 14})$$

$$(1.26) \quad \mathbf{r}_{\text{CM}} = \frac{\int \mathbf{r} dm}{M} \quad (\text{p. 15})$$

$$(1.27) \quad \mathbf{r}_{\text{CM}} = \frac{\sum_i \mathbf{r}_i m_i}{M} \quad (\text{p. 15})$$

$$(1.28) \quad L(q, \dot{q}, t) = T - U \quad (\text{p. 16})$$

$$(1.29) \quad \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q} \quad (\text{p. 17})$$

$$(1.30) \quad p_i \equiv \frac{\partial L}{\partial \dot{q}_i} : \text{momentum conjugate to } q \quad (\text{p. 18})$$

$$(1.31) \quad H(p, q) = \sum_i p_i \dot{q}_i - L \quad (\text{p. 18})$$

$$(1.32) \quad H = T + U \quad (\text{if } U \text{ does not depend explicitly on velocities or time}) \quad (\text{p. 18})$$

$$(1.33) \quad \dot{p} = -\frac{\partial H}{\partial q}, \quad \dot{q} = \frac{\partial H}{\partial p} \quad (\text{p. 18})$$

$$(1.34) \quad L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\phi}^2 - U(r) \quad (\text{p. 20})$$

$$(1.35) \quad l = mr^2\dot{\phi} \quad (\text{p. 20})$$

$$(1.36) \quad V(r) = \frac{l^2}{2mr^2} + U(r) \quad (\text{p. 20})$$

$$(1.37) \quad \mu = \frac{m_1m_2}{m_1 + m_2} \quad (\text{p. 20})$$

$$(1.38) \quad E = T + V = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} + U(r) \quad (\text{p. 20})$$

$$(1.39) \quad F = m\ddot{x} = -kx \quad (\text{p. 22})$$

$$(1.40) \quad \omega = \sqrt{\frac{k}{m}} \quad (\text{p. 22})$$

$$(1.41) \quad x(t) = Ae^{i\omega t} \quad (\text{p. 22})$$

$$(1.42) \quad q_k(t) = a_k e^{i\omega t} \quad (\text{p. 23})$$

$$(1.43) \quad \det(A_{jk} - \omega^2 m_{jk}) = 0 \quad (\text{p. 23})$$

$$(1.44) \quad m\ddot{x} + b\dot{x} + kx = 0 \quad (\text{p. 25})$$

$$(1.45) \quad \omega_R = \sqrt{\omega_0^2 - 2\beta^2} \quad (\text{p. 25})$$

$$(1.46) \quad m\ddot{x} = -mgx/L \quad (\text{p. 25})$$

$$(1.47) \quad \omega = \sqrt{\frac{g}{L}} \quad (\text{p. 25})$$

$$(1.48) \quad \omega = \sqrt{\frac{mgR}{I}} \quad (\text{p. 26})$$

$$(1.49) \quad \frac{v^2}{2} + gz + \frac{p}{\rho} = \text{constant} \quad (\text{p. 27})$$

$$(1.50) \quad v_1 A_1 = v_2 A_2 \quad (\text{p. 28})$$

$$(1.51) \quad \frac{v_1^2}{2} + gz_1 + \frac{p_1}{\rho} = \frac{v_2^2}{2} + gz_2 + \frac{p_2}{\rho} \quad (\text{p. 28})$$

Electricity and Magnetism

$$(2.1) \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (\text{p. 35})$$

$$(2.2) \quad \nabla \times \mathbf{E} = 0 \quad (\text{electrostatics}) \quad (\text{p. 35})$$

$$(2.3) \quad \mathbf{F}_E = q\mathbf{E} \quad (\text{p. 35})$$

$$(2.4) \quad \mathbf{E} = -\nabla V \quad (\text{p. 35})$$

$$(2.5) \quad V(b) = -\int_a^b \mathbf{E} \cdot d\mathbf{l} \quad (\text{p. 36})$$

$$(2.6) \quad \nabla^2 V = -\frac{\rho}{\epsilon_0} \quad (\text{p. 36})$$

$$(2.7) \quad \nabla^2 V = 0 \quad (\text{empty space}) \quad (\text{p. 36})$$

$$(2.8) \quad V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' \quad (\text{p. 36})$$

$$(2.9) \quad \oint_S \mathbf{E}(\mathbf{r}) \cdot d\mathbf{S} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad (\text{p. 36})$$

$$(2.10) \quad \oint_C \mathbf{E}(\mathbf{r}) \cdot d\mathbf{l} = 0 \quad (\text{electrostatics}) \quad (\text{p. 36})$$

$$(2.11) \quad \mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}} \quad (\text{p. 37})$$

$$(2.12) \quad V(\mathbf{r}) = \frac{q}{4\pi\epsilon_0 r} \quad (\text{p. 37})$$

$$(2.13) \quad \mathbf{E} = \frac{\sigma}{2\epsilon_0} \hat{\mathbf{n}} \quad (\text{p. 38})$$

$$(2.14) \quad \mathbf{E}_{\text{out}}^{\parallel} - \mathbf{E}_{\text{in}}^{\parallel} = 0 \quad (\text{p. 40})$$

$$(2.15) \quad E_{\text{out}}^{\perp} - E_{\text{in}}^{\perp} = \frac{\sigma}{\epsilon_0} \quad (\text{p. 40})$$

$$(2.16) \quad W = \frac{1}{2} \sum_{i=1}^n q_i V(\mathbf{r}_i) \quad (\text{p. 42})$$

$$(2.17) \quad W = \frac{1}{2} \int \rho(\mathbf{r}) V(\mathbf{r}) d^3\mathbf{r} \quad (\text{p. 43})$$

$$(2.18) \quad U_E = \frac{\epsilon_0}{2} \int |\mathbf{E}|^2 d^3\mathbf{r} \quad (\text{p. 43})$$

$$(2.19) \quad Q = CV \quad (\text{p. 43})$$

$$(2.20) \quad C = \frac{\epsilon_0 A}{d} \quad (\text{parallel-plate capacitor}) \quad (\text{p. 44})$$

$$(2.21) \quad U_C = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 \quad (\text{p. 44})$$

$$(2.22) \quad \nabla \cdot \mathbf{B} = 0 \quad (\text{p. 45})$$

$$(2.23) \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad (\text{magnetostatics}) \quad (\text{p. 45})$$

$$(2.24) \quad \oint_S \mathbf{B} \cdot d\mathbf{S} = 0 \quad (\text{p. 45})$$

$$(2.25) \quad \oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \quad (\text{magnetostatics}) \quad (\text{p. 45})$$

$$(2.26) \quad \nabla \times \mathbf{A} = \mathbf{B} \quad (\text{p. 45})$$

$$(2.27) \quad \mathbf{F}_B = q\mathbf{v} \times \mathbf{B} \quad (\text{p. 46})$$

$$(2.28) \quad d\mathbf{F}_B = I d\mathbf{l} \times \mathbf{B} \quad (\text{p. 46})$$

$$(2.29) \quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l} \times \hat{\mathbf{r}}}{r^2} \quad (\text{p. 46})$$

$$(2.30) \quad |\mathbf{B}|(2\pi r) = \mu_0 I \implies \mathbf{B} = \frac{\mu_0 I}{2\pi r} \hat{\boldsymbol{\phi}} \quad (\text{p. 47})$$

$$(2.31) \quad B = \mu_0 nI \quad (\text{solenoid}) \quad (\text{p. 47})$$

$$(2.32) \quad B = \frac{\mu_0 NI}{2\pi r} \quad (\text{toroid}) \quad (\text{p. 47})$$

$$(2.33) \quad B_{\text{out}}^{\perp} - B_{\text{in}}^{\perp} = 0 \quad (\text{p. 48})$$

$$(2.34) \quad \mathbf{B}_{\text{out}}^{\parallel} - \mathbf{B}_{\text{in}}^{\parallel} = \mu_0 \mathbf{K} \times \hat{\mathbf{n}} \quad (\text{p. 48})$$

$$(2.35) \quad U_B = \frac{1}{2\mu_0} \int |\mathbf{B}|^2 d^3\mathbf{r} \quad (\text{p. 48})$$

$$(2.36) \quad R = \frac{mv}{qB} \quad (\text{p. 48})$$

$$(2.37) \quad \omega = \frac{qB}{m} \quad (\text{p. 49})$$

$$(2.38) \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (\text{p. 49})$$

$$(2.39) \quad \nabla \cdot \mathbf{B} = 0 \quad (\text{p. 49})$$

$$(2.40) \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (\text{p. 49})$$

$$(2.41) \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (\text{p. 49})$$

$$(2.42) \quad \mathcal{E} = -\frac{d\Phi_B}{dt} \quad (\text{p. 50})$$

$$(2.43) \quad \Phi_{21} = M_{12}I_1 \quad (\text{p. 50})$$

$$(2.44) \quad \Phi_B = LI \quad (\text{p. 50})$$

$$(2.45) \quad \mathcal{E} = -L\frac{dI}{dt} \quad (\text{p. 51})$$

$$(2.46) \quad L = \frac{\mu_0 N^2 A}{l} \quad (\text{solenoid}) \quad (\text{p. 51})$$

$$(2.47) \quad U_L = \frac{1}{2}LI^2 \quad (\text{p. 51})$$

$$(2.48) \quad \mathbf{p} = q\mathbf{r}_1 - q\mathbf{r}_2 = q\mathbf{d} \quad (\text{p. 52})$$

$$(2.49) \quad \mathbf{p} = \int \mathbf{r}\rho(\mathbf{r})d^3\mathbf{r} \quad (\text{p. 52})$$

$$(2.50) \quad V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2} \quad (\text{p. 52})$$

$$(2.51) \quad \mathbf{N} = \mathbf{p} \times \mathbf{E} \quad (\text{p. 52})$$

$$(2.52) \quad U = -\mathbf{p} \cdot \mathbf{E} \quad (\text{p. 52})$$

$$(2.53) \quad \mathbf{m} = I\mathbf{A} \quad (\text{p. 52})$$

$$(2.54) \quad \mathbf{N} = \mathbf{m} \times \mathbf{B} \quad (\text{p. 53})$$

$$(2.55) \quad U = -\mathbf{m} \cdot \mathbf{B} \quad (\text{p. 53})$$

$$(2.56) \quad \sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}} \quad (\text{p. 54})$$

$$(2.57) \quad \rho_b = -\nabla \cdot \mathbf{P} \quad (\text{p. 54})$$

$$(2.58) \quad \epsilon_0 \mapsto \epsilon = \kappa\epsilon_0 \quad (\text{p. 54})$$

$$(2.59) \quad C = \frac{\epsilon A}{d} = \kappa \frac{\epsilon_0 A}{d} \quad (\text{p. 54})$$

$$(2.60) \quad c = 1/\sqrt{\epsilon_0\mu_0} \quad (\text{p. 55})$$

$$(2.61) \quad \tilde{\mathbf{E}}(\mathbf{r}) = \tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} \hat{\mathbf{n}} \quad (\text{p. 55})$$

$$(2.62) \quad \tilde{\mathbf{B}}(\mathbf{r}) = \frac{1}{c} \tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} (\hat{\mathbf{k}} \times \hat{\mathbf{n}}) \quad (\text{p. 55})$$

$$(2.63) \quad \mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B}) \quad (\text{p. 55})$$

$$(2.64) \quad \mathbf{S} = \frac{1}{2\mu_0} \text{Re}(\tilde{\mathbf{E}} \times \tilde{\mathbf{B}}^*) \quad (\text{p. 55})$$

$$(2.65) \quad I = \langle S \rangle = \frac{1}{2} c\epsilon_0 E_0^2 \quad (\text{p. 55})$$

$$(2.66) \quad P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3} = \frac{\mu_0 q^2 a^2}{6\pi c} \quad (\text{p. 56})$$

$$(2.67) \quad \langle S \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \right) \frac{\sin^2 \theta}{r^2} \quad (\text{p. 56})$$

$$(2.68) \quad \langle P \rangle_E = \frac{\mu_0 p_0^2 \omega^4}{12\pi c} \quad (\text{p. 56})$$

$$(2.69) \quad \langle P \rangle_B = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3} \quad (\text{p. 56})$$

$$(2.70) \quad V_R = IR \quad (\text{p. 57})$$

$$(2.71) \quad V_C = \frac{Q}{C} \quad (\text{p. 57})$$

$$(2.72) \quad V_L = L\frac{dI}{dt} \quad (\text{p. 57})$$

$$(2.73) \quad R_{\text{eq}} = \sum_i R_i \quad (\text{series}) \quad (\text{p. 57})$$

$$(2.74) \quad \frac{1}{C_{\text{eq}}} = \sum_i \frac{1}{C_i} \quad (\text{series}) \quad (\text{p. 57})$$

$$(2.75) \quad L_{\text{eq}} = \sum_i L_i \quad (\text{series}) \quad (\text{p. 57})$$

$$(2.76) \quad \frac{1}{R_{\text{eq}}} = \sum_i \frac{1}{R_i} \quad (\text{parallel}) \quad (\text{p. 57})$$

$$(2.77) \quad C_{\text{eq}} = \sum_i C_i \quad (\text{parallel}) \quad (\text{p. 57})$$

$$(2.78) \quad \frac{1}{L_{\text{eq}}} = \sum_i \frac{1}{L_i} \quad (\text{parallel}) \quad (\text{p. 57})$$

$$(2.79) \quad R = \frac{\rho \ell}{A} \quad (\text{p. 57})$$

$$(2.80) \quad \sum_k I_k = 0 \quad (\text{p. 57})$$

$$(2.81) \quad \sum_k V_k = 0 \quad (\text{p. 57})$$

$$(2.82) \quad P = IV = \frac{V^2}{R} = I^2 R \quad (\text{p. 57})$$

$$(2.83) \quad \tau_{RL} = L/R \quad (\text{p. 58})$$

$$(2.84) \quad \tau_{RC} = RC \quad (\text{p. 58})$$

$$(2.85) \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad (\text{p. 58})$$

Optics and Waves

$$(3.1) \quad \frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2} \quad (\text{p. 63})$$

$$(3.2) \quad f(x, t) = A \cos(kx - \omega t + \delta) \quad (\text{p. 63})$$

$$(3.3) \quad \lambda = \frac{2\pi}{k}, \quad T = \frac{2\pi}{\omega}, \quad \omega = 2\pi f \quad (\text{p. 64})$$

$$(3.4) \quad \omega = vk \quad (\text{p. 65})$$

$$(3.5) \quad \text{Phase velocity: } \frac{\omega}{k} \quad (\text{p. 65})$$

$$(3.6) \quad \text{Group velocity: } \frac{d\omega}{dk} \quad (\text{p. 65})$$

$$(3.7) \quad v = \sqrt{\frac{T}{\mu}} \quad (\text{p. 65})$$

(3.8) $\omega/k = c/n$ (for light waves) (p. 65)

(3.9) $\lambda \rightarrow \frac{\lambda}{n}$ (p. 65)

(3.10) $I = I_0 \cos^2 \theta$ (p. 66)

(3.11) $\theta_B = \arctan\left(\frac{n_2}{n_1}\right)$ (p. 66)

Constructive interference \iff phase difference

(3.12) of $2m\pi$ (p. 67)

Destructive interference \iff phase difference

(3.13) of $(2m + 1)\pi$ (p. 67)

(3.14) $\delta = k\Delta x$ (p. 67)

(3.15) Maxima: $d \sin \theta = m\lambda$ (p. 67)

(3.16) Minima: $d \sin \theta = (m + 1/2)\lambda$ (p. 67)

(3.17) $a \sin \theta = m\lambda$, $m = 1, 2, \dots$ (p. 68)

(3.18) $\Delta x = nd$ (optical path length) (p. 69)

(3.19) $n_2 > n_1$: phase shift of π (p. 69)

(3.20) $n_2 < n_1$: no phase shift (p. 69)

(3.21) First circular diffraction minimum: $D \sin \theta = 1.22\lambda$ (p. 70)

(3.22) Maxima: $d \sin \theta = n\lambda/2$ (p. 70)

Reflection: $\theta_i = \theta_r$ (angle of incidence equals angle of

(3.23) reflection) (p. 71)

(3.24) Refraction: $n_1 \sin \theta_1 = n_2 \sin \theta_2$ (Snell's law) (p. 71)

(3.25) $c_2 \sin \theta_1 = c_1 \sin \theta_2$ (p. 71)

(3.26) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ (p. 71)

(3.27) $f = R/2$ (p. 71)

(3.28) $\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (p. 71)

Positive distances \iff same side as light rays (incoming for s ,

(3.29) outgoing for s') (p. 71)

Negative distances \iff opposite side as light rays (incoming for s ,

(3.30) outgoing for s') (p. 71)

(3.31) $I \propto I_0 \lambda^{-4} a^6$ (p. 72)

(3.32) $f = \left(\frac{v + v_r}{v - v_s}\right)f_0$ (p. 73)

Thermodynamics and Statistical Mechanics

(4.1) $p_i = \frac{e^{-\beta E_i}}{\sum_j e^{-\beta E_j}}$ (p. 78)

(4.2) $\beta = \frac{1}{k_B T}$ (p. 78)

(4.3) $\langle \mathcal{O} \rangle = \sum_i p_i \mathcal{O}_i$ (p. 79)

(4.4) $p_i = \frac{e^{-\beta E_i}}{Z}$ (p. 79)

(4.5) $Z = \sum_j e^{-\beta E_j}$ (p. 79)

(4.6) $\langle E \rangle = \sum_i p_i E_i = \frac{\sum_i E_i e^{-\beta E_i}}{Z} = -\frac{\partial}{\partial \beta} \ln Z$ (p. 79)

(4.7) $S = k_B \ln \Omega$ (p. 79)

(4.8) $S = -k_B \sum_i p_i \ln p_i = \frac{\partial}{\partial T} (k_B T \ln Z)$ (p. 79)

(4.9) $S = Nk_B \left(\ln \frac{V}{N} + \frac{3}{2} \ln T + \frac{5}{2} + \frac{3}{2} \ln \frac{2\pi m k_B}{h^2} \right)$ (p. 80)

(4.10) $S = Nk_B \ln \frac{VT^{3/2}}{N} + \text{constants}$ (p. 80)

(4.11) $Z_N = \frac{1}{N! h^{3N}} \int e^{-\beta H(\mathbf{p}_1, \dots, \mathbf{p}_N; \mathbf{x}_1, \dots, \mathbf{x}_N)} d^3 \mathbf{p}_1 \dots d^3 \mathbf{p}_N d^3 \mathbf{x}_1 \dots d^3 \mathbf{x}_N$ (p. 80)

(4.12) $\binom{N}{M} = \frac{N!}{(N-M)!M!}$ (p. 80)

(4.13) $\ln(n!) \approx n \ln n - n$ (p. 80)

(4.14) $\Delta U = Q - W$ (p. 81)

(4.15) $\Delta S \geq \int \frac{\delta Q}{T}$ (p. 82)

(4.16) $PV = Nk_B T$ (p. 82)

(4.17) $\delta W = P dV$ (reversible) (p. 83)

(4.18) $\delta Q = T dS$ (reversible) (p. 83)

(4.19) $\Delta S = \int \frac{\delta Q}{T}$ (p. 83)

(4.20) $PV^\gamma = \text{constant}$ (p. 83)

$$(4.21) \quad dU = TdS - PdV \quad (\text{p. 84})$$

$$(4.22) \quad T = \left(\frac{\partial U}{\partial S} \right) \bigg|_V \quad (\text{p. 84})$$

$$(4.23) \quad P = - \left(\frac{\partial U}{\partial V} \right) \bigg|_S \quad (\text{p. 84})$$

$$(4.24) \quad \left(\frac{\partial P}{\partial S} \right) \bigg|_V = - \left(\frac{\partial T}{\partial V} \right) \bigg|_S \quad (\text{p. 84})$$

$$(4.25) \quad dH = TdS + VdP \quad (\text{p. 84})$$

$$(4.26) \quad dA = -SdT - PdV \quad (\text{p. 84})$$

$$(4.27) \quad dG = -SdT + VdP \quad (\text{p. 84})$$

$$(4.28) \quad \left(\frac{\partial T}{\partial P} \right) \bigg|_S = \left(\frac{\partial V}{\partial S} \right) \bigg|_P \quad (\text{p. 84})$$

$$(4.29) \quad \left(\frac{\partial S}{\partial V} \right) \bigg|_T = \left(\frac{\partial P}{\partial T} \right) \bigg|_V \quad (\text{p. 84})$$

$$(4.30) \quad - \left(\frac{\partial S}{\partial P} \right) \bigg|_T = \left(\frac{\partial V}{\partial T} \right) \bigg|_P \quad (\text{p. 84})$$

$$(4.31) \quad \left(\frac{\partial Q}{\partial T} \right) \bigg|_V = C_V \quad (\text{p. 84})$$

$$(4.32) \quad \left(\frac{\partial Q}{\partial T} \right) \bigg|_P = C_P \quad (\text{p. 84})$$

$$(4.33) \quad \left(\frac{\partial Q}{\partial T} \right) \bigg|_V = \frac{\partial U}{\partial T} \quad (\text{p. 85})$$

$$(4.34) \quad C_P - C_V = Nk_B \quad (\text{p. 85})$$

$$(4.35) \quad Q = mc\Delta T \quad (\text{p. 85})$$

$$(4.36) \quad e = 1 - \left| \frac{Q_C}{Q_H} \right| \quad (\text{p. 85})$$

$$(4.37) \quad e = 1 - \frac{T_C}{T_H} \quad (\text{p. 85})$$

$$(4.38) \quad U = \frac{3}{2}Nk_B T \quad (\text{p. 86})$$

$$(4.39) \quad v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} \quad (\text{p. 87})$$

$$(4.40) \quad c = \sqrt{\gamma \frac{P}{\rho}} \quad (\text{p. 87})$$

$$(4.41) \quad c = \sqrt{\gamma \frac{k_B T}{m}} \quad (\text{p. 87})$$

$$(4.42) \quad F_{\text{FD}}(\epsilon_i) = \frac{1}{e^{(\epsilon_i - \mu)/k_B T} + 1} \quad (\text{p. 88})$$

$$(4.43) \quad F_{\text{BE}}(\epsilon_i) = \frac{1}{e^{(\epsilon_i - \mu)/k_B T} - 1} \quad (\text{p. 88})$$

$$(4.44) \quad \langle N \rangle = \sum_i g(\epsilon_i) F(\epsilon_i) \quad (\text{p. 88})$$

$$(4.45) \quad \langle N \rangle = \int \rho(\epsilon) F(\epsilon) d\epsilon \quad (\text{p. 88})$$

Quantum Mechanics and Atomic Physics

$$(5.1) \quad \langle A \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{A} \Psi dx \quad (\text{p. 92})$$

$$(5.2) \quad \int_{-\infty}^{\infty} |\Psi(x, t)|^2 dx = 1 \quad (\text{p. 92})$$

$$(5.3) \quad \int_{-\infty}^{\infty} f(x)^* (\hat{A}g(x)) dx = \int_{-\infty}^{\infty} (\hat{A}f(x))^* g(x) dx \quad (\text{p. 93})$$

$$(5.4) \quad \hat{x} = x, \quad \hat{p} = -i\hbar \frac{\partial}{\partial x} \quad (\text{p. 93})$$

$$(5.5) \quad c_n = \int_{-\infty}^{\infty} f_n(x)^* \Psi(x, t) dx \quad (\text{p. 94})$$

$$(5.6) \quad \langle A \rangle = \sum_k \lambda_k |c_k|^2 \quad (\text{p. 94})$$

$$(5.7) \quad \text{Inner product of } |a\rangle \text{ and } |b\rangle \equiv \langle b|a\rangle \quad (\text{p. 94})$$

$$(5.8) \quad \langle a|b\rangle := \langle b|a\rangle^* \quad (\text{p. 94})$$

$$(5.9) \quad \langle a|\hat{A}b\rangle := \langle \hat{A}^\dagger a|b\rangle \quad (\text{p. 94})$$

$$(5.10) \quad \langle x|f\rangle := f(x)$$

$$(5.11) \quad \langle f|g\rangle := \int_{-\infty}^{\infty} f(x)^* g(x) dx \quad (\text{p. 94})$$

$$(5.12) \quad i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \hat{H} \Psi(x, t) \quad (\text{p. 95})$$

$$(5.13) \quad \hat{H} = \frac{\hat{p}^2}{2m} + \hat{V}(x) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \hat{V}(x) \quad (\text{p. 95})$$

$$(5.14) \quad i\hbar \frac{\partial}{\partial t} \Psi(x, t) = E_n \Psi(x, t) \quad (\text{p. 95})$$

$$(5.15) \quad [\hat{x}\hat{p}] = i\hbar \quad (\text{p. 96})$$

$$(5.16) \quad \sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2 \quad (\text{p. 97})$$

$$(5.17) \quad \sigma_A^2 := \langle A^2 \rangle - \langle A \rangle^2 \quad (\text{p. 97})$$

$$(5.18) \quad \sigma_x \sigma_p \geq \frac{\hbar}{2} \quad (\text{p. 97})$$

$$(5.19) \quad \Delta x \Delta p \approx \hbar \quad (\text{p. 97})$$

$$(5.20) \quad \Delta E \Delta t \approx \hbar \quad (\text{p. 97})$$

$$(5.21) \quad H = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega^2 \hat{x}^2 \quad (\text{harmonic oscillator}) \quad (\text{p. 99})$$

$$(5.22) \quad H = \hbar \omega \left(a^\dagger a + \frac{1}{2} \right) \quad (\text{p. 99})$$

$$(5.23) \quad [a, a^\dagger] = 1 \quad (\text{p. 99})$$

$$(5.24) \quad H|n\rangle = \hbar \omega \left(n + \frac{1}{2} \right) |n\rangle, \quad n = 0, 1, 2, \dots \quad (\text{p. 99})$$

- (5.25) $\langle T \rangle = \langle V \rangle = \frac{E_n}{2}$ (p. 100)
- (5.26) $\psi_N(x, y, z) = \psi_{n_1}(x)\psi_{n_2}(y)\psi_{n_3}(z); \quad E_N = \left(N + \frac{3}{2}\right)\hbar\omega$ with
 $N = n_1 + n_2 + n_3$ (p. 100)
- (5.27) $\psi(x) = e^{\pm ikx}, \quad E = \frac{\hbar^2 k^2}{2m}$ (p. 102)
- (5.28) $p = \hbar k$ (p. 102)
- (5.29) $E = \hbar\omega$ (p. 102)
- (5.30) $H = -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})$ (three dimensions) (p. 105)
- (5.31) $[\hat{x}, \hat{p}_x] = i\hbar, \quad [\hat{y}, \hat{p}_y] = i\hbar, \quad [\hat{z}, \hat{p}_z] = i\hbar$ (p. 105)
- (5.32) $[\hat{x}, \hat{y}] = 0, \quad [\hat{x}, \hat{p}_y] = 0, \quad [\hat{x}, \hat{p}_z] = 0, \dots$ (p. 105)
- (5.33) $\int_0^\infty |R(r)|^2 r^2 dr = 1, \quad \int_0^{2\pi} \int_0^\pi |Y(\theta, \phi)|^2 \sin\theta d\theta d\phi = 1$ (p. 105)
- (5.34) $\hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y$ (p. 105)
- (5.35) $\hat{L}_y = \hat{z}\hat{p}_x - \hat{x}\hat{p}_z$ (p. 105)
- (5.36) $\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$ (p. 105)
- (5.37) $[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z$, and cyclic permutations of x, y, z (p. 105)
- (5.38) $\hat{L}^2 := \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$ (p. 105)
- (5.39) $\hat{L}_z Y_l^m = m\hbar Y_l^m$ (p. 106)
- (5.40) $\hat{L}^2 Y_l^m = l(l+1)\hbar^2 Y_l^m$ (p. 106)
- (5.41) $m = l, l-1, l-2, \dots, -l$ (p. 106)
- (5.42) $\int_0^{2\pi} \int_0^\pi (Y_l^m(\theta, \phi))^* Y_{l'}^{m'}(\theta, \phi) \sin\theta d\theta d\phi = \delta_{ll'} \delta_{mm'}$ (p. 106)
- (5.43) $H = -\frac{\hbar^2}{2\mu}\nabla^2 - \frac{e^2}{4\pi\epsilon_0 r}$ (hydrogen atom) (p. 106)
- (5.44) $a = \frac{4\pi\epsilon_0\hbar^2}{\mu e^2}$ (p. 107)
- (5.45) $\psi_1(r) \propto e^{-r/a}$ (p. 107)
- (5.46) $-E_1 = \frac{\hbar^2}{2\mu a^2} = \frac{\mu e^4}{2(4\pi\epsilon_0)^2 \hbar^2} = 13.6 \text{ eV for hydrogen}$ (p. 107)
- (5.47) $-E_n = \frac{\hbar^2}{2\mu a^2} \frac{1}{n^2}, \quad n = 1, 2, 3, \dots$ (p. 107)
- (5.48) $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \approx 1/137$ (p. 108)
- (5.49) $|\uparrow\rangle_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad |\downarrow\rangle_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (p. 109)
- (5.50) $|\uparrow\rangle_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \quad |\downarrow\rangle_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$ (p. 109)
- (5.51) $\hat{S}_+ := \hat{S}_x + i\hat{S}_y, \quad \hat{S}_- := \hat{S}_x - i\hat{S}_y$ (p. 109)
- (5.52) $\hat{S}_+ |\uparrow\rangle = 0, \quad \hat{S}_- |\uparrow\rangle = \hbar |\downarrow\rangle$ (p. 109)
- (5.53) $\hat{S}_+ |\downarrow\rangle = \hbar |\uparrow\rangle, \quad \hat{S}_- |\downarrow\rangle = 0$ (p. 109)
- (5.54) Spin s and spin s' : $s_{\text{tot}} = s + s'$,
 $s + s' - 1, s + s' - 2, \dots, |s - s'|$ (p. 110)
- (5.55) $m_{\text{tot}} = m_s + m_{s'}$ (p. 110)
- (5.56) $s = 0, m_s = 0: \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$ (p. 111)
- (5.57) $E_n = E_n^0 + \lambda \langle \psi_n^0 | H' | \psi_n^0 \rangle$ (p. 113)
- (5.58) $E_n = E_n^0 + \lambda^2 \sum_{m \neq n} \frac{|\langle \psi_m^0 | H' | \psi_n^0 \rangle|^2}{E_n^0 - E_m^0}$ (p. 113)
- $J^2 = (\mathbf{L} + \mathbf{S})^2$
 $= L^2 + 2\mathbf{L} \cdot \mathbf{S} + S^2$
- (5.59) $\Rightarrow \mathbf{L} \cdot \mathbf{S} = \frac{1}{2}(J^2 - L^2 - S^2)$ (p. 115)
- (5.60) $\Delta H = e\mathbf{E} \cdot \mathbf{r}$ (p. 116)
- (5.61) $\Delta H = \frac{e}{2m}(\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B}$ (p. 116)
- (5.62) $I(\omega) \propto \frac{\hbar\omega^3}{c^2} \frac{1}{e^{\hbar\omega/k_B T} - 1}$ (p. 118)
- (5.63) $\frac{dP}{dA} \propto T^4$ (p. 118)
- (5.64) $\lambda_{\text{max}} = (2.9 \times 10^{-3} \text{ K} \cdot \text{m}) T^{-1}$ (p. 118)

Special Relativity

- (6.1) $t' = \gamma \left(t - \frac{v}{c^2} x \right)$ (p. 123)
- (6.2) $x' = \gamma (x - vt)$ (p. 123)
- (6.3) $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ (p. 124)
- (6.4) $t = \gamma \left(t' + \frac{v}{c^2} x' \right)$ (p. 124)
- (6.5) $x = \gamma (x' + vt')$ (p. 124)
- (6.6) $\Delta t = \gamma \Delta t'$ (fixed x') (p. 124)
- (6.7) $L' = \gamma L$ (fixed t) (p. 125)

$$(6.8) \quad w = \frac{v+u}{1+vu/c^2} \quad (\text{p. 125})$$

$$(6.9) \quad x^0 = ct, \quad x^1 = x, \quad x^2 = y, \quad x^3 = z \quad (\text{p. 125})$$

$$(6.10) \quad x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z) \quad (\text{p. 125})$$

$$(6.11) \quad \beta = v/c \quad (\text{p. 125})$$

$$(6.12) \quad \begin{pmatrix} x^{0'} \\ x^{1'} \\ x^{2'} \\ x^{3'} \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} \quad (\text{p. 125})$$

$$(6.13) \quad \text{Energy-momentum: } p^\mu = (E/c, \mathbf{p}) \quad (\text{p. 126})$$

$$(6.14) \quad \text{Current density: } j^\mu = (c\rho, \mathbf{J}) \quad (\text{p. 126})$$

$$(6.15) \quad \text{Wavevector: } k^\mu = (\omega/c, \mathbf{k}) \quad (\text{p. 126})$$

$$(6.16) \quad \mathbf{p} = \gamma m \mathbf{v} = \frac{m \mathbf{v}}{\sqrt{1 - |\mathbf{v}|^2/c^2}} \quad (\text{p. 126})$$

$$(6.17) \quad E_0 = mc^2 \quad (\text{p. 126})$$

$$(6.18) \quad T = E - mc^2 \quad (\text{p. 126})$$

$$(6.19) \quad E = \gamma mc^2 \quad (\text{p. 126})$$

$$(6.20) \quad T = (\gamma - 1)mc^2 \quad (\text{p. 126})$$

$$(6.21) \quad a \cdot b \equiv a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3 \quad (\text{p. 126})$$

$$(6.22) \quad \text{Timelike: } (\Delta x)^2 > 0 \quad (\text{p. 127})$$

$$(6.23) \quad \text{Spacelike: } (\Delta x)^2 < 0 \quad (\text{p. 127})$$

$$(6.24) \quad \text{Lightlike or null: } (\Delta x)^2 = 0 \quad (\text{p. 127})$$

$$(6.25) \quad E^2 = \mathbf{p}^2 c^2 + m^2 c^4 \quad (\text{p. 127})$$

$$(6.26) \quad \sum_i p_i^\mu = \sum_f p_f^\mu \quad (\text{p. 127})$$

$$(6.27) \quad \frac{\lambda'}{\lambda} = \sqrt{\frac{1+\beta}{1-\beta}} \quad (\text{p. 129})$$

$$(6.28) \quad \beta = 0.6 \implies \gamma = 1.25 \quad (\text{p. 129})$$

$$(6.29) \quad \beta = 0.8 \implies \gamma = 5/3 \quad (\text{p. 129})$$

Laboratory Methods

$$(7.1) \quad \sigma_S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \quad (\text{p. 135})$$

$$(7.2) \quad \sigma_{\text{tot}} = \sqrt{\sigma_{\text{stat}}^2 + \sigma_{\text{sys}}^2} \quad (\text{p. 135})$$

$$(7.3) \quad \sigma_z^2 = \sum_{i=1}^n \left(\frac{\partial z}{\partial x_i} \right)^2 \sigma_{x_i}^2 \quad (\text{p. 135})$$

$$(7.4) \quad X = \frac{x/\sigma_x^2 + y/\sigma_y^2}{1/\sigma_x^2 + 1/\sigma_y^2} \quad (\text{p. 136})$$

$$(7.5) \quad \sigma_{\text{tot}}^2 = \frac{1}{1/\sigma_x^2 + 1/\sigma_y^2} \quad (\text{p. 136})$$

$$(7.6) \quad P(n) = \frac{\lambda^n e^{-\lambda}}{n!} \quad (\text{p. 136})$$

$$(7.7) \quad \text{Capacitor: } Z = \frac{1}{i\omega C} \quad (\text{p. 137})$$

$$(7.8) \quad \text{Inductor: } Z = i\omega L \quad (\text{p. 137})$$

$$(7.9) \quad \text{Resistor: } Z = R \quad (\text{p. 137})$$

$$(7.10) \quad \text{Series: } Z_{\text{tot}} = Z_1 + Z_2 + \cdots Z_n \quad (\text{p. 137})$$

$$(7.11) \quad \text{Parallel: } Z_{\text{tot}}^{-1} = Z_1^{-1} + Z_2^{-1} + \cdots Z_n^{-1} \quad (\text{p. 137})$$

$$(7.12) \quad \overline{A \cdot B} = \overline{A} \cdot \overline{B} \quad (\text{p. 139})$$

$$(7.13) \quad \overline{A + B} = \overline{A} \cdot \overline{B} \quad (\text{p. 139})$$

$$(7.14) \quad E_{\text{max}} = E_\gamma - \phi \quad (\text{p. 140})$$

$$(7.15) \quad \lambda = \frac{h}{mc} \quad (\text{p. 140})$$

$$(7.16) \quad \Delta\lambda = \frac{h}{mc}(1 - \cos\theta) \quad (\text{p. 140})$$

$$(7.17) \quad N = N_0 e^{-t/\tau} \quad (\text{p. 141})$$

$$(7.18) \quad t_{1/2} = \tau \ln 2 \quad (\text{p. 141})$$

$$(7.19) \quad \frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \cdots \quad (\text{p. 141})$$

Specialized Topics

$$(8.1) \quad k_F = (3\pi^2 n)^{1/3} \quad (\text{p. 151})$$

$$(8.2) \quad E_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3} \quad (\text{p. 151})$$

$$(8.3) \quad \rho(E) = \frac{V\sqrt{2}}{\pi^2 \hbar^3} m^{3/2} \sqrt{E} \quad (\text{p. 151})$$

$$(8.4) \quad N = \int_0^{E_F} \rho(E) dE \quad (\text{p. 151})$$

$$(8.5) \quad \rho(E_F) = \frac{3}{2} \frac{N}{E_F} \quad (\text{p. 151})$$

$$(8.6) \quad N_C \approx \rho(E_F)(k_B T) \sim N \frac{k_B T}{E_F} \quad (\text{p. 151})$$

$$(8.7) \quad \frac{\lambda_0}{\lambda_T} = \frac{a(\text{today})}{a(T)} \quad (\text{p. 152})$$

$$(8.8) \quad v = H_0 D \quad (\text{p. 152})$$

$$(8.9) \quad z(T) = \frac{\lambda_0}{\lambda_T} - 1 \quad (\text{p. 152})$$