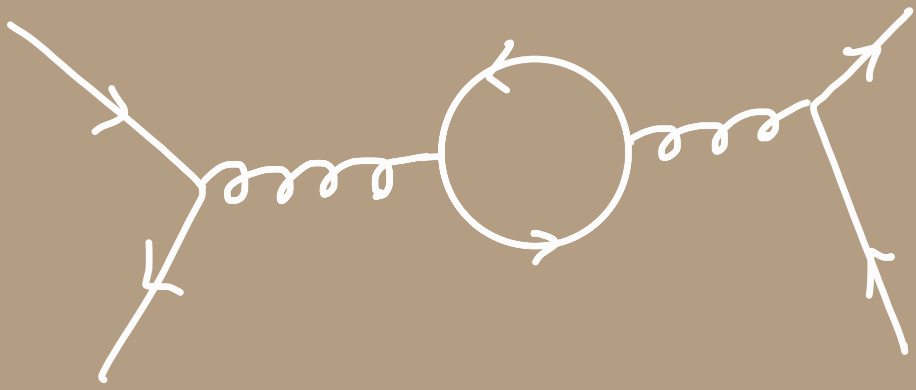
STANDARD MODEL II



Course '22-'23

2nd semester.

• INTRODUCTION

1.- Overview of the SM Lagrangian and the big picture

The SM is very simple. It contains scalars, spin 1/2 fermions and spin 1 bosons (both massive and massless).
We can describe it by a simple Lagrangian, which is often printed in t-shirts:

$$
\begin{aligned}
\mathcal{L}_{SM} = & -\frac{1}{4} F_{\mu\nu}^2 + \bar{\Psi}_i \, i \not{D} \Psi_i + |D_\mu H|^2 \\
& - y_{ij} \bar{\Psi}_L^i \Psi_R^j H + m^2 |H|^2 - \lambda |H|^4 \\
& + \underbrace{\frac{\theta g^2}{32\pi} F_{\mu\nu} \tilde{F}_{\mu\nu}}_{\text{QCD } \theta\text{-term}} + \underbrace{\partial_\mu \bar{c} D_\mu c - \frac{1}{2\xi} (\partial_\mu A)^2}_{\substack{\text{not so oftenly printed in t-shirts. Ghosts and} \\ \text{gauge fixing terms. For good reasons!}}} + \underbrace{\mathcal{O}\left(\frac{p}{\Lambda}, \frac{m}{\Lambda}\right)}_{\text{to be discussed.}}
\end{aligned}
$$

* Ghosts are relevant for loop processes.
They enter quadratically (i.e. $\sim \bar{c} \partial_\mu c A^\mu$), and do not belong to the set of physical states (i.e. they are not asymptotic scattering states). Therefore they do not enter in tree-level processes. Their job in life is to make sure that unphysical degrees of freedom (w/ negative norm) are not propagated in loops:

This is a course on the physical consequences of this theory.
Let's unpack a bit further $\mathcal{L}_{SM}$ and review the basic features.

The SM gauge group is $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

the Label "c" is to remind us of "color" or "chromo-dynamics", QCD

This label reminds us that this force acts on Left-handed fermions, but not on the right-handed.

Hypercharge historically associated to the quantum number "Y".

After specifying the gauge group, the structure constants "f_{abc}" of $\mathcal{L}_{\text{Gauge Boson}}$ are fixed.

$$\mathcal{L}_{\text{Gauge Boson}} = -\frac{1}{4} F_{\mu\nu}^2 = -\frac{1}{4}\left(\partial_\mu A_\nu^c - \partial_\nu A_\mu^c\right)^2 - g f^{abc}\left(\partial_\mu A_\nu^a\right) A_\mu^b A_\nu^c$$
$$-\frac{1}{4} g^2 \left(f^{eab} A_\mu^a A_\nu^b\right)\left(f^{ecd} A_\mu^c A_\nu^d\right).$$

Recall that there is an underlying Lie algebra structure.

$$[T_a, T_b] = i \sum_c f_{ab}{}^c \, T_c$$

in this course it doesn't matter if indices are "up" or "down".

the structure constants f_{abc} are totally antisymmetric.

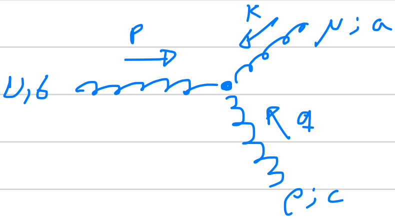
For $SU(2)_L$ we take $i f_{abc} = (t_G^b)_{ac} = \epsilon^{abc}$,

$SU(3)_c$ $f_{123} = 1$, $f_{458} = f_{678} = \sqrt{3}/2$, $f_{147} = f_{246} = f_{257} = f_{345} = -f_{156} = -f_{367} = 1/2$.

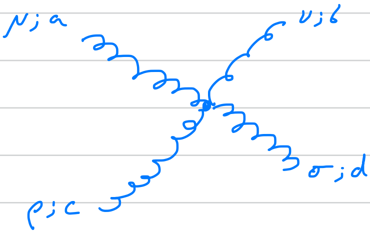
$U(1)_Y$ $f_{abc} = 0$

The couplings are $\{g', g, g_s\} \simeq \{0.4, 0.6, 1.2\}$ @ 100 GeV

with the Lagrangian above $\mathcal{L}_{\text{gauge bosons}}$ we can compute any quantum amplitude using the Feynman rules

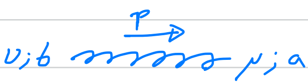


$$= g\, f^{abc}\left[g^{\mu\nu}(k-p)^{\rho} + g^{\nu\rho}(p-q)^{\mu} + g^{\rho\mu}(q-k)^{\nu} \right].$$



$$= -ig^2\left[f^{abe} f^{cde}\left(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right) + f^{ace} f^{bde}\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}\right) + f^{ade} f^{bce}\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma}\right)\right]$$

The propagator of the Gauge bosons is given by



$$= i\, \frac{-g^{\mu\nu} + (1-\xi)\frac{p^\mu p^\nu}{p^2}}{p^2 + i\epsilon}\, \delta^{ab}$$

it comes from the term $-\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2\xi}(\partial_\mu A)^2$ and describes the propagation
gauge fixing.

of either the photon γ, the gluons, and the high energy limit of massive $W^\pm, Z$ (more about this later on).

The gauge bosons of the gauge group are "force carriers" and mediate weak, strong and QED interactions among the fermions (i.e. the matter) and the Higgs boson.

these are tensors
i.e. collection of numbers.

From the Lagrangian $g A^a_\mu \bar{\Psi}_i \gamma^\mu \Psi_j T^a_{ij} + ig A^a_\mu R^a_{ij} (H^*_i \partial_\mu H_j - H_j \partial_\mu H^*_i)$

$$+ g^2 H^*_i A^a_\mu R^a_{ik} R^b_{kj} A^b_\mu H_j \equiv A^a_\mu J^a_\mu$$

a sum over $\{a, i, j, k\}$ is implicit.

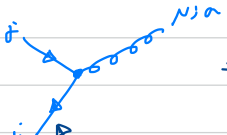
$$\text{w/ } J^a_\mu = g \bar{\Psi}_i \gamma^\mu T^a_{ij} \Psi_j + ig H^+ \overleftrightarrow{\partial_\mu} R H$$

we get the following interactions

$= ig(k^\mu + q^\mu) R^a_{ij}$;

$= ig^2 R^a_{ik} R^b_{kj} g^{\mu\nu}$

Higgs, a scalar boson.



$= ig \gamma^\mu T^a_{ij}$

Fermion (leptons & quarks)

Now, to fully specify these interaction we need to know more than the gauge group G_{SM} and the numerical values of the couplings $\{g, g_s, g'\}$. We need to know the "charges" or "group representation" of the SM fermions and the Higgs.

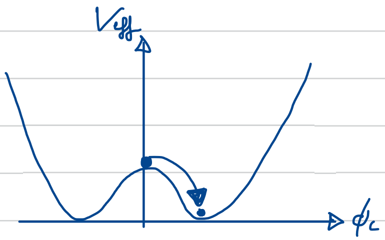
Field	Names	Spin	$U(1)_Y$	$SU(2)_L$	$SU(3)_c$
q^i_L $(\equiv Q)$	quarks	1/2	1/6	2	3
u^i_R $(\equiv u)$			2/3	1	3
d^i_R $(\equiv d)$			−1/3	1	3
e^i_L $(\equiv L)$	leptons	1/2	−1/2	2	1
e^i_R $(\equiv e)$			−1	1	1
H	Higgs	0	1/2	2	1

(e.g. $\bar{L} e H$ is an invariant; $\bar{Q} u H$ is not an invariant.)

So far we have covered

Next we will briefly review the Higgs sector.

$$\mathcal{L}_{SM} = -\frac{1}{4} F_{\mu\nu}^2 + \bar{\Psi}_i i \not{D} \Psi_i + |D_\mu H|^2$$
$$- y_{ij} \bar{\Psi}_L^i \Psi_R^j H + m^2 |H|^2 - \lambda |H|^4$$
$$+ \frac{\theta g^2}{32\pi} F_{\mu\nu} \tilde{F}_{\mu\nu} + \partial_\mu \bar{c} D_\mu c - \frac{1}{2\xi} (\partial_\mu A)^2 + O\left(\frac{p}{\Lambda}, \frac{m}{\Lambda}\right)$$



vacuum state of SM.

$$\langle 0 | \mathcal{H} | 0 \rangle = V_{eff}(\phi_c)\big|_{min} \quad \text{subject to}$$

$$\langle 0 | h | 0 \rangle = \phi_c . \quad \left(\text{Higgs doublet } H = \begin{pmatrix} G^+ \\ \frac{h + i G^0}{\sqrt{2}} \end{pmatrix} \right)$$

In the SM $\alpha_i / 4\pi \ll 1$ w/ $\alpha_i = \{\lambda, y^2, g'^2, g^2, g_s^2\}/4\pi$,
therefore $V_{eff} \simeq V(\phi_c) = -m^2 \phi_c^2 + \frac{\lambda}{4} \phi_c^4$.
Using this potential we get $\langle 0 | \mathcal{H} | 0 \rangle = V_{eff}(\phi_c)_{min}$ for $\phi_c = \langle 0 | h | 0 \rangle \equiv v$
and we say

"the Higgs acquires a vacuum expectation value".

It turns out $v = 246$ GeV.

There is more than that: the expectation value of the Higgs originates electroweak symmetry breaking. The SM Lagrangian is invariant under G_{SM} but the SM vacuum is invariant under a subgroup $SU(3)_c \otimes U(1)_{QED} \subset G_{SM}$.

$$T^i \cdot \langle 0 | H^T | 0 \rangle = T^i \cdot (0, v/\sqrt{2})^T \neq 0$$

where T^i is any of the $SU(2)_L$ or $U(1)_Y$ generators. However,

$$(T^3 + Q_Y) | 0 \rangle = Q_{QED} | 0 \rangle = 0$$

END OF LECTURE 1

is a symmetry of the vacuum and therefore $U(1)_{QED} \subset SU(2)_L \otimes U(1)_Y$ is not spontaneously broken by the Higgs vev (vacuum expectation value).

On the vacuum, the Higgs mechanism provides mass for the Z and $W^\pm$ gauge bosons as well as for the leptons and quarks.

Indeed, using

$$H = \begin{pmatrix} G^+ \\ \frac{v+h+iG^0}{\sqrt{2}} \end{pmatrix}$$

$$|D_\mu H|^2 \supset g^2 W_\mu^i W_\mu^i H^\dagger H \sim \underline{\frac{g^2 v^2}{4}} W_\mu^2$$

(also $g'^2 B_\mu B^\mu H^\dagger H$).

$$y^e_{ij} \bar{L}^i e^j H + y^d_{ij} \bar{Q}^i d^j H + y^u_{ij} \bar{Q}^i u^j \tilde{H} + h.c.$$

$\tilde{H} \equiv i\sigma_2 H^*$

$$\supset \frac{y^e_i v}{\sqrt{2}} \bar{e}^i e^i + \frac{y^d_i v}{\sqrt{2}} \bar{d}^i d^i + \frac{y^u_i v}{\sqrt{2}} \bar{u}^i u^i$$

	Gauge Bosons				Leptons			
	m_γ^2	m_Z^2	$m_{W^\pm}^2$	m_g^2	$m_{e^\pm}$	$m_{\mu^\pm}$	$m_{\tau^\pm}$	m_ν^i
SM (tree-level)	0	$\frac{g^2+g'^2}{4} v^2$	$\frac{g^2}{4} v^2$	0	$\frac{y_e v}{\sqrt{2}}$	$\frac{y_\mu v}{\sqrt{2}}$	$\frac{y_\tau v}{\sqrt{2}}$	$\frac{v^2}{\Lambda}$
Value	0	$(91)^2$	$(80)^2$	0	$5\cdot 10^{-4}$	0.1	0.7	$3\cdot 10^{-10}$

more about this soon.

The masses of the quarks span five orders of magnitude $m_{top}/m_{up} \approx 10^5$, while the masses of the neutrinos is much smaller than the electroweak scale. $m_\nu \simeq 10^{-1} eV \simeq 10^{-12} v$. There is <u>no</u> explanation for this hierarchies in the SM.

	Quarks						Higgs
	m_u	m_d	m_c	m_s	m_t	m_b	m_h^2
SM (tree-level)	$\frac{y_u v}{\sqrt{2}}$	$\frac{y_d v}{\sqrt{2}}$	$\frac{y_c v}{\sqrt{2}}$	$\frac{y_s v}{\sqrt{2}}$	$\frac{y_t v}{\sqrt{2}}$	$\frac{y_b v}{\sqrt{2}}$	$2\lambda v^2$
Value	$2\cdot 10^{-3}$	$4\cdot 10^{-3}$	1	0.1	173	4	$(125)^2$

Further explanations on EWSB later on.

Next we should mention few more things on the Yukawa sector. Recall that the Fermion - Gauge interactions arise from

$$\mathcal{L}_{F-G} = \overline{Q}_i i \not{D} Q_i + \bar{u}\, i \not{D} u + \bar{d}\, i \not{D} d + \overline{L}\, i \not{D} L + \bar{e}\, i \not{D} e$$

$$\mathcal{L}_{Yukawa} = -\, i\, \overline{Q}_i (y_d)_{ij}\, d_j H - \overline{Q}_i\, (y_u)_{ij}\, u_j \widetilde{H} - \overline{L}_i (y_e)_{ij}\, e_j H + h.c.$$

Yukawa matrices

$i, j = 1, 2, 3$, because there are three families of Quarks & Leptons.

(recall that $\widetilde{H} = i\sigma_2 H^*$)

$\hookrightarrow \sigma_2$ Pauli matrix

Recall also that $Q = (u_L, d_L)$, $L = (\nu_L, e_L)$ are doublets of left handed fermions and e, u, d are right handed $SU(2)_L$ singlets.

$e \equiv e_R$; $u \equiv u_R$; $d \equiv d_R$.

The Yukawa matrices are generic 3×3 complex matrices. As such, they admit the decomposition

$$y_f = L_f\, y_D^f\, R_f^\dagger \quad \text{w/ } f = u, d \text{ or } e.$$

$\hookrightarrow$ diagonal matrix.

where L_f and R_f are unitary matrices. Next we set $\langle H\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ and perform the rotation

$$Q \to L_d Q \;;\quad u \to R_u u \;;\quad d \to R_d d \;;\quad L \to L_e L \;;\quad e \to R_e e$$

and we are lead to

$$\mathcal{L}_{Yukawa} = -\left(\bar{d}_L^i (y_D^d)_{ij} d_R^j + \bar{u}_L \underbrace{L_d^\dagger L_u}_{\equiv V_{CKM}^\dagger} y_D^u u_R + \bar{e}_L y_D^\ell e_R \right) \frac{v}{\sqrt{2}} + h.c.$$

$$= -(\bar{d}_L, \bar{s}_L, \bar{b}_L) \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} + h.c.$$

$$-(\bar{u}_L, \bar{c}_L, \bar{t}_L)\, V_{CKM}^\dagger \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} + h.c.$$

$$-(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} + h.c.$$

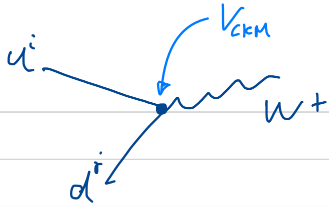
While the kinetic terms $\bar{Q}_i i \not{D} Q_i + \bar{d}_i i \not{D} d_i + \dots$ are invariant under unitary transformations of the fermion fields.

$V_{CKM} = L_u^\dagger L_d$ is the Cabbibo-Kobayashi-Maskawa matrix. It is a 3×3 unitary matrix. It is useful to work in "mass eigenstates". For that, we rotate u_L independently from d_L:

$$\left(\bar{d}_L^i (y_D^d)_{ij} d_R^j + \bar{u}_L^i (V_{CKM}^\dagger\, y_D^u)_{ij} u_R^j \right) \frac{v}{\sqrt{2}} \xrightarrow{u_L \to V_{CKM}^\dagger u_L} \left(\bar{d}_L^i y_{D_{ij}}^d d_R^j + \bar{u}_{L_i} y^u_{D_{ij}} u_R \right) \frac{v}{\sqrt{2}}$$

and

$$\bar{Q}_i i \not{D} Q_i \supset \frac{g}{\sqrt{2}} \bar{u}_L^i W_\mu^+ \gamma^\mu d_L^i + h.c. \xrightarrow{u_L \to V_{CKM}^\dagger u_L} \frac{g}{\sqrt{2}} \bar{u}_L^i (V_{CKM})_{ij} W_\mu^+ \gamma^\mu d_L^j + h.c.$$



induces processes that mix the SM quark families as well as C and P violation.

It can be parametrized as:

$$V_{CKM} = R_x(\theta_{23}) \cdot R_y(\theta_{13}) \, \mathrm{diag}(1, e^{i\delta}, 1) \cdot R_z(\theta_{12})$$

It turns out that

$$|V_{CKM}| \equiv \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} .97 & .23 & .0035 \\ .23 & .97 & .041 \\ .0087 & .04 & 1 \end{pmatrix} = \mathcal{O}(1) \times \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$$

w/ $\epsilon = 0.2$.

I will say more about U_{PMNS}, i.e. Pontecorvo – Maki – Nakagawa – Sakata matrix later on. It provides a source for neutrino mixing.

Regarding neutrino masses, note that there are these terms in the SM EFT.

$$\begin{aligned} \mathcal{L}_{SM} = & -\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}_i \, i \not{D} \, \psi_i + |D_\mu H|^2 \\ & - y_{ij} \bar{\psi}_L^i \psi_R^j H + m^2 |H|^2 - \lambda |H|^4 \\ & + \frac{\theta g^2}{32\pi} F_{\mu\nu} \tilde{F}_{\mu\nu} + \partial_\mu \bar{c} D_\mu c - \frac{1}{2\xi} (\partial_\mu A)^2 + \mathcal{O}\left(\frac{p}{\Lambda}, \frac{m}{\Lambda}\right) \end{aligned}$$

These denote any operator of dimension $\Delta > d$. Note that all the operators we wrote above have dimension $\Delta \leq 4$.
There is single operator that we can write with the SM fields and $\Delta = 5$. This is the so called Weinberg operator:

$$\mathcal{L}_{\Delta=5} = \frac{1}{\Lambda} y_\nu^{ij} (\bar{L}_i \tilde{H})(\tilde{H}^T i\sigma_2 L^c_j) + h.c.$$

Breaks lepton number.

upon EWSB $\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$, $\mathcal{L}_{\Delta=5} = \frac{y_\nu^{ij}}{\Lambda} \bar{\nu}_i \nu^c_j v^2$,
providing neutrino masses, $m_\nu \sim \frac{v^2 y_\nu}{\Lambda}$.

$$\begin{aligned}\mathcal{L}_{SM} = & -\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}_i i \not{D} \psi_i + |D_\mu H|^2 \\ & - y_{ij} \bar{\psi}_L^i \psi_R^j H + m^2 |H|^2 - \lambda |H|^4 \\ & + \frac{\theta g^2}{32\pi} F_{\mu\nu} \tilde{F}_{\mu\nu} + \partial_\mu \bar{c} D_\mu c - \frac{1}{2\xi} (\partial_\mu A)^2 + \frac{1}{\Lambda} HHLL + O\left(\frac{v^2}{\Lambda^2}, \frac{p^2}{\Lambda^2}\right)\end{aligned}$$

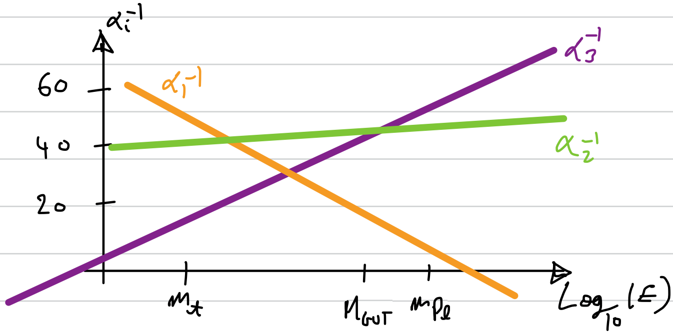
we will discuss this operator later on. Θ_{QCD} provides a source of CP violation in the strong interactions $\Theta_{QCD} < 10^{-10}$!

we will discuss operators of dimension $\Delta \geq 6$ later on in the course.

(Θ is not physical / observable, only
$\bar{\Theta} = \Theta - \arg\det y_u y_d < 10^{-10}$)

Besides the gauge symmetry $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, $\mathcal{L}_{SM}$ is invariant under many global accidental symmetries. This will be discussed later on.

Last introductory comment: the SM is weakly coupled at high energies $E \gtrsim 200\,\mathrm{MeV}$. $\alpha_s(Q^2 = 200\,\mathrm{MeV}) \simeq 1$.



$$\alpha_i = \left\{ \frac{5}{3}\frac{g'^2}{4\pi}, \frac{g^2}{4\pi}, \frac{g_s^2}{4\pi} \right\}.$$

END OF L2

• THE HIGGS MECHANISM IN THE SM

3.- Introduction to the Higgs mechanism

The Glashow - Weinberg - Salam model provides a unified description of electro-magnetism and weak interactions. The model is based on the concept of spontaneously broken gauge theory. But first of all,

Q: what are the weak interactions?

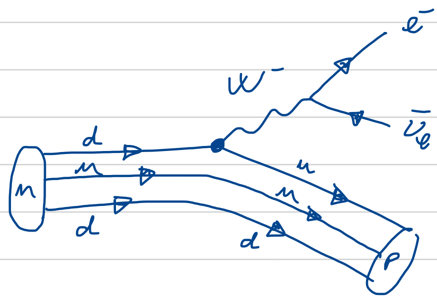
most famously it is responsible for β - decay

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

$$\underset{\frac{2}{3}\ -\frac{1}{3}\ -\frac{1}{3}}{u\ d\ d} \qquad \underset{\frac{2}{3}\ \frac{2}{3}\ -\frac{1}{3}}{u\ u\ d}$$

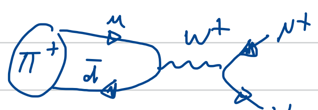
who remembers the mass and lifetime of the neutron?

$$d \longrightarrow u + e^- + \bar{\nu}_e$$



There is ~~no~~ interaction vertex in the strong force ($SU(3)_c$) or in electro-magnetism that would allow one type of quark to convert into another.

Other famous decays mediated by the weak force:



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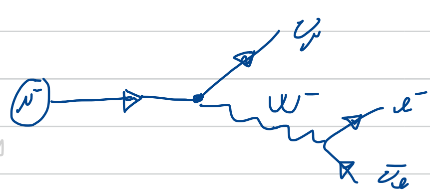


,

and so on.

mass and lifetime of $\pi^\pm$, π^0 and $\mu^\pm$?

* There is one defining property of $SU(2)_L \otimes U(1)_Y$ that differentiates this force from the strong force and electromagnetism ($U(1)_Q \subset SU(2)_L \otimes U(1)_Y$):

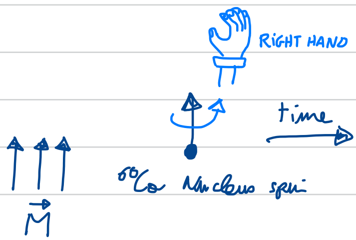
Weak and hypercharge force do not respect parity!

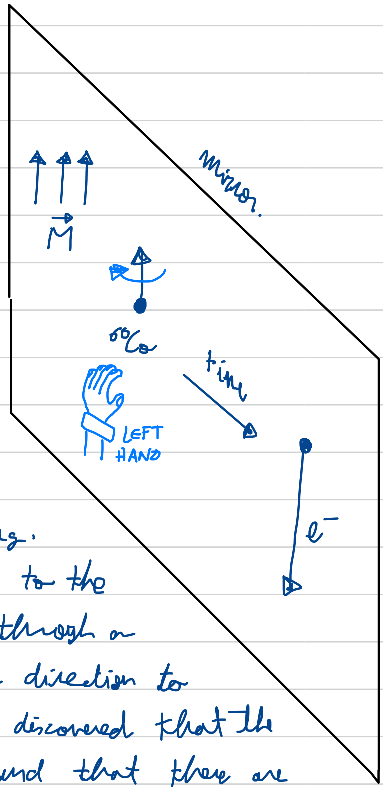
This fact was discovered by Chien-Shiung Wu in 1956. Wu's experiment is one of the most important experiments in particle physics. She placed a bunch of Cobalt atoms in a magnetic field and measured their decay:

$$^{60}Co \longrightarrow {}^{60}Ni^* + e^- + \bar{\nu}_e \longrightarrow {}^{60}Ni + e^- + \bar{\nu}_e + 2\gamma$$

$\tau_{1/2} = 5.3\,yr.$

It turns out that if ^{60}Co is placed in a magnetic field, the electrons are preferentially emitted in the opposite direction of the magnetic field.





When we say ^{60}Co spin is aligned with the magnetic field is meant according to the "right-hand-rule": take the right hand and curl the fingers in the direction ^{60}Co is spinning. Then, your thumb points in the direction opposite to the electrons. IF the same experiment is viewed through a mirror, then the electrons move in the same direction to where your thumb would be pointing! Wu discovered that the fundamental laws of nature break parity and that there are processes that can happen if viewed reflected in a mirror that can not happen in our real world!

How can we build a theory that breaks parity (i.e. a theory that describes a world which looks different if reflected in a mirror)?

Recall that any massless spin 1/2 particle decomposes into two pieces called "left-handed" and "right-handed". The right-handed particle is one whose spin is aligned with its momentum, while a left-handed particle has its spin and momentum anti-aligned. This definition only makes sense for massless particles: since they travel at the speed of light, all observers, regardless of their motion, would agree on the direction of their spin and momentum.

Thus to formulate a theory that breaks parity, we simply need to give different charges to the left and right-handed fermions. The SM achieves this in a quite radical manner: only left-handed fermions are charged under the weak force, while right handed particles do not feel it at all. Recall:

Field	Names	Spin	$U(1)_Y$	$SU(2)_L$	$SU(3)_c$
$q_L^i \; (\equiv Q)$	quarks	$1/2$	$1/6$	2	3
$u_R^i \; (\equiv u)$			$2/3$	1	3
$d_R^i \; (\equiv d)$			$-1/3$	1	3
$e_L^i \; (\equiv L)$	leptons	$1/2$	$-1/2$	2	1
$e_R^i \; (\equiv e)$			-1	1	1

$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $\quad$ $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

uncharged.

Left handed and charged under W_i's.

A remarkable and shocking consequence of parity violation is that, at the fundamental level, all elementary spin 1/2 particles are massless! This seems contradicting the basic facts that the electron has a mass of

5 MeV, or that the top quark is very massive (173 GeV).
The second surprising observation is that the force carriers of the Weak interactions mediate short range interactions, how can this be if gauge bosons are massless?

It turns out that both problems are resolved through the Higgs mechanism: the higgs mechanism provides mass to fermions (whose left and right components have different charges) and mass to gauge bosons.

4.- Basics of the Higgs mechanism:

Example I:

We start by presenting an Abelian example. Consider a complex scalar field coupled to itself and to an electromagnetic field. The Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu \phi|^2 - V(\phi)$$

where $D_\mu = \partial_\mu + ieA_\mu$. This Lagrangian is invariant under the local $U(1)$ gauge transformation:

$$\begin{cases} \phi(x) \to e^{i\alpha(x)}\phi(x) \\ A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x) \end{cases}$$

(Please check by yourself that this transformation leaves the Lagrangian above invariant!)

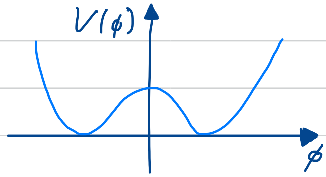
If we choose the potential in $\mathcal{L}$ to be of the form

$$V(\phi) = -\mu^2 \phi^* \phi + \frac{\lambda}{2}(\phi^*\phi)^2 ,$$

with $\mu^2 > 0$, then the field ϕ acquires a vacuum expectation value (VEV) and the $U(1)$ global symmetry is spontaneously broken.

SSB: spontaneous symmetry breaking is the phenomena where the Lagrangian posses certain symmetry but the vacuum of the theory does not.

The minimum of this potential occurs at



$$\langle \Omega | \phi | \Omega \rangle \equiv \phi_0 = \left(\frac{\mu^2}{\lambda} \right)^{1/2}$$

or at any other value related by a $U(1)$ symmetry.
Next we expand the Lagrangian around the vacuum state by decomposing the field $\phi(x)$ as

$$\phi(x) = \phi_0 + \frac{1}{\sqrt{2}} \left(\phi_1(x) + i\, \phi_2(x) \right).$$

a constant.

Then the potential is rewritten as $V(\phi) = -\frac{1}{2\lambda} \mu^4 + \frac{1}{2}\, 2\mu^2 \phi_1^2 + O(\phi_i^3)$,
therefore ϕ_1 acquires a mass, while ϕ_2 would be a goldstone boson.
It is not a simple Goldstone bosons! Let's look at the kinetic term:

$$|D_\mu \phi|^2 = \frac{1}{2} (\partial_\mu \phi_1)^2 + \frac{1}{2} (\partial_\mu \phi_2)^2 + \sqrt{2} e\, \phi_0 A_\mu \partial^\mu \phi_2 + e^2 \phi_0^2 A_\mu A^\mu + O(\phi_i^3, A_\mu^3).$$

ϕ_2 would be a Goldsone associated to the SSB by $\phi_0 \neq 0$ if it wasn't for this term!

photon mass $\frac{1}{2} m_A^2 A_\mu A^\mu$
w/ $m_A^2 = 2e^2 \phi_0^2$

$= i\sqrt{2}\, e\, \phi_0 (-i k^\mu) = m_A k^\mu.$


$= i m_A^2 + m_A k^\mu \frac{i}{k^2} (-m_A k^\nu) = i m_A^2 \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right)$,

transverse vacuum polarization!

ϕ_2 is not an independent degree of freedom. It accounts for the longitudinal polarizations of the massive photon. This is clearly seen in the "unitary gauge": pick a value of $\alpha(x)$ in such a way that $\phi(x)$ becomes real valued at all points. Then the Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (\partial_\mu \phi)^2 + e^2 \phi^2 A_\mu A^\mu - V(\phi).$$

	$\phi_0 \to 0$	$\phi_0 \neq 0$
# d.o.f.	• 1 complex scalar $= 2$ real scalars • 1 massless photon $= 2$ transverse polarizations $2 + 2 = 4$	• 1 real scalar • 1 massive gauge boson $= 3$ polarizations $3 + 1 = 4$

number of degrees of freedom

$2 + 2 = 3 + 1$

Example II:

Consider a model with a $SU(2)$ gauge field coupled to a complex scalar field Φ that transforms as a spinor of $SU(2)$, i.e. a doublet of $SU(2)$. The covariant derivative is

$$D_\mu \Phi = (\partial_\mu - i g A_\mu^a \tau^a)\Phi$$

where $\tau^a = \sigma^a/2$.

pauli matrices $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$; $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

The Lagrangian is

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D_\mu \Phi) - \frac{1}{4}(F_{\mu\nu})^2 - V(\Phi).$$

The potential is given by

$$V(\Phi) = -m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2.$$

The Lagrangian is invariant under

$$\begin{cases} \Phi \to U(x)\,\Phi \\ \tau^a A^a_\mu \to \frac{i}{g} U(x)\,\partial_\mu\, U^\dagger(x) \end{cases}$$ where $U(x) = e^{i\vec{\tau}\cdot\vec{\alpha}(x)}$.

(recall $[\tau^i, \tau^j] = i\,\epsilon^{ijk}\tau^k$)

The field Φ acquires a VEV.

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

, or at any other value related by an SU(2) transformation. Next we expand the kinetic term around the VEV:

$$|\partial_\mu \Phi|^2 = \frac{1}{2} g^2 (0, v)\, \tau^a \tau^b \begin{pmatrix} 0 \\ v \end{pmatrix} A^a_\mu A^{b\mu} + \ldots =$$

$\{\tau^a, \tau^b\} = \frac{1}{2}\delta^{ab}$

$$= \frac{g^2 v^2}{8} A^a_\mu A^{a\mu} \qquad \Longrightarrow \qquad m_A = \frac{gv}{2}.$$

Therefore all the generators are broken by $\langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$, all three gauge bosons acquire a mass m_A.

	$v = 0$	$v \neq 0$
# d.o.f	• 1 SU(2) doublet = 4 real scalars • 3 massless gauge bosons = 2 × 3 transverse polarizations	• 1 real scalar • 3 massive gauge bosons = (2 + 1) × 3 polarizations (transverse, Longitudinal)

i.e. we count the number of real scalar fields.

$$6 + 4 = 1 + (2+1) \times 3$$

In the SM the electroweak symmetry $SU(2)_L \otimes U(1)_Y$ is spontaneously broken. The model is a combination of the previous two examples.

GWS theory:

Recall that the quantum numbers of the Higgs are given by

Field	Names	Spin	$U(1)_Y$	$SU(2)_L$	$SU(3)_c$
H	Higgs	0	1/2	2	1

Therefore the Higgs field transforms as

"w/" means "with"

$$H(x) \longrightarrow U(x)H(x) \quad \text{w/} \quad U(x) = e^{i\vec{\alpha}(x)\vec{\tau}} e^{i\beta/2}$$

$SU(2)_L \otimes U(1)_Y$

i.e. it is a doublet of $SU(2)_L$ and has charge $+1/2$ under $U(1)_Y$.
The covariant derivative is given by

$$D_\mu H = \Big(\partial_\mu - i\underbrace{g\vec{A}_\mu}_{\text{3 gauge bosons of } SU(2)_L}\vec{\tau} - i\underbrace{g'\tfrac{1}{2}B_\mu}_{\text{1 gauge boson of } U(1)_Y}\Big)H$$

The Higgs potential is such that $H(x)$ acquires a VEV given by

$$\langle H \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}.$$

Note that the gauge transformation w/ $\alpha^1 = \alpha^2 = 0$ and $\alpha^3 = \beta$ leaves $\langle H \rangle$ invariant. Therefore not all of the global $SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken by the VEV. The theory contains 1 massless gauge boson, while 3 gauge bosons acquire a mass. Let's see the mechanism in detail!
The gauge boson mass comes from the term

$$(D_\mu H)^\dagger (D_\mu H)\Big|_{H=\langle H\rangle} = \frac{1}{2}(0, v)\left(g A_\mu^a \tau^a + \frac{1}{2}g' B_\mu\right)\left(g A^{b\mu}\tau^b + \frac{1}{2}g' B^\mu\right)\begin{pmatrix} 0 \\ v \end{pmatrix} + \dots$$

$$= \frac{1}{2}\frac{v^2}{4}\left[g^2 (A_\mu^1)^2 + g^2 (A_\mu^2)^2 + (-g A_\mu^3 + g' B_\mu)^2\right] + \dots$$

Therefore, there are 3 massive gauge bosons:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(A_\mu^1 \mp i A_\mu^2\right) \qquad \text{with mass} \qquad m_W^2 = \frac{g^2 v^2}{4}$$

$$Z_\mu^0 = \frac{1}{\sqrt{g^2+g'^2}}\left(g A_\mu^3 - g' B_\mu\right) \qquad \text{with mass} \quad m_Z^2 = \frac{g^2+g'^2}{4} v^2 .$$

The 4th vector, orthogonal to Z is massless

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g' A_\mu^3 + g B_\mu\right) \qquad \text{with mass} \quad m_A = 0$$

→ this is the photon!

END OF L3

From now on we will find it more convenient to work with mass eigenstates.
Consider a fermion belonging to an $SU(2)_L$ irrep and with $U(1)_Y$ charge "Y".
Then, the covariant derivative is

matrices fullfilling the su(2) algebra.

$$D_\mu = \partial_\mu - i g A_\mu^a T^a - i g' Y B_\mu$$

We rotate the covariant derivative into

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}\left(W_\mu^+ T^+ + W_\mu^- T^-\right) - i\frac{1}{\sqrt{g^2+g'^2}} Z^\mu\left(g^2 T^3 - g'^2 Y\right) - i\frac{g g'}{\sqrt{g^2+g'^2}} A_\mu\left(T^3 + Y\right)$$

The normalization is such that in the spinor irrep $T^\pm = \frac{1}{2}(\sigma^1 \pm i\sigma^2) = \sigma^\pm$.
Why is the last term proportional to T^3+Y? Well, this precisely corresponds to the symmetry operation $\alpha^1 = \alpha^2 = 0$; $\alpha^3 = \beta$!
Now we can identify the electric charge in terms of the more "fundamental" parameters

$$e = \frac{g g'}{\sqrt{g^2+g'^2}} \qquad \text{and} \qquad Q = T^3 + Y\mathbb{1}$$

electron charge

electric charge quantum number, e.g. $Q = -1$ for the electron

It is also convenient to define the "weak" or "weinberg" mixing angle:

$$\begin{pmatrix} Z^0 \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A^3 \\ B \end{pmatrix}$$

where $\cos\theta_W = g/\sqrt{g'^2+g^2}$ and $\sin\theta_W = g'/\sqrt{g^2+g'^2}$. Therefore the couplings of the Z boson are given by:

$$g^2 T^3 - g'^2 Y = (g^2+g'^2)T^3 - g'^2 Q .$$

All in all the covariant derivative in the mass eigenstates can be written as:

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}\left(W_\mu^+ T^+ + W_\mu^- T^-\right) - i\frac{g}{c_W} Z_\mu\left(T^3 - s_W^2 Q\right) - ieA_\mu Q$$

where $g = e/s_W$ and $c_W = \cos\theta_W$, $s_W = \sin\theta_W$.

Therefore all the couplings of $W^\pm$'s, Z and A are controlled by the parameters

$$e \quad , \quad \theta_W$$

well measured electron charge

new parameter

+ gauge boson masses are related as

$$m_W = m_Z \cos\theta_W$$

at tree level. Loop corrections give a small modification.

Therefore all (tree level) phenomenology of W and Z exchange is parametrised by $\{m_W, e, \theta_W\}$*

* and V_{CKM}.

Next we need to discuss further the coupling of the electroweak vector bosons to the SM fermions.

EW boson couplings to fermions:

We'd like to couple fermions to the gauge bosons. In order to achieve this, it is enough to specify the quantum numbers, or representations. Also, we'd like to couple differently left and right handed fermions.

This is straightforward at the level of the Lagrangian. Indeed, recall that the kinetic term of the Dirac fields splits as

$$\bar{\psi}\, i\not{\partial}\, \psi = \bar{\psi}_L\, i\not{\partial}\, \psi_L + \bar{\psi}_R\, i\not{\partial}\, \psi_R$$

where $\psi_{L,R} = \left(\frac{1 \mp \gamma_5}{2}\right)\psi$ w/ $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$; $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.

Only left handed field couple to W bosons! Left handed quarks and leptons are doublets of $SU(2)_L$, while right handed fields are singlets of $SU(2)_L$. The hypercharge Y assignments (charge under $U(1)_Y$) are such that

$$Q = T_3 + Y\mathbb{1}$$

matches. The right-handed fields have $T_3 = 0$ so $Q = Y$.

Therefore $Y_u = +\frac{2}{3}$, while e_R has $Y = -1$.

Left handed fields

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad ; \quad Q = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$Y = -\tfrac{1}{2} \; ; \; T_3 = \pm\tfrac{1}{2} \qquad\qquad Y = +\tfrac{1}{6} \; ; \; T_3 = \pm\tfrac{1}{2}.$$

Knowing the quantum numbers, we can immediately write down the kinetic terms and covariant derivatives:

$$\mathcal{L} = \bar{L}\, i\not{D} L + \bar{e}_R\, i\not{D}\, e_R + \bar{Q}_L\, i\not{D}\, Q_L + \bar{u}_R\, i\not{D}\, u_R + \bar{d}_R\, i\not{D}\, d_R$$

e.g. $$\bar{Q}_L\, i\not{D}\, Q_L = \bar{Q}_L\, i\gamma^\mu\big(\partial_\mu - i g A^a_\mu \tau^a - i \underbrace{Y_{Q_L}}_{=1/6}\, g' B_\mu\big) Q_L$$

Next, using the covariant derivative and the mass eigenstates for the gauge bosons, we can work out

$$\mathcal{L} = \sum_{\psi = Q,u,d,L,e} \bar{\psi}\, i\not{\partial}\, \psi + g\left(W_\mu^+ J_{W^+}^\mu + W_\mu^- J_{W^-}^\mu + Z_\mu J_Z^\mu\right) + e A_\mu J_{EM}^\mu$$

where

- $$J_{W^+}^\mu = \frac{1}{\sqrt{2}}\left(\bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L\right)$$

only left handed fermions!

- $$J_{W^-}^\mu = \frac{1}{\sqrt{2}}\left(\bar{e}_L \gamma^\mu \nu_L + \bar{d}_L \gamma^\mu u_L\right)$$

$\frac{g'^2}{g^2+g'^2}$

- $$\begin{aligned} J_Z^\mu = \frac{1}{c_W}\Big[& \bar{\nu}_L \gamma^\mu \left(\tfrac{1}{2}\right) \nu_L + \bar{e}_L \gamma^\mu \left(-\tfrac{1}{2} + s_W^2\right) e_L + \bar{e}_R \gamma^\mu s_W^2 e_R \\ & + \bar{u}_L \gamma^\mu \left(\tfrac{1}{2} - \tfrac{2}{3} s_W^2\right) u_L + \bar{u}_R \gamma^\mu \left(-\tfrac{2}{3} s_W^2\right) u_R \\ & + \bar{d}_L \gamma^\mu \left(-\tfrac{1}{2} + \tfrac{1}{3} s_W^2\right) d_L + \bar{d}_R \gamma^\mu \left(\tfrac{1}{3} s_W^2\right) d_R \Big] \end{aligned}$$

- $$J_{EM}^\mu = \bar{e}\gamma^\mu e\, \underset{-1}{Q_e} + \bar{u}\gamma^\mu u\, \underset{+2/3}{Q_u} + \bar{d}\gamma^\mu d\, \underset{-1/3}{Q_d}$$

(So far in this theory there are no mass terms for fermions. $\Delta\mathcal{L} = -m_e(\bar{e}_L e_R + \bar{e}_R e_R)$ is forbidden by symmetries. It is useful to think of L and e_R as different particles)

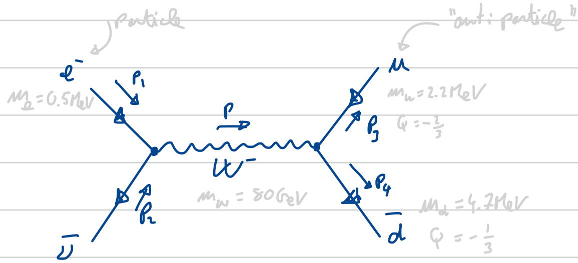
We'll cover fermion mass terms later on. At the moment, knowing the gauge couplings to fermions we are ready to discuss some experimental consequences

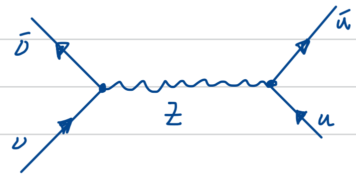
* ## Experimental consequence of GWS, part I:

We have a theory of massive gauge bosons and $U(1)_{EM}$. We also know the couplings to fermions (1st family so far)

We can work out any process mediated by gauge bosons.

For instance:

(a) = 

(b) = 

The propagators of the W's and Z is given by

$$\langle W^{+\mu}(p)\, W^{-\nu}(-p)\rangle = \frac{-i}{p^2 - m_W^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2 - \xi m_W^2}(1-\xi)\right)$$

$$\overset{\xi = 1}{=} \frac{-i g^{\mu\nu}}{p^2 - m_W^2}$$

$$\langle Z^\mu(p) Z^\nu(-p)\rangle = \text{iden w/ } m_W \leftrightarrow m_Z .$$

It is straightforward to compute these Feynman diagrams. For instance:

$$(a) = \underbrace{\frac{ig}{\sqrt{2}}\, \bar{v}_\nu(p_2)\gamma^\mu u_e(p)}_{\text{Fermion current}}\ \underbrace{\frac{-i g^{\mu\nu}}{p^2 - m_W^2}}_{\text{Propagator}}\ \underbrace{\frac{ig}{\sqrt{2}}\, \bar{u}_d(p_4)\gamma^\nu v_u(p_3)}_{\text{Fermion current.}}$$

Are there left handed fields? Easy to check.

$$u(p) \to u_{\text{boosted}} = \sqrt{2E}\begin{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\\ \begin{pmatrix}0\\0\end{pmatrix}\end{pmatrix}$$ under a large boost in the z-direction.

The helicity operator is given by $h = \hat{p}\cdot\hat{S} = \frac{1}{2}\begin{pmatrix}\sigma_3 & 0_{2\times2}\\ 0_{2\times2} & \sigma_3\end{pmatrix}$.

It turns out $h \cdot u_{\text{boosted}} = -\frac{1}{2} u_{\text{boosted}}$. Therefore it is left-handed!

(I am following Peskin & Schroeder page 47).

Now, at low energies

$$(a) \simeq + \frac{g^2}{2m_W^2}\underbrace{(\bar{\nu}_L \gamma_\mu e_L)(\bar{d}_L \gamma^\mu u_L)}_{\text{what is the dimension?!}} + O\left(E^4/m_W^4\right)$$

This is nothing but Fermi's theory of 4-fermions. Therefore

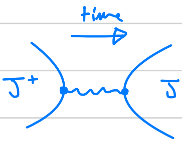
$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

Q: What is the full Effective Field Theory Lagrangian?

A: We can work it out in full generality starting with the SM Lagrangian:

$$\mathcal{L}_{SM} \supset g W_\mu^+ J^\mu_{W^+} + g W_\mu^- J^\mu_{W^-}$$

Next, we compute a process mediated by the $W^\pm$'s:



$$= \int d^4x\, d^4y\, (-1) g^2 \langle p_1 p_2 | J_\mu^+(x) \langle W^{\mu+}(x) W^{-\nu}(y)\rangle J_\nu^-(y) | p_3 p_4\rangle$$

$$= (-1)\, g^2 \int d^4x\, d^4y \, \langle P_1 P_2 | \, J_\mu^+(x) \int \frac{d^4p}{(2\pi)^4} \frac{-i g^{\mu\nu}}{p^2 - m_W^2 + i\epsilon} e^{ip\cdot(x-y)} J_\nu^-(y) \, | P_3 P_4 \rangle =$$

$$= (-1)\, g^2 \left(\langle P_1 P_2 | \, J_\mu^+(P_1, P_2) \right) e^{i(P_1+P_2)x} \left(\int \frac{d^4p}{(2\pi)^4} \frac{-i g^{\mu\nu}}{p^2 - m_W^2} e^{iP(x-y)} \right) e^{-i(P_3+P_4)y} \left(J_\nu^-(P_3, P_4) | P_3 P_4 \rangle \right)$$

$$= (-1)\, g^2 \int \frac{d^4p}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(P - P_1 - P_2)\, (2\pi)^4 \delta^{(4)}(P - P_3 - P_4) \left(\langle P_1 P_2 | \, J_\mu^+(P_1 P_2) \right) \frac{-i g^{\mu\nu}}{p^2 - m_W^2 + i\epsilon} \left(J_\nu^-(P_3, P_4) | P_3 P_4 \rangle \right)$$

$$= \frac{-i g^2}{m_W^2} (2\pi)^4 \delta^{(4)}(P_1 + P_2 - P_3 - P_4) \left(\langle P_1 P_2 | \, J_\mu^+(P_1 \cdot P_2) \right) \cdot \left(J^{-\mu}(P_3, P_4) | P_3 P_4 \rangle \right)$$

$$= -\, i \int d^4x \, \langle P_1 P_2 | \, \frac{g^2}{m_W^2} J_\mu^+(x)\, J^{-\nu}(x) \, | P_3, P_4 \rangle$$

Therefore $\Delta \mathcal{L}_W = \frac{g^2}{m_W} J_\mu^+ J_\nu^-$.

Thus we have derived the Fermi theory for β-decay. In a similar fashion we can work out the effective field theory interactions due to the exchange of the Z boson.

$$\Delta \mathcal{L}_Z = \frac{g^2}{2 m_Z^2} J_Z^\mu J_{Z\mu} = \frac{4 G_F}{\sqrt{2}} \Big(\sum_\psi \bar{\psi} \gamma^\mu (T^3 - s_W^2 Q) \psi \Big)^2$$

Adding up both Lagrangians:

$$\Delta \mathcal{L}_W + \Delta \mathcal{L}_Z = \frac{4 G_F}{\sqrt{2}} \left((J_\mu^1)^2 + (J_\mu^2)^2 + (J_\mu^3 - s_W^2 J_{EM}^\mu)^2 \right)$$

where $J_\mu^a = \sum_\psi \bar{\psi} \gamma^\mu T^a \psi$. We observe that in the limit $\theta \to 0$, we have $s_W^2 = \sin^2\theta_W \to 0$ (i.e. $g' \to 0$). Then $\Delta \mathcal{L}_W + \Delta \mathcal{L}_Z$ is invariant under $SO(3) = SU(2)$ rotations of the vector (J^1, J^2, J^3). This is called custodial symmetry, we will comment more about it later on. Recall $s_W^2 \approx 0.22 = 1 - \left(\frac{m_W}{m_Z}\right)^2$.

$\Delta \mathcal{L}$ violates parity! Simply because it is not invariant under

$$\bar{\psi}(x) \gamma^\mu (1 - \gamma_5) \psi(x) \longrightarrow (-1)^\mu \bar{\psi}(-x) \gamma^\mu (1 + \gamma_5) \psi(-x)$$

This is the only interaction that violates parity at the fundamental level.

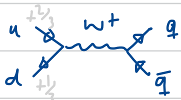
In the GWS theory the only two parameters are G_F and $\sin^2\theta_W$, and many predictions!

Beyond the EFT, many tests of the theory are possible at high energy.

For instance, the process $e^+e^- \longrightarrow f\bar{f}$ is affected in an essential way since the theory contains a new diagram w/ s-chanel Z-boson exchange which interferes with the photon exchange diagram

We leave as homework exercise to compute the interference between these processes.

The Z boson appears as a resonance as we increase the energy and so does $W^\pm$ in a process like

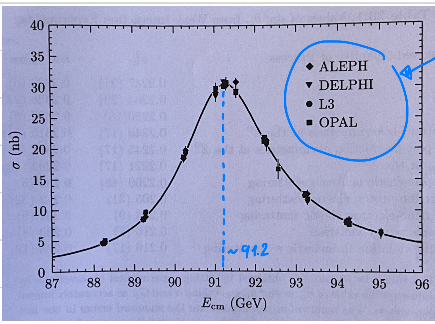


The position of the resonance is predicted to be at

$$m_Z^2 = \frac{\pi\alpha}{\sqrt{2}\, G_F\, s_W^2\, c_W^2} \qquad \& \qquad m_W^2 = \frac{\pi\alpha}{\sqrt{2}\, G_F\, s_W^2}$$

Experimental measurement of the Z-boson resonance:

Measurement at Large Electron Positron (LEP) collider, from $e^+e^- \to$ hadrons.

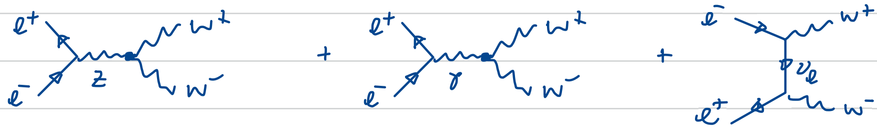


experiments inside the LEP facility.

[Phys. Rev. D50 (1994) Fig 32.14]

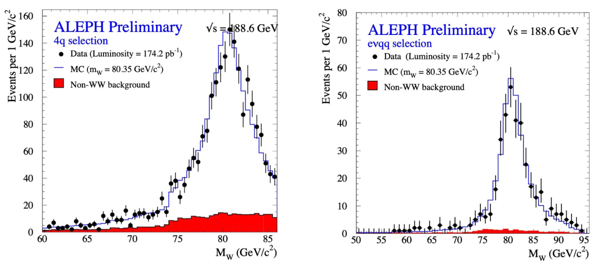
W's and Z were discovered at the UA1 and UA2 experiments @ CERN in 1983!

LEP I worked at the Z-boson mass C.O.M. energy in order to measure with precission the Z-boson resonance shape. Afterwards LEP I was upgraded into LEP II in order to measure the W mass. The LEP II energy was ~161 GeV so that it could produce a pair of W's



The $W^\pm$ pair subsequently decay into 4 quarks $q\bar{q}q\bar{q}$ ($Br \sim 46\%$), or $q\bar{q}\,\ell\nu$ ($Br \sim 44\%$) or $\ell\nu\ell\nu$ (10%).

For instance, these are measurements of the AEPH experiment.



[hep/ex-9912055]

These were very important precision measurements at the time, now there are independent precise measurements from CDF and DØ (at TeVatron) and ATLAS and CMS (at CERN).

END OF L4

A crucial test of the GWS theory is the following: the theory treats different the Left and Right handed fermions. One manifestation of this is in the "polarization asymmetry": a net polarization in the $Z \to f\bar{f}$ decays:

$$A_{LR}^f \equiv \frac{\Gamma(Z \to f_L \bar{f}_R) - \Gamma(Z \to f_R \bar{f}_L)}{\Gamma(Z \to f_L \bar{f}_R) + \Gamma(Z \to f_R \bar{f}_L)} =$$

$$= \frac{\left(\frac{1}{2} - |Q_f|\, s_W^2\right)^2 - \left(Q_f\, s_W^2\right)^2}{\left(\frac{1}{2} - |Q_f|\, s_W^2\right)^2 + \left(Q_f\, s_W^2\right)^2} \,,$$

which can be easily derived from $Z^\mu \bar{f}_{L,R}\, \gamma^\mu f_{L,R}$. It turns out that, for $s_W^2 \equiv \sin^2\theta_W = 0.23$, $A_{LR}^{\ell^+} \simeq 15\%$ and $A_{LR}^{d,s,b} \simeq 95\%$.

The theory is highly predictive. For instance, fix α and G_F, then:

Observed Quantity or Process	s_W^2
m_Z	0.2247 (21)
m_W	0.2264 (25)
Γ_Z	0.2250(18)
Lepton f-b asymmetries at the Z^0	0.2243 (17)
All pair-production asymmetries at the Z^0	0.2245 (17)
A^e_{LR} at the Z^0	0.2221 (17)
Deep inelastic neutrino scattering	0.2260 (48)
Neutrino-proton elastic scattering	0.205 (31)
Neutrino-electron elastic scattering	0.224 (9)
Atomic parity violation	0.216 (8)
Parity violation in inelastic e^- scattering	0.216 (17)

Some of the most relevant measurements of

$$s_W^2 = 1 - \frac{m_W^2}{m_Z^2} \,,$$

a prediction of GWS theory.

We will have more to say about EW precision tests later on. For the moment we are done with "Experimental consequences of GWS, part I", and now we move on to discuss the Yukawa couplings and Fermion mass terms.

FLAVOR

Next we'll discuss the Higgs coupling to fermions and mass terms. We assumed a scalar doublet with VEV $\langle H(x)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$ and with $Y = 1/2$, so that we can give mass to the W's and Z.

With these quantum numbers we can write the following invariant under G_{SM} and the Lorentz group:

Lorentz contracted

$$\Delta \mathcal{L}_e = - y_e\, \bar{L}\cdot e_R \cdot H + h.c.$$

a new dimensionless coupling: the "Yukawa"

$SU(2)_L$ contracted

Next we set $H \to \langle H\rangle$ and we get

$$\Delta \mathcal{L}_e = -\frac{1}{\sqrt{2}}\, y_e v\, \bar{e}_L \cdot e_R + h.c.$$

Therefore $m_e = \frac{1}{\sqrt{2}} y_e v$. Note that $\frac{m_e}{m_W} \simeq \frac{0.5\,\text{MeV}}{80\,\text{GeV}} = \frac{1\ 10^{-3}\,\text{GeV}}{2\cdot 80\,\text{GeV}} = 6\cdot 10^{-6}$. This ratio can be accomodated by taking a tiny value for $y_e \simeq 3\cdot 10^{-6}$!

The smallness of y_e has no deep explanation within the SM.

We can follow a similar logic for quarks:

$$\Delta \mathcal{L}_q = - y_d\, \bar{Q}\cdot H d - y_u\, (\bar{Q})_a (H^\dagger)_b\, \varepsilon^{ab} u_R + h.c.$$

or using $\tilde{H} \equiv i\sigma_2 H^*$ we can write the up-quark sector as $y_u \bar{Q}\cdot\tilde{H} u_R + h.c.$ $\Delta\mathcal{L}_q$ is invariant under G_{SM}. These are <u>all</u> the operators we can write with dimension $\Delta \le 4$. Again, substituting $\langle H(x)\rangle = 1/\sqrt{2}\begin{pmatrix}0\\ v\end{pmatrix}$ we get masses for the up and down quarks:

$$\Delta \mathcal{L}_q = -\frac{1}{\sqrt{2}} y_d v\, \bar{d}_L d_R - \frac{1}{\sqrt{2}} y_u v\, \bar{u}_L u_R + h.c.$$

thus $m_u = \frac{1}{\sqrt{2}} y_u v$ and $m_d = \frac{1}{\sqrt{2}} y_d v$. Similar to what happens with the electron, the theory "parametrizes" the quark masses in terms of couplings and the VEV but does not "explain" why they are much smaller than v.

We wrote down Yukawa interactions and masses of quarks. <u>But</u> we should remember that there are 3 families, thus 3 left-handed doublets

$$Q^i = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} c \\ s \end{pmatrix}_L , \begin{pmatrix} t \\ b \end{pmatrix}_L \right)$$

$Y = +\frac{1}{6}$

and 6 right-handed $SU(2)_L$ singlets:

$$u_R^i = (u_R, c_R, t_R) \qquad \& \qquad d_R^i = (d_R, s_R, b_R)$$

We couple them to G_{SM} gauge bosons through $\partial_\mu \to D_\mu$ which is flavour diagonal, i.e.

$$\sum_{j=1}^{3} \bar{Q}^j i \not\partial Q^j \to \sum_{j=1}^{3} \bar{Q}^j i \not D Q^j ,$$

and similarly for the right handed quarks. Therefore, no mixing between families ?? Not quite. Note that the couplings of the Higgs to the quarks is a priori an arbitrary 3×3 complex matrix.

($\rightarrow$ we could declare a new <u>global</u> symmetry like for instance the discrete transformation: $(d_1, d_2, d_3) \longrightarrow (e^{i\frac{2\pi}{3}} d_1, e^{i\frac{4\pi}{3}} d_2, 1\, d_3)$
This strategy does not work.)

Therefore, in general

$$\mathcal{L}_q = - y_d^{ij} \bar{Q}_L^i H d_R^j - y_u^{ij} \bar{Q}_L^i \tilde{H} u_R^j + h.c.$$

where y_d and y_u are generic complex-valued matrices (not symmetric, not hermitian). Under CP

$$\bar{Q}_L \cdot H d_R \longrightarrow h.c.$$

thus if y_d was real then the interaction is CP-symmetric. In general however $y_d^{ij} \bar{Q}_L^i H d_R^j$ violates CP and all flavour symmetries!

↳ we'll see what are they soon.

In order to proceed and clarify the number of independent physical parameters, we perform the singular value decomposition $M = U D V^+$

complex matrix. unitary matrices. diagonal matrix.

In the introduction I used different notation $y_f = L_f \, y_f^d R_f^+$

Then $y_f = U_f \, y_f^d \, V_f^+$ and perform the rotations

$$u_R^i \to (V_u)^i{}_j \, u_R^j \; ; \; d_R^i \to (V_d)^i{}_j \, d_R^j \; ; \; u_L^i \to (U_u)^i{}_j \, u_L^j \; ; \; d_L^i \to (U_d)^i{}_j \, d_L^j$$

which completely eliminates flavour changing couplings in the Yukawa sector. Indeed, taking the unitary gauge:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} ,$$

we get

$$\mathcal{L}_q = -m_d^i \, \bar{d}_L^i d_R^i \left(1 + \frac{h}{v}\right) - m_u^i \, \bar{u}_L^i u_R^i \left(1 + \frac{h}{v}\right)$$

this interactions conserve P, C, T and are diagonal in flavour!

The left handed u_L and d_L quarks have identical couplings to QCD. Therefore the rotations $u_L^i \to (U_u)^i{}_j \, u_L^j$; $d_L^i \to (U_d)^i{}_j \, d_L^j$ commute with the QCD covariant derivative.

However, u_L and d_L are part of a doublet and get mixed by weak interactions:

$$\bar{Q}_L i \not{D} Q_L = \underbrace{\bar{Q}_L i \not{\partial} Q_L + e A_\mu J^\mu_{EM} + g Z_\mu J^\mu}_{} + \underbrace{g W^+_\mu J^+_\mu + g W^-_\mu J^-_\mu}_{} .$$

Invariant under $u_L \to U_u u_L$ and $d_L \to U_d d_L$ because $\bar{u}_L^i \gamma^\mu u_L^i \to \bar{u}_L^i U_u^+ \gamma^\mu U_u u_L = \bar{u}_L \gamma^\mu u_L$ // and similarly for d_L current.

However, the J_μ^+ current is not invariant:

$$J_\mu^+ = \frac{1}{\sqrt{2}} \bar{u}_L \gamma_\mu d_L \longrightarrow \frac{1}{\sqrt{2}} \bar{u}_L \gamma^\mu (U_u^\dagger U_d) d_L$$

Therefore, charged currents of the weak interactions can mix quark families!

The matrix is called Cabbibo-Kobayashi-Maskawa mixing matrix.

$$V_{CKM} = U_u^\dagger U_d$$

The CKM matrix is a priori a generic unitary matrix. We can remove some of the parameters by rotations. Let's analyze the 2×2 case, i.e. two families (u,d) and (c,s). Then V_{CKM} has a priori 4 parameters

Cabbibo

$$V_c = \begin{pmatrix} \cos\theta_c \, e^{i\alpha} & \sin\theta_c \, e^{i\beta} \\ -\sin\theta_c \, e^{i(\alpha+\gamma)} & \cos\theta_c \, e^{i(\beta+\gamma)} \end{pmatrix},$$

one rotation angle θ_c and 3 phases. We can remove the phases by $q_L^i \longrightarrow e^{i\alpha^i} q_L^i$, $i = u, d, c, s$. thus 4 phases. A phase rotation that is equal for the 4 left-handed quarks leaves invariant $\bar{U}_L^i (V_c)_{ij} d_L^j$. The remaining 3 phases can be used to remove α, β and γ. Therefore

$$V_c = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}, \quad \text{and} \quad J^{\mu+} \overset{\text{quarks}}{=} \frac{1}{\sqrt{2}}\Big(\cos\theta_c \, \bar{u}_L \gamma^\mu d_L + \sin\theta_c \, \bar{u}_L \gamma^\mu s_L - \sin\theta_c \, \bar{c}_L \gamma^\mu d_L + \cos\theta_c \, \bar{c}_L \gamma^\mu s_L\Big).$$

It turns out $\theta_c \simeq 13°$.

We have eliminated the complex phases. Therefore there is no CP-violation in processes that involve only two families!

The story for the lepton sector is slightly simpler because we only have one type of right handed field $e_R^i = (e_R, \mu_R, \tau_R)^i$. Therefore

$$\bar{L}\, i\not{\partial} L + \bar{e}_R i\not{\partial} e_R - y_e \bar{L} e_R H + h.c. \xrightarrow{y_e = U_e y_e^d V_e^\dagger} \bar{L}\, i\not{\partial} L + \bar{e}_R i \not{\partial} e_R - y_e^d \bar{L} e_R H + h.c.$$

$e_R^i \longrightarrow (V_e)^i_{\ j} e_R^j$

$L^i \longrightarrow (U_e)^i_{\ j} L^j$

Let's next analyze the 3×3 flavour structure of the quark sector by counting all the independent parameters:

$$\mathcal{L} = \sum_i (\bar{Q}_L^i, u_R^i, d_R^i, L_L^i, e_R^i)\, i\not{D}\, (Q_L^i, u_R^i, d_R^i, L_L^i, e_R^i)^T$$

$$- y_d^{ij}\, \bar{Q}_L^i H d_R^j - y_u^{ij}\, \bar{Q}_L^i \tilde{H} u_R^j - y_e^{ij}\, \bar{L}_L^i H e_R^j + h.c.$$

The first line is invariant under

$$\underbrace{G_F}_{\text{Flavour}} = \mathcal{U}(3)^{\otimes 5} = \mathcal{U}(3)_q^3 \otimes \mathcal{U}(3)_e^2 \quad , \text{ globally.}$$

where

$$\begin{cases} \mathcal{U}(3)_q^3 = \mathcal{U}(3)_Q \otimes \mathcal{U}(3)_u \otimes \mathcal{U}(3)_d \\ \mathcal{U}(3)_e^2 = \mathcal{U}(3)_L \otimes \mathcal{U}(3)_e \end{cases}$$

Namely, each different type of fermion can be rotated independently:

e.g.: $M \in \mathcal{U}(3)_Q \qquad \bar{Q}_L i\not{D} Q_L \longrightarrow \bar{Q}_L M^\dagger i\not{D} M Q_L = \bar{Q}_L i\not{D} Q_L$.

The Yukawa sector, in the second line, breaks explicitly the symmetry

$$G_F \longrightarrow \underbrace{\mathcal{U}(1)_B}_{\substack{\text{Baryon} \\ B = \frac{1}{3}(n_q - n_{\bar{q}})}} \otimes U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau \otimes U(1)_Y$$

END OF L5

there remains an accidental symmetry $G_F \supset U(1)^{\otimes 5}$ in the SM, for the operators of Dimension $\Delta \leq 4$! $U(1)_Y$ is gauged, so it is not an accidental symmetry after all.

$$\mathcal{L} = \mu^2 \mathcal{L}_2 + \mathcal{L}_4 + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \dots$$

$m_\nu = 0$

$U(1)_{L_1} \otimes U(1)_{L_2} \otimes U(1)_{L_3} \otimes U(1)_B$

~~Flavor~~ & ~~CP~~ $\longleftrightarrow Y_u, Y_d$

GIM mechanism

$\cancel{U(1)_L}$	$m_\nu \neq 0$
~~Flavor~~	$\mu \to e\gamma$, Δm_K
~~CP~~	edms
$\cancel{U(1)_B}$	$p \to \pi_0 e^+$

Counting parameters:

Physical parameters are those that can not be rotated away by performing field redefinitions (i.e. changing coordinates in the path integral). In particular phase transformations or flavor rotations:

$$\mathcal{L}_Y \supset -y_e^{ij}\,\bar{L}^i H e_R^j + h.c. \xrightarrow[\substack{L \to UL \\ e \to Ve}]{} -(y_e^d)_{ii}\,\bar{L}^i H e_R^i + h.c.$$

Let's count parameters: Y_e has 9 real and 9 phases.

U and V are unitary matrices, thus contain

$2 \times (3 \text{ real} + 6 \text{ phases})$

$SO(3) \subset SU(3)$
↳ 3 angles of rotation.
The rest of $SU(3)$ are phases.

$$9 \text{ real} + 9 \text{ phases} - 2 \times (3 \text{ real} + 6 \text{ phases} - 3) = 3 \text{ real masses.}$$

↑ phases of $U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau$.

Next let's do the same for the quark sector and V_{CKM}. We did three unitary transformations and were left with the V_{CKM} unitary matrix.

Y_u and Y_d have $2 \times (9 \text{ real} + 9 \text{ phases})$ parameters. The three 3×3 unitary matrices have $3 \times (3 \text{ real} + 6 \text{ phases})$ + unbroken global phase transformation $(Q_L, u_R, d_R) \to e^{i\phi}(Q_L, u_R, d_R)$. Therefore

$$2 \times (9 \text{ real} + 9 \text{ phases}) - 3 \times 3 \text{ real} - 3 \times 6 \text{ phases} + 1 \text{ phase}$$

$$= 9 \text{ real} + 1 \text{ phase} = 6 \text{ quark masses} + 3 \text{ mixing angles} + 1 \text{ CP phase}$$

Conventionally

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The CKM matrix is the product of 3 rotations with one phase in the θ_{13} mixing. Experimentally $\theta_{12} \gg \theta_{23} \gg \theta_{13}$ and $\delta \sim O(1)$.

We said that $G_F = U(3)_q^3 \otimes U(3)_\ell^2 \xrightarrow[\text{to}]{\text{broken}} U(1)_B \otimes U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau \otimes U(1)_Y$

by the Yukawa couplings. Let's turn one at a time:

$*$ $Y_e \not\propto \mathbb{1} \Rightarrow U(3)_L \otimes U(3)_e \longrightarrow U(1)_e \otimes U(1)_\tau \otimes U(1)_\mu$

i.e. accidentally charged lepton family number is conserved!

$*$ $Y_u \not\propto \mathbb{1} \Rightarrow U(3)_q \otimes U(3)_u \longrightarrow U(1)_u \otimes U(1)_c \otimes U(1)_t$

i.e. up-quark family number

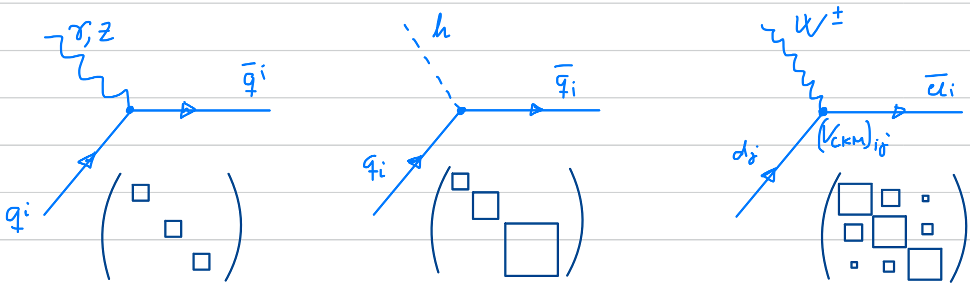
$*$ $Y_d \not\propto \mathbb{1} \Rightarrow U(3)_q \otimes U(3)_d \longrightarrow U(1)_d \otimes U(1)_s \otimes U(1)_b$

i.e. down-quark family number.

$*$ Collective symmetry breaking

since $[Y_u, Y_d] \neq 0$ $\quad U(1)_q^6 \longrightarrow U(1)_B$

Summary of fermion couplings:

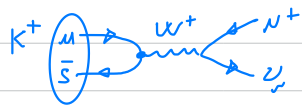


Very important distinction between flavour changing <u>neutral</u> and <u>charged</u> currents.

* FCNC's are processes in which the quark flavor changes but the charge doesn't.
* Charged currents can change both charge and flavour.

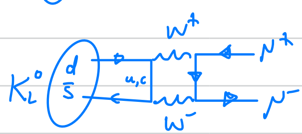
<u>Examples of charged currents v.s. neutral</u>

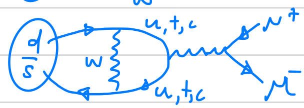
* $s \longrightarrow u\,\mu^-\bar{\nu}_\mu$: $B_r(K^+ \to \mu^+\nu) \simeq 64\%$



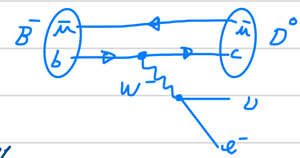
v.s.

* $s \longrightarrow d\,\mu^+\mu^-$: $Br(K_L \to \mu^+\mu^-) \simeq 7\cdot 10^{-9}\%$



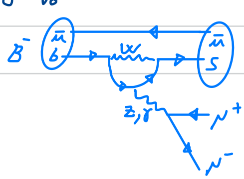


* $b \longrightarrow c\,e^-\nu_e$: $B_r(B^- \to D^0\ell\bar{\nu}) \simeq 2.3\%$

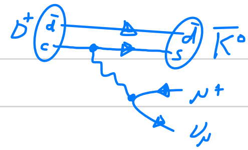


v.s.

* $b \longrightarrow d\,\mu^+\mu^-$: $Br(B^- \to K^*\ell^+\ell^-) \simeq 5\cdot 10^{-9}\%$

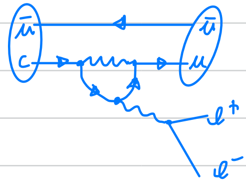


* $c \longrightarrow s\,\mu^+ \nu_\mu$: $Br(D^\pm \longrightarrow K^0 \mu^\pm \nu) \simeq 9\%$



V.S.

* $c \longrightarrow u\, \ell^+ \ell^-$: $Br(D^0 \longrightarrow \pi^0 \ell^+ \ell^-) \simeq 1.8 \cdot 10^{-4}\,\%$



V_{CKM} STRUCTURE & MEASUREMENTS

Wolfenstein parametrization of the CKM matrix.

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \quad ;$$

where $\lambda \equiv |V_{us}| \simeq 0.22$ and $\lambda, A, \rho, \eta = O(1)$. A global fit gives

$(A, \lambda, \bar{\rho}, \bar{\eta}) = (0.82, 0.22, 0.16, 0.35)$ w/ $\bar{\rho} + i\bar{\eta} \equiv -\dfrac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}$

and $\bar{\rho} = \rho(1 - \lambda^2/2) + O(\lambda^4)$; $\bar{\eta} = \eta(1 - \lambda^2/2) + O(\lambda^4)$.

The Wolfstein parametization encodes $V_{CKM}^\dagger V_{CKM} = V_{CKM} V_{CKM}^\dagger = \mathbb{1}$ in simple terms.

As we saw, in the SM there is CP violation because the number of families is larger than two. Furthermore, if Y_u and Y_d

were alligned, i.e. if we could diagonalize them with the same left-handed quarks rotation, then $V_{CKM} = \mathbb{1}$. Therefore if there is no flavour violation then there is no CP violation (ignoring the flavour universal θ_{QCD} parameter). These insights can be encoded in a measure of CP violation using the Jarlskog invariant:

$$J_Y \equiv \text{Im} \det [Y_d Y_d^\dagger , Y_u Y_u^\dagger] .$$

J_Y is invariant under flavor transformations G_F and therefore it is basis independent. CP is conserved iff $J_Y = 0$ (ignoring higher dim. ops.). Indeed

$$J_Y = J_{CP} \prod_{i>j} (m_i^2 - m_j^2)\, 2/v^2 = \mathcal{O}(10^{-22}) ,$$

where $J_{CP} = \text{Im}[V_{us} V_{cb} V_{ub}^* V_{cs}^*] = c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} \sin\delta_{CKM} \simeq \lambda^6 A^2 \eta \simeq \mathcal{O}(10^{-5})$.
The product of the masses is given by

$$\prod_{i>j} m_i^2 - m_j^2 = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) ,$$

and therefore if any of the pairs of masses is equal CP would be conserved.

All flavor transitions in the SM depend only on 4 parameters (λ, A, ρ, η). We can test V_{CKM} by performing <u>many</u> different measurements to over-constrain the system.
One way to visualize a subset of experimental constraints is through the CKM unitary triangle, which tests one of the nine unitary equations $V_{CKM} V_{CKM}^\dagger = \mathbb{1}$, obtained by taking the product of the 1st row and third column:

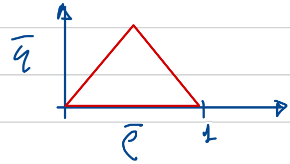
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 ,$$

which we can write as

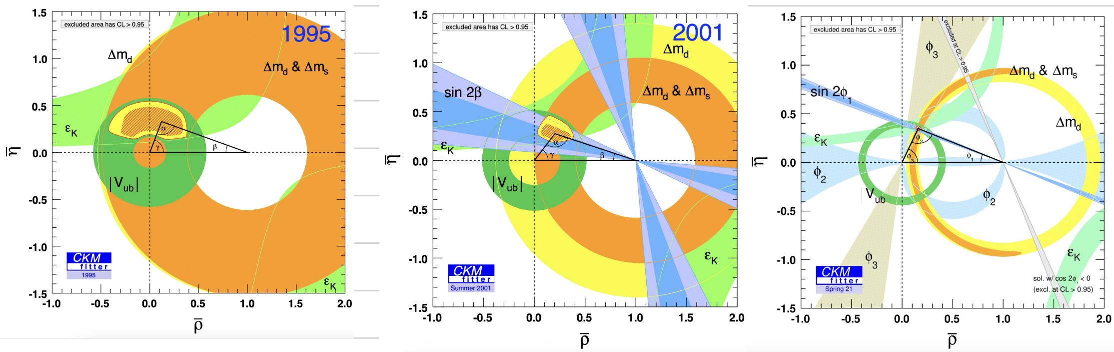
Wolfenstein parametrization.

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0 \quad \text{or} \quad -(\bar\rho + i\bar\eta) + 1 + (-1 + \bar\rho + i\bar\eta) = 0$$

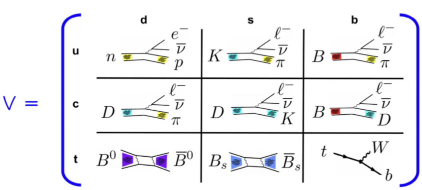
This equation can be interpreted as the sides of a triangle in the complex plane

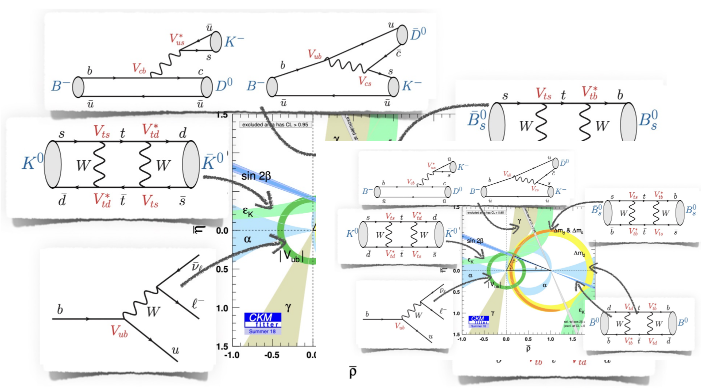


Evolution of CKM triangle measurements:



Main obs. in V_{CKM}:



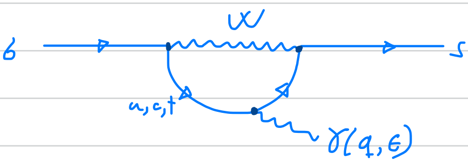


END OF L6

* FCNC's

Flavor Changing Neutral Currents (FCNC) is used more generally to mean Flavour Changing Neutral Transitions, not necessarily "currents". By this we mean an interaction that changes flavor (i.e. family identity) but does not change electric charge. For instance, a transition from b-quark to an s- or d-quark is flavor changing neutral, while a transition of the type $b \to c, u$ changes flavor and charge.

GIM-mechanism: a fancy name that summarises the fact that in many instances FCNC's are further suppressed by mass ratios m_i^2/m_W^2 and CKM factors. Let's see how it works in the context of $b \to s\gamma$:



$$= e\, q_\mu \epsilon_\nu \bar{u}(p_s)\sigma^{\mu\nu}\left(\frac{1+\gamma_5}{2}\right)u(p_b)\, \frac{m_b}{M_W^2}\, \frac{g^2}{16\pi^2} \times I$$

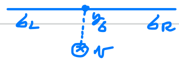
where $I = \sum_{i=u,c,t} V_{ib} V_{is}^* F\left(\frac{m_i^2}{M_W^2}\right)$

The function $F(x)$ results from doing the loop integral and is expected to be $O(1)$, roughly. The rest of the expression above is easy to justify:

* There are the obvious coupling factors $g^2/16\pi^2$ from the loop and W vertices, the factor "e" for the photon coupling and "$V_{ib} V_{is}$" arising from from the flavor changing charged interactions.

* Next, in order to produce γ on-shell we must produce an operator like $F_{\mu\nu}\bar{s}\sigma^{\mu\nu}b$, i.e. a dipole. (The monopole interaction $A_\mu \bar{b}\gamma^\mu s$ is not is not Lorentz/Gauge invariant).
$F_{\mu\nu}\bar{s}\sigma^{\mu\nu}b \Longrightarrow q_\mu \epsilon_\nu \bar{u}(p_s)\sigma^{\mu\nu}u(p_b)$.

* But now we have a "problem": the external quarks interact with

the rest of the diagram through W's, so they must be left-
-handed. This would suggest getting an operator of the form $\bar u_L \sigma^{\mu\nu} u_L = \bar u \left(\frac{1+\gamma_5}{2}\right)\sigma^{\mu\nu}\left(\frac{1-\gamma_5}{2}\right) u = 0$! Therefore we must change the chirality of one of the quark lines. The chirality flip costs a mass factor, diagrammatically: 

. We change the chirality of b because $m_b \gg m_s$.
This explains both the "m_b" factor and the structure $\bar u(p_s)\sigma_{\mu\nu}\left(\frac{1+\gamma_5}{2}\right)u(p_b)$.

* Finally to get the correct units we must divide by a quantity with $[E]^2$ dimensions. The natural one is "$1/m_W^2$" because this process must vanish in the limit $m_W^2 \to \infty$.

Old GIM mechanism: for pedagogical purposes let us assume that $m_u < m_c < m_t \ll m_W$. Of course this is not a correct assumption, because as all of you know $m_u \ll m_c \ll m_W \simeq m_t/2$. Let us however proceed under this assumption. Then, we can expand in Taylor series $F(x) = F(0) + x F'(0) + \cdots$,

$$I = \left(\sum_{i=u,c,t} V_{ib}V_{is}^*\right)F(0) + \left(\sum_{i=u,c,t} V_{ib}V_{is}^* \frac{m_i^2}{m_W^2}\right)F'(0) + \ldots$$

Unitarity of the CKM matrix predicts $\sum_{i=u,c,t} V_{ib}V_{is}^* = 0$, so the first term vanishes. Furthermore, using $V_{tb}V_{ts}^* = -\sum_{i=u,c} V_{ib}V_{is}^*$, we get $I \simeq -F'(0)\sum_{i=u,c} V_{ib}V_{is}^*(m_t^2-m_i^2)/m_W^2$. Therefore we have uncovered that FCNC's have additional supression beyond the loop factors:

$$I \sim V_{ub}V_{us}^*\frac{m_t^2-m_u^2}{m_W^2} + V_{cb}V_{cs}^*\frac{m_t^2-m_c^2}{m_W^2} \sim \epsilon^4\frac{m_t^2}{m_W^2} + \epsilon^2\frac{m_t^2}{m_W^2}.$$

Recall that $V_{CKM} = \mathcal{O}(1) \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$ w/ $\epsilon = 0.2$.

V_{us} V_{ub} V_{cs} V_{cb}

Therefore, in addition to the one loop supression factor there is a mass supression m_t^2/m_W^2 and a mixing angle supression.

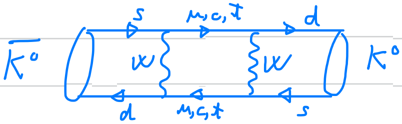
MODERN GIM mechanism: we have to revisit the above story because $m_t \ll m_W$ is not a good approximation to estimate F as a taylor series when the argument is $F(m_t/m_W)$. Note however that F is invariant under $F(x) \longrightarrow F(x) + \text{constant}$. Therefore without loss of generality we take $F(0) = 0$. Then,

$$I = -V_{cb} V_{cs}^* \left(F\left(\frac{m_t^2}{m_W^2}\right) - F'(0) \frac{m_c^2}{m_W^2} \right) - V_{ub} V_{us}^* \left(F\left(\frac{m_t^2}{m_W^2}\right) - F'(0) \frac{m_u^2}{m_W^2} \right) + \dots$$

$$= F\left(\frac{m_t^2}{m_W^2}\right) V_{tb} V_{ts}^* + F'(0) \sum_{i=u,c} V_{ib} V_{is}^* \frac{m_i^2}{m_W^2} + \dots \sim \epsilon^2 F\left(\frac{m_t^2}{m_W^2}\right)$$

It turns out $F(x) \simeq \mathcal{O}(1)\, x$, the contributions from u- and c-quarks to "I" are negligible.

The true story: the combination of mass and CKM mixing supression factors was discovered by Glashow, Iliopoulos and Maiani (GIM) back in the days when only the existence of u, d and s was known. GIM studied neutral K^0 mixing



which involves a FCNC for $s \leftrightarrow d$ transition. They realized that the theory would grossly over estimate the mixing rate unless a fourth quark (the charm) existed that would produce the above type

ϵ^5 ?

of cancelation, restricted to two generations: $V_{su} V_{du}^* + V_{sc} V_{dc}^* = 0$. Not only they did explain Kaon mixing and predicted the exitence of a forth quark, they also gave a rough upper bound for the c-quark, which they could because the FCNC grows rapidly with the mass.
It turns out the the top contribution to Kaon mixing is approximately as big as the charm contribution! GIM got a bit lucky: a different CKM structure could had easily favored top quark mediation dominance in kaon mixing.

Bounds on new physics:

It turns out that $b \to s\gamma$ places some of the most stringent constraints on new physics. We can model the contribution of Beyond the SM physics by a higher dimension operator

After rotation

$$\Delta \mathcal{L} = \frac{C'}{\Lambda^2} g' B^{\mu\nu} \bar{Q}_L H \sigma^{\mu\nu} b_R \supset \frac{e v C}{\sqrt{2}\Lambda^2} \underbrace{F_{\mu\nu} \bar{s}_L \sigma^{\mu\nu} b_R}_{\text{A dipole operator.}} + \cdots$$

$\frac{1}{4}[\gamma^\mu, \gamma^\nu]$

Where I have assumed that the doublet belongs to the 2d generation. The coefficient of the operator is "C/Λ^2": "C" is assumed to be $O(1)$ while Λ indicates the scale of new physics. The dimension of the operator is $\Delta = 6$, which explains the power $1/\Lambda^2$.

It is easy to compute the contribution of the dipole to the amplitude

$$\frac{M_{NP}}{M_{SM}} \sim \frac{\frac{vC}{\sqrt{2}\Lambda^2}}{|V_{tb} V_{ts}^*| \frac{\alpha_e}{4\pi s_W^2} \frac{m_b}{m_W^2}}$$

We require this ratio to be less than 10% because the SM agrees at that level with the measurement. Therefore

$$\Lambda^2 C^{-1} \gtrsim \frac{v\, M_W^2\, S_W^2\, 4\pi}{\sqrt{2}\, m_b\, |V_{tb} V_{ts}^*|\, \alpha_2} \times \frac{1}{0.1} \implies \Lambda \gtrsim 60 \text{ TeV}$$

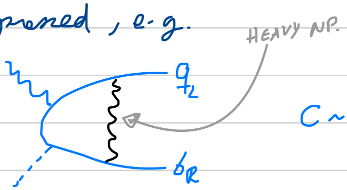
$C = O(1)$; $m_W = 80$; $m_b = 4.5$; $S_W^2 = 0.23$

$\alpha_2 = 0.6/4\pi$; $|V_{tb} V_{ts}^*| = \epsilon^2 = (0.2)^2$; $v = 246$.

This bound is extraordinarily large if compared with current highest CoM energy of colliders (LHC 13.6 TeV).

In the numerical bound above we took $C \sim O(1)$. Is this a good assumption? Clearly if "C" was smaller, then the bound on "Λ" would decrease.

For instance if the new physics is weakly coupled, we expect "C" to be loop suppressed, e.g.



$C \sim O(1) \to C \sim O(1/16\pi^2)$.

Note that his diagram is even smaller than our estimate due to the yukawa y_b.

We can formalize this pattern with the help of the Flavor Group.

We impose that Yukawas are spurions:

$$y_u \to U_Q\, y_u\, V_u^+$$

$$y_d \to U_Q\, y_d\, V_d^+$$

and require that interactions are invariant under G_F. The SM Yukawas are now invariant

$$y_d \bar{Q}_L H d_R + y_u \bar{Q}_L \tilde{H} u_R$$

under G_F, thanks to the spurious transformation of the Y_u and Y_d.
But what about the dipole operator above?

$$\frac{C'}{\Lambda^2} g' B^{\mu\nu} \bar{Q}_L H \sigma^{\mu\nu} b_R \longrightarrow \frac{C'}{\Lambda^2} g' B^{\mu\nu} \bar{Q}_L H \sigma^{\mu\nu} y_d b_R$$

Now this interaction brings a small m_b/v suppression like the SM amplitude. Under flavor diagonalization the dipole behaves like the mass terms.
But the SM amplitude contains additional V_{CKM} suppression! In order to achieve this we can add further spurious factors:

$$\frac{C'}{\Lambda^2} g' B^{\mu\nu} \bar{Q}_L H \sigma^{\mu\nu} y_u y_u^\dagger y_d b_R$$

is also G_F invariant. Then, because of V_{CKM} factors now the new physics amplitude is suppressed like in the SM and our estimate changes into

$$C^{-1} \Lambda^2 \gtrsim \frac{m_W^2 \, s_W^2}{\sqrt{2} \; \alpha_2/4\pi} \cdot \frac{1}{0.1} \implies \sqrt{C}\,\Lambda \gtrsim O(1) \cdot \text{TeV}.$$

"Λ" can be interpreted as the mass of the new particles running through the loop. Then this bound would be within the reach of LHC, barely. If new physics is weakly coupled, we add the loop factor and get $\Lambda \gtrsim 100\,\text{GeV}$.
The moral is that models of Beyond the SM that break G_F in arbitrary direction are tipically very much constrained. We can get around this by imposing (spurious) flavor symmetries.
In these lectures I did not cover neutral meson mixing. You should know that: (i) CP violation was discovered through neutral-K mixing; (ii) some of the best constraints on new physics arise from neutral meson mixing. (iii) it is an active field of research. (LHCb, Belle-II, ...)

END OF L7

• THE GOLDSTONE EQUIVALENCE THEOREM

We saw that $\quad -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu H|^2 + \dots \quad =$

8 d.o.f. $\quad$ 4 d.o.f.

4 massless spin 1 bosons $\quad (8+4=12)$

$\langle H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ h+v\end{pmatrix}$

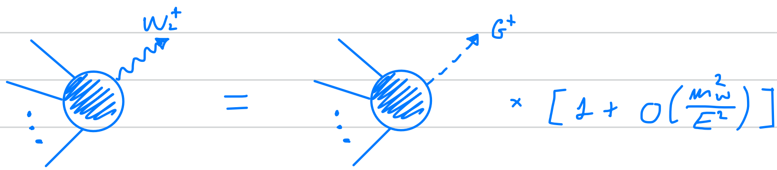
$$= \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - m_W^2 W_\mu^+ W^{-\mu} - \frac{1}{2}m_Z^2 Z_\mu Z^\mu}_{\pm\text{photon};\ \pm,0\ Z, W^+, W^-} + \frac{1}{2}\partial_\mu h\,\partial^\mu h + \dots$$

1 d.o.f.

$2 + 3\times 3 = 11 \qquad (11+1=12).$

When W's and Z's are at rest, the 3 polarizations are equivalent. However when moving relativistically the longitudinal component is distinct from the two transverses. One would imagine that the longitudinal is controlled by the Goldstones.

<u>Goldstone equivalence theorem:</u>



where recall that the Higgs doublet is $H = \begin{pmatrix}G^+\\ h+iG^0\end{pmatrix}$. G^+ is a charged pseudo goldstone "eaten" by W^+. G^0 is a neutral pseudo-goldstone "eaten" by the Z.

Kinematics: recall that a massive vector boson at rest has 4-momentum $K^\mu = (m,0,0,0)$, and the polarization $\epsilon^\mu(K)$ is a linear combination of:

$$(0,1,0,0)\ ,\ (0,0,1,0) \text{ and } (0,0,0,1).$$

If we boost the massive vector in the 3rd direction $K^\mu \longrightarrow (E_K,0,0,K)$.
Polarization "vectors" are boosted similar but satisfy:

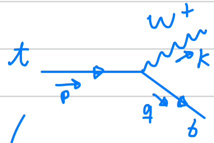
$$\epsilon\cdot K = 0 \quad ; \quad \epsilon^2 = -1$$

and $\epsilon_L(K) = \left(\frac{E_K}{m},0,0,\frac{K}{m}\right) \simeq \frac{K^\mu}{m} + O\left(\frac{E_K}{m}\right)$; $\epsilon^\pm = \begin{matrix}(0,0,1,0)\\ (0,1,0,0).\end{matrix}$

Let's see some examples of this equivalence.

<u>Top quark decay</u>: $t \longrightarrow W^+ + b$, Kinematically allowed.

$+\frac{2}{3}$, 173 GeV $\quad$ $+1$, 80 GeV $\quad$ $-\frac{1}{3}$, 4.5 GeV



Let us try to guess the answer: $\Gamma \propto \frac{g^2}{4\pi} m_t$. This is correct, but the actual answer is enhanced by $(m_t/m_W)^2$.

$$i\mathcal{M} = \frac{ig}{\sqrt{2}}\, \bar{u}(q)\, \gamma^\mu \left(\frac{1-\gamma_5}{2}\right) u(p)\, \epsilon^*_\mu(k) \qquad (\text{recall } V_{tb} = 1.013 \pm 0.030 \simeq 1)$$

Next we square the amplitude, average over initial spins (polarizations), and sum over final polarizations:

$$\frac{1}{2}\sum_{\text{spins}} |\mathcal{M}|^2 \overset{(*)}{=} \frac{g^2}{2}\left[q^\mu p^\nu + q^\nu p^\mu - g^{\mu\nu}\, q\cdot p\right] \underbrace{\sum_{\text{polarizations}} \epsilon^*_\mu(k)\,\epsilon_\nu(k)}_{= -\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_W^2}\right)} =$$

$$= \frac{g^2}{2}\left[q\cdot p + 2\,\frac{(k\cdot p)(k\cdot q)}{m_W^2}\right]$$

In (*) we used $\sum_s u^s_a(p)\,\bar{u}^s_b(p) = (\not{p} + m)_{ab}$. Indeed:

$$\bar{u}(q)\,\gamma^\mu (1-\gamma_5)\, u(p) \times u^\dagger(p)\, \underset{\gamma_5^\dagger = \gamma_5}{(1-\gamma_5)}\, \gamma^{\mu\dagger}\, \gamma_0^\dagger\, u(q) \overset{\{\gamma^\mu,\gamma_5\}=0}{=}$$

$$= \bar{u}(q)\,\gamma^\mu (1-\gamma_5)\, u(p) \times u^\dagger(p)\, \gamma^{\mu\dagger}\gamma_0^\dagger\, (1-\gamma_5)\, u(q) \overset{\gamma_0\gamma^\mu\gamma_0^\dagger = \gamma^\mu}{=}$$

$$= \bar{u}(q)\,\gamma^\nu (1-\gamma_5)\, u(p) \times \bar{u}(p)\, \gamma^\mu (1-\gamma_5)\, u(q)$$

Next put indices everywhere:

$$\sum_{s,s'} \bar{u}^s_a(q)\, A^\mu_{ab}\, u^{s'}_b(p)\, \bar{u}^{s'}_c(p)\, A^\nu_{cd}\, u^s_d(q) = \qquad \left(\text{where } A^\mu = \gamma^\mu\left(\frac{1-\gamma_5}{2}\right)\right)$$

$$= A^\mu_{ab} A^\nu_{cd}\, (\not{q} + \overset{0}{\cancel{m_b}})_{da}\, (\not{p} + m_t)_{bc} =$$

$$= \frac{1}{4}\mathrm{Tr}\left[\gamma^\mu (1-\gamma_5)(\not{p} + m_t)\, \gamma^\nu (1-\gamma_5)\, \not{q}\right] =$$

$$= \frac{1}{4}\mathrm{Tr}\left[\gamma^\mu \not{p} \gamma^\nu \not{q} + \gamma^\mu\gamma^5 \not{p}\gamma^\nu\gamma_5\not{q} - \gamma^\mu\gamma^5\not{p}\gamma^\nu\not{q} + \gamma^\mu\not{p}\gamma^\nu\gamma_5\not{q}\right] =$$

$$= \underbrace{2 p_\alpha q_\beta \left(g^{\mu\alpha} g^{\nu\beta} - g^{\mu\nu} g^{\alpha\beta} + g^{\mu\beta} g^{\nu\alpha}\right)}_{= (p^\mu q^\nu + p^\nu q^\mu - p\cdot q\, g^{\mu\nu})} - \underbrace{2i\,\epsilon^{\mu\alpha\nu\beta} p^\alpha p^\beta}_{0 \text{ because } \epsilon \text{ is antisymmetric.}}$$

In order to simplify the computation of $1/2 \sum_{spins} |M|^2$ further, we need to work out the following invariants:

$$2q\cdot p = 2q\cdot k = m_t^2 - m_W^2 \quad ; \quad 2k\cdot p = m_t^2 + m_W^2$$

$\hookrightarrow (q-p)^2 = k^2$

$-2q\cdot p + m_t^2 = m_W^2$

where we made use of the approximation $m_W, m_t \gg m_b \simeq 0$. Thus, all in all, we get

$$\frac{1}{2}\sum_{spins} |M|^2 = \frac{g^2}{4}\frac{m_t^4}{m_W^2}\left(1 - \frac{m_W^2}{m_t^2}\right)\left(1 + 2\frac{m_W^2}{m_t^2}\right)$$

Next, to compute the decay rate, we use the formula $\Gamma = \frac{1}{2m_t}\int d\Pi_f \, |M(p\to f)|^2$, where $d\Pi_f = \prod_i \frac{d^3 p_i}{(2\pi)^3}\frac{1}{2E_i}\delta^{(4)}(p-q-k)$ is the Lorentz invariant phase space measure.

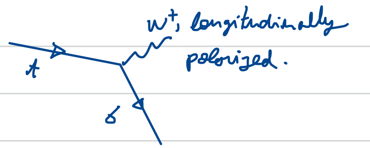
$$\Gamma = \frac{g^2}{8}\frac{m_t^3}{m_W^2}\left(1 - \frac{m_W^2}{m_t^2}\right)\left(1 + \frac{2m_W^2}{m_t^2}\right)\int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{\vec{k}^2 + m_W^2}}\frac{d^3q}{(2\pi)^3}\frac{1}{\sqrt{\vec{q}^2 + \underbrace{\cancel{m_b^2}}_{0}}}\cdot(2\pi)^4\delta^{(4)}(p-q-k)$$

$$= \frac{g^2}{64\pi}\frac{m_t^3}{m_W^2}\left(1 - \frac{m_W^2}{m_t^2}\right)^2\left(1 + \frac{2m_W^2}{m_t^2}\right) \simeq \frac{g^2}{64\pi}\frac{m_t^3}{m_W^2}$$

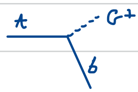
in the limit $m_t \gg m_W$.

Therefore, in the limit $m_t \gg m_W$ we do find an m_t^2/m_W^2 enhancement w.r.t. the very naive estimate. This makes sense, after all we expect $\Gamma\downarrow$ if $m_W\uparrow$.

Let's next use the Goldstone equivalence theorem:



$$\Delta\mathcal{L} = -y_t\,\epsilon^{ab}(\overline{Q}_L)_a (H^+)_b\, t_R + h.c. \quad ; \quad H = \begin{pmatrix} G^+ \\ \frac{h + iG^0}{\sqrt{2}} \end{pmatrix}.$$

$\hookrightarrow \Delta\mathcal{L} = -y_t\,\overline{b}_L t_R G^+ \implies iM =$  $= -y_t\,\bar{u}(q)\left(\frac{1+\gamma_5}{2}\right)u(p)$;

Therefore $\frac{1}{2}\sum_{spins}|M|^2 = y_t^2\, q\cdot p$.

Next we perform the integral over phase space to get

$$\Gamma = \frac{y_t}{32\pi}m_t = \frac{g^2}{64\pi}\frac{m_t^3}{m_W^2}.$$

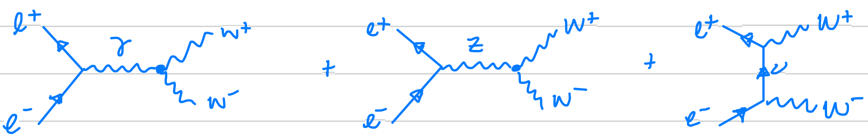
Nicely the result we get using the "Goldstone equivalence theorem" agrees with the exact tree-level computation we did of $t \to W_b^+$ in the limit $m_t \gg m_W$. This makes sense! In this limit it is favorable to produce energetic W's through the decay, and in particular it is more favorable to produce the longitudinal polarized ones because their polarization grows with the energy.

By the way this decay mode of the top is very important because $Br(t \to W^+ b) \simeq 1$, and therefore the top quark does not hadronize! Namely it decays through EW interactions before it has the time to reach longer distances where the QCD flux tubes acquire large energy and become energetically favorable to pair-produce on-shell quarks and gluons.

END OF L8

* Another instance of the Goldstone equivalence theorem:

Next we want to compute the following process



in the SM and using the equivalence theorem. First of all we note that naively these diagrams grow with the energy:

$$\frac{d\sigma}{d\cos\theta} \sim \frac{\pi\alpha^2}{4s}\left|\epsilon(k_+)\cdot\epsilon(k_-)\right|^2 \sim \frac{\pi\alpha^2}{4s}\left(\frac{s}{4m_W}\right)^2$$

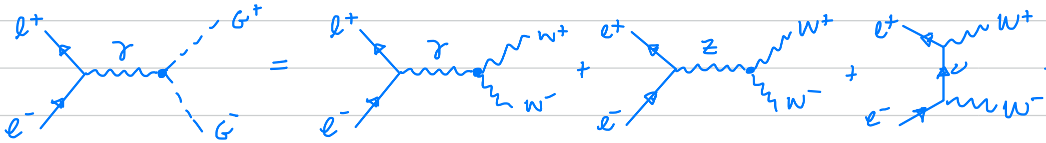
For longitudinals $\epsilon(k_+)\cdot\epsilon(k_-) = \frac{k_+\cdot k_-}{m_W^2}$ for $s \equiv (k_+ + k_-)^2 \gg m_W^2$.

On the other hand,

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to G^+G^-) = \frac{\pi\alpha^2}{4s}\sin\theta$$

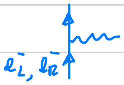
decays with $1/s$. How are these two claims compatible? We will see that there is a nice cancelation between the diagrams above and it turns out that $\frac{d\sigma}{d\cos\theta}(e^+e^- \to W^+W^-) \propto 1/s$!

We want to show

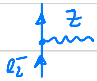


$+\, O\left(\frac{m_W^2}{E^2}\right)$

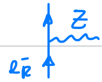
We will need the following Feynman rules:



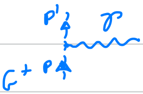
$= -ie\gamma^\mu$,



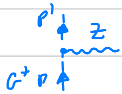
$= i\frac{e}{s_W c_W}\gamma_\mu\left(-\frac{1}{2}+s_W^2\right)$,



$= i\frac{e}{s_W c_W}\gamma_\mu\left(s_W^2\right)$,

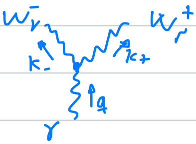


$= ie(p+p')^\mu$

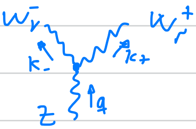


$= i\frac{e}{s_W c_W}\gamma_\mu\left(\frac{1}{2}-s_W^2\right)$

as well as the usual triple gauge vertices



$$= ie\left[g^{\mu\nu}(k_- - k_+)^\lambda + g^{\mu\lambda}(-q-k_-)^\mu + g^{\lambda\mu}(q+k_+)^\nu\right]$$



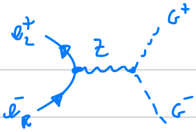
$$= igc_W\left[g^{\mu\nu}(k_- - k_+)^\lambda + g^{\mu\lambda}(-q-k_-)^\mu + g^{\lambda\mu}(q+k_+)^\nu\right]$$

All right, we start by computing the left hand side of the equation above. The photon exchange contribution is given by

$$i\mathcal{M}(e^+e^- \to \gamma^* \to G^+G^-) = (-ie)\,\bar{v}\gamma_\mu u\,\frac{-ig^{\mu\nu}}{q^2+i\epsilon}(k_+ - k_-)^\nu(-ie)$$

$$= ie^2\,\bar{v}\gamma_\mu u\,\frac{1}{q^2+i\epsilon}(k_+ - k_-)^\mu.$$

Next we compute the Z-exchange contribution. We will distinguish between right and left handed electrons:



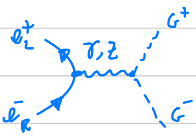
$$= \frac{ie}{c_W \cancel{s_W}} \bar{v}_R \gamma_\nu u_R \cancel{s_W^2} \frac{-ig^{\mu\nu}}{q^2-m_Z^2+i\epsilon} (k_+-k_-)^\nu \frac{-ie}{c_W \cancel{s_W}} \left(\frac{1}{2} - s_W^2\right)$$

$$= -ie^2 \bar{v}_R \gamma_\mu u_R \frac{1}{q^2-m_Z^2} (k_+-k_-)^\mu \frac{1}{c_W^2} \left(\frac{1}{2} - s_W^2\right)$$

$$= \frac{-ie}{c_W s_W} \bar{v}_L \gamma_\nu u_L \left(\frac{1}{2} - s_W^2\right) \frac{-ig^{\mu\nu}}{q^2-m_Z^2+i\epsilon} (k_+-k_-)^\nu \frac{-ie}{c_W s_W} \left(\frac{1}{2} - s_W^2\right)$$

$$= -ie^2 \bar{v}_L \gamma_\mu u_L \frac{1}{q^2-m_Z^2} (k_+-k_-)^\mu \frac{1}{c_W^2 s_W^2} \left(\frac{1}{2} - s_W^2\right)^2$$

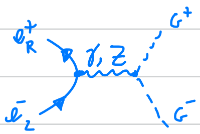
Next we combine the photon and Z exchange amplitudes. For the right handed electrons we get



$$\simeq \frac{i e^2}{2 c_W^2} \bar{v}_R \gamma_\mu u_R \frac{1}{q^2} (k_+-k_-)^\mu$$ which is just the amplitude

@ high energy

for e_R^- with $Y=-1$, to couple to a G^+ with $Y=1/2$, through the $U(1)$ gauge boson B_μ with coupling constant $g' = e/c_W$.
This expression reflects the fact that e_R has no direct coupling to $SU(2)_L$.
Instead the high energy limit for the left handed amplitude:



$$\simeq ie^2 \left(\frac{1}{4c_W^2} + \frac{1}{4s_W^2}\right) \cdot \bar{v}_L \gamma^\mu u_L \frac{1}{q^2} (k_+-k_-)^\mu$$ which is the sum of amplitudes with B_μ and A_μ^3 exchange.

@ high energy

Next, let's compute the right hand side, i.e. the diagrams in the GWS theory.
For right handed electrons we only need to compute the γ and Z exchange diagrams:

$$iM(e_R^- e_L^+ \to W^+W^-) = \bar{v}_R \gamma_\nu u_R \left[(-ie)\frac{-i}{q^2+i\epsilon}(ie) + \frac{ie \cancel{s_W}}{\cancel{c_W}} \frac{-i}{q^2-m_Z^2} \frac{ie \cancel{c_W}}{\cancel{s_W}}\right] \times$$

$$\times \left(g^{\mu\nu}(k_--k_+)^\lambda + g^{\lambda\nu}(-q-k_-)^\mu + g^{\lambda\mu}(k_++q)^\nu\right) \epsilon^*_\mu(k_+) \epsilon^*_\nu(k_-)$$

Note that the last equation is valid in any gauge because $q^\lambda \bar{v}_R \gamma_\lambda u_R = 0$. Note also that the second line contains and enhancement at high energies for the longitudinal gauge bosons. Indeed, using $\epsilon_L^\mu = \frac{k^\mu}{m} + O(m/E_k)$, we have

$$\left(g^{\mu\nu}(k_- - k_+)^\lambda + g^{\lambda\nu}(-q-k_-)^\mu + g^{\lambda\mu}(k_+ + q)^\nu \right) \epsilon_\mu^*(k_+)\, \epsilon_\nu^*(k_-) = \frac{s}{2m_W^2}(k_+ - k_-)^\lambda + O(1)\cdot(k_+ - k_-)^\lambda$$

Using $\frac{1}{q^2} - \frac{1}{q^2 - m_Z^2} = -\frac{m_Z^2}{q^2(q^2-m_Z^2)} \simeq -\frac{m_Z^2}{q^4}$, we get

$$i\mathcal{M}(e_R^- e_R^+ \to W^+W^-) \simeq \bar{v}_R \gamma_\mu u_R \,\frac{ie^2 m_Z^2}{s} \frac{1}{2m_W^2} (k_+ - k_-)^\lambda$$

which agrees with the high energy limit of $i\mathcal{M}(e_R^- e_L^+ \to G^+G^-)$ that we computed above (use $m_Z^2/m_W^2 = 1/c_W^2$). Equivalence theorem at work!

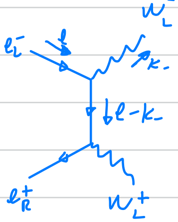
Next we compute the amplitude for the left handed electrons. We have the γ, Z and ν-exchange diagrams. The first two are given by

$$i\mathcal{M}(e_R^- e_L^+ \to W^+W^-) = \bar{v}_L \gamma_\mu u_L \left[(-ie)\frac{-i}{q^2+i\epsilon}(ie) + \frac{ie\left(-\frac{1}{2}+s_W^2\right)}{s_W \cancel{c_W}} \frac{-i}{q^2 - m_Z^2} \frac{ie \cancel{c_W}}{s_W} \right] \times$$
$$\times \left(g^{\mu\nu}(k_- - k_+)^\lambda + g^{\lambda\nu}(-q-k_-)^\mu + g^{\lambda\mu}(k_+ + q)^\nu \right) \epsilon_\mu^*(k_+)\, \epsilon_\nu^*(k_-)$$

Next we perform similar simplifications to what we did for the high energy limit of the e_R^- amplitude, and we are lead to

$$\simeq \bar{v}_L \gamma_\mu u_L \; ie^2 \left(\frac{m_Z^2}{s(s-m_Z^2)} - \underbrace{\frac{1}{2s_W^2} \frac{1}{s - m_Z^2}}_{} \right) \frac{s}{2m_W^2} (k_+ - k_-)^\mu$$

Potentially dangerous term. It grows with the energy, thus it would lead to a loss of perturbativity. It better be cancelled by the neutrino exchange diagram:



$$= \left(\frac{ig}{\sqrt{2}}\right)^2 \bar{v}_L \gamma^\mu \frac{i(\not{\ell} - \not{k}_-)}{(\ell - k_-)^2} \gamma^\nu u_L(\ell) \underbrace{\epsilon_\mu^*(k_+)\, \epsilon_\nu^*(k_-)}_{\simeq \frac{k_+^\mu k_-^\nu}{m_W^2} + \cdots}$$

$$\simeq -\frac{ig^2}{2} \bar{v}_L \frac{\not{k}_+}{m_W} \frac{\not{\ell} - \not{k}_-}{(\ell - k_-)^2} \frac{\not{k}_-}{m_W} u_L .$$

In order to simplify further the last expression we note that $u_L(\ell)$ satisfies the Dirac equation in momentum space: $(\not\ell-\not k_-)\not k_- u_L(\ell) = -(\not\ell-\not k_-)^2 u_L(\ell) = -(\ell-k_-)^2 u_L$

$\not\ell\, u_L(\ell) = 0 + \mathcal{O}(m_e)$

Therefore:

$$= \frac{i g^2}{2} \bar{\nu}_L \frac{\not k_+}{m_W^2} u_L(\ell) = \frac{i e^2}{2 s_W^2} \frac{1}{2 m_W^2} \bar{\nu}_L \gamma_\mu u(\ell)(k_+ - k_-)^\mu$$

$(k_+ + k_-)^\mu \bar{\nu}_L \gamma^\mu u(\ell) = q^\mu \underbrace{\bar{\nu}_L \gamma^\mu u_L(\ell)}_{J^\mu} = 0$

Thus, the neutrino exchange diagram <u>does</u> cancel the leading high energy term of the γ, Z-exchange diagrams!

In order to work out the ν-exchange diagram to the same order as the γ, Z-exchange, we need to keep more powers in the High energy expansion of ϵ_L^μ. This has the net effect of multiplying the γ, Z-exchange diagrams by $(1+\frac{2m_W^2}{s})$ (after removing the leading high energy growing piece that we cancelled):

$$i\mathcal{M}(e_L^- e_R^+ \to W_L^+ W_L^-) = i e^2 \bar{\nu}_L \gamma^\mu u_L (k_+ - k_-)^\mu \frac{1}{s} \underbrace{\left(\frac{1}{2c_W^2} - \frac{1}{4c_W^2 s_W^2} + \frac{1}{2 s_W^2}\right)}_{\frac{2s_W+2c_W-1}{4c_W^2 s_W^2} = \frac{1}{4s_W^2} + \frac{1}{4c_W^2} = \frac{1}{4s_W^2 c_W^2}}$$

which is equal to $\mathcal{M}(e_L^- e_R \to G^+ G^-)$ that we computed!

The cancellation of the $e^+e^- \to W^+W^-$ naive high energy growth is a crucial feature of a theory of Spontaneous Symmetry Breaking.

At the beginning of these lectures we saw "SSB of gauge invariance" as a mechanism that can generate masses for gauge bosons. Now we have argued the opposite: that only the theories with SSB have amplitudes with a nice high energy behaviour, i.e. without strong coupling. We'll now see this feature in more generality, and expose the high energy behaviour at the level of the Lagrangian. The high energy behaviour is more obscure when working with the ϵ_L^μ's.

END OF L9

⊛ THE HIGGS MECHANISM II

A dialogue with questions (Q) and answers (A).

Q1: The Higgs particle "h" is a scalar, but it is not <u>any</u> scalar... right? What makes the scalar "h" a Higgs boson? For instance, imagine adding an extra singlet scalar to the SM: $\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\partial_\mu s(x)\partial^\mu s(x) - V(s)$. The field "s" is a scalar but not a Higgs particle.

A1: Well, "h" is inside the Higgs doublet $H = (G^+, (h + iG_0)/\sqrt{2})$, which takes a non-zero VEV, that's why "h" is a Higgs boson!

Q2: Well... this does not sound very invariant... is there a more physical way in which we can describe the role of "h" by referring to observables and not to "mechanisms of Lagrangians written with dummy integration variables"?

A2: Yes! this is what today's lecture is about.

Next, for pedagogical purposes, to make the argument simpler I will set

* $g' = 0$, * $y_f = 0$, so that for the time being we can ignore fermions.
* Let's also imagine that we do not know about the Higgs boson because the energy that we have explored is $E_{cm} < m_h$.

We do know however of the existence of massive gauge bosons. Therefore the Lagrangian of this hypothetical universe is:

$$\mathcal{L}_{SU(2)} = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} + m_W^2 \, \mathrm{Tr}\left[W_\mu W^\mu\right] \qquad \left(\text{where } W_\mu \equiv \frac{\sigma^a}{2} W_\mu^a\right)$$

$$\sim \frac{1}{2}(\partial W)^2 - \frac{1}{2} m_W^2 W^2 - g\, W^2 \partial W + g^2 W^4$$

This theory makes perfect sense as an Effective Field Theory, i.e. observables computed with this Lagrangian are perfectly consistent as long as the energies that are probed are $E_{cm} < \Lambda$, where Λ is the physical cutoff.

By physical cutoff we mean the value of the energy at which new phenomena beyond the weakly coupled Lagrangian emerges.

<u>What is the value of Λ in this EFT?</u>

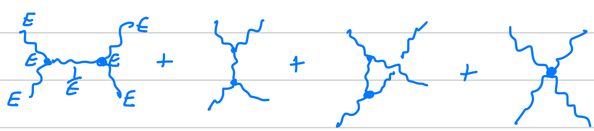
Recall that $\epsilon_L^\mu(P) = P^\mu/m_W + O(m_W/E)$, therefore perturbation theory Feynman diagrams in this theory can be trusted as long as $E \lesssim m_W/g \sim \Lambda$. This can be seen order by order computing Feynman diagrams, as we will show next.

(This is very different from e.g. QED! The E.M. coupling runs with the energy

, but only logarithmically $\alpha(\mu_f) = \dfrac{\alpha(\mu_i)}{1+\left(\beta\,\frac{\alpha(\mu_i)}{4\pi}\log\left(\frac{\mu_f}{\mu_i}\right)\right)}$ @ 1-loop.)

Let's compute an elastic $2\to 2$ process involving the W_L^μ's:

$$\mathcal{M}(W_L + W_L \longrightarrow W_L + W_L) =$$



Individually each of these diagrams grows like E^4. The sum however grows like E^2.

$$\mathcal{M}(W_L + W_L \longrightarrow W_L + W_L) \simeq \frac{g^2}{4 m_W^2}(s+t).$$

This is a milder growth, but still poses a problem of perturbativity at large $E \gg m_W$ energies. One manifestation of the loss of perturbativity (i.e. of our ability to compute observables by evaluating the first few Feynman diagrams) can be found in the naive violation of the unitary equation.

A quick recapitulation of unitarity and the partial waves unitarity bound:

Consider an elastic scattering in the C.o.M. frame

$$A(p_1) + A(p_2) \longrightarrow A(p_3) + A(p_4)$$

then,

$$\sigma_{tot}(AB \to AB) = \frac{1}{32\pi E_{cm}^2} \int d\cos\theta \, |M(\theta)|^2 .$$

To derive a useful bound, it is convenient to decompose the amplitude in partial waves. We can allways perform the decomposition

$$\mathcal{M}(\theta) = 16\pi \sum_{j=0}^{\infty} a_j (2j+1) P_j(\cos\theta)$$

where $P_j(\cos\theta)$ are the Legendre Polynomials. They satisfy:

$$P_j(1) = 1 \qquad \& \qquad \int_{-1}^{1} P_j(x) P_i(x)\, dx = \frac{2}{2j+1} \delta_{ji} .$$

Therefore we can integrate over "$d\cos\theta$" and write

$$\sigma_{tot} = \frac{16\pi}{E_{cm}^2} \sum_{j=0}^{\infty} (2j+1) |a_j|^2$$

p.w.
partial waves.

Now, the optical theorem says

$$\sum_{j=0}^{\infty} (2j+1) \, \mathrm{Im}(a_j) = 2 E_{cm} |\vec{p}_i| \sum_{x} \sigma_{tot}(AB \to X) \geq 2 E_{cm} |\vec{p}_i| \, \sigma_{tot}(AB \to AB) .$$

Therefore

$$\sum_{j=0}^{\infty} (2j+1) \, \mathrm{Im}(a_j) \geq \frac{2|\vec{p}_i|}{E_{cm}} \sum_{j=0}^{\infty} (2j+1) |a_j|^2$$

It turns out that the sum can be dropped by considering angular momentum eigenstates (see e.g. Izykson & ZUBER) and we are lead to

$$\mathrm{Im}\, a_j \geq |a_j| \quad , \quad \text{for } E_{cm} \gg m_A, m_B \; ; \; |\vec{p}_i| = \tfrac{1}{2} E_{cm}$$

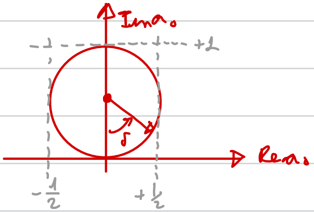
(A nice way to write the p.w. unitarity is using $S_\ell \equiv 1 + i\sqrt{\frac{s-4m_W^2}{s}}\, a_\ell$, then $|S_\ell| \leq 1$ for $s \geq 4m_W^2$.)

Let's then evaluate the partial wave unitary equation for our $W_L + W_L \to W_L + W_L$ amplitude. We have to compute $a_\ell = \frac{1}{32\pi}\int_{-1}^{+1} d\cos\theta\, \mathcal{M}(s,\theta)\, P_\ell(\cos\theta)$, in particular

$$a_0(W_L^+ W_L^- \to W_L^+ W_L^-) \simeq \frac{g^2}{32\pi\, 4m_W^2}\int_{-1}^{+1} d\cos\theta \left(s - \frac{s-4m^2}{2}(1-\cos\theta)\right) =$$

$$= \frac{s}{32\pi}\frac{g^2}{4m_W^2}$$

[recall $t = -\frac{(s-4m^2)}{2}(1-\cos\theta)$]



$(\mathrm{Re}\, a_\ell)^2 + (\mathrm{Im}\, a_\ell - \frac{1}{2})^2 = 1/4$

The loss of perturbativity can be estimated to be at $\pi \simeq \delta \simeq 2\mathrm{Re}(a_0)$, i.e. for

← when real & Im are of the same order.

$\delta = \frac{1}{2}\left|\log\left(\frac{i + a_\ell}{-i + a_\ell^*}\right)\right| \simeq$
$\simeq 2\,\mathrm{Re}\, a_\ell$

$$\sqrt{s} \simeq \Lambda = 4\pi v \simeq 3\ \mathrm{TeV}.$$

where we introduced $v \equiv m_W^2\, 4/g^2$. Sometimes the estimate used is $\mathrm{Re}(a_0) \leq 1/2$, this is also a valid estimate, reflecting the theoretical uncertainty of the estimate.

We have found Λ by inspecting a particular process. Next we would like to expose this "EFT-like" behaviour in the Lagrangian description.
In order to do so we perform the following field redefinition in the Lagrangian above:

$$W_\mu \longrightarrow \frac{i}{g}\mathcal{U}(D_\mu \mathcal{U})^+ = \mathcal{U} W_\mu \mathcal{U}^+ + \frac{i}{g}\,\mathcal{U}\,\partial_\mu \mathcal{U}^+$$

where $\mathcal{U}(x) = e^{i g \tau^a \pi^a / m_W}$, $T^a \in SU(2)$. Then, the Lagrangian becomes

$$\mathcal{L}_{SU(2)} = -\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{m_W^2}{g}\,\mathrm{Tr}\left[(D_\mu \mathcal{U})^+ (D^\mu \mathcal{U})\right]$$

$$= -\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \left[D_\mu \mathcal{U}\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}\right]^+ \left[D_\mu \mathcal{U}\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}\right]$$

with $D_\mu = \partial_\mu - i\,\vec{\tau}\cdot\vec{W}$ and $v \equiv 2m_W/g$. Several comments:

→ Note that we have introduced a new field $\pi^a(x)$! But we also declare a new gauge redundancy:

$$\mathcal{U} \sim L(x)\,\mathcal{U}\,L^+(x)$$
$$W_\mu \sim L(x)\,D_\mu\,L^+(x) \quad ; \text{ with } L(x) \in SU(2).$$

→ "emergent" gauge invariance in this description. By taking the so called unitarity gauge we can set $\pi^a(x) = 0$ everywhere and then we are back to the original Lagrangian.

→ the second line make it clear that $SU(2)$ is broken: $T^a \langle U(0, v/\sqrt{2})^T \rangle \neq 0$.

The new Lagrangian is equivalent, but has a great advantage: it makes the high energy limit of the theory manifest! At high energies $W_L^\pm$ is described by the π^a fields.

This can be seen by taking the following limit:

$$m_W \to 0 \quad \& \quad g \to 0 \qquad \text{with} \qquad \frac{m_W}{g} \text{ fixed}.$$

which is called the "decoupling limit". Then, the Lagrangian becomes:

decoupled

$$\mathcal{L}_{SU(2)} = \frac{m_W^2}{g} \operatorname{Tr} \partial_\mu U^\dagger \partial^\mu U \quad - \frac{1}{4} W_{\mu\nu}^2$$

The most important lesson is that this Lagrangian is not perturbative at energies $E \gtrsim m_W/g$. More accurately at $E = 4\pi m_W/g$ the loop expansion breaks down. Why so?

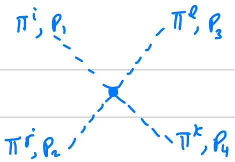
$$\begin{aligned}
\mathcal{L}_{SU(2)} &= \frac{v^4}{4} \operatorname{Tr}\left[\partial_\mu \left(-i\frac{\pi\cdot\sigma}{v} - \frac{(\pi\cdot\sigma)^2}{2v^2} + i\frac{(\pi\sigma)^3}{3!\,v^3} \right) \partial^\mu \left(\frac{i\pi\cdot\sigma}{v} - \frac{(\pi\cdot\sigma)^2}{2v^2} - \frac{i(\pi\sigma)^3}{3!\,v^3} \right) \right] + O(v^{-4}) \\
&= \frac{1}{4} \partial_\mu \pi^a \partial^\mu \pi^b \underbrace{\operatorname{Tr}[\sigma^a \sigma^b]}_{2\delta^{ab}} + \cancelto{0}{\underbrace{\operatorname{Tr}[\sigma^a \sigma^b \sigma^c]}_{\sim \epsilon^{abc}} \partial_\mu(\pi^a \pi^b) \partial^\mu \pi^c \, O(1)/v^3} \\
&\quad + \frac{1}{v^2} \underbrace{\operatorname{Tr}[\sigma^a \sigma^b \sigma^c \sigma^d]}_{= 2\{\delta_{ab}\delta_{cd} \underbrace{- \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}}_{\text{antisymmetric in } c \leftrightarrow d}\}} \left\{ \frac{1}{4\times 4} \partial_\mu(\pi^a \pi^b) \partial^\mu(\pi^c \pi^d) - \frac{1}{3!\,4}\left(\partial_\mu \pi^a \cancel{\partial_\mu(\pi^b \pi^c \pi^d) + \partial_\mu(\pi^a \pi^b \pi^c) \partial_\mu \pi^d} \right) \right\}
\end{aligned}$$

0, by Eqs. of motion

Thus, all in all:

$$\mathcal{L}_{SU(2)} = \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{1}{8v^2} \partial_\mu (\vec{\pi}\cdot\vec{\pi}) \partial^\mu (\vec{\pi}\cdot\vec{\pi}) + O(\pi^6)$$

OK, let's now do some simple calculations with this Lagrangian.



$$= \frac{i}{8v^2} \langle \pi^i \pi^j | \int d^4x \, \partial_\mu (\pi^a \pi^a) \partial^\mu (\pi^b \pi^b) | \pi^k \pi^l \rangle$$

or

$$= \frac{1}{8v^2} \Big\{ \delta_{ij} \, (p_1+p_2)_\mu \, \delta_{kl} (p_3+p_4)^\mu \times 8$$

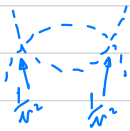
$$+ \delta_{kj} \delta_{il} \, (p_1+p_3)\cdot(p_2+p_4) \times 8$$

$$+ \delta_{ki} \delta_{jl} \, (p_1+p_4)\cdot(p_2+p_3) \times 8 \Big\} \times (2\pi)^4 \delta^{(4)}(p_1+p_2-p_3-p_4)$$

Note $3 \times 8 = 4!$, as it should.

$$= \frac{1}{v^2} \Big\{ \delta_{ij} \delta_{kl} \, s + \delta_{il} \delta_{kj} \, t + \delta_{ik} \delta_{jl} \, u \Big\} \times \text{"4-momentum conservation"}$$

This computation is valid as long as $s/v^2 \ll 1$. Indeed:



$$\sim \frac{1}{16\pi^2} \frac{s^2}{v^4} \ll \underbrace{\frac{s}{v^2}}_{\text{tree level}} \Rightarrow \frac{s}{(4\pi)^2 v^2} \ll 1 \quad \text{i.e. } E \ll 4\pi v \equiv \Lambda$$

We can project the computation into charged states:

$$P^{+-}_{ij} \pi^i \pi^j = \sum_{i,j} \sigma^+_i \sigma^-_j \pi^i \pi^j$$

with $\sigma^+ = (1, i)/\sqrt{2}$; $\sigma^- = (1, -i)/\sqrt{2}$.

$$\mathcal{M}(\pi^+\pi^- \to \pi^+\pi^-) = \sum_{i,j,k,l} P^{+-}_{ij} P^{+-}_{kl} \mathcal{M}(\pi_i \pi_j \to \pi_k \pi_l) = \frac{1}{v^2}(s+t)$$

$$= \frac{g^2}{4 m_W^2} (s+t)$$

like the $W_L^+ W_L^- \to W_L^+ W_L^-$ amplitude!

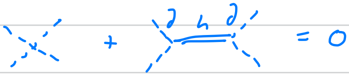
Now the "Higgs mechanism" comes in as a particularly simple way to UV complete this Lagrangian. By promoting

$$U \to H \equiv U \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

the Lagrangian becomes

$$\mathcal{L}_{SU(2)} \longrightarrow -\frac{1}{4} W_{\mu\nu}W^{\mu\nu} + (\partial_\mu H)^\dagger(\partial^\mu H)$$

which is UV complete because it does not have an offending growth at high energies. Indeed use the gauge equivalence to work with the variables $H = \left(G^+, \frac{h+iG_0}{\sqrt{2}}\right)^T$ then the decoupling limit of $(\partial_\mu H)^\dagger \partial^\mu H$ is the free theory



[Notice that for the symmetry pattern $U(1) \otimes U(1) \longrightarrow U(1)_{QED}$ there is no loss of perturbativity at large energies. A theory with only photons and Z's would not necessarily need a Higgs to remain weakly coupled]

⊛ Bottom up SM Higgs model: