SM key concepts and data

$$N_S = \frac{N_A N_B}{A} \sigma \quad \rightsquigarrow \quad \Gamma = \frac{dN_S}{dt} = n_B \sigma v_A$$

Phase space measure: $\quad d\phi_n = \prod_{i=1}^{n} \frac{d^3 q_i}{(2\pi)^3 2E_i}$

2→n scattering

- $$\boxed{d\sigma = \frac{|\mathcal{M}_{2\to n}|^2}{4 E_A E_B \underbrace{|v_A - v_B|}_{\text{transforms as an area}}} (2\pi)^4 \delta^4(p_A + p_B - q_{tot})\, d\phi_n}$$

- 2→2 $$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{|\mathcal{M}|^2}{64\pi^2 \underbrace{s}_{(p_A+p_B)^2}} \frac{|\vec{q}_1|}{|\vec{p}_A|}$$

Optical theorem

$$2\,\mathrm{Im}\left(A \succ\!\bigcirc\!\prec A\right) = \sum_c \left| A \succ\!\bigcirc\!\equiv c \right|^2 (2\pi)^4 \delta(\ldots)$$

Renormalized propagator:

$$\frac{i\, Z}{p^2 - m^2 + i\underbrace{\Gamma}_{\text{decay rate}} m}$$

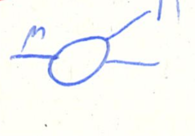
Retarded green function gives decay rate:

$$G_R \propto \int \frac{dp^0}{(2\pi)} \frac{i}{p^2 - m^2 + i m \Gamma} e^{-ip^0 t} \sim 2\pi i \frac{e^{-\frac{\Gamma}{2}\frac{m}{E_p}} e^{i\ldots}}{2E_p}$$

$\frac{1}{\tau} \xrightarrow{\text{rest}} \tau = \frac{1}{\Gamma}$

$$\boxed{d\Gamma = \frac{1}{2M} |\mathcal{M}_{1\to n}|^2 d\phi_n (2\pi)^4 \delta^4(P - q_{tot})}$$

for 1→2 $\quad \boxed{\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{|\vec{p}_1|^{CM}}{m^2}}$



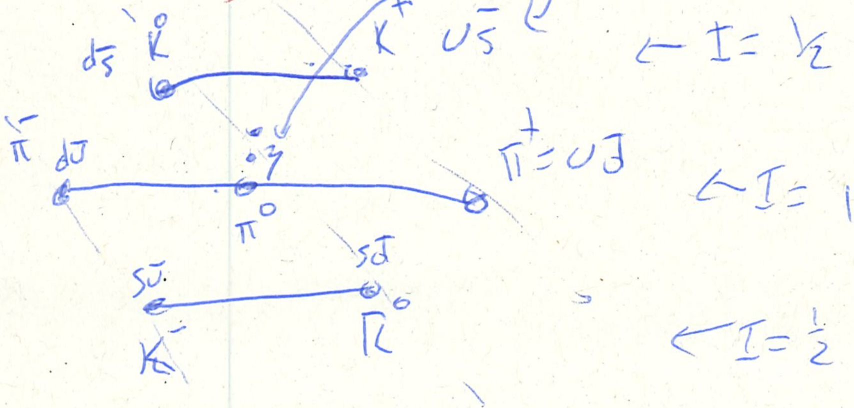
Isospin (SU(2)) $q = \begin{pmatrix} u \\ d \end{pmatrix}$

generators $\sigma^i \rightsquigarrow \begin{pmatrix} \pi^0 & \pi^+ \\ \pi^- & -\pi^0 \end{pmatrix} \ni$ $\pi^0\sigma_3$ $\pi^a\sigma_a = \pi^0\sigma_3 + \pi^+\sigma_+ + \pi^-\sigma_-$

Isospin 1 $\rightsquigarrow$ π^- π^0 π^+
spin 0 $\bar{u}d$ $u\bar{u}-d\bar{d}$ $\bar{d}u$

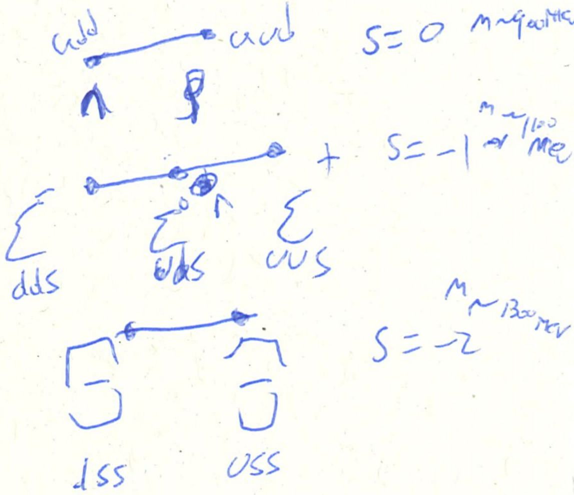
Isospin
spin-1 $\rho^- = \bar{u}\gamma^\mu d$ ... $u\gamma^\mu\bar{d} = \rho^+$

(SU(3)) $(q\bar{q}) \leftarrow 3\otimes\bar{3} = 1\oplus 8$

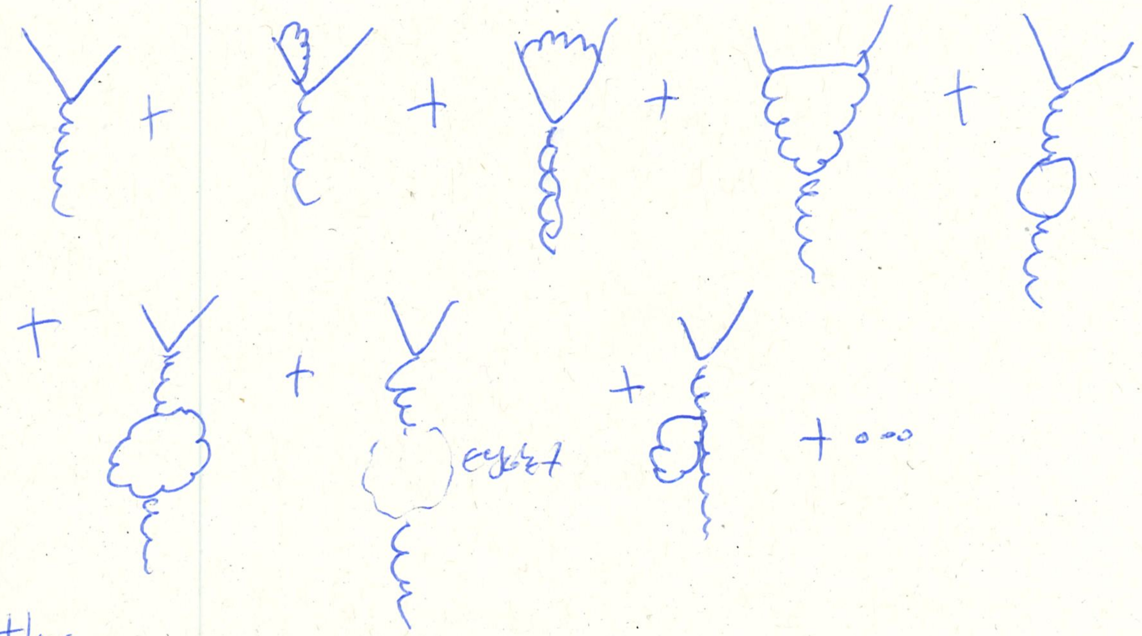


charge $= +1$

$qqq \leftarrow 3\otimes 3\otimes 3 = 1\oplus 8\oplus 8\oplus 10$



For QCD we consider



this gives

$$g_{s\,\text{bare}} = g_s - \frac{g_s^3}{2\,4\pi^3}\left[N_f \cdot T_f - \frac{11}{4} C_A\right] \frac{\Gamma(\varepsilon)}{(\mu^2)^\varepsilon}(4\pi)^\varepsilon$$

$$\underbrace{\qquad}_{\left(\frac{1}{\varepsilon} - \gamma + \log 4\pi - \log(\mu^2)\right) + O(\varepsilon)}$$

The β term is then

$$\beta = \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} = \alpha_s^2\left[b_0 + b_1 \alpha_s + b_2 \alpha_s^2 + \cdots\right]$$

- $b_0 = N_f T_f - \frac{11}{4} C_A \overset{C_A=3,\ T_f=\frac{1}{2}}{=} \frac{2N_f - 33}{12\pi} < 0$ for $N_f \lesssim 16$

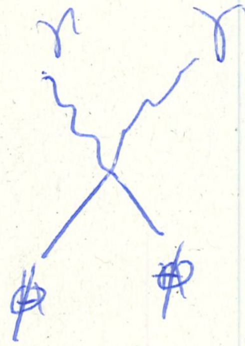
for QED $b_0^{(QED)} = \frac{\sum_f Q_f^2}{3\pi} \overset{\text{only electron}}{=} \frac{1}{3\pi} > 0$

$$\boxed{\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 - \alpha_s(\mu_0)\, b_0 \log\left(\frac{\mu^2}{\mu_0^2}\right)}} = \frac{1}{1 - \log\left(\frac{\mu^2}{\Lambda_{QCD}^2}\right) b_0}$$

Landau pole divergence defines Λ_{QCD}.

Only scale dimensionfull is Λ_{QCD}.

Type of interactions and dependences



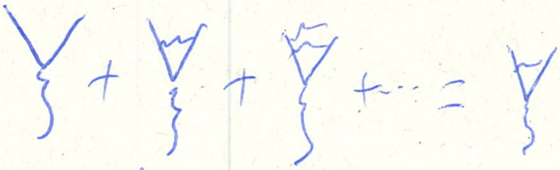
$\leftarrow |\phi|^2 F_{\mu\nu}F^{\mu\nu}$

$\sim \sigma = g_5^2 E^2$, $E^2 = s$

$\alpha = \frac{e^2}{4\pi}$

$\leftarrow g_8 (F_{\mu\nu}F^{\mu\nu})^2 \rightsquigarrow g_8 \sim \frac{1}{\Lambda^4} \sim c\frac{\alpha^2}{m_e^4}$

$\sim \sigma \sim \frac{\alpha^4}{m_e^8} E^6 \leftarrow$ dimensions of Area.

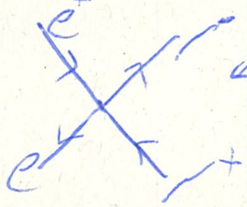


$\left(1 + \frac{\alpha}{2\pi} + \cdots\right)$

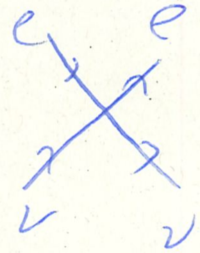
breaks parity
item proportional to $\boxed{m_e}\,\bar e_R e_L$

$\leftarrow g_5 \bar e \gamma^{\mu\nu} e F_{\mu\nu} = \frac{g_5}{2}(\bar e_L \gamma^{\mu\nu} e_R F_{\mu\nu} + \cdots)$

$\sim g_5 \sim \alpha^2 \frac{e}{2m_e}\frac{m_e}{M_W^2} \sim \frac{1}{\Lambda} \sim \alpha^2 \frac{m_e}{M_W^2}$

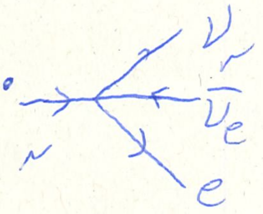


$\leftarrow \frac{G_F}{\sqrt{2}} \underbrace{\bar e \gamma^\mu (1-\gamma_5) e}_{\propto J_L^{0\mu}}\ \underbrace{\bar\mu \gamma_\mu (1-\gamma_5)\mu}_{\propto J_L^{0\mu}}$



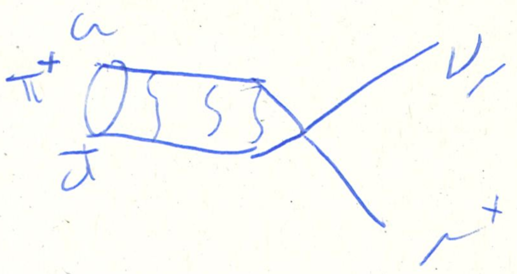
$\leftarrow \frac{G_F}{\sqrt{2}} \bar e \gamma^\mu (1-\gamma_5) \nu_e\, \bar\nu_e \gamma_\mu (1-\gamma_5) e$

$\sim \mathcal{M} = G_F E^2 \to \sigma = \frac{(G_F E^2)^2}{E^2 \text{ area}} = G_F^2 E^2$



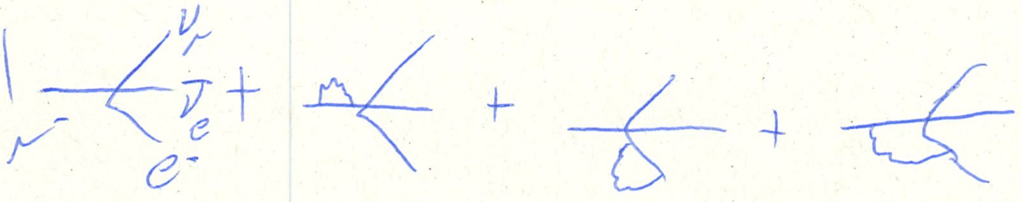
$\leftarrow \frac{G_F}{\sqrt{2}} \bar e \gamma^\rho (1-\gamma_5) \nu_e\, \bar\nu_\mu \gamma_\rho (1-\gamma_5)\mu$

$\sim \Gamma = \frac{G_F^2}{192\pi^3} m_\mu^5 \left(1 - 8\frac{m_e^2}{m_\mu^2} + \cdots\right)$



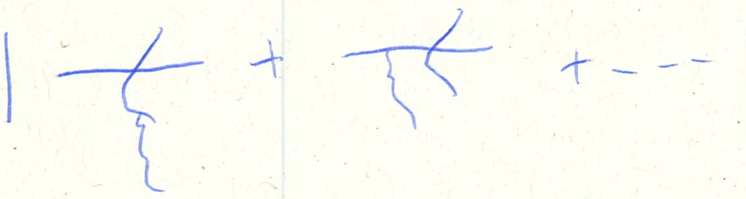
$$\frac{G_F}{\sqrt{2}}\left[\bar{\nu}_\mu \gamma^\rho (1-\gamma_5)\mu^-\right] \bar{d}\gamma_\rho(1-\gamma_5)u\, V_{ud}^*$$

$$\Gamma = \frac{G_F^2}{4\pi}|V_{ud}|^2 f_\pi^2 m_\pi m_\mu^2\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2$$



$$+ \cdots \Big|^2 \neq |M_{tree}|^2 e^{-\alpha_{em} \gamma \log\left(\frac{m_\mu^2}{m_\gamma^2}\right)} \left(1 + \cdots\right)$$

regulator mass photon



$$+ \cdots \Big|^2 = |M_{tree}|^2 e^{\alpha_{em} \gamma \log\left(\frac{m_\mu^2}{m_\gamma^2}\right)}$$

So  + all radiative corrections $\Big|^2 = |M|^2 (1 + \cdots)$

(use optical theorem)

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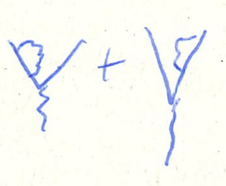
what about in $g-2$ 

?

In the electron it was ok as $\sim \frac{m_e^2}{m_\pi^2} \left(\frac{\alpha}{\pi}\right)^2$ is suppressed

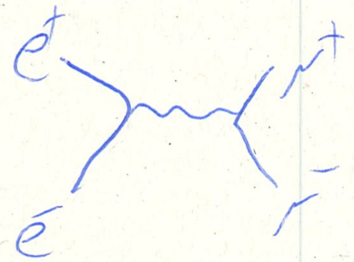
but $\frac{m_\mu^2}{m_\pi^2} \left(\frac{\alpha}{\pi}\right)^2$ is not so much. $\rightsquigarrow$ optical theorem 

QED α_{em} is corrupted by

 + gluon corrections $\Rightarrow$  + ... etc

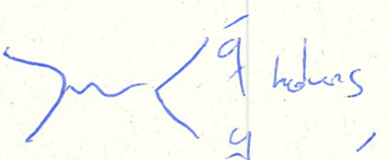
$\rightsquigarrow$ we find $\alpha_{em} =$

Hadron prod (116)



$$\sigma = \frac{4\pi}{3}\frac{\alpha_{em}^2}{s}$$

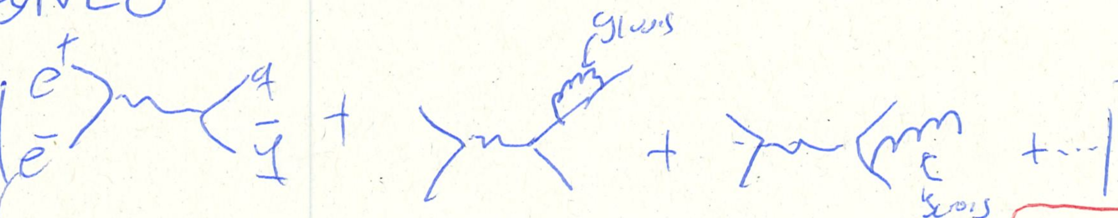
$$R = \frac{\sigma(e^+e^- \to \text{Had})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \left(\sum_f Q_f^2\right) N_c$$



replicas

$$\sigma = \times \left(\sum_f Q_f^2 N_c\right)$$

@NLO



which α_s?

harder $\rightsquigarrow$ optical Im() $\Rightarrow$

$$\boxed{R = \left(\sum_f Q_f^2\right) N_c \left(1 + \frac{\alpha_s}{\pi}\right)}$$

only sensible scale is $s = (p_1 + p_2)^2$.

@NLO

$$\Rightarrow R = \left(\sum_f Q_f^2\right) N_c \left[1 + \frac{\alpha_s}{\pi} - \frac{\alpha_s^2}{\pi}\Big(b_0 \left(\frac{1}{\varepsilon} - \gamma_E + \log(4\pi) - \log(\mu^2)\right) - b_0 \log(s) - \frac{c_1}{\pi}\Big]$$

$$\boxed{= \left(\sum_f Q_f^2\right) N_c \left(1 + \frac{\alpha_s^R(\mu^2)}{\pi} + \frac{(\alpha_s^R(\mu))^2}{\pi}\left(b_0 \log\left(\frac{s}{\mu^2}\right) + \frac{c_1}{\pi}\right) + O(\alpha_s^3)\right)}$$

$$\mathcal{L}_{QCD} = -\frac{1}{2g^2}\,\mathrm{tr}(G_{\mu\nu}G^{\mu\nu}) + \bar{q}\, i\not{D}\, q - \bar{q}(s+ip\gamma_5)\, q$$

$$\qquad \parallel$$

$$+ \bar{q}_L \gamma^\mu A^L_\mu q_L + \bar{q}_R \gamma^\mu A^R_\mu q_R \qquad \bar{q}_L(s+ip)q_R + \bar{q}_R(s-ip)q_L$$

$A_\mu^{L/R}$, $s+ip$ functions associated w/ its transformation under symmetries.

i.e. $A_\mu^{L/R}$ same as G_μ and $s+ip\gamma_5$ same as mass term.

we get

$$\mathcal{L}_{\chi PT} = \frac{f_\pi^2}{4}\Big[\mathrm{tr}\left(D^\mu\Sigma^\dagger D_\mu\Sigma\right) + \mathrm{tr}\left(\chi\Sigma^\dagger + \Sigma\chi^\dagger\right)\Big]$$

$$(s+ip)\,2B_0$$

$$D_\mu\Sigma = \partial_\mu\Sigma - iA^L_\mu\Sigma + i\Sigma A^R_\mu$$

Ex/ $\dfrac{\delta\mathcal{L}_{QCD}}{\delta A^L_\mu} \sim (\bar{q}^i_L \gamma^\mu q^j_L)$

$\rightsquigarrow$

$$\frac{\delta\mathcal{L}_{\chi PT}}{\delta(A^L_\mu)_{ij}} \sim \frac{f_\pi^2}{4}\left(-i\Sigma D^\mu\Sigma^\dagger\right)_{ij}$$

$$(\bar{q}^i\gamma^\mu\gamma_5 q^j) = \langle j^\mu_{A_{ij}}\rangle = i\frac{f_\pi^2}{2}\left(\Sigma D^\mu\Sigma^\dagger - \Sigma^\dagger D^\mu\Sigma\right)_{ij}$$

$$= f_\pi\,\partial^\mu\Pi(x)_{ji} + O(\pi^3)$$

$$\langle 0|\, \bar{d}\gamma^\mu\gamma_5 u\,|\pi^+_p\rangle = \langle 0|\sqrt{2}\, f_\pi\, \partial_\mu\pi^+|\pi^+_p\rangle = -i\sqrt{2}\, f_\pi\, p^\mu$$

$$\langle 0|\, q^i\bar{q}^j\, |0\rangle = -f_\pi^2 B_0\,\delta^{ij} \rightsquigarrow \quad B_0 = -\frac{\langle u\bar{u}\rangle}{f_\pi^2}$$

1) $\mathcal{L}_1 = \mathcal{L}_{QCD} + g_6 (\bar{s}\gamma^\mu \underbrace{(1-\gamma^5)d)(\bar{s}\gamma_\mu (1-\gamma^5)}_{\text{Only left handed fields}} d) + \text{h.c.}$

$$M = \begin{pmatrix} m_u & & 0 \\ & m_d & \\ 0 & & m_s \end{pmatrix}$$

$$\mathcal{L}_{\chi PT} = \frac{1}{4} f_\pi^2 \, tr(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) + \frac{f_\pi^2}{2} B_0 \, tr(M(\Sigma^\dagger + \Sigma))$$

infinity interactions

$$\Sigma = \exp(i\Pi), \quad f_\pi \Pi = \begin{pmatrix} \pi^0 + \frac{\eta}{\sqrt{3}} & \sqrt{2}\pi^+ & \dots \\ & & \\ - & - & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

$$+ \, O(\partial^n, m^n)$$

$$g_6 (\bar{s}_L \gamma^\mu d_L)(\bar{s}_L \gamma_\mu d_L) = g_6 (\bar{q}_L^i \gamma^\mu q_L^j)(\bar{q}_L^m \gamma_\mu q_L^n) V^{ij}_{mn}$$

$\delta^{i3}\delta^{j2}\delta^{m3}\delta^{n2}$

generalize V^{ij}_{mn} to get spurion trick.

$q_L \to U_L q_L$

$q_R \to U_R q_R$ $\quad$ $\mathcal{L}_6$ is inv under $SU(3)_L \times SU(3)_R$

to preserve the symmetry $\quad V^{ij}_{mn} \to U_L^{ii'} U_L^{mm'} V^{i'j'}_{m'n'} (U_L^\dagger)^{j'j} (U_L^\dagger)^{n'n}$

Symmetry is restored!

In the UV theory V has quarks to be contracted, in the χPT, V has to be matched with things that transform the same as left quarks, for example $\Sigma \to L\Sigma R^\dagger \Rightarrow \Sigma\Sigma^\dagger \to L\Sigma R^\dagger R \Sigma^\dagger L^\dagger = L\Sigma\Sigma^\dagger L^\dagger$

Good but not quite as is trivial.

Hence, use next one, $\Sigma\partial_\mu\Sigma^\dagger$ transforms as $L\dots L^\dagger$ ✓!!

→ not trivial ✓

So our term is $\quad \mathcal{L}_{\chi PT} \supset a(\Sigma\partial_\mu\Sigma^\dagger)^{ij} V^{ij}_{mn} (\Sigma\partial^\mu\Sigma)^{mn} + \text{h.c}$

we could have put $M\Sigma^\dagger$ as $M \to LMR^\dagger$

and $M\Sigma^\dagger \to LM\Sigma^\dagger L^\dagger$ but this is order mass so

not so relevant. (why?)

putting back our V^{ij}_{mn} expression

$$\mathcal{L}_{\chi PT} = a\left(\Sigma\partial_\mu\Sigma^\dagger\right)_{3,2}\left(\Sigma\partial^\mu\Sigma^\dagger\right)_{3,2} + \text{h.c.}$$

$$= a\left(\Sigma_{3b}\partial_\mu\Sigma^*_{2b}\right)\left(\Sigma_{3c}\partial^\mu\Sigma^*_{2c}\right)$$
$$+ a^*\left(\Sigma^*_{2b}\partial_\mu\Sigma_{3b}\right)\left(\Sigma^*_{2c}\partial^\mu\Sigma_{3c}\right)$$

Check a is real:

Back in $\mathcal{L}_1 \supset g_1(\bar{s}_L\gamma^\mu d_L)(\bar{s}_L\gamma_\mu d_L) + \text{h.c.}$

breaks P, C (C, P exchanges $L \leftrightarrow R$) but preserves T.

and PC is ok.

$$\Sigma \xrightarrow{P} \Sigma^\dagger \qquad B_0\Sigma^{ij} = \langle \bar{q}^j_R q^i_L\rangle$$
$$\Sigma \xrightarrow{C} \Sigma^T$$
$$\Sigma \xrightarrow{T} \Sigma^*$$

So since $*$ is a symmetry $\to$ (T)

then $\mathcal{L}_{\chi PT}$ first term goes to the second $\Rightarrow a = a^*$.

$\Sigma = e^{i\Pi} = 1 + i\Pi - \frac{1}{2}\Pi^2 + \cdots$

only keep quadratic order

$$\overset{\Downarrow}{=} a\left(\left(\Pi_{3b}\,(-i\partial\Pi)_{2b}\right)\left(\Pi_{3c}\,(-i\partial\Pi)_{c2}\right)\right) + \text{c.c.}$$

only quadratic order

$$\overset{\downarrow}{=} -a\left(\partial_\mu \Pi_{23}\partial^\mu \Pi_{23} + \partial_\mu \Pi_{32}\partial^\mu \Pi_{32}\right) = -a2\left(\partial_\mu K^0 \partial^\mu K^0 + \partial_\mu \bar{K}^0 \partial^\mu \bar{K}^0\right)$$

only correction for kinetic piece of $K^0, \bar{K}^0$.

$$\mathcal{L}_{\chi PT} = \frac{f_\pi^2}{4} \mathrm{tr}\left(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma\right) - \frac{1}{2} f_\pi^2 B_0 \mathrm{tr}\left(M\left(\Sigma^\dagger + \Sigma\right)\right)$$

$$- 2a\left(\partial_\mu K^0 \partial^\mu K^0 + \partial_\mu \bar{K}^0 \partial^\mu \bar{K}^0\right)$$

only care quadratic terms!

$$\overset{\downarrow}{=} \underbrace{\partial_\mu K^0 \partial^\mu \bar{K}^0 - B_0(m_d + m_s) K_0 \bar{K}_0}_{\text{this is like } \partial_\mu \phi \partial^\mu \phi^\dagger - m\phi\phi^\dagger} +$$

but this term mixes masses so diagonalize. (mixes degrees of freedom)

$$= \frac{1}{2}\partial_\mu \begin{pmatrix} K_0 \\ \bar{K}^0 \end{pmatrix}^T \begin{pmatrix} -a & 1 \\ 1 & -a \end{pmatrix} \partial^\mu \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix} + \frac{1}{2}(K^0\ \bar{K}^0)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix} B_0(m_d+m_s)$$

Redefine in new basis.

$$\overset{!}{=}$$

$$K^0{}' = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+a}}(K^0 + i\bar{K}^0) \leftarrow CP \text{ even}$$

as expected.

$$\bar{K}^0{}' = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1-a}}(K^0 - i\bar{K}^0) \swarrow CP \text{ odd}$$

2)

Remark: 3 quarks to work in perturbation theory $\Lambda < \Lambda_{QCD} \ll m_u^{-1}, m_d^{-1}$

and $\Lambda_{QCD} < m_s^{-1} <$ borderline.

Consider $\mathcal{L}_2 = \mathcal{L}_{QCD} + \underbrace{i d_5 \bar{u}\gamma^{\mu\nu}\tilde{G}_{\mu\nu} u}_{\mathcal{L}_{New}}$ $\epsilon^{\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}}$

to use spurion ~~ana~~ needed to (as we want to recover symmetry $SU(3)_L \times SU(3)_R$)

we need to express this in this rep

$$\mathcal{L}_{new} \sim i d_5\left(\bar{u}_L \gamma\cdot\tilde{G}\, u_R + \bar{u}_R \gamma\cdot\tilde{G}\, u_L\right)$$

where we used $\bar{u}_L u_L = 0$, $\bar{u}_L \gamma^\mu u_R = 0$, $\bar{u}_L \gamma^\mu \gamma^\nu u_L = 0$.

$$= i d_5 \bar{u}_L (\gamma \cdot \tilde{\sigma}) u_R + h.c.$$

what this preserves?

$$\bar{u} \gamma^{\mu\nu} u \xrightarrow{P} \mathbb{P}^\mu{}_\sigma \mathbb{P}^\nu{}_\rho (\ ) = (-1)^\mu (-1)^\nu (\ )$$

$$\xrightarrow{T} -\mathbb{P}^\mu \mathbb{P}^\nu (\ ) = -(-1)^\mu (-1)^\nu (\ )$$

$$\xrightarrow{C} -(\ ) = -(\ )$$

$$\tilde{\sigma}_{\mu\nu} \xrightarrow{P} -(\mathbb{P})^\mu \mathbb{P}^\nu (\ ) = -(-1)^\mu (-1)^\nu (\ )$$

$$\xrightarrow{T} (-1)^\mu (-1)^\nu (\ )$$

$$\rightarrow -(\ )$$

P	×
T	~~✓~~ ×
C	~~×~~ ✓

$$= \tilde{M}^{ij} \bar{q}_L^i \gamma \cdot \tilde{\sigma} q_R^j + h.c., \quad \tilde{M}^{ij} = i d_5 \delta^{iu} \delta^{ju}$$

so $\tilde{M} \to L \tilde{M} R^+$ ← transforms like Σ.

so in the χPT, $\mathcal{L}_{\chi PT} = \frac{f_\pi^2}{4} \mathrm{tr}(\partial_\mu \Sigma^+ \partial^\mu \Sigma) - \frac{f_\pi^2 B_0}{2} \mathrm{tr}(M(\Sigma^+ + \Sigma))$

$$+ \frac{f_\pi^2}{2} B_0' \,\mathrm{tr}(\underbrace{\tilde{M} \Sigma^+ + \tilde{M}^+ \Sigma}_{i d_5 (\Sigma^+)_{11} - i d_5 \Sigma_{11}})$$

$$\propto \frac{f_\pi^2}{2} B_0 \, i d_5 ((\Sigma^+)_{11} - \Sigma_{11})$$