Quantum measurements on relativistic quantum field theory

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Outline

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3. Causally disjoint sets → no particular order of measure.
4. ¿Sorkin's protocol?

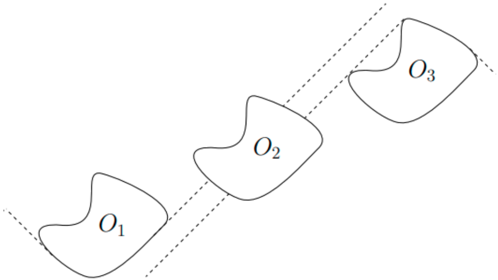

Figure: Sorkin's protocol[?]

Definitions and notation I

Let $x \in M, S \subseteq M$

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3. Compatibility: $N \subset M, N = ch(N)$
   then, exist a (Compatibility map) $\alpha : \mathscr{A}(M\big|_N) \to \mathscr{A}(M)$ with

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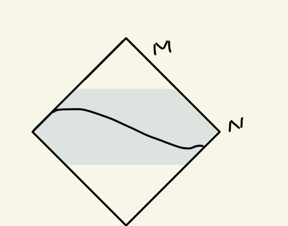
4. Time-slice property (existence of dynamics)



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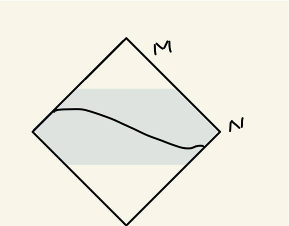

The measurement and the probe I

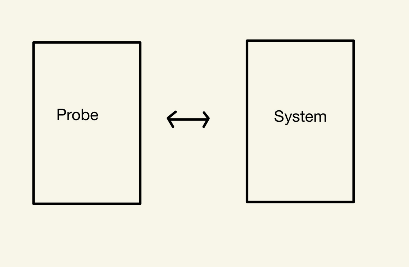

Figure: The experimental setup

The probe and the system are coupled in a compact region K. Outside the region they behave uncoupled.

The measurement and the probe II

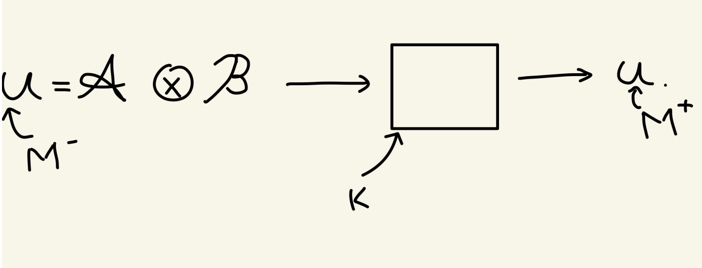

Figure: In and out algebras

On **M** the algebra of observables $\mathscr{U}(\mathrm{M}) = \mathscr{A}(\mathrm{M}) \otimes \mathscr{B}(\mathrm{M})$ with Compatibility map $\alpha_{M;N} \otimes \alpha_{M;N}$

The measurement and the probe III

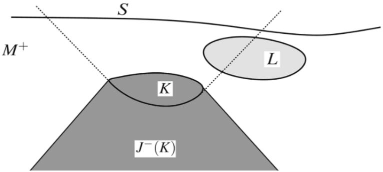

Figure: Compatibility

$L \subset L'$ open the following diagram commutes:

$$\begin{array}{ccc} \mathscr{A}(L') \otimes \mathscr{B}(L') & \xrightarrow{\alpha_{L;L'} \otimes \beta_{L;L'}} & \mathscr{A}(L) \otimes \mathscr{B}(L) \\ \Big\downarrow \chi_{L'} & & \Big\downarrow \chi_L \\ \mathscr{C}(L') & \xrightarrow{\gamma_{L;L'}} & \mathscr{C}(L) \end{array}$$

The regions $M^{\pm}$ are big enough to have Cauchy surfaces $\xrightarrow{time-slice\ pptty}$ The morphisms α, β, γ and χ are isomorphism on $M^{\pm}$.

$$\tau^{\pm} : \mathcal{U}(M) \to \mathcal{C}(M)$$

$$\tau^{\pm} = \kappa^{\pm} \circ (\alpha^{\pm} \circ \beta^{\pm})^{-1} = \gamma^{\pm} \circ \chi^{\pm} \circ (\alpha^{\pm} \circ \beta^{\pm})^{-1}$$

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$$\Theta = (\tau^{-})^{-1} \circ \tau^{+}$$

an automorphism.

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- $L \subset K \perp$ then Θ acts trivially on $\mathcal{U}(M; L)$-
- $L^{\pm} \subset M^{\pm}$ open causually convex and $L^{+} \subset D(L^{-})$. Then $\Theta(\mathcal{U}(M; L^{+})) \subset \mathcal{U}(M; L^{-})$

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Measurement scheme I

Prepare the system at a state ω of $\mathscr{A}$ and the probe at a state σ of $\mathscr{B}$ we want to describe the measurements of the coupled $\mathcal{C}(M)$ in terms of the uncupled observables (QMT).

Measurement scheme II

Measure $\tilde{B}$ at ω but since the information of an observable A must be on the probe we want to read information of A from B. That is,

$$\omega(A) = \underset{\sim}{\omega}_\sigma(\tilde{B}) = (\omega \otimes \sigma)(\Theta(1 \otimes B)) \quad \forall\, \omega \text{ state of } \mathscr{A}(M)$$

Consider $\eta_\sigma : \mathscr{A} \otimes \mathscr{B} \to \mathscr{A}$ be the map $\eta_\sigma(A \otimes B) = \sigma(B)A$

$$A\eta_\sigma(C) = \eta_\sigma((A \otimes 1)C), \quad \eta_\sigma(C)A = \eta_\sigma(C(A \otimes 1))$$

Measurement scheme III

Then,

$$A = \varepsilon_\sigma(B) := \eta_\sigma(\Theta(1 \otimes B))$$

where $\eta_\sigma(B)$ is called the **induced system observable**

$$\omega(A) = \omega(\eta_\sigma(\Theta(1 \otimes B))) = (\omega \otimes \sigma)(\Theta(1 \otimes B)) = \underset{\sim}{\omega}_\sigma(\tilde{B})$$

The system $(\mathscr{B}, \mathcal{C}, \chi(\tau^\pm), \sigma)$ is called the **measurment scheme** of the induced system observable.

Properties of the induced system observable

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- $\varepsilon_\sigma(B^*B) - \varepsilon_\sigma(B)^*\varepsilon_\sigma(B)$ is a positive operator
- $$\begin{aligned}\mathrm{Var}(\tilde{B};\omega_\sigma) &= \underset{\sim}{\omega}_\sigma(\tilde{B}^2) - \underset{\sim}{\omega}_\sigma(\tilde{B})^2 = \omega(\varepsilon_\sigma(B^2)) - \omega(\varepsilon_\sigma(B))^2 \\ &\geq \omega(A^2) - \omega(A)^2 = \mathrm{Var}(A;\omega)\end{aligned}$$

An **effect** is an observable such that B and (1-B) are both positive.

$$\mathrm{Prob}(B|\omega) = \omega(B), \quad \mathrm{Prob}(B^c|\omega) = \omega(1 - B)$$

Effect-valued measures (EVMs) are maps $E : \chi \to \mathrm{Effects}(\mathscr{B}(M))$ where χ is a $\sigma - algebra$ the values $\omega(E(X))$ are the probability of measure in a set X is observed in state ω

Suppose a probe-effect B is tested when the system is in a state ω The post slected system state conditioned on the effect observed should correctly predict the probability of any system effect being observed given that B was observed.

$$(Prob)(A \cap B) = (\omega \otimes \sigma)(\Theta(A \otimes B)) = (\mathscr{I}_\sigma(B)(\omega))(A)$$

so

$$(Prob)(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{(\mathscr{I}_\sigma(B)(\omega))(A)}{(\mathscr{I}_\sigma(B)(\omega))(A)}$$

We call $\mathscr{I}_\sigma(B)$ a pre-instrument. If defined, the normalized post-selected satem conditioned on B, is

$$\omega' = \frac{\mathscr{I}_\sigma(B)(\omega)}{\mathscr{I}_\sigma(B)(\omega)(1)}$$

A non-selective measurment results in $\omega_B = \mathscr{I}_\sigma(B)(\omega) + \mathscr{I}_\sigma(1 - B)(\omega) = \mathscr{I}_\sigma(1)(\omega)$ wich is independent of B.

Theorem

For a localizable in $K^{\perp}$, $\omega'(A) = \frac{\omega(A\varepsilon_\sigma(B))}{\omega(\varepsilon_\sigma(B))}$

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If ωhas a Reeh-Schlider property (e.g. Minkowski vacuum state)

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for observables localizable in $k^{\perp}$ Post selection on any nontricial measurments alters expectation values on $K^{\perp}$ This is due the correlations. We have not assumed any rule that ω changes ω' across a sruface in M. nor have we found any indication that such a rule is desirable.

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Succesive measurments I

Consider $\mathscr{B}_i$ with coupling regions K_i and scattering morphisms Θ_i. The probe system consist of $\mathscr{B}_1 \otimes \mathscr{B}_2$ with coupling region $K_1 \cup K_2$ and morphism Θ-

Sorkin's protocol I

Model A and B using probes.

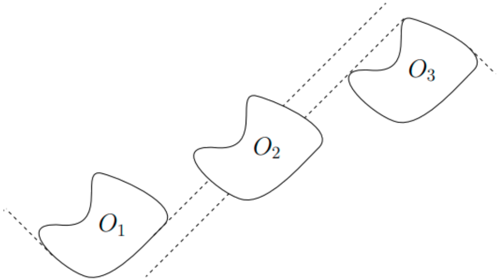

Figure: Sorkin's protocol[?]

Sorkin's protocol II

The scattering morhpism gives $\Theta_2 C \otimes_2 Id \in \mathcal{U}(M;N)$ for
$N \subset K_A^{\perp} \cap M_B^{-}$
Hence, C cannot determine what A has measured:

$$\begin{aligned}\omega_{AB}(C) &= (\omega \otimes \sigma_1 \otimes \sigma_2)((\Theta_1 \otimes_3 Id) \circ (\Theta_2 \otimes_2 Id)(C \otimes_2 1)) \\ &= (\omega \otimes \sigma_1 \otimes \sigma_2)(\Theta_2(C \otimes 1 \otimes 1)) = \omega_B(C)\end{aligned}$$

A Specific Probe Model I

Let the probe and the system both free scalar fields with a linear coupoling on a bounded region.

$$\mathcal{L}_0 = -(\nabla_a \phi)(\nabla^a \phi) + m_\phi^2 \phi^2 - (\nabla_a \psi)(\nabla^a \psi) + m_\psi^2 \psi^2 := P\phi + Q\psi$$

$$\mathcal{L}_{\text{int}} = \rho\phi\psi, \quad \rho \in C_0^\infty(M), \quad K = \operatorname{supp}\rho$$

$E_p^\pm : C_0^\infty(M)(M) \to C_0^\infty(M)(M)$

$$E_P^\pm P f = f, P E_p^\pm f = f, \mathrm{E_P}^\pm f \subset J^\pm(\mathrm{suppf})$$

$E_P = E_P^- - E_P^+$ then every ϕ can be written as $\phi = E_P f$.

$$\underbrace{\begin{pmatrix} P & R \\ R & Q \end{pmatrix}}_{T} \underbrace{\begin{pmatrix} \phi \\ \psi \end{pmatrix}}_{\zeta} = 0$$

Quantization I

Let f, h be a (smooth) compactly supported functions on M

- $f \to \psi(f)$
- $\psi(f) = \psi(f)^*$,
- $\psi(Pf) = 0$
- $[\psi(f), \psi(h)] = iE_p[f, h]1$

$$E_p(f, h) := \int_M dvol\, f E_p h$$

Quantization II

If the region $L \subset K^{\perp}$ the algebras $\mathscr{A}(L) \otimes \mathscr{B}(L)$ and $\mathscr{C}(L)$ coincide as we saw earlier (commutative diagram).
By the time-slice property it is enough to consider the action of the scattering map on a generator $\zeta_0(F)$ of $\mathscr{A}(L) \otimes \mathscr{B}(L)$ with F (compactly) supported on M^+ Giving

$$\Theta\zeta_0(F) = \zeta_0(F - \tilde{R}E_T F) = \zeta_0(F - \tilde{R}E_T^- F)$$

where $\tilde{R} = T - P \oplus Q$

A Specific Probe Model I

$$\Theta(1 \otimes \psi(h)) = \phi(f^-) \otimes 1 + 1 \otimes \psi(h^-)$$

$$\begin{pmatrix} f^- \\ h^- \end{pmatrix} = \begin{pmatrix} 0 \\ h \end{pmatrix} - \begin{pmatrix} 0 & R \\ R & 0 \end{pmatrix} E_T^- \begin{pmatrix} 0 \\ h \end{pmatrix}$$

f^- supported in K and h^- in $M^+ \cup K \implies$

$$\Theta(1 \otimes e^{i\psi(h)}) = e^{i\phi(f^-)} \otimes e^{i\psi(h^-)}$$

Induced observables I

$$\varepsilon_\sigma(\psi(h)) = \eta_\sigma(\Theta(1 \otimes \psi(h))) = \phi(f^-) + \sigma(\psi(h^-1))1$$

$\Longrightarrow$

$$\varepsilon_\sigma(e^{i\psi(h)}) = \eta_\sigma(\Theta(1 \otimes e^{i\psi(h)})) = \sigma(e^{i\psi(h^-1)})e^{i\phi(f^-)}$$

Sharpness of the observable $\tilde{\psi}(h)$ on the state σ

$$\begin{aligned}
\mathrm{Var}(\tilde{\psi}(h); \underset{\sim}{\omega}_\sigma) &= \underset{\sim}{\omega}_\sigma(\tilde{\psi}^2(h)) - \underset{\sim}{\omega}_\sigma(\tilde{\psi}(h))^2 \\
&= \omega(\varepsilon_\sigma(\psi(h)^2)) - \omega(\varepsilon_\sigma(\psi(h)))^2 \\
&= \omega(\phi(f^-)^2) + 2\omega(\phi(f^-))\sigma(\psi(h^-)) + \sigma(\psi(h^-)^2) \\
&- (\omega(\phi(f^-))^2 + 2\omega(\phi(f^-))\sigma(\psi(h^-)) + \sigma(\psi(h^-)))^2 \\
&= \mathrm{Var}(\phi(f^-); \omega) + \mathrm{Var}(\tilde{\psi}(h^-); \sigma)
\end{aligned}$$

To do:

- σ a quasifree state
- Localisation of induced observables
- Perturbative treatment of the detector response

References I

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Henning Bostelmann, Christopher J. Fewster and Maximilian H. Ruep
Impossible measurements require impossible apparatus

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