Department of Physics, Universidad de los Andes

Paper Tube

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Outline

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Roll a long paper strip into a tight tube and put it vertically on a table. Why does it often unwind in jerks? What determines the period of the jerks?

- Plastic and elastic deformation.
- Curvature of the paper $\rightarrow$ Potential energy

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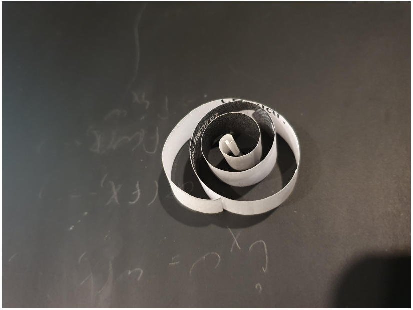
- Plastic and elastic deformation.

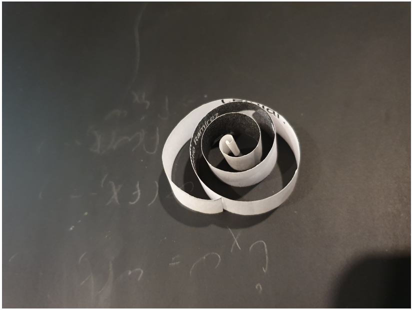
Paper before rolling.

- Plastic and elastic deformation.



Paper after rolling.

- Plastic and elastic deformation.



Paper after rolling.

The longer the paper the greatest elastic behaviour (rather than plastic behaviour)!!!

- Curvature of the paper $\rightarrow$ Potential energy

The paper has two properties, **Elastic** and **Plastic** behaviour.

- Elastic $\rightarrow$ Potential energy.
- Plastic $\rightarrow$ New equilibrium position

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- Elastic $\longrightarrow$ Potential energy.
- Plastic $\longrightarrow$ New equilibrium position

For a beam with normal vector from the cross section pointing to the x coordinate

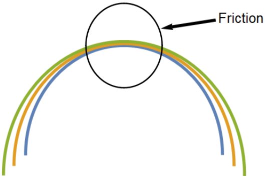
$$\frac{1}{2}EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 \tag{1}$$

$\boxed{\textbf{Least energy principle changed!}}$

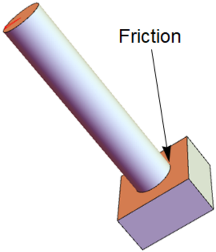
Frictional Forces

- Friction due to ground (Wood).
- Friction due to layers of paper.

Those contribute to a force against the motion.



n, $n+1$ and $n+2$ winding's of the paper tube seen from above



Paper tube standing on a wood ground

Energy stages

- Energy in motion (During the jerk).
- Energy in static state.

We can apply conservation of energy in all the system.

Energy equation:

$$\boxed{V_i = V_f(\theta) + T(\theta) + d_1(t,\theta)Fr_{P-P}(t,\theta) + d_2(t,\theta)Fr_{P-W}(t,\theta)} \qquad (2)$$

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Friction dependence's

- $\mu_{k_{P-P}}, \mu_{s_{P-P}}$
- $\mu_{k_{P-W}}, \mu_{s_{P-W}}$

Remember, the coefficients $\mu_{k_{P-W}}, \mu_{s_{P-W}}$ may fluctuate!

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Friction dependence's

- $\mu_{k_{P-P}}, \mu_{s_{P-P}}$
- $\mu_{k_{P-W}}, \mu_{s_{P-W}}$
- $d_1(\theta)$
- $d_2(\theta)$

In general we do not know how d_1 and d_2 changes but we could find some constrains.
For d_1, d_2 we have the following inequalities for a given θ:

$$d_1 \leq \ell - 2\pi r$$

$$d_2 \leq \int_0^{\theta_i - \theta} r(\theta')d\theta'$$

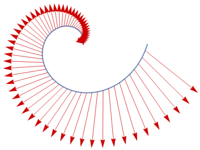
Initial conditions determine:

1. Tightness $\rightarrow$ Change of initial total radius $\rightarrow$ change of the graph
2. Deformation of the paper (Maximum potential energy)
3. d_1 and d_2.

Initial conditions determine the curvature. We have:

$$\kappa(\theta) = \frac{\left|2\,(f'(\theta))^2 + (f(\theta))^2 - f(\theta)f''(\theta)\right|}{\left[(f'(\theta))^2 + (f(\theta))^2\right]^{3/2}}$$

Types of spirals and energy:

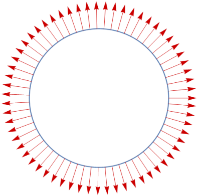


Spiral with increasing curvature

$$\kappa(\theta) = \frac{\left|2\,(f'(\theta))^2 + (f(\theta))^2 - f(\theta)f''(\theta)\right|}{\left[(f'(\theta))^2 + (f(\theta))^2\right]^{3/2}}$$

Types of spirals and energy:

- Increasing curvature



Spiral with constant curvature

$$\kappa(\theta) = \frac{\left|2\left(f'(\theta)\right)^2 + \left(f(\theta)\right)^2 - f(\theta)f''(\theta)\right|}{\left[\left(f'(\theta)\right)^2 + \left(f(\theta)\right)^2\right]^{3/2}}$$

Types of spirals and energy:

- Constant curve

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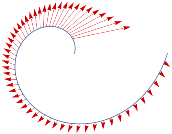

Spiral with decreasing curvature

$$\kappa(\theta) = \frac{\left|2\,(f'(\theta))^2 + (f(\theta))^2 - f(\theta)f''(\theta)\right|}{\left[(f'(\theta))^2 + (f(\theta))^2\right]^{3/2}}$$

Types of spirals and energy:

- Decreasing curvature

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Determinat factors

1. Tightness $\rightarrow$ Variation in initial energy $\rightarrow$ Variation in Fr_{P-P}
2. Variations on $\mu_{k_{P-W}}$ $\rightarrow$ Different unrolling $\rightarrow$ Variation in Fr_{P-P}

The friction allows the tube to have a differences of forces which generates torque as the video may suggest

We first analyse the tortion in half of a hollow cylinder.
Define ϕ as the torsion angle, $\phi = M/C$, where C given by
$C = \mu\pi R^4/2$
The free energy is given by $F = \sigma_{ik}u_{ik}/2$
The potential energy

$$F_R = \int C\phi^2 dz = \frac{1}{4}\mu\pi R^4\phi^2 L$$

.

Conservation of energy gives:

$$\frac{1}{2}C\phi_0^2 L = \frac{1}{2}I\left(\frac{d\phi}{dt}\right)^2 + \frac{1}{2}C\phi_f^2$$

Given that $\phi(0) = 0, \phi'(0) = \omega_0$ the solution is:

$$\phi(t) = \sqrt{\frac{I}{CL}}\omega_0 \sin\left(\sqrt{\frac{CL}{I}}t\right)$$

Second law of motion gives:

$$\frac{\phi}{C} - \mu_k mgR = \frac{d}{dt}\left(I(R)\frac{d\phi}{dt}\right)$$

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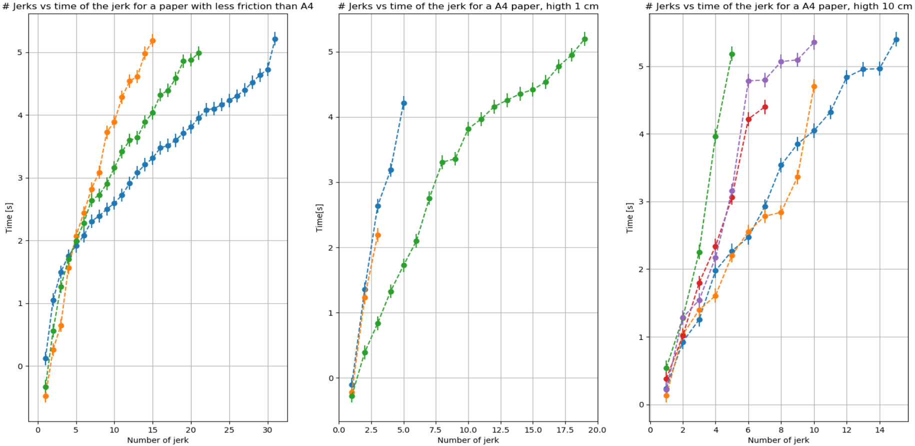
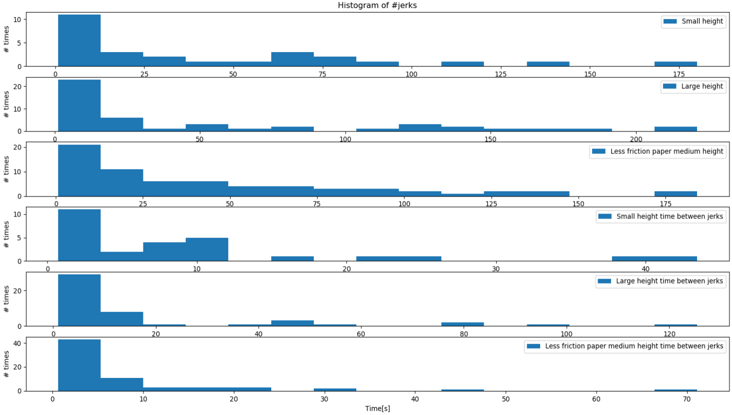


Comparison between different paper tubes.



Histogram of times of jerks and time between jerks in different paper tubes

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Coefficients of friction for a A4 paper.

1. $\mu_{s_{P-P}} = 0.79 \pm 0.06$
2. $\mu_{s_{P-W}} = 0.78 \pm 0.05$

Unrolling paper tube without friction paper-paper.

Unrolling paper tube without friction paper-wood.

Fit of the model: ¿Why?

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Things that do fit

1. The less friction paper-paper the frequent the jerks $\longrightarrow$ period changes with friction paper-paper.
2. Jerks with only Wood- paper friction and with only Paper-paper friction.
3. Deformation of paper makes less jerks.

Things that do not fit

1. Not much dependence of height of the paper.