A Holographic Approach to Boundary CFTs

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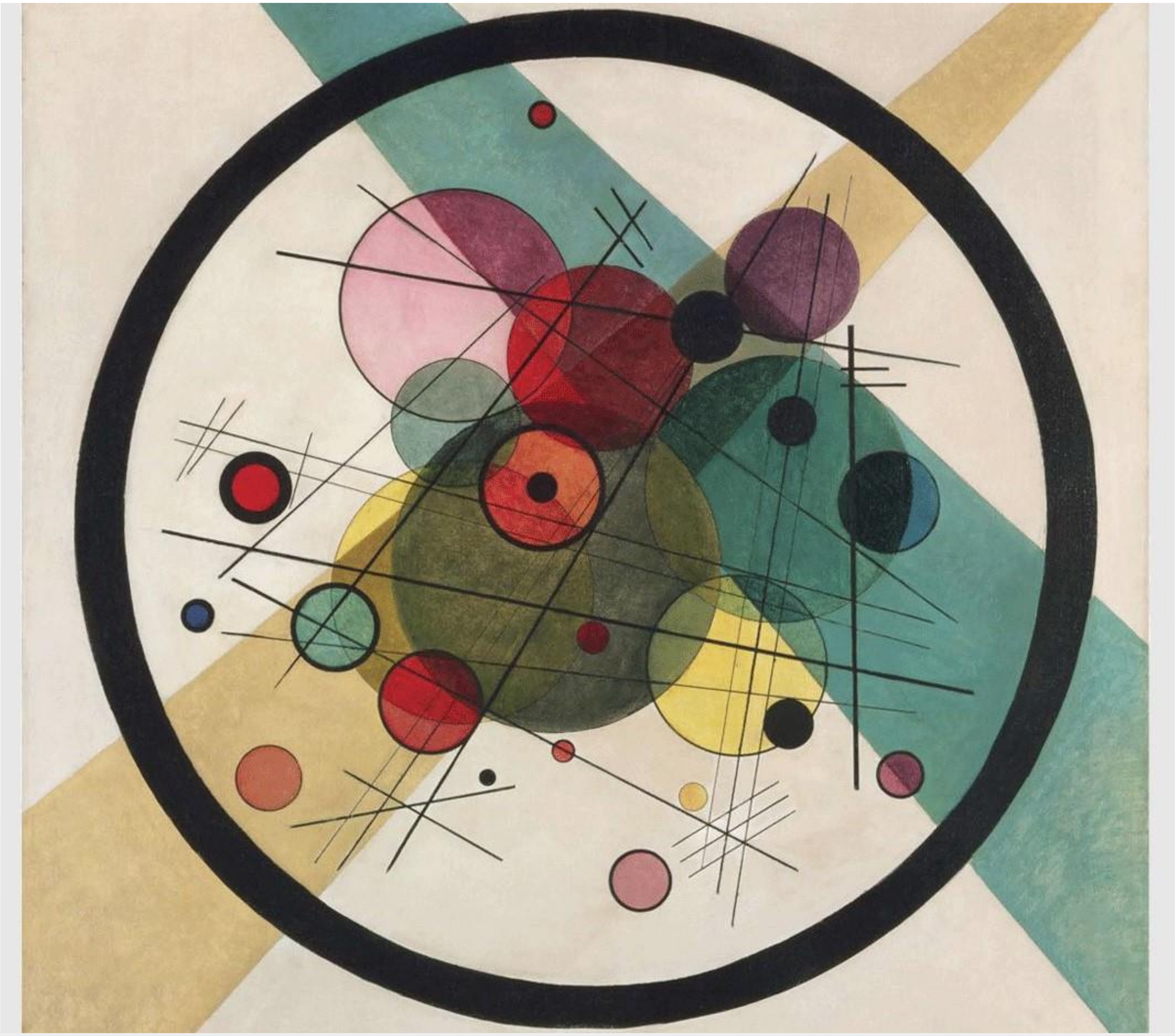


Conformal Field Theories and boundaries

CFTs are **locally** characterized by the so-called CFT data

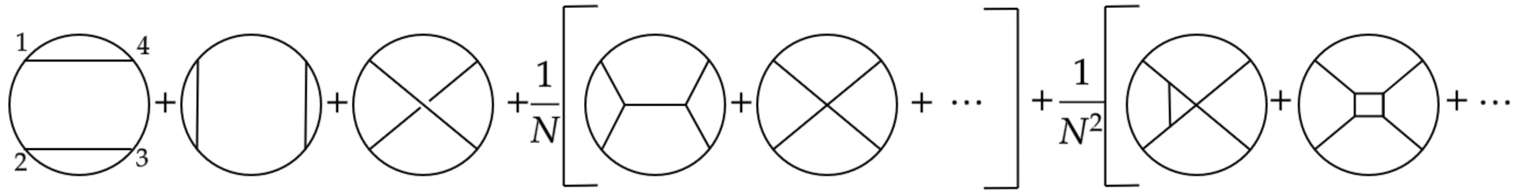
$$\{\Delta_i, C_{ijk}\}_{i,k,l\in I},$$

$$\text{Poincare invariance} + \{D, K_\mu\} \sim SO(d+1,1)$$

$$\Big\downarrow \text{Defect}$$

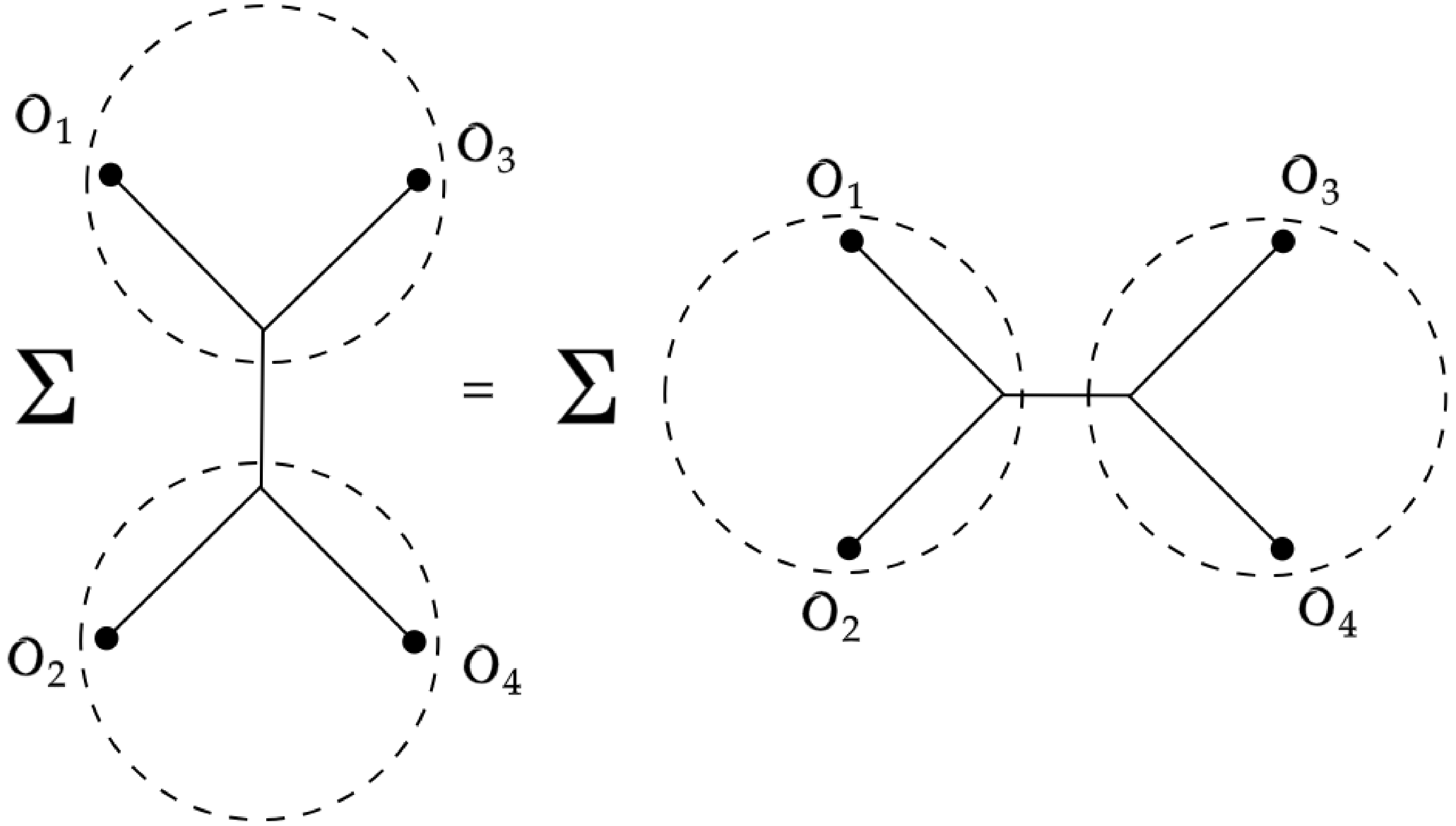
$$SO(p+1,1)\times SO(q).$$

- Useful when conformal symmetry is broken by experimental limitations or boundary conditions.
- Quantum gravity, where a boundary can be interpreted as a D-brane in string theory or the edge of an AdS space.

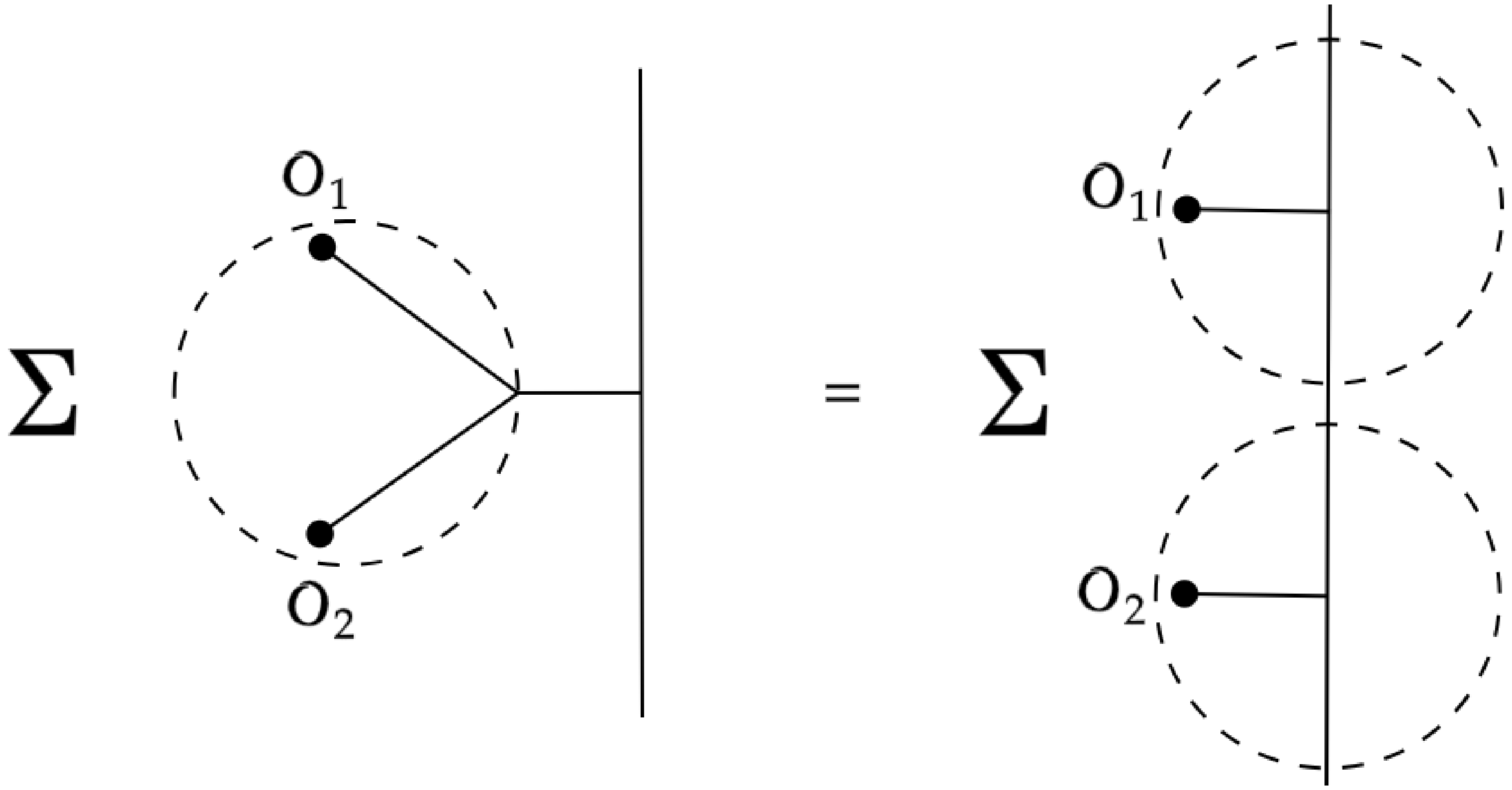


- Simplest instance of enhancing the CFT data due to nonlocal operators (the codimension $q=1$ defect).

$$\{\Delta_i, \hat{\Delta}_i, \hat{C}_{ijk}, C_{ijk}\}.$$



$$1+\sum \lambda^2_{\Delta,\ell} G_{\Delta,\ell}(u,v) = \left(\frac{u}{v}\right)^{\Delta}\left(1+\sum \lambda^2_{\Delta,\ell} G_{\Delta,\ell}(v,u)\right)$$



$$\sum_{\Delta,I} \lambda_{\Delta,I} a_\Delta G^{\text{bulk}}_{\Delta,I}(\xi_1) = \sum_{\hat{\Delta},I} \lambda^2_{\hat{\Delta},I} G^{\text{defect}}_{\Delta,I}(\xi_1).$$

Light cone embedding

$$x^\mu \to (1, x^2, x^i) \equiv (X^+, X^-, X^i)$$

The conformal group acts as $SO(d+1,1)$ under the identification:

$$J_{\mu\nu} = M_{\mu\nu}\,, \quad J_{\mu+} = P_\mu, \quad J_{\mu-} = K_\mu, \quad J_{+-} = D.$$

This implements the conformal transformations by

$$X' = R(\Lambda X) = (1, x'^2, x'^\mu)$$

- Scaling: $O(\lambda X) = \lambda^{-\Delta} O(X)$
- Gauge fixing: $X^\mu O_{\mu \ldots}(X) = 0$

Projection back to physical space is given by:

$$O_{\mu\nu\cdots}(x) = O_{MN\cdots} \frac{\partial X^M}{\partial x^\mu} \frac{\partial X^N}{\partial x^\nu} \cdots .$$

This allows us to write general operators as:

$$F_J(Z, X) = Z^{\mu_1} \cdots Z^{\mu_J} F_{\mu_1 \ldots \mu_J}(X).$$

Light cone embedding and the Defect

We write $x^\mu = (x^a, r)$.

$$P \cdot Q = P^a Q^a \eta_{ab} \text{ and } P \circ Q = r_P\, r_Q .$$

The boundary is spatially embedded in the light cone by

$$X^A = (1, x^2, x^a), \quad X^r = 0.$$

Two types of operators:

$$\hat{\mathcal{O}}(Z^A, W^I) \text{ and } \mathcal{O}(Z^M)$$

Examples without defect

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{k}{((x-y))^{2\Delta}}$$

$$\langle \mathcal{O}(X)\mathcal{O}(Y)\rangle = \frac{k'}{(-2X\cdot Y)^{\Delta}}$$

$$\langle \mathcal{O}_\mu(x)\mathcal{O}_\nu(y)\rangle = C_{\mathcal{O}\mathcal{O}}\frac{\delta_{\mu\nu} - 2\frac{(x-y)_\nu (x-y)_\mu}{(x-y)^2}}{(x-y)^{2\Delta}}$$

$$\langle \mathcal{O}_M(X)\mathcal{O}_N(Y)\rangle = C_{\mathcal{O}\mathcal{O}}\frac{1}{(X\cdot Y)^{\Delta}}\left(\eta_{MN} + \alpha\frac{X_N Y_M}{X\cdot Y}\right)$$

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_{3\mu}(z)\rangle = \frac{C_{123}}{(x-y)^{\alpha_{123}}(x-z)^{\alpha_{132}}(z-y)^{\alpha_{321}}}\frac{|y-z||x-z|}{|x-y|}\left(\frac{(x-z)_\mu}{|x-z|^2} - \frac{(y-z)_\mu}{|y-z|^2}\right)$$

$$\langle \mathcal{O}_1(X)\mathcal{O}_2(Y)\mathcal{O}_{3M}(Z)\rangle = \frac{C_{123}}{(-X\cdot Y)^{\alpha_{123}}(-X\cdot Z)^{\alpha_{132}}(-Z\cdot Y)^{\alpha_{321}}}\left(\frac{(X\cdot Z)Y_M - (Y\cdot Z)X_M}{(X\cdot Y)^{\frac{1}{2}}(Y\cdot Z)^{\frac{1}{2}}(X\cdot Z)^{\frac{1}{2}}}\right)$$

Examples with defect

$$\langle \hat{\mathcal{O}}_{\hat{\Delta},0,0}(x_1^a)\hat{\mathcal{O}}_{\hat{\Delta},0,0}(x_2^a)\rangle = \frac{1}{|x_1^a - x_2^a|^{2\hat{\Delta}}}$$

$$\langle \hat{\mathcal{O}}_{\hat{\Delta}}(X_1, W_1)\hat{\mathcal{O}}_{\hat{\Delta}}(X_2, W_2)\rangle = \frac{1}{(-2X_1\bullet X_2)^{\hat{\Delta}}}$$

$$\langle \mathcal{O}_{\Delta,0}(x)\rangle = \frac{a_{\mathcal{O}}^{1,0}}{|x^i|^{\Delta}}$$

$$\langle \mathcal{O}(X, Z)\rangle = \frac{a_{\mathcal{O}}}{(X\circ X)^{\frac{\Delta}{2}}}$$

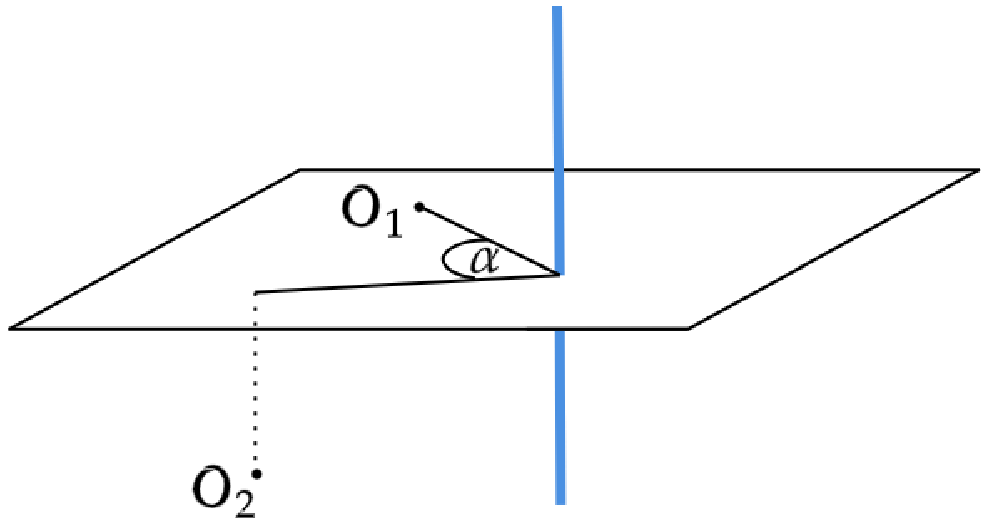
$$\left\langle \mathcal{O}_\Delta(x)\hat{\mathcal{O}}_{\hat{\Delta}}(0)\right\rangle = \frac{b_{\mathcal{O}\hat{\mathcal{O}}}}{|x|^{\Delta-\hat{\Delta}}|x^a|^{2\hat{\Delta}}}$$

$$\left\langle \mathcal{O}(X_1)\hat{\mathcal{O}}(X_2)\right\rangle = \frac{b_{\mathcal{O}\hat{\mathcal{O}}}}{(-2X_1\bullet X_2)^{\hat{\Delta}}(X_1\circ X_1)^{\frac{\Delta-\hat{\Delta}}{2}}}$$

Defect crossing equation

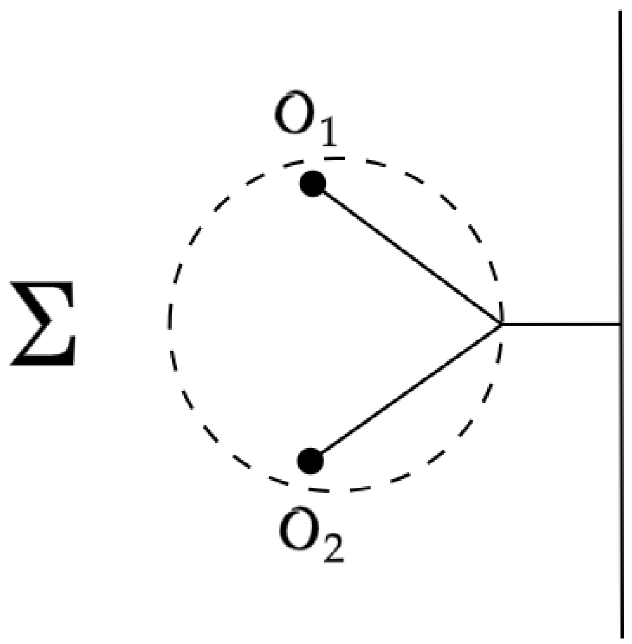
We consider two point functions of two bulk operators in the presence of the defect.
In this case, we have two invariants:

$$\xi_1 = \frac{(x_1 - x_2)^2}{4|x_1^i||x_2^i|}, \quad \xi_2 = \cos\alpha = \frac{x_1^i \cdot x_2^i}{|x_1^i||x_2^i|},$$



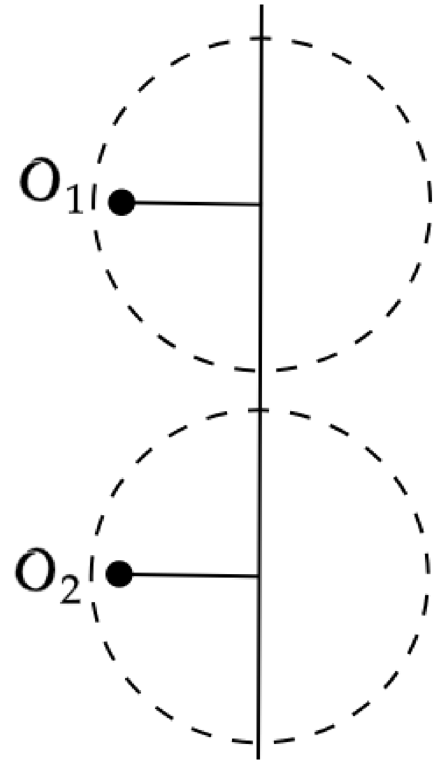
Embedding in the light cone implies the two point function is:

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\rangle = \frac{f(\xi_1,\xi_2)}{|x_1^i|^{\Delta_1}|x_2^i|^{\Delta_2}}.$$



$$= \frac{1}{r_1^{\Delta_1} r_2^{\Delta_2}}\left(\sum_{\hat{\Delta},I}\lambda^2_{\hat{\Delta},I} G^{\text{defect}}_{\Delta,I}(\xi_1,\xi_2)\right) = \sum_{\hat{\Delta},I}\lambda^2_{\hat{\Delta},I}\left[\hat{C}_{\hat{\Delta},I}|_{y=0}(r_1,\partial_y) C_{\hat{\Delta},I}|_{z=0}(r_2,\partial_z)\right]\langle\hat{\mathcal{O}}_{\hat{\Delta},I}(y)\hat{\mathcal{O}}_{\hat{\Delta},I}(w)\rangle$$

$$= \sum_{\Delta,I}\lambda_{\Delta,I} C_{\Delta,I}(x_{12},\partial_y)|_{y=0}\frac{a_\Delta}{r_y^\Delta} = \frac{1}{r_1^{\Delta_1} r_2^{\Delta_2}}\left(\sum_{\Delta,I}\lambda_\Delta a_\Delta G^{\text{bulk}}_{\Delta,I}(\xi_1,\xi_2)\right) = \sum$$



CFTs at Large N and the AdS/CFT correpondence

We say that this theory enjoy large N factorization if the planar 2-pt function contribution is independent of N while connected higher point functions are suppressed by powers of N. This implies that the 2-pt function of multitrace operators is dominated by the product of the two-point functions of its single-trace constituents:

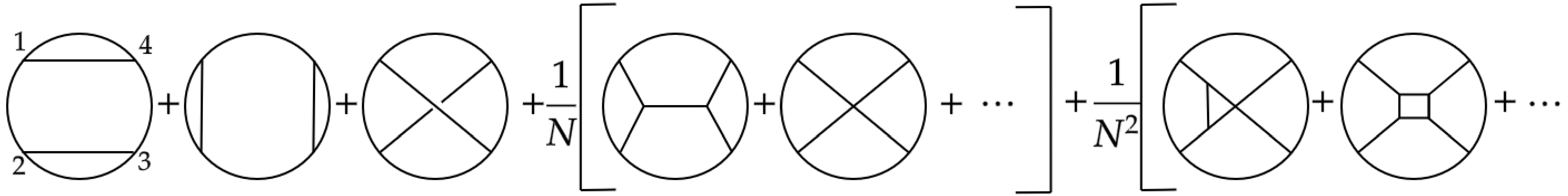
$$\langle \tilde{\mathcal{O}}(x)\tilde{\mathcal{O}}(y)\rangle \approx \prod_i \langle \mathcal{O}_i(x)\mathcal{O}_i(y)\rangle = \frac{1}{(x-y)^{2\sum_i \Delta_i}}.$$

We conclude that the CFT data of scaling dimension of the multi-trace operator are given, in terms of $g = 1/N$, by

$$\Delta_i = \sum_i \Delta_i^{(0)} + g\Delta_i^{(1)} + \dots \qquad\qquad C_{ij}^{k} = C_{ij}^{k\,(0)} + gC_{ij}^{k\,(1)} + \cdots$$

This implies that crossing equation must hold order by order in $\frac{1}{N}$. This is the form of large N factorization that admits a dual interpretation by the AdS/CFT prescription.

Witten diagrams



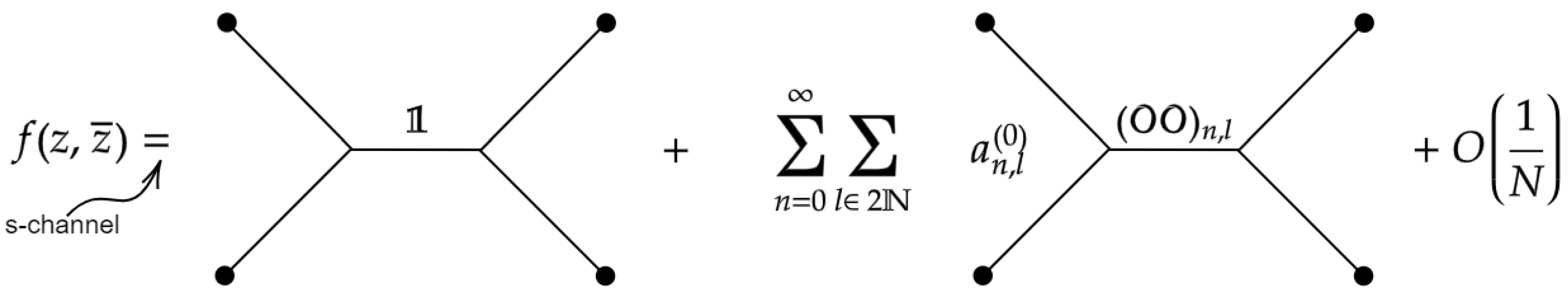
(a) s-channel (b) t-channel (c) u-channel (d) exchange (e) contact (f) sand clock (g) box

Harnessing information from Witten diagrams

$$
\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{f(u,v)}{|x_{12}|^{2\Delta}|x_{34}|^{2\Delta}}
$$
$$
= \frac{1}{x_{12}^{2\Delta}x_{34}^{2\Delta}} + \frac{1}{x_{14}^{2\Delta}x_{32}^{2\Delta}} + \frac{1}{x_{13}^{2\Delta}x_{24}^{2\Delta}} + O\left(\frac{1}{N^2}\right) = \frac{1}{x_{12}^{2\Delta}x_{34}^{2\Delta}}\left[1 + \left(\frac{u}{v}\right)^{\Delta} + u^{\Delta}\right] + O\left(\frac{1}{N^2}\right)
$$

Setting $u \to 0$ we get, to leading order, the s –channel diagrams, $v \to 0$ sets the t –channel and $u, v \to \infty$ the u –channel.

Expanding in powers of u, the leading order is a single trace contribution (the identity) and, to first order in u, a double trace contribution (the operator of dimension 2Δ). More precisely, the conformal block decomposition gives:

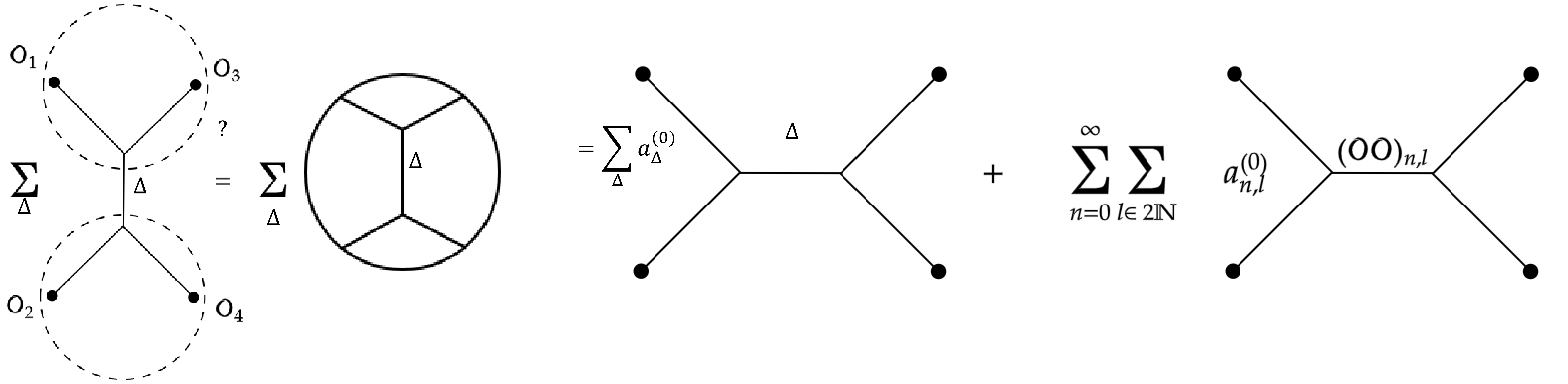


This already gives very interesting information!

OPE from holography

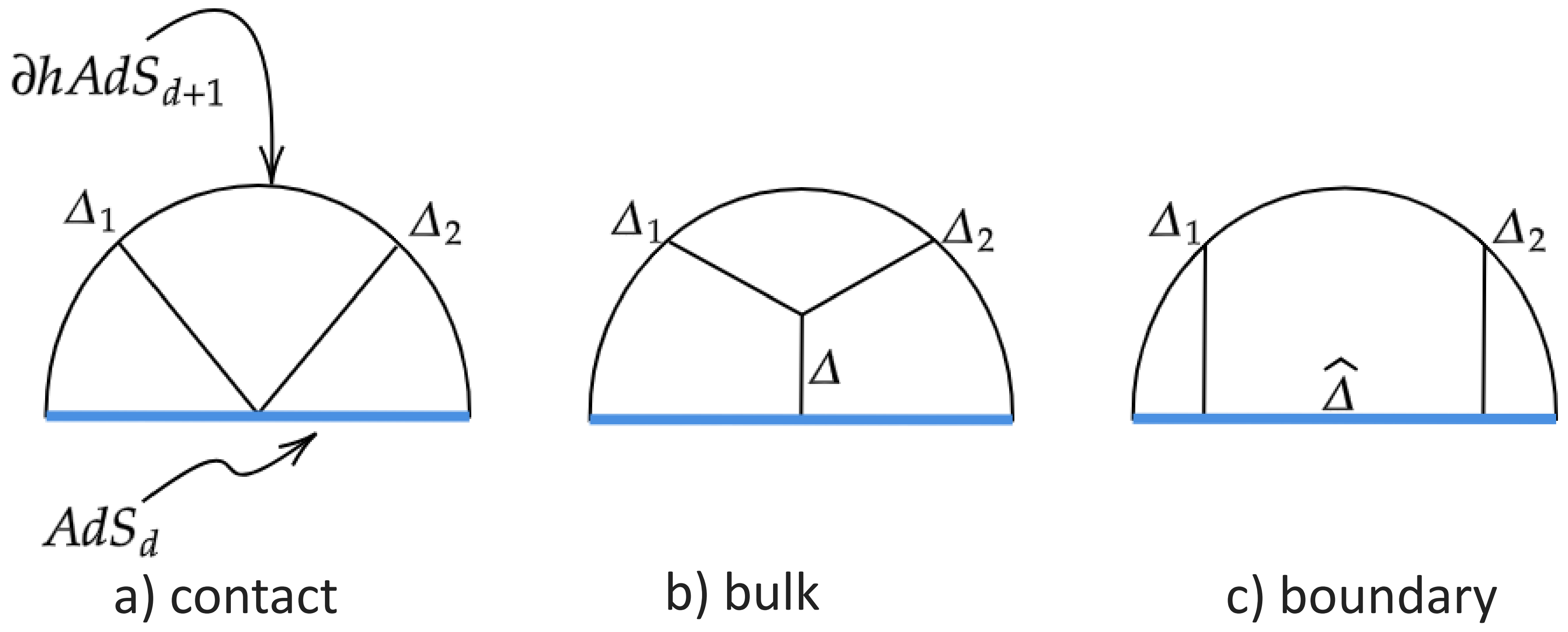
$$\mathcal{O}_1(x)\mathcal{O}_2(0) \sim \frac{1}{x^{\Delta_1+\Delta_2}}\left[g\sum_k C_k \mathcal{O}_k x^{\Delta_k} + \sum_{i,j,n,l}\left(\delta_{i(1}\delta_{2)j}\left(C'^{(i,j)}_{n,l} + g^2 C''^{(i,j)}_{n,l}\log x\right) + g^2\right) C^{(i,j)}_{n,l}\mathcal{O}^{(ij)}_{n,l}\, x^{\Delta_i+\Delta_j+2n+l}\right].$$

Crossing equations and matching



We consider an AdS_{d+1} whose boundary is realized by a AdS_d space. The space is given by the line

$$ds^2 = \frac{dz_0^2 + d\vec{\mathbf{z}}^2 + dz_\perp^2}{z_0^2}, \qquad z_\perp \geq = 0,$$

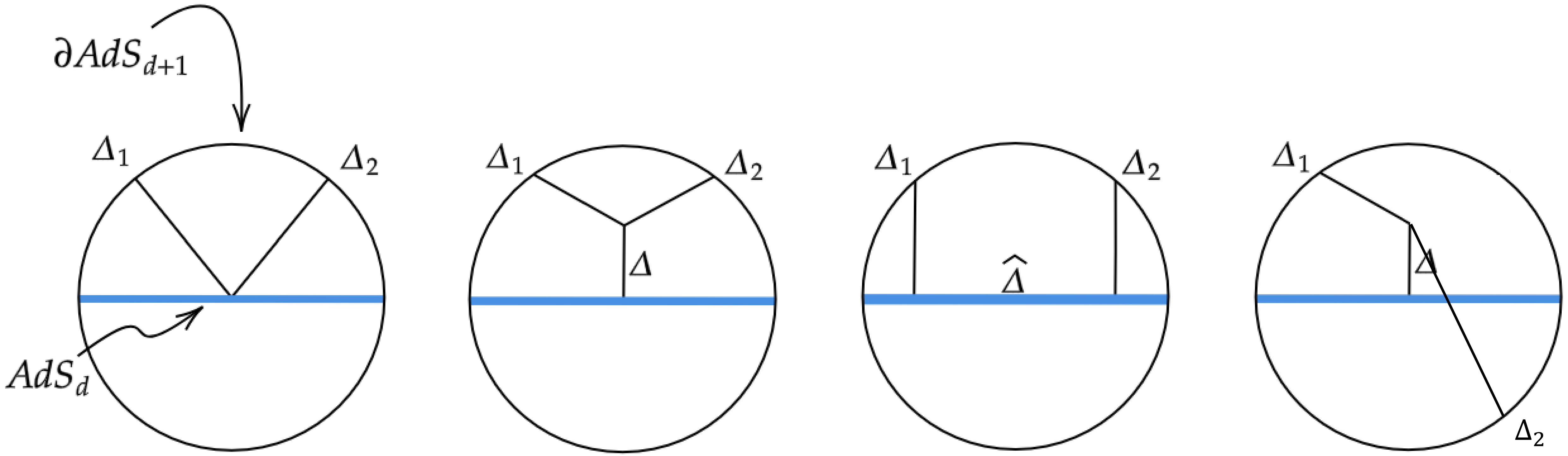


$$W^{\text{contact}}_{\text{Neum}}(x,y) = \int_{AdS_d} \frac{d^d w}{w_0^d} \widetilde{G}^{\Delta_1}_{B\partial}(w,x)\widetilde{G}^{\Delta_2}_{B\partial}(w,y),$$

$$W^{\text{bulk}}_{\text{Neum}}(x,y) = \int_{AdS_d} \frac{d^d w}{w_0^d} \int_{hAdS^+_{d+1}} \frac{d^{d+1} z}{z_0^{d+1}} \widetilde{G}^{\Delta}_{BB}(w,z)\widetilde{G}^{\Delta_1}_{B\partial}(z,x)\widetilde{G}^{\Delta_2}_{B\partial}(z,y),$$

$$W^{\text{boundary}}_{\text{Neum}}(x,y) = \int_{AdS_d} \frac{d^d w_1}{w_{10}^d} \frac{d^d w_2}{w_{20}^d} \widetilde{G}^{\widehat{\Delta}}_{BB}(w_1,w_2)\, \widetilde{G}^{\Delta_1}_{B\partial}(w_1,x)\, \widetilde{G}^{\Delta_2}_{B\partial}(w_2,y)\,,$$

Two point functions in the boundary

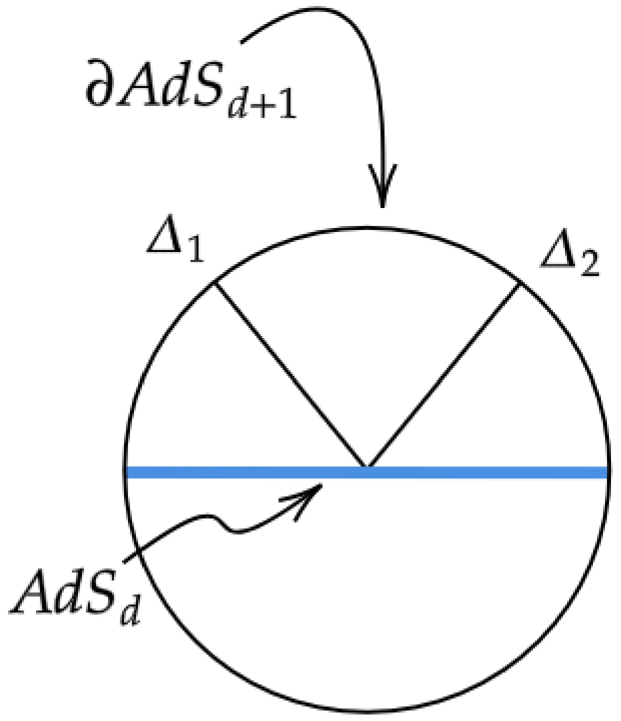


a) Contact b) Bulk c) Boundary d) Boundary mirror

$$W^{\text{Contact}}_{\text{Neuman}}(x,y) = 4W^{\text{Contact}}(x,y),$$
$$W^{\text{Bulk}}_{\text{Neuman}}(x,y) = 2\big(W^{\text{Bulk}}(x,y) + W^{\text{Bulk}}(x,\bar{y})\big),$$
$$W^{\text{boundary}}_{\text{Neumann}}(x,y) = 8W^{\text{boundary}}(x,y),$$

$$W^{\text{contact}}(x,y) = \int_{AdS_d} \frac{d^d w}{w_0^d} G^{\Delta_1}_{B\partial}(w,x) G^{\Delta_2}_{B\partial}(w,y),$$
$$W^{\text{bulk}}(x,y) = \int_{AdS_d} \frac{d^d w}{w_0^d} \int_{AdS_{d+1}} \frac{d^{d+1} z}{z_0^{d+1}} G^{\Delta}_{BB}(w,z) G^{\Delta_1}_{B\partial}(z,x) G^{\Delta_2}_{B\partial}(z,y),$$
$$W^{\text{boundary}}(x,y) = \int_{AdS_d} \frac{d^d w_1}{w_{10}^d} \frac{d^d w_2}{w_{20}^d} G^{\widehat{\Delta}}_{BB}(w_1,w_2)\, G^{\Delta_1}_{B\partial}(w_1,x)\, G^{\Delta_2}_{B\partial}(w_2,y)\,.$$

Contact diagram

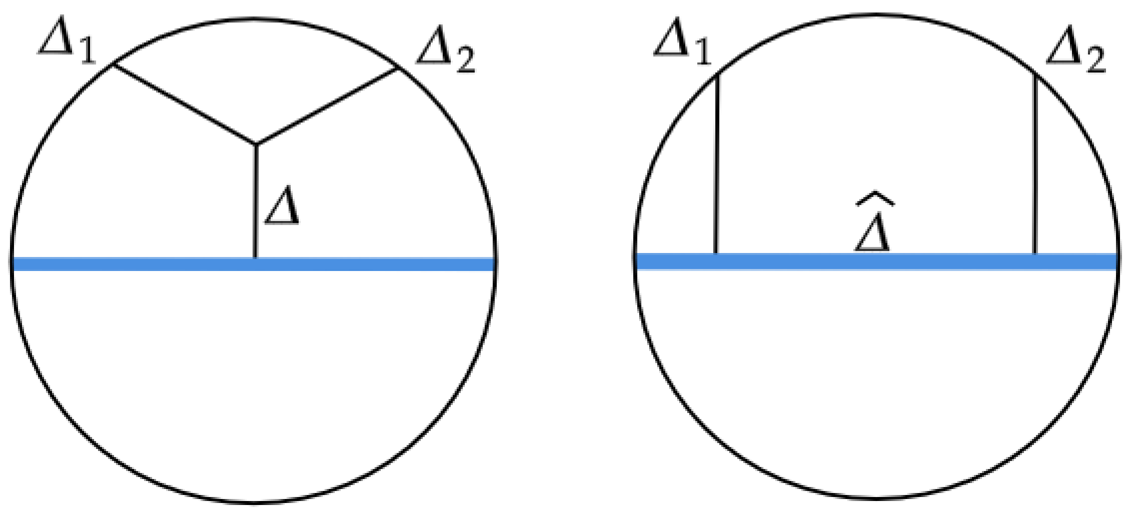


$$\mathcal{W}^{\text{contact}}(\xi)=\sum_{N=0}^{\infty}a'_N g^B_{\Delta_1+\Delta_2+2N}(\xi)=\sum_{N=0}^{\infty}a_N g^B_{\Delta_1+2N}(\xi)+b_N g^B_{\Delta_1+2N}(\xi),$$

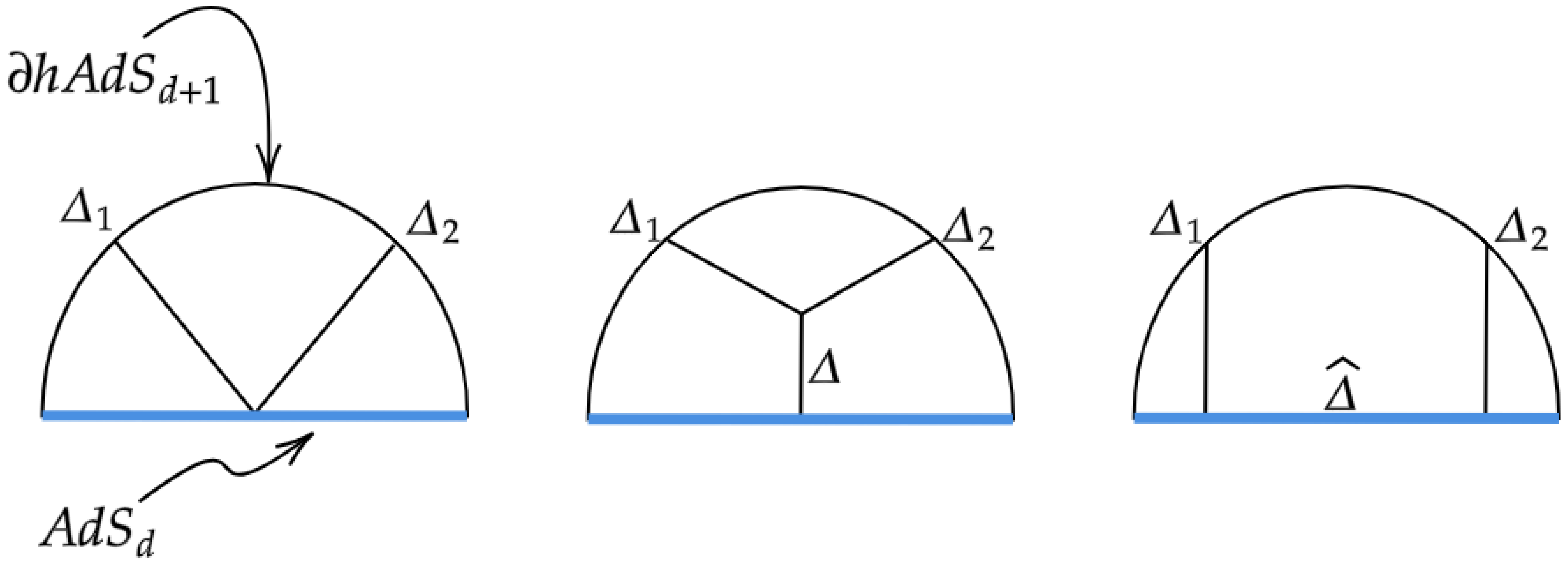
$$\left(\frac{1}{2}\left(\mathbf{L}_1+\mathbf{L}_2\right)^2+\Delta(\Delta-d)\right)W^{\text{bulk}}(x,y)=W^{\text{contact}}(x,y),$$

$$\left(\frac{1}{2}\left(\mathbf{L}_1+\widehat{\mathbf{L}}_2\right)^2+\Delta(\Delta-d)\right)W^{\text{bulk}}(x,y)=W^{\text{contact}}(x,y),$$

$$\left(\frac{1}{2}\hat{\mathbf{L}}_1^2+\hat{\Delta}(\hat{\Delta}-(d-1))\right)W^{\text{boundary}}(x,y)=W^{\text{contact}}(x,y),$$



Conformal Block decomposition



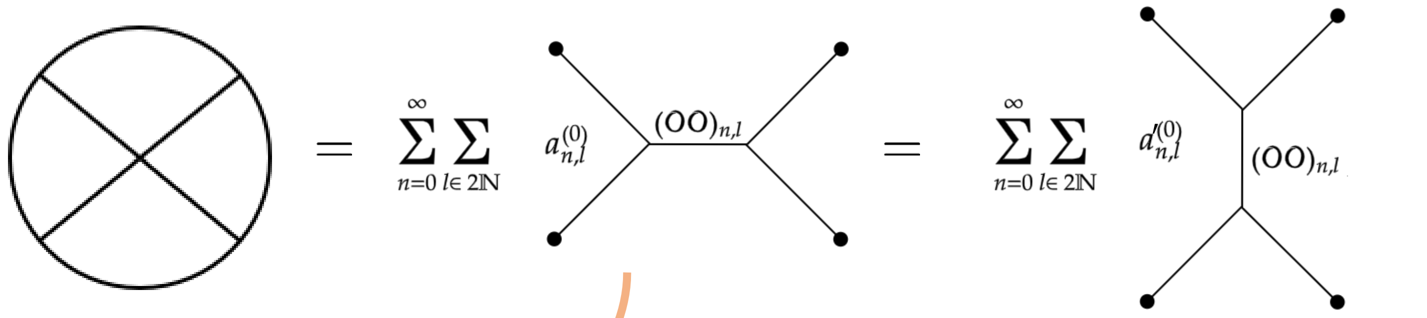
a) contact b) bulk c) boundary

$$\boxed{\mathcal{W}^{\text{contact}}_{\text{Neum}}(\xi)} = 4\left(\sum_{N=0}^{\infty} a'_N g^B_{\Delta_1+\Delta_2+2N}(\xi)\right) = 4\left(\sum_{N=0}^{\infty} a_N g^B_{\Delta_1+2N}(\xi) + b_N g^B_{\Delta_2+2N}(\xi)\right),$$

$$\mathcal{W}^{\text{Bulk}}_{\text{Neum}}(\xi) = 2\left(\boxed{A^B g^B_{\Delta}(\xi)} + \sum_{n\in 2\mathbb{N}+1} A^B_n g^B_{\Delta_1+\Delta_2+2n}(\xi)\right) = 2\sum_{n\in 2\mathbb{N}+1} \hat{A}^{B,(1)}_n g^b_{\Delta_1+n}(\xi) + \hat{A}^{B,(2)}_n g^b_{\Delta_2+n}(\xi)$$

$$\mathcal{W}^{\text{Boundary}}_{\text{Neum}}(\xi) = 8\left(\sum_N A^b_n g^B_{\Delta_1+\Delta_2+2N}(\xi)\right) = 8\left(\boxed{A^b g^b_{\hat{\Delta}}(\xi)} + \sum_N A^b_N g^b_{\Delta_1+2N}(\xi) + B^b_N g^b_{\Delta_2+2N}(\xi)\right)$$

Conclusions



- Which diagrams should I consider?
- Which information is relevant?