Electromagnetic and T-dualities

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I. ELECTROMAGNETIC DUALITY

Consider an abelian Yang-Mills theory in the presence of an "electric" charge current to which we use the $U(1)$ connection A. All the allowed terms are:

$$S = -\frac{1}{4g}\int F \wedge *F + F \wedge F - J_e \wedge A, \quad J_e = 2\pi i q \delta(x) \tag{1}$$

for some charge q that requires Dirac charge quantization. From the equations of motion of A, it follows $d*F = 4gJ_e = 8\pi i q\delta(x)$ so indeed we have (also due to the Bianchi identity $dF = 0$) Maxwell equations in the presence of an electric charge at $x = 0$.

Equivalently, we could have described the theory in the dual form $*F$ described by the local gauge connection $\tilde{A}$ such that $\tilde{F} = *F = d\tilde{A}$. To do this, we consider an action in terms of $\tilde{a}$ and F, where is now a general 2−form (i.e. it might be non-closed). To recover the Maxwell equations, we introduce a lagrange multiplier for the constraint on F being exact. The most general action is then

$$S = \int -\frac{1}{4g}F \wedge *F + \frac{i}{2\pi}d\tilde{a} \wedge F - J \wedge \tilde{a} \tag{2}$$

where the second term is the lagrange multiplier and the third the "Magnetic" current. Under the equation of motion of F we have $*F \sim d\tilde{a}$ and for $\tilde{a}$, we get $dF \sim J_m$, so indeed the roles of $*F$ and F have swapped. Completing the square and, integrating out the F fields, we get

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II. T DUALITY

A. Example