Universidad de los Andes

Thermodynamics

# Energy Transport Mechanisms

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October 16, 2022

# 1 Introduction

The processes that occur within a star, such as energy transport due by convection, radiation and conduction, are one of the most important ones that we can analyze with thermodynamics. These following pages will explain how can this first process can be describe in an idealistic way, taking into account how are the conditions and assumption that need to be taken. Before analyzing the thermodynamic processes, we need establish that, as stars are very complicated systems, the assumption will be a bit harsh (compared with a good scientific model).

To model a thermodynamic system of a star, we must see the process of energy flow that the star involves, convection, radiation and conduction are the three main process for such systems. First, we will analyze stars in a simplified model where the energy flow produced by radiation and conduction may be neglected which will be explained later why this assumption is enough accurate to the purpose of this work. Furthermore, in the simplified model we assume a spherical symmetry that's because in stars like the sun the angular speed does not produce a centrifugal force that changes the structure of the star (Sphere), this assumption is very important because in this way, we can analyze the medium where it evolves not as a three dimensional(x,y,x) space but as a one dimensional space with coordinate radial (r). Second, we want to model the convection of a star in such models that the complex behavior produces some changes in the flow energy by describing better and fill the assumptions that the first idealized model has. In conclusion we describe the idealized thermodynamic convection process of a star and then we specialized for some special cases. Finally, we compare those computer results with theoretical to give an idea of the process relationship and accuracy.

# 2 Model

We assume peculiarities characterizing stars (turbulence, non-homogeneous media, etc.) while describing the energy transport inside a star in order to avoid any mathematical barrier. Various in-stellar processes may dominate depending on the region of study, which can be catalogued in three main mechanisms: Radiation, convection, and conduction. The latter is usually non-dominant, that is because of the size of stars, the process of conduction in that way has a very slow develop which affects a relatively small neighbourhood, hence, it may be neglected in the analysis of the complete stars. For the former mechanism (radiation), we begin to analyze the pressure gradient.

Molecules collide to generate pressure under a specified area, so it must be proportional to the negative flux of the radiative energy, in other words, a big change in the radius will have a greater flux and in that way the star will have less pressure. Denoting the outward radiative flux as $F_{rad}$, we can express this relation of the pressure gradient as:

$$\frac{dP_{rad}}{dr} = -\frac{\kappa\rho}{c} F_{rad} \tag{1}$$

On the other hand, the radiation, as emission of photons, obeys the black body radiation with Stefan-Boltzman law, which gives a proportion of pressure $P$ with the temperature $T$ to the forth power, this can be derived by integrating the pressure along a sphere and the legth waves from 0 to infinity. So the pressure also may be expressed as follows

$$P_{rad} = aT^4/3 \tag{2}$$

Differencing equation 2 and using equation 1 to get the change in temperature while changing $F_{rad} = L_r/(4\pi r^2)$, we have:

$$\frac{dT}{dr} = \frac{-3\bar{\kappa}\rho L_r}{16\pi ac T^3 r^2} \tag{3}$$

Where $L_r$ is the local radiative luminosity of the star, $r$ the radius at which the change in temperature, $dT$, is occurring, $\bar{\kappa}$ the Rosseland mean opacity, $\rho$ the density of the Star, and $a$ and $c$ are constants. We see from equation 2 that $a$ by the Stefan-Boltzmann law equals 4 $\sigma/c$ where $\sigma$ is the Stefan-Boltzmann constant and the constant $c$ is the speed of light. Even though this mechanism plays a big roll on energy transport, convection will reign at deep regions within the star. For instance, non-ancient red dwarfs will not, in general, have a radiation zone due to its low mass condition. Then, for our purpose, we shall describe the convection process more carefully in a complete thermodynamic view. This process will take place in non-exotic stars[1] and in later sections will be discussed the conditions where the process is realized. Let us start with the following:

## 2.1 Essentials.

First, to analyze convection we must have in count that the hole process must have conservation of energy and because the velocity of the process it will be a adiabatic process, we must develop some general aspects to analyze this. For the energy conservation, we use the first law of thermodynamics which states:

$$dU = đQ + đW \tag{4}$$

Where $U, Q, W$ are the internal energy, the heat, and the work realized, respectively.

The total energy in a mono-atomic ideal gas is the kinetic energy per unit mass. As we know, it is expressed as

$$U = \textit{average energy per particle * number of particles}$$
$$= \frac{3}{2} nRT$$

In order to express the adiabatic process we have the specific heats for pressure $C_p$ and $C_v$ temperature constant, we therefore have as the definition of $C_p$ and $C_v$

[1] Exotic stars such as Quark stars, Strange stars, Preon stars, along with others.

the amount of heat required to rise the temperature by a unit temperature interval.

$$C_p = \left(\frac{\partial Q}{\partial T}\right)_p$$

$$C_v = \left(\frac{\partial Q}{\partial T}\right)_v$$

We can then find the $C_v$, $C_p$ relation by noticing that $dU$, keeping $v$ constant, equals $C_v$

$$dU = C_v dT \tag{5}$$

Deriving implicitly we have:

$$dU = \left(\frac{\partial Q}{\partial T}\right)_p dT + đW$$

$$dU = \left(\frac{\partial Q}{\partial T}\right)_p dT - P\left(\frac{\partial V}{\partial T}\right)_p dT \tag{6}$$

and differentiating the ideal gas law ( $PV = nRT$)

$$PdV + VdP = nRdT + RTdn \tag{7}$$

then by substituting 5, 6 and 7, keeping in mind that $n$ and $p$ are constant

$$C_v dT = C_p dT - nRdT$$

$$C_p = C_v + nR \tag{8}$$

.

### 2.1.1 The adiabatic process

To describe the convection process as we said we must to analyze the adiabatic process of a bubble that is moving upward towards the surface of the star.
The adiabatic process has no interchange of heat between other systems, in other words, the bubble does not interact in a thermodynamic way with other systems. In this way, in a adiabatic process we have that the change in energy equals the work done and by equation 5, $dU = C_v dT$. Now we first find the relation between Pressure and Temperature which is a standard result in thermodynamics even though we deduce here.

$$dT = \frac{dU}{C_v} = -\frac{PdV}{C_v} \tag{9}$$

And then with equation 7 keeping constant n

$$PdV + VdP = nRdT = \frac{-nRPdV}{C_v} \tag{10}$$

By the following definition $\gamma := C_p/C_v$, and some relations of $C_v$ and $C_p$ (Eq 8)

$$PdV + VdP = \frac{-nRPdV}{C_v}$$

$$PdV + \frac{nRPdV}{C_v} = VdP$$

$$\left(1 + \frac{nR}{C_v}\right)_p dV = VdP$$

$$\frac{C_v + nR}{C_v} PdV = VdP$$

$$\frac{C_p P}{C_v dP} = \frac{V}{dV}$$

$$-\frac{\gamma P}{dP} = \frac{V}{dV}$$

If we put $\rho = 1/V$ then it follow :

$$\frac{dP}{P} = \gamma \frac{P}{\rho}\frac{d\rho}{dr} \tag{11}$$

Solving the differential equation to see the P-V relation and using the ideal gas law for expressing the relation between and $T$

$$PV^\gamma = K$$

$$V = \frac{nrT}{P}(PV = nrT)$$

$$PV^\gamma = P(\frac{nrT}{P})^\gamma$$

$$P(\frac{nrT}{P})^\gamma = P^{1-\gamma}(nRT)^\gamma$$

$$P^{1-\gamma}(nRT)^\gamma = K$$

$$P = kT^{\frac{\gamma}{\gamma-1}} \tag{12}$$

With this information we can calculate the speed o sound in the material, that is related to its inertia and its compressibility, this is given by:

$$\nu_s = \sqrt{\frac{B}{\rho}}$$

where $B$ is the bulk modulus of the gas that a constant temperature it is equal to $\frac{1}{k_t}$, therefore, replacing $B$ we obtain that

$$\nu_s = \sqrt{\frac{-V}{\rho}\frac{\partial P}{\partial V}}$$

and using previoys equations we conclude that:

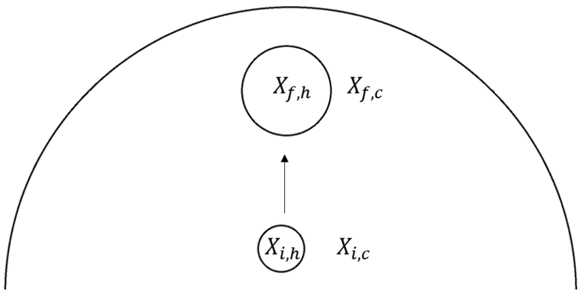
$$\nu_s = \sqrt{\frac{-\gamma P}{\rho}} \tag{13}$$

# 3 The convection process

In the previous section we develop the essential results in order to analyze the simplified model of convection as a adiabatic process. The hot convective bubbles at first sight rises and expands in a adiabatic process. In that way as we mentioned above, the model its given by certain aspects. First, the density changes with radius of the pressure and in doing so changes its volume.

The change in density also affects the buoyant force. We begin to describe the convection, to describe the mechanism lets consider a bubble of hot gas (lets us call it B) which is composed by ideal gas, a assumption that in such temperatures fits very well to the theory. The bubble which develops by virtue of the proximity of the gas to the nucleus of the star, rising from the interior of the star due to the density difference $\delta\rho$ between the media and the gas using the Archimedes' principle. Let us denote, for simplicity, on some values the initial and final conditions with an $i$ and an $f$ as subscript respectively and a second subscript $c$ and $h$ to identify the exterior and the interior of B respectively (as the image suggest) . Then, the Bouyant force excerted by the outsides of B and the gravitational force acting on B due to the innards of the star would turn out to be:

$$\vec{F}_{total} = \vec{F}_{buoyant} - \vec{F}_{grav} = \rho_{i,c}\text{g} - \rho_{i,h}\text{g}$$




Now, because B forms in the depths of the star, the initial temperature, as well as the initial density, of B is nearly the same as its surroundings, and the pressures within and outward B are very similar (if it was not the case, the ball would abruptly explode, or implode). The assumptions of setting $T_{i,h} = T_{i,c}$, $\rho_{i,h} < \rho_{i,c}$, and $P_h = P_c$ would still describe in a very neat way this process. Having this, we want to see the change in density of a material inside a star as a function of the radius, for that, when increased a distance $\delta r$ from the center we have a cahnge in density, This is:

$$\delta\rho = \frac{d\rho}{dr}\delta r \tag{14}$$

This rising would cause an adiabatic expansion on the ball, which can be traced with usual thermodynamic equations (the ones that were deduced previously).

First we see the change in density of B and in its surroundings, as we said, we can develop a Taylor expansion of equation 14 of first order, we have for B and its surroundings respectively

$$\rho_{f,h} = \rho_{i,h} + \frac{d\rho}{dr}\Big|_h dr$$

$$\rho_{f,c} = \rho_{i,c} + \frac{d\rho}{dr}\Big|_c dr$$

Keeping in mind that in the initial point the density of bubble is less of its surroundings, then the change in density flows:

$$\begin{aligned} \rho_{i,c} &< \rho_{i,h} \\ \rho_{f,h} &= \rho_{i,h} + \frac{d\rho}{dr}\Big|_h dr \\ & and \\ \rho_{f,c} &= \rho_{i,c}\frac{d\rho}{dr}\Big|_c dr \\ \frac{d\rho}{dr}\Big|_h dr &< \frac{d\rho}{dr}\Big|_c dr \end{aligned}$$

$$\frac{d\rho}{dr}\Big|_h < \frac{d\rho}{dr}\Big|_c \tag{15}$$

We want to see this on a more plausible way and the consequences that this gives in pressure and temperature, in that way, using the ideal gas equation in the density form,

$$P = \rho kT/\mu m_h \tag{16}$$

where $\rho$ is the density, $\mu$ the mean molecular weight. We can obtain the pressure gradient:

$$\frac{dP}{dr} = -\frac{P}{\mu}\frac{d\mu}{dr} + \frac{P}{\rho}\frac{d\rho}{dr} + \frac{P}{T}\frac{dT}{dr} \tag{17}$$

Which will give us a relation between pressure, density and temperature.

Now we can solve for density and then re-express the equation 15 in variables such as temperature or pressure which are extensive parameters that can easily be measured. We have by the previous equation keeping constant $\mu$:

$$\begin{aligned} \frac{dP}{dr} &= -\frac{P}{\mu}\frac{d\mu}{dr} + \frac{P}{\rho}\frac{d\rho}{dr} + \frac{P}{T}\frac{dT}{dr} \\ \frac{dP}{dr} &= \frac{P}{\rho}\frac{d\rho}{dr} + \frac{P}{T}\frac{dT}{dr} \\ \frac{dP}{dr} - \frac{P}{T}\frac{dT}{dr} &= \frac{P}{\rho}\frac{d\rho}{dr} \end{aligned}$$

$$\frac{\rho}{P}\frac{dP}{dr} - \frac{P}{T}(\frac{dT}{dr} = \frac{d\rho}{dr}) \tag{18}$$

Therefore, using this results and equations 11 and 15 we get:

$$\begin{aligned} \frac{d\rho}{dr}\Big|_h &< \frac{d\rho}{dr}\Big|_c \\ By\ 11,\ \frac{\rho_{i,h}}{\gamma P_{i,h}}\frac{dP}{dr}\Big|_h &< \frac{d\rho}{dr}\Big|_c \end{aligned}$$

Finally using equation 18 on the right side of the inequality gives:

$$\frac{\rho_{i,h}}{\gamma P_{i,h}}\frac{dP}{dr}\Big|_h < \frac{\rho_{i,c}}{P_{i,c}}\left(\frac{dP}{dr}\Big|_c - \frac{P_{i,c}}{T_{i,c}}\frac{dT}{dr}\Big|_c\right)$$

Evoking, we assumed that the pressures inside and outside the hot rising gas would be the same, therefore we can ignore some subscripts and simplify to obtain:

$$\frac{1}{\gamma}\frac{dP}{dr} < \frac{dP}{dr} - \frac{P_{i,c}}{T_{i,c}}\frac{dT}{dr}\Big|_c$$

therefore re-arranging terms,

$$\left(\frac{1}{\gamma}-1\right)\frac{dP}{dr} < \frac{-P}{T}\frac{dT}{dr}\Big|_{\text{actual}}$$

which is the same as:

$$\left(1-\frac{1}{\gamma}\right)\frac{dP}{dr}\frac{T}{P} > \frac{dT}{dr}\Big|_{\text{actual}} \tag{19}$$

This equation express explicitly the condition for a bubble raising in a convection process, the change in pressure with respect to temperature must be positive along the radius of the convective process.
Now, having this, we can use some deductions on the change in temperature with respect to the radius to see a dependency of the temperature with respect to radius. First, we use the equation 17 and the equation 11 so it gives:

$$\begin{aligned}
\frac{dP}{dr} &= \frac{P}{\rho}\frac{d\rho}{dr} + \frac{P}{T}\frac{dT}{dr}\\
\frac{dP}{dr} - \frac{P}{\rho}\frac{d\rho}{dr} &= \frac{P}{T}\frac{dT}{dr}\\
\frac{T}{P}\left(\frac{dP}{dr} - \frac{P}{\rho}\frac{d\rho}{dr}\right) &= \frac{dT}{dr}\\
\frac{T}{P}\left(\frac{dP}{dr} - \frac{1}{\gamma}\frac{dP}{dr}\right) &= \frac{dT}{dr}\\
\frac{T}{P}\frac{dP}{dr}\left(1-\frac{1}{\gamma}\right) &= \frac{dT}{dr}
\end{aligned}$$

$$\frac{dT}{dr}\Big|_{\text{adiabatic}} = \left(1-\frac{1}{\gamma}\right)\frac{dP}{dr}\frac{T}{P} \tag{20}$$

In order to make easier our calculations, we can express the previous equation, we can use the ideal gas law, $\gamma = C_p/C_v$, $C_p - C_v = nR$ and keeping in mind that $P = -\rho g r$, the differentiating with respect to $r$ and replacing in the previous equation, we have:

$$\frac{dT}{dr}|_{ad} = \left(1-\frac{C_v}{C_p}\right)\frac{dP}{dr}\frac{T}{P} \tag{21}$$

$$\frac{dT}{dr}|_{ad} = -\Big(\frac{nR}{C_p}\Big)\rho g\frac{T}{P} \tag{22}$$

$$\frac{dT}{dr}|_{ad} = -\Big(\frac{1}{C_p}\Big)\rho g\frac{PV}{P} \tag{23}$$

$$\frac{dT}{dr}|_{ad} = -\Big(\frac{1}{C_p}\Big)g\frac{V}{V} \tag{24}$$

$$\frac{dT}{dr}|_{ad} = -\Big(\frac{1}{C_p}\Big)g \tag{25}$$

$$\frac{dT}{dr}\Big|_{\text{adiabatic}} = \left(1-\frac{1}{\gamma}\right)\frac{dP}{dr}\frac{T}{P}\frac{dP}{dr}$$

then back in equation 19 and the later equation, we get:

$$\frac{dT}{dr}\Big|_{\text{adiabatic}} > \frac{dT}{dr}\Big|_{\text{actual}}$$

from which follow that, due the fact that when the gas is further from the center (this is, a bigger radius), the temperature decreases, the following holds:

$$\Big|\frac{dT}{dr}\Big|_{\text{actual}} > \Big|\frac{dT}{dr}\Big|_{\text{adiabatic}}$$

We can use the equation 20 to get a relation between pressure and temperature which is seen as follows:

$$\begin{aligned}
\frac{T}{P}\frac{dP}{dr}\left(1-\frac{1}{\gamma}\right) &= \frac{dT}{dr}\\
\frac{T}{P}\frac{dP}{dr}\frac{dr}{dT} &= \left(1-\frac{1}{\gamma}\right)^-1
\end{aligned}$$

$$\frac{T}{P}\left(\frac{dT}{dr}\right)^{-1}\frac{dP}{dr} < -\frac{1}{\gamma^{-1}-1}$$

Solving the differential equation, we get:

$$\frac{TdP}{PdT} = \frac{d\ln P}{d\ln T} < \frac{\gamma}{\gamma-1}$$

In this way, we have a explicit condition when the process of convection is to be realized, furthermore, this gives us the region where the inequality holds will present convection process. Most of these process as the values of gamma and composition of the stars are will have convection process near the interior of stars but not all ways is this the case.

## 3.1 The mixing-Length Theory of Super adiabatic Convection

The problem to analyze a general convective process its still unsolved and its simulations may involve several variables and time, the three dimensional space, temperature and the large distance difficult this process, in this way we must simplify our assumptions to relate just the length it takes to a bubble to rise and dissolve in its surroundings, therefore, we change our process to one dimensional analysis. Taking into account the fundamental criterion for convection, $\rho_f^{(b)} < \rho_f^{(s)}$, and knowing that the pressure of the bubble is always equal to its surroundings, when using the ideal gas law, we realise that $T_f^{(s)} < T_f^{(b)}$ assuming that there was thermal equilibrium initially. Hence the temperature of the surrounding gas must decrease more rapidly, then

$$\left|\frac{dT}{dr}\right|^{(s)} - \left|\frac{dT}{dr}\right|^{(b)} > 0$$

having in mind that the temperture gradients are negative, we have

$$\frac{dT}{dr}^{(b)} - \frac{dT}{dr}^{(s)} > 0$$

Designating the temperature gradient of the surroundings as the average temperature gradient of the star and assuming that the bubble moves adiabatically, then

$$\frac{dT^{(s)}}{dr} = \frac{dT}{dr}\Big|_{ave} \; and \; \frac{dT^{(b)}}{dr} = \frac{dT}{dr}\Big|_{ad}$$

The temperature of the bubble will be greater than the temperature of the surrounding gas after having travelled a distance $dr$ by

$$\delta T = \left( \frac{dT}{dr}\Big|_{ad} - \frac{dT}{dr}\Big|_{ave} \right) dr = \delta \left( \frac{dT}{dr} \right) dr \qquad (26)$$

Now, we define an adjustable parameter

$$\alpha = \frac{\ell}{H_p} \qquad (27)$$

where $\ell$ is called the mixing length and it is the distance the bubble travels before dissipating and $H_p$ is defined as:

$$H_p = \frac{P}{\rho g} \qquad (28)$$

After having travelled a mixing length, the excess heat flow per unit volume from the bubble into its surroundings is

$$\delta q = (C_p \delta T)\rho$$

with $\delta T$ as in 26. The convective flux, which is the amount of energy per unit area per unit time carried by the bubble can be obtained by multiplying by the average velocity $\bar{\nu}_c$ of the convective bubble

$$F_c = \delta q \bar{\nu}_c = (C_p \delta T)\rho \bar{\nu}_c \qquad (29)$$

We can find the average velocity by using the net force per unit volume $f_{net}$ acting on the bubble. Using the ideal gas law and assuming constant $\mu$, we can write

$$\delta P = \frac{P}{\rho}\delta\rho + \frac{P}{T}\delta T$$

But we know that $\delta P = P^{(b)} - P^{(s)} = 0$ so

$$\delta\rho = -\frac{\rho}{T}\delta T$$

Using the buoyant force we have that

$$f_{net} = \frac{\rho g}{T}\delta T$$

Now, recalling our initial assumptions, the initial temperature between the bubbles and its surroundings is zero, so $\delta T_i = 0$. Therefore at the inital point the buoyant force at a initial point must be equal to zero and knowing that it is a linear function we can take the average value over a distance $\ell$, that is:

$$\langle f_{net} \rangle = \frac{1}{2}\frac{\rho g}{T}\delta T$$

If we don't take into account the viscous forces, the work is equal to the kinetic energy:

$$\langle f_{net} \rangle \ell = \frac{1}{2}\rho v_f^2$$

Then we can get an average value of the velocity knowing that:

$$\bar{v}_c^2 = \beta v^2$$

$$\frac{1}{2}\bar{v}_c^2 \rho = \langle F_{net} \rangle \ell$$

$$\bar{v}_c = \Big( \frac{2\beta \langle f_{net} \rangle \ell}{\rho} \Big)^{1/2}$$

Now using 26 with $dr = \ell$

$\langle f_{net} \rangle = \frac{1}{2}\frac{\rho g}{T}\delta T$

$\delta T = \delta \Big( \frac{dT}{dr} \Big) l$

$\langle f_{net} \rangle = \frac{1}{2}\frac{\rho g}{T}\delta \frac{dT}{dr}\ell$

$\bar{v}_c = \Big( \frac{\beta g \ell^2}{T} \Big)^{1/2} \Big[ \delta \Big( \frac{dT}{dr} \Big) dr \Big]$

and replacing $\ell = \alpha H_p$ and $H_p = \frac{P}{g\rho}$ and $\rho = \frac{k}{\mu m_h}$

$$\bar{v}_c = \beta^{1/2} \Big( \frac{T}{g} \Big)^{1/2} \Big( \frac{k}{\mu m_h} \Big) \Big[ \delta \Big( \frac{dT}{dr} \Big) \Big]^{1/2} \alpha \qquad (30)$$

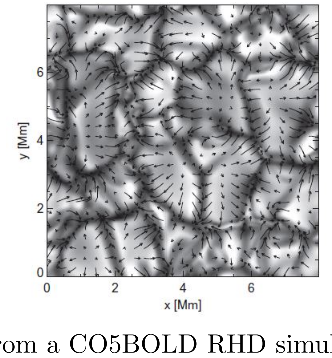
Now using 29 and 30 we can deduce an expression for the convective flux:

$$F_c = \rho C_p \beta^{1/2} \Big( \frac{T}{g} \Big)^{3/2} \Big( \frac{k}{\mu m_h} \Big)^2 \Big[ \delta \Big( \frac{dT}{dr} \Big) \Big]^{3/2} \alpha^2 \qquad (31)$$

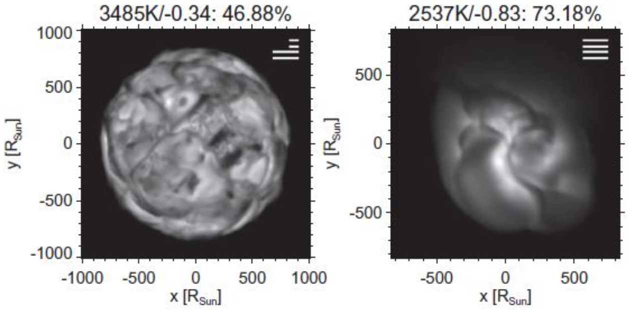
# 4 Results

We can now compare our results with some simulations and some data taken from stars that has been published in some journals.

At first we will look at some results taken from the *Journal of Computational Physics* which model convection with **CO5BOLD**. With the model that we have described, the convection cells would rise and generate a granulation of the surface of the star. The following figure shows a simulation of this process with some other details taken into account, indicating with arrows the horizontal speed of the plasma.




Snapshot from a CO5BOLD RHD simulation at the bottom of the photo-sphere. Taken from [4]




Global models of a red supergigant and an AGB star with low surface gravity. Taken from [4]

In the first figure, it is clear that one would get these irregular patterns of the convective cells in the surface of the star, while in the second figure, it is appreciable that this granulation is not so clear. This is because the stars do not fulfill our assumptions.

Another example of this process can be seen in a letter published in *Nature*, where it is seen large granulation cells in the surface of a gigant star ($\pi^1$ Gruis), as shown:

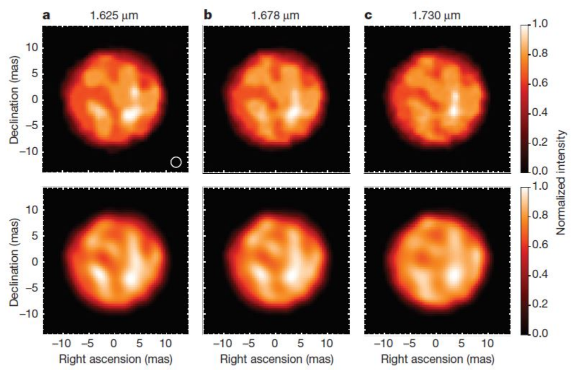
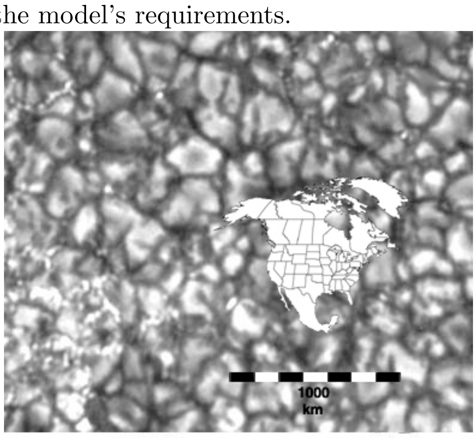



Image of the stellar surface of $\pi^1$ Gruis reconstructed from interferometric data. Taken from [5]

This star shows a really clear consequence of the convective cells.

We also see this process in our own sun. This due because our sun is an average star which will indeed fulfill the model's requirements.




**Photospheric granulation, G. Scharmer**
**Swedish Vacuum Solar Telescope**
**10 July 1997**

Solar granulation NASA photo, modified 2004/02/07 by I. Gillan

We can also find the value of the average velocity of this cells when rising. Assuming that all the energy transport is realized by convection at the sun, we have:

$$F_c = \frac{L_r}{4\pi r^2}$$

With values of $\alpha = 1$ and $\beta = 1/2$ for the sun and using the following data:
$M_r = 0,976 M_{sun}$, $L_r = 1 L_{sun}$, $r = 0.714 R_{sun}$, $g = 525 ms^{-2}$, $C_p = 5nR/2$, $P = 5.59x10^{12} Nm^{-2}$, $\rho = 187 kgm^{-3}$, $\mu = 0.606$ y $T = 2.18x10^6 K$ (Taken from [1]), now using equation 25 we have:

$$\left.\frac{dT}{dr}\right|_{ad} = 0.015 K/m \tag{32}$$

Now solving for $\delta\left(\frac{dT}{dr}\right)$ in 31 we have:

$$\delta\left(\frac{dT}{dr}\right) = 6.7x10^{-9} K/m \tag{33}$$

Dividing the last two equations:

$$\frac{\delta(dT/dr)}{|dT/dr|_{ad}} = 4.4x10^{-7}$$

Now, we can obtain the average velocity for a convective process with 30:

$$\bar{v}_c = 50 m/s$$

Having this, we now may ask how does turbulence caused by the rotation of the star may affect this process. How does the convection cells get to follow the granulation pattern which is observed. For a complete theory of convection cells, we must account for the rotational speed, deal with nuclear processes being active in the nuclear core, how much mass does the star have, if perhaps a binary system might affect its cells, how does the location at the Main sequence will affect it, and many other details that weren't cover in this very simple and idealistic model we took into account.

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