

Hennessy Milner Logic

- temporal properties of processes
- Hennessy-Milner logic: syntax and semantics
- denotational semantics
- correspondence with strong bisimilarity

Verifying Correctness of Reactive Systems

Let *Impl* be an implementation of a system (e.g. in CCS syntax).

Equivalence Checking Approach

$$\textcolor{red}{Impl} \equiv \textcolor{red}{Spec}$$

- \equiv is an abstract equivalence, e.g. \sim or \approx
- *Spec* is often expressed in the same language as *Impl*
- *Spec* provides the full specification of the intended behaviour

Model Checking Approach

$$\textcolor{red}{Impl} \models \textcolor{red}{Property}$$

- \models is the satisfaction relation
- *Property* is a particular feature, often expressed via a logic
- *Property* is a partial specification of the intended behaviour

Model Checking of Reactive Systems

Our Aim

- Develop a logic in which we can express interesting properties of reactive systems.
- Better if it is somehow connected with bisimulation semantics.

Logical Properties of Reactive Systems

Modal Properties – what can happen now (possibility, necessity)

- drink a coffee (can drink a coffee now)
- does not drink tea
- drinks both tea and coffee
- drinks tea after coffee

Temporal Properties – behaviour in time

- never drinks any alcohol
(**safety property**: nothing bad can happen)
- eventually will have a glass of wine
(**liveness property**: something good will happen)

Can these properties be expressed using equivalence checking?

Hennessy-Milner Logic – Syntax

Syntax of the Formulae ($a \in Act$)

$$F, G ::= tt \mid ff \mid F \wedge G \mid F \vee G \mid \langle a \rangle F \mid [a]F$$

Intuition:

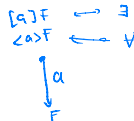
tt all processes satisfy this property

ff no process satisfies this property

\wedge, \vee usual logical AND and OR

$\langle a \rangle F$ there is at least one a -successor that satisfies F

$[a]F$ all a -successors have to satisfy F



Remark

Temporal properties like *always/never in the future* or *eventually* are not included.

Hennessy-Milner Logic – Semantics

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS.

Validity of the logical triple $p \models F$ ($p \in Proc$, F a HM formula)

$p \models tt$ for each $p \in Proc$

$p \models ff$ for no p (we also write $p \not\models ff$)

$p \models F \wedge G$ iff $p \models F$ and $p \models G$

$p \models F \vee G$ iff $p \models F$ or $p \models G$

$p \models \langle a \rangle F$ iff $p \xrightarrow{a} p'$ for some $p' \in Proc$ such that $p' \models F$

$p \models [a]F$ iff $p' \models F$, for all $p' \in Proc$ such that $p \xrightarrow{a} p'$

We write $p \not\models F$ whenever p does not satisfy F .

What about Negation?

For every formula F we define the formula F^c as follows:

- $tt^c = ff$
- $ff^c = tt$
- $(F \wedge G)^c = F^c \vee G^c$
- $(F \vee G)^c = F^c \wedge G^c$
- $(\langle a \rangle F)^c = [a]F^c$
- $([a]F)^c = \langle a \rangle F^c$

Theorem (F^c is equivalent to the negation of F)

For any $p \in Proc$ and any HM formula F

- 1 $p \models F \implies p \not\models F^c$
- 2 $p \not\models F \implies p \models F^c$

Therefore, **negation is a derived operator**.

Hennessy-Milner Logic – Denotational Semantics

For a formula F let $\llbracket F \rrbracket \subseteq Proc$ contain all states that satisfy F .

Denotational Semantics: $\llbracket _ \rrbracket : Formulae \rightarrow 2^{Proc}$

- $\llbracket tt \rrbracket = Proc$
- $\llbracket ff \rrbracket = \emptyset$
- $\llbracket F \vee G \rrbracket = \llbracket F \rrbracket \cup \llbracket G \rrbracket$
- $\llbracket F \wedge G \rrbracket = \llbracket F \rrbracket \cap \llbracket G \rrbracket$
- $\llbracket \langle a \rangle F \rrbracket = \langle \cdot a \cdot \rrbracket \llbracket F \rrbracket$ $\langle \cdot a \cdot \rangle S$
- $\llbracket [a] F \rrbracket = [\cdot a \cdot] \llbracket F \rrbracket$

where $\langle \cdot a \cdot \rangle, [\cdot a \cdot] : 2^{(Proc)} \rightarrow 2^{(Proc)}$ are defined by

S satisfies F

$$\langle \cdot a \cdot \rangle S = \{p \in Proc \mid \exists p'. p \xrightarrow{a} p' \text{ and } p' \in S\}$$

$$[\cdot a \cdot] S = \{p \in Proc \mid \forall p'. p \xrightarrow{a} p' \implies p' \in S\}.$$

The Correspondence Theorem

Theorem

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS, $p \in Proc$ and F a formula of Hennessy-Milner logic. Then

$$p \models F \quad \text{if and only if} \quad p \in \llbracket F \rrbracket.$$

Proof: by structural induction on the structure of the formula F .

Image-Finite Labelled Transition Systems

Image-Finite System

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS. We call it **image-finite** iff for every $p \in Proc$ and every $a \in Act$ the set

$$\{p' \in Proc \mid p \xrightarrow{a} p'\}$$

is finite.

Relationship between HM Logic and Strong Bisimilarity

Theorem (Hennessy-Milner)

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an image-finite LTS and $p, q \in Proc$. Then

$$p \sim q$$

if and only if

for every HM formula F : $(p \models F \iff q \models F)$.