

## Exercises for Analysis of Distributed and Concurrent Systems

Resource Analysis of Distributed and Concurrent Systems: Lecture 2

**Exercise 1:** Consider the following ABS code:

```

class C {
    Unit m (Int a, Int b) {
        while (a > 0) {
            this!p(b);
            a = a - 1;
        }
    }
    Unit main (Int a, Int b) {
        C c = new C();
        c!m(a, b);
    }
    Unit p (Int b) {
        while (b < 10) {
            b = b + 1;
        }
    }
}

```

Give the ABS code above, please, write the **cost relation system** of this ABS code. Additionally, write the **upper bound** of the program.

- (1)  $\text{while}(a > 0) \text{ this!p}(b); a = a - 1;$
- (2)  $\text{while}(b < 10) \text{ b} = b + 1;$

In order to know how much (2) cost, we have to know if variable  $b$  is modified by external process. In this case doesn't occurs, therefore (2) cost:

$$\begin{aligned} \text{while}(b < 10) &= k_1 && \{ b \geq 10 \} \\ \text{while}(b < 10) &= k_2 + \text{while}(b+1) && \{ b < 10 \} \end{aligned}$$

Let us consider  $d: (\mathbb{Z}, \mathbb{Z}) \rightarrow \mathbb{N}^*$  the distance function, so the cost of  $\text{while}(b+1)$  is  $d(b+1, 10)$ . Therefore.

$$\begin{aligned} \text{while}(b < 10) &= k_1 && \{ b > 10 \} \\ \text{while}(b < 10) &= k_2 + |b+1 - 10| && \{ b < 10 \} \end{aligned} \quad \Rightarrow \quad \text{while}(b < 10) = k_1 + k_2 + |b+1 - 10|$$

On the other hand, we have another while loop that containing the above loop (2) nested. If we analyse the loop perse as follow:

$\text{while}(a > 0) \text{ this!p}(b); a = a - 1;$

We obtain:

$$(1.1) \text{ while } (a > 0) = k_3 \quad \text{if } a \leq 0 \}$$

$$(1.2) \text{ while } (a > 0) = k_4 + k_5 + \text{while } (a-1) \quad \text{if } a > 0 \}$$

where  $k_5 = \text{cost of (2)}$ . Therefore, we obtain that (1) :

$$\text{while } (a > 0) = k_3 \quad \text{if } a \leq 0 \}$$

$$\text{while } (a > 0) = k_4 + \text{while } (b) + \text{while } (a-1) \quad \text{if } a > 0 \} \Leftrightarrow$$

$$\Leftrightarrow \text{while } (a > 0) = k_4 + (k_1 + k_2 + \text{while } (b+1)) + \text{while } (a-1) \quad \text{if } a > 0 \} \Leftrightarrow \text{Analogous to (2)}$$

$$\Leftrightarrow \text{while } (a > 0) = (k_4 + (k_1 + k_2 + |b+1-10|)) \cdot \text{dist}(0, a-1) \quad \text{if } a > 0 \} \Leftrightarrow$$

$$\Leftrightarrow \text{while } (a > 0) = (k_4 + (k_1 + k_2 + |b+1-10|)) \cdot |a-1| \quad \text{if } a > 0 \}$$

$$\Rightarrow \text{while } (a > 0) = k_3 + |a-1| \cdot (k_4 + k_1 + k_2 \cdot |b+1-10|) \quad \text{if } a > 0 \}$$

$$\left. \begin{array}{l} (1.1) \\ (1.2) \end{array} \right\} \Rightarrow \text{while } (a > 0) \leftarrow k_3 + |a-1| \cdot (k_4 + k_1 + k_2 \cdot |b+1-10|)$$

$$\Rightarrow \text{main } (a, b) = k_0 + k_3 + |a-1| \cdot (k_4 + k_1 + k_2 \cdot |b+1-10|)$$

**Exercise 2:** Consider the following ABS code:

```

7   class C {
8     Unit m (Int a, Int b) {
9       while (a > 0) {
10      this!p(b);
11      a = a - 1;
12    }
13  }
14  Unit p (Int b) {
15    while (b < 10) {
16      b = b + 1;
17    }
18  }
19}

```

Give the ABS code above, please, write the **object sensitive cost relation system** of this ABS code. Additionally, write the **upper bound** of the program including the corresponding **cost centers**.

We can observe main's code has two different instances of C, so we could consider that our program has three cost-centers.

Let us consider the code above, we know that for points-to analysis, we have  $O_2, O_3$  where  $O_2 \in A(u, \text{main})$  and  $O_3 \in A(s, \text{main})$  also  $O_{\text{main}}$ . By exercise ① we know that

$$\text{cost}(O_2) = k_3 + \underbrace{(a-1)}_{\text{while}} \cdot (k_4 + k_1 + k_2 \cdot |b+1-10| + k_p) + k_m ; \quad \text{and}$$

$$\text{cost}(O_3) = k_1 + k_2 \cdot |b+1-10| + k_p$$

$$\Rightarrow U = U_m + U_{O_2} + U_{O_3} \Leftrightarrow$$

$$U = C(O_{\text{main}}) \cdot k_{\text{main}} + C(O_2) \cdot \left( k_3 + |a-1| \cdot (k_p + k_4 + k_1 + k_2 \cdot |b+1-10|) + k_m \right) + C(O_3) \cdot (k_p + k_4 + k_1 + k_2 \cdot |b+1-10|)$$

**Exercise 3:** Consider the following ABS code:

```

1 main (int n) {
2   C z = new C(); // m1
3   C y = new C(); // m1
4   f = y!p(n); // m1
5   await f?; // m2
6   ...
7   y!p(n); // m3
8   ...
9   y!q(z, n); // m4
10  ...
11 }

```

```

class C {
  void p (Int n) {
    ...
  } // p1
  void q (C z, Int n) {
    z!s(n); // q1
    ...
  } // q2
  void s (Int n) {
    ...
  } // s1
}

```

Draw the **distributed flow graph** of the program and compute all possible paths to *terminal nodes* from the entry node ( $m_1$ ).

