A Calculus of Communicating Systems (CCS)

- labelled transition systems
- process algebras
- informal introduction to CCS
- syntax of CCS
- semantics of CCS

Labelled Transition System

Definition

A labelled transition system (LTS) is a triple $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ where

- $Proc$ is a set of states (or processes),
- Act is a set of labels (or actions), and
- for every $a \in Act$, $\xrightarrow{a} \subseteq Proc \times Proc$ is a binary relation on states called the transition relation.

We will use the infix notation $s \xrightarrow{a} s'$ meaning that $(s, s') \in \xrightarrow{a}$.

Sometimes we distinguish the initial (or start) state.

Sequencing, Nondeterminism and Parallelism

LTS explicitly focuses on interaction.

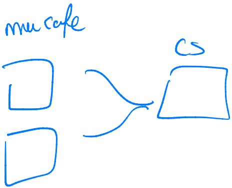
LTS can also describe:

- sequencing $(a; b)$
- choice (nondeterminism) $(a + b)$
- limited notion of parallelism (by using interleaving) $(a|b)$

aquí hay un combinación de decisión no determinista

similar pero con alguna toma aparte.

Interleaving
a follow



Labelled Transition Systems – Notation

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS.

- we extend $\xrightarrow{a}$ to the elements of Act^*
- $\longrightarrow = \bigcup_{a \in Act} \xrightarrow{a}$
- $\longrightarrow^*$ is the reflexive and transitive closure of $\longrightarrow$
- $s \xrightarrow{a}$ and $s \not\xrightarrow{a}$ ← something is not possible
- reachable states

Process Algebra

Basic Principle

1. Define a few atomic processes (modelling the simplest process behaviour).
2. Define compositionally new operations (building more complex process behaviour from simple ones).

Example

1. atomic instruction: assignment (e.g. x:=2 and x:=x+2)
2. new operators:
   - sequential composition ($P_1; P_2$)
   - parallel composition ($P_1 \mid P_2$)

   Now e.g. (x:=1 | x:=2); x:=x+2; (x:=x-1 | x:=x+5) is a process.

CCS Basics (Sequential Fragment)

- *Nil* (or 0) process (the only atomic process)
- action prefixing ($a.P$) la a es la acción, no el proceso a
- names and recursive definitions ($\stackrel{\mathrm{def}}{=}$)
- nondeterministic choice ($+$)

This is Enough to Describe Sequential Processes

Any finite LTS can be (up to isomorphism) described by using the operations above.

CCS Basics (Parallelism and Renaming)

- parallel composition ($|$)
  (synchronous communication between two components = handshake synchronization)
- restriction ($P \setminus L$) esto es para restar semántica en el conjunto o proceso haciendo que se restrinja L sobre P.
- relabelling ($P[f]$)

re-etiquetado (?)

$\Downarrow$

de aquí sale un corolario que te dice que no puedes incluir L' sobre $P \setminus L$ cuando $L \cap L'$ (≠ ∅) porque $P \setminus L$ es un conjunto ya consolidado y para usar L' hay que reprocesar P para relacionar P con L'.

Definition of CCS (channels, actions, process names)

Let

- $\mathcal{A}$ be a set of channel names (e.g. *tea*, *coffee* are channel names)

- $\mathcal{L} = \mathcal{A} \cup \overline{\mathcal{A}}$ be a set of labels where
  - $\overline{\mathcal{A}} = \{\bar{a} \mid a \in \mathcal{A}\}$
    ($\mathcal{A}$ are called names and $\overline{\mathcal{A}}$ are called co-names)
  - by convention $\bar{\bar{a}} = a$

esta es la sincronización → la veremos como medio del sistema interno para evitar la sincronización con el exterior, pero resulta de manera implícita (invisible) existe las τ de sincronización.

- $Act = \mathcal{L} \cup \{\tau\}$ is the set of actions where
  - τ is the internal or silent action

  (e.g. τ, *tea*, $\overline{coffee}$ are actions)

- $\mathcal{K}$ is a set of process names (constants) (e.g. CM).

Definition of CCS (expressions)

$P :=$	K	$\mid$	process constants ($K \in \mathcal{K}$)
	$\alpha.P$	$\mid$	prefixing ($\alpha \in Act$)
	$\sum_{i \in I} P_i$	$\mid$	summation (I is an arbitrary index set)
	$P_1 \mid P_2$	$\mid$	parallel composition
	$P \smallsetminus L$	$\mid$	restriction ($L \subseteq \mathcal{A}$)
	$P[f]$	$\mid$	relabelling ($f : Act \to Act$) such that • $f(\tau) = \tau$ • $f(\overline{a}) = \overline{f(a)}$

The set of all terms generated by the abstract syntax is called CCS process expressions (and denoted by $\mathcal{P}$).

Notation

$$P_1 + P_2 = \sum_{i \in \{1,2\}} P_i \qquad Nil = 0 = \sum_{i \in \emptyset} P_i$$

reglas de precedencia y orden de operaciones

Precedence

1. restriction and relabelling (tightest binding)
2. action prefixing
3. parallel composition
4. summation

Example: $R + a.P|b.Q \setminus L$ means $R + \big((a.P)|(b.(Q \setminus L))\big)$.

Definition of CCS (defining equations)

CCS program

A collection of defining equations of the form

$$K \stackrel{\mathrm{def}}{=} P$$

where $K \in \mathcal{K}$ is a process constant and $P \in \mathcal{P}$ is a CCS process expression.

- Only one defining equation per process constant.
- Recursion is allowed: e.g. $A \stackrel{\mathrm{def}}{=} \overline{a}.A \mid A$.

$Clock = tick.Clock \leftarrow$ aquí hay que visualizar que la propia definición de Clock provee semántica al segundo término sin tener que definir Clock previamente

Semantics of CCS

y aquí aún podemos ver cada X como una variable libre, donde cada proceso asociado a ese término estamos haciendo probado cosas $\forall X$.

Syntax		Semantics
CCS (collection of defining equations)	$\longrightarrow$	LTS (labelled transition systems)

HOW?

Structural Operational Semantics for CCS

Structural Operational Semantics (SOS) – G. Plotkin 1981

Small-step operational semantics where the behaviour of a system is inferred using syntax driven rules.

Given a collection of CCS defining equations, we define the following LTS $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$:

- $Proc = \mathcal{P}$ (the set of all CCS process expressions)
- $Act = \mathcal{L} \cup \{\tau\}$ (the set of all CCS actions including τ)
- transition relation is given by SOS rules of the form:

$$\text{RULE} \quad \frac{premises}{conclusion} \quad conditions$$

SOS rules for CCS ($\alpha \in Act$, $a \in \mathcal{L}$)

ACT $\dfrac{}{\alpha.P \xrightarrow{\alpha} P}$

SUM$_j$ $\dfrac{P_j \xrightarrow{\alpha} P'_j}{\sum_{i \in I} P_i \xrightarrow{\alpha} P'_j} \quad j \in I$

COM1 $\dfrac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q}$

COM2 $\dfrac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'}$

COM3 $\dfrac{P \xrightarrow{a} P' \qquad Q \xrightarrow{\overline{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$

RES $\dfrac{P \xrightarrow{\alpha} P'}{P \smallsetminus L \xrightarrow{\alpha} P' \smallsetminus L} \quad \alpha, \overline{\alpha} \notin L$

REL $\dfrac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$

CON $\dfrac{P \xrightarrow{\alpha} P'}{K \xrightarrow{\alpha} P'} \quad K \stackrel{\mathrm{def}}{=} P$

Deriving Transitions in CCS

Let $A \stackrel{\mathrm{def}}{=} a.A$. Then

$$\big((A \mid \overline{a}.Nil) \mid b.Nil\big)[c/a] \xrightarrow{c} \big((A \mid \overline{a}.Nil) \mid b.Nil\big)[c/a].$$

$$\text{REL}\ \dfrac{\text{COM1}\ \dfrac{\text{COM1}\ \dfrac{\text{CON}\ \dfrac{\text{ACT}\ \dfrac{}{a.A \xrightarrow{a} A}}{A \xrightarrow{a} A}\ A \stackrel{\mathrm{def}}{=} a.A}{A \mid \overline{a}.Nil \xrightarrow{a} A \mid \overline{a}.Nil}}{(A \mid \overline{a}.Nil) \mid b.Nil \xrightarrow{a} (A \mid \overline{a}.Nil) \mid b.Nil}}{\big((A \mid \overline{a}.Nil) \mid b.Nil\big)[c/a] \xrightarrow{c} \big((A \mid \overline{a}.Nil) \mid b.Nil\big)[c/a]}$$

LTS of the Process $a.Nil \mid \overline{a}.Nil$

$$
\begin{tikzcd}
 & a.Nil \mid \overline{a}.Nil \arrow[dl, "a"'] \arrow[dr, "\overline{a}"] \arrow[dd, "\tau"] & \\
Nil \mid \overline{a}.Nil \arrow[dr, "\overline{a}"] & & a.Nil \mid Nil \arrow[dl, "a"'] \\
 & Nil \mid Nil &
\end{tikzcd}
$$