From Strong Bisimulation to Weak Bisimulation

- applying strong bisimilarity: a simple buffer
- weak bisimilarity and weak bisimulation games
- properties of weak bisimilarity
- example: a communication protocol and its modelling in CCS
- concurrency workbench (CWB)

Example – Buffer

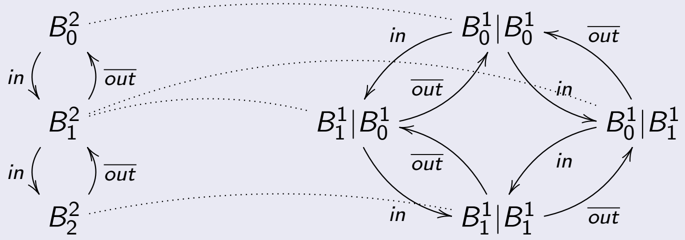
Buffer of Capacity 1

$$B_0^1 \stackrel{\text{def}}{=} in.B_1^1$$
$$B_1^1 \stackrel{\text{def}}{=} \overline{out}.B_0^1$$

Buffer of Capacity n

$$B_0^n \stackrel{\text{def}}{=} in.B_1^n$$
$$B_i^n \stackrel{\text{def}}{=} in.B_{i+1}^n + \overline{out}.B_{i-1}^n \quad \text{for } 0 < i < n$$
$$B_n^n \stackrel{\text{def}}{=} \overline{out}.B_{n-1}^n$$

Example: $B_0^2 \sim B_0^1 | B_0^1$



Example – Buffer

Theorem

For all natural numbers n: $B_0^n \sim \underbrace{B_0^1 | B_0^1 | \cdots | B_0^1}_{n \ \textit{times}}$

Proof.

Construct the following binary relation, where $i_1, i_2, \ldots, i_n \in \{0, 1\}$:

$$R = \{ \left(B_i^n, \ B_{i_1}^1 | B_{i_2}^1 | \cdots | B_{i_n}^1 \right) \mid \sum_{j=1}^{n} i_j = i \}$$

- $\left(B_0^n, \ B_0^1 | B_0^1 | \cdots | B_0^1 \right) \in R$
- R is a strong bisimulation

$\square$

But still Internal Actions must be Abstracted away

Question

Does $a.\tau.Nil \sim a.Nil$ hold? NO!

Problem

Strong bisimilarity does not abstract away from τ actions.

Example: SmUni $\not\sim$ Spec

$$
\begin{tikzcd}
\mathrm{SmUni} \arrow[d, "\overline{pub}"] & \not\sim & \mathrm{Spec} \arrow[loop below, "\overline{pub}"] \\
(CM \mid CS_1) \setminus \{coin, coffee\} \arrow[d, "\tau"] & & \\
(CM_1 \mid CS_2) \setminus \{coin, coffee\} \arrow[d, "\tau"] & & \\
(CM \mid CS) \setminus \{coin, coffee\} \arrow[uu, bend right, "\overline{pub}"'] & &
\end{tikzcd}
$$

Weak Transitions will (mostly) hide τ actions

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS such that $\tau \in Act$.

Definition of Weak Transition Relation

$$\overset{a}{\Longrightarrow} = \begin{cases} (\xrightarrow{\tau})^* \circ \xrightarrow{a} \circ (\xrightarrow{\tau})^* & \text{if } a \neq \tau \\ (\xrightarrow{\tau})^* & \text{if } a = \tau \end{cases}$$

What does $s \overset{a}{\Longrightarrow} t$ informally mean?

- If $a \neq \tau$ then $s \overset{a}{\Longrightarrow} t$ means that
  from s we can get to t by doing zero or more τ actions, followed by the action a, followed by zero or more τ actions.
- If $a = \tau$ then $s \overset{\tau}{\Longrightarrow} t$ means that
  from s we can get to t by doing zero or more τ actions.

Weak Bisimilarity

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS such that $\tau \in Act$.

Weak Bisimulations

A binary relation $R \subseteq Proc \times Proc$ is a weak bisimulation iff whenever $(s, t) \in R$ then for each $a \in Act$ (including τ):

- if $s \xrightarrow{a} s'$ then $t \overset{a}{\Longrightarrow} t'$ for some t' such that $(s', t') \in R$
- if $t \xrightarrow{a} t'$ then $s \overset{a}{\Longrightarrow} s'$ for some s' such that $(s', t') \in R$.

Weak Bisimilarity

Two processes $p_1, p_2 \in Proc$ are weakly bisimilar ($p_1 \approx p_2$) if and only if there exists a weak bisimulation R such that $(p_1, p_2) \in R$.

$$\approx \; = \; \cup\{R \mid R \text{ is a weak bisimulation}\}$$

Weak Bisimulation Game

Definition

Just as the Strong game except that

- defender can now reply using $\overset{a}{\Longrightarrow}$ moves.

The attacker is still using only $\xrightarrow{a}$ moves.

Theorem

- States s and t are weakly bisimilar if and only if the defender has a universal winning strategy starting from the configuration (s, t).
- States s and t are not weakly bisimilar if and only if the attacker has a universal winning strategy starting from the configuration (s, t).

Properties of Weak Bisimilarity

Properties of $\approx$

- $\approx$ is an equivalence relation
- $\approx$ is the largest weak bisimulation
- validates lots of natural laws, e.g.
  - $a.\tau.P \approx a.P$
  - $P + \tau.P \approx \tau.P$
  - $a.(P + \tau.Q) \approx a.(P + \tau.Q) + a.Q$
  - $P + Q \approx Q + P \qquad P|Q \approx Q|P \qquad P + Nil \approx P \qquad \ldots$
- strong bisimilarity is included in weak bisimilarity ($\sim \subseteq \approx$)
- $\approx$ (totally) abstracts τ loops

Is Weak Bisimilarity a Congruence for CCS?

Theorem

Let P and Q be CCS processes such that $P \approx Q$. Then

- $\alpha.P \approx \alpha.Q$ *for each action* $\alpha \in Act$
- $P \mid R \approx Q \mid R$ *and* $R \mid P \approx R \mid Q$ *for each CCS process* R
- $P[f] \approx Q[f]$ *for each relabelling function* f
- $P \setminus L \approx Q \setminus L$ *for each set of labels* L.

But what about choice?

$\tau.a.Nil \approx a.Nil$ but $\tau.a.Nil + b.Nil \not\approx a.Nil + b.Nil$

Conclusion

Weak bisimilarity is not a congruence for CCS.

Case Study: A simple Communication Protocol

$$\begin{aligned}
\mathsf{Send} &\stackrel{\text{def}}{=} \mathsf{acc}.\mathsf{Sending} \\
\mathsf{Sending} &\stackrel{\text{def}}{=} \overline{\mathsf{send}}.\mathsf{Wait} \\
\mathsf{Wait} &\stackrel{\text{def}}{=} \mathsf{ack}.\mathsf{Send} + \mathsf{error}.\mathsf{Sending}
\end{aligned}
\qquad
\begin{aligned}
\mathsf{Rec} &\stackrel{\text{def}}{=} \mathsf{trans}.\mathsf{Del} \\
\mathsf{Del} &\stackrel{\text{def}}{=} \overline{\mathsf{del}}.\mathsf{Ack} \\
\mathsf{Ack} &\stackrel{\text{def}}{=} \overline{\mathsf{ack}}.\mathsf{Rec}
\end{aligned}$$

$$\begin{aligned}
\mathsf{Med} &\stackrel{\text{def}}{=} \mathsf{send}.\mathsf{Med}' \\
\mathsf{Med}' &\stackrel{\text{def}}{=} \tau.\mathsf{Err} + \overline{\mathsf{trans}}.\mathsf{Med} \\
\mathsf{Err} &\stackrel{\text{def}}{=} \overline{\mathsf{error}}.\mathsf{Med}
\end{aligned}$$

$$\mathsf{Impl} \stackrel{\text{def}}{=} (\mathsf{Send} \mid \mathsf{Med} \mid \mathsf{Rec}) \smallsetminus \{\mathsf{send}, \mathsf{trans}, \mathsf{ack}, \mathsf{error}\}$$

$$\mathsf{Spec} \stackrel{\text{def}}{=} \mathsf{acc}.\overline{\mathsf{del}}.\mathsf{Spec}$$

Question

$$\mathsf{Impl} \stackrel{?}{\approx} \mathsf{Spec}$$

1. Draw the LTS of Impl and Spec and prove (by hand) their equivalence.
2. This could be done (automatically) using the Concurrency WorkBench (CWB).