# Firms' Rollover Risk and Macroeconomic Dynamics

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**Abstract**

This paper analyzes the macroeconomic implications of firms' rollover risk. I develop a heterogeneous-firms macroeconomic model with rollover crises emerging from coordination failures among creditors. Rollover crises are events in which a firm defaults because creditors fail to roll over its debt, but would have repaid otherwise. I assess the quantitative relevance of rollover crises by employing a model-based identification strategy which argues that their incidence is informed by the observed distribution of firms' bankruptcy outcomes, and find that roughly half of corporate bankruptcy events are due to rollover crises. I then use the model to assess the aggregate implications of rollover risk for the U.S. economy and find that rollover risk can significantly amplify the impact of recessions. Lastly, I show that imperfectly targeted credit policies can mitigate rollover crises but can exacerbate firms' future debt overhang.

*Keywords*: financial frictions, heterogeneous firms, coordination failures, rollover risk, bankruptcy, business cycles, economic crises.

*JEL classifications*: E32, E44, E58, G01, G33.

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# 1 Introduction

I study the impact of firms' rollover crises on macroeconomic dynamics. Rollover crises are events in which an *economically solvent* firm — i.e., with positive net present value — defaults because its creditors fail to roll over its debt. Although the notion of firms' rollover crises is often alluded to by policymakers during recessions and by top managers of bankrupt firms, we know little about their quantitative and aggregate implications.[1]

To assess the aggregate implications of firms' rollover risk, I develop a quantitative macroeconomic framework in which firms' rollover crises can be identified and quantified. In the framework, there is feedback between interest rates on newly issued corporate debt and firms' incentives to default on outstanding debt. Rollover crises are events in which a firm defaults because creditors fail to roll over its debt, but would have repaid otherwise. To assess the incidence of rollover crises, I use an approach that combines the model and data on firms' bankruptcy outcomes and bankrupt firms' characteristics. I find that roughly half of the corporate bankruptcy events are driven by rollover crises. I then conduct a quantitative analysis of the U.S. economy to assess the impact of firms' rollover risk on macroeconomic dynamics. I find that rollover risk can explain between 10% and 30% of the drop in aggregate output during large recessions. Finally, I study the effectiveness of an imperfectly targeted credit policy — akin to those used extensively during the Covid crisis and other recessions — and find that in the short term the policy can prevent rollover crises but can backfire if it exacerbates firms' future debt overhang.

The framework is a general equilibrium model populated by heterogeneous firms which use internal resources and/or issue debt to finance investment and production. There are three key ingredients. First, firms have no commitment to repay their debt; thus, endogenous default risk limits their borrowing capacity. Second, using tools from the literature that studies self-fulfilling sovereign debt crises [Cole and Kehoe (2000)], I incorporate potential coordination failures among firms' creditors — rollover crises. Third, emulating the U.S. bankruptcy code, firms can decide to liquidate and exit, or restructure

[1] For example, this notion was alluded to in discussions around financial institutions and firms bailouts during recent recessions [see, for example, Financial Times (09/30/2014) and BIS Bulletin by Banerjee, Noss and Vidal Pastor (2021)]. The notion of rollover crises is also alluded to in regulations, for example, in Section 13(3) of the Dodd Frank Act, which delimits the lending powers of the Fed. Lastly, top managers of bankrupt firms and other actors involved in the bankruptcy procedure (for example, the judge of the case) often mention financial problems akin to rollover crises as the main cause of bankruptcy [see, for example, Ayotte and Skeel (2013) and references within].

their liabilities and continue operating.

The environment considered creates complementarities between debt prices and firms' default choices, which can lead to multiple equilibria for a firm. I characterize the default choice and find that there are three types of firms in the economy. First, there are safe firms with strong fundamentals that will not default even if creditors fail to rollover the debt. On the other extreme, there are insolvent firms with weak fundamentals that default even if creditors would be willing to rollover. Finally, there are risky firms, which are exposed to rollover problems, and default or not depending on creditors' coordination, thus their equilibrium outcome is undetermined. To construct the equilibrium for these firms, I assume that an *idiosyncratic* and stochastic sunspot variable selects the default equilibrium with a given probability *common* to all risky firms. This probability — jointly with the share of firms exposed — captures the rollover risk in the economy.

To measure the incidence of rollover crises, I design a model-based identification strategy that learns from firms' bankruptcy choices and bankrupt firms' characteristics. In the U.S. bankruptcy code, bankrupt firms can choose to use Chapter 7 provisions and be liquidated, or use Chapter 11 provisions and renegotiate their debt with creditors while they continue operating and, importantly, debt payments are suspended and new debt issuance is facilitated.[2] To exploit these features of the U.S. bankruptcy code I embed in the model a bankruptcy procedure in which bankrupt firms can choose between the liquidation and restructuring chapters. I argue that I can infer indirectly the incidence of rollover crises from firms' bankruptcy chapter choices and bankrupt firms' financial characteristics (for example, debt recovery rates and firms' leverage). I find that in the model, ceteris paribus, a larger share of firms in the restructuring process, rather than liquidation, implies a greater incidence of rollover crises. The intuition is that the restructuring process provides little benefit to insolvent firms, since observed debt haircuts in the restructuring process are small, but provide large benefits to firms under a rollover crisis, since the restructuring process can operate as a way to (temporarily) force the rollover of debt (for example, by suspending debt payments) and coordinate creditors.[3]

[2] It is widely argued in the bankruptcy law literature [see, for example, the seminal work by Jackson (1986)] that several provisions in the U.S. bankruptcy code (such as the one preventing creditors from collecting debt payments) are designed to solve credit coordination failures for bankrupt firms (or, in other words, coerce debt rollover).

[3] Anecdotal evidence suggests that firms' managers consider the restructuring process, (and the debtor-in-possession (DIP) protection they provide) as a way to buy some time to make payments due.

My first main quantitative result shows that roughly half of the corporate bankruptcy events in the U.S. economy are driven by rollover crises. I find that 1.6% of the firms are subject to rollover crises: 21% of the firms are exposed with a 7% (conditional) probability of a coordination failure implied by the quantitative model.

As a validation exercise, I then assess the model's ability to reproduce untargeted patterns in the data. I simulate a panel of firms and find that the model is able to match the observed investment heterogeneity patterns in recent recessions and how firms' characteristics predict the occurrence of a restructuring event. Consistent with the data, the investment heterogeneity patterns during the last two recessions in U.S. (the Great Recession and the Covid crisis) show that firms with less internal resources (cash-on-hand) cut their investment by more. Moreover, in the model and data, publicly-held firms with less total assets, fewer internal resources, lower sales growth, and less leverage are more likely to enter the restructuring process next period.

I then conduct a quantitative study of how firms' rollover crises function as an amplification mechanism in large recessions. I simulate a prototypical large recession (5% drop in output) and study the dynamics of aggregate output with and without rollover crises. For different types of shocks driving the recession — total factor productivity (TFP), cash and credit shocks — I find that rollover risk can significantly amplify the impact of recessions, explaining 10% to 30% of output losses during the episode.[4] Moreover, rollover risk makes recessions more persistent. A sudden reduction in firms' cash flows temporarily exposes more firms to rollover crises, leading to more failures (bankruptcy and liquidation) of healthy firms, which creates greater misallocation in the extensive margin and extra persistence since new firms take some time to grow.

Finally, I study the policy implications of rollover crises. I focus on direct lending policies, which resemble those deployed by the Fed during the Covid crisis and other credit interventions in previous recessions.[5] I simulate an imperfectly targeted credit policy during a recession for different scales of the policy. By scale, I mean how many firms

In addition, the DIP protection facilitates new debt issuance (known as DIP financing).

[4]In my experiments a credit shock is a reduction in the recovery rate of creditors during a firm's liquidation event, which is equivalent to a sudden reduction in the firms' debt collateral's value.

[5]For example, the Fed provided credit to corporate firms through the Primary Market Corporate Credit Facility (PMCCF) and Secondary Market Corporate Credit Facility (SMCCF) during the Covid crisis.

are eligible to participate. First, I find that small-scale policies significantly reduce the short-term impact of rollover crises and promote a swift recovery. The intuition is that the credit policy works as insurance for creditors, which precludes coordination failures, even if firms do not draw funds from the government's credit facilities. Lastly, I find that large-scale policies can backfire. While they mitigate more rollover crises and have greater short-term benefits than small-scale policies, they subsidize credit to too many firms which exacerbates resource misallocation and future debt overhang.

**Literature and Contributions.** The paper fits in the broad research agenda which aims to incorporate firm-level corporate financing considerations in quantitative macroeconomic models to study their aggregate (positive and normative) implications.[6] My paper's main contributions can be placed in the following strands of literature in macroeconomics and finance:

*Financial heterogeneity and default risk in macroeconomics.* This paper is related to the broad literature that works with general equilibrium models of firms with default risk [see, for example, Cooley and Quadrini (2001) for an early reference]. More specifically, this paper is related to the literature that studies the implications of aggregate shocks in macro models with firm default risk [see, for example, Cooley, Marimon and Quadrini (2004); Jermann and Quadrini (2012); Arellano, Bai and Kehoe (2019); Khan, Senga and Thomas (2020); Ottonello and Winberry (2020)]. I contribute to this literature by studying the aggregate implications of rollover risk (i.e., default driven by credit coordination failures) in recessions.

Corbae and D'Erasmo (2021) studies the long-term implications of changes to the U.S. bankruptcy code in a general equilibrium model with heterogeneous firms. Distinctively, I study the interaction between firms' rollover crises and bankruptcy provisions aimed to prevent coordination failures among creditors.[7]

*Rollover crises and multiple equilibria in macroeconomics.* An ample literature studies rollover crises and multiple equilibria in macroeconomics.[8] My paper is closely connected

[6] See, for example, the review by Brunnermeier and Krishnamurthy (2020).

[7] The resolution of creditors' coordination problems are widely discussed in the bankruptcy law literature. See, for example, the seminal work Jackson (1986) or, more recently, Ayotte and Skeel (2013).

[8] Another related literature studies self-fulfilling expectations and business cycles. See, for example, Benhabib and Wang (2013); Harrison and Weder (2013); Liu and Wang (2014); Azariadis, Kaas and Wen (2015); Cui and Kaas (2021); Schmitt-Grohé and Uribe (2020).

to the work on sovereign debt self-fulfilling crises [see, for example, Cole and Kehoe (2000); Bocola and Dovis (2019); Aguiar, Chatterjee, Cole and Stangebye (2021)]. In my model, the Cole and Kehoe (2000) timing generates the possibility of firms' rollover crises caused by creditors' coordination failures. My paper is also connected to the literature that studies bank runs in macroeconomic models [see, for example, Diamond and Dybvig (1983); Gertler and Kiyotaki (2015); Gertler, Kiyotaki and Prestipino (2019); Amador and Bianchi (2024)]. The contribution of my paper to this literature is twofold. First, I develop a macroeconomic model with heterogeneous firms that can be subject to rollover crises which is applied to assess the quantitative relevance of non-financial firms' rollover crises and, second, I use an identification strategy which argues that salient features of the bankruptcy process — jointly with firms' choices and characteristics — inform the incidence of rollover crises and coordination failures.

*Rollover (coordination) crises in corporate finance theory.* A strand of literature in corporate finance theory studies firms' rollover crises caused by creditors' coordination failures. Salient examples can be found in Morris and Shin (2004, 2016), who adopt a global games approach to study the relationship between firms' rollover crises (coordination) and corporate debt pricing. In related work, He and Xiong (2012a,b); Cheng and Milbradt (2012) study the relationship between rollover crises and corporate debt maturity, and Zhong (2021) studies the relation between the risk of credit coordination failures and firms' creditors concentration choice. My contribution to this strand of literature is the analysis of the interaction between rollover crises and bankruptcy provisions in a general equilibrium infinite horizon economy.

*Corporate credit policy intervention and recessions.* Sparked by the Covid crisis — and the policy response that followed — a recent body of work studies the effectiveness of corporate credit policies during large recessions using structural models. Elenev, Landvoigt and Van Nieuwerburgh (2021) finds that credit policy during large recessions can prevent firms' bankruptcy; Ebsim, Faria e Castro and Kozlowski (2021) shows that policy effectiveness may depend on the source of the crisis shock; and Crouzet and Tourre (2021) finds that subsidizing credit to firms may induce future debt overhang. My paper is in line with this recent line of work. In addition, I show that credit policy also works through a coordination channel, akin to a deposit insurance for banks, and may be potent even if (in equilibrium) few of the eligible firms borrow from the credit policy program.[9]

*Rollover crises quantification.* Rollover crises are hard to disentangle from crises driven by fundamentals. Bocola and Dovis (2019) uses a model-based identification strategy that exploits the time-series of government's maturity choices to identify the how much rollover risk drove the increase in credit spreads of Italian bonds during the Euro Crises. On the other hand, Foley-Fisher, Narajabadand and Verani (2020) uses an empirical identification strategy that exploits particularities of insurers' debt contracts to assess the incidence of rollover crises on life insurance companies. In the same spirit as those papers, I identify and quantify indirectly the incidence of rollover crises using a model-based strategy, which relies on salient features of the U.S. bankruptcy code and which learns from observed cross-sectional patterns of firms' bankruptcy choices and bankrupt firms' characteristics.

**Paper's Organization.** The paper is organized as follows: Section 2 develops a macroeconomic model in which heterogeneous firms can be subject to rollover crises; Section 3 identifies and quantifies firms' rollover crises; Section 4 quantifies the amplification mechanism of rollover risk in large recessions and studies the effectiveness of direct credit policies; and Section 5 concludes. Lastly, the Appendix contains further details on the theory, data, other exercises and extensions.

# 2 Macroeconomic Model of Firms' Rollover Crises

In this section, I describe the theoretical framework developed to study the macroeconomic consequence of firms' rollover crises. The model is an heterogeneous firms general equilibrium model with three key ingredients. First, following Khan *et al.* (2020) and Ottonello and Winberry (2020), firms cannot commit to repay their debt, then endogenous default risk limits their borrowing capacity. Second, firms can default because of fundamentals or as a result of rollover crises driven by creditors' coordination failures à la Cole and Kehoe (2000). Third, there is a bankruptcy procedure which allows bankrupt

[9] This last result is related to observations made by Cox, Greenwald and Ludvigson (2021) regarding the workings of the Fed's corporate credit facilities during Covid. They observe limited participation in Federal Reserve's Primary and Secondary Market Corporate Credit Facility programs, and argue that credit policy (announcements) affected asset prices through non-fundamentals (analogous to the workings of the credit policy in my paper).

firms to liquidate and exit, or restructure their liabilities and continue operating [see, for example, Corbae and D'Erasmo (2021)].

## 2.1 Environment

The economy has an infinite horizon and is in discrete time. It is inhabited by four types of agents: (i) heterogeneous firms that invest, produce the unique final good and make financial choices in order to maximize the present value of their dividends (i.e., firm's market value); (ii) atomistic and perfectly competitive creditors that lend to the firms; (iii) a representative capital producer that sells capital to the firms; and (iv) a representative household that consumes, saves and works, and owns all the firms in the economy. The price of the final good is the numeraire, and the price of capital good $q_t$ and wages $w_t$ are endogenous.

## 2.2 Firm's Setup

The objective of firm $i$ is to maximize its value $V_{it} = \mathbb{E}_t\left[\sum_{h \geq t} \Lambda_{t,h} d_{ih}\right]$ where $\Lambda_{t,h}$ is the stochastic discount factor of the households from period $h$ to period $t$ and $d_{ih}$ are the dividends issued by firm $i$ in period $h$.

The firm has three types of idiosyncratic state variables: (i) exogenous fundamental state variables $\mathbf{s}^f_{it}$, (ii) an exogenous non-fundamental state variable $s^n_{it}$, and (iii) endogenous state variables $\mathbf{s}^e_{it}$. The idiosyncratic state vector of the firm is defined as $\mathbf{s}_{it} = \left(\mathbf{s}^f_{it}, s^n_{it}, \mathbf{s}^e_{it}\right)$. I will explain which are the relevant state variables when I describe the firm's problem.

Firms are perfectly competitive and there is a continuum of them producing in each period. The distribution $\Omega_t$ of operating firms is normalized to $\int \mathrm{d}\Omega = 1$ in steady state. There is no aggregate risk and the firm's problem can be written recursively; thus, for clarity of exposition I will drop subscripts for firm $i$ and period $t$, and adopt the recursive notation convention.

**Technology.** Firms combine capital $k$ and labor $l$ to produce a unique final good using a Cobb-Douglas production function

$$y = f(z, \omega, k, l) = z(\omega k)^{\alpha} l^{\nu},$$

where $\alpha \in (0,1)$ is the share of capital and $\nu$ is the share of labor. I assume the firm operates with decreasing returns to scale, i.e., $\alpha + \nu < 1$. The firm's production is subject to two idiosyncratic shocks: (i) a persistent idiosyncratic productivity process $\ln z^{'} = \rho \ln z + \epsilon_z$ with $\epsilon_z \sim^{\text{iid}} (0, \sigma_z^2)$; and (ii) an idiosyncratic iid capital quality shock $\omega$, which is drawn from a log-normal truncated distribution with $\ln \omega \in [\underline{\omega}, 0]$. The capital quality shock $\omega$ is standard in the literature and is helpful to match the default rates observed in the data. Firms own capital $k$, which is inherited from the previous period, and hire labor $l$ at given wage $w$. The firms' labor choice problem is static, so that operating profits of the firm are

$$\pi(z, \omega, k) = \max_{l} z (\omega k)^{\alpha} l^{\nu} - wl \tag{1}$$

with labor demand $l^{*} = \left[\frac{\nu z (\omega k)^{\alpha}}{w}\right]^{\frac{1}{1-\nu}}$.

**Resources.** Firms can issue new one-period debt $b'$ or use internal resources $n$ to finance themselves. The new debt is issued at endogenous price schedule $Q$, which is offered by the creditors. The firm's internal resources after production, or cash-on-hand holdings, are

$$n = \pi(z, \omega, k) + (1-\delta) q \omega k - b. \tag{2}$$

where $\pi(z, \omega, k)$ are the operational profits, $\delta \in [0,1]$ is the capital's depreciation rate, $(1-\delta) q\omega k$ the current value of the firm's productive capital after depreciation, and $b$ the maturing inherited debt from the previous period. The external and own resources are used to issue dividends $d$ and make capital purchases, i.e.,

$$d + qk^{'} = n + Qb^{'}. \tag{3}$$

In the presence of financial frictions—such as costly bankruptcy and limits to equity issuance—the financial structure of the firm can matter for the firm's investment decisions.

**Entry.** The mass of entrant firms $\bar{\mu}$ equals the mass of exiting firms in steady state. I assume entrants are endowed with capital $k = k_0$ and debt $b = 0$, and draw their initial productivity level $z$ from invariant distribution $\Omega^{e}(z)$ which has an average productivity $m \leq 0$ percent lower than the mean of the ergodic distribution of $z$. This assumption is consistent with evidence that young firms have lower measured productivity, as pointed out by Ottonello and Winberry (2020), and it is instrumental to matching firms' life-cycle moments.

**Exit.** Following Khan *et al.* (2020), at the beginning of each period, firms can receive an *exogenous* exit shock with probability $\gamma \in [0,1]$, which forces them to exit after production. This assumption precludes all firms from eventually overcoming the financial frictions in steady state. On the other hand, the firms which don't receive the exogenous exit shock can *endogenously* exit if they decide to stop operating and liquidate. The bankruptcy decision of the firms is described below and characterized in Section 2.4.

**Financial frictions.** There are two forms of firm-level financial frictions: limits to equity issuance and a lack of commitment to repay the debt. First, firms are precluded from issuing equity, i.e.,

$$d \geq 0. \tag{4}$$

This assumption is consistent with the scarce issuance of equity by corporate firms in the data and provides greater tractability to the model. In Appendix C.2, I characterize the firm's liquidation choice in a model where there is costly equity issuance and show that, as in the baseline model, a multiplicity region can exist. The other financing friction arises from firms' lack of commitment to repay their debt and the associated bankruptcy costs. In each period, firms have the option to file for bankruptcy and default on their debt. As creditors internalize this risk, they will charge a premium for the default risk, accepting a lower debt price, thereby limiting the borrowing capacity of the firms.

**Bankruptcy choices.** Emulating the U.S. bankruptcy code, firms can decide to liquidate and exit, or restructure their liabilities and continue operating.

If the firm decides to liquidate and exit, then there is undiscriminated default. Importantly, the liquidation choice takes place after the new debt issuance $b'$, so that the firm defaults on both its inherited debt $b$ and new debt issuance $b'$. The firm's exit value is $V^{\text{exit}} = 0$. Furthermore, creditors of inherited debt $b$ recover a fraction $R(b,k,\omega) = \min\left\{1, \alpha_7 \frac{(1-\delta)q\omega k}{b}\right\}$ of debt $b$, where $\alpha_7 \in [0,1]$ is a parameter that indicates the creditors' recovery rate of capital when the firm is liquidated. On the other hand, creditors of new debt $b'$ do not recover anything from a contemporaneous liquidation. This assumption is made for technical reasons and captures the fact that existing creditors tend to have seniority over new creditors. Liquidation events such as *Chapter 7* liquidations and *Chapter 11* piecemeal liquidations through the U.S. Bankruptcy Code are the empirical counterpart of the liquidation events in the model.

If the firm decides to restructure its debt $b$, then the creditors and the firm go through a Nash Bargaining protocol to determine the debt recovery rate $\alpha_{11} \in [0,1]$ of inherited debt $b$. I assume that the outside option of the bargaining problem is to continue as if the firm never filed for bankruptcy. This assumption captures the notion that the restructuring procedure is a joint decision of the firm, creditors and the court. For the restructuring to be feasible, both the firm and its creditors should be willing to participate in the negotiations; thus, both have to be better off. Additionally, restructuring firms pay bankruptcy cost $c_{11} \in [0,1]$ which is proportional to the firms' the current value of capital. The restructuring cost captures bankruptcy costs such as legal fees, administrative costs, reputation deterioration, among others. After the haircut and paying the restructuring costs, the internal resources left are

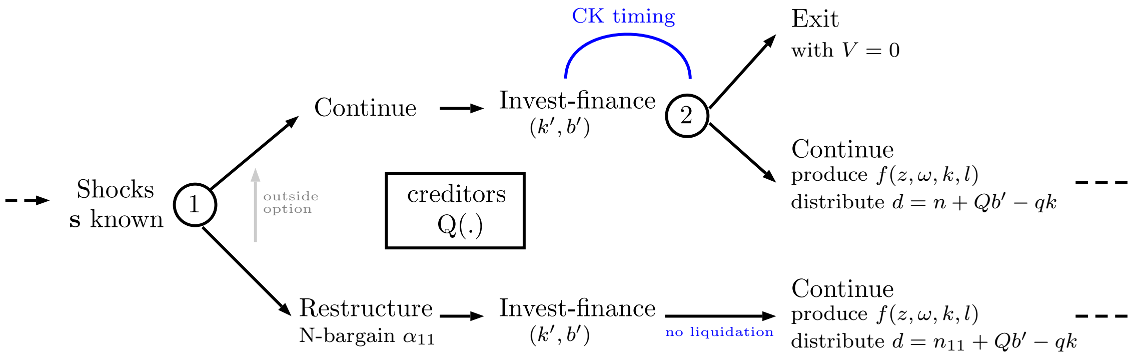
$$n_{11} = \underbrace{\pi\left(z,\omega,k\right) + \left(1-\delta\right) q\omega k - b}_{n} + \left(1-\alpha_{11}\right) b - c_{11}\left(1-\delta\right) q\omega k.$$

When the firm and the creditors meet to renegotiate the debt, I assume that creditors act as a unity. As a consequence of this, rollover crises, which are driven by creditors' coordination failures, are precluded in the restructuring process. This feature of the restructuring process captures the provisions in Chapter 11 of the U.S. Bankruptcy Code where, for example, creditors committees are facilitated, debt payments are suspended and new debt issuances are facilitated. The empirical counterpart of the restructuring process includes events such as those covered by *Chapter 11* of the U.S. Bankruptcy Code, excluding de facto liquidations such as '363' sales.

Section 2.8 includes a thorough discussion of the assumptions related to the bankruptcy process and Appendix D provides further institutional details of the Chapter 7 and the Chapter 11 of the U.S. Bankruptcy Code.

**Timing.** Figure 1 shows the within-period timing of the firm problem for firms which are not subject to an exogenous exit shock $\gamma$. At the beginning of the period, the idiosyncratic states are known, i.e., fundamental and non-fundamental shocks are revealed. After uncertainty is resolved, there is no within period source of uncertainty. All shocks and states are known, i.e., $\mathbf{s}$ is known and is common knowledge for all.

**Figure 1:** Within-period timing




Note: timing is conditioned on a firm that doesn't receive an exit shock $\gamma$. In Appendix A.2.1, I describe and characterize the exiting firm's problem.

In Figure 1, the first dot indicates the firms' choice to either continue or restructure. If the firm decides to continue, it makes its investment and financing choices $(k', b')$. Debt is issued taking as given the pricing schedule $Q$ offered by the creditors, which is determined before the investment-financing choice. After the firm issues new debt $b'$, the firm decides either to liquidate and exit, or produce and continue to the next period. The liquidation choice (second dot in Figure 1) takes place after the new debt $b'$ is priced and issued. The lack of within period commitment to repay the debt can create multiple equilibria for the firm. The timing follows Cole and Kehoe (2000) seminal paper (CK timing), which is widely used in the international macroeconomics literature that studies rollover crises in sovereign debt models.[10]

On the other hand, if the firm decides to restructure, then it renegotiates its inherited debt $b$ with its creditors. Formally, they Nash-bargain to determine the debt recovery rate $\alpha_{11}$. The outside option of the bargaining protocol, for both, is to continue, which is indicated by the gray arrow up in Figure 1. After restructuring the debt, if the negotiations are successful, the firm makes the investment-financing choice, and then produces and

[10]The multiplicity generated by this timing convention is sometimes known as *static* multiplicity in sovereign debt models [Aguiar and Amador (2021)].

distributes the dividends. Importantly, in the restructuring process there is no possibility of current coordination failures among creditors. I assume that creditors act as a unity in the restructuring process which implies that there is no liquidation choice after issuing the new debt, i.e., visually, in Figure 1, there is no dot after entering the restructuring process.

## 2.3 Corporate Debt Prices

To characterize the liquidation and restructuring choices, it is necessary to describe the endogenous debt price schedule. Creditors are perfectly competitive and atomistic; thus, the no-profit condition pins down the debt prices. They borrow from households at risk-free rate $r$ and lend to firms at price schedule $Q\left(\mathbf{s},k^{'},b^{'}\right)$. All intermediaries (creditors) are owned by the representative household, hence they discount future flows using the one-period household's stochastic discount factor (SDF) $\Lambda$. The SDF is determined by the household problem, which is described in Section 2.6. Thus, the debt price schedule offered by the creditors for a firm that doesn't restructure today with idiosyncratic state vector $\mathbf{s}$ and investment-financing choice $(k', b')$ is

$$\begin{aligned}
Q\left(\mathbf{s},k^{'},b^{'}\right) = {} & \left[1-\mathbf{1}_{\{\text{ch7}\}}\left(z,n,s^{n}\right)\right]\left(1-\gamma\right)\Big\{\mathbb{E}_{\left(z^{'}|z;\omega^{'};s^{n'}\right)}\left[\Lambda\mathbf{1}_{\{\text{cont}\}}\left(\mathbf{s}^{'}\right)\right] \\
& +\mathbb{E}_{\left(z^{'}|z;\omega^{'};s^{n'}\right)}\left[\Lambda\mathbf{1}_{\{\text{ch11}\}}\left(\mathbf{s}^{'}\right)\alpha_{11}\left(z^{'},\omega^{'},b^{'},k^{'}\right)\right] \\
& +\mathbb{E}_{\left(z^{'}|z;\omega^{'};s^{n'}\right)}\left[\Lambda\tilde{\mathbf{1}}_{\{\text{ch7}\}}\left(\mathbf{s}^{'}\right)R\left(b^{'},k^{'},\omega^{'}\right)\right]\Big\} \\
& +\left[1-\mathbf{1}_{\{\text{ch7}\}}\left(z,n,s^{n}\right)\right]\gamma\tilde{Q}_{\text{exit}}\left(z,k^{'},b^{'}\right),
\end{aligned} \tag{5}$$

with

$$\begin{aligned}
\tilde{Q}_{\text{exit}}\left(z,k^{'},b^{'}\right) = {} & \mathbb{E}_{\left(z^{'}|z;\omega^{'}\right)}\left[\Lambda\mathbf{1}_{\{\text{cont}\,|\,\text{exit}\}}\left(\mathbf{s}^{'}\right)\right] \\
& +\mathbb{E}_{\left(z^{'}|z;\omega^{'}\right)}\left[\Lambda\mathbf{1}_{\{\text{ch11}\,|\,\text{exit}\}}\left(\mathbf{s}^{'}\right)\alpha_{11}^{\text{exit}}\left(z^{'},\omega^{'},b^{'},k^{'}\right)\right] \\
& +\mathbb{E}_{\left(z^{'}|z;\omega^{'}\right)}\left[\Lambda\mathbf{1}_{\{\text{ch7}\,|\,\text{exit}\}}\left(\mathbf{s}^{'}\right)R\left(b^{'},k^{'},\omega^{'}\right)\right],
\end{aligned} \tag{6}$$

where the indicator function $\mathbf{1}_{\{\text{ch7}\}}\left(z,n,s^{n}\right)$ indicates if the firm is liquidated today given that it doesn't restructure, and indicators $\tilde{\mathbf{1}}_{\{\text{ch7}\}}\left(\mathbf{s}^{'}\right)$, $\mathbf{1}_{\{\text{ch11}\}}\left(\mathbf{s}^{'}\right)$, and $\mathbf{1}_{\{\text{cont}\}}\left(\mathbf{s}^{'}\right)$ indicate if the firm is liquidated, restructures, or continues in equilibrium the next period, respectively. Notice that $\mathbf{1}_{\{\text{cont}\}}\left(\mathbf{s}^{'}\right)+\mathbf{1}_{\{\text{ch11}\}}\left(\mathbf{s}^{'}\right)+\tilde{\mathbf{1}}_{\{\text{ch7}\}}\left(\mathbf{s}^{'}\right)=1$ conditional on not receiving the exit shock.[11] Furthermore, the indicators conditioned on the firm receiving the exit shock — i.e., indicator functions $\{\mathbf{1}_{\{\text{cont} \mid \text{exit}\}}\left(\mathbf{s}'\right), \mathbf{1}_{\{\text{ch11} \mid \text{exit}\}}\left(\mathbf{s}'\right), \mathbf{1}_{\{\text{ch7} \mid \text{exit}\}}\left(\mathbf{s}'\right)\}$ — are defined analogously. The bankruptcy choices for firms that do not receive an exit shock are characterized in Section 2.4 and the bankruptcy choices for firms that receive an exit shock are characterized in Appendix A.2.1.

The no-profit condition implies that the debt pricing schedule depends on both the default choice today and the probability of default in the next period. The contemporaneous liquidation decision shows up in the debt pricing because of the CK timing — i.e., within the period, the liquidation choice happens after the new debt issuance — and this is the source of potential multiplicity as explained in the following Section. In the RHS of (5), the first line shows the component of the price when the firm repays fully next period, the second line shows the component of the price when the firm restructures next period and creditors recover $\alpha_{11}\left(z', \omega', b', k'\right)$, and the third line shows the component of the price when the firm is liquidated next period and creditors recover $R\left(b', k', \omega'\right)$. The last line shows the component of the price when the firm receives the exogenous exit shock next period, and equation (6) shows the determinants of the debt price when the firm receives the exogenous exit shock next period and is not liquidated today. The description of the pricing schedule when the firm receives the exogenous exit shock tomorrow is derived analogously to the pricing function for non-exiting firms.

For the firms that decide to restructure, the creditors offer the same debt price schedule but conditional on $\mathbf{1}_{\{\text{ch7}\}}\left(z, n, s^n\right) = 0$ . Thus, it is useful to define the *fundamental debt price* $\tilde{Q}\left(z, b', k'\right)$ as the price of debt whenever there is no contemporaneous liquidation. This price is then determined solely by future default probabilities, recovery rates, and creditors' discounting, as in standard endogenous default models, i.e.,

$$Q\left(\mathbf{s}, k', b'\right) = \left[1 - \mathbf{1}_{\{\text{ch7}\}}\left(z, n, s^n\right)\right] \tilde{Q}\left(z, b', k'\right). \tag{7}$$

[11] As a technical note, the liquidation choice doesn't depend on $(k', b')$ because the policy function imposes a constraint redundant to $d \geq 0$ for the financing-investment choice, creditors of $b'$ have 0 recovery rate, and there is no within period uncertainty after shocks are realized at the beginning of the period.

## 2.4 Liquidation and Restructuring

In this subsection, I characterize the liquidation and restructuring choices. I characterize the choices backwards since the payoffs of the liquidation choice will affect incentives to restructure, within the period. First, I show that there can be multiple equilibria for a firm, then I characterize the solution of the liquidation choice and find that there are three endogenous types of firms across the state space: firms safe from (current) rollover crises, firms exposed to rollover crises, and fundamentally insolvent firms. Finally, I characterize which firms enter the restructuring process.

### 2.4.1 Liquidation

**Multiple equilibria intuition.** Firms are subject to the no-equity issuance constraint $d \geq 0$ and their exiting value is $V^{\text{exit}} = 0$. Thus, the firm decides to liquidate and exit if it cannot issue (weakly) positive dividends. Creditors price the debt according to equation (5). The pricing schedule shows that debt prices depend on expectations of future default and conjectures about the firm's liquidation choice today. The debt price schedule and dividends are

$$Q\left(\mathbf{s}, k^{'}, b^{'}\right) = \underbrace{\mathbf{1}_{d\geq 0}}_{\text{no liquidation choice}} \quad \underbrace{\tilde{Q}\left(z, k^{'}, b^{'}\right)}_{\text{debt price if no liquidation}}$$

$$d = n - qk^{'} + Q\left(\mathbf{s}, k^{'}, b^{'}\right),$$

where $\mathbf{1}_{d\geq 0} = 0$ indicates that the firm cannot satisfy $d \geq 0$. There is feedback between dividends and current debt prices, which could generate multiple equilibria. To illustrate this, assume the firm has negative cash-on-hand $n < 0$ and creditors conjecture liquidation today $\mathbf{1}_{d\geq 0} = 0$. Then they offer price $Q\left(\mathbf{s}, k^{'}, b^{'}\right) = 0$ and the firm cannot satisfy the constraint on dividends, since $d = n + \max_{k^{'}\geq 0}\left\{-qk^{'}\right\} < 0$, which implies that the firm is liquidated. Thus, the outcome is consistent with the conjecture. On the contrary, if creditors conjecture that there is no liquidation today, then $Q\left(\mathbf{s}, k^{'}, b^{'}\right) = \tilde{Q}\left(z, k^{'}, b^{'}\right)$, and assume it exists $b^{'}$ such that $\tilde{Q}\left(z, k^{'}, b^{'}\right) b^{'} > n$, then the firm can satisfy $d \geq 0$ and it is not liquidated. Thus, this outcome is also consistent with the conjecture. Therefore, under certain conditions—which I show next—the outcomes could depend on creditors' conjectures of the current liquidation choice. Creditors are atomistic, identical and move simultaneously, so they could coordinate on either debt price.[12] I adopt the convention that if creditors coordinate on $Q\left(\mathbf{s}, k', b'\right) = 0$, then they coordinate on the rollover crisis equilibrium for a firm.

**Proposition 1.** *The state-space $(z, n)$ can be divided into three regions that depend on the firms' liquidation choice:*

1. *Safe region $\mathcal{S}$: if $n \geq 0$ then $\forall z$ the firm does not liquidate and exit,*
2. *Liquidation region $\mathcal{L}$: if $n < \underline{n}(z)$ then the firm does liquidate and exit,*
3. *Risky region $\mathcal{R}$: if $n \in [\underline{n}(z), 0)$ then the firm equilibrium is undetermined. There is a repayment equilibrium in which the debt price is positive and a rollover crisis equilibrium in which the debt price is zero,*

*where $\underline{n}(z) = -\max_{k',b'}\left\{\tilde{Q}\left(z, k', b'\right) b' - qk'\right\}$. The proof is in Appendix A.1.1.*

**Characterization.** Proposition 1 shows that the *fundamental* state-space $(z, n)$ can be divided into three regions: a safe region $\mathcal{S}$, an insolvency region $\mathcal{L}$, and a risky region $\mathcal{R}$. Firms in the region $\mathcal{S}$ will repay independently of their creditors' coordination. The intuition is that firms with positive internal resources ($n \geq 0$) do not need external resources to rollover their debt. On the other extreme, firms in $\mathcal{L}$ will liquidate and exit independently of their creditors' coordination. The intuition is that firms with very weak fundamentals cannot get enough external resources even if creditors were willing to provide credit today. Finally, firms in the intermediate region $\mathcal{R}$ will repay if creditors coordinate in the repayment equilibrium and will liquidate and exit if creditors coordinate in the rollover crisis equilibrium (as previously explained).

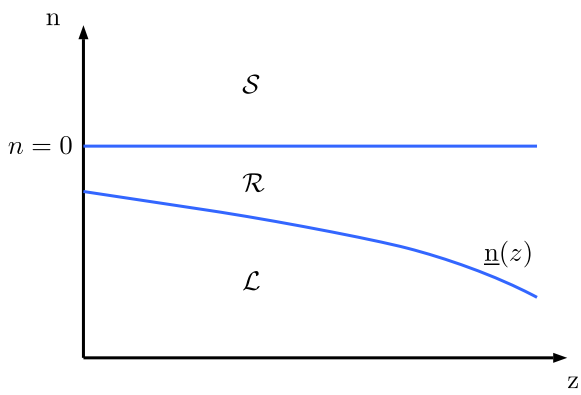
Figure 2 illustrates the regions in the state-space according to Proposition 1. The threshold $\underline{n}(z)$ is weakly decreasing in $z$ if the fundamental debt schedule $\tilde{Q}\left(z, k', b'\right)$ is weakly increasing in $z$.

To construct the equilibrium in region $\mathcal{R}$, I define an *idiosyncratic* and stochastic sunspot variable $\phi \sim [0, 1]$, iid across time and firms, that is drawn at the beginning of every

[12] The atomicity assumption is not necessary for coordination failures to happen. For example, Halac, Kremer and Winter (2020); Chaumont, Gordon and Sultanum (2023) show that financing coordinations failures can happen even with non atomistic and heterogeneous investors.

period. The sunspot variable will be the non-fundamental state variable for the firm problem, then $s^n = \phi$. I assume that creditors of firms in $\mathcal{R}$ coordinate on the rollover crisis equilibrium whenever $\phi \leq \eta$, where $\eta$ is a parameter *common* across firms. The parameter $\eta$ is the probability of a rollover crisis for an exposed firm. In the aggregate, the rollover risk likelihood will depend on the distribution of firms, which is endogenous, and the parameter $\eta$. Thus, the identification and quantification of $\eta$ is central for the main quantitative results of the paper.

**Figure 2:** Rollover and solvency: regions across $(z, n)$ state-space




Notes: the figure shows the state-space $(z, n)$ and the relevant regions for the liquidation choice.

**Solvency and insolvency.** The liquidation choice for firms that do not restructure (i.e., the choice illustrated in the second dot in Figure 1 from left to right) is

$$\underbrace{\mathbf{1}_{\{\text{ch7}\}}(z, n, \phi)}_{\text{insolvent}} = \underbrace{\mathbf{1}_{\{n<\underline{\mathrm{n}}(z)\}}}_{\text{fundamentally insolvent}} + \underbrace{\mathbf{1}_{\{n\in[\underline{\mathrm{n}}(z),0) \text{ and } \phi<\eta\}}}_{\text{rollover crisis}}.$$

The characterization of the liquidation choice provides a clear distinction between firms that are *fundamentally insolvent* [i.e., $(z, n) \in \mathcal{L}$] and firms that are *fundamentally solvent* [i.e., $(z, n) \in \mathcal{R} \cup \mathcal{S}$]. Within the group of fundamentally solvent firms, there are firms that are *solvent* [i.e., $\{(z, n) \in \mathcal{R}$ and $\phi > \eta\}$ or $(z, n) \in \mathcal{S}$] and those that are subject to a *rollover crises* [i.e., $(z, n) \in \mathcal{R}$ and $\phi \leq \eta$]. I define the set of *insolvent* firms as the complement of the set of solvent firms, then insolvent firms are either fundamentally insolvent or under a rollover crises. In Appendix A.2.1, I characterize the liquidation choice for the firms that receive the exogenous exit shock.

In addition, in Appendix C.1, I characterize the liquidation choice for various extensions of the model, such as long-term debt, costly equity issuance, and others, and find that firms' could be subject to rollover crises and their cash-on-hand determines the exposure to them, as in the baseline framework.

In this section, I have characterized the liquidation choice of the firms across the relevant state-space, but due to the timing convention of the model (see the timing in Figure 1) some of the insolvent firms may not be liquidated in equilibrium. Thus, in the next Section, I characterize the restructuring choice to determine which insolvent firms liquidate.

### 2.4.2 Restructuring

**Characterization.** When firms choose to enter the restructuring process, firms and creditors bargain over the debt recovery rate $\alpha_{11} \in [0,1]$ of debt $b$. To participate in the restructuring process, both creditors and the firm must be better off than in their outside option (i.e., they must both have a positive surplus). I assume that the renegotiation follows a standard Nash Bargaining protocol.

**Assumption 1.** *The outside option of the debt renegotiation problem is to continue as if the firm never filed for restructuring.*

Assumption 1 implies that creditors of firms that are solvent get fully repaid in the outside option. Moreover, the recovery rate is $\alpha_{11} \in [0,1]$, then creditors are (weakly) better off if they do not participate of the restructuring process and the firm continues. Thus, solvent firms cannot restructure their debt since creditors would not participate in the restructuring process. Additionally, we assume that firms pay a fixed cost if they restructure and there are no TFP losses/gains from restructuring.[13]

Let $V(z,n)$ be the value of a solvent firm when making the financing-investment choice (this will be defined in the firm's problem in Section 2.5), then the Nash Bargaining protocol for an insolvent firm is

$$\alpha_{11}(z,\omega,b,k) \equiv \min\left\{1, \arg\max_{\alpha_{11}\in[0,1]} \left[V(n_{11},z) - 0\right]^{1-\Xi} \left[b\alpha_{11} - bR(b,k,\omega)\right]^{\Xi}\right\} \tag{8}$$

[13]This assumption is in line with the evidence in Maksimovic and Phillips (1998) regarding the TFP dynamics of firms that restructure through Chapter 11.

subject to

$$n_{11} = \pi\left(z,\omega,k\right) + \left(1-c_{11}\right)\left(1-\delta\right)q\omega k - \alpha_{11}b \geq \underline{\mathrm{n}}\left(z\right) \tag{9}$$

$$\alpha_{11} \geq R\left(b,k,\omega\right). \tag{10}$$

where $\Xi \in (0,1)$ is the bargaining power of creditors. By definition, insolvent firms are liquidated if they don't restructure, then their outside option is $V^{\text{exit}} = 0$. Constraint (9) shows that firms will participate if they are fundamentally solvent after restructuring (i.e., $n_{11} \geq \underline{\mathrm{n}}(z)$), constraint (10) shows that creditors will participate if they recover more than under liquidation, and equation (8) shows that the Nash Bargaining protocol is a sharing rule of the firms' and creditors' surpluses. For the bargaining process to be feasible, we need that the maximum recovery rate that the firm is willing to pay is greater than the minimum recovery rate that creditors are willing to accept, i.e.,

$$\alpha_{11}^{\max}\left(z,\omega,k,b\right) > \alpha_{11}^{\min}\left(\omega,k,b\right),$$

where

$$\alpha_{11}^{\max}\left(z,\omega,k,b\right) = \min\left\{1, \frac{\pi\left(z,\omega,k\right) + \left(1-c_{11}\right)\left(1-\delta\right)q\omega k - \underline{\mathrm{n}}\left(z\right)}{b}\right\}$$
$$\alpha_{11}^{\min}\left(\omega,k,b\right) = R\left(b,k,\omega\right).$$

Now I can characterize the restructuring choice of the firms. Letting $\mathbf{s} = (z,\omega,\phi,k,b)$ denote the idiosyncratic states of the firm, then the restructuring choice is

$$\underbrace{\mathbf{1}_{\{\text{ch11}\}}\left(\mathbf{s}\right)}_{\text{restructure}} = \underbrace{\mathbf{1}_{\{\text{ch7}\}}\left(z,n,\phi\right)}_{\text{insolvent}} \times \underbrace{\mathbf{1}_{\left\{\alpha_{11}^{\max} > \alpha_{11}^{\min}\right\}}\left(z,\omega,k,b\right)}_{\text{renegotiation feasible}}, \tag{11}$$

thus the firms that restructure are those which are insolvent, either fundamentally insolvent or had a rollover crises, for which the negotiation is feasible.

**Liquidation in equilibrium.** From the characterization of the restructuring choice it follows that the firms that are liquidated *in equilibrium* are insolvent firms which cannot successfully restructure their debt, i.e.,

$$\underbrace{\tilde{\mathbf{1}}_{\{ch7\}}(\mathbf{s})}_{\text{liquidated in equilibrium}} = \underbrace{\left(1-\mathbf{1}_{\{ch11\}}(\mathbf{s})\right)}_{\text{don't restructure}} \times \underbrace{\mathbf{1}_{\{ch7\}}(z,n,\phi)}_{\text{insolvent}}. \tag{12}$$

Finally, the firms that continue without going through the bankruptcy process (restructuring or liquidation) are the solvent ones, i.e.,

$$\underbrace{\mathbf{1}_{\{cont\}}(\mathbf{s})}_{\text{don't file for bankruptcy}} = 1-\mathbf{1}_{\{ch11\}}(\mathbf{s})-\tilde{\mathbf{1}}_{\{ch7\}}(\mathbf{s}) = \underbrace{1-\mathbf{1}_{\{ch7\}}(z,n,\phi)}_{\text{solvent}}. \tag{13}$$

The characterization of the restructuring and liquidation choices of exiting firms is straightforward and it is relegated to Appendix A.2.1.

## 2.5 Firms' Problem

Given the structure of the problem, I can write the firm problem in the recursively. The idiosyncratic states of the firm at the beginning of the period are $\mathbf{s}=(z,\omega,\phi,b,k)$, where the fundamental exogenous states are $\mathbf{s}^f=(z,\omega)$, the non-fundamental exogenous state is $s^n=\phi$, and endogenous states are $\mathbf{s}^e=(b,k)$.

Let $V(z,n)$ be the value of a solvent firm when making the investment-financing decision, let $\tilde{V}(\mathbf{s})$ be the value of a firm that didn't receive the exit shock, and let $V_{\text{exit}}(\mathbf{s})$ be the value of a firm that received an exit shock. Thus, the investment-financing problem for a solvent firm is

$$V(z,n) = \max_{d,k',b'} d + \mathbb{E}_{(z'|z;\omega';\phi')}\left[\Lambda\left\{(1-\gamma)\tilde{V}\left(\mathbf{s}'\right)+\gamma V_{\text{exit}}\left(\mathbf{s}'\right)\right\}\right] \tag{14}$$

subject to

$$d = n - qk' + \tilde{Q}\left(z,b',k'\right)b' \geq 0$$
$$\mathbf{s}' = \left(z',\omega',\phi',b',k'\right)$$

where the continuation value $\tilde{V}(s)$ and $V_{\text{exit}}(s)$ are

$$\tilde{V}(\mathbf{s}) = \mathbf{1}_{\{ch11\}}(\mathbf{s})V(z,n_{11}) + \mathbf{1}_{\{cont\}}(\mathbf{s})V(z,n)$$

$$V_{\text{exit}}(\mathbf{s}) = \mathbf{1}_{\{\text{ch11}|\text{exit}\}}(\mathbf{s})\, n_{11|\text{exit}} + \mathbf{1}_{\{\text{cont}|\text{exit}\}}(\mathbf{s})\, n$$

with

$$\begin{aligned} n &= \pi(z,\omega,k) + (1-\delta)\, q\omega k - b \\ n_{11} &= \pi(z,\omega,k) + (1-c_{11})(1-\delta)\, q\omega k - \alpha_{11}(z,\omega,k,b)\, b \\ n_{11|\text{exit}} &= \pi(z,\omega,k) + (1-c_{11})(1-\delta)\, q\omega k - \alpha_{11|\text{exit}}(z,\omega,k,b)\, b, \end{aligned}$$

where the fundamental debt price $\tilde{Q}(z, b', k')$ is defined in Section 2.3, the indicator functions $\mathbf{1}_{\{.\}}(\mathbf{s})$ are described in Section 2.4 and the indicator functions $\mathbf{1}_{\{.|\text{exit}\}}(\mathbf{s})$ are described in Appendix A.2.1, $\alpha_{11}(z,\omega,k,b)$ solves problem (8) which determines the restructuring firm's cash-on-hand $n_{11}$, and the recovery rate $\alpha_{11|\text{exit}}(z,\omega,k,b)$ solves problem (22) described in Appendix A.2.1 and determines the exiting firm's cash-on-hand $n_{11|\text{exit}}$. The investment-financing policy functions $\{b'(z,n), k'(z,n)\}$ solve problem (14).

It is easy to see that the liquidation choice's policy function is redundant since it imposes the same constraint as $d \geq 0$, so it is excluded from the recursive formulation of the firm's problem. Finally, the firms that successfully restructure their debt also solve problem (14), but with cash-on-hand $n = n_{11}$.

## 2.6 Capital Producers and Households

To close the model, I describe the problem of the representative capital producer that sells capital to firms, and the representative household that owns all firms, works, consumes the final good, and saves.

### 2.6.1 Capital Producers.

There is a representative aggregate capital producer that solves

$$\max_{I} q\Phi\left(\frac{I}{K}\right) - I$$

where $I$ is the amount of final goods used to produce capital, $K$ is the aggregate capital stock, and $\Phi$ is the aggregate capital adjustment cost function. The first order condition of the problem is such that

$$q = \frac{1}{\Phi'\left(\frac{I}{K}\right)} \tag{15}$$

where $q$ is the price of capital. The time-varying price of capital $q$ changes the recovery rate $\mathcal{R}$ which impacts debt prices, and thus maps to the financial accelerator mechanism (Bernanke, Gertler and Gilchrist, 1999) in the transitions periods after an aggregate shock. I assume a standard functional form such that $\Phi'\left(\frac{I}{K}\right) = \left[\frac{I/K}{\hat{I}}\right]^{-\psi}$ where $\hat{I}$ is the steady-state investment to capital ratio.

### 2.6.2 Households.

There is a unit mass of identical households that consume $C$ and make labor decision $L$ taking wages $w$ and risk-free interest rate $r$ as given. Households own all the firms in the economy. The one-period stochastic discount factor $\Lambda$ is defined as the ratio of today's and tomorrow's marginal utilities of $C$ in present terms, the Euler Equation holds, and the optimal labor-leisure choice is determined by the marginal rate of substitution, i.e.,

$$\Lambda' = \beta \frac{U_C\left(C', L'\right)}{U_C\left(C, L\right)} \tag{16}$$

$$1 = E\left[\beta \frac{U_C\left(C', L'\right)}{U_C\left(C, L\right)}\left(1+r\right)\right] \tag{17}$$

$$w = -\frac{U_L\left(C, L\right)}{U_C\left(C, L\right)} \tag{18}$$

with quasi-linear utility function $U\left(C, L\right) = \ln C - \Phi L$.

## 2.7 Equilibrium

I define the equilibrium for this economy in the steady state and in the transition to an unexpected temporary aggregate shock.

**Law of motion of the distribution of firms.** Before defining the equilibrium, I need to characterize the law of motion of the distribution of firms. I will characterize the law of motion in steady state. The law of motion of the distribution in the transition path are characterized analogously. Let $\Omega(z, n)$ be the distribution of producing firms that has a total mass of 1 in steady state, $\tilde{\Omega}\left(\mathbf{s}\right)$ the distribution of incumbent firms at the beginning of the period, $g(\omega)$ and $\hat{g}(\phi)$ the pdfs of $\omega$ and $\phi$ respectively, $p(\epsilon_z \mid z')$ the conditional pdf of the productivity shocks $\epsilon_z$, and $\Omega^e(z)$ the distribution of $z$ for entrant firms. The distribution of firms that produce is determined by

$$\Omega(z,n) = \Omega^s(z,n) + \Omega^f(z,n), \tag{19}$$

where $\Omega^s(z,n)$ is the distribution of firms that produce and continue to the next period, which is determined by

$$\begin{aligned}\Omega^s(z,n) = (1-\gamma)\int_{(\omega,\phi,k,b)} &\left[\mathbf{1}_{\{\text{ch11}\}}(\mathbf{s})\,\mathbf{1}_{\{z,n_{11}(z,\omega,k,b)=n\}} + \mathbf{1}_{\{\text{cont}\}}(\mathbf{s})\,\mathbf{1}_{\{z,n(z,\omega,k,b)=n\}}\right] \mathrm{d}\tilde{\Omega}(\mathbf{s})\\ &+ \bar{\mu}(1-\gamma)\int_{(\omega,\phi)} \left[\mathbf{1}_{\{\text{ch11}\}}(\mathbf{s})\,\mathbf{1}_{\{z,n_{11}(z,\omega,k_0,0)=n\}}\right.\\ &\left.+\ \mathbf{1}_{\{\text{cont}\}}(\mathbf{s})\,\mathbf{1}_{\{z,n(z,\omega,k_0,0)=n\}}\right] \hat{g}(\phi)\,g(\omega)\,\mathrm{d}\phi\mathrm{d}\omega\mathrm{d}\Omega^e(z),\end{aligned}$$

and $\Omega^f(z,n)$ is the distribution of firms that produce and exit, which is determined by

$$\begin{aligned}\Omega^f(z,n) = \gamma\int_{(\omega,\phi,k,b)} &\left[\mathbf{1}_{\{\text{ch11}|\text{exit}\}}(\mathbf{s})\,\mathbf{1}_{\{z,n_{11}^{\text{exit}}(z,\omega,k,b)=n\}} + \mathbf{1}_{\{\text{cont}|\text{exit}\}}(\mathbf{s})\,\mathbf{1}_{\{z,n(z,\omega,k,b)=n\}}\right] \mathrm{d}\tilde{\Omega}(\mathbf{s})\\ &+ \bar{\mu}\gamma\int_{(\omega,\phi)} \left[\mathbf{1}_{\{\text{ch11}|\text{exit}\}}(\mathbf{s})\,\mathbf{1}_{\{z,n_{11}^{\text{exit}}(z,\omega,k_0,0)=n\}}\right.\\ &\left.+\ \mathbf{1}_{\{\text{cont}|\text{exit}\}}(\mathbf{s})\,\mathbf{1}_{\{z,n(z,\omega,k_0,0)=n\}}\right] \hat{g}(\phi)\,g(\omega)\,\mathrm{d}\phi\mathrm{d}\omega\mathrm{d}\Omega^e(z).\end{aligned}$$

Finally, the distribution of incumbent firms $\tilde{\Omega}(\mathbf{s})$ at the beginning of the period evolves according to

$$\tilde{\Omega}\left(\mathbf{s}'\right) = \int_{(z,n)} \mathbf{1}_{\{k'(z,n)=k'\}}\mathbf{1}_{\{b'(z,n)=b'\}}\hat{g}\left(\phi'\right) g\left(\omega'\right) p\left(\epsilon_z \mid \rho_z z + \epsilon_z = z'\right) \mathrm{d}\Omega^s(z,n). \tag{20}$$

**Steady State Equilibrium.** The *steady-state equilibrium* in this economy is defined as a set of value functions $\left\{V(z,n), \tilde{V}(s)\right\}$, the firm's decision rules of capital purchases and new debt issuance $\left\{k'(z,n), b'(z,n)\right\}$, bankruptcy decisions for firms which have not received the exit shock $\left\{\mathbf{1}_{\{\text{ch11}\}}(\mathbf{s}), \tilde{\mathbf{1}}_{\{\text{ch7}\}}(\mathbf{s}), \mathbf{1}_{\{\text{ch7}\}}(z,n,\phi)\right\}$ and for firms which received the exit shock $\left\{\mathbf{1}_{\{\text{ch11}|\text{exit}\}}(\mathbf{s}), \mathbf{1}_{\{\text{ch7}|\text{exit}\}}(\mathbf{s})\right\}$, aggregates $\{Y, C, I\}$, the corporate debt price schedule $Q\left(\mathbf{s}, b', k'\right)$, fundamental corporate debt price schedule $\tilde{Q}(z,k',b')$, interest rate $r$, prices $\{q, w\}$, distributions $\Omega(z,n)$ and $\tilde{\Omega}(\mathbf{s})$, and debt haircuts $\{\alpha_{11}(z,\omega,b,k)\}$ in the restructuring process such that:

1. Household choices are determined by (16), (17) and (18).

2. The price of capital is determined by the solution to (15).

3. The debt price satisfies (5) and $Q\left(\mathbf{s}, b^{'}, k^{'}\right) = \left[1 - \mathbf{1}_{\{\text{ch7}\}}\left(z, n, \phi\right)\right] \tilde{Q}\left(z, b^{'}, k^{'}\right)$ defines implicitly the fundamental price $\tilde{Q}$.

4. Given prices, the firm's decision rules solve the firm problem for firms that produce (14), continuing firms' bankruptcy decisions are consistent with (11)–(13) and exiting firms bankruptcy decisions are consistent with equations (21) and (23) in Appendix A.2.1, and the recovery rates are solved by negotiation protocols (8) and (22).

5. Markets clear: investment is implicitly determined by the law of motion

$$K^{'} = \varPhi\left(I/K\right)K + \left(1-\delta\right)K - \left(k_0 - \left(1-\delta\right)\mathbb{E}\left[\omega\right]k_0\right)\bar{\mu}$$

   with $K = \int k \mathrm{d}\tilde{\Omega}\left(s\right)$ and aggregate resource constraint is

$$C = Y - I - \mu_{11}$$

   where $\mu_{11}$ is the aggregate cost of firms filing to Chapter 11.

6. The distribution of firms that produce $\Omega(z, n)$ and incumbents before the bankruptcy choice $\tilde{\Omega}\left(\mathbf{s}\right)$ satisfy (19) and (20).

In steady state, the distribution's laws of motions imply a fixed point, the household's stochastic discount factor is $\Lambda = \beta = \frac{1}{1+r}$ and the capital price is $q = 1$ since $K = K'$.

**Transitions.** The equilibrium in the perfect foresight transition path from an unexpected aggregate shock is defined analogously. I focus on temporary shocks that eventually converge to the initial steady state (the one before the shock). Moreover, I assume that the mass of entrants is fixed over the transition, which allows for the measure of firms that operate $\Omega_t(z, n)$ to change along the transition path. This happens since the mass of firms exiting endogenously is changing after the shock.

## 2.8 Discussion of Assumptions

*Liability structure.* In the model, I assume that firms borrow from several creditors (i.e., atomistic creditors) and use short-term financing. In Appendix B.2.2, I show that, in the data, U.S. firms tend to borrow from several creditors, especially large corporate firms. Bankruptcy data from the Federal Judicial Center's (FJC) Integrated Database shows that 85% of firms with assets over $50 million (which represent more than 90% of total sales in the manufacturing sector) have more than a hundred creditors, and two thirds of them have over a thousand creditors. Furthermore, it is well documented that corporate firms' financial debt is mostly composed by corporate debt (rather than bank loans). Thus, corporate firms' liabilities tend to have a dispersed ownership, which could exacerbate coordination problems among creditors. Moreover, in Appendix B.2.2, I show that an average firm in Compustat has a large fractions of its debt maturing in the short term. On average, around one third of financial debt matures in less than 1 year and more than half of all liabilities are due in less than 1 year. For quantitative purposes, in the calibration in Section 3.1, I will abstract from long-term debt financing and match moments using the firms' liabilities that mature in the short term.

Furthermore, I do not allow firms to manage their liability structure, such as extending their maturity to reduce short-term payments or concentrating their creditors to preclude coordination failures. In Appendix A.3.6, I show that, for the baseline steady state calibration, ex-ante costs of firm's rollover risk are negligible for most firms and, when comparing to the costs of managing the firm's liabilities typically used in the literature, most firms would not modify their liability structure (in steady state) even if allowed.

Finally, in Appendix C.1, I characterize the liquidation choice for various extensions of the model, such as long-term debt, costly equity issuance, and others, and find that there can be a region of multiplicity as in the baseline framework. In this alternative setups, the liquidation value of the firm's assets still determines the firm's vulnerability to rollover crises.

*Coordination failures and the restructuring process.* I assume that firms in the restructuring process benefit from the removal of coordination failures (i.e., $Q = \tilde{Q}$) during the restructuring process. This assumption captures various provisions of the Chapter 11 process aimed at resolving coordination issues among creditors. Ayotte and Skeel (2013) observe that "the dominant normative theory of bankruptcy" [see for example, Jackson (1986)] states that the sole purpose of bankruptcy provisions are to solve "coordination problems caused by multiple creditors." In this spirit, the restructuring process chapter, Chapter 11 of the U.S. bankruptcy, for example, temporarily prevents creditors from individually collecting their debt (Automatic Stay provision, 11 U.S. Code § 362), allows firms to issue new debt and continue operating (usually known as Debtor-In-Possession (DIP) financing), and arranges official and ad-hoc committees of creditors.

# 3 Identifying Firms' Rollover Crises

The incidence of rollover crises depends on how many firms are exposed to them (i.e., share of firms in region $\mathcal{R}$) and the likelihood that exposed firms are subject to a rollover crises (i.e., value of $\eta$). I cannot observe directly how many firms are exposed, nor the conditional probability of a rollover crisis, so I use a model-based identification strategy to infer them indirectly.

In this section, I describe the identification strategy and estimate the incidence of rollover crises in steady state. Furthermore, I validate the model by comparing the model simulated data with the empirical evidence regarding investment heterogeneity in the past two large recessions and firm-level bankruptcy predictors.

**Identification strategy.** To estimate the incidence of rollover crises, first, I fix a set of parameters to standard values in the literature and calibrate the parameters unrelated to the bankruptcy process to fit several moments of the U.S. economy. Next, I infer the value of $\eta$, and other bankruptcy parameters, using bankrupt firms' characteristics and firms' bankruptcy choices. Finally, using the steady-state distribution of firms I determine how many firms are exposed.

## 3.1 Standard Calibration

Now, I focus on parameters and moments unrelated to the bankruptcy process. The standard calibration consists of 9 fixed parameters and 4 fitted parameters. To evaluate the empirical fitness of the model I contrast the moments in the model to a wide range of moments (16) in the data. The calibration is done at a quarterly frequency. I use national accounts data from NIPA, firms' balance sheet micro data from Compustat, firms' life-cycle data from the Longitudinal Business Database (LBD), and moments computed in other papers. Further details on the data sources, samples and definitions are included in Appendix B.

**Table 1:** Standard calibration

| Parameter | Value | Calibration |
|---|---|---|
| *a. fixed* | | |
| $\beta = 1/(1+r)$ | 0.99 | fixed to $r = 0.05$ annual |
| $\Phi$ | 1.16 | fixed to match 58% emp rate |
| $\nu$ | 0.64 | fixed labor share |
| $\alpha$ | 0.21 | fixed capital share |
| $\delta$ | 0.025 | fixed to match BEA quarterly data |
| $\rho_z$ | 0.90 | fixed |
| $b_0$ | 0 | fixed |
| $\gamma$ | 0.02 | fixed to 10% annual firm exit rate |
| $\psi$ | 1/4 | fixed to standard values in literature |
| *b. fitted* | | |
| $\sigma_z$ | 0.032 | internally calibrated |
| $\underline{\omega}$ | −0.33 | internally calibrated |
| $k_0$ | 0.16 | internally calibrated |
| $m$ | −0.24 | internally calibrated |

**Fixed parameters.** Panel (a) in Table 1 shows the value of fixed parameters. The subjective discount rate $\beta = 0.99$ is set to fit an annual real interest rate of 5%. The labor disutility parameter $\Phi = 1.16$ is set to match an employment rate of 58%. The parameters of the production function scale of production $\nu = 0.21$ and $\alpha = 0.64$ are set to fit the labor and the capital share, respectively; and the capital depreciation rate of $\delta = 0.025$ is set to match estimates from BEA. Following Ottonello and Winberry (2020), I fix the persistence of the idiosyncratic productivity process to $\rho = 0.9$. Further, I fix the initial inherited debt to $b_0 = 0$ and exogenous exit rate to $\gamma = 0.02$ to fit the annual exit rate of 10%. Finally, I fix the aggregate capital adjustment cost parameter $\psi = 1/4$ to a standard value in the literature.

**Table 2:** Standard calibration's moments

| Moment | Data | Model | Data source |
|---|---|---|---|
| *Aggregates* | | | |
| K/Y | 3.00 | 2.59 | NIPA |
| I/Y | 0.17 | 0.15 | NIPA |
| *Credit spreads* | | | |
| default rate: $\mathbb{E}[\tilde{\mathbf{1}}_{\{\text{ch7}\}} + \mathbf{1}_{\{\text{ch11}\}}]$ | 0.03 | 0.03 | Annual rate from Dun and Bradstreet |
| cred spread: $\mathbb{E}[r^Q - r]$ | 2.2% | 0.7% | Moody's BAA corporate bonds |
| *Investment heterogeneity* | | | |
| average investment rate: $\mathbb{E}[i/k]$ | 0.12 | 0.20 | Cooper and Haltiwanger (2006) |
| SD investment rate: $\text{SD}[i/k]$ | 0.34 | 0.36 | Cooper and Haltiwanger (2006) |
| *Life-cycle* | | | |
| share of firms that exit | 0.10 | 0.11 | LBD |
| share of labor at age 1 | 0.03 | 0.04 | LBD |
| share of firms at age 1 | 0.10 | 0.11 | LBD |
| share of firms at age 2 | 0.08 | 0.09 | LBD |
| *Balance sheet* | | | |
| average leverage: $\mathbb{E}[1_{b>0} b'/k']$ | 0.39 | 0.72 | Compustat |
| fraction of firms with $\frac{n}{k'} < 0$ | 0.21 | 0.18 | Compustat |
| fraction of firms with $\frac{n}{k'} \in [0, 1]$ | 0.65 | 0.77 | Compustat |
| fraction of firms with $\frac{n}{k'} > 1$ | 0.15 | 0.05 | Compustat |

**Fitted parameters and moments.** Panel (b) in Table 1 shows the value of fitted parameters unrelated to the bankruptcy process. The fitted parameters are the volatility of idiosyncratic productivity shocks $\sigma_z$, the lower bound of the truncated normal process of capital quality shocks (in logs) $\underline{\omega}$, initial capital level $k_0$, and the relative scale of the initial productivity draw $m$ which are set to fit 16 empirical moments. The moments are related to aggregates, firms' credit spreads and default rates, investment heterogeneity, life-cycle of firms, and balance sheet moments.

Table 2 shows the moments targeted in the calibration. The model fits fairly well the life-cycle of firms (exit rate and share of labor and firms at the early stages), and investment rates heterogeneity (average and standard deviation) moments. These moments are from the LBD and Cooper and Haltiwanger (2006). Further, it fits the annual default rate of 3% — from Dun and Bradstreet — which includes liquidations and restructures. On the other hand, in the model steady-state equilibrium the average annual credit spread is 0.01, which is lower than in the data (0.02). This discrepancy can be explained by the absence of aggregate risk in the model.

Also, the model fits well the distribution of cash-on-hand $n/k'$ — shares of firms with negative values, between 0 and 1, and greater than 1 — which are relevant moments to estimate the incidence of rollover crises. Cash-on-hand $n$ is measured using data from Compustat and the asset-specific liquidation rates from Kermani and Ma (2021). There is no balance sheet data on non-Compustat firms; thus, I extrapolate this values the rest of the cash-on-hand distribution. Finally, as a consequence of targeting cash-on-hand $n/k$, the model exhibits significantly higher leverage compared to the data.[14] Leverage is measured as short-term liabilities to capital. Further details of the measurement of these variables can be found in Appendix B.

## 3.2 Identification and Incidence of Rollover Crises

The identification is model-based and utilizes observed cross-sectional moments. Specifically, neither the parameter $\eta$, nor threshold function $\underline{\mathrm{n}}(z)$, which defines the regions in the state space, are directly observable, thus I will infer them indirectly using observed firms' bankruptcy choices, bankrupt firms' characteristics and the financial distribution of firms.[15]

**Proposition 2.** *Assume that in the restructuring process the debt haircut $\alpha_{11}$ is exogenous and bankruptcy cost is a lump-sum fixed cost $c_{11} \in (0, -\underline{n}(z_{max}))$ with $z_{max}$ the highest productivity firm in the economy. Then for a given distribution of firms*

*(a) if $\alpha_{11} \to 1$ then firms that are fundamentally insolvent do not restructure their debt,*

*(b) if $\alpha_{11} \to 1$ then the share of firms that restructure their debt (i.e., $(z, n) \in \mathcal{R}$ with $n_{11} \geq n$) identifies $\eta$,*

*(c) if $\alpha_7 < \alpha_{11} < 1$ then firms with higher debt require a smaller $c_{11}$ to restructure.*

*The proof is in Appendix A.1.2.*

[14] One potential solution is to introduce capital frictions that would lead to some firms to hold a much higher capital level for low levels of cash-on-hand in the model. To maintain tractability, I assume that capital in the model is fully reversible.

[15] A salient and related example of the indirect inference of rollover crises is Bocola and Dovis (2019) who infer indirectly the rollover risk faced by the government through the time series of observed debt maturity choices. In this paper I use the cross-section of bankruptcy choices to infer the relevance of firms' rollover crises.

**Identification's intuition.** Proposition 2 provides insight into the relationship between bankruptcy outcomes and the incidence of rollover crises. Specifically, it shows that firms' bankruptcy choices are more informative about the probability $\eta$ when debt haircuts in the restructuring process $(1-\alpha_{11})$ are relatively low, and the cost of restructuring $c_{11}$ is relatively large. Moreover, it suggests that the leverage of the firms' when restructuring and the share of firms that restructure are informative about the costs of restructuring and the incidence of rollover crises.

In the model, the restructuring process provides different benefits to different firms. Fundamentally insolvent firms only benefit from reducing their debt, while firms under a rollover crises have the extra benefit of preventing the rollover crises. Proposition 2 (a) shows that if we assume that debt haircuts are close to zero, then firms under a rollover crises are the only ones that benefit from restructuring. Thus, the share of firms that restructure can identify the incidence of rollover crises in the economy (Proposition 2 (b)). Furthermore, in an intermediate case, where debt haircuts are not negligible, such as the one in Proposition 2 (c), I show that firms with higher leverage require lower $c_{11}$ to restructure. In other words, a larger $c_{11}$ will select fewer fundamentally insolvent firms to the restructuring process even if debt haircuts are non-negligible. Then, if cost $c_{11}$ is large enough, I conjecture that we can approximate $\eta$ in way similar to point (b) of the proposition. Finally, Proposition 2 (c) also suggests that a relevant moment to identify $c_{11}$, given the rest of parameters, is the leverage of the firms in the restructuring process. Overall, Proposition 2 shows that large bankruptcy costs and low debt haircuts dissuade insolvent firms from restructuring, then the observed firms' bankruptcy choice between restructuring and liquidation is informative about the value of $\eta$.

**Model-based identification.** I follow two steps to quantify $\eta$ using the model and data. First, I fit use observed bankrupt firms' characteristics and bankruptcy outcomes to infer the parameters related to the bankruptcy procedure and, then, I infer $\eta$ from the observed bankruptcy choices of firms.[16]

First, I want to identify the parameters $\left(\alpha_7, \tilde{\Xi}, c_{11}\right)$ of the bankruptcy process using moments related to the bankruptcy procedure and bankrupt firms' characteristics. Table 3

[16]For exposition purposes, I explain sequentially the identification of $\eta$, but, technically, the problem solves a fixed point given that $\eta$ could also shift other moments.

shows the value of the bankruptcy parameters, and targeted moments in the data and model. The capital recovery rate of creditors during liquidation $\alpha_7 = 0.29$ is set to match the debt recovery rate $\mathbb{E}[R(b,k,\omega)] = 0.27$ in Chapter 7 liquidations reported by Acharya, Bharath and Srinivasan (2007). *Approximate* bargaining power of creditors $\tilde{\Xi} = 0.89$ is set to match the debt recovery rate $\mathbb{E}[\alpha_{11}] = 0.69$ in Chapter 11 restructuring, reported by Acharya *et al.* (2007).[17] The parameter that represents the costs of the Chapter 11 process $c_{11} = 0.40$ and is set to fit the leverage of firms under Chapter 11 $\mathbb{E}[b'/k' \mid \text{Ch } 11] = 0.73$, reported by Antill (2021).

**Table 3:** Parameters and targeted moments of bankruptcy process

| Parameters | Value | Moment targeted | Data | Model | Data source |
|---|---|---|---|---|---|
| $\alpha_7$ | 0.38 | $\mathbb{E}[R(b,k,\omega)]$ | 0.27 | 0.29 | Acharya *et al.* (2007) |
| $\tilde{\Xi}$ | 0.89 | $\mathbb{E}[\alpha_{11}]$ | 0.69 | 0.79 | Acharya *et al.* (2007) |
| $c_{11}$ | 0.40 | $\mathbb{E}[b'/k' \mid \text{Ch } 11]$ | 0.73 | 0.67 | Antill (2021) |
| $\eta$ | **0.07** | $\mathbb{E}[1_{\text{ch}11}]/\mathbb{E}[1_{\text{ch}7}]$ | 2.0 | 1.9 | Antill (2021) |

Notes: Panel (a) shows the parameters and moments of the bankruptcy process in the baseline calibration. Panel (b) figure shows the relation between $\eta$ and the share of firms in Chapter 11 relative to Chapter 7, i.e., $\mathbb{E}[1_{\text{ch}11}]/\mathbb{E}[1_{\text{ch}7}]$, in the model.

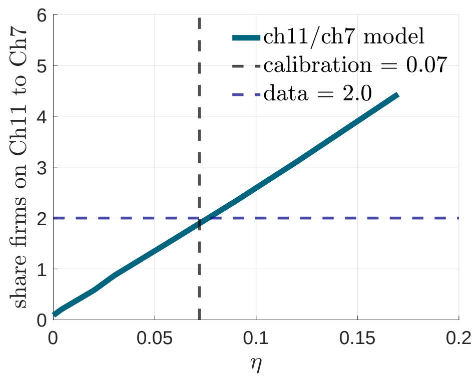
Next, I want to infer $\eta$. The data on recovery rates and leverage of Chapter 11 firms suggest that restructuring the debt is costly and haircuts are relatively low. Therefore, *relatively few* insolvent firms decide to restructure their debt, since gains from renegotiation are low (recall Proposition 2). In the model, the distribution of firms is shifting with $\eta$. The share of restructuring firms drops if $\eta$ increases because firms in the long run are able to stay far from the risky region. To control for this, I use the share of firms that restructure relative to those that liquidate.[18] Consistent with previous observations, Figure 3 shows that, in the quantitative model, a higher $\eta$ implies a larger share of firms that restructure relative to those that liquidate in steady state, which provides the identification of $\eta$.

[17] For computational efficiency, I use a convex-pricing function to approximate the bargaining outcome. Details are included in Appendix A.2.2.

[18] Although I do not prove this analytically, my numerical exercises suggest the distribution of firms that are insolvent and risky shift roughly proportional with $\eta$.

To approximate better the incidence of liquidation and restructuring in the data, I use the summary statistics provided by Antill (2021). Using Chapter 11 outcomes from the Moody's Ultimate Recovery database, Antill (2021) is able to identify how many Chapter 11 cases end in acquisition, piecemeal and full liquidations. When considering this, the ratio of firms that restructure to those that liquidate is around 2. Matching the model to the data, see Figure 3, I find $\eta = 0.07$. Furthermore, in the calibration, less than 10% of firms that restructure their debt are insolvent firms, which is consistent with the intuition provided by Proposition 2.

**Figure 3:** Identification of $\eta$




Notes: The figure shows the relation between $\eta$ and the share of firms in Chapter 11 relative to Chapter 7, i.e., $\mathbb{E}[1_{\text{ch11}}]/\mathbb{E}[1_{\text{ch7}}]$, in the model.

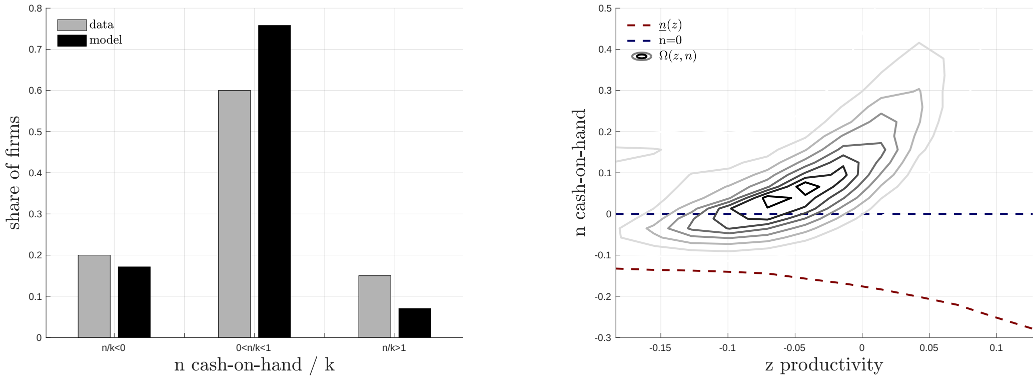
To validate the results of the quantitative model, in Section 3.3 and Appendix A.3.5, I show that the model fits untargeted moments related to firm-level bankruptcy predictors and investment dynamics.

**Incidence of rollover crises.** Given the value of $\eta$, it is straightforward to calculate how many firms are subject to rollover crises. First, I compute the distribution of firms at the beginning of the period across productivity $z$ and cash-on-hand $n$— i.e., $\Omega^{\text{bop}}(z,n) = (1-\gamma)\int \mathbf{1}_{\{z,n(z,\omega,k,b)=n\}} d\tilde{\Omega}(s)$ — then I estimate the number of firms exposed to rollover crises by computing the share of firms in risky region $\mathcal{R}$ — i.e., $\int_{(z,n)\in\mathcal{R}} d\Omega^{\text{bop}}(z,n)$ — and, finally, I multiply this share by the conditional probability of rollover failure $\eta$ to estimate the incidence of rollover crises. Figure A.9 shows the financial distribution of firms in the model and data. I find that around 20% of firms are in the risky region and with $\eta = 0.07$ probability of a coordination failure, then 1.6% of firms are subject to rollover crises each period.[19] To illustrate how large is the incidence of rollover crises for firms is, I calculate that roughly half of bankrupt firms (in restructuring and liquidation process) had a rollover crisis.

**Result 1.** *Roughly half of bankruptcy events (liquidations and restructures) are driven by rollover crises. The quantitative model infers that 1.6% of the firms are exposed to rollover crises, where 20% are exposed and the probability of a rollover crisis is 7%.*

**Figure 4:** Observed and model's distribution of firms

(a) Cash-on-hand ($n$) model and data (b) Incumbent $\Omega^{\text{bop}}(z,n)$ model




Notes: Panel (a) compares the distribution of cash-on-hand in the model and the data. Panel (b) shows the contour plot (darker line = higher mass) of the distribution of incumbent firms at the beginning of the period (bop), which do not receive the exit shock across productivity $z$ (x-axis) and cash-on-hand $n$ (y-axis), i.e., $\Omega^{\text{bop}}(z,n) = (1-\gamma)\int \mathbf{1}_{\{z,n(z,\omega,k,b)=n\}} d\tilde{\Omega}(s)$. The dashed red line is the $\underline{n}(z)$ threshold and the dashed blue line is the $n=0$ threshold.

## 3.3 Validation Exercises

As a validation exercise, I assess the model's ability to reproduce (untargeted) patterns in the data regarding bankruptcy events and investment dynamics.

**Bankruptcy Predictors.** First, I study how firms' characteristics predict a restructuring event in the data and the model. Using Compustat data, following Corbae and D'Erasmo (2021), I identify which firms filed for bankruptcy under Chapter 11. I explain the details in Appendix B. To estimate what predicts that a firm files for bankruptcy, I run the regression specified in Appendix A.3.1 for the observed and the model-simulated data. Table A.1 shows the results. I find that, in the model and the data, lower sales

[19]Notice that thus number includes firms that may not be liquidated because they enter the restructuring process.

growth, smaller size, low cash-on-hand, and high leverage predict a higher restructuring likelihood the next period. The magnitudes are similar in the model and data.[20]

**Investment Heterogeneity in Recessions** Next, I study how firms' investment responded during past recessions in the data and model. Figure A.3, in Appendix A.3.1, shows the peak-to-trough investment responses across the firms' cash-on-hand $n/k$ distribution. Results are quantitatively similar in the data and the model. Firms with the lowest group of cash-on-hand adjust investment the most, by around percentage points more than firms with high levels of cash-on-hand. Details of the data and the empirical specification are relegated to the Appendix A.3.1.

# 4 Macroeconomic and Policy Implications

In this section, we study the aggregate and policy implications of firms' rollover crises during large recessions. Firms' rollover risk has negligible consequences over macroeconomic outcomes over the long run (see Appendix A.3.4), then I focus on recessions episodes which suddenly expose firms to rollover crises and could motivate transitory policy interventions.[21]

## 4.1 Aggregate Consequences of Rollover Crises

Using the quantitative model, I simulate a prototypical large recession episode — a recession driven by a large unexpected aggregate shock — and assess the role of rollover crises by comparing its macroeconomic dynamics with the macro dynamics of a counterfactual large recession without rollover crises. I show that rollover crises can significantly amplify aggregate output losses during a large recession.

**Aggregate Shocks.** I focus on the transition paths after large and unexpected aggregate shocks. I assume that once an aggregate shock happens there is perfect foresight

[20] Interestingly, the relation between leverage and restructuring is reverted once we control for cash-on-hand [see specification (3) in Table A.1]. One potential explanation for this counter-intuitive relation is that firms suffering from rollover crises given their cash-on-hand ($n$) are more likely to restructure if they are in a better shape once they pay the bankruptcy cost (proxied by lower leverage).

[21] Over the long-run firms are able to accumulate internal resources to stay far from the risky region and in equilibrium they improve their financial position, reducing the overall impact of rollover crises on aggregate outcomes.

about its future path. The shocks are temporary, then I study the transitional dynamics where the economy returns to the baseline steady state equilibrium. To calibrate the shocks, I target a peak-to-trough 5% reduction in aggregate output $Y$. I simulate three types of aggregate shocks: (i) an aggregate TFP shock $A_t$, which shifts all firms' productivity proportionally such that the firm-level production is $y_{it} = A_t z_{it} f(z_{it}, \omega_{it} k_{it}, l_{it})$ along the transition path and $A = 1$ in steady state; (ii) a cash shock which suddenly reduces each firm's cash on hand $n_{it}$ by $N_t k_{it}$ where $N = 0$ in steady state; and (iii) a credit shock which suddenly decreases the creditors' recovery rate $\alpha_{7t}$.[22] Appendix A.2.4 contains more details about the aggregate shocks and their simulated paths.

**Recessions and rollover crises.** To assess the role of rollover risk in recessions, I simulate a prototypical large recessions episode — large and unexpected aggregate shock as explained before — and perform a counterfactual experiment where coordination failures do not happen during the recession episode.

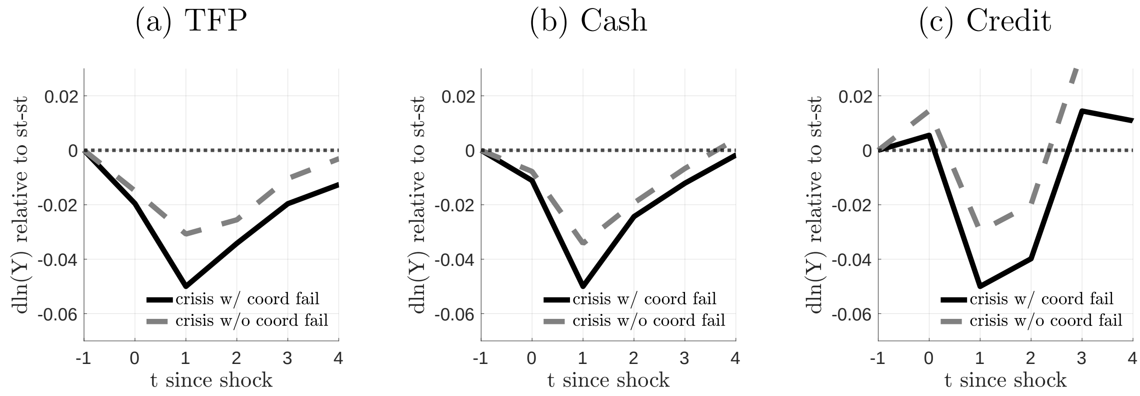
Figure 5 shows the response of aggregate output $Y$ to different types of aggregate shocks with coordination failures (black line) and without them (gray dashed line). In all panels the absence of firm's rollover crises —i.e., no extra rollover crises at $t = 0, 1$ — significantly reduces the depth of the trough in the crises. The counterfactual exercises indicate that rollover crises can explain between 10% and 30% of total output losses in large recessions.[23]

Although the relevance of rollover risk is similar across shocks, on impact, the dynamics are different. In Appendix A.3.2, I show the dynamics of firm (net) exit, debt and capital accumulation during large recessions. Weaker fundamentals — due to a TFP or a cash shock — expose (temporarily) several solvent firms to rollover crises, which increases the exit of healthy firms during the recession. Moreover, new firms entering the economy are smaller and take time to grow. Therefore, recessions driven by a TFP or an aggregate cash shock which are exacerbated by coordination failures have a much slower recovery than crises driven by the credit shock. In the crisis driven by the credit shock, firms are

[22] Notice the credit shock in this model is a reduction in the collateral value when the firms are liquidated, which is different from the credit shock in Khan *et al.* (2020). Their credit shock is qualitatively closer to the cash shock in my paper.

[23] Numbers depend on the driver of recessions and on how I compute them (e.g., as a present discounted value or a simple sum of output losses).

**Figure 5:** Recession Shock and Aggregate Output




Notes: Panels (a), (b) and (c) show the response of aggregate output $Y$ to an aggregate TFP shock, cash shock and credit shock with coordination failures and without coordination failures (for t=0 and t=1), respectively. The economy at $t = -1$ is in steady state. The definition of the shocks are in the text and in Appendix A.2.4.

induced to deleverage quickly and reduce their investment initially to preclude liquidation, which makes the recovery relatively stronger (compared to other shocks), although the initial impact is similar.

**Result 2.** *Firms' rollover risk significantly amplifies the impact of recessions. Rollover risk explains 10% to 30% of output losses in large recession experiments.*

## 4.2 Credit Policy during Recessions

In the previous section, I show that firms' rollover crises, by driving healthy firms to bankruptcy, can augment the depth and persistence of recessions. Motivated by this, in this section, I study how effective are *imperfectly-targeted* direct lending policies in reducing the impact of creditors' coordination failures during large recessions. I assume the credit policies are imperfectly targeted to capture the notion that the government may not observe all the characteristics of individual firms and other policy implementation restrictions (e.g., regulation on lending powers).

In my quantitative exercises, I study how effective are these credit policies across different scales of the policy. I find that low-scale credit policies, which target few firms and offer a relatively expensive credit, are the most effective. They undo much of the amplification of rollover crises because they are able to preclude coordination failures ex ante and, since credit is relatively expensive, the government credit facilities remain mostly unused (in equilibrium). On the contrary, credit programs that are very ample and subsidize credit to many firms, can mitigate firms' rollover crises in the short term, but exacerbate debt overhang problems in the future, amplifying the overall impact of the recession.

**Direct credit policy.** A direct lending policy in the model is promised unexpectedly at $t = 0$, and implemented at period $J_0$ for $J$ periods. When the policy is active, the government offers an alternative pricing schedule $Q_j^g$ for the new debt issuance of the firm at each period $j = J_0, J_0 + 1, .., J$ to a set of eligible firm $\mathcal{P}$ which depends on observable, either current or past, characteristics of the firms. For simplicity, I assume that the set of eligible firms $\mathcal{P}$ is fixed over time, once the policy is implemented, and creditors know about the policy. Then, for a eligible firm with idiosyncratic states $\mathbf{s}$ and invest-financing choice $(k', b')$ the external resources from new debt issuance are

$$\max \left\{Q_j^g, Q_j\left(\mathbf{s}, k', b'\right)\right\} b'.$$

Given there are fixed costs of entering the program, the firms choose the most favorable pricing schedule between the government program and the market. In the quantitative model, I assume the policy is financed by the representative household that pays a lump-sum tax to the government, as described in Appendix A.2.5.

**Proposition 3.** *Assume the government implements the credit policy the current period and lasts one period then:*

1. *First best policy: if $Q_j^g = \tilde{Q}_j\left(z, k^{'}, b^{'}\right)$ then no rollover crises happen today and firms are indifferent between using market credit or government's credit facilities.*

2. *Trade-off: the more generous is the imperfectly-targeted credit policy the lower the incidence of rollover crises, but the greater the share of firms with a subsidized credit (adverse selection).*

*The definition of the imperfectly-targeted credit policy and the proofs are in Appendix A.2.5.*

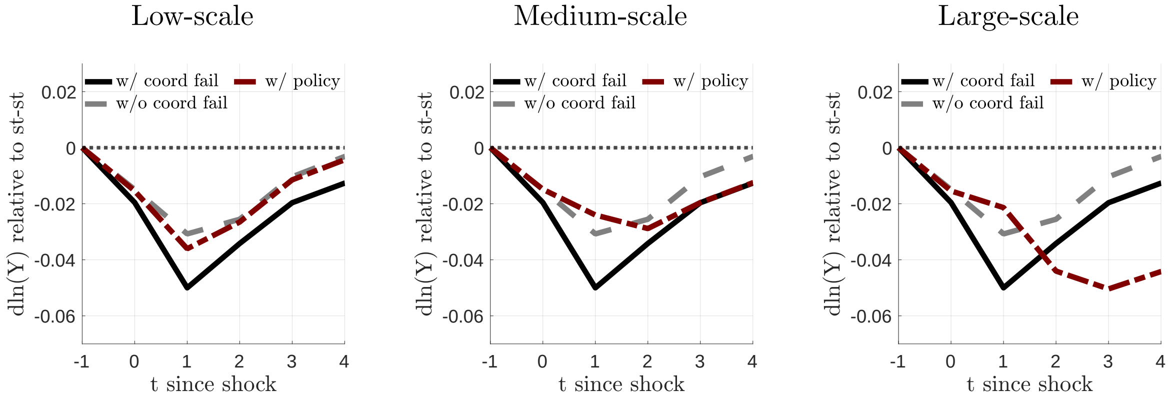
**Workings and trade-off.** Proposition 3 shows how the direct policy can mitigate rollover crises and how the policy faces a potential trade-off when it is imperfectly targeted. The first point of the proposition shows that if the government has perfect information about firms states $(z, n)$ and sets its pricing schedule identical to the market price, then rollover crises are precluded and in equilibrium firms do not use the government's credit facilities. The intuition is that the credit facility provides insurance to creditors. They know that if the firm has a rollover crises it can access an alternative source of financing and rollover its debt, then they coordinate in the repay equilibrium.

The credit policy when imperfectly targeted is such that the government offers to buy the debt at $Q_j^g\left(z^g, k^{'}, b^{'}\right)$ where $z^g$ is a parameter chosen by the government (exogenous). Parameter $z^g$ also parameterizes the set of eligible firms. The greater is $z^g$ the cheaper is the credit from the government's facilities and more firms are eligible for the policy. In Appendix A.2.5, I provide further details of the imperfectly-targeted policy.

The second point in Proposition 3 shows that the imperfectly-targeted credit policy can mitigate rollover crises, but some firms will receive a subsidized credit (i.e., higher debt prices compared to fundamental market prices). For a firm under a rollover crisis that is eligible to the policy, the rollover crisis is precluded if the maximal amount of resources when using the government credit facilities is $\max_{b^{'},k^{'}}\left\{n + Q_j^g\left(z^g, k^{'}, b^{'}\right)b^{'} - qk^{'}\right\} \geq 0$. If it is the case that $Q_j^g\left(z^g, k^{'}, b^{'}\right) \leq \tilde{Q}_j\left(z, k^{'}, b^{'}\right)$, then the rollover crisis is precluded by coordinating creditors in the good equilibrium. On other hand, if $Q_j^g\left(z^g, k^{'}, b^{'}\right) > \tilde{Q}_j\left(z, k^{'}, b^{'}\right)$, then the firm precludes the rollover crisis by borrowing from the government facilities at a subsidized borrowing cost. Moreover, in a similar way, the imperfectly-targeted policy may subsidize the credit for fundamentally insolvent firms, that are eligible for the policy, whenever the credit terms of the government facilities are good enough for them to continue operating. Therefore, the greater $z^g$ the lower is the incidence of rollover crises, but more credit is subsidized and allocated to fundamentally insolvent firms.

**Quantification.** The effectiveness of the policies will depend on the distribution of firms and how much they distort the incentives to accumulate capital and debt, then it is fundamentally a quantitative question. Figures A.6 and 6 show the cost and benefits of the credit program for different levels of scale $(z^g)$ when applied at $t = 0, 1$, i.e., peak-to-trough of the recession. I focus on the TFP shock, results for the cash shock are quantitatively similar and are relegated to Appendix A.2.5.

**Figure 6:** Macro Dynamics With and Without Policy




Notes: The Figure shows the response of aggregate output $Y$ to an aggregate cash shock with perfectly targeted policy (same as without coordination failures, dashed gray line), imperfectly-targeted policy (red dashed line), and without policy intervention (solid black line) for different policy scale. The economy at $t = -1$ is in steady-state. The definition of the shocks and crises experiments are in Section 4.1 and Appendix A.2.4. Further description of the policy in the text.

Figure A.6 panel (a) shows the fiscal cost of the policy for different scales of the policy. The fiscal cost is determined by how much credit the government subsidizes in total, i.e., the sum of $(Q - Q^g)$ integrated across all participant firms. I show a low-scale policy that has a cost close to 0, a medium-scale policy that has a low cost (0.05% of output) and a large-scale policy that has a large cost (around 0.3% of output). On the other hand, Figure A.6 panel (b) and Figure 6 show the short- and medium-term costs and benefits of the credit programs, and the macro dynamics, for different scales of the policy. I find that in the short-term the large-scale program improves aggregate outcomes and even provides some extra stimulus (relative to the counterfactual without rollover crises), but in the medium-term it creates greater output losses.

The intuition, for this result, is that large-scale programs subsidize credit to many financially exposed and fundamentally weak firms, which eventually backfires by exacerbating debt overhang in the future.[24] On the other hand, the low-scale program has significant short-term benefits with a fiscal cost close to 0, and provides a swift recovery (similar to the counterfactual without rollover crises). The intuition, for this result, is that small-scale programs can mitigate rollover crises (even if firms do not use the credit facilities ex-post), and can rescue several healthy firms in the economy, which contributes to a stronger recovery.

[24] This mechanism is similar to the one studied by Crouzet and Tourre (2021).

**Result 3.** *Imperfectly-targeted credit policies can mitigate the amplification created by rollover crises (even if the policy is just announced), but can backfire if they subsidize credit to many firms. The trade-off is quantitatively relevant.*

# 5 Concluding Remarks

In this paper, I develop a framework where firms' rollover crises can be identified and quantified. Salient features of the U.S. bankruptcy code allow me to quantify the incidence of rollover crises using observed bankruptcy choices and bankrupt firms' characteristics. I find that rollover crises can explain roughly half of the bankruptcy events.

My quantitative results suggest that firms' rollover crises, through the failure of healthy firms, have a significant impact during large recessions. On the other hand, direct credit policies can act as insurance for creditors and can prevent coordination failures from happening, but, if policy is imperfectly-targeted, then the government faces a trade-off between short-run mitigation of rollover crises and future debt overhang problems. Quantitative results suggest that, during large recessions, the benefits of direct credit policies are ambiguous.

In the model, I focus on the problem of firms which have homogeneous and atomistic creditors, and no active management of its liability structure. Potential extensions could allow for heterogeneous and non atomistic investors [see, for example, Halac *et al.* (2020); Chaumont *et al.* (2023)]; endogenous debt maturity structure [for example, Bocola and Dovis (2019) for sovereign debt or Cheng and Milbradt (2012); Crouzet (2017b) for firms]; and endogenous number of creditors [for example, Bris and Welch (2005); Bolton and Scharfstein (1996)].

In this paper, bankruptcy outcomes are (indirectly) informative about why firms fail. One potential avenue for future is to collect and study data on bankruptcy procedures from legal documents where the managers of the bankrupt firm, the creditors and the courts provide rich information about the potential bankruptcy causes. This could provide more direct evidence on why firms fail and the role of creditors' coordination failures. Furthermore, my paper provides insights into the relationship between rollover crises and bankruptcy provisions during large recessions, which can be applied to other contexts. For example, these insights can inform studies on lender-of-last-resort policies in economies prone to runs, where the lender may face constraints, or the design of supranational sovereign debt restructuring frameworks involving multiple economies. I leave these extensions to the framework and alternative applications for future research.

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# Appendices

The Appendix is organized as follows: Appendix A shows proofs, further details and exercises related to the model; Appendix B includes details of the data sources and further empirical results; Appendix C includes extensions of the model; and Appendix D provides a brief description of the U.S. bankruptcy code.

# A Model

This section contains the main proofs, details of the model, and additional results.

## A.1 Proofs

### A.1.1 Proof Liquidation Choice

*Proof.* Firm's exit with value $V^{\text{exit}} = 0$ and requires $d \geq 0$ then the firm decides to liquidate and exit if $d \geq 0$ is not feasible. The default decision can be characterized by regions of state-space $(z, n)$. First, there is a Safe region $\mathcal{S}$ where firms in this region do not liquidate even if creditors conjecture they are liquidated today. This means that if $Q = 0$ they can satisfy $d = n + \max_{k' \geq 0} \left\{-qk'\right\} = n \geq 0$. Thus, firms with $n \geq 0$ can always satisfy $d \geq 0$ and are in $(z, n) \in \mathcal{S}$. Next, there is a Liquidation region $\mathcal{L}$ where firms are liquidated even if creditors conjecture no liquidation today. This means that even if $Q = \tilde{Q}$ then firms cannot satisfy the $d \geq 0$, i.e., $d = n + \max_{b', k' \geq 0} \left\{\tilde{Q}b' - qk'\right\} = n < 0$. Since $\tilde{Q} = \tilde{Q}\left(z, b', k'\right)$, then it follows that firms $(z, n) \in \mathcal{L}$ are those with cash-on-hand $n$ bellow a threshold $\underline{\mathrm{n}}\left(z\right)$ where the threshold is defined by the negative of the maximum amount of external resouces the firm can raise, i.e., $n < \underline{\mathrm{n}}\left(z\right) = -\max_{b', k' \geq 0} \left\{\tilde{Q}b' - qk'\right\}$. Finally, there is a Risky region $\mathcal{R}$ where firms can either be liquidated or not depending on creditors' conjecture. This means that if $Q = 0$ then they cannot satisfy $d \geq 0$ so we need $n < 0$, but if $Q = \tilde{Q} > 0$ then firms satisfy $d \geq 0$ so we need $n \geq \underline{\mathrm{n}}\left(z\right)$. Thus, firms $(z, n) \in \mathcal{R}$ whenever $n \in \left[\underline{\mathrm{n}}\left(z\right), 0\right)$. □

### A.1.2 Proof Identification of $\eta$

*Proof.* Assume $\alpha_{11}$ is fixed, $c > 0$ is a fixed cost and the distribution of firms at the beginning of the period $\tilde{\Omega}$ is fixed.

Consider the case $\alpha_{11} \to 1$ we have that resources after restructuring are $n_{11} \to n - c$, then insolvent firms will have $\underline{\mathrm{n}}(z) > n > n - c$ then they will not restructure their debt. On the other hand, firms with a rollover crisis restructure if $n > n - c \geq \underline{\mathrm{n}}(z)$ since the firm is in $\mathcal{R}$ we have $n < 0$ we need the cost is $c < -\underline{\mathrm{n}}(z_{\max})$, with $z_{\max}$ the highest productivity draw in the economy, for at least on firm to participate of the restructuring process. Next, without knowledge of $\eta$ we can compute the share of firms that *would* restructure if they receive the sunspot shock and are exposed, which is $\int_{(z,n)\in\mathcal{R}} \mathbf{1}_{\{n-c>\underline{\mathrm{n}}(z)\}} \mathrm{d}\Omega^{\mathrm{bop}}(z,n)$ with the beginning of the period distribution of non-exiting firms across $(z,n)$ is $\Omega^{\mathrm{bop}}(z,n) = (1-\gamma)\int \mathbf{1}_{\{z,n(z,\omega,k,b)=n\}} \mathrm{d}\tilde{\Omega}(\mathbf{s})$. Therefore, if we have the values of $c$ (which could be inferred from other moments, e.g., bankrupt firms' characteristics) and distribution of firms, then we can identify $\eta$ using the fact the share of firms that restructure equates the share of firms that would restructure if $\phi < \eta$ times the probability $\eta$.

Consider the case $\alpha_7 < \alpha_{11} < 1$ assume, without loss of generality, no recovery rate in liquidation $\alpha_7 = 0$, price of capital $q = 1$, the variance of $\omega$ shocks is 0 and $\delta = 0$ then internal resources are $n = \pi(z,k) + k - b$ and internal resources when restructuring are $n_{11} = n + (1-\alpha_{11})b - c$ with $(1-\alpha_{11})b$ the gains from restructuring and $c$ the cost. Now, for insolvent firms we have now that the net benefits of the debt restructuring process can be ambiguous, so there can be insolvent firms restructuring. In particular, for firms to participate they need that the gains from the haircut are large enough to cover the bankruptcy costs and resources needed to become solvent $(1-\alpha_{11})b > c + (\underline{\mathrm{n}}(z) - n)$. Replacing with the definition of $n$ then we can find for a firm with $(z,k,b)$ that $b < \frac{\underline{\mathrm{n}}(z)+\pi(z,k)+k-c}{\alpha_{11}}$ for the firm to restructure.

□

The last condition in the proposition shows that firms with higher $b$, given $(k,z)$ are less likely to restructure. The intuition is that while leveraged firms benefit more from the haircut, they have lower internal resources which requires them larger gains from the haircut. Furthermore, we can interpret it as higher cost $c$ will create less leveraged firms file for restructuring (using the leverage of the firm we can identify $c$, which is the identification strategy in the quantitative model). The higher $c$ then will make firms with higher debt become less likely to restructure, therefore we can approximate $\eta$ if $c$ is large enough such that few insolvent firms can restructure even if $\alpha_{11} < 1$.

## A.2 Model Details

### A.2.1 Exiting firms problem

Incumbents firms at the beginning of the period receive with probability $\gamma$ a shock that force them to exit after production. I allow for exiting firms to make also the liquidation choice and restructuring choice. Notice that since they exit at the end of the period these firms don't choose $(b', k')$ then they are not subject to coordination failures such as the ones described for nonexiting firms. Exiting firms choose to liquidation choice is

$$\mathbf{1}_{\{\text{ch7}|\text{exit}\}}(\mathbf{s}) = \tilde{\mathbf{1}}_{\{\text{ch7}|\text{exit}\}}(z,\omega,b,k) = \begin{cases} 1 & \text{if } \max\left\{n, n_{11|\text{exit}}\right\} < 0 \\ 0 & \text{otherwise} \end{cases}. \tag{21}$$

where $n$ defined as before and

$$n_{11|\text{exit}} = \pi(z,\omega,k) + (1-c_{11})(1-\delta)q\omega k - \alpha_{11|\text{exit}}(z,\omega,k,b)\,b.$$

Since the outside option is to continue then only firms with $n<0$ will restructure their debt and the debt recovery $\alpha_{11|\text{exit}}$ is determined by

$$\alpha_{11|\text{exit}}(z,\omega,b,k) = \max_{\alpha_{11}}\left[n_{11|\text{exit}} - 0\right]^{1-\Xi}\left[b\alpha_{11|\text{exit}} - bR(b,k,\omega)\right]^{\Xi} \tag{22}$$

subject to

$$\begin{aligned} n_{11|\text{exit}} &> 0 \\ \alpha_{11} &\geq R(b,k,\omega). \end{aligned}$$

The restructuring choice is

$$\begin{aligned} \mathbf{1}_{\{\text{ch11}|\text{exit}\}}(\mathbf{s}) &= \tilde{\mathbf{1}}_{\{\text{ch11}|\text{exit}\}}(z,\omega,b,k) \\ &= \begin{cases} 1 & \text{if } \{n<0\} \cap \left\{n_{11|\text{exit}} > 0\right\} \cap \left\{\alpha^{\max}_{11|\text{exit}} > \alpha^{\min}_{11}\right\} \\ 0 & \text{otherwise} \end{cases}, \end{aligned} \tag{23}$$

where $\alpha^{\max}_{11|\text{exit}} = \frac{\pi(z,\omega,k)+(1-c_{11})(1-\delta)q\omega k}{b}$ and $\alpha^{\min}_{11} = R(b,k,\omega)$. The firms that continue are defined as $\mathbf{1}_{\{\text{continue}|\text{exit}\}}(\mathbf{s}) = 1 - \mathbf{1}_{\{\text{ch11}|\text{exit}\}}(\mathbf{s}) - \mathbf{1}_{\{\text{ch7}|\text{exit}\}}(\mathbf{s}) = \mathbf{1}_{\{n\geq 0\}}$.

### A.2.2 Computational solution of Bargaining Problem

To solve the bargaining problem I adopt a very simple convex-pricing function to approximate the result from the Nash Bargaining problem.[25] Although this is a reduced form solution to the bargaining problem, it provides better computational speed since we don't need the value function of the firm to compute it. I proceed as follows: I compute the maximum and minimum recovery rates, $\alpha^{\max}_{11}(z,\omega,k,b)$ and $\alpha^{\min}_{11}(\omega,k,b)$, respectively. Using these bounds, for the restructuring processes that are feasible I compute the approximate

[25] In their robustness exercises Guntin and Kochen (2021) adopt this function to solve computationally for a complex bargaining problem.

recovery rate $\tilde{\alpha}_{11}(z,\omega,k,b)$ as

$$\tilde{\alpha}_{11}(z,\omega,k,b) = \tilde{\Xi}\alpha_{11}^{\max}(z,\omega,k,b) + \left(1-\tilde{\Xi}\right)\alpha_{11}^{\min}(\omega,k,b)$$

where $\tilde{\Xi} \in [0,1]$ is the approximate bargaining power of the creditors. There is no one-on-one mapping, but to check for robustness I solve for the exact solution and find similar results. Therefore, I adopt this convex-pricing function, which is computationally significantly more efficient than the exact solution.

### A.2.3 Number of firms

In steady state, the number of firms that produce $F = \int \mathrm{d}\Omega(n,z)$ is normalized to 1. Along the transition path, after an unexpected aggregate shock, I assume that the entry of firms is fixed, but the firms exiting changes endogenously. Thus, the number of firms that operate $F_t$ will change along the transition path. Now I will characterize analytically how the number of firms evolve over time. For simplicity assume that the there is no restructuring process and no exogenous exit shock, then the law of motion of then, using the characterization of Proposition 1, the number of firms that produce at $t$ $(F_t)$ is

$$F_t = F_{t-1}\left[\int_{(n,z)\in\mathcal{S}_t} \mathrm{d}\hat{\Omega}_t^{\mathrm{bop}}(n,z) + (1-\eta)\int_{(n,z)\in\mathcal{R}_t} \mathrm{d}\hat{\Omega}_t^{\mathrm{bop}}(n,z)\right] + \bar{\mu},$$

where $\hat{\Omega}_t^{\mathrm{bop}} = \Omega_t^{\mathrm{bop}}/F_{t-1}$ such that $\int \mathrm{d}\hat{\Omega}_t^{\mathrm{bop}} = 1$ and $\Omega_t^{\mathrm{bop}}(z,n) = \int \mathbf{1}_{\{z,n(z,\omega,k,b)=n\}}\mathrm{d}\tilde{\Omega}_t(s)$ the measure of incumbent firms with $(z,n)$ at $t$. In steady state $F_t = F_{t-1} = 1$ then the outflow of firms (all endogenous in this version of the model) has to equate to the measure of entrants, i.e.,

$$\int_{(n,z)\in\mathcal{L}} \mathrm{d}\hat{\Omega}^{\mathrm{bop}}(n,z) + \eta\int_{(n,z)\in\mathcal{R}} \mathrm{d}\hat{\Omega}^{\mathrm{bop}}(n,z) = \bar{\mu}.$$

Let $\hat{\mu}_t^{\mathrm{exit}} \equiv \int_{(n,z)\in\mathcal{L}_t} \mathrm{d}\hat{\Omega}^{\mathrm{bop}}(n,z) + \eta\int_{(n,z)\in\mathcal{R}_t} \mathrm{d}\hat{\Omega}^{\mathrm{bop}}(n,z)$ be the share of incumbent firms that exit at $t$ then we can iterate backwards the law of motion of $F_t$ such that the number of firms $F_t$ can be expressed as

$$F_t = \Pi_{j=0}^{t}\left(1-\hat{\mu}_{t-j}^{\mathrm{exit}}\right) + \bar{\mu}\left[1 + \mathbf{1}_{\{t>0\}}\sum_{i=1}^{t}\Pi_{j=1}^{i}\left(1-\hat{\mu}_{t+1-j}^{\mathrm{exit}}\right)\right].$$

The number of firms is going to be a function of the share of exiting incumbent firms. The first term is the compound of the number of firms that exit since the aggregate shock happens and the second term determines the weight of the sequence of entrants since the shock. I assume that $F_\infty = 1$ then endogenous exit will decrease at some point along the path if it increases initially.

### A.2.4 Crises shocks and counterfactual

I work with 3 different types of crisis shocks: a TFP shock, cash shock and credit shock. Shock are unforeseen and I study the perfect foresight transitions from $t \geq 0$ where $t = 0$ is the initial impact of the shock (at the beginning of the period). The initial impact is calibrated to match a 5% drop in aggregate output from peak-to-trough (large aggregate shock) and the persistence of all shocks is $\rho_{\text{shock}} = 0.5$ (i.e., short lived).[26] I assume the process are

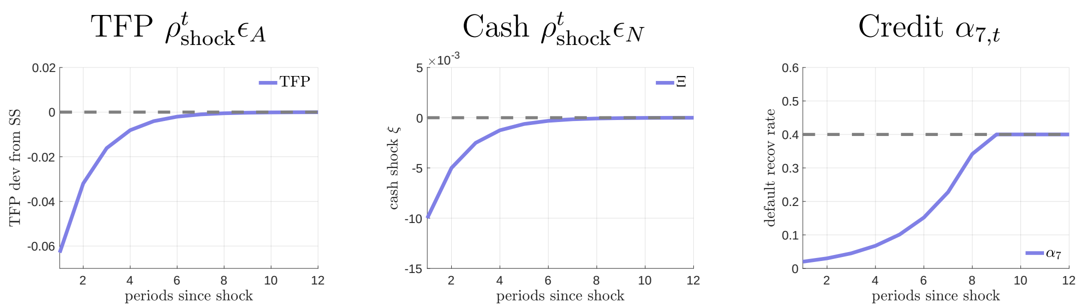
1. TFP shock: firms production function is now $y_{it} = A_t f(z_{it}, \omega_{it}, k_{it})$ for $t \geq 0$ where $A_t = \exp(\rho_{\text{shock}}^t \epsilon_A)$ with $\epsilon_A < 0$ the initial shock at $t = 0$.

2. Cash shock: firms cash-on-hand is $n_{it} = \pi_t(z_{it}, \omega_{it}, k_{it}) + (1-\delta) q_t \omega_{it} k_{it} - b_{it} - N_t k_{it}$ for $t \geq 0$ where $N_t = \rho_{\text{shock}}^t \epsilon_N$ with $\epsilon_N > 0$ initial shock to cash proportional to capital.

3. Credit shock: recovery rate when liquidated $\alpha_{7t}$ is time-varying for $t \geq 0$ where $\alpha_{7t} = \alpha_7 - \rho_{\text{shock}}^t \epsilon_7$ where $\epsilon_7 > 0$ initial decrease in liquidation recovery rate.

Figure A.1 shows the path for the baseline counterfactuals.

**Figure A.1:** Crises Shocks Path




Notes: panel (a), (b) and (c) show the path of the shocks. Shocks happen at $t = 0$. Further description of the shocks in the text.

For computing the aggregates during the transitions I assume that the distribution of firms is no longer a fixed point and allow for *net exit* by fixing the amount of new firms created each period to the initial steady-state calibration, i.e., $\bar{\mu}_t = \bar{\mu}$.

[26] More than 95% of the shocks fades away in an year.

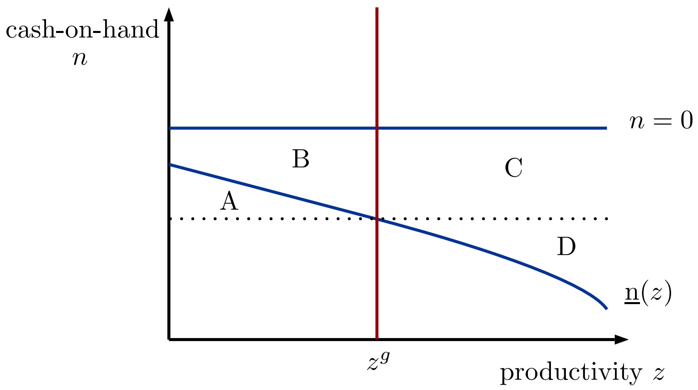
### A.2.5 Credit policy program

**Imperfectly-targeted credit policy description.** A policy $Q^g = \tilde{Q}$ that precludes all rollover crises without being used in equilibrium requires the government to observe perfectly each firm's productivity and cash-on-hand $(z, n)$. To make the policy more realistic, I assume that the government observes only $n$ and sets the policy according to a simple rule, i.e., *imperfect credit policy*, which has the following parametrization:

1. The set of eligible firms depends on $n$ only and requires that $n < 0$ (i.e., firms need external resources to satisfy $d \geq 0$). Therefore, the set $\mathcal{P}$ is composed by firms with $n$ such that $n \in [\underline{n}^g, 0)$, where $\underline{n}^g < 0$ is a parameter chosen by the government.

2. All eligible firms receive enough funds such that they can satisfy $d \geq 0$, but the government can't discriminate across the $n$ position of eligible firms, i.e., $Q^g\left(z, k', b'\right)$ is such that $\underline{n}^g = -\max_{b',k'} Q^g\left(z^g, k', b'\right) b' - qk' = \underline{n}\left(z^g\right)$ which implies that $n^g$ determines the choice of $z = z^g$ for the pricing schedule offered by the government.[27]

**Figure A.2:** Imperfect Credit Policy Eligibility




Notes: figure shows an illustration of the eleigbility and firms participation in the program for a one period example.

Figure A.2 shows what firms are eligible and the static choice of the participating or not in the program for a 1 period policy.[28] Eligible firms are those in the area $A \cup B \cup C$. In the case of $A$, in absence of the credit program the firm would be insolvent, then these firms receive subsidized credit. On the other hand, in $B$, firms will find the credit in the program cheaper than the market then they participate so they receive a subsidized credit. On the contrary, firms in region $C$ will have a more expensive credit than the market then

[27] The assumption that the government pricing function doesn't depend on the firm's cash-on-hand simplifies greatly the computational problem.

[28] If the policy lasts more than one period or is implemented with a lag, then it will affect the solvency thresholds (even in partial equilibrium) since they depend on future prospects of the firm.

they don't participate in the program. Thus, firms in $A \cup B$ receive a subsidized credit and firms in $C$ don't. Moreover, the credit program will preclude those firms under a rollover crisis in $B \cup C$ from being liquidated. Notice that firms under a rollover crisis in $B$ will participate in the program and those in $C$ will not participate in the program but the mere existence of the program will preclude coordination failures. Therefore, if the scale of the policy increases — i.e., lower $n^g$ or, equivalently, greater $z^g$ — more coordination failures are precluded and more firms are subsidized. The subsidized credit could exacerbate future debt overhang problems and has fiscal costs, so the policy faces a potential trade-off when incrementing the scale of the program.

**Proofs credit policy.**

*Proof. First best policy - Proposition 3.* The government sets for 1 period $Q^g = \tilde{Q}\left(z, k', b'\right)$. Assume by contradiction that a firm with $(z, n) \in \mathcal{R}$ and $\phi < \eta$ then $Q = 0$ in equilibrium. Since $(z, n) \in \mathcal{R}$ then $n \geq \underline{n}(z)$ and since $\tilde{Q} = Q^g$ then $\max_{b',k'} \left\{n + \tilde{Q}b' - qk'\right\} = \max_{b',k'} \left\{n + Q^g b' - qk'\right\} \geq 0$ so it cannot be true that $Q = 0$ in equilibrium.

□

*Proof. Trade-off - Proposition 3.* The government offers $Q(z^g, k', b')$ then firms with $z < z^g$ and $n \geq n^g$ receive a subsidized credit. On the other hand, from the first point is trivial that if $Q(z^g, k', b')$ increasing inf $z^g$ more rollover crises are precluded.

□

**Further details quantitative setup.** The baseline credit policy experiment consist of a parameter $z^g$ that determines the pricing schedule $Q_t^g = \tilde{Q}_t\left(z^g, b', k'\right)$ and set of eligible firms $[\underline{n}\left(z^g\right), 0)$ and lasts two periods (implemented at $t = 0$ and $t = 1$). The policy is computed backwards, since the presence of the policy at $t + 1$ will affect the solvency thresholds $\underline{n}_t\left(z\right)$. To estimate the cost of the policy I compute the aggregate credit subsidy as the difference between the price offered by the private sector relative to the government credit program times the amount borrowed for the firms that choose to participate in the program, i.e.,

$$G_t = \int_{(z,n)\in\mathcal{P}} \max\left\{0, \tilde{Q}_t\left(z, k', b'\right) - \tilde{Q}_t\left(z^g, k', b'\right)\right\} b' \mathrm{d}\tilde{\Omega}_t\left(\mathbf{s}\right).$$

The subsidy is financed through a lump-sum transfer $T_t = G_t$ such that the aggregate output net of government expenditure is $\tilde{Y}_t = Y_t - G_t$.

## A.3 Additional Results

### A.3.1 Validation Exercises

In this section, I show the results for the validation exercises, which are explained in Section 3.3 in the paper.

**Bankruptcy Predictors.** To study how firms' characteristics predict a restructuring event, I make the following regression estimation in the data and model

$$\mathbf{1}^{\text{ch11}}_{i,t} = \beta X_{i,t-1} + \alpha_t + \alpha_i + \alpha_s + \epsilon_{i,t},$$

where $\mathbf{1}^{\text{ch11}}_{i,t}$ indicates if the firm $i$ in period $t$ is in Chapter 11 and operating (restructuring rather than liquidate), $\alpha_i$ are firm FE, $\alpha_s$ sector FE, $\alpha_t$ time fixed effects and $X_{i,t-1}$ is a vector of characteristics (predictors) of interest lagged one period. In my baseline specification I include in $X_{i,t}$ the size of the firm (assets in logs), real quarterly growth of sales, the cash-on-hand and leverage positions. I standardize all variables in $X_{i,t-1}$ and winsorize the cash-on-hand $n/k'$ and leverage $b'/k'$ at level 0.5% and 99.5%.

**Table A.1:** Predictors of Chapter 11 - data and model

| | dependent variable: $\mathbf{1}^{\text{ch11}}_{i,t}$ | | | | | |
|---|---|---|---|---|---|---|
| | (1) | | (2) | | (3) | |
| | data | model | data | model | data | model |
| $n_{i,t-1}/k_{i,t}$ | -0.39 | -0.05 | | | -0.39 | -0.45 |
| | (0.03) | | | | (0.10) | |
| $b_{i,t}/k_{i,t}$ | | | 0.11 | 0.03 | -0.29 | -0.41 |
| | | | (0.04) | | (0.09) | |
| $\log(k_{i,t-1})$ | -0.50 | -0.06 | -0.52 | -0.06 | -0.49 | -0.10 |
| | (0.12) | | (0.12) | | (0.12) | |
| $d\log(\text{sales}_{i,t-1})$ | -0.04 | -0.03 | -0.04 | -0.02 | -0.04 | -0.01 |
| | (0.00) | | (0.00) | | (0.00) | |
| Sector FE | Y | | Y | | Y | |
| Firm FE | Y | Y | Y | Y | Y | Y |
| Year FE | Y | | Y | | Y | |
| Observations | 370,973 | | 373,362 | | 370,973 | |

Notes: This table shows the baseline results of the regression using bankruptcy outcomes in the data and the model simulations. All variables are standardized, and leverage and cash-on-hand are also winsorized at level 0.5% and 99.5% and demeaned relative to the sector's average. Standard errors (in parenthesis) are clustered by firm. Coefficients are times 100.
*Data source*: Compustat quarterly.

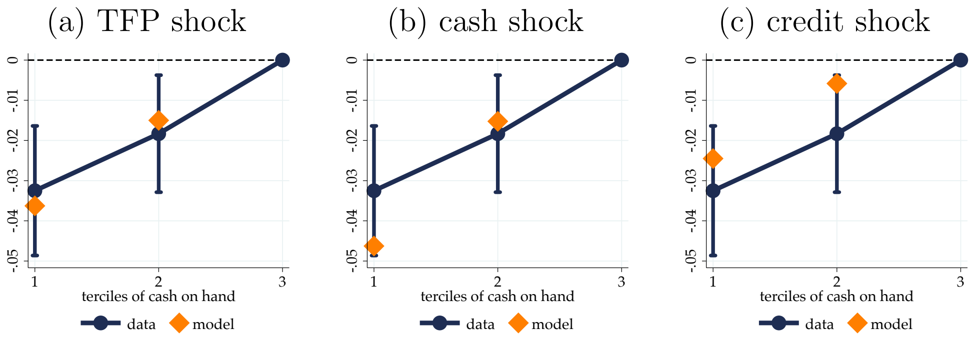
**Investment Heterogeneity in Recessions** I study the investment dynamics in the model-simulated data and observed data during the last recessions: Great Recession and Covid crisis. To estimate the *on impact* heterogeneous response — both in the model and the data — from peak-to-trough of recessions, I will proceed as follows. First, to account for permanent sectoral heterogeneity I will demean each of the firm-quarter observations of cash-on-hand over capital $n_{it}/k_{it}$ for firm $i$ in period $t$ of interest by its sectoral average, i.e. $\hat{n}_{it} = n_{it}/k_{it} - \mathbb{E}_s[n_{it}/k_{it}]$ for firm $i$ in sector $s$. Next, I will assign each firm-quarter observation of $\hat{n}$ to each tercile (for each period's distribution). Lastly, I run the following panel regression episode analysis to estimate the heterogeneous responses of investment across cash-on-hand $n/k$ during the recession:

$$\Delta_h \log(k_{it}) = \underbrace{\sum_{j=1}^{J} \beta_j^n \left(Q_{it}^{nj} \times \text{crisis}_t\right)}_{\text{heterogeneity across } n/k} + \underbrace{\Lambda' Z_{it}}_{\text{controls}} + \varepsilon_{it}, \tag{24}$$

where $Q_{it}^{nj}$ indicates if $\hat{n}_{it}$ belongs to tercile $j$, $\Delta_h \log(k_{it}) = \log(k_{it+h}) - \log(k_{it})$ is firm's $i$ capital accumulation over a period equivalent to the recession episode studied (i.e., the extension from peak-to-trough of episode studied $h$), $\text{crisis}_t$ indicates if a recession happens during the period considered (from $t$ to $t+h$) and $Z_{i,t}$ includes the control variables. For baseline specifications controls, I include firm's fixed effects, sectoral fixed effects, log assets as proxy for size, last quarter sales growth and heterogeneity across firm's leverage.[29]

[29] The empirical specification is similar to the one used in Kalemli-Özcan, Laeven and Moreno (2020).

**Figure A.3:** Investment heterogeneity during recessions: $\beta_j^n$

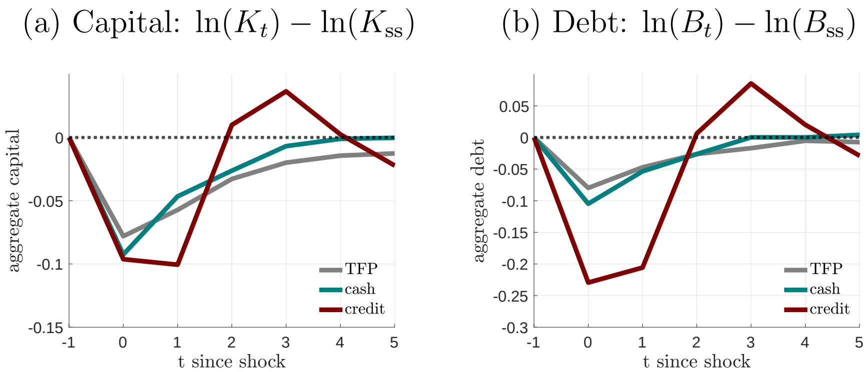



Notes: Panel (a), (b) and (c) show the estimates of $\beta_j^n$ for the Great Recession and Covid Crisis average, for the data and different aggregate shocks simulated in the model. The blue connected line shows the data and the 90% confidence interval. The diamond dots show the estimates from the model's simulated panel data. Since I focus on the heterogeneity, the coefficient values are relative to the tercile of firms with the highest cash-on-hand. All estimates (in model and data) are from empirical specification (24). Estimates are in semester frequency to make them comparable across episodes.

Coefficients $\beta_j^n$ are the estimates of interest and can be interpreted as difference-in-difference estimates of the recession episode impact on capital accumulation for firms in tercile $j$ of $\hat{n}$. The results are normalized relative to the group with the highest cash-on-hand. In the empirical application I use data from Compustat (limited to publicly traded firms) and in the model I select firms that approximate this set of firms. Figure A.3 shows the results. Further details of the data, estimates and other results are in Appendix B.

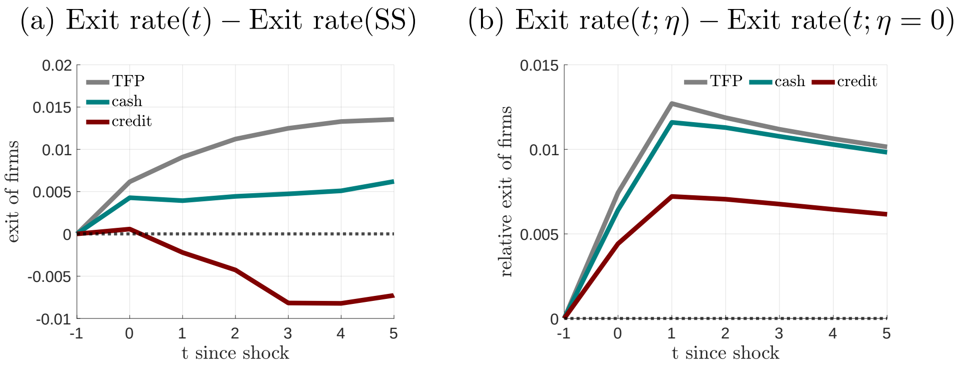
### A.3.2 Firm Exit and Spreads during Crises Experiment

**Figure A.4:** Capital and Debt during Crises




Notes: Figures show the dynamics of capital and debt accumulation for the three crisis shocks studied. In both panels, the variables are in terms of log difference relative to steady state — $\ln X_t - \ln X_{SS}$. Panel (a) shows the dynamics of aggregate capital accumulation. Panel (b) shows the dynamics of aggregate debt accumulation.

**Figure A.5:** Firm Exit during Crises

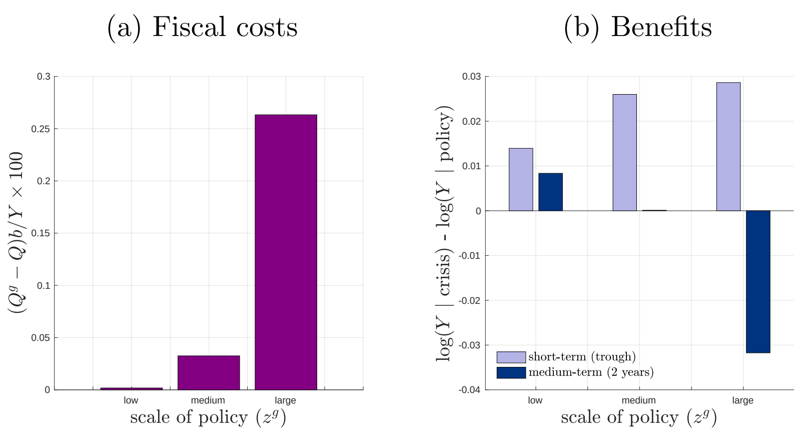



Notes: Figures show the dynamics of firm exit for the three crisis shocks studied. Panel (a) shows the difference between firm exit rates (exogenous and endogenous) relative to pre-crisis steady-state levels during the crisis episode. Panel (b) shows the difference between firm exit rates with coordination failures relative to the counterfactual without coordination failures during the crisis episode.

### A.3.3 Credit policy program

Figure  summarizes the gains and losses from the credit policy after a large negative aggregate TFP shock.

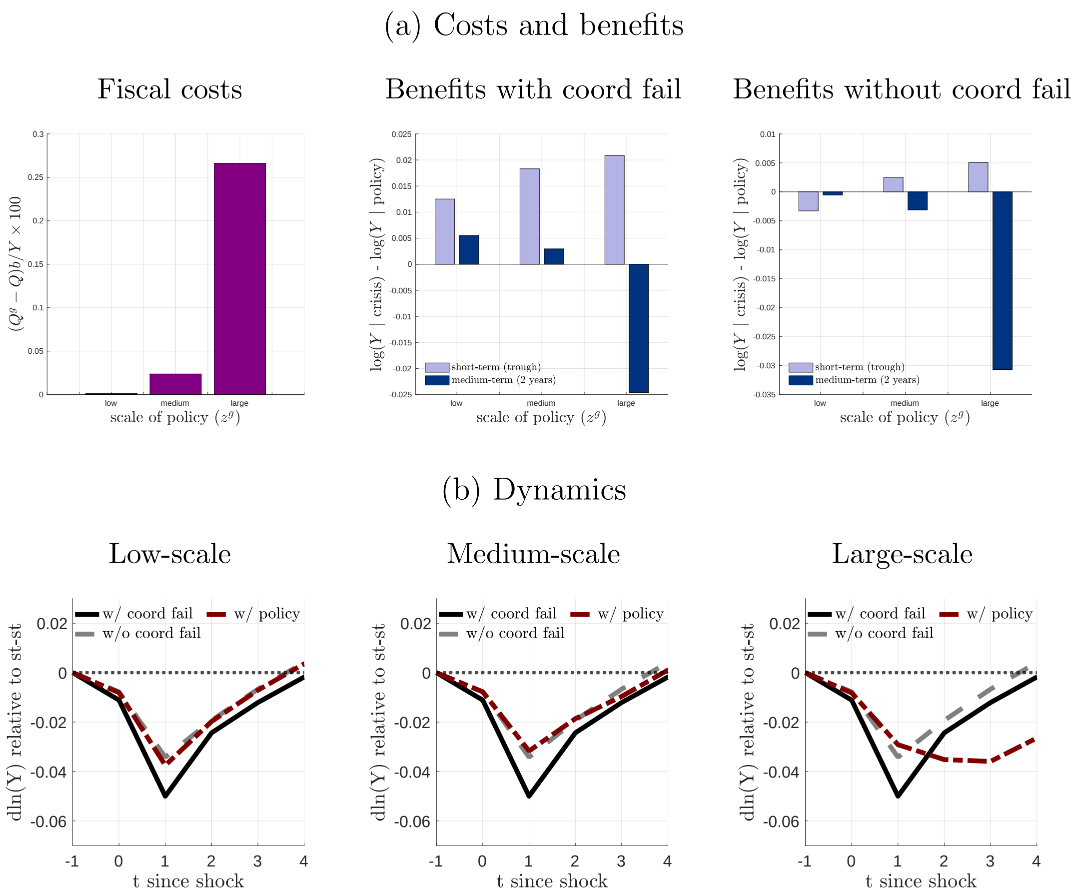
**Figure A.6:** Costs and Benefits of Imperfect Credit Policy by Scale




Notes: The Figure shows the policy costs and benefits for different policies during a large recession driven by a TFP shock. Panel (a) shows the fiscal cost of the policy in GDP terms, panel (b) the short (at the crisis' trough) and medium (2 years after the trough) term benefits of the policy. The economy at $t = -1$ is in steady-state. The definition of the shocks and crises experiments are in Section 4.1 and Appendix A.2.4. Further description of the policy in the text.

**Further quantitative results.** Figure A.7 shows the results for the policy experiments when the driving shock is a cash shock.

**Figure A.7:** Imperfect Credit Policy by Scale (cash shock)

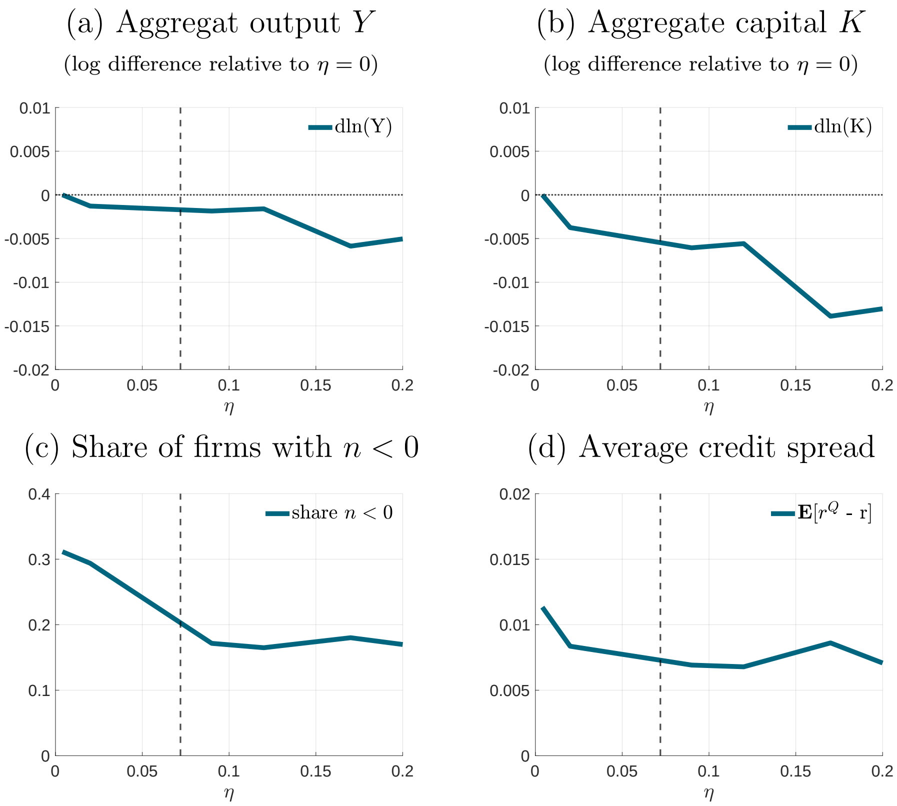



Notes: the Figure shows the policy costs and benefits for different policies during a crises driven by a TFP shock. Figure (b) shows the fiscal cost of the policy in GDP terms, the short (in the crisis trough) and medium (2 years) term benefits of the policy with and without coordination failures. Figures in Panel (b) show the response of aggregate output $Y$ to an aggregate cash shock with perfectly targeted policy (same as without coordination failures, dashed gray line), imperfectly-targeted policy (red dashed line), and without policy intervention (solid black line) for different policy scale. The economy at $t = -1$ is in steady-state. The definition of the shocks and crises expermients are in Section 4.1 and Appendix A.2.4. Further description of the policy in the text.

### A.3.4 Steady-state comparative statics

To study the long run implications of firms' rollover risk I make some simple comparative statics with $\eta$. Figure A.8 shows for different values of $\eta$ the output and capital level, and the share of firms with negative cash-on-hand and the average spread rate. I find that the importance of rollover crises in the long run is relatively low. First, in the long run, aggregate output $Y$ is 0.2% lower, see panel (a), and aggregate capital $K$ is 0.5% lower, see panel(b), because of creditors' coordination failures. Second, the higher is $\eta$ less firms have a weak balance sheet position in steady-state, see panel (c). Rollover risk shifts (improves) the financial distribution of firms significantly. The increase in the risk of rollover failure, for a given financial position, incentivize firms to save away; thus, accumulating internal resources to preclude the coordination failure. The improvement in the financial position is reflected on the little change observerd in credit spreads across $\eta$, even if the risk of rollover crises is greater (given the financial position). Overall, the likelihood of coordination failures for exposed firms $\eta$ shifts the financial position of firms, but don't impact significanly aggregate outcomes over the long run.

**Figure A.8:** Steady-state Comparison



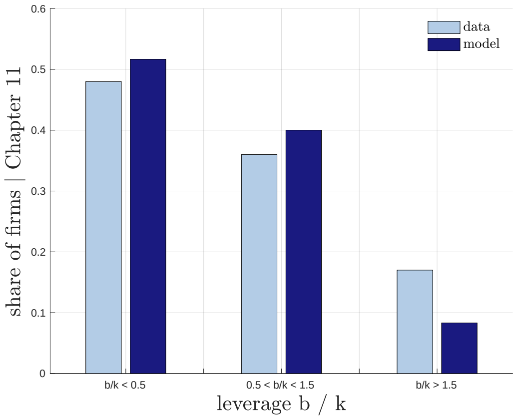

Notes : Panel (a) and (b) show the log difference in aggregate output $Y$ and capital $K$, respectively, across different values of $\eta$ in steady-state. Panel (c) and (d) show the share of firms with negative cash-on-hand $n$ and average credit spread rate across, respectively, across different values of $\eta$ in steady-states. In all the plots, the vertical dashed line indicates the calibrated value of $\eta$.

### A.3.5 Leverage Distribution of Bankrupt Firms

Although I target the average leverage ratio of firms in Chapter 11, I don't target it's distribution. In this section, I compare the share of bankrupt firms according to their leverage (liabilities over capital) position. I split in three groups, those bankrupt firms that have choose low leverage (less than 0.5), medium to high leverage (0.5 to 1.5) and extremely high leverage (more than 1.5). Figure A.9 shows that the model fits well the distribution of leverage for firms in Chapter 11 that continue operating.

**Figure A.9:** Leverage distribution of bankrupt (restructuring) firms

(a) Leverage ($b'/k'$) model and data



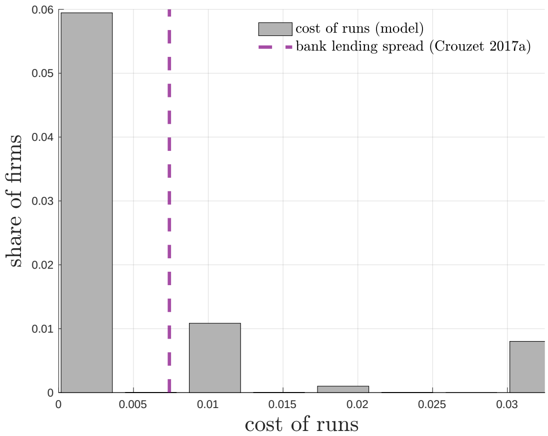

Notes: Figure shows the distribution of leverage for firms in the restructuring process in the model and the data (in Chapter 11 and operating the next period). Data source: Compustat.

Further details of the sample selection and data processing are in Appendix B.1.1.

### A.3.6 How costly is firm's rollover risk (ex-ante)?

In this section to assess how costly is rollover risk. For this, I explore the spread distribution between the pricing schedule with and without coordination failures, i.e., $\tilde{Q}\left(z, k', b'; \eta\right) - \tilde{Q}\left(z, k', b'; 0\right)$. As a benchmark, I compute how many firms would pay the bank's markup over market borrowing soley to preclude future creditors' coordination failures if they could. I use the spread in intermediation costs estimated by Crouzet (2017a) of 0.74% (annual). I find that only 2.2% of the firms face a cost of rollover risk higher than intermediation spread. Figure A.10 shows the distribution of the cost of rollover risk across firms that produce and don't exit at the end of the period. The figure shows that most firms face a cost close to 0 since many firms become exposed tomorrow only in case of an extremely bad shock, therefore the average cost is negligible. The small cost of rollover risk ex-ante in steady state suggests that it can be optimal for firms choose a liability structure where they are exposed to rollover crises.

**Figure A.10:** Cost of Rollover Risk (in annual spread terms)




Notes : Figure shows the distribution of the cost of firms' rollover risk — i.e., $\tilde{Q}\left(z, k^{'}, b^{'}; \eta\right) - \tilde{Q}\left(z, k^{'}, b^{'}; 0\right)$ — for producing firms that don't exit at the end of the period. Exclude from plot the ones with 0 cost and truncated distribution at 3% cost. The spread of intermediation between bank and market lending is from the calibration of Crouzet (2017a).

# B Data

In this section, I provide details of the data sources, sample selection and definitions of the variables, and provide additional empirical results.

## B.1 Sources, Sample and Variables

In this section I describe the details (definitions and sample construction) of the main data sources used to compute moments related to the balance sheet of firms and empirical exercises in the paper.

### B.1.1 Compustat

I use Compustat data to compute moments related to the balance sheet of firms and bankruptcy process, and study the patterns of investment in recent large crises. Compustat is limited to publicly held firms, therefore I assume the balance sheet distribution replicates in the rest of the firms.[30] To construct the sample I follow standard practices in the empirical investment literature.

**Balance Sheet Data.** I explain how I construct the sample and the variables for the balance sheet data used for calibration and empirical exercises. The sample selection

[30] An alternative approach is to fit the model to a subset of firms that can be defined as the Compustat firms. For simplicity I use the assumption described in the text.

criteria follows a firm level filter and firm-date filter. Table B.1 shows the number of observations and those dropped by each filtering step. I drop firms from finance, insurance, and real estate sectors ($\texttt{sic} \in [6000, 6799]$), utilities ($\texttt{sic} \in [4900, 4999]$), nonoperating establishments ($\texttt{sic} = 9995$) and industrial conglomerates ($\texttt{sic} = 9997$), and those not incorporated in U.S. and not operate in USD. I drop firm-date observations that with negative capital or total assets, observations with acquisitions of more than 5% of firm's assets, bottom 0.5% and top 99.5% investment rate across the distribution, investment spells of less than 20 quarters, drop if net liquid leverage (net current liquid debt/total assets) is greater than 10 in absolute value, drop if log sales growth is greater than 1 in absolute value, and negative sales or negative liquid assets.

Due to changes in the accounting data of Compustat, I split the sample for the Great Recession (period 1983-2017) and Covid-19 Crisis (period 2019-2020) [see Ma (2020) notes on the accounting changes after 2019].[31] The sample criteria for the 2019-2020 period differs slightly from the 1983-2017 sample. Since the 2019-2020 sample is smaller I exclude filters related to investment outliers and spells, and select firm-quarter observations that register they changed they updated their accounting criteria.[32] Lastly, for the annual data we follow a similar sample selection criteria (we exclude the intermittent observations filter).

**Table B.1:** Sample Selection Compustat

| | # Drop | # Obs |
|---|---|---|
| *Annual* | | |
| 1983-2017 | | 437,226 |
| Non-financial sector | 126,425 | 310,801 |
| U.S. incorporated and USD currency | 25,680 | 285,121 |
| Exclude outliers | 52,172 | 232,949 |
| *Quarterly (Pre-Covid)* | | |
| 1983-2017 | | 1,484,973 |
| Non-financial sector | 474,327 | 1,010,646 |
| U.S. incorporated and USD currency | 216,697 | 793,949 |
| Exclude outliers and intermittent | 367,476 | 426,473 |
| *Quarterly (Covid)* | | |
| Change in accounting | | 39,532 |
| 2019-2020 | 1,895 | 37,637 |
| Other filters | 23,663 | 13,974 |

[31] An alternative approach is to use Compustat Snapshot to remove the operation leases from various entries in the balance sheet, but access to this dataset is restricted.

[32] The variable `acctchgq` is `ASU16-02` or `IFRS16` the quarter the firm changes it's accounting criteria.

The final sample — pre-Covid — has 426,465 firm-date observations for the quarterly sample and 232,949 firm-year observations for the annual sample, and the Covid sample has 13,974 firm-date observations for the quarterly sample.

The definition of the main variables used for the calibration and regressions are:

1. Capital stock $k$: is constructed using the perpetual inventory method, following the usual convention in the investment literature.[33] I compute the initial capital level using the level of gross plant, property and equipment `ppegtq`, and using the quarterly change of net plant, property and equipment `ppentq`. The deprecation rates $\delta$ are calculated using the BEA accounts to compute investment rates (i.e., change in capital $k$ net of capital depreciation).

2. Net debt stock $b$: different from other papers in the literature I assume $b$ corresponds to the short-term liabilites. Liabilities include financial debt, debt with suppliers and other firms, accounts and tax payables, and others. `lctq` minus cash holdigns `cheq`. On the other hand, the gross debt position $\max b, 0$ is defined as the short-term liabilities `lctq`.

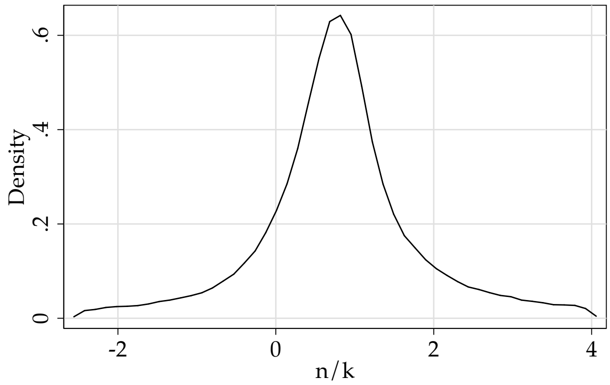
3. Operating profits $\pi$: corresponds to the variable `ibdpq`

4. Liquid value of assets $q\omega k\,(1-\delta)$: to compute this I use the assets of the firm (excluding cash) as follows: for asset category $a_{ij}$ we can compute the liquid value of firms' assets as $\sum_j lr_j \times a_{ij}$ where $lr_j$ is the liquidation rate. The liquidation rates used by asset category are 44% inventories, 63% receivables and 35% physical capital from Kermani and Ma (2021).

5. Cash-on-hand $n$: is computed as the sum of $\pi$ and $q\omega k\,(1-\delta)$ minus $b$. It is assumed that all liabilities can be collected each period. Figure B.1

[33]See for example, Mongey and Williams (2017); Jeenas (2019); Ottonello and Winberry (2020) for recent references.

**Figure B.1:** Empirical distribution of firms' cash-on-hand ($n/k$)




Notes: Figure shows the distribution of the cash-on-hand for all firms in the Compustat baseline sample. Data source: Compustat.

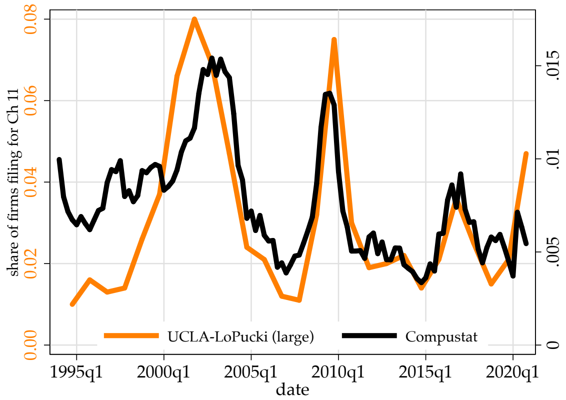
6. Size: log of total assets `atq`.

7. Sales growth: quarterly growth of sales `saleq`.

Nominal variables are deflated using the BLS implicit price deflator, unless specified. Percentiles of variables used are constructed by year (not quarter). When specified variables are standarized, winzorized and/or demeaned.

**Bankruptcy Data.** To identify when and what firms operate under Chapter 11 in Compustat I use the same criteria as Corbae and D'Erasmo (2021). I use the footnote to total assets (`atq`) and deletion information variables `dlrsn` and `dldte`. A firm is under Chapter 11 (i) if the firm's footnote reports adoption of new accounting under Chapter 11 bankruptcy; (ii) if the firm shows as bankrupt but is not deleted from the data; (iii) if the firm shows as bankrupt and is deleted but this is not due to liquidation; or (iv) if the firm's last observation in the sample is bankruptcy but there is no bankruptcy information.

**Figure B.2:** Filings to Chapter 11




Notes : The figure shows the filings to Chapter 11 in the last 12 months. Filings to Chapter 11 are identified through the steps detailed in the text. Data sources: Compustat-Quarterly and UCLA-LoPucki.

Figure *B*.2 shows that the Compustat data evolution is consistent with UCLA-LoPucki estimates which are for large firms in US. Next, I show some summary statistics that compare firms in Chapter 11 and outside Chapter 11. Table B.2 shows that firms operating in Chapter 11 have a lower size, lower investment, lower sales growth, are more leveraged and more of them have negative internal resources.

**Table B.2:** Compustat Chapter 11 Firms' Summary Statistics

| | Chapter 11 | All |
|---|---|---|
| Leverage: $\mathbb{E}[b/k]$ | 0.68 | 0.37 |
| Negative cash-on-hand: $\mathbb{E}[1_{\{n<0\}}]$ | 0.38 | 0.21 |
| Investment rate (annualized, median): $\mathcal{P}_{50}[i/k]$ | -0.9% | 12.6% |
| Real sales growth (annualized): $\mathbb{E}[\log(\text{sales}_t/\text{sales}_{t+1})]$ | -8.8% | 9.5% |
| Size (2017 USD millions): $\mathbb{E}[\text{total assets}]$ | 1,625 | 2,181 |
| Observations (firm $\times$ year) | 2,519 | 228,212 |

*Notes*: This table compares firms in Chapter 11 with all the firms in the economy across several characteristics. Real quantities are calculated using the GDP implicit price deflator.
*Data source*: Compustat.

### B.1.2 Federal Judicial Center - Integrated Database (FJC-IDB).

FJC-IDB bankruptcy data includes all petitions filed under the Bankruptcy Code (any of the Chapters) on or after October 1, 2007 and any petitions filed before October 1, 2007 that are still pending. This dataset provides information of the fillings, closures and several firm characteristics.

I will focus on a sample of corporate firms filings to Chapter 11 and Chapter 7. This includes public and privately held firms. Table B.3 shows the sample selection criteria.

**Table B.3:** Sample Selection FJC-IDB

| | # Drop | # Obs |
|---|---|---|
| All | - | 32,084,867 |
| Fillings | 18,259,254 | 13,825,613 |
| Business | 13,654,287 | 171,326 |
| Chapter 11 and Chapter 7 | 1,975 | 169,351 |
| Exclude missing size data | 5,124 | 164,227 |

*Notes*: This table shows the number of observations resulting from the sample selection for the FJC. The first line, *All*, shows the original number of entries from in the dataset. from 2008 to 2020, and the following lines the observations dropped after applying different filters to the sample and the resulting number of observations.
*Data source*: FJC-IDB.

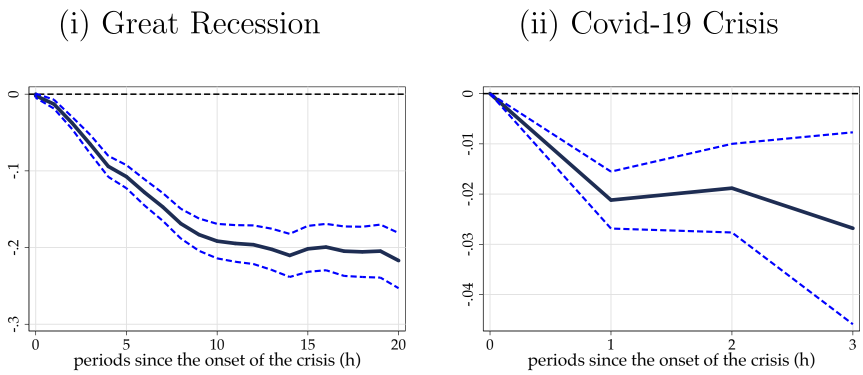
## B.2 Additional Results

### B.2.1 Heterogeneous Investment Responses During Recent Crises

In this section I will study the heterogeneous investment response of firms during the Great Recession and Covid-19 crisis. First, I will show the aggregate dynamics of the crises. Second, I will show the heterogeneity across the balance sheet positions, focusing on cash-on-hand and leverage positions.

**Average Dynamics** Figure B.3 shows that using firm level data of publicly listed firms the capital accumulation rate drop significantly in both episodes — Great Recession and Covid-19 crisis.

**Figure B.3:** Corporate Investment in Recent Crises Episodes
$\Delta k^{\text{crisis}}_{t+h} - \Delta k^{\text{no crisis}}_{t+h}$




Notes: figures show the dynamics of the capital stock relative accumulation during the Great Recession and Covid crisis. The change in capital accumulation comes from the followin specification using firm-level data: $\log(k_{it+h}) - \log(k_{it}) = \alpha_i + \beta_h \text{crisis}_t + \varepsilon_{it+h}$, where $\text{crisis}_t$ indicates the pre-crisis peak and $\beta_h$ is the h-periods ahead change in the accumulation of capital during the crisis episode relative to no crisis periods. Drop $t$ such that for $\text{crisis}_{t+i} = 1$ for at least one $i \in \{0, .., h\}$, i.e. capital accumulation before the crisis overlaps with the crisis. Panels (a) and (b) show coefficients $\beta_h$ and their 90% confidence interval. Standard errors are clustered at firm level.
Data source: Compustat.

**Heterogeneous Responses** To estimate the *on impact* heterogeneous response — from peak-to-trough of the crisis — I will proceed as follows. First, to account for permanent sectoral heterogeneity — in my baseline estimations — I will demean each of the firm-quarter observations of variable $x$ of interest by its sectoral average, i.e. $\hat{x}_{it} = x_{it} - \mathbb{E}_s[x_{it}]$. Next, I will assign each firm-quarter observation of $x$ to different quartiles (terciles if Covid sample) relative to the annual distribution. Lastly, I run the following panel regression to estimate the heterogeneous responses of investment across cash-on-hand $n/k$ and leverage $b/k$ during the crisis:

$$\Delta \log(k_{it}) = \underbrace{\sum_{j=1}^{J} \beta^n_j \left(Q^{nj}_{it} \times \text{crisis}_t\right)}_{\text{het across } n/k} + \underbrace{\sum_{j=1}^{J} \beta^b_j \left(Q^{bj}_{it} \times \text{crisis}_t\right)}_{\text{het across } b/k} + \underbrace{\Lambda' Z_{it}}_{\text{controls}} + \varepsilon_{it}, \tag{25}$$

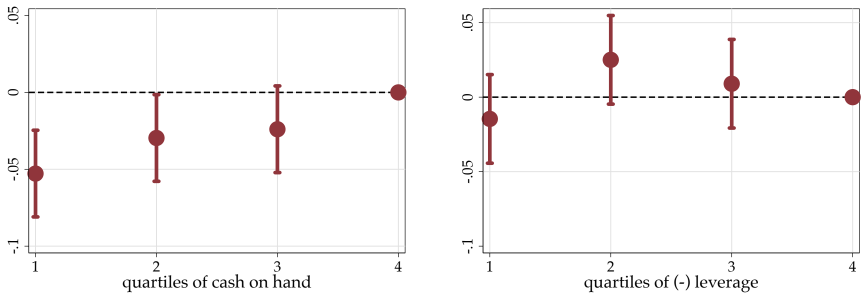
where $Q^{xj}_{it}$ indicates if $\hat{x}_{it}$ belongs to quartile or tercile $j$, $\Delta \log(k_{it}) = \log(k_{it+h}) - \log(k_{it})$ is the capital accumulation over a period as long as the crisis studied (i.e., the extension from peak-to-trough of episode studied $h$), $\text{crisis}_t$ indicates if a crisis happens during the period considered and $Z_{i,t}$ includes the control variables. For the baseline specifications I include as controls firm's fixed effects, sectoral fixed effects, log assets as proxy for size and last quarter sales growth. The coefficients $\beta^x_j$ are interpreted as the diff-in-diff estimates of the crisis impact on capital accumulation for firms in quartile or tercile $j$ of $\hat{x}$.

The empirical strategy is close to the one used in other work that studies investment adjustment heterogeneity on recent crises episodes. Salient examles are Almeida, Campello, Laranjeira and Weisbenner (2012) for the Great Recession in U.S. and Kalemli-Özcan *et al.* (2020) for the EU crisis.

**Figure B.4:** Heterogeneous Investment Response during Crises

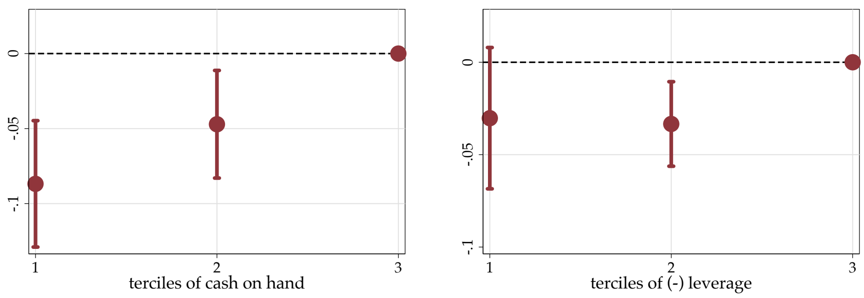
(a) Great Recession

(i) Across cash-on-hand: $n/k$ (ii) Across leverage: $b/k$




(b) Covid Crisis

(i) Across cash-on-hand: $n/k$ (ii) Across leverage: $b/k$




Notes: Figures show the change in the capital accumulation from peak to trough in both episodes. For the Great Recession the episode is from 2007q4 to 2009q4, and for the Covid-19 crisis is from 2019q4 to 2020q2. Figures in panel (a) show the coefficient $\beta_j^n$ and Figures in panels (b) shows $\beta_j^b$ for the Great Recession and Covid-19 crisis in a joint estimation of specification (25). Coefficients are normalized to 0 with respect to the highest quartile or tercile coefficient. The interval is at 90% confidence level and standard errors are clustered at firm level for the Great Recession and sector level for the Covid-19 crisis. Balance sheet variables are demeaned at sectoral level. Because of data limitations the estimates of the Covid-19 crisis don't include firm's FE. Coefficients are in annual terms.
Data sources: calculations using Compustat data.

Figure B.4 shows the investment response across different levels of cash-on-hand and leverage during the Great Recession and Covid crisis. For both episodes, panel (a) and (b) figure (i) show that firms with low levels of cash-on-hand adjust substantially more their investment, around 5-10 p.p. points in annual terms relative to the firms with the highest levels of cash-on-hand. On the other hand, panel (a) and (b) figure (ii) show that the heterogeneity across leverage is not significant. In Section 3.3, I contrast these results with simulations from the model.

These findings are related to a recent literature that studies empirically the heterogeneity of investment responses across firm's financial positions during recent large recession episodes. Two salient examples are Almeida *et al.* (2012) for the Great Recession in U.S. and Kalemli-Özcan *et al.* (2020) for the EU crisis. Almeida *et al.* (2012) explore the relevance of long term debt that matured in the short-term during the Great Recession in U.S. and find that firms with more long term debt maturing in the short-term reduced more their investment. Moreover, Kalemli-Özcan *et al.* (2020) find evidence of debt overhang problems and rollover risk being relevant during the EU crisis. These patterns were greater in peripheral europe countries, which were hardly hit by the crisis. In addition, Ebsim *et al.* (2021) show that cash holdings were relevant explaining the heterogeneity of credit spreads dynamics during the Covid crisis, but they weren't relevant during the Great Recession.

### B.2.2 Firm's Balance Sheet

In this section, I study what is the firms' debt maturity structure and how many creditors do firms have. Table B.4 shows that corporate firms use extensively short-term liabilities to finance their investments and operations, and Table B.5 shows that the great majority of medium to large corporate firms (i.e., with more than 50 million assets) in U.S. borrow from hundreds of creditors. These results are consistent with Crouzet (2017a), who observes that corporate firms' leverage is mostly composed by bonds, which tend to have a very dispersed ownership.

Furthermore, in Appendix A.3.6, I study the ex-ante costs of rollover crises and find that they are moderate to small in steady state. Thus, the observed liability structure could be consistent with a theory where changing the liability structure is moderately costly. In this setup, most firms' liability structure would remain unchanged, even if their liability structure exposed them to problems such as rollover crises.

**Table B.4:** Firms' Debt Maturity

| | Time to mature (share) | | |
|---|---|---|---|
| | $< 1$ year | 1 to 4 years | $\geq 5$ years |
| Debt | 0.29 | 0.33 | 0.38 |
| | (0.32) | (0.28) | (0.34) |
| | $< 1$ year | $> 1$ years | |
| Liabilities | 0.61 | 0.39 | |
| | (0.29) | (0.29) | |

*Notes*: the table shows the share of debt or liabilities maturing at different time horizons. The summary statistic is computed for the average firm, in parenthesis is the standard deviation. Short term liabilities are `lct` and long-term `lt - lct`. Debt maturing in less than one year is `dlc`, in one to four years is `dd2` + `dd3` + `dd4`, and maturing at 5 or more years is `dltt-dd2-dd3-dd4`. Total debt is `dlc` + `dltt` and total liabilities is `lct`.
*Data source*: Compustat.

**Table B.5:** Number of Creditors When Filing to Bankruptcy

| | # Creditors | | |
|---|---|---|---|
| | 1 to 100 | 101 to 1,000 | >1,000 |
| Small (< 50 million assets) | 0.88 | 0.10 | 0.02 |
| Medium (> 50 million and < 1 billion assets) | 0.16 | 0.19 | 0.65 |
| Large (> 1 billion assets) | 0.03 | 0.04 | 0.93 |
| All | 0.73 | 0.10 | 0.16 |

*Notes*: the table shows the share of firms with by creditor number groups and size when filing to Chapter 11 bankruptcy. Shares are relative to the total filings of each size group. Asset value correspond to the one declared when filing for bankruptcy.
*Data source*: FJC-IDB.

# C Model Extensions

In this section, I will study two extensions of the model, one that uses more general functional forms for the operational profits function, capital adjustment idiosyncratic frictions and long-term debt, and other that allows firms to issue equity (costly).

## C.1 Long-term debt and firm-level capital frictions.

I assume profits are a function $\pi(\mathbf{z}, k) \in \mathbb{R}$ strictly increasing in both arguments, where $\mathbf{z} = (\mathbf{z}^p, \mathbf{z}^{\text{iid}})$ is a vector of shocks that contain a set of persistent shocks $\mathbf{z}^p$ follow a markov process and $\mathbf{z}^{\text{iid}}$ follow an iid process. Both are related to idiosyncratic productivity and cost shocks. Next, I assume that $\iota(\omega k, k') \in \mathbb{R}$ is the investment expenditure function of the firm that is decreasing on $k$ and increasing on $k'$, where $-\iota(\omega k, 0) \geq 0$ is the liquidation value of capital.[34] Last, I assume that the firm can issue long-term debt, which fraction $m^b \in (0, 1]$ matures randomly each period and pays $c^b \geq 0$ cupon payments on non-maturing debt. The rest of the model it follows as the baseline model.

I focus on the characterization of the liquidation choice. For the extended setup, firms dividends now can be defined as

$$d = \pi\left(\mathbf{z}, k\right) - \iota\left(k, k^{'}\right) - b\left[m^b + \left(1 - m^b\right)c^b\right] + Q\left(b^{'} - \left(1 - m^b\right)b\right) \geq 0$$

where $Q\left(b^{'} - \left(1 - m^b\right)b\right)$ is the amount of new debt issued. Analogous to the baseline model, firms can default after issuing the new debt. The firm never default whenever

$$\max_{k^{'}} \pi\left(\mathbf{z}, k\right) - \iota\left(k, k^{'}\right) - b\left[m^b + \left(1 - m^b\right)c^b\right] =$$

[34] I assume no capital quality shock $\omega$ for notational clarity.

$$\underbrace{\pi(\mathbf{z},k)-\iota(k,0)-b\left[m^b+\left(1-m^b\right)c^b\right]}_{n(\mathbf{z},k,b)}\geq 0. \tag{26}$$

where I can define $n$ as the cash-on-hand of the firm is the sum of operational profits, liquidation value of capital, and maturing debt and cupon payments. On the other hand, we have that the firm will always default whenever

$$\begin{aligned}\pi(\mathbf{z},k)-b\left[m^b+\left(1-m^b\right)c^b\right]+\max_{k',b'}\left\{-\iota\left(k,k'\right)+\tilde{Q}\left(\mathbf{z}^p,k',b'\right)\left(b'-\left(1-m^b\right)b\right)\right\}&=\\ n(\mathbf{z},k,b)+\underbrace{\max_{k',b'}\left\{-\iota\left(k,k'\right)+\iota(k,0)+\tilde{Q}\left(\mathbf{z}^p,k',b'\right)\left(b'-\left(1-m^b\right)b\right)\right\}}_{-\underline{\mathrm{n}}\left(\mathbf{z}^p,k,b\right)}&<0\end{aligned} \tag{27}$$

For multiplicity to exists we need that conditions (26) and (27) don't hold, i.e.,

$$0>n(\mathbf{z},k,b)\geq\underline{\mathrm{n}}\left(\mathbf{z}^p,k,b\right). \tag{28}$$

Notice $\underline{\mathrm{n}}\left(\mathbf{z}^p,k,b\right)$ bounded below by 0 (we can always implement $\{k'=0,b'=b\}$). Moreover, there is the possibility of multiple equilibrium whenever the firm can have strictly positive external resources in this region of the state-space. Analogous to the baseline model, the firms default decision is determined by the firm's cash-on-hand and a threshold that depends on the fundamentals of the firm (shocks and financial position).

Further, assume there is no bankruptcy, $c^b=0$ and creditors have no recovery for clarity, then the fundamental pricing schedule $\tilde{Q}$ (without coordination problem today) is pinned down by creditors no profit condition and is

$$\tilde{Q}\left(\mathbf{z}^{\mathbf{p}},b',k'\right)=\mathbb{E}\left[\Lambda\left(\mathbf{1}_{\{n\geq\underline{\mathrm{n}}\}}-\eta\mathbf{1}_{\{0>n\geq\underline{\mathrm{n}}\}}\right)\left(m^b+\left(1-m^b\right)\tilde{Q}'\right)\right]. \tag{29}$$

The pricing schedule with long-term becomes recursive. Also tomorrow's coordination failures show up in the pricing schedule. These two observations suggest, in the firm problem with long-term debt, rollover crises could even be greater than in the baseline model. With long-term debt the pricing schedule is affected by the future stream of expected rollover crises, which can augment their impact.

## C.2 Equity issuance

In the baseline specification, I don't allow firms to issue equity — $d\geq 0$. This assumption is consistent with the relatively low equity issuance observed in the data, and helps on the tractability of the characterization and computational solution of the model. In this section, I will relax this assumption and show how this affects the characterization of the liquidation choice (equilibrium multiplicity). Moreover, I provide a discussion on the model concepts of rollover and solvency in the model.

Firms issue equity $e < 0$ at cost $\phi(e)$, which is decreasing in $e$ and unbounded. I assume that equity is raised at the end of the period. Therefore, firms that never default are those when $Q = 0$ they don't default, i.e.,

$$V^{Q=0}(z, n) \geq 0$$

where $V^{Q=0}(z, n)$ is determined by

$$V^{Q=0}(z, n) = d + \mathbb{E}\left[\Lambda \tilde{V}\left(\mathbf{s}^{'}\right)\right]$$

subject to

$$d = \begin{cases} e & \text{if } e \geq 0 \\ e - \phi(e) & \text{if } e < 0 \end{cases}$$
$$e = n - qk^{'}$$

where continuation value $\tilde{V}\left(\mathbf{s}^{'}\right)$ is analogous to one defined in the baseline firm problem. Thus, we can define safe region

$$\mathcal{S} = \{(z, n) : V^{Q=0}(z, n) \geq 0\}. \tag{30}$$

On the other hand, firms that default are those default even if $Q > 0$, i.e.,

$$V^{Q>0}(z, n) < 0$$

where $V^{Q>0}(z, n)$ is determined by

$$V^{Q>0}(z, n) = d + \mathbb{E}\left[\Lambda \tilde{V}\left(\mathbf{s}^{'}\right)\right]$$

subject to

$$d = \begin{cases} e & \text{if } e \geq 0 \\ e - \phi(e) & \text{if } e < 0 \end{cases}$$
$$e = n + \tilde{Q}\left(z, b^{'}, k^{'}\right) b^{'} - qk^{'}$$

where $\tilde{Q}$ fundamental pricing schedule (no liquidation today) and continuation value $\tilde{V}\left(\mathbf{s}'\right)$ (this analogous as the one in the baseline firm problem). Thus, we can define liquidation region

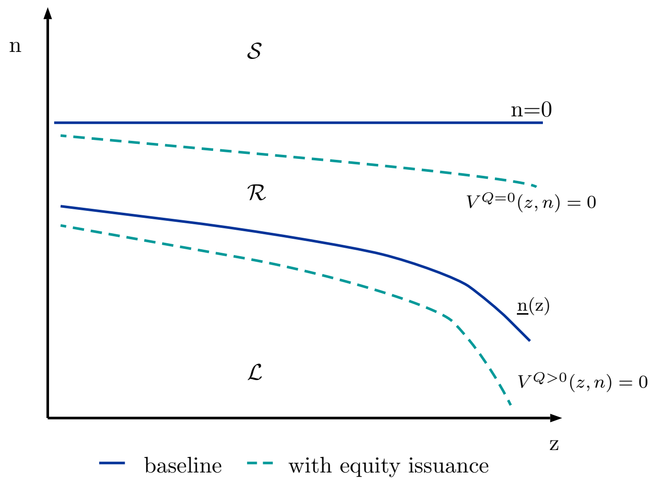
$$\mathcal{L} = \{(z, n) : V^{Q>0}\,(z, n) < 0\}. \tag{31}$$

Last, it's straightforward to show that $V^{Q>0}(z, n) \geq V^{Q=0}(z, n)$, then under certain conditions it can be the case that firm is in a region that is undetermined, i.e.,

$$\mathcal{R} = \{(z, n) : V^{Q>0}\,(z, n) \geq 0 \quad \text{and} \quad V^{Q=0}\,(z, n) < 0\}. \tag{32}$$

Similar to the baseline mode, we have that firms can be exposed to coordination failure even if they can issue equity. Assume the equity issuance function $\phi(e) = \lambda|e|$ with $\lambda > 0$. In Figure C.1, I illustrate how these affects the characterization of the regions.

**Figure C.1:** Rollover and solvency regions across $(z, n)$
Baseline vs Equity Issuance model




Notes: figures shows the state-space $(z, n)$ and the relevant regions for the liquidation choice for the baseline model (solid blue lines) and the model with equity issuance (dashed cyan lines).

Finally, it's worth noticing that in the model with unbounded equity issuance firms in $\mathcal{L}$ threshold — $V^{Q>0}\,(z, n) = 0$ — have 0 value, which is the standard notion of economic insolvency. On the other hand, in the baseline model, or with bounded equity issuance, firms in $\mathcal{L}$ threshold $(\underline{\text{n}}(z))$ could have strictly positive value. For my calibration, I find that firms in the insolvency threshold have values close to 0 — $V(z, \underline{\text{n}}(z)) \approx 0$ —; therefore, it approximates well the standard notion of insolvency.

# D U.S. Bankruptcy Code

In this section, I provide a brief review of some institutional details of the bankruptcy process for firms in the U.S. bankruptcy code. Chapter 7 and 11 are the typically used to liquidate or restructure the firm's liabilities. Chapter 7 is associated with firm liquidations, and Chapter 11 with restructurings (or sometimes called "reorganizations") and liquidations through piecemeal sales of firms.

**Chapter 7 bankruptcy.** Firms can enter a Chapter 7 liquidation process by filing directly to this chapter or being redirected by court ruling from other chapters (e.g., a judge may rule that a Chapter 11 case is switched to a Chapter 7 one). In this process, a case impartial trustee is appointed by the court to gather and sell the bankrupt firms assets to pay the firm's creditors.

**Chapter 11 bankruptcy.** Cases begin usually with the voluntary filling of the debtor (firm). Involuntary petitions (done by creditors) are very rare. When filling the firm automatically assumes an additional identity as the "debtor in possession." by 11 U.S.C. § 1101. The DIP provisions can provide access to new credit for the firm (DIP financing) and the automatic stay of firm's debt payments by 11 U.S.C. § 362(a) preclude (most) creditors from collecting the firm's debt. Further, when filing automatically a creditor's committee is appointed, which typically consists of the unsecured creditors who hold the seven largest unsecured claims against the debtor. Further, is common that creditors form ad hoc committees to coordinate their actions and have further surveillance over the debtor-in-possession's management of the firm.

The firm usually files a written disclosure statement and a reorganization plan. The disclosure statement contains information of the firms' assets, liabilities and other business affairs. Typically, the disclosure statements contains a counterfactual analysis of the credit recovery rates under liquidation (liquidation analysis) and other information relevant for the judge to decide if the reorganization chapter is appropriate. Lastly, the plan presented by the creditors needs to be approved by the creditors for the restructuring to be executed.

Moreover, Chapter 11 process are sometimes used by large firms to piecemeal liquidate the firm. The provisions provided by 11 U.S.C. § 363(b), "363 sales", allow firms to liquidate part of the firm's assets without the creditors' consent. This process is closer to a Chapter 7 "piecemeal" liquidation of the firm, instead of a restructuring or reorganization.