Firms' Rollover Risk and Macroeconomic Dynamics

Rafael Guntin[*]

New York University

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Abstract

This paper analyzes the macroeconomic implications of firms' rollover risk. I develop a heterogeneous-firms macroeconomic model with rollover crises emerging from coordination failures among creditors. Rollover crises are events in which a firm defaults because creditors fail to roll-over its debt, but would have repaid otherwise. I assess the quantitative relevance of rollover crises by employing a model-based identification strategy which argues that their incidence is informed by the observed distribution of firms' bankruptcy outcomes, and find that roughly half of the bankruptcy events are due to rollover crises. I validate the model using individual firms' observed investment dynamics during the last recessions and then use the model to assess the aggregate implications of rollover risk for the U.S. economy. I find that rollover risk can significantly amplify the impact of recessions. Lastly, I show that imperfectly-targeted credit policies can mitigate rollover crises but can exacerbate firms' debt overhang in the future.

1 INTRODUCTION

I study the impact of firms' rollover crises on macroeconomic dynamics. Rollover crises are events in which an *economically solvent* firm — i.e., with positive net present value — defaults because its creditors fail to roll-over the debt. Although the notion of firms' rollover crises is often alluded by policymakers during recessions and top managers of bankrupt firms, we know little about their quantitative and aggregate implications.[1]

To assess the aggregate implications of firms' rollover risk, I develop a quantitative macroeconomic framework where firms' rollover crises can be identified and quantified. In the framework, there are complementarities between creditors' decisions. Rollover crises are events in which a firm defaults because creditors fail to roll-over its debt, but would have repaid otherwise. Provisions in the U.S. bankruptcy code are designed to mitigate these types of crises. I use an approach that combines model and data on firms' bankruptcy outcomes and bankrupt firms' characteristics to assess the incidence of rollover crises. I find that roughly half of the bankruptcy events are driven by rollover crises.

I then conduct a quantitative analysis of the U.S. economy to assess the impact of firms' rollover risk on macroeconomic dynamics. I find that rollover risk can explain between 10% and 30% of the drop in aggregate output during large recessions. Finally, I study the effectiveness of an imperfectly-targeted credit policy — akin to those used extensively during the Covid crisis and other recessions — and find that in the short term the policy can prevent rollover crises but can backfire if it exacerbates firms' future debt overhang.

The framework is a general equilibrium model populated by heterogeneous firms which use internal resources and/or issue debt to finance investment and production. There are three key ingredients. First, firms have no commitment to repay their debt; thus, endogenous default risk limits their borrowing capacity. Second, using tools from

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[1]See, for example, Financial Times (09/30/2014) describes the discussion around the debt rollover and solvency issues of Lehman Brothers and other financial institutions during the 2008 financial crisis. More recently, in the Covid crisis, the Federal Reserve intervention in the corporate debt markets was limited to solvent but "illiquid" non-financial firms by Section 13(3) of the Dodd Frank act. Where illiquidity is usually interpreted as a problem akin to a firm's rollover crisis. Moreover, Ayotte and Skeel (2013), and references within, argue frequently the bankrupt firm and/or judge, during bankruptcy procedures, mention financial problems akin to rollover crises as the main driver of the firm's bankruptcy filing. Some salient examples are the recent bankruptcy cases of Kodak and Hertz.

the literature that studies self-fulfilling sovereign debt crises [Cole and Kehoe (2000)], I incorporate potential coordination failures among firms' creditors — rollover crises. Third, emulating the U.S. bankruptcy code, firms can decide to liquidate and exit, or restructure their liabilities and continue operating.

The timing in the model creates complementarities between debt prices and firms' default choices, which can lead to multiple equilibria in the firm problem. I characterize the solution of the model and find that there are three types of firms in the economy. First, there are safe firms with strong fundamentals that will not default even if creditors fail to rollover the debt. On the other extreme, there are insolvent firms with weak fundamentals that default even if creditors would be willing to rollover. Finally, there are risky firms exposed to rollover problems that default or not depending on the creditors' coordination, then the equilibrium outcome for the firm is undetermined. Formally, to construct the equilibrium for the last type of firms, I assume that an *idiosyncratic* and stochastic sunspot variable selects the equilibrium with a given probability *common* to all risky firms.The probability — jointly with the share of firms exposed — captures the rollover risk in the economy.

To estimate this probability, along with other key model parameters, I design a model-based identification strategy that learns from firms' bankruptcy choices and bankrupt firms' characteristics. In the U.S. bankruptcy code, bankrupt firms can choose to use the chapter 7 provisions and be liquidated, or use chapter 11 provisions and renegotiate its debt with creditors while they continue operating and, importantly, debt payments are suspended and new debt issuance is facilitated.[2] To exploit these features of the U.S. bankruptcy code I embed in the model a bankruptcy procedure where bankrupt firms can choose between the liquidation and restructuring chapters. I argue that I can infer indirectly the incidence of rollover crises from the firms bankruptcy chapter choices and bankrupt firms' financial characteristics (for example, leverage). I find that in the model, a larger share of firms in the restructuring process implies a greater incidence of rollover crises. The basic intuition is that the restructuring process provides little benefit to insolvent firms since observed debt haircuts in the restructuring process are small, but provide large benefits to firms under a rollover crises since the

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[2]It is widely argued in the bankruptcy law literature [see, for example, the seminal work by Jackson (1986)] that several provisions in the U.S. bankruptcy code (such as the one preventing creditors from collecting debt payments) are designed to solve credit coordination failures for bankrupt firms (or, in other words, coerce debt rollover).

restructuring process can operate as a way to (temporarily) force the rollover of debt (for example, by suspending debt payments).[3]

My first main quantitative result shows that, in the U.S. economy, 1.6% of the firms are subject to rollover crises: 21% of the firms are exposed with 7% likelihood of a coordination failure. Roughly half of the bankruptcy events in the economy are driven by rollover crises.

As a validation exercise, I then assess the model's ability to reproduce the observed investment decisions at the cross section during crises. I simulate a panel of firms during a large recession and do a difference-in-difference estimation of the investment heterogeneity patterns, and find that the patterns in the model are quantitatively similar to the ones I estimate for last recessions in U.S. (Great Recession and Covid crisis). Firms with less internal resources (cash-on-hand) — thus, more vulnerable to rollover crises — adjust their investment by more.

Next, I conduct a quantitative study of the amplification mechanism of firms' rollover crises in large recessions. I simulate a prototypical large recession and study the transition of aggregate output with and without coordination failures. For different types of shocks driving the recession — total factor productivity (TFP), cash and credit shocks — I find that rollover risk can significantly amplify the impact of recessions, explaining around one fifth of output losses during the episode.[4] Moreover, rollover risk makes recessions more persistent, especially when crises are driven by aggregate TFP or cash shocks. A sudden reduction in the firms' cash-flows temporarily exposes more firms to rollover crises, leading to more failures (bankruptcy and liquidation) of healthy firms, which creates the extra persistence.

Finally, I study the policy implications of rollover crises. I focus on direct lending policies, which resemble those deployed by the Fed during the Covid crisis and other credit interventions in previous recessions.[5] I simulate an imperfectly-targeted credit policy in the recession experiments. I find that low scale policies are the most potent in reducing the short term impact of rollover crises and providing a swift recovery. The

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[3]Anecdotal evidence suggests that firms' managers tend to think of the restructuring process, (and the Debtor-in-possession (DIP) protection they provide) as a way to get some time for payments. In addition, the DIP protection facilitates new debt issuance (known as DIP financing).

[4]In my experiments a credit shock is a reduction in the recovery rate of creditors during a firm's liquidation event.

[5]For example, the Fed provided credit to corporate firms through the Primary Market Corporate Credit Facility (PMCCF) and Secondary Market Corporate Credit Facility (SMCCF) during the Covid crisis.

intuition is that the credit policy works as an insurance for creditors which precludes coordination failures, even if firms are not drawing funds from the government's credit facilities. On the contrary, more ample credit programs, while they mitigate more coordination failures and have greater short term benefits, they subsidize credit to many firms and create future debt overhang that extend the recession.

LITERATURE. The paper fits in the broad research agenda described by Brunnermeier and Krishnamurthy (2020). This research agenda aims to incorporate firm-level corporate financing considerations in quantitative macroeconomic models to study their aggregate (positive and normative) consequences. My paper's main contributions can be placed in the following strands of literature in macroeconomics and finance:

Financial heterogeneity and default risk in macroeconomics. Broadly, this paper is related to the literature that works with general equilibrium models of firms with default risk [see, Cooley and Quadrini (2001); Hennessy and Whited (2005) for early references]. More specifically, this paper is related to the literature that studies the implications of aggregate shocks in macro models with firms' default risk [see, for example, Cooley, Marimon and Quadrini (2004); Jermann and Quadrini (2012); Arellano, Bai and Kehoe (2019); Khan, Senga and Thomas (2020); Ottonello and Winberry (2020)] I contribute to this literature by studying the aggregate implications of rollover risk (i.e., default driven by credit coordination failures) in recessions.

Corbae and D'Erasmo (2021) studies the long-term implications of changes to the bankruptcy procedures in a general equilibrium model with heterogeneous firms. Distinctively, in my paper, I study the interaction between firms' rollover crises and bankruptcy provisions aimed to prevent the hold out problem (coordination failures) of creditors during bankruptcy.[6]

Rollover crises and multiple equilibrium in macroeconomics. There is an ample literature that studies rollover crises and multiple equilibrium in macroeconomics. My paper is closely connected to the work on sovereign debt self-fulfilling crises. In my model, the Cole and Kehoe (2000) timing creates rollover risk trough creditors' coordination failures. Similar to my paper, a major challenge for the (quantitative) literature on sovereign debt self-fulfilling crises [see, for example, Aguiar, Chatterjee, Cole and

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[6]This mechanism is widely argued in the bankruptcy law literature. See, for example, the seminal work Jackson (1986) or more recently Ayotte and Skeel (2013).

Stangebye (2021)] is to disentangle rollover crises from fundamental/solvency crises. In the same spirit as Bocola and Dovis (2019), I try to identify and quantify indirectly the incidence of rollover crises. My paper is also connected to the literature on bank runs in macroeconomics, which builds on the seminal work by Diamond and Dybvig (1983) and studies bank runs in macroeconomic models [see, for example, Gertler and Kiyotaki (2015); Gertler, Kiyotaki and Prestipino (2019); Amador and Bianchi (2021)]. The contribution of my paper to this literature is twofold. First, I do a quantitative assessment of rollover crises on firms and, second, I use an identification strategy which argues that salient features of the bankruptcy process — jointly with firms' choices and characteristics — inform the incidence of rollover crises and coordination failures.

Rollover (coordination) crises in corporate finance theory. There is a theoretical literature in corporate finance that studies firms' rollover crises due to creditors' coordination failures. Salient examples are Morris and Shin (2004, 2016), which adopt a global games approach to study the relation between firm's rollover crises (coordination) and corporate debt pricing. In related work, He and Xiong (2012a,b); Cheng and Milbradt (2012) study the relation between rollover crises and corporate debt maturity, and Zhong (2021) studies the relation of coordination failures with creditor's concentration. My contribution to this literature is to study the interaction of rollover crises and bankruptcy provisions in a general equilibrium infinite horizon model.

Corporate credit policy intervention and recessions. Sparked by the Covid crisis — and the policy response that followed — a recent body of work studies the effectiveness of corporate credit policies during large recessions using structural models. Elenev, Landvoigt and Van Nieuwerburgh (2021) find that credit policy during large recessions can prevent firms' bankruptcy, Ebsim, Faria e Castro and Kozlowski (2021) show policy effectiveness may depend on the source of the crisis shock and Crouzet and Tourre (2021) find subsidize credit may induce future debt overhang. My paper is in line with these recent line of work. In addition, I show that credit policy also works through a coordination channel, akin to a deposit insurance for banks, and may be potent even if few firms participate on the credit policy program (in equilibrium). This last result is related to observations made by Cox, Greenwald and Ludvigson (2021) regarding the workings of the Fed's corporate credit facilities during Covid.[7]

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[7]Cox *et al.* (2021) observe limited participation in Federal Reserve's Primary and Secondary Market Corporate Credit Facility programs, and argue that credit policy (announcements) affected asset prices through non-fundamentals (analogous to the workings of the credit policy in my paper).

Rollover crises empirics. Kalemli-Özcan, Laeven and Moreno (2020); Almeida, Campello, Laranjeira and Weisbenner (2012) study individual investment dynamics and financial heterogeneity during recent recessions, in U.S. and Europe, and find suggestive evidence that firms were subject to rollover risk. My paper is line with these findings. Foley-Fisher, Narajabadand and Verani (2020) use an empirical identification strategy to assess the incidence of rollover crises for life insurance companies that exploits particularities of insurers debt contracts. Similarly, my paper identifies the incidence of rollover crises for firms, but I use a model-based strategy which relies on salient features of the U.S. bankruptcy code, and uses observed cross-sectional patterns of firms' bankruptcy choices and bankrupt firms' characteristics.

PAPER'S ORGANIZATION. The paper is organized as follows: Section 2 develops a macroeconomic model where firms can be subject to rollover crises; Section 3 identifies and quantifies firms' rollover crises; Section 4 quantifies the amplification mechanism of rollover risk in large recessions; Section 5 studies the effectiveness of credit policies in the presence of rollover risk; and Section 6 concludes. Lastly, the Appendix contains further details on the theory, data, other exercises and extensions.

2   A MACRO MODEL OF FIRMS' ROLLOVER CRISES

In this section I describe the theoretical framework. The framework is used to identify the incidence of firms' debt rollover crises and conduct the baseline quantitative exercises.

I develop a heterogeneous-firms macroeconomic model with rollover risk. Firms use internal and/or external resources to invest and produce. Building on Khan *et al.* (2020) and Ottonello and Winberry (2020), firms cannot commit to repay their debt, then endogenous default risk limits their borrowing capacity. Firms can default because of fundamental reasons or rollover crises driven by creditors' coordination failures à la Cole and Kehoe (2000). Furthermore, the model features a realistic bankruptcy procedure where firms can decide to liquidate and exit, or restructure their liabilities and continue operating, as in Corbae and D'Erasmo (2021).

To describe the model, I follow several steps: Section 2.1 is an overview of the environment; Section 2.2 describes the non-financial firms' setup; Section 2.3 shows how

creditors determine debt prices given the choices of the firm; Section 2.4 characterizes non-financial firms' bankruptcy (liquidation and restructuring) choices, which depend on debt prices; Section 2.5 shows the non-financial firms' recursive problem formulation; Section 2.6 briefly describes the rest of the agents: capital producers and households; Section 2.7 defines the equilibrium for this economy; and Section 2.8 discusses the main assumptions.

2.1 Environment

The economy has an infinite horizon and is in discrete time, i.e., $t = 0, 1, 2, ...$ It is inhabited by four types of agents: (i) non-financial heterogeneous firms that invest, produce and make financial choices in order to maximize the present value of their dividends (i.e., firm value); (ii) atomistic and perfectly competitive creditors that lend to non-financial firms; (iii) a representative capital producer that sells capital to non-financial firms; and (iv) a representative household that consumes, saves and works, and owns all the firms in the economy. The price of the final good is normalized to 1, and the price of capital good is q and wages w are determined in general equilibrium. I will assume there is no aggregate risk.

2.2 Firms Setup

Firm i objective is to maximize its value $V_{it} = \mathbb{E}_t \left[\sum_{t \geq s} \Lambda_s d_{ij} \right]$ where Λ_s is the stochastic discount factor of the households and d_{is} are the dividends issued by firm i in period j. The firm has three types of idiosyncratic state variables: (i) exogenous fundamental state variables $\mathbf{s}_{it}^f$; (ii) an exogenous non-fundamental state variable s_{it}^n ; and (iii) endogenous state variables $\mathbf{s}_{it}^e$. The idiosyncratic state vector of the firm is defined as $\mathbf{s} = \left(\mathbf{s}_{it}^f, s_{it}^n, \mathbf{s}_{it}^e \right)$. Firms are perfectly competitive, and there is a continuum of them producing each period with a distribution $\Omega\left(.\right)$, which is normalized to $\int d\Omega\left(.\right) = 1$.

There is no aggregate risk and the firm's problem can be written recursively (shown later); thus, for clarity of exposition I will drop subscripts for firm i and period t, and adopt the recursive timing convention.

Technology and operational profits. Firms combine capital k and labor l to produce a unique final good using a Cobb-Douglas production function

$$y = f(z, \omega, k, l) = z (\omega k)^\alpha l^\nu,$$

where $\alpha \in (0, 1)$ is the share of capital and ν is the share of labor. I assume the firm operates with decreasing returns to scale, i.e., $\alpha + \nu < 1$. The firm is subject to two idiosyncratic shocks: (i) a persistent idiosyncratic productivity process $\ln z' = \rho \ln z + \epsilon_z$ with $\epsilon_z \sim^{\text{iid}} (0, \sigma_z^2)$; and (ii) idiosyncratic iid capital quality shock ω, which is drawn from a log-normal truncated distribution where $\ln \omega \in [\underline{\omega}, 0]$. The sole purpose of including the ω shock is to match quantitatively the default rates observed in the data.

Firms own capital k, which is inherited from the previous period, and hire labor l at given wage w. The labor choice problem of firms is static, and I define the operating profits function as

$$\pi(z, \omega, k) = \max_l z (\omega k)^\alpha l^\nu - wl \tag{1}$$

with labor demand of firms $l = \left[\frac{\nu z (\omega k)^\alpha}{w} \right]^{\frac{1}{1-\nu}}$.

Resources. Each period firms can raise external resources by issuing one-period debt b' given price schedule $Q(.)$ offered by creditors. Furthermore, firms internal resources are cash-on-hand n, which is the sum of operational profits $\pi(z, \omega, k)$ and current value of the firm's productive capital after depreciation $(1 - \delta) q \omega k$ — where $\delta \in [0, 1]$ is the capital depreciation rate — minus the maturing inherited debt b, i.e.,

$$n = \pi(z, \omega, k) + (1 - \delta) q \omega k - b. \tag{2}$$

External and own resources are used to issue dividends d and make capital purchases, i.e.,

$$d + q k' = n + Q(.) b'. \tag{3}$$

As I show later the financial structure of the firm is going to matter in the presence of financial frictions.

Exit and entry. The mass of entrant firms $\bar{\mu}$ equates the mass of exiting firms in steady-state. I assume entrants are endowed with a capital $k = k_0$ and debt $b = 0$,

and draw their initial productivity level z from an invariant distribution $\Omega^e(z)$ with an average productivity $m \leq 0$ percent lower than z ergodic distribution's average. This assumption is consistent with evidence that young firms have lower measured productivity, as pointed out by Ottonello and Winberry (2020), and is useful to match the firms' life-cycle moments.

I assume there is endogenous exit (explained later) and exogenous exit. Following Khan *et al.* (2020), firms at the beginning of each period receive an exogenous exit shock with probability $\gamma \in [0,1]$ which forces them to exit after production. This assumption precludes that all firms overcome the financial frictions in steady-state. Furthermore, I assume, exiting firms can decide to liquidate or restructure their inherited debt. The bankruptcy choice of exiting firms is explained in Appendix A.1.

Financial frictions.   There are two forms of firm-level financial frictions.

First, firms are precluded from issuing equity, i.e.,

$$d \geq 0. \tag{4}$$

This assumption is standard in the literature, is consistent with the scarce issuance of equity by corporate firms in the data and provides greater tractability to the model. In Appendix C.3, I extend the model to allow for costly equity issuance and show how the characterization of the firm's liquidation choice changes.

Second, firms' debt is defaultable. Each period, firms can decide to file for bankruptcy and default their debt. Emulating the U.S. bankruptcy code, I assume that firms can decide to liquidate and exit, or restructure their liabilities and continue operating. An important assumption that I will revisit when describing the whole timing is that firms' restructuring decision occurs before making new debt issuance b' and firms' liquidation decision occurs after issuing b'. As I will explain later, the timing assumption is a key element of the model.

If the firm decides to liquidate and exit, then all debt — inherited b and new issuance b' — is defaulted, and firms exit with value $V = 0$. On the other hand, creditors of inherited debt b recover a fraction $R(b,k,\omega) = \min\left\{1, \alpha_7 \frac{q\omega k}{b}\right\}$ of b, where $\alpha_7 \in [0,1]$ is a parameter that indicates the creditors' recovery rate of capital when the firm is liquidated. Furthermore, I assume that creditors of new debt issuances b' don't recover

anything from a contemporaneous liquidation. This last assumption is done for technical reasons and captures the fact that inherited creditors tend to have seniority over new creditors. *Chapter 7* liquidations and *Chapter 11* piecemeal liquidations of the U.S. Bankruptcy Code are the empirical counterpart of the liquidation events in the model.

If the firm decides to restructure its debt b and continue operating, then the creditors and the firm use a Nash Bargaining protocol to negotiate the debt recovery rate $\alpha_{11} \in [0,1]$ of inherited debt b. I assume that the outside option of the bargaining problem is to continue as if the firm never filed for bankruptcy. We can interpret this assumption as "the judge" in charge of the restructuring process dismissing the filing. Moreover, we need that the firm and its creditors are willing to participate in the negotiations; thus, the bankruptcy decision is a joint decision. Additionally, firms pay bankruptcy costs $c_{11} \in [0,1]$ which are proportional to the firms' capital. This assumption captures bankruptcy losses such as legal fees, administrative costs, reputational deterioration. After the haircut and paying the cost the internal resources left are $n_{11} = \pi(z, \omega, k) + (1 - c_{11})(1 - \delta) q \omega k - \alpha_{11} b$.

Furthermore, a key assumption is that in the restructuring process creditors' coordination failures are precluded. This assumption captures the provisions in Chapter 11 of the U.S. bankruptcy code where, for example, debt payments are suspended and new debt issuance is facilitated. The empirical counterpart of the restructuring process is *Chapter 11* of the U.S. Bankruptcy Code, excluding Chapter 11 "liquidations" or "363" sales.

In Section 2.8, there is a thorough discussion of the assumptions related the bankruptcy process and, in Appendix D, I provide further institutional details of the Chapter 7 and Chapter 11 of the U.S. bankruptcy code.

Timing. Figure 1 shows the within period timing of the firm problem for firms that are not subject to the exogenous exit shock.

At the beginning of the period idiosyncratic states are realized, i.e., the fundamental and non-fundamental shocks are revealed. After uncertainty is resolved, there is no more within period uncertainty, which means that all shocks and states (including non-fundamental) are known, i.e., s is know, and they are common knowledge for all agents.

In Figure 1, the first gray dot indicates the restructuring choice where firms choose to

either continue or restructure. If the firm decides to continue, it makes the investment and financing choice, i.e., chooses (k', b') given pricing schedule $Q(.)$ and price q. After the firm issues new debt b', the firm has a liquidation choice (i.e., second gray dot in Figure 1), where it decides to liquidate and exit, or continue and produce. The fact that the liquidation choice takes place after issuing new debt will be the source of multiple equilibria. This timing is the well-known Cole and Kehoe (2000) (CK) timing in the international macroeconomics literature. If the firm decides to restructure its liabilities, then it enters the bargaining process for α_{11} — with outside option to continue (gray arrow up in Figure 1). In the restructuring process there are no current coordination failures since there is no liquidation choice after issuing new debt, i.e., visually, in Figure 1, there is no gray dot after entering the restructuring process.

Figure 1: Within period timing
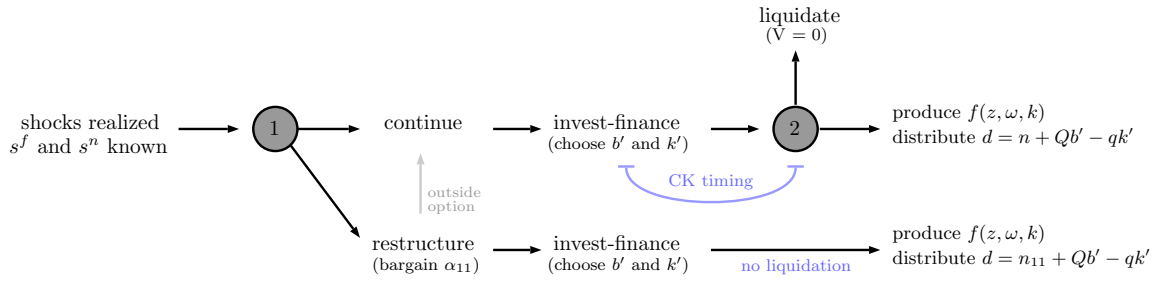
Note: timing is conditioned on a firm that doesn't receive an exit shock. In Appendix A.1, I describe and characterize the exiting firm's problem.

Next, I will define the pricing schedule for creditors, which is relevant for the characterization of the liquidation and restructuring choice.

2.3 Creditors and the Debt Pricing Schedule

Creditors borrow from households at the risk free rate r and lend to the firms at price $Q(.)$. They are perfectly competitive and atomistic; thus, the no-profit condition holds which prices the debt. All intermediaries are owned by the household, hence they discount future flows using stochastic discount factor Λ (defined in the household problem in Section 2.6). Thus, the price of debt is determined according to

$$Q\left(\mathbf{s}, k', b'\right) = \left[1 - \mathbf{1}_{\{\text{ch7}\}}(\mathbf{s})\right] \mathbb{E}_{(\mathbf{s'}|\mathbf{s})}\left[\Lambda(1-\gamma)\mathbf{1}_{\{\text{continue}\}}\left(\mathbf{s'}\right)\right]$$
$$+ \left[1 - \mathbf{1}_{\{\text{ch7}\}}(\mathbf{s})\right] \mathbb{E}_{(\mathbf{s'}|\mathbf{s})}\left[\Lambda(1-\gamma)\mathbf{1}_{\{\text{Ch11}\}}\left(\mathbf{s'}\right)\alpha_{11}\left(\mathbf{s'}\right)\right]$$

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$$+ \left[1 - 1_{\{ch7\}} (\mathbf{s})\right] \mathbb{E}_{(\mathbf{s}'|\mathbf{s})} \left[\Lambda \left(1 - \gamma\right) \mathbf{1}_{\{Ch7\}} \left(\mathbf{s}'\right) R\left(b', k', \omega'\right)\right]$$

$$+ \gamma \left[1 - 1_{\{ch7\}} (\mathbf{s})\right] \tilde{Q}_{exit} \left(z, k', b'\right), \tag{5}$$

with

$$\tilde{Q}_{exit} \left(z, k', b'\right) = \mathbb{E}_{(\mathbf{s}'|\mathbf{s})} \left[\Lambda \left\{ \mathbf{1}_{\{continue \,|\, exit\}} \left(\mathbf{s}'\right) + \mathbf{1}_{\{ch11 \,|\, exit\}} \left(\mathbf{s}'\right) \alpha_{11}^{exit} \right\}\right]$$

$$+ \mathbb{E}_{(\mathbf{s}'|\mathbf{s})} \left[\Lambda \mathbf{1}_{\{ch7 \,|\, exit\}} \left(\mathbf{s}'\right) R\left(\omega', b', k'\right)\right], \tag{6}$$

where $\mathbf{1}_{\{Ch7\}} (\mathbf{s})$ indicates if the firm is liquidated, $\mathbf{1}_{\{Ch11\}} (\mathbf{s})$ is indicates if the firm restructures the debt and $\mathbf{1}_{\{continue\}} (\mathbf{s})$ indicates which firms continue and is defined as $\mathbf{1}_{\{continue\}} (\mathbf{s}) = 1 - \mathbf{1}_{\{Ch11\}} (\mathbf{s}) - \mathbf{1}_{\{Ch7\}} (\mathbf{s})$ for firms that don't receive the exit shock.[8] Furthermore, the indicators conditioned on the firm receiving the exit shock, i.e. $1_{\{.|exit\}}$, are defined analogously. The liquidation and restructuring choices are characterized in Section 2.4 for firms that don't receive the exit shock and in Appendix A.1 for firms that receive the exit shock. On the RHS of (5), the first line shows the component of the price where the firm repays fully next period, the second line shows the component of the price where the firm restructures next period and creditors recover $\alpha_{11} \left(\mathbf{s}'\right)$, and the third line shows the component of the price where the firm is liquidated next period and creditors recover $R\left(b', k', \omega'\right)$. The last line shows the component of the price related to exiting firms, and equation (6) shows the determinants of the debt price given no liquidation today and firm receiving the exit shock tomorrow, it is analogous to the pricing function for non exiting firms.

It is useful to define the fundamental price $\tilde{Q} (.)$ as the price of debt whenever there is no contemporaneous liquidation, i.e.,

$$Q\left(\mathbf{s}, k', b'\right) = \left[1 - 1_{\{ch7\}} (\mathbf{s})\right] \tilde{Q}\left(z, b', k'\right).$$

The contemporaneous liquidation decision shows up in the debt pricing because of the CK timing assumption — i.e., the liquidation choice happens after the new debt issuance — and this is the source of potential multiplicity as I explain next.

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[8]As a technical note, the liquidation choice doesn't depend on $\left(k', b'\right)$ because the policy function imposes a constraint redundant to $d \geq 0$ for the financing-investment choice, and creditors of b' have 0 recovery rate and there is no within period uncertainty after shocks are realized at the beginning of the period.

2.4 Firms' Liquidation and Restructuring Choices

In this subsection, I will characterize liquidation and restructuring choices. I will start backwards, according to Figure 1 timing, since the payoffs of the liquidation choice will affect the decisions in the restructuring choice. First, I show that there can be multiple equilibria, then I characterize the solution of the liquidation choice and find that there are three types of firms across the state space: firms safe from current rollover crises, insolvent firms and firms exposed to rollover crises. Finally, I characterize what firms enter the restructuring process.

2.4.1 Liquidation Choice

First, I show how the liquidation choice timing can create the possibility of multiple equilibria. Next, I briefly discuss some of the assumptions necessary for coordination failures to happen and, lastly, I characterize the liquidation choice across the relevant state space.

Multiple equilibria intuition. Firms are subject to the no-equity issuance constraint $d \geq 0$ (feasibility) and their exiting value is 0. Tt follows that firms liquidate if they can't issue (weakly) positive dividends.[9] Creditors price the debt according to equation (5). The pricing schedule shows that firms price the debt by calculating the expectations of future default and making conjectures about the firm's liquidation choice today. The price of debt and dividends can be written as

$$Q(.) = \underbrace{\mathbf{1}_{d \geq 0}}_{\text{no liquidation choice}} \underbrace{\tilde{Q}(.)}_{\text{debt price if no liquidation}}$$

$$d = n - qk' + Q(.)b',$$

where $\mathbf{1}_{d \geq 0}$ is the today's no liquidation indicator — i.e., if $\mathbf{1}_{d \geq 0} = 0$, the firm cannot satify $d \geq 0$ and is liquidated, and $\mathbf{1}_d = 1$ otherwise — and $\tilde{Q}(.)$ the pricing schedule without liquidation today.

There is feedback between dividends and current debt prices, which could create multiple outcomes. To illustrate this, assume the firm has negative cash-on-hand $n < 0$ and creditors conjecture liquidation today $\mathbf{1}_{d \geq 0} = 0$, then they offer price $Q = 0$ and

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[9]This simple result is straightforward from the assumptions.

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the firm cannot satisfy the constraint on dividends $d = n + \max_{k' \geq 0} \left\{ -qk' \right\} < 0$, which implies the firm is liquidated. Thus, the outcome is consistent with the conjecture. On the contrary, if creditors conjecture that there is no liquidation today $Q = \tilde{Q}$ and $\exists b'$ such that $\tilde{Q}(.) b' > n$ then the firm can satisfy $d \geq 0$ and is not liquidated. Thus, this outcome is als consistent with the conjecture. Therefore, under certain conditions — which I show next — outcomes could depend on creditor's conjectures of the current liquidation choice and creditors could coordinate on either debt price. I adopt the convention that if creditors coordinate on $Q = 0$, then they coordinate in the rollover crisis outcome.

Characterization of the liquidation choice. Figure 2 shows that the *fundamental state-space* (z, n) can be divided in three regions.[10] First, there is a Safe region $\mathcal{S}$ where firms in this region don't liquidate even if creditors conjecture liquidation today. This means that if $Q = 0$ then they can still satisfy $d = n + \max_{k' \geq 0} \left\{ -qk' \right\} = n \geq 0$. Thus, firms with $n \geq 0$ can always satisfy $d \geq 0$ and are in $(z, n) \in \mathcal{S}$. Next, there is a Liquidation region $\mathcal{L}$ where firms are liquidated even if creditors conjecture no liquidation today. This means that even if $Q = \tilde{Q}$ then firms cannot satisfy the $d \geq 0$, i.e., $d = n + \max_{b', k' \geq 0} \left\{ \tilde{Q}(.) b' - qk' \right\} = n < 0$. Since $\tilde{Q}(.) = \tilde{Q}\left(z, b', k' \right)$, then it follows that firms $(z, n) \in \mathcal{L}$ are those with cash-on-hand n bellow a threshold $\underline{n}(z)$ where the threshold is defined by the negative of the maximum amount of external resouces the firm can raise, i.e., $n < \underline{n}(z) = -\max_{b', k' \geq 0} \left\{ \tilde{Q}(.) b' - qk' \right\}$. Finally, there is a Risky region $\mathcal{R}$ where firms can either be liquidated or not depending on creditors' conjecture. This means that if $Q = 0$ then they cannot satisfy $d \geq 0$ so we need $n < 0$, but if $Q = \tilde{Q} > 0$ then firms satisfy $d \geq 0$ so we need $n \geq \underline{n}(z)$. Thus, firms $(z, n) \in \mathcal{R}$ whenever $n \in [\underline{n}(z), 0)$.

_____

[10]The characterization into three regions is common in models of sovereign debt self-fulfilling debt crises that use the Cole and Kehoe (2000) timing convention. See, for example, Bocola and Dovis (2019).

Figure 2: Rollover and solvency: regions across (z, n) state-space

Notes: figures shows the state-space (z, n) and the relevant regions for the liquidation choice.

To construct the equilibrium in region $\mathcal{R}$, I define an *idiosyncratic* and stochastic sunspot variable $\phi \sim^{\text{iid}} U[0,1]$ that is drawn every period at the beginning of the period (i.e., the non-fundamental state variable). I assume that creditors coordinate on the rollover crisis equilibrium whenever $\phi \leq \eta$, where η is *common* across firms and — jointly with the share of firms exposed — summarizes the incidence of rollover risk in the economy.

Formally, the liquidation choice for firms after they decide to continue (i.e., second red dot in Figure 1 from left to right) — $\tilde{\mathbf{1}}_{\{\text{ch7}\}}(.)$ — depends only on states (z, n, ϕ) and is characterized formally as follows

$$
\tilde{\mathbf{1}}_{\{\text{ch7}\}}(z, n, \phi) = \begin{cases} 1 & \text{if } n < \underline{n}(z) \\ 1 & \text{if } \{\underline{n}(z) \leq n < 0\} \cap \{\phi \leq \eta\} \\ 0 & \text{if } \{\underline{n}(z) \leq n < 0\} \cap \{\phi > \eta\} \\ 0 & \text{if } n \geq 0 \end{cases}.
$$

This characterization provides a clear distinction between firms that are liquidated because of solvency issues (those in $\mathcal{L}$) and rollover crises (those in $\mathcal{R}$ and coordination failure). Furthermore, I will consider solvent firms as those in the risky $\mathcal{R}$ and safe $\mathcal{S}$ regions. In Appendix A.1, I characterize the liquidation choice of exiting firms. Notice that to determine what firms are liquidated in equilibrium we also need to characterize what firms enter the restructuring process (which happens before the liquidation choice).

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2.4.2 Restructuring Choice

Now I will characterize the restructuring choice of firms and conjecture which types of firms — according to the liquidation choice characterization — enter the restructuring process.

Characterization. When entering the restructuring process, firms and creditors will bargain a debt recovery rate $\alpha_{11} \in [0,1]$. From the characterization of the liquidation choice, we know the payoffs of continuing without restructuring.

I assume that the outside option of the bargaining problem is to continue without restructuring. Creditors participate in the debt renegotiation if they receive a higher payoff than the firm continuing (which is the outside option). For firms that are solvent (i.e., either safe or risky) without a rollover crisis, their creditors will get fully paid if they continue; thus, in this case creditors will not accept a recovery rate lower than 1 on their debt. The outside option assumption rules out cases where solvent firms without a rollover crisis restructure their debt, then I can focus on the cases where firms are liquidated — from solvency or rollover crises — if they continue (i.e., as the outside option).[11]

Let $V(.)$ be the value of the firm when making the financing-investment choice of firms that are solvent without a rollover crises, then the Nash Bargaining protocol for an insolvent or risky firm in a rollover crisis is

$$\alpha_{11}(z,\omega,b,k) = \max_{\alpha_{11}} \left[V(n_{11},z) - 0\right]^{1-\Xi} \left[b\alpha_{11} - bR(b,k,\omega)\right]^{\Xi} \tag{7}$$

where recovery rate α_{11} depends on states (z,ω,b,k) and $\Xi \in (0,1)$ is the bargaining power of creditors. The protocol is subject to the participation constraints

$$n_{11} = \pi(z,\omega,k) + (1 - c_{11})(1 - \delta) q\omega k - \alpha_{11}b \geq \underline{n}(z) \tag{8}$$

$$\alpha_{11} \geq R(b,k,\omega). \tag{9}$$

Insolvent firms or those risky in a rollover crisis have an outside option of $V = 0$ (liquidated if continuing without restructuring their debt). Equation (8) shows that firms will participate if they are solvent after the restructuring process (i.e., $n_{11} \geq \underline{n}(z)$),

---

[11]In a previous version of the model, I allowed for safe firms to restructure their debt and I found that the share of firms that do this was negligible for the baseline calibration.

equation (9) shows that creditors will participate if they recover more than under liquidation, and (7) shows the objective function of the protocol is a sharing rule of the firm's and creditors' surpluses. For the bargaining process to be feasible we need that the maximum recovery rate that the firm is willing to pay is greater than the minimum creditors are willing to accept, i.e.,

$$\frac{\pi\left(z,\omega,k\right)+\left(1-c_{11}\right)\left(1-\delta\right)q\omega k}{b} \equiv \alpha_{11}^{\max} > \alpha_{11}^{\min} \equiv R\left(b,k,\omega\right).$$

It is easy to see that this is also a sufficient condition for firms to restructure their debt (if they are insolvent or risky with rollover crises). Then, the firms that restructure their liabilities are those that are either insolvent or risky in a rollover crisis where the bargaining process is feasible, i.e.,

$$\mathbf{1}_{\{\text{ch}11\}}\left(z,\omega,\phi,b,k\right) = \begin{cases} 1 & \text{if } \{(z,n)\in\mathcal{L}\}\cup\{\{(z,n)\in\mathcal{R}\}\cap\{\phi\leq\eta\}\}\cap\{\alpha_{11}^{\max}>\alpha_{11}^{\min}\}, \\ 0 & \text{otherwise.} \end{cases}$$

(10)

Then the firms that are liquidated are those insolvent or risky in a rollover crisis that don't restructure their debt, i.e.,

$$\mathbf{1}_{\{\text{ch}7\}}\left(z,\omega,\phi,b,k\right) = \begin{cases} 1-\mathbf{1}_{\{\text{ch}11\}}\left(z,\omega,\phi,b,k\right) & \text{if } \{(z,n)\in\mathcal{L}\}\cup\{\{(z,n)\in\mathcal{R}\}\cap\{\phi\leq\eta\}\}, \\ 0 & \text{otherwise.} \end{cases}$$

(11)

Finally, the firms that continue without going through the bankruptcy process are those that are in the safe region or in the risky region without rollover crises (solvent without rollover crises), i.e.,

$$\begin{aligned} \mathbf{1}_{\{\text{continue}\}}\left(z,\omega,\phi,b,k\right) &= 1-\mathbf{1}_{\{\text{ch}11\}}\left(z,\omega,\phi,b,k\right)-\mathbf{1}_{\{\text{ch}7\}}\left(z,\omega,\phi,b,k\right) \\ &= \begin{cases} 1 & \text{if } \{(z,n)\in\mathcal{S}\}\cup\{\{(z,n)\in\mathcal{R}\}\cap\{\phi>\eta\}\}, \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

(12)

Furthermore, firms that receive an exit shock can decide to liquidate or restructure before production. The characterization of the restructuring and liquidation choices of exiting firms — i.e., $\mathbf{1}_{\{.|\text{exit}\}}\left(\mathbf{s}\right)$ indicator functions — is simple and relegated to Appendix A.1.

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Which firms restructure their debt?   Intuitively, insolvent firms may enter the restructuring process if debt haircuts are large and costs are low. On the contrary, firms in a rollover crisis may restructure their debt even if there are low haircuts and the process is very costly, since they are able to mitigate the rollover crisis and likely continue operating. Therefore, if the bankruptcy process features low haircuts and is costly, it follows that firms in the restructuring process are *mostly* the ones subject to a rollover crisis (instead of being insolvent). This simple observation is formally shown in Section 3.2 and is crucial for the identification strategy in the quantitative model.

2.5   Firms' Problem

In this section, I describe recursive formulation of the firm's problem. The idiosyncratic states of the firm at the beginning of the period are $\mathbf{s} = (z, \omega, \phi, b, k)$ where the fundamental exogenous states are $\mathbf{s}^f = (z, \omega)$, the non-fundamental exogenous state is $\mathbf{s}^n = \phi$, and endogenous states are $\mathbf{s}^e = (b, k)$. Let $V(.)$ be the value of the firm that is solvent and not under a rollover crises when making the investment-financing decision[12], and let $\tilde{V}(.)$ be the value of the firm at the beginning of the period (before exit shock). Then the problem of invest-financing for solvent firms without rollover crises today is as follows

$$V(z, n) = \max_{d, k', b'} d + \mathbb{E}_{(z'|z; \omega'; \phi')} \left[\Lambda \tilde{V} \left(\mathbf{s}' \right) \right] \tag{13}$$

subject to

$$d = n - qk' + \tilde{Q} \left(z, b', k' \right) b' \geq 0$$
$$\mathbf{s}' = \left(z', \omega', \phi', b', k' \right)$$

where the continuation value $\tilde{V}(s)$ is defined as

$$\tilde{V}(\mathbf{s}) = (1 - \gamma) \left[\mathbf{1}_{\{ch11\}}(\mathbf{s}) V(z, n_{11}) + \mathbf{1}_{\{continue\}}(\mathbf{s}) V(z, n) \right] \tag{14}$$
$$+ \gamma \left[\mathbf{1}_{\{ch11|exit\}}(\mathbf{s}) n_{11}^{exit} + \mathbf{1}_{\{continue|exit\}}(\mathbf{s}) n \right]$$

---

[12]Notice these firms can be firms that went through the restructuring process and are solvent now, or firms that continue, are solvent and are not under a rollover crisis.

with

$$n = \pi(z, \omega, k) + (1 - \delta) q\omega k - b$$

$$n_{11} = \pi(z, \omega, k) + (1 - c_{11})(1 - \delta) q\omega k - \alpha_{11}(z, \omega, k, b) b$$

$$n_{11}^{\text{exit}} = \pi(z, \omega, k) + (1 - c_{11})(1 - \delta) q\omega k - \alpha_{11}^{\text{exit}}(z, \omega, k, b) b,$$

where $\mathbf{1}_{\{.\}}(\mathbf{s})$ indicator functions are described in Section 2.4 and $\mathbf{1}_{\{.|\text{exit}\}}(\mathbf{s})$ are described in Appendix A.1, $\alpha_{11}(z, \omega, k, b)$ solves problem (7) which determines continuing firm's cash-on-hand n_{11} and recovery rate $\alpha_{11}^{\text{exit}}(z, \omega, k, b)$ solves problem (23) described in Appendix A.1 which determines exiting firm's cash-on-hand n_{11}^{exit}. Investment-financing policy functions $\{b'(z, n), k'(z, n)\}$ solve problem (13).

2.6 Capital Producers and Households

To close the model, I describe the problem of the representative capital producer that sells capital to the firms, and the representative household that owns all firms, works, consumes the final good, and saves.

2.6.1 Capital Producers.

There is a representative aggregate capital producer that solves

$$\max_{I} q\Phi\left(\frac{I}{K}\right) - I$$

where I is the amount of final goods used to produce capital, K is the aggregate capital stock, and $\Phi(.)$ is the aggregate capital adjustment cost function. The first order condition of the problem is such that

$$q = \frac{1}{\Phi'\left(\frac{I}{K}\right)} \tag{15}$$

where q is the price of capital. The time-varying price of capital q changes the recovery rate $\mathcal{R}(.)$ which impacts debt prices, then maps to the financial accelerator mechanism (Bernanke, Gertler and Gilchrist, 1999) in the transitions after the shock. I assume a standard functional form such that $\Phi'\left(\frac{I}{K}\right) = \left[\frac{I/K}{\hat{I}}\right]^{-\psi}$ where $\hat{I}$ is the steady-state investment to capital ratio.

2.6.2 Households.

There is a unit mass of identical households that make the consumption-saving C and labor-leisure L decisions taking wages w, interest rate r and own all the firms in the economy. Then the household determine the stochastic discount factor Λ, the Euler equation holds, and the optimal labor-leisure choice is determined by the marginal rate of substitution, i.e.,

$$\Lambda' = \beta \frac{U_C\left(C',L'\right)}{U_C\left(C,L\right)} \tag{16}$$

$$1 = E\left[\beta \frac{U_C\left(C',L'\right)}{U_C\left(C,L\right)}\left(1+r\right)\right] \tag{17}$$

$$w = -\frac{U_L\left(C,L\right)}{U_C\left(C,L\right)} \tag{18}$$

with utility function $U\left(C,L\right) = \ln C - \Phi L$.

2.7 Equilibrium

Now I define the equilibrium of the model. I focus on a steady-state definition, the definition of the equilibrium during transitions follows straightforward from this. First, I describe the law of motion of the distribution of firms and then define the steady-state equilibrium.

Law of motion of the distribution of firms. Let Ω be the distribution of producing firms that has a mass of 1, $\tilde{\Omega}$ the distribution of incumbent firms at the beginning of the period, g and $\hat{g}$ the pdf of ω and ϕ respectively, p the conditional pdf of the productivity shocks ϵ_z, and Ω^e the distribution z of entrant firms. To define the equilibrium first we need to determine the law of motion of the distribution. Distribution of firms that produce is determined by

$$
\begin{aligned}
\Omega\left(z,n\right) = {}& \left(1-\gamma\right)\int\left[\mathbf{1}_{\{\text{ch11}\}}\left(\mathbf{s}\right)\mathbf{1}_{\{z,n_{11}(z,\omega,k,b)=n\}} + \mathbf{1}_{\{\text{cont}\}}\left(\mathbf{s}\right)\mathbf{1}_{\{z,n(z,\omega,k,b)=n\}}\right]\mathrm{d}\tilde{\Omega}\left(\mathbf{s}\right) \\
& + \gamma\int_z\left[\mathbf{1}_{\{\text{ch11}|\text{exit}\}}\left(\mathbf{s}\right)\mathbf{1}_{\left\{z,n_{11}^{\text{exit}}(z,\omega,k,b)=n\right\}} + \mathbf{1}_{\{\text{cont}|\text{exit}\}}\left(\mathbf{s}\right)\mathbf{1}_{\{z,n(z,\omega,k,b)=n\}}\right]\mathrm{d}\tilde{\Omega}\left(\mathbf{s}\right) \\
& + \bar{\mu}\left(1-\gamma\right)\int_z\left[\mathbf{1}_{\{\text{ch11}\}}\left(\mathbf{s}\right)\mathbf{1}_{\{z,n_{11}(z,\omega,k_0,0)=n\}}\right. \\
& \left. + \mathbf{1}_{\{\text{cont}\}}\left(\mathbf{s}\right)\mathbf{1}_{\{z,n(z,\omega,k_0,0)=n\}}\right]\hat{g}\left(\phi\right)g\left(\omega\right)\mathrm{d}\phi\mathrm{d}\omega\mathrm{d}\Omega^e\left(z\right)
\end{aligned}
$$

$$+ \bar{\mu} \gamma \int_z \left[\mathbf{1}_{\{\text{ch11|exit}\}} (\mathbf{s}) \, \mathbf{1}_{\{z, n_{11}^{\text{exit}}(z, \omega, k_0, 0) = n\}} \right.$$

$$\left. + \mathbf{1}_{\{\text{cont|exit}\}} (\mathbf{s}) \, \mathbf{1}_{\{z, n(z, \omega, k_0, 0) = n\}} \right] \hat{g} (\phi) \, g (\omega) \, \mathrm{d}\phi \mathrm{d}\omega \mathrm{d}\Omega^e (z) . \tag{19}$$

The distribution of incumbent firms $\tilde{\Omega} (z, \omega, k, b, \phi)$ at the beginning of the period evolves according to

$$\tilde{\Omega} \left(\mathbf{s}' \right) = \int \mathbf{1}_{\{k'(z,n)=k'\}} \mathbf{1}_{\{b'(z,n)=b'\}} \hat{g} \left(\phi' \right) g \left(\omega' \right) p \left(\epsilon_z \mid \rho_z z + \epsilon_z = z' \right) \mathrm{d}\epsilon_z \mathrm{d}\Omega (z, n) . \tag{20}$$

Equilibrium definition. *Steady-state equilibrium* in this economy is defined as a set of value functions $\{ V (z, n), \tilde{V} (s) \}$, firm's decision rules of capital purchases and new debt issuance $\{ b' (z, n), k' (z, n) \}$, bankruptcy decisions for firms without the exit shock $\{ \mathbf{1}_{\{\text{ch11}\}} (\mathbf{s}), \mathbf{1}_{\{\text{ch7}\}(\mathbf{s})} \}$ and with the exit shock $\{ \mathbf{1}_{\{\text{ch11|exit}\}} (\mathbf{s}), \mathbf{1}_{\{\text{ch7|exit}\}} (\mathbf{s}) \}$, aggregates $\{ Y, C, I \}$, corporate debt price schedule $Q \left(\mathbf{s}, b', k' \right)$, fundamental corporate debt price schedule $\tilde{Q}(z, k', b')$, interest rate r, prices $\{ q, w \}$, distributions $\Omega (\mathbf{s})$ and $\tilde{\Omega} (\mathbf{s})$, and debt haircuts $\{ \alpha_{11} (z, \omega, b, k) \}$ in the restructuring process such that:

1. Households choices are determined by (16), (17) and (18).

2. The price of capital is determined by the solution to (15).

3. The debt price satisfy (5) and fundamental price $\tilde{Q}$ is implicit in
   $$Q \left(\mathbf{s}, b', k' \right) = \left[1 - \mathbf{1}_{\{\text{ch7}\}} (\mathbf{s}) \right] \tilde{Q} \left(z, b', k' \right) .$$

4. Given prices, firm's decision rules solve the firm problem for firms that produce (13), continuing bankruptcy decisions are consistent with (10) (11) (12) and exiting firms bankruptcy decisions are consistent with equations (22) (24) in Appendix A.1, and the recovery rates are solved by negotiation protocols (7) (23).

5. Markets clear: investment is implicitly determined by the law of motion

$$K' = \Phi (I / K) K + (1 - \delta) K - (k_0 - (1 - \delta) \mathbb{E} [\omega] k_0) \bar{\mu}$$

with $K = \int k \mathrm{d}\tilde{\Omega} (s)$ and aggregate resource constraint is

$$C = Y - I - \mu_{11}$$

where μ_{11} is the aggregate cost of firms filing to Chapter 11.

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6. The distribution of firms that produce $\Omega(z, n)$ and before bankruptcy $\tilde{\Omega}(\mathbf{s})$ satisfy (19) and (20).

In steady state the distribution's law of motions is a fixed point, and the households stochastic discount factor is $\Lambda = \beta = \frac{1}{1+r}$ and capital prices are $q = 1$.

2.8 Discussion of Model Assumptions

Now I provide a brief discussion of key assumptions of the theory.

In the model, I assume the firm borrows from several creditors (i.e., atomistic creditors) and uses short term financing. In Appendix B.3, I show that, in the data, firms tend to borrow from several creditors, especially large corporate firms. Further, it is well documented that corporate firms' financial debt is mostly composed of corporate debt (instead of bank loans). Thus, corporate firms liabilities tend to have a dispersed ownership, which could create simple coordination problems among them. Moreover, in Appendix B.3, I show that the average firm in Compustat has large fractions of their debt maturing in the short term. On average, around one third of the financial debt matures in less than 1 year and more than half of the liabilities are due in less than 1 year. For quantitative purposes, I will abstract from long-term debt financing and match moments using liabilities that mature in the short term in the calibration in Section 3.1.

Furthermore, I do not allow firms to manage their liability structure (e.g., extend maturity or concentrate creditors). In Appendix C.2, I show that for the baseline calibration the ex-ante costs of firm rollover risk is negligible for most firms and when comparing to reasonable costs of managing their liabilities in the literature most firms would not modify their liability structure (in steady state) even if allowed. Finally, in Appendix C.3, I characterize the liquidation choice and the conditions for equilibrium multiplicity in various extensions of the model (with long-term debt, costly equity issuance, etc.), and find that the qualitative results are unaffected.

Another key assumption of the model is that firms in the restructuring process benefit from the removal of coordination failures (i.e., $Q = \tilde{Q}$) during the restructuring process. This captures various provisions of the Chapter 11 process aimed at resolving coordination issues among creditors. Ayotte and Skeel (2013) observed that "the dominant normative theory of bankruptcy" (see for example, Jackson (1986)) states that the

sole purpose of bankruptcy provisions is to solve "coordination problems caused by multiple creditors." In this spirit, the restructuring process chapter (Chapter 11) of the U.S. bankruptcy, for example, temporarily prevents creditors from individually collecting their debt (Automatic Stay provision, 11 U.S. Code § 362), allows firms to issue new debt and continue operating (usually known as Debtor-In-Possession (DIP) financing), and arranges official and ad-hoc committees of creditors.

3   IDENTIFYING FIRMS' ROLLOVER CRISES

The incidence of rollover crises depends on how many firms are exposed to them — i.e., share of firms in region $\mathcal{R}$ — and the likelihood that exposed firms are subject to a rollover crises — i.e., value of η. I cannot observe directly how many firms are exposed neither the probability they have a rollover crisis, then I use a model-based identification strategy to infer them indirectly. In this section, I describe the identification strategy and the estimation of firms' rollover crises incidence. Finally, as a validation exercise, I assess the model's ability to reproduce the observed investment decision at the cross section during crises.

Identification strategy.   To estimate the incidence of rollover crises, first, I fix a set of parameters to standard values in the literature and calibrate the parameters unrelated to the bankruptcy process to fit several moments of the U.S. economy. Next, I infer the value of η, and other bankruptcy parameters, using bankrupt firms' characteristics and firms' bankruptcy choice distribution. Finally, using the steady-state distribution of firms I determine how many firms are exposed. I find that 1.6% of the firms are subject to coordination failures each period where 21% of them are exposed and the conditional probability is 7%. Roughly half of the bankruptcy events are due to rollover crises.

3.1   Standard Calibration

Now, I focus on parameters and moments unrelated to the bankruptcy process. The standard calibration consists of 9 fixed parameters and 4 fitted parameters. To evaluate the empirical fitness of the model I contrast the moments in the model to a wide range of moments (16) in the data. The calibration is done at a quarterly frequency. I use

national accounts data from NIPA, firms' balance sheet micro data from Compustat, firms' life-cycle data from the Longitudinal Business Database (LBD), and moments computed in other papers. Further details on the data sources, samples and definitions are in Appendix B.

Table 1: Standard calibration

Parameter	Value	Calibration
a. fixed		
$\beta = 1/(1+r)$	0.99	fixed to $r = 0.05$ annual
Φ	1.16	fixed to match 58% emp rate
ν	0.64	fixed labor share
α	0.21	fixed capital share
δ	0.025	fixed to match BEA quarterly
ρ_z	0.90	fixed
b_0	0	fixed to no inherited debt for entrants
γ	0.02	fixed to exit rate 10% annual
ψ	1/4	fixed to standard values in literature
b. fitted		
σ_z	0.032	internally calibrated
$\underline{\omega}$	−0.33	internally calibrated
k_0	0.16	internally calibrated
m	−0.24	internally calibrated

Fixed parameters.   Panel (a) in Table 1 shows the value of the fixed parameters. The subjective discount rate $\beta = 0.99$ is set to fit an annual real interest rate of 5%. The labor disutility parameter $\Phi = 1.16$ is set to match an employment rate of 58%. The parameters of the curvature of the production $\nu = 0.21; \alpha = 0.64$ are set to fit the labor and capital share, respectively, and the capital depreciation rate of $\delta = 0.025$ is set to match estimates from BEA. Following Ottonello and Winberry (2020), I fix the persistence of the idiosyncratic productivity process to $\rho = 0.9$. Further, I fix the initial inherited debt is $b_0 = 0$ and exogenous exit rate $\gamma = 0.02$ to fit the total the annual exit rate of 10%. Finally, I fix the aggregate capital adjustment cost parameter $\psi = 1/4$ to a standard value in the literature.

Fitted parameters and moments.   Panel (b) in Table 1 shows the value of the fitted parameters unrelated to the bankruptcy process. The fitted parameters are the volatility of the idiosyncratic productivity shocks σ_z, the lower bound of the truncated nor-

mal process of capital quality shocks (in logs) $\underline{\omega}$, initial capital level k_0, and the relative scale of the initial productivity draw m which are set to fit 16 empirical moments. The moments are related to aggregates, firms' credit spreads and default rates, investment heterogeneity, life-cycle of firms, and balance sheet moments.

Table 2: Moments standard calibration

Moment	Data	Model	Data source
Aggregates			
K/Y	3.00	2.59	NIPA
I/Y	0.17	0.15	NIPA
gross debt: $\mathbb{E}[\,1_{b>0}b]/Y$	1.05	1.83	NIPA and Flow of Funds
Credit spreads			
default rate: $\mathbb{E}[\mathbf{1}_{\{Ch7\}} + \mathbf{1}_{\{Ch11\}}]$	0.03	0.03	Annual rate from Dun and Bradstreet
cred spread: $\mathbb{E}[r^Q - r]$	2.2%	0.7%	Moody's BAA corporate bonds
Investment heterogeneity			
average investment rate: $\mathbb{E}[i/k]$	0.12	0.20	Cooper and Haltiwanger (2006)
SD investment rate: $\text{SD}[i/k]$	0.34	0.36	Cooper and Haltiwanger (2006)
Life-cycle			
share of firms that exit	0.10	0.11	LBD
share of labor at age 1	0.03	0.04	LBD
share of firms at age 1	0.10	0.11	LBD
share of firms at age 2	0.08	0.09	LBD
Balance sheet			
average leverage: $\mathbb{E}[1_{b>0}b'/k']$	0.39	0.72	Compustat
correlation between n and k'	0.74	0.23	Compustat
fraction of firms with $\frac{n}{k'} < 0$	0.21	0.18	Compustat
fraction of firms with $\frac{n}{k'} \in [0,1]$	0.65	0.77	Compustat
fraction of firms with $\frac{n}{k'} > 1$	0.15	0.05	Compustat

Table 2 shows the moments targeted in the calibration. The model fits fairly well the life-cycle of firms (exit rate, and share of labor and firms at the early stages), and investment rates heterogeneity (average and standard deviation) moments. These moments are from the LBD and Cooper and Haltiwanger (2006). Further, it fits the annual default rate of 3% — from Dun and Bradstreet — which includes defaults by liquidation (Chapter 7 liquidation and Chapter 11 piecemeal liquidation) and restructuring (Chapter 11 restructure). On the other hand, in the model steady-state equilibrium the average annual credit spread is 0.7%, which is lower than in the data (2.2%). This discrepancy can be explained by the absence of aggregate risk in the model. Also the model fits well the distribution of cash-on-hand n/k' — shares of firms with negative

values, between 0 and 1, and greater than 1 — which are particularly relevant moments to estimate the relevance of rollover crises. The cash-on-hand n is measured using data from Compustat. There is no data on non-Compustat firms; thus, I extrapolate the rest of the cash-on-hand distribution. Finally, the model shows lower correlation between cash-on-hand and capital, and larger average and aggregate leverage than in the data. Leverage is measured as short term liabilities to capital. Further details of the measurement can be found in Appendix B.

3.2   Identification and Incidence of Rollover Crises

In this section, I show how I identify the likelihood of coordination failures that lead to rollover crises and what is the share of firms exposed to them. Neither the parameter η nor the threshold function $\underline{n}(z)$ that defines the regions in the state space are directly observable, thus I will infer them indirectly using observed firms' bankruptcy choices and financial distribution of firms.[13]

Qualitative identification of η.   Now, I will show a simple qualitative result in a simplified version of the model that provides intuition about the identification in the quantitative model.

Proposition 1. *Assume that in the restructuring process the debt haircut haircut is α_{11} fixed and bankruptcy costs are a fixed cost $c_{11} \in (0, -\underline{n}(z_{max}))$ with z_{max} highest productivity firm in the economy. Then for a given distribution of firms*

1. *if $\alpha_{11} \to 1$ then firms that are insolvent don't restructure their debt,*

2. *if $\alpha_{11} \to 1$ then the share of firms that restructure their debt (i.e., $(z,n) \in \mathcal{R}$ with $n_{11} \geq n$) identifies η,*

3. *if $\alpha_7 < \alpha_{11} < 1$ then firms with higher debt require a smaller c_{11} to restructure.*

The proof is in Appendix A.3.

Proposition 1 shows that η can be identified whenever debt haircuts are low in the restructuring process, and suggests the identification is stronger if the cost c_{11} is larger

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[13]A salient and related example of indirect inference of rollover crises, is Bocola and Dovis (2019) who infer indirectly the rollover risk faced by the government through the time series of observed debt maturity choices. In this paper I use the cross-section of bankruptcy choices to infer the relevance of firm's rollover crises.

(since less leveraged firms restructure). The intuition is that whenever the haircuts are low or close to 0, and cost $c_{11} > 0$, then there are low benefits for insolvent firms to restructure the debt. On the other hand, firms that have a rollover crises they benefit from filing — and preventing liquidation through a rollover crises — even if the debt haircut gains are low and the process costly. In the extreme (case 1 and 2), I find that only firms in a rollover crisis restructure, then the identification is trivial given c_{11}. To find η we only need to divide the observed share of firms that restructure by the share of firms in the risky region that *would be* solvent when they restructure, which is straightforward to compute given c_{11}.

In an intermediate case, where haircuts are not negligible, such as the one in point 3, I show that firms with higher leverage (which are more likely to be insolvent) require lower c to restructure, which suggests that with a large c few firms with high debt will restructure even if the haircut is non-negligible. Then, if the cost c is large, I conjecture that we can approximate η in a similar way that in the other case. Point 3 also suggests that a relevant moment to find c (given the rest of the parameters) is the leverage of the firms in the restructuring process. I use this insight in the quantitative model.

Overall, Proposition 1 suggests that large bankruptcy costs and low debt haircuts dissuade insolvent firms from restructuring; thus, allowing for identification of the probability η using the observed bankruptcy choice of firms between restructuring and liquidation.

Quantitative identification of η. I follow two steps to identify η using the model, first, I use bankrupt firms' characteristics and some outcomes of the bankruptcy process to infer the parameters related to the bankruptcy procedure, and then I infer η from the observed bankruptcy choices of firms'.[14]

First, I want to identify the parameters $(\alpha_7, \tilde{\Xi}, c)$ of the bankruptcy process using moments related to the bankruptcy procedure and bankrupt firms' characteristics. Table 3 panel (a) shows the value of the bankruptcy parameters, and targeted moments in the data and model. The capital recovery rate of creditors during liquidation $\alpha_7 = 0.29$ is set to match the debt recovery rate $\mathbb{E}[R(b, k, \omega)] = 0.27$ in Chapter 7 liquidations reported by Acharya, Bharath and Srinivasan (2007). The *approximate* bargaining power of creditors $\tilde{\Xi} = 0.89$ is set to match the debt recovery rate $\mathbb{E}[\alpha_{11}] = 0.69$ in Chapter

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[14]For exposition purposes, I explain sequentially the identification of η, but, technically, the problem solves a fixed point given that η could also shift other moments.

11 restructuring reported by Acharya *et al.* (2007).[15] The parameter that represents the costs of the Chapter 11 process $c_{11} = 0.40$ and is set to fit the leverage of firms under Chapter 11 $\mathbb{E}[b'/k' \mid \text{Ch 11}] = 0.73$ reported by Antill (2021).

Table 3: Identifying the likelihood of rollover crises

(a) Parameters and targeted moments of bankruptcy process

Parameters	Value	Moment targeted	Data	Model	Data source
α_7	0.38	$\mathbb{E}[R(b,k,\omega)]$	0.27	0.29	Acharya *et al.* (2007)
$\tilde{\Xi}$	0.89	$\mathbb{E}[\alpha_{11}]$	0.69	0.79	Acharya *et al.* (2007)
c_{11}	0.40	$\mathbb{E}[b'/k' \mid \text{Ch 11}]$	0.73	0.67	Antill (2021)
η	**0.07**	$\mathbb{E}[1_{\text{Ch11}}]/\mathbb{E}[1_{\text{Ch7}}]$	2.0	1.9	Antill (2021)

(b) Rollover crises likelihood and restructuring vs liquidation choice
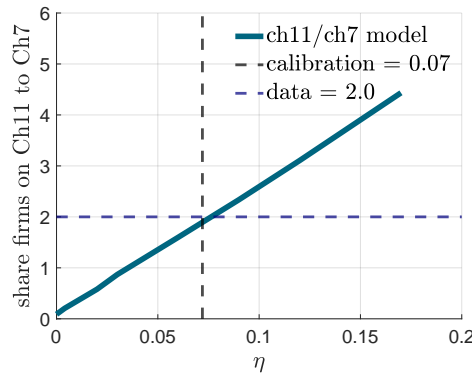
Notes: panel (a) shows the parameters and moments of the bankruptcy process in the baseline calibration. Panel (b) figure shows the relation between η and the share of firms on Chapter 11 relative to Chapter 7, i.e., $\mathbb{E}[1_{\text{Ch11}}]/\mathbb{E}[1_{\text{Ch7}}]$, in the model.

Next, I want to infer η. The data on recovery rates and leverage of Chapter 11 firms suggest that restructuring the debt is costly and haircuts are relatively low. Therefore, *relatively few* insolvent firms decide to restructure their debt, since gains from renegotiation are low (recall Proposition 1). In the model also the distribution of firms is shifting, then the share of restructuring firms can also drop if η increases (for example, firms in the long run want to stay far from the risky region). To solve for this, I use the share of firms that restructure relative to those that liquidate.[16] Consistent with previous observations, in the quantitative model, panel (b) in Table 3 shows that the

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[15] For computational efficiency, I use a convex-pricing function to approximate the bargaining outcome. Details are in Appendix A.2.

[16] Although I can't prove this analytically (yet), my numerical exercises suggest the distribution of firms that are insolvent and risky shift roughly proportionally with η.

higher η the larger is the share of firms that restructure relative to those that liquidate in steady state, which provides the identification of η.

To approximate better the incidence of liquidation and restructuring in the data, I use the summary statistics provided by Antill (2021). Using Chapter 11 outcomes from the Moody's Ultimate Recovery database Antill (2021) identifies how many Chapter 11 cases end in acquisition, piecemeal and full liquidations. When considering this, the ratio of firms that restructure to those that liquidate is 2. Matching the model to the data, [see figure in Table 3 panel (b)], I find a probability $\eta = 0.07$ of rollover crises for firms that are exposed, i.e. in region $\mathcal{R}$. Furthermore, in the calibration, less than 10% of the firms which restructure their debt are insolvent firms, this is consistent with Proposition 1.

To validate the results of the calibration, in Appendix C.4, I show that the model fits other (untargeted) moments of bankrupt firms in the data. The model, not only matches the average leverage of bankrupt firms, but also the distribution. Moreover, I study the predictors of a restructuring event in the data and model, and find that smaller firm size (assets), lower sales growth, lower cash-on-hand and lower leverage increase the likelihood of entering the restructuring process. I further discuss the results in Appendix C.4.

Figure 3: Financial distribution of firms

(a) Cash-on-hand (n) model and data

(b) Incumbents $\Omega^{\mathrm{bop}}(z,n)$ model
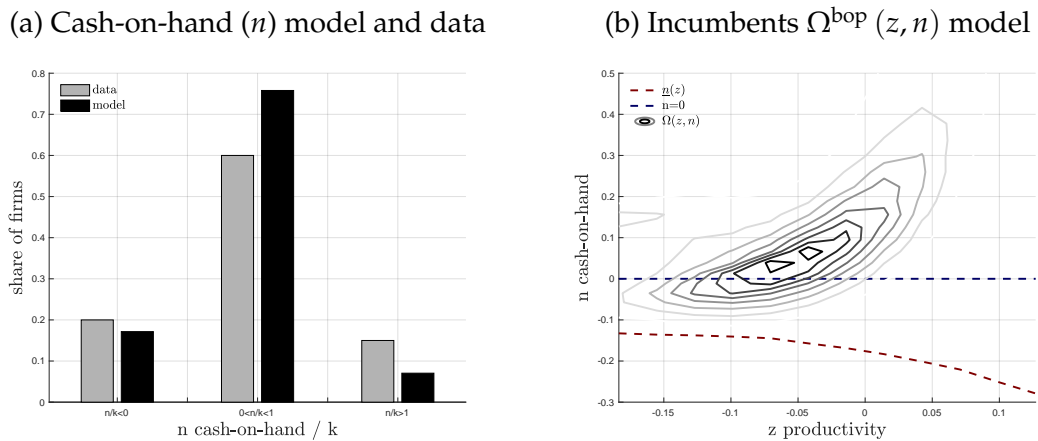
Notes: Panel (a) compares the distribution of cash-on-hand in the model and the data. Panel (b) shows the contour plot (darker line = higher mass) of the distribution of incumbents firms at the beginning of the period (bop) which doesn't receive the exit shock across productivity z (x-axis) and cash-on-hand n (y-axis), i.e., $\Omega^{\mathrm{bop}}(z,n) = (1-\gamma)\int \mathbf{1}_{\{z,n(z,\omega,k,b)=n\}}\mathrm{d}\tilde{\Omega}(s)$. The dashed red line is the $\underline{n}(z)$ threshold and the dashed blue line is the $n = 0$ threshold.

Incidence of rollover crises. Given the value of η, it is straightforward to calculate how many firms are subject to rollover crises. First, compute the distribution of firms at the beginning of the period across productivity z and cash-on-hand n— i.e., $\Omega^{\text{bop}}(z, n) = (1 - \gamma) \int \mathbf{1}_{\{z, n(z, \omega, k, b) = n\}} d\tilde{\Omega}(s)$ — then I estimate the number of firms exposed to rollover crises by computing the share of firms in the risky region $\mathcal{R}$ — i.e., $\int_{(z,n)\in\mathcal{R}} d\Omega^{\text{bop}}(z, n)$ — and, finally, I multiply this share by the conditional probability of a rollover failure η to estimate the incidence of rollover crises. Figure C.4 shows the financial distribution of firms in the model and data. I find that around 20% of the firms are in the risky region and with $\eta = 0.07$ probability of coordination failure, then 1.6% of the firms are subject to rollover crises each period.[17] To illustrate how large is the incidence of rollover crises for firms, I calculate that roughly half of the bankrupt firms (in restructuring plus liquidation processes) had a rollover crisis.

Result I:

I estimate 1.6% of the firms are exposed to rollover crises, where 20% are exposed and the probability of a rollover crisis is 7%. Moreover, roughly half of the bankruptcy events (liquidations and restructures) are driven by rollover crises.

3.3 Investment Heterogeneity and Model's Validation

As a validation exercise, I assess the model's ability to reproduce the observed investment decision in the cross section during large recessions. In the model, I simulate a panel of firms during a large recession. A large recession in the model can be driven by a sudden drop in aggregate TFP, drop in firms' cash-on-hand or a lower recovery rate for creditors in liquidation (credit shock). Further details of the recession shocks in Appendix A.4.

Using an empirical specification similar to Kalemli-Özcan *et al.* (2020), I study the investment dynamics in the simulated data and empirical data of last recessions (Great Recession and Covid crisis). To estimate the *on impact* heterogeneous response — in the model and data — from peak-to-trough of recessions, I will proceed as follows. First, to account for permanent sectoral heterogeneity I will demean each of the firm-quarter observations of cash-on-hand over capital n_{it}/k_{it} for firm i in period t of interest by its

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[17]Notice this number includes firms that may not be liquidated because they enter the restructuring process.

sectoral average, i.e. $\hat{n}_{it} = n_{it}/k_{it} - \mathbb{E}_s[n_{it}/k_{it}]$ for firm i in sector s. Next, I will assign each firm-quarter observation of $\hat{n}$ to each tercile (for each period's distribution). Lastly, I run the following panel regression episode analysis to estimate the heterogeneous responses of investment across cash-on-hand n/k during the recession:

$$\Delta \log(k_{it}) = \underbrace{\sum_{j=1}^{J} \beta_j^n \left(Q_{it}^{nj} \times \text{crisis}_t \right)}_{\text{heterogeneity across } n/k} + \underbrace{\Lambda' Z_{it}}_{\text{controls}} + \varepsilon_{it}, \tag{21}$$

where Q_{it}^{nj} indicates if $\hat{n}_{it}$ belongs to tercile j, $\Delta \log(k_{it}) = \log(k_{it+h}) - \log(k_{it})$ is firm's i capital accumulation over a period as long as the recession studied (i.e., the extension from peak-to-trough of episode studied h), crisis_t indicates if a recession happens during the period considered (from t to $t + h$) and $Z_{i,t}$ includes the control variables. For the baseline specifications controls, I include firm's fixed effects, sectoral fixed effects, log assets as proxy for size, last quarter sales growth and heterogeneity across firm's leverage.

The coefficients β_j^n are the estimates of interest, and can be interpreted as the difference-in-difference estimates of the recession episode impact on capital accumulation for firms in tercile j of $\hat{n}$. Results are shown relative to the highest group (i.e., the one with highest cash-on-hand or lowest leverage). In the empirical application I use data from Compustat (limited to publicly traded firms) and in the model I select firms that approximate this set of firms. Further details of the data, estimates and other results are in Appendix B.

Figure 4 shows the results across cash-on-hand n/k. In all plots, the blue connected line corresponds to the data estimates and the diamond dots indicate the estimates of the model-simulated data. The heterogeneity of investment responses across cash-on-hand is similar in the data and model , surprisingly, for different types of shocks in both episodes. For firms in the lowest group of cash-on-hand we observe that the drop in investment (differences in capital accumulation) is 3.5 percentage points greater compared to firms with high levels of cash-on-hand.[18] These results shows that the model performs well matching the heterogeneity of investment in crises (which is not targeted), but doesn't provide a clear identification of the source of the aggregate shock.

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[18]Further, in Appendix B, I show the empirical results across leverage levels and for individual episodes.

Figure 4: Investment Heterogeneity During Recessions: β_j^n

| (a) TFP shock | (b) cash shock | (c) credit shock |
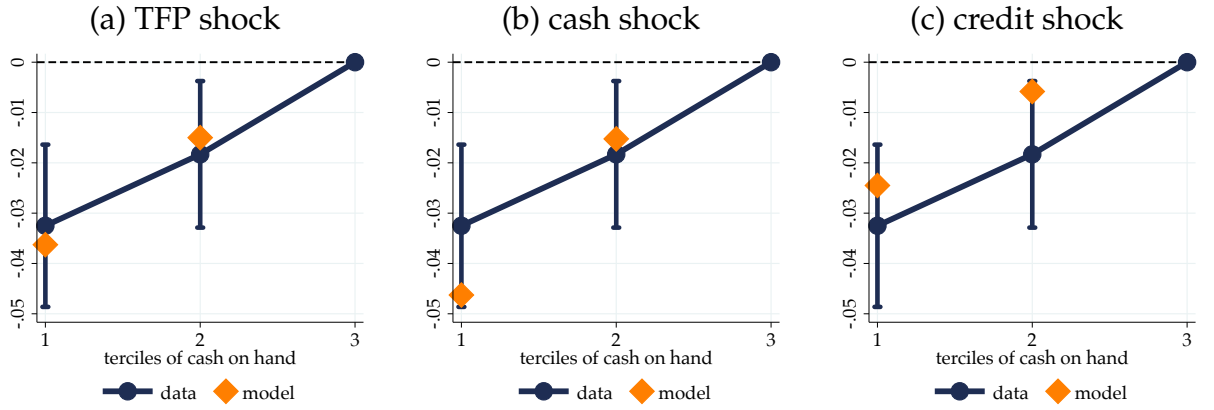
Notes: Panel (a), (b) and (c) show the estimates of β_j^n for the Great Recession and Covid Crisis average, for the data and different aggregate shocks simulated in the model. The blue connected line shows the data and the 90% confidence interval. The diamond dots show the estimates from the model's simulated panel data. Since I focus on the heterogeneity, the coefficient values are relative to the tercile of firms with the highest cash-on-hand. All estimates (in model and data) are from empirical specification (21). Estimates are in semester frequency to make them comparable across episodes.

4 MACROECONOMIC IMPLICATIONS

In this section, I study the relation of firms' rollover crises and macroeconomic dynamics during large recessions.[19] I simulate a prototypical large recession episode — i.e., crisis driven by large aggregate shocks that are standard in the literature — and assess the role of rollover crises by comparing with a counterfactual recession without coordination failures.

To assess the role of rollover risk in recessions I simulate a prototypical large recessions episode — large and unexpected shock — driven by different types of shocks and perform a counterfactual where coordination failures don't happen during the recession episode. The three types of recession shocks studied are: (i) aggregate TFP shock; (ii) sudden reduction in firm's cash; (iii) a decrease in liquidation recovery rate α_7, i.e., credit shock.[20] Shocks happen unexpectedly, they return to the initial steady state in the long run and there is perfect foresight of their path. All shocks match a

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[19]In Appendix C.1, I show that firms' rollover risk has negligible consequences over macroeconomic outcomes over the long run. This happens because firms accumulate internal resources to stay far from the risky region and in equilibrium they improve their financial position, reducing the overall impact of rollover crises on aggregate outcomes.

[20]Notice the credit shock in this model is a reduction in the collateral value when liquidated, which is different from a credit shock in Khan *et al.* (2020). Their credit shock is closer to the cash shock in this paper.

peak-to-trough drop of aggregate output Y of 5%, and have a transitory nature, i.e., persistence of the shocks is set to 0.5. Shocks happen at $t = 0$, where the economy was in steady-state the previous period. Further details of the definition of the shocks and computation of transitions in Appendix A.4.

Figure 5 shows the response of aggregate output Y to different types of aggregate shocks with coordination failures (black line) and without them (gray dashed line). In all panels the absence of firm rollover crises —i.e., no extra rollover crises at $t = 0, 1$ — significantly reduces the depth of the trough in the crises. The counterfactual exercises indicate that rollover crises during recessions can explain from 10% to 30% of the total output losses.[21]

Figure 5: Crisis Shock and Aggregate Output
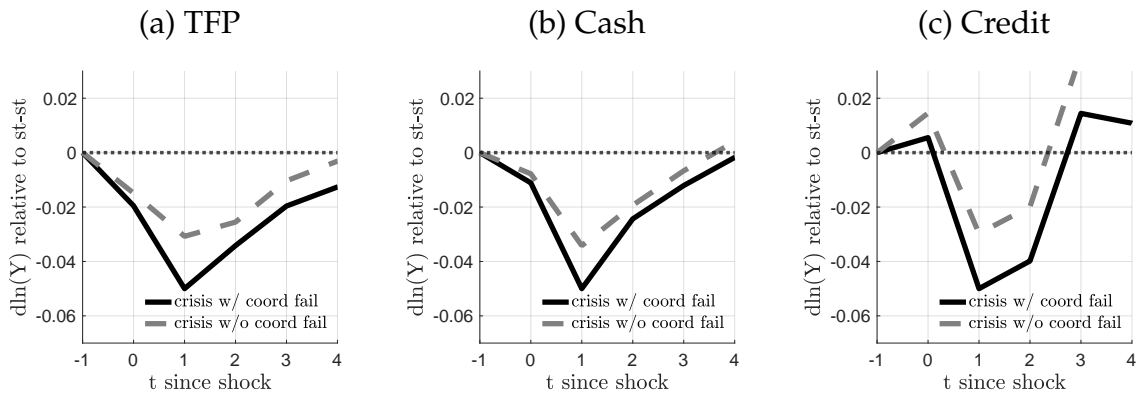
Notes: panel (a), (b) and (c) show the response of aggregate output Y to an aggregate TFP shock, cash shock and credit shock with coordination failures and without coordination failures (for t=0 and t=1), respectively. The economy at $t = -1$ is in steady-state. The definition of the shocks are in the text and Appendix A.4.

Although, the relevance of rollover risk is similar across shocks, on impact, the dynamics are different. In Appendix A.5, I show the dynamics of firm (net) exit, debt and capital accumulation during large recessions. Weaker fundamentals — due to a TFP or cash shock — expose (temporarily) several solvent firms to rollover crises, which increases firm exit of healthy firms during the recession. Further, new firms entering the economy are smaller and take time to grow. Therefore, recessions driven by a TFP or cash shock — exacerbated by coordination failures — slow the recovery down significantly. On the other hand, a crisis driven by a credit shock will induce firms to deleverage quickly and reduce their investment initially to preclude liquidation, which

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[21]Numbers depend on the driver of recessions and how I compute them (e.g., as a present discounted value or simple sum of output losses).

makes the recovery relatively stronger (compared to other shocks), although the initial impact is the similar.

Result II:

Firms' rollover risk significantly amplify the impact of recessions. It explains between 10% to 30% of output losses during large recession episodes.

5 POLICY IMPLICATIONS

Firms' rollover crises increase the bankruptcy and liquidation of healthy firms, and — as shown in the previous section — they have significant (negative) macroeconomic consequences (i.e., greater depth and slower recovery) during large recession episodes. Arguably a variety of the policies displayed during the recent Covid crises had the motivation of preventing healthy firms from being liquidated. For example, the Federal Reserve's Primary and Secondary Lending Programs provided direct credit access to a set of seemingly sound corporate firms —i.e., those with a good financial position before the recession. Moreover, direct credit interventions in other recessions (e.g., firm bailouts during the Great Recession) had a similar motivation.

 Motivated by this, in this section, I study how effective are *imperfectly-targeted* direct lending policies deployed to reduce the impact of creditors' coordination failures during large recessions. Policies are imperfectly targeted because I assume the government cannot target the firms that participate using all relevant characteristics. This assumption captures the notion that the government may not observe all the characteristics of individual firms or the policy can't be restricted for other reasons (e.g., regulation on lending powers). In my quantitative exercises, I find that credit programs that operate mostly through the insurance channel (precluding coordination failures) and government credit facilities remain mostly unused (in equilibrium) are very potent. On the contrary, credit programs that are very ample and subsidize credit to many firms, can mitigate firms' rollover crises in the short term, but exacerbate debt overhang problems in the future, amplifying the overall impact of the recession.

Credit policy workings. A direct lending policy in the model is promised unexpectedly at $t = 0$, and implemented at period J_0 for J periods. When the policy is active, the government offers an alternative pricing schedule $Q_j^g(.)$ for the new debt issuance

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of the firm at each period $j = J_0, J_0 + 1, .., J$ to a set of eligible firm $\mathcal{P}$ which depends on observable, current or past, characteristics of the firms. The set of eligible firms $\mathcal{P}$ is assumed to be fixed over the time the policy is implemented and I assume creditors know about the policy. Now the external resources from new debt issuance of eligible firms are

$$\max \left\{ Q^g \left(. \right), Q \left(s, k', b' \right) \right\} b',$$

where firms choose the best pricing schedules between the government program and the market.

To understand how this policy works, let me assume that the government sets for 1 period $Q^g \left(. \right) = \tilde{Q} \left(z, k', b' \right)$ and a firm with $(z, n) \in \mathcal{R}$ and $\phi < \eta$ — i.e., under a rollover crisis — is eligible for the government program. Without the policy creditors would offer $Q = 0$ to the firm. With the policy, creditors know that since firm are in $\mathcal{R}$ they are solvent under $Q^g = \tilde{Q}$. Thus, if creditors conjecture all other private creditors will not lend to the firm ($Q = 0$) still the firm will be able to satistify $d \geq 0$ at the pricing schedule $Q^g = \tilde{Q}$. The presence of this alternative pricing schedule will coordinate creditors in the $Q = \tilde{Q} > 0$ equilibrium, even if the firm doesn't participate of the policy (i.e., use Q^g) in equilibrium. The intuition is that this policy operates as a form of insurance to creditors, which prevents the $Q = 0$ equilibrium from happening.

Imperfect credit policy quantification. A policy $Q^g = \tilde{Q}$ that precludes all rollover crises without being used in equilibrium requires the government to observe perfectly each firm's productivity and cash-on-hand (z, n). To make the policy more realistic, I assume that the government observes only n and sets the policy according to a simple rule, i.e., *imperfect credit policy*:

1. The set of eligible firms depends on n only and requires that $n < 0$ (i.e., firms need external resources to satisfy $d \geq 0$). Therefore, the set $\mathcal{P}$ is composed by firms with n such that $n \in [\underline{n}^g, 0)$, where $\underline{n}^g < 0$ is a parameter chosen by the government.

2. All eligible firms receive enough funds such that they can satisfy $d \geq 0$, but the government can't discriminate across the n position of eligible firms, i.e., $Q^g \left(z, k', b' \right)$ is such that $\underline{n}^g = - \max_{b', k'} Q^g \left(z^g, k', b' \right) b' - qk' = \underline{n} \left(z^g \right)$ which implies that n^g determines the choice of $z = z^g$ for the pricing schedule offered

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by the government.[22]
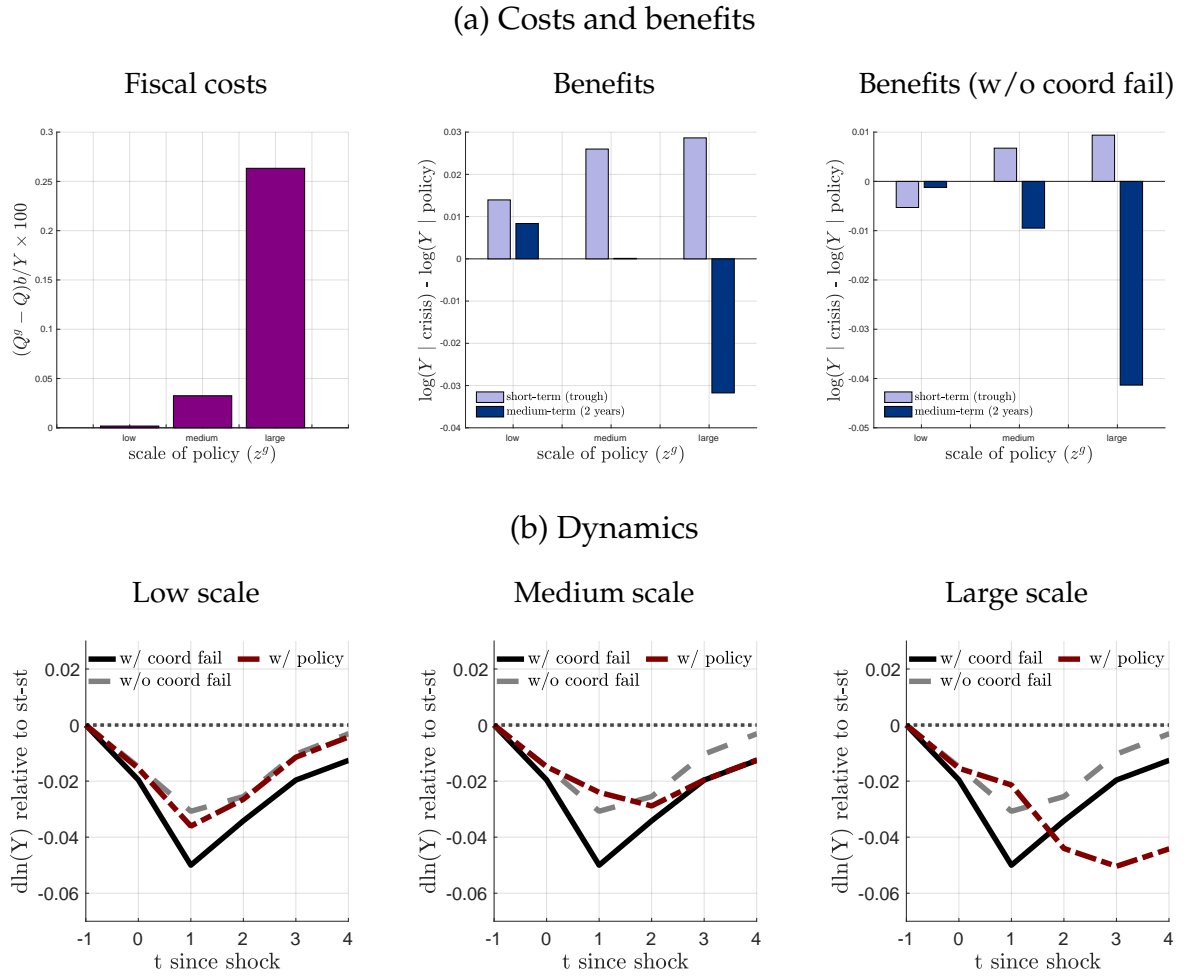
Figure 6: Imperfect Credit Policy Eligibility
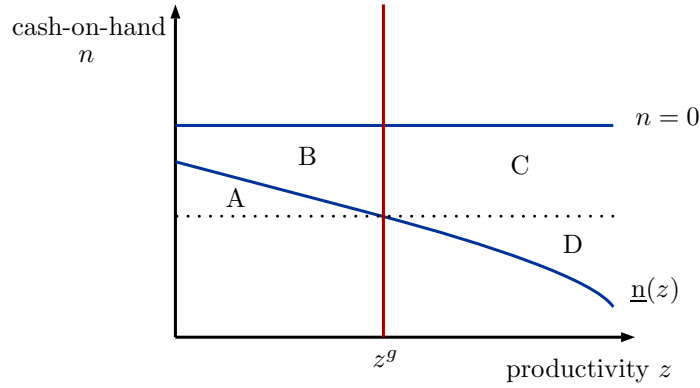
Notes: figure shows an illustration of the eleigbility and firms participation in the program for a one period example.

Figure 6 shows what firms are eligible and the static choice of the participating or not in the program for a 1 period policy.[23] Eligible firms are those in the area $A \cup B \cup C$. In the case of A, in absence of the credit program the firm would be insolvent, then these firms receive subsidized credit. On the other hand, in B, firms will find the credit in the program cheaper than the market then they participate so they receive a subsidized credit. On the contrary, firms in region C will have a more expensive credit than the market then they don't participate in the program. Thus, firms in $A \cup B$ receive a subsidized credit and firms in C don't. Moreover, the credit program will preclude those firms under a rollover crisis in $B \cup C$ from being liquidated. Notice that firms under a rollover crisis in B will participate in the program and those in C will not participate in the program but the mere existence of the program will preclude coordination failures. Therefore, if the scale of the policy increases — i.e., lower n^g or, equivalently, greater z^g — more coordination failures are precluded and more firms are subsidized. The subsidized credit could exacerbate future debt overhang problems and has fiscal costs, so the policy faces a potential trade-off when incrementing the scale of the program.

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[22]The assumption that the government pricing function doesn't depend on the firm's cash-on-hand simplifies greatly the computational problem.

[23]If the policy lasts more than one period or is implemented with a lag, then it will affect the solvency thresholds (even in partial equilibrium) since they depend on future prospects of the firm.

Figure 7: Imperfect Credit Policy by Scale

(a) Costs and benefits

Fiscal costs   Benefits   Benefits (w/o coord fail)

(b) Dynamics

Low scale   Medium scale   Large scale

Notes: The Figure shows the policy costs and benefits for different policies during a large recession driven (crises) by a cash shock. Panel (b) shows the fiscal cost of the policy in GDP terms, the short (in the crisis trough) and medium (2 years) term benefits of the policy with and without coordination failures. Graphs in Panel (b) show the response of aggregate output Y to an aggregate cash shock with perfectly targeted policy (same as without coordination failures, dashed gray line), imperfectly targeted policy (red dashed line), and without policy intervention (solid black line) for different policy scale. The economy at $t = -1$ is in steady-state. The definition of the shocks and crises experiments are in Section 4 and Appendix A.4. Further description of the policy in the text.

Figure 7 shows the evolution of aggregate Y for different levels of z^g for programs implemented during $t = 0, 1$ — i.e., peak-to-trough of the recession without the policy. I focus on the cash shock, results for the TFP shock are quantitatively similar and relegated to Appendix A.6 for clarity.

First graph in panel (a) shows the fiscal cost of the policy for different scales, where the fiscal cost is how much credit is the government subsidizing in total, i.e., sum of $(Q - Q^g)$ for participant firms. In this exercise, I show a low scale policy that has a cost close to 0 (can be interpreted as an announcement), a medium scale policy that has a

low cost (0.05% of output) and a large scale policy that has a large cost (around 0.3% of output). Next, panel (a) and (b) show the short and long term costs and benefits of credit programs with different scales. I find that in the short term the large scale program improves aggregate outcomes and even provide some extra stimulus (relative to the counterfactual without rollover crises), but in the medium term it aggravates the recession and creates greater output losses. The intuition, for this result, is that large scale programs subsidize credit to many financially exposed and fundamentally weak firms, which eventually backfire by exacerbating debt overhang in the future.[24] On the other hand, the low scale program has significant short term benefits with a fiscal cost close to 0, and provide a swift recovery (similar to the counterfactual without rollover crises). The intuition. for this result, is that small scale programs can mitigate rollover crises (even if firms don't use actively the credit facilities), and rescue several healthy firms in the economy which creates a stronger recovery.

Result III:

Imperfectly-targeted credit policies can mitigate the amplification created by rollover crises (even if the policy is just announced), but can backfire if they subsidize credit to many firms.

6 CONCLUDING REMARKS

In this paper, I develop a framework where firms' rollover crises can be identified and quantified. Salient features of the U.S. bankruptcy code allow me to quantify the incidence of rollover crises using observed bankruptcy choices and bankrupt firms' characteristics. I find that rollover crises can explain roughly half of the bankruptcy events.

My quantitative results suggest that firms' rollover crises, through the failure of healthy firms, have a significant impact during large recessions. On the other hand, direct credit policies can act as insurance for creditors and prevent coordination failures from happening, but, if policy is imperfectly-targeted, then the government faces a trade-off between short run mitigation of rollover crises and future debt overhang problems. Quantitative results suggest that, during large recessions, the benefits of direct credit policies are ambiguous.

In the model, I focus on the problem of firms which have homogeneous and atomistic

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[24]This mechanism is similar to the one studied by Crouzet and Tourre (2021).

creditors, and without active management of its liability structure. Potential extensions could allow for, investors' heterogeneity [for example, Halac, Kremer and Winter (2020)]; endogenous debt maturity structure [for example, Bocola and Dovis (2019) for sovereign debt or Cheng and Milbradt (2012); Crouzet (2017b) for firms]; and endogenous number of creditors [for example, Bris and Welch (2005); Bolton and Scharfstein (1996)].

The evidence presented in this paper suggests that firm-level financial frictions can play a relevant role during crises, which complements the view that creditor-level financial frictions were important during recent crises [see, for example, Gertler and Gilchrist (2018)]. One potential avenue of future research is to explore the relation between these two forms of credit frictions and how can we separately identified them. Furthermore, my paper provides insights on the relationship between rollover crises and bankruptcy provisions during large recessions, which can be applied in other contexts. For example, other potential avenue of future research is to study — in a sovereign debt model with rollover crises — the costs and benefits of adopting provisions analogous to those of the U.S. bankruptcy code in a supranational sovereign debt bankruptcy process. I leave these extensions and alternative applications for future work.

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The Appendix is organized as follows: Appendix A shows additional derivations of the baseline model and further details on the computations; Appendix B includes details of the data sources (sample selection and definitions) and details of the empirical exercises; and Appendix C includes further exercises and extensions of the model.

A   Appendix: Model

A.1   Exiting firms problem

Incumbents firms at the beginning of the period receive with probability γ a shock that force them to exit after production. I allow for exiting firms to make also the liquidation choice and restructuring choice. Notice that since they exit at the end of the period these firms don't choose $\left(b', k'\right)$ then they are not subject to coordination failures such as the ones described for nonexiting firms. Exiting firms choose to liquidation choice is

$$\mathbf{1}_{\{\text{ch7}|\text{exit}\}}\left(\mathbf{s}\right) = \tilde{\mathbf{1}}_{\{\text{ch7}|\text{exit}\}}\left(z, \omega, b, k\right) = \begin{cases} 1 & \text{if max}\left\{n, n_{11}^{\text{exit}}\right\} < 0 \\ 0 & \text{otherwise} \end{cases}. \tag{22}$$

where n defined as before and $n_{11}^{\text{exit}} = \pi\left(z, \omega, k\right) + \left(1 - c_{11}\right)\left(1 - \delta\right)q\omega k - \alpha_{11}^{\text{exit}}\left(z, \omega, k, b\right)b$. Since the outside option is to continue then only firms with $n < 0$ will restructure their debt and the debt recovery $\alpha_{11}^{\text{exit}}$ is determined by

$$\alpha_{11}^{\text{exit}}\left(z, \omega, b, k\right) = \max_{\alpha_{11}}\left[n_{11}^{\text{exit}} - 0\right]^{1-\Xi}\left[b\alpha_{11}^{\text{exit}} - bR\left(b, k, \omega\right)\right]^{\Xi} \tag{23}$$

subject to

$$n_{11}^{\text{exit}} > 0$$

$$\alpha_{11} \geq R\left(b, k, \omega\right).$$

The restructuring choice is

$$\mathbf{1}_{\{\text{ch11}|\text{exit}\}}\left(\mathbf{s}\right) = \tilde{\mathbf{1}}_{\{\text{ch11}|\text{exit}\}}\left(z, \omega, b, k\right) = \begin{cases} 1 & \text{if } \{n < 0\} \cap \{n_{11}^{\text{exit}} > 0\} \cap \left\{\alpha_{11}^{\text{max}|\text{exit}} > \alpha_{11}^{\text{min}}\right\} \\ 0 & \text{otherwise} \end{cases}, \tag{24}$$

where $\alpha_{11}^{\text{max}|\text{exit}} = \frac{\pi(z,\omega,k)+(1-c_{11})(1-\delta)q\omega k}{b}$ and $\alpha_{11}^{\text{min}} = R\left(b, k, \omega\right)$. The firms that continue are defined as $\mathbf{1}_{\{\text{continue}|\text{exit}\}}\left(\mathbf{s}\right) = 1 - \mathbf{1}_{\{\text{ch11}|\text{exit}\}}\left(\mathbf{s}\right) - \mathbf{1}_{\{\text{ch7}|\text{exit}\}}\left(\mathbf{s}\right) = \mathbf{1}_{\{n \geq 0\}}$.

A.2 Computational solution of Bargaining Problem

To solve the bargaining problem I adopt a very simple convex-pricing function to approximate the result from the Nash Bargaining problem.[25] Although this is a reduced form solution to the bargaining problem, it provides better computational speed since we don't need the value function of the firm to compute it. I proceed as follows: I compute the maximum and minimum recovery rates, $\alpha_{11}^{\max}(z, \omega, k, b)$ and $\alpha_{11}^{\min}(\omega, k, b)$, respectively. Using these bounds, for the restructuring processes that are feasible I compute the approximate recovery rate $\tilde{\alpha}_{11}(z, \omega, k, b)$ as

$$\tilde{\alpha}_{11}(z, \omega, k, b) = \tilde{\Xi}\alpha_{11}^{\max}(z, \omega, k, b) + \left(1 - \tilde{\Xi}\right)\alpha_{11}^{\min}(\omega, k, b)$$

where $\tilde{\Xi} \in [0, 1]$ is the approximate bargaining power of the creditors. There is no one-on-one mapping, but to check for robustness I solve for the exact solution and find similar results. Therefore, I adopt this convex-pricing function, which is computationally significantly more efficient than the exact solution.

A.3 Proof Identification of η

Proof. Assume α_{11} is fixed, $c > 0$ is a fixed cost and the distribution of firms at the beginning of the period $\tilde{\Omega}$ is fixed.

Consider the case $\alpha_{11} \to 1$ we have that resources after restructuring are $n_{11} \to n - c$, then insolvent firms will have $\underline{n}(z) > n > n - c$ then they will not restructure their debt. On the other hand, firms with a rollover crisis restructure if $n > n - c \geq \underline{n}(z)$ since the firm is in $\mathcal{R}$ we have $n < 0$ we need the cost is $c < -\underline{n}(z_{\max})$, with $z_{\max}$ the highest productivity draw in the economy, for at least on firm to participate of the restructuring process. Next, without knowledge of η we can compute the share of firms that *would* restructure if they receive the sunspot shock and are exposed, which is $\int_{(z,n)\in\mathcal{R}} \mathbf{1}_{\{n-c>\underline{n}(z)\}} d\Omega^{\text{bop}}(z, n)$ with the beginning of the period distribution of non-exiting firms across (z, n) is $\Omega^{\text{bop}}(z, n) = (1 - \gamma) \int \mathbf{1}_{\{z, n(z,\omega,k,b)=n\}} d\tilde{\Omega}(\mathbf{s})$. Therefore, if we have the values of c (which could be inferred from other moments, e.g., bankrupt firms' characteristics) and distribution of firms, then we can identify η using the fact the share of firms that restructure equates the share of firms that would restructure if $\phi < \eta$ times the probability η.

Consider the case $\alpha_7 < \alpha_{11} < 1$ assume, without loss of generality, no recovery rate in liquidation $\alpha_7 = 0$, price of capital $q = 1$, the variance of ω shocks is 0 and $\delta = 0$ then internal resources are $n = \pi(z, k) + k - b$ and internal resources when restructuring

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[25] In their robustness exercises Guntin and Kochen (2021) adopt this function to solve computationally for a complex bargaining problem.

are $n_{11} = n + (1 - \alpha_{11}) b - c$ with $(1 - \alpha_{11}) b$ the gains from restructuring and c the cost. Now, for insolvent firms we have now that the net benefits of the debt restructuring process can be ambiguous, so there can be insolvent firms restructuring. In particular, for firms to participate they need that the gains from the haircut are large enough to cover the bankruptcy costs and resources needed to become solvent $(1 - \alpha_{11}) b > c + (\underline{n}(z) - n)$. Replacing with the definition of n then we can find for a firm with (z, k, b) that $b < \frac{\underline{n}(z) + \pi(z,k) + k - c}{\alpha_{11}}$ for the firm to restructure. This condition shows that firms with higher b, given (k, z) are less likely to restructure. The intuition is that while leveraged firms benefit more from the haircut, they have lower internal resources which requires them larger gains from the haircut. Furthermore, we can interpret it as higher cost c will create less leveraged firms file for restructuring (using the leverage of the firm we can identify c, which is the identification strategy in the quantitative model). The higher c then will make firms with higher debt become less likely to restructure, therefore we can approximate η if c is large enough such that few insolvent firms can restructure even if $\alpha_{11} < 1$.

$\square$

## A.4   Crises shocks and counterfactual

I work with 3 different types of crisis shocks: a TFP shock, cash shock and credit shock. Shock are unforeseen and I study the perfect foresight transitions from $t \geq 0$ where $t = 0$ is the initial impact of the shock (at the beginning of the period). The initial impact is calibrated to match a 5% drop in aggregate output from peak-to-trough (large aggregate shock) and the persistence of all shocks is $\rho_{\text{shock}} = 0.5$ (i.e., short lived).[26] I assume the process are

1. TFP shock: firms production function is now $y_{it} = A_t f(z_{it}, \omega_{it}, k_{it})$ for $t \geq 0$ where $A_t = \exp\left(\rho_{\text{shock}}^t \epsilon_A\right)$ with $\epsilon_A < 0$ the initial shock at $t = 0$.

2. Cash shock: firms cash-on-hand is $n_{it} = \pi_t(z_{it}, \omega_{it}, k_{it}) + (1 - \delta) q_t \omega_{it} k_{it} - b_{it} - N_t k_{it}$ for $t \geq 0$ where $N_t = \rho_{\text{shock}}^t \epsilon_N$ with $\epsilon_N > 0$ initial shock to cash proporitional to capital.

3. Credit shock: recovery rate when liquidated α_{7t} is time-varying for $t \geq 0$ where $\alpha_{7t} = \alpha_7 - \rho_{\text{shock}}^t \epsilon_7$ where $\epsilon_7 > 0$ initial decrease in liquidation recovery rate.

Figure A.1 shows the path for the baseline counterfactuals.

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[26]More than 95% of the shocks fades away in an year.

Figure A.1: Crises Shocks Path
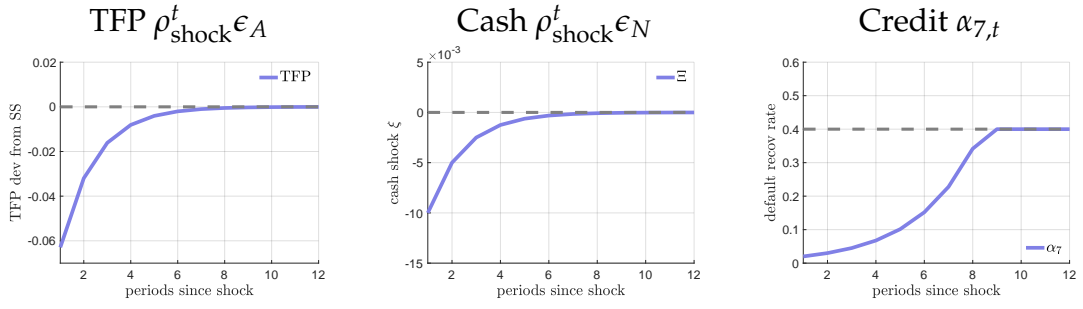
Notes: panel (a), (b) and (c) show the path of the shocks. Shocks happen at $t = 0$. Further description of the shocks in the text.

For computing the aggregates during the transitions I assume that the distribution of firms is no longer a fixed point and allow for *net exit* by fixing the amount of new firms created each period to the initial steady-state calibration, i.e., $\bar{\mu}_t = \bar{\mu}$.

A.5 Firm Exit and Spreads during Crises Experiment

Figure A.2: Capital and Debt during Crises

(a) Capital: $\ln(K_t) - \ln(K_{ss})$ (b) Debt: $\ln(B_t) - \ln(B_{ss})$
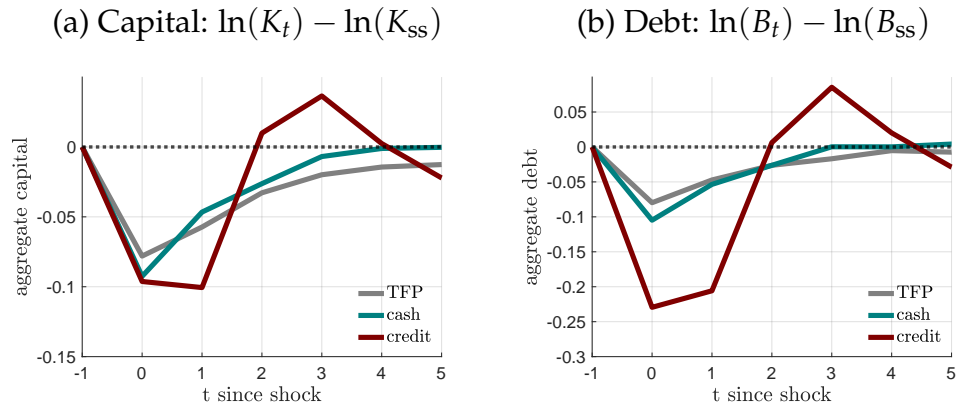
Notes: Figures show the dynamics of capital and debt accumulation for the three crisis shocks studied. In both panels, the variables are in terms of log difference relative to steady state — $\ln X_t - \ln X_{SS}$. Panel (a) shows the dynamics of aggregate capital accumulation. Panel (b) shows the dynamics of aggregate debt accumulation.

Figure A.3: Firm Exit during Crises

(a) Exit rate(t) − Exit rate(SS) (b) Exit rate$(t;\eta)$ − Exit rate$(t;\eta=0)$
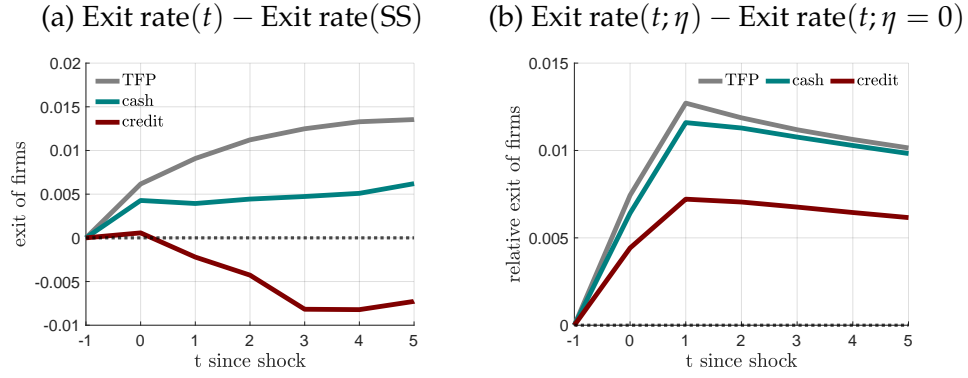
Notes: Figures show the dynamics of firm exit for the three crisis shocks studied. Panel (a) shows the difference between firm exit rates (exogenous and endogenous) relative to pre-crisis steady-state levels during the crisis episode. Panel (b) shows the difference between firm exit rates with coordination failures relative to the counterfactual without coordination failures during the crisis episode.

A.6 Credit policy program

Further details quantitative setup. The baseline credit policy experiment consist of a parameter z^g that determines the pricing schedule $Q_t^g = \tilde{Q}_t\left(z^g, b', k'\right)$ and set of eligible firms $[\underline{n}\left(z^g\right), 0)$ and lasts two periods (implemented at $t = 0$ and $t = 1$). The policy is computed backwards, since the presence of the policy at $t + 1$ will affect the solvency thresholds $\underline{n}_t\left(z\right)$. To estimate the cost of the policy I compute the aggregate credit subsidy as the difference between the price offered by the private sector relative to the government credit program times the amount borrowed for the firms that choose to participate in the program, i.e.,

$$ G_t = \int_{(z,n)\in\mathcal{P}} \max\left\{0, \tilde{Q}_t\left(z, k', b'\right) - \tilde{Q}_t\left(z^g, k', b'\right)\right\} b' \, d\tilde{\Omega}_t\left(\mathbf{s}\right). $$

The subsidy is financed through a lump-sum transfer $T_t = G_t$ such that the aggregate output net of government expenditure is $\tilde{Y}_t = Y_t - G_t$.

Further quantitative results. Figure A.4 shows the results for the policy experiments when the driving shock is a cash shock.

Figure A.4: Imperfect Credit Policy by Scale (cash shock)

(a) Costs and benefits

(b) Dynamics
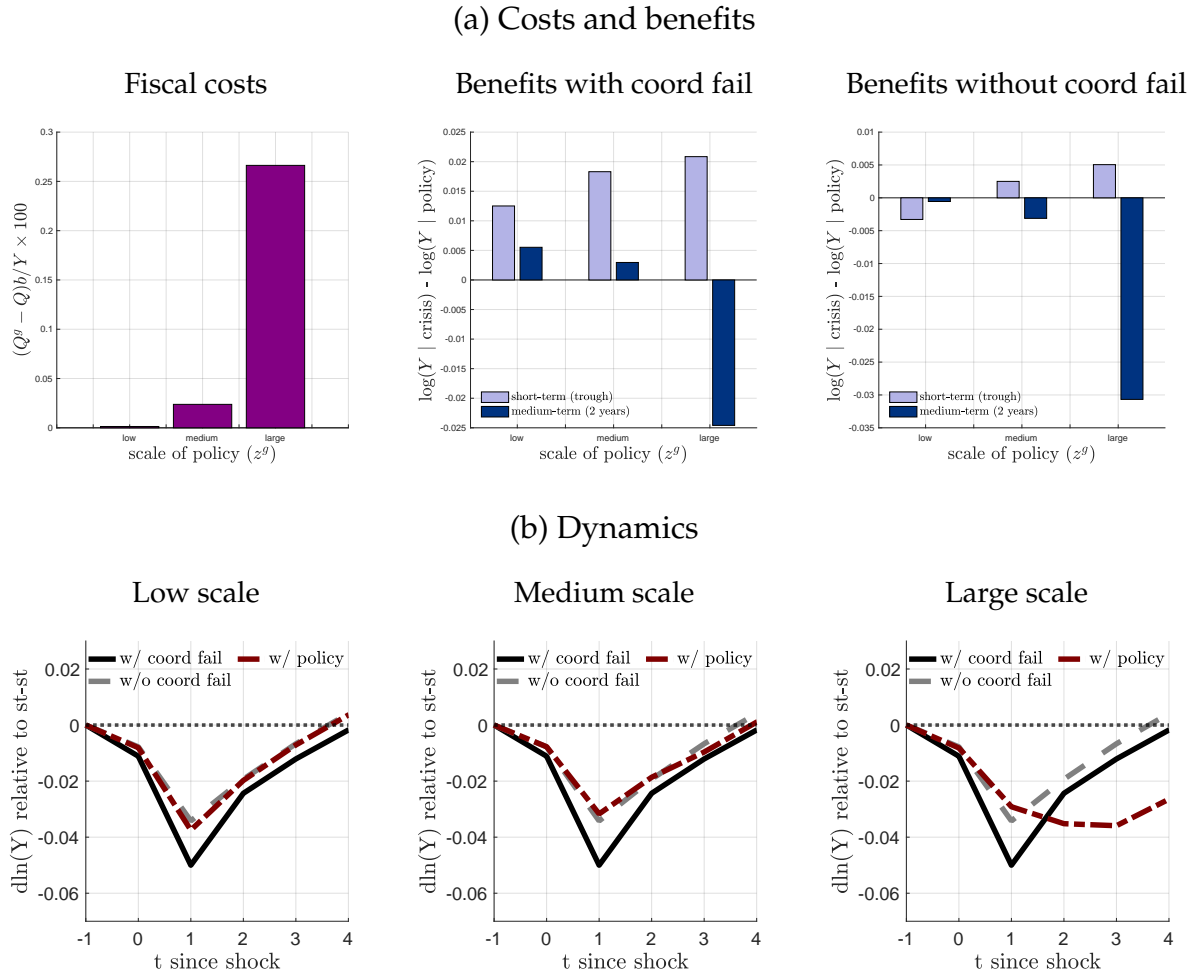
Notes: the Figure shows the policy costs and benefits for different policies during a crises driven by a TFP shock. Figure (b) shows the fiscal cost of the policy in GDP terms, the short (in the crisis trough) and medium (2 years) term benefits of the policy with and without coordination failures. Figures in Panel (b) show the response of aggregate output Y to an aggregate cash shock with perfectly targeted policy (same as without coordination failures, dashed gray line), imperfectly targeted policy (red dashed line), and without policy intervention (solid black line) for different policy scale. The economy at $t = -1$ is in steady-state. The definition of the shocks and crises experimients are in Section 4 and Appendix A.4. Further description of the policy in the text.

B   Appendix: Data and Empirical Exercises

In this section I outline the data sources, variable definitions, and further empirical exercises and results. I show how the balance sheet moments for the calibration are computed, how it is estimated the heterogeneity in investment dynamics during during the Great Recession and Covid-19 crisis, and study the characteristics of bankrupt firms using microdata.

B.1 Data Sources, Sample Selection and Variable Definitions

In this section I describe the details (definitions and sample construction) of the main data sources used to compute moments related to the balance sheet of firms and empirical exercises in the paper.

B.1.1 Compustat

I use Compustat data to compute moments related to the balance sheet of firms and bankruptcy process, and study the patterns of investment in recent large crises. Compustat is limited to publicly held firms, therefore I assume the balance sheet distribution replicates in the rest of the firms.[27] To construct the sample I follow standard practices in the empirical investment literature.

Balance Sheet Data. Now I explain how I construct the sample and the variables for the balance sheet data used for calibration and empirical exercises. The sample selection criteria follows a firm level filter and firm-date filter. Table **B.1** shows the number of observations and those dropped by each filtering step. I drop firms from finance, insurance, and real estate sectors ($\texttt{sic} \in [6000, 6799]$), utilities ($\texttt{sic} \in [4900, 4999]$), non-operating establishments ($\texttt{sic} = 9995$) and industrial conglomerates ($\texttt{sic} = 9997$), and those not incorporated in U.S. and not operate in USD. I drop firm-date observations that with negative capital or total assets, observations with acquisitions of more than 5% of firm's assets, bottom 0.5% and top 99.5% investment rate across the distribution, investment spells of less than 20 quarters, drop if net liquid leverage (net current liquid debt/total assets) is greater than 10 in absolute value, drop if log sales growth is greater than 1 in absolute value, and negative sales or negative liquid assets.

Due to changes in the accounting data of Compustat, I split the sample for the Great Recession (period 1983-2017) and Covid-19 Crisis (period 2019-2020) (see Ma (2020) notes on the accounting changes after 2019).[28] The sample criteria for the 2019-2020 period differs slightly from the 1983-2017 sample. Since the 2019-2020 sample is smaller I exclude filters related to investment outliers and spells, and select firm-quarter observations that register they changed they updated their accounting criteria.[29]

The final sample — pre-Covid — has 426,465 firm-date observations, and the Covid sample has 13,974 firm-date observations.

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[27] An alternative approach is to fit the model to a subset of firms that can be defined as the Compustat firms. For simplicity I use the assumption described in the text.

[28] An alternative approach is to use Compustat Snapshot to remove the operation leases from various entries in the balance sheet, but access to this dataset is restricted.

[29] The variable $\texttt{acctchgq}$ is "ASU16-02" or "IFRS16" the quarter the firm changes it's accounting criteria.

Table B.1: Sample Selection Compustat - Quarterly Data

	# Drop	# Obs
Baseline		
1983-2017	-	1,484,973
Non-financial sector	474,327	1,010,646
U.S. incorporated and USD currency	212,680	797,966
>20 quarter investment spell	105,503	585,286
No outliers	158,821	426,465
Covid sample		
2019-2020	-	75,712
Change in accounting	38,446	37,266
Other filters	23,292	13,974

The definition of the main variables used for the calibration and regressions are:

1. Capital stock k: is constructed using the perpetual inventory method, following the usual convention in the investment literature.[30] I compute the initial capital level using the level of gross plant, property and equipment `ppegtq`, and using the quarterly change of net plant, property and equipment `ppentq`. The deprecation rates δ are calculated using the BEA accounts to compute investment rates (i.e., change in capital k net of capital depreciation).

2. Net debt stock b: different from other papers in the literature I assume b corresponds to the short term liabilites. Liabilities include financial debt, debt with suppliers and other firms, accounts and tax payables, and others. `lctq` minus cash holdigns `cheq`. Complementary, the gross debt position I define it as the short term liabilities `lctq` only.

3. Operating profits π: corresponds to the variable `ibdpq`

4. Liquid value of assets $q\omega k (1-\delta)$: to compute this I use the assets of the firm (excluding cash) as follows: for asset category a_{ij} we can compute the liquid value of firms' assets as $\sum_j lr_j \times a_{ij}$ where lr_j is the liquidation rate. The liquidation rates used by asset category are 44% inventories, 63% receivables and 35% physical capital from Kermani and Ma (2021).

5. Cash-on-hand n: is computed as the sum of π and $q\omega k (1-\delta)$ minus b. It is assumed that all liabilities can be collected each period.

6. Size: log of total assets `atq`.

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[30]See for example, Mongey and Williams (2017); Jeenas (2019); Ottonello and Winberry (2020) for recent references.

7. Sales growth: quarterly growth of sales `saleq`.

Nominal variables are deflated using the BLS implicit price deflator, unless specified. Percentiles of variables used are constructed by year (not quarter). When specified variables are standarized, winzorized and/or demeaned.

Banckurptcy Data. To identify when and what firms operate under Chapter 11 in Compustat I use the same strategy as Corbae and D'Erasmo (2021). I use the footnote to total assets (`atq`) and deletion information variables `dlrsn` and `dldte`. A firm is in Chapter 11 if (i) footnote (next period) reports adoption of new accounting under Chapter 11 bankruptcy; (ii) if firm shows as bankrupt but is not deleted; (iii) if the firm shows as bankrupt and deleted but this is not due to liquidation; (iv) and if firm's last observation in the sample is bankrupcy but there is no bankruptcy information.

Figure B.1: Filings to Chapter 11
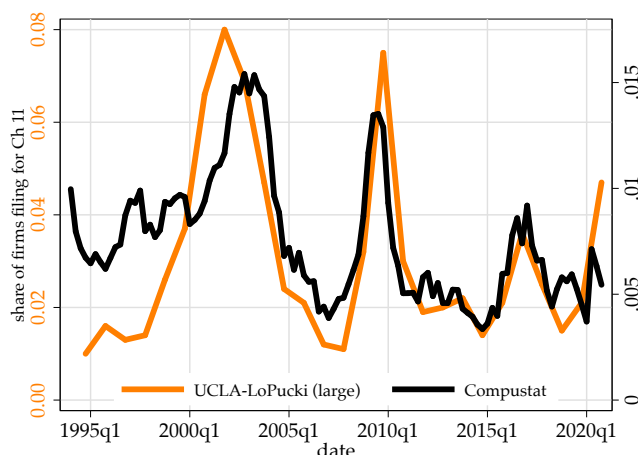
Notes : Figures shows the flow of filings to Chapter 11 in the last 12 months. Filings to Chapter 11 are identified through the steps detailed in the text. Data sources: Compustat-Quarterly and UCLA-LoPucki.

Figure B.1 shows that the Compustat data evolution is consistent with UCLA-LoPucki estimates which are for large firms in US.

B.1.2 Federal Judicial Center - Integrated Database (FJC-IDB).

FJC-IDB bankruptcy data includes all petitions filed under the Bankruptcy Code (any of the Chapters) on or after October 1, 2007 and any petitions filed before October 1, 2007 that are still pending. This dataset provides information of the fillings, closures and several firm characteristics.

I will focus on a sample of corporate firms filings to Chapter 11 and Chapter 7. This includes public and privately held firms. Table B.2 shows the sample selection criteria.

Table B.2: Sample Selection FJC-IDB

	# Drop	# Obs
All		32,084,867
Corporations	31,546,855	538,012
Chapter 11 and 7	9,771	528,241
Filings	358,890	169,351
Closures	357,023	171,218

Notes: This table shows the number of observations resulting from the sample selection for the FJC. The first line, *All*, shows the original number of entries from in the dataset. from 2008 to 2020. Two samples are used, one that includes only the filings and other that includes only the closures (closure sample entries include filing characteristics). The following lines detail the set of observations dropped from different filters applied to the sample and the resulting number of observations. More details on these filters can be found in the text.
Data source: FJC-IDB.

B.2   Heterogeneous Investment Responses During Recent Crises

In this section I will study the heterogeneous investment response of firms during the Great Recession and Covid-19 crisis. First, I will show the aggregate dynamics of the crises. Second, I will show the heterogeneity across the balance sheet positions, focusing on cash-on-hand and leverage positions.
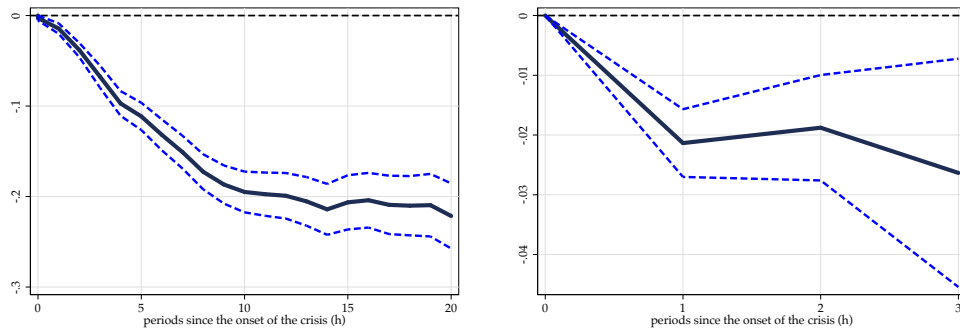

B.2.1   Aggregate Dynamics

Figure B.2 shows that using firm level data of publicly listed firms the capital accumulation rate drop significantly in both episodes — Great Recession and Covid-19 crisis.

Figure B.2: Corporate Investment in Recent Crises Episodes
$$\Delta k_{t+h}^{\text{crisis}} - \Delta k_{t+h}^{\text{no crisis}}$$

(i) Great Recession   (ii) Covid-19 Crisis

Notes: figures show the dynamics of the capital stock relative accumulation during the Great Recession and Covid crisis. The change in capital accumulation comes from the followin specification using firm-level data:
$\log(k_{it+h}) - \log(k_{it}) = \alpha_i + \beta_h \text{crisis}_t + \varepsilon_{it+h}$, where crisis_t indicates the pre-crisis peak and β_h is the h-periods ahead change in the accumulation of capital during the crisis episode relative to no crisis periods. Drop t such that for $\text{crisis}_{t+i} = 1$ for at least one $i \in \{0, .., h\}$, i.e. capital accumulation before the crisis overlaps with the crisis. Panels (a) and (b) show coefficients β_h and their 90% confidence interval. Standard errors are clustered at firm level..
Data sources: Compustat.

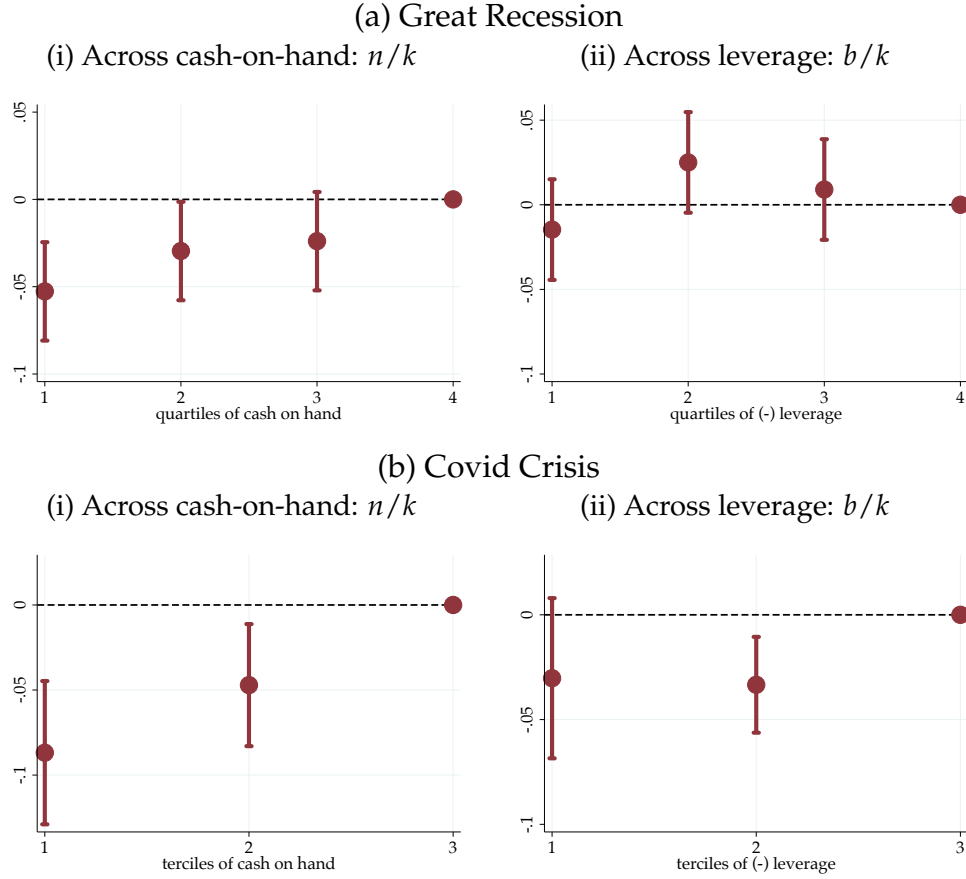### B.2.2 Investment Heterogeneous Response Estimation

To estimate the *on impact* heterogeneous response — from peak-to-trough of the crisis — I will proceed as follows. First, to account for permanent sectoral heterogeneity — in my baseline estimations — I will demean each of the firm-quarter observations of variable x of interest by its sectoral average, i.e. $\hat{x}_{it} = x_{it} - \mathbb{E}_s[x_{it}]$. Next, I will assign each firm-quarter observation of x to different quartiles (terciles if Covid sample) relative to the annual distribution. Lastly, I run the following panel regression to estimate the heterogeneous responses of investment across cash-on-hand n/k and leverage b/k during the crisis:

$$\Delta \log(k_{it}) = \underbrace{\sum_{j=1}^{J} \beta_j^n \left(Q_{it}^{nj} \times \text{crisis}_t \right)}_{\text{het across } n/k} + \underbrace{\sum_{j=1}^{J} \beta_j^b \left(Q_{it}^{bj} \times \text{crisis}_t \right)}_{\text{het across } b/k} + \underbrace{\Lambda' Z_{it}}_{\text{controls}} + \varepsilon_{it}, \qquad (25)$$

where Q_{it}^{xj} indicates if $\hat{x}_{it}$ belongs to quartile or tercile j, $\Delta \log(k_{it}) = \log(k_{it+h}) - \log(k_{it})$ is the capital accumulation over a period as long as the crisis studied (i.e., the extension from peak-to-trough of episode studied h), crisis_t indicates if a crisis happens during the period considered and $Z_{i,t}$ includes the control variables. For the baseline specifications I include as controls firm's fixed effects, sectoral fixed effects, log assets as proxy for size and last quarter sales growth. The coefficients β_j^x are interpreted as the diff-in-diff estimates of the crisis impact on capital accumulation for firms in quartile or tercile j of $\hat{x}$.

The empirical strategy is close to the one used in other work that studies investment adjustment heterogeneity on recent crises episodes. Salient examles are Almeida *et al.* (2012) for the Great Recession in U.S. and Kalemli-Özcan *et al.* (2020) for the EU crisis.

Figure B.3: Heterogeneous Investment Response during Crises

(a) Great Recession

(i) Across cash-on-hand: n/k (ii) Across leverage: b/k

(b) Covid Crisis

(i) Across cash-on-hand: n/k (ii) Across leverage: b/k

Notes: Figures show the change in the capital accumulation from peak to trough in both episodes. For the Great Recession the episode is from 2007q4 to 2009q4, and for the Covid-19 crisis is from 2019q4 to 2020q2. Figures in panel (a) show the coefficient β_j^n and Figures in panels (b) shows β_j^b for the Great Recession and Covid-19 crisis in a joint estimation of specification (25). Coefficients are normalized to 0 with respect to the highest quartile or tercile coefficient. The interval is at 90% confidence level and standard errors are clustered at firm level for the Great Recession and sector level for the Covid-19 crisis. Balance sheet variables are demeaned at sectoral level. Because of data limitations the estimates of the Covid-19 crisis don't include firm's FE. Coefficients are in annual terms.
Data sources: calculations using Compustat data.

Figure B.3 shows the investment response across different levels of cash-on-hand and leverage during the Great Recession and Covid crisis. For both episodes, panel (a) and (b) figure (i) show that firms with low levels of cash-on-hand adjust substantially more their investment, around 5-10 p.p. points in annual terms relative to the firms with the highest levels of cash-on-hand. On the other hand, panel (a) and (b) figure (ii) show that the heterogeneity across leverage is not significant. In Section 3.3, I contrast these results with simulations from the model.

These findings are related to a recent literature that studies empirically the heterogeneity of investment responses across firm's financial positions during recent large recession episodes. Two salient examples are Almeida *et al.* (2012) for the Great Recession in U.S. and Kalemli-Özcan *et al.* (2020) for the EU crisis. Almeida *et al.* (2012) explore the relevance of long term debt that matured in the short term during the Great Recession in U.S. and find that firms with more long term debt maturing in the short term reduced more their investment. Moreover, Kalemli-Özcan *et al.* (2020) find evi-

dence of debt over-hang problems and rollover risk being relevant during the EU crisis. These patterns were greater in peripheral europe countries, which were hardly hit by the crisis. In addition, Ebsim *et al.* (2021) show that cash holdings were relevant explaining the heterogeneity of credit spreads dynamics during the Covid crisis, but they weren't relevant during the Great Recession.

B.3 Other Observations on the Firm's Balance Sheet

In this section I show further facts related to the liability structure (e.g. maturity and number of creditors) of firms that complement the baseline analysis. Table B.3 shows that corporate firms use extensively short term liabilities to finance their investments and operations, and Table B.4 shows that the great majority of medium to large corporate firms (i.e., with more than 50 million assets) in U.S. borrow from hundreds of creditors. This is complementary to the observation of Crouzet (2017a) that corporate firms financial leverage is mostly in bonds, which they are very likely to have a disperssed ownership. These observations supports the idea that the firms' creditors are likely dispersed and difficult to coordinate, unless the firm wants to incur in costs. Further, in Appendix C.2, I show that the benefits of being able to manage the liability structure in the model are not large (ex-ante), therefore for moderate costs of changing their liabilities most firms will remain inactive.

Table B.3: Firms' Debt Maturity

Time to mature (share)

	< 1 year	1 to 4 years	≥ 5 years
Debt	0.29	0.33	0.38
	(0.32)	(0.28)	(0.34)

	< 1 year	> 1 years
Liabilities	0.61	0.39
	(0.29)	(0.29)

Notes: the table shows the share of debt or liabilities maturing at different time horizons. The summary statistic is computed for the average firm, in parenthesis is the standard deviation. Short term liabilities are `lct` and long-term `lt - lct`. Debt maturing in less than one year is `dlc`, in one to four years is `dd2 + dd3 + dd4`, and maturing at 5 or more years is `dltt-dd2-dd3-dd4`. Total debt is `dlc + dltt` and total liabilities is `lct`.
Data source: Compustat.

Table B.4: Number of Creditors When Filing to Bankruptcy

	# Creditors		
	1 to 100	101 to 1,000	>1,000
Small ($<$ 50 million assets)	0.88	0.10	0.02
Medium ($>$ 50 million and $<$ 1 billion assets)	0.16	0.19	0.65
Large ($>$ 1 billion assets)	0.03	0.04	0.93
All	0.74	0.10	0.16

Notes: the table shows the share of firms with by creditor number groups and size when filing to Chapter 11 bankruptcy. Shares are relative to the total filings of each size group. Asset value correspond to the one declared when filing for bankruptcy.
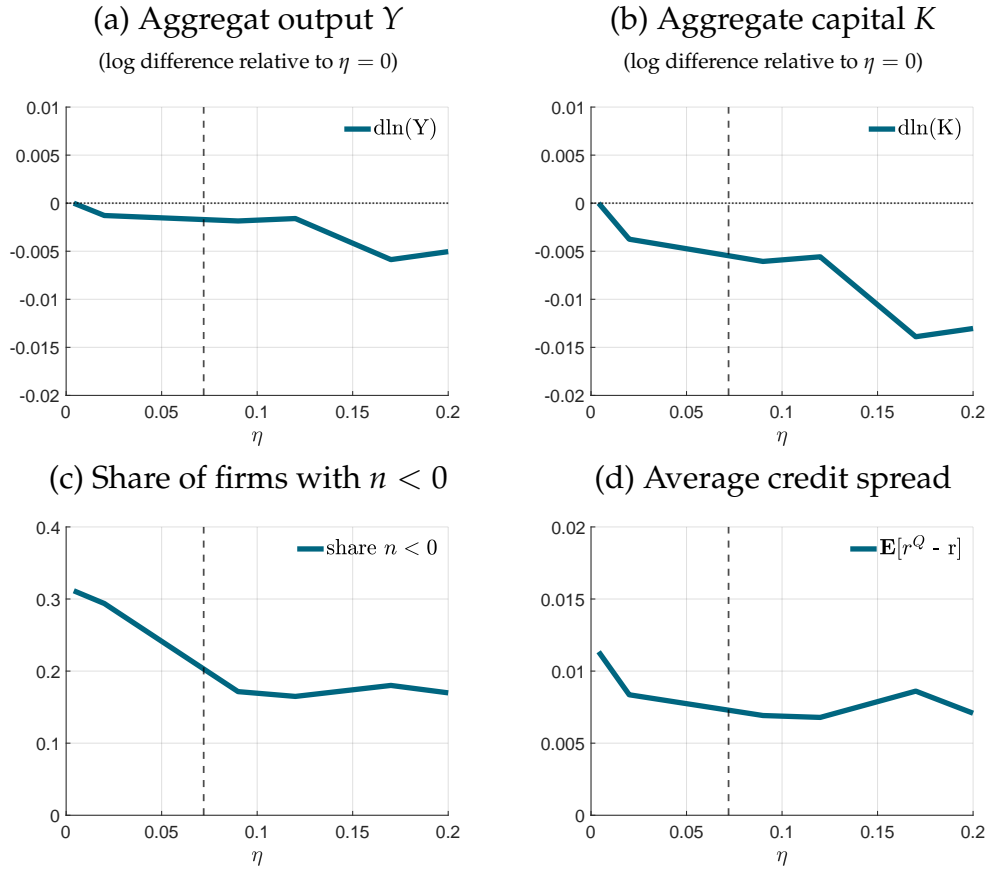*Data source*: FJC-IDB.

C   Appendix: Further Exercises and Extensions

C.1   Steady-state comparative statics

To study the long run implications of firms' rollover risk I do some simple comparative statics with η. Figure C.1 shows for different values of η the output and capital level, and the share of firms with negative cash-on-hand and the average spread rate. I find that the importance of rollover crises in the long run is relatively low. First, in the long run, aggregate output Y is 0.2% lower, see panel (a), and aggregate capital K is 0.5% lower, see panel(b), because of creditors' coordination failures. Second, the higher is η less firms have a weak balance sheet position in steady-state, see panel (c). Rollover risk shifts (improves) the financial distribution of firms significantly. The increase in the risk of rollover failure, for a given financial position, incentivize firms to save away; thus, accumulating internal resources to preclude the coordination failure. The improvement in the financial position is reflected on the little change observerd in credit spreads across η, even if the risk of rollover crises is greater (given the financial position). Overall, the likelihood of coordination failures for exposed firms η shifts the financial position of firms, but don't impact significanly aggregate outcomes over the long run.

Figure C.1: Steady-state Comparison

(a) Aggregat output Y

(log difference relative to $\eta = 0$)

(b) Aggregate capital K

(log difference relative to $\eta = 0$)

(c) Share of firms with $n < 0$

(d) Average credit spread

Notes : Panel (a) and (b) show the log difference in aggregate output Y and capital K, respectively, across different values of η in steady-state. Panel (c) and (d) show the share of firms with negative cash-on-hand n and average credit spread rate across, respectively, across different values of η in steady-states. In all the plots, the vertical dashed line indicates the calibrated value of η.

C.2 How costly is firm's rollover risk (ex-ante)?

In this section to assess how costly is rollover risk. For this, I explore the spread distribution between the pricing schedule with and without coordination failures, i.e., $\tilde{Q}\left(z, k', b'; \eta\right) - \tilde{Q}\left(z, k', b'; 0\right)$. As a benchmark, I compute how many firms would pay the bank's markup over market borrowing soley to preclude future creditors' co-ordination failures if they could. I use the spread in intermediation costs estimated by Crouzet (2017a) of 0.74% (annual). I find that only 2.2% of the firms face a cost of rollover risk higher than intermediation spread. Figure C.2 shows the distribution of the cost of rollover risk across firms that produce and don't exit at the end of the period. The figure shows that most firms face a cost close to 0 since many firms become exposed tomorrow only in case of an extremely bad shock, therefore the average cost is negligible. The small cost of rollover risk ex-ante in steady state suggests that it can be optimal for firms choose a liability structure where they are exposed to rollover crises.

Figure C.2: Cost of Rollover Risk (in annual spread terms)
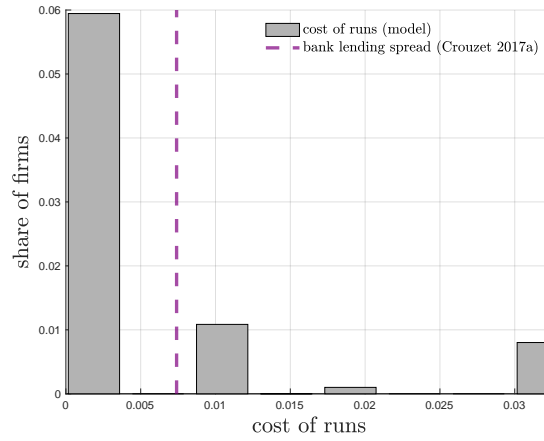
Notes : Figure shows the distribution of the cost of firms' rollover risk — i.e., $\tilde{Q}\left(z,k',b';\eta\right) - \tilde{Q}\left(z,k',b';0\right)$ — for producing firms that don't exit at the end of the period. Exclude from plot the ones with 0 cost and truncated distribution at 3% cost. The spread of intermediation between bank and market lending is from the calibration of Crouzet (2017a).

C.3   Model Extensions and Multiple Equilibrium

In this section, I will study two extensions of the model, one that uses more general functional forms for the operational profits function, capital adjustment idiosyncratic frictions and long-term debt, and other that allows firms to issue equity (costly).

Long-term debt and capital adjustment frictions.   I assume profits are a function $\pi(\mathbf{z}, k) \in \mathbb{R}$ strictly increasing in both arguments, where $\mathbf{z} = (\mathbf{z}^p, \mathbf{z}^{\text{iid}})$ is a vector of shocks that contain a set of persistent shocks $\mathbf{z}^p$ follow a markov process and $\mathbf{z}^{\text{iid}}$ follow an iid process. Both are related to idiosyncratic productivity and cost shocks. Next, I assume that $\iota(\omega k, k') \in \mathbb{R}$ is the investment expenditure function of the firm that is decreasing on k and increasing on k', where $-\iota(\omega k, 0) \geq 0$ is the liquidation value of capital.[31] Last, I assume that the firm can issue long-term debt, which fraction $m^b \in (0,1]$ matures randomly each period and pays $c^b \geq 0$ cupon payments on non-maturing debt. The rest of the model it follows as the baseline model.

I focus on the characterization of the liquidation choice. For the extended setup, firms dividends now can be defined as

$$d = \pi\left(\mathbf{z}, k\right) - \iota\left(k, k'\right) - b\left[m^b + \left(1 - m^b\right) c^b\right] + Q\left(.\right)\left(b' - \left(1 - m^b\right) b\right) \geq 0$$

where $Q\left(.\right)\left(b' - \left(1 - m^b\right) b\right)$ is the amount of new debt issued. Analogous to the baseline model, firms can default after issuing the new debt. The firm never default

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[31]I assume no capital quality shock ω for notational clarity.

whenever

$$\max_{k'} \pi(\mathbf{z},k) - \iota\left(k,k'\right) - b\left[m^b + \left(1 - m^b\right)c^b\right] =$$

$$\underbrace{\pi(\mathbf{z},k) - \iota(k,0) - b\left[m^b + \left(1 - m^b\right)c^b\right]}_{n(\mathbf{z},k,b)} \geq 0. \tag{26}$$

where I can define n as the cash-on-hand of the firm is the sum of operational profits, liquidation value of capital, and maturing debt and cupon payments. On the other hand, we have that the firm will always default whenever

$$\pi(\mathbf{z},k) - b\left[m^b + \left(1 - m^b\right)c^b\right] + \max_{k',b'}\left\{-\iota\left(k,k'\right) + \tilde{Q}\left(\mathbf{z}^p,k',b'\right)\left(b' - \left(1 - m^b\right)b\right)\right\} =$$

$$n(\mathbf{z},k,b) + \underbrace{\max_{k',b'}\left\{-\iota\left(k,k'\right) + \iota(k,0) + \tilde{Q}\left(\mathbf{z}^p,k',b'\right)\left(b' - \left(1 - m^b\right)b\right)\right\}}_{-\underline{n}(\mathbf{z}^p,k,b)} < 0$$

$$\tag{27}$$

For multiplicity to exists we need that conditions (26) and (27) don't hold, i.e.,

$$0 > n(\mathbf{z},k,b) \geq \underline{n}(\mathbf{z}^p,k,b). \tag{28}$$

Notice $\underline{n}(\mathbf{z}^p,k,b)$ bounded below by 0 (we can always implement $\{k' = 0, b' = b\}$). Moreover, there is the possibility of multiple equilibrium whenever the firm can have strictly positive external resources in this region of the state-space. Analogous to the baseline model, the firms default decision is determined by the firm's cash-on-hand and a threshold that depends on the fundamentals of the firm (shocks and financial position).

Further, assume there is no bankruptcy, $c^b = 0$ and creditors have no recovery for clarity, then the fundamental pricing schedule $\tilde{Q}(.)$ (without coordination problem today) is pinned down by creditors no profit condition and is

$$\tilde{Q}\left(\mathbf{z}^p,b',k'\right) = \mathbb{E}\left[\Lambda\left(\mathbf{1}_{\{n \geq \underline{n}\}} - \eta\mathbf{1}_{\{0 > n \geq \underline{n}\}}\right)\left(m^b + \left(1 - m^b\right)\tilde{Q}'\right)\right]. \tag{29}$$

The pricing schedule with long-term becomes recursive. Also tomorrow's coordination failures show up in the pricing schedule. These two observations suggest, in the firm problem with long-term debt, rollover crises could even be greater than in the baseline model. With long-term debt the pricing schedule is affected by the future stream of expected rollover crises, which can augment their impact.

Equity issuance. In the baseline specification, I don't allow firms to issue equity — $d \geq 0$. This assumption is consistent with the relatively low equity issuance observed in the data, and helps on the tractability of the characterization and computational solution of the model. In this section, I will relax this assumption and show how this affects the characterization of the liquidation choice (equilibrium multiplicity). Moreover, I provide a discussion on the model concepts of rollover and solvency in the model.

Firms issue equity $e < 0$ at cost $\phi(e)$, which is decreasing in e and unbounded. I assume that equity is raised at the end of the period. Therefore, firms that never default are those when $Q = 0$ they don't default, i.e.,

$$V^{Q=0}(z, n) \geq 0$$

where $V^{Q=0}(z, n)$ is determined by

$$V^{Q=0}(z, n) = d + \mathbb{E}\left[\Lambda \tilde{V}\left(\mathbf{s}'\right)\right]$$

subject to

$$d = \begin{cases} e & \text{if } e \geq 0 \\ e - \phi(e) & \text{if } e < 0 \end{cases}$$

$$e = n - qk'$$

where continuation value $\tilde{V}\left(\mathbf{s}'\right)$ is analogous to one defined in the baseline firm problem. Thus, we can define safe region

$$\mathcal{S} = \{(z, n) : V^{Q=0}(z, n) \geq 0\}. \tag{30}$$

On the other hand, firms that default are those default even if $Q > 0$, i.e.,

$$V^{Q>0}(z, n) < 0$$

where $V^{Q>0}(z, n)$ is determined by

$$V^{Q>0}(z, n) = d + \mathbb{E}\left[\Lambda \tilde{V}\left(\mathbf{s}'\right)\right]$$

subject to

$$d = \begin{cases} e & \text{if } e \geq 0 \\ e - \phi(e) & \text{if } e < 0 \end{cases}$$

$$e = n + \tilde{Q}\left(z, b', k'\right) b' - q k'$$

where $\tilde{Q}$ fundamental pricing schedule (no liquidation today) and continuation value $\tilde{V}\left(s'\right)$ (this analogous as the one in the baseline firm problem). Thus, we can define liquidation region

$$\mathcal{L} = \{(z, n) : V^{Q>0}(z, n) < 0\}. \tag{31}$$

Last, it's straightforward to show that $V^{Q>0}(z, n) \geq V^{Q=0}(z, n)$, then under certain conditions it can be the case that firm is in a region that is undetermined, i.e.,

$$\mathcal{R} = \{(z, n) : V^{Q>0}(z, n) \geq 0 \text{ and } V^{Q=0}(z, n) < 0\}. \tag{32}$$

Similar to the baseline mode, we have that firms can be exposed to coordination failure even if they can issue equity. Assume the equity issuance function $\phi(e) = \lambda|e|$ with $\lambda > 0$. In Figure C.3, I illustrate how these affects the characterization of the regions.

Figure C.3: Rollover and solvency regions across (z, n)
Baseline vs Equity Issuance model
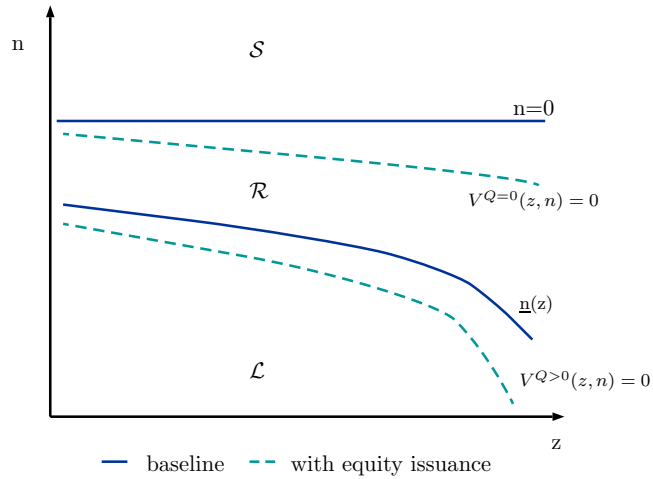
Notes: figures shows the state-space (z, n) and the relevant regions for the liquidation choice for the baseline model (solid blue lines) and the model with equity issuance (dashed cyan lines).

Finally, it's worth noticing that in the model with unbounded equity issuance firms in $\mathcal{L}$ threshold — $V^{Q>0}(z, n) = 0$ — have 0 value, which is the standard notion of economic insolvency. On the other hand, in the baseline model, or with bounded

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equity issuance, firms in $\mathcal{L}$ threshold ($\underline{n}(z)$) could have strictly positive value. For my calibration, I find that firms in the insolvency threshold have values close to 0 — $V(z, \underline{n}(z)) \approx 0$ —; therefore, it approximates well the standard notion of insolvency.

C.4   Bankruptcy Analysis: Data and Model

In this section, I compute moments related to bankrupt firms, which are not targeted in the baseline calibration. I use Compustat data, and construct the data and identify bankrupt firms following the steps described in Appendix B.1.1. For this analyisis I will focus on Chapter 11 events where the firm is operating next period.[32] First, I will compare in the data and model the leverage position distribution of bankrupt firms and, next, I will compare how the firm's financial characteristics — in the data and model — predict a bankruptcy event.

Leverage (b'/k') distribution.   Although I target the average leverage ratio of firms in Chapter 11, I don't target it's distribution. I compare the share of bankrupt firms according to their leverage (liabilities over capital) position. I split in three groups, those bankrupt firms that have choose low leverage (less than 0.5), medium to high leverage (0.5 to 1.5) and extremely high leverage (more than 1.5). Figure C.4 shows that the model fits well the distribution of leverage for firms in Chapter 11 that continue operating.

Figure C.4: Leverage distribution of bankrupt (restructuring) firms
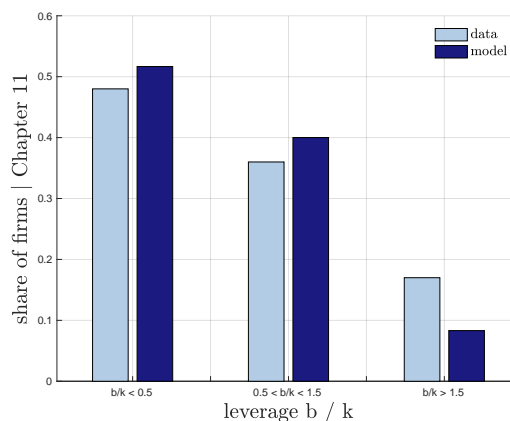
Notes: Figure shows the distribution of leverage for firms in the restructuring process in the model and the data (in Chapter 11 and operating the next period). Data source: Compustat.

Bankruptcy predictors.   Now I study how the firm's lagged financial variables predict the firm's bankruptcy. For this I ran the following regression (in the data and

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[32]There are few events of Chapter 7 for publicly traded firms and Chapter 11 events that lead to liquidation are difficult to identify.

model)

$$\mathbf{1}_{i,t}^{\text{ch11}} = \beta X_{i,t-1} + \alpha_t + \alpha_i + \alpha_s + \epsilon_{i,t}$$

where $\mathbf{1}_{i,t}^{\text{ch11}}$ indicates if the firm i in period t is in Chapter 11 and operating (restructuring instead of liquidation), α_i are firm FE, α_s sector FE, α_t time fixed effects and $X_{i,t-1}$ is a vector of characteristics (predictors) of interest lagged one period. In my baseline specification I include in $X_{i,t}$ the size of the firm (assets in logs), real quarterly growth of sales, the cash-on-hand and leverage positions. I standarize all variables in $X_{i,t-1}$ and winzorized the cash-on-hand n/k' and leverage b'/k' at level 0.5% and 99.5%.

Table C.1 shows the results. For all the specifications the coefficients in the model and data have the same sign. In the full specification (3) I find that lower sales growth, smaller size, low cash-on-hand and (surprisingly) low leverage predicts a higher restructuring likelihood next period. Further, the magnitudes are relatively similar between the model and data. The relation between leverage and bankruptcy could be explained is reverted if we exclude cash-on-hand from the specification (in (2)). One potential explanation for this counter-intuitive relation is that firms suffering from rollover crises given their financial position — summarized by n — are more likely to restructure if they are in better shape given the costs of bankruptcy.

Table C.1: Predictors of Chapter 11 - Data vs Model

dependent variable: $\mathbf{1}^{\text{ch11}}_{i,t}$

	(1) data	(1) model	(2) data	(2) model	(3) data	(3) model
$n_{i,t-1}/k_{i,t}$	-0.39	-0.05			-0.39	-0.45
	(0.03)				(0.10)	
$b_{i,t}/k_{i,t}$			0.11	0.03	-0.29	-0.41
			(0.04)		(0.09)	
$\log(k_{i,t-1})$	-0.50	-0.06	-0.52	-0.06	-0.49	-0.10
	(0.12)		(0.12)		(0.12)	
$d\log(\text{sales}_{i,t-1})$	-0.04	-0.03	-0.04	-0.02	-0.04	-0.01
	(0.00)		(0.00)		(0.00)	
Sector FE	Y		Y		Y	
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y		Y		Y	
Observations	370,973		373,362		370,973	

Notes: This table shows the baseline results of the regression using bankruptcy outcomes in the data and the model simulations. All variables are standarized, and leverage and cash-on-hand are also winsorized at level 0.5% and 99.5% and demeaned relative to the sectoral average. Standard errors (in parenthesis) are clustered by firm. Coefficients are times 100.
Data source: Compustat quarterly.

D Appendix: U.S. Bankruptcy Code Institutional Details

In this section, I provide a brief review of some institutional details of the bankruptcy process for firms in the U.S. bankruptcy code. Chapter 7 and 11 are the typically used to liquidate or restructure the firm's liabilities. Chapter 7 is associated with firm liquidations, and Chapter 11 with restructurings (or sometimes called "reorganizations") and liquidations through piecemeal sales of firms.

Chapter 7 bankruptcy. Firms can enter a Chapter 7 liquidation process by filing directly to this chapter or being redirected by court ruling from other chapters (e.g., a judge may rule that a Chapter 11 case is switched to a Chapter 7 one). In this process, a case impartial trustee is appointed by the court to gather and sell the bankrupt firms assets to pay the firm's creditors.

Chapter 11 bankruptcy. Cases begin usually with the voluntary filling of the debtor (firm). Involuntary petitions (done by creditors) are very rare. When filling the firm au-

tomatically assumes an additional identity as the "debtor in possession." by 11 U.S.C. § 1101. The DIP provisions can provide access to new credit for the firm (DIP financing) and the automatic stay of firm's debt payments by 11 U.S.C. § 362(a) preclude (most) creditors from collecting the firm's debt. Further, when filing automatically a creditor's committee is appointed, which typically consists of the unsecured creditors who hold the seven largest unsecured claims against the debtor. Further, is common that creditors form ad hoc committees to coordinate their actions and have further surveillance over the debtor-in-possession's management of the firm.

The firm usually files a written disclosure statement and a reorganization plan. The disclosure statement contains information of the firms' assets, liabilities and other business affairs. Typically, the disclosure statements contains a counterfactual analysis of the credit recovery rates under liquidation (liquidation analysis) and other information relevant for the judge to decide if the reorganization chapter is appropriate. Lastly, the plan presented by the creditors needs to be approved by the creditors for the restructuring to be executed.

Moreover, Chapter 11 process are sometimes used by large firms to do a piecemeal liquidation of the firm. The provisions provided by 11 U.S.C. § 363(b), "363 sales", allow firms to liquidate part of the firm's assets without the creditors' consent. This process is closer to a Chapter 7 "piecemeal" liquidation of the firm, instead of a restructuring or reorganization.