Quantifying Firm Runs

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October 28, 2021

Motivation

- Firms with NPV>0 can have liquidity problems such as **rollover problems** and fail
  - a concern for policy-makers → shape regulations and (credit) policies
  - frequently cited by owners/managers of bankrupt firms
  - firms with higher rollover risk adjust more investment in recent crises
- We know little about (nofinancial) firms' rollover problems **macro consequences**
  - challenging to disentangle from solvency problems
- Build **quantitative framework** where rollover problems can be identified and quantified
  - mechansim akin to bank runs and sovereign debt crises $\Rightarrow$ **firm runs**

Research question

How relevant are **firm runs**, and what are their macroeconomic and policy implications?

What I Do

- Build on GE models of heterogeneous firms with default
  Khan Senga Thomas 2016; Ottonello Winberry 2020; and others

- Borrow technics from international macro literature to allow for **runs**
  Cole Kehoe 2000; Bocola Dovis 2020

- **Identify** incidence of runs exploiting heterogeneity in firm's bankruptcy outcomes (liquidiation vs restructuring)
  insights from Corp Law literature, e.g., Jackson 1986; Corbae D'Erasmo 2021

- Conduct **quantitative** analysis of U.S. economy to asses aggregate relevance of firm runs

related papers related papers on runs

What I Find

1. How **relevant** are firm runs?

   + **1.5%** firms rollover prob = 20% exposed $\times$ 7% probability
     (more than **60%** of bankruptcy events are driven by runs)
   - indirect inference using bankruptcy outcomes and financial distribution of firms
   - key for identification: restructuring process provides coord tool to preclude runs

2. What are the **macroeconomic** implications?

   + runs **amplify** significantly impact of crises (more output drop and persistence)
   - similar across different types of shocks

3. What are the **policy** implications?

   + credit policy can **undo runs** amplification of crises, but can **backfire**
   - imperfect credit policy precludes runs, but exacerbate future debt overhang

Outline

- Theoretical Framework
- Identifying Firm Runs
- Macroeconomic Consequences of Firm Runs

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- **Theoretical Framework**
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Overview of the Model

- Quantitative GE models of heterogeneous firms with default

  Khan Senga Thomas 2017; Ottonello Winberry 2020

- Extend model to

  1. possibility of coordination failures among creditors á la Cole Kehoe 2000
  2. allow debt restructuring similar to Corbae D'Erasmo 2021

- Study unforseen crises and policy shocks (MIT shocks)

Environment

- Infinite horizon and discrete time
- Four types of agents
  1. **nonfinancial firms**: invest and produce to maximize their value
  2. **creditors**: lend to nonfinancial firms, and are perfectly competitive and atomistic
  3. **capital producer**: sell capital to nonfinancial firms
  4. **representative HH**: consumes, saves and works. Owns all firms in the economy

Nonfinancial Firms' Environment

- Firm i objective is to max

$$\sum_{t\geq 0} \mathbb{E}_0[\Lambda_t d_{it}]$$

with Λ_t HH's SDF and d_t firm's dividends

- Idiosyncratic state variables:

  1. s^f_{it} exogenous
  2. s^{nf}_{it} exogenous non-fundamental
  3. s^e_{it} endogenous

  where $s_{it} = (s^f_{it}, s^{nf}_{it}, s^e_{it})$

- For clarity, drop i and t subscripts

Overview of Nonfinancial Firms' Timing

Within period timing is as follows (firms with no exit shock)

1. **All** uncertainty about fundamentals and nonfundamentals is realized
2. **Restructure** choice
3. Investment and **new debt** issuance choice
4. **Liquidation** choice (if continue in 2)
5. Firms produce and distribute dividends (if don't liquidated in 4)

Nonfinancial Firms' Production

- Operate with technology

$$f(z, \omega, k, l) = z(\omega k)^{\alpha} l^{\nu}$$

  - decreasing returns to scale $\nu + \alpha < 1$
  - idiosyncratic persisent productivity shocks $\ln z' = \rho_z \ln z + \epsilon_z$ with $\epsilon_z \sim N\left(0, \sigma_z^2\right)$
  - idiosyncratic capital quality shock ω iid log-normal trunc. where $\ln \omega \in \left[\underline{\omega}, 0\right]$
    (fit quantitative default rate)

- Own capital k and hire labor l at wage w, then operating profits are

$$\pi(z, \omega, k) = \max_{l} f(z, \omega, k, l) - wl$$

Nonfinancial Firms' Financial Resources

- **Internal** resources (cash-on-hand)
  - inherit k at price q which depreciates at $\delta \in [0,1]$ and maturing b, and has $\pi(z,\omega,k)$

$$n = \underbrace{\pi(z,\omega,k)}_{\text{operational profits}} + \underbrace{(1-\delta)q\omega k}_{\text{selling value of capital}} - \underbrace{b}_{\text{maturing debt}}$$

- (net) **External** resources
  - issues one-period debt b' at price schedule $Q(.)$ and buys k' at price q

- **Dividends**
  - distributed at end of period

$$d = \underbrace{n}_{\text{cash-on-hand}} + \underbrace{Q(.)b'}_{\text{new debt issuance resources}} - \underbrace{qk'}_{\text{capital purchases}}$$

Nonfinancial Firms' Financial Frictions

1. No-equity issuance constraint $d \geq 0$ (data low eq issuance, standard and simplify)

2. Debt is defaultable in two ways

   (a) liquidation (Chapter 7)

   - non selective default on b and b'
   - firm exits with $V = 0$ and creditors of b recover $\alpha_7 \in [0, 1]$ of the liquidated capital

   (b) restructuring (Chapter 11)

   - (Nash) bargain debt recovery rate $\alpha_{11} \in (0, 1)$ over debt b detail
   - firm pays exogenous cost $c_{11} \in [0, 1]$, which is proportional to capital
   - **no firm runs** (bankruptcy provisions, Corp Law)
   - resources after restructuring are: $n_{11} = \pi(z, \omega, k) + (1 - c_{11})(1 - \delta)q\omega k - \alpha_{11} b$

US bankruptcy code

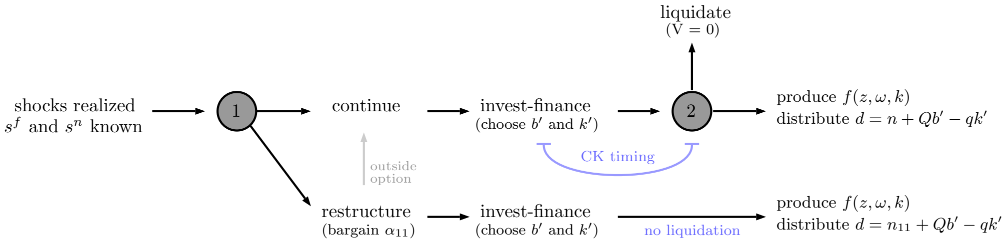
Nonfinancial Firms' Entry/Exit

Technical and quantitative assumptions

- Exogenous exit probability γ (KST 2016, stationary dist)
  - if receive shock the firm exits after production
- Entrants enter on average productivity $m\%$ below ergodic distribution average (OW 2020, life-cycle firms)

details

Within Period Timing Nonfinancial Firms' Problem



- Timing for non-exiting firms
- Cole Kehoe 2000 (CK) timing for liquidation choice

Multiple Equilibrium Intuition

- Liquidate if they can't satisfy $d \geq 0$ (result of assumptions)
- Debt price schedule from no-profit condition of creditors and dividends are

$$Q = \underbrace{\mathbb{I}_{d \geq 0}}_{\text{liquidation choice}} \underbrace{\tilde{Q}}_{\text{price if no liquidation}}$$

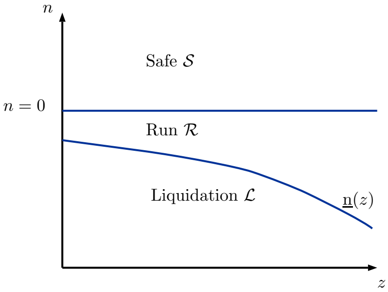
$$d = n - k' + Qb'$$

- Feedback between liquidation choice and prices today could create multiple outcomes

$$Q = 0 \iff d < 0$$
$$Q > 0 \iff d \geq 0$$

Liquidation

- **Fundamental** state-space (z, n) is divided in three regions liquidation proposition



  - $\mathcal{S}$: $Q = 0$ then continue if $d = n + \underbrace{\max_{k'}\{-k'\}}_{0} > 0$
  - $\mathcal{L}$: $Q = \tilde{Q}$ then liquidate if $d = n + \underbrace{\max_{k',b'}\{-k' + \tilde{Q}b'\}}_{-\underline{n}(z)} < 0$
  - $\mathcal{R}$: liquidate if $Q = 0$, continue if $Q = \tilde{Q} > 0$

- Define **idiosyncratic sunspot** shock $\phi \sim^{iid} U[0,1]$ draw every period, such that if $(z, n) \in \mathcal{R}$ and $\phi < \eta$ then coord in $Q = 0$ (run)

more general setup costly equity issuance

Restructure

Bargain outside option is to continue bargain protocol then conditions are

- necessary condition: firms are under a run (in $\mathcal{R}$ with $\varphi < \eta$) or insolvent (in $\mathcal{L}$)
- sufficient condition: both better-off participating
  1. creditors: $\alpha_{11} > \min\{1, \alpha_7 \frac{(1-\delta) q \omega k}{b}\}$
  2. firm: $n_{11} > \underline{n}(z)$

Costs and **benefits** for firms

1. c_{11} cost (proportional to capital)
2. $1 - \alpha_{11}$ debt haircut
3. no coord failure, i.e., $Q = \tilde{Q}$

observation

if c_{11} large and $(1 - \alpha_{11})$ low $\Rightarrow$
firms with rollover problems **restructure**

Nonfinancial Firm's Recursive Problem

- V value of firm before exit shock and restructure choice with $s = (z, \omega, \phi, k, b)$

$$\begin{aligned} V(s) = (1-\gamma) \left[1_{\{ch11\}}(s) \tilde{V}(z, n_{11}) + 1_{\{cont\}}(s) \tilde{V}(z, n) + 1_{\{ch7\}}(s) \times 0 \right] \\ + \gamma V_{exit}(s) \end{aligned} \tag{1}$$

where indicators follow from previous results, $V_{exit}(s)$ value of exiting firm details and

$$\begin{aligned} n &= \pi(z, \omega, k) + (1-\delta) q \omega k - b \\ n_{11} &= \pi(z, \omega, k) + (1 - c_{11})(1-\delta) q \omega k - \alpha_{11}(s) b \end{aligned}$$

- $\tilde{V}(z, n)$ value of the solvent firm and without run is

$$\tilde{V}(z, n) = \max_{d, k', b'} d + \mathbb{E}_{(z' | z; \omega'; \phi')} \left[\Lambda V\left(s'\right)\right] \tag{2}$$

subject to $d = n - qk' + \tilde{Q}\left(z, b', k'\right) b' \geq 0$, where $\tilde{Q}(.)$ debt price without coord failure

Corporate Debt Prices

- $Q = [1 - \mathbf{1}_{ch7}(s)]\tilde{Q}$ from creditor's no profit condition
- $\tilde{Q}$ determined by (discounted) $\mathbb{E}$[prob tomorrow's bankruptcy events]

$$
\begin{aligned}
\tilde{Q}\left(z, k', b'\right) = & (1-\gamma)\, \mathbb{E}_{(z'|z,\omega',\phi')} \left[\Lambda 1_{\{\text{continue}\}}\left(s'\right) \times 1\right] \\
& + (1-\gamma)\, \mathbb{E}_{(z'|z,\omega',\phi')} \left[\Lambda 1_{\{\text{Ch11}\}}\left(s'\right) \times \alpha_{11}\left(s'\right)\right] \\
& + (1-\gamma)\, \mathbb{E}_{(z'|z,\omega',\phi')} \left[\Lambda 1_{\{\text{Ch7}\}}\left(s'\right) \times R\left(k', b', \omega'\right)\right] \\
& + \gamma \tilde{Q}_{\text{exit}}\left(z, k', b'\right)
\end{aligned}
$$

where

- $\alpha_{11}(s)$ recovery rate of creditors if restructure bargain protocol
- $R(k, b, \omega) = \min\{1, \alpha_7 (1-\delta)\, q\omega k / b\}$ recovery rate if liquidated
- $\tilde{Q}_{\text{exit}}\left(z, k', b'\right)$ debt price conditional on exit shock Q with exogenous exit

long-term debt example

Other Agents and Equilibrium

Agents

1. **HH**'s choices are determined by Euler eq, SDF Λ and labor supply eq detail
2. **K producer** problem sells capital at price q and has a standard aggregate capital adjustment function detail
3. **Creditors** price debt through no-profit condition (SDF Λ) detail

Equilibrium

- Steady-state (law of motion fixed point) full definition law of motion firm distribution

Outline

- Theoretical Framework
- **Identifying Firm Runs**
- Macroeconomic Consequences of Firm Runs

Identification

Questions

1. How many firms are in $\mathcal{R}$?
2. Value of η?

Steps

- Calibration of **standard** parameters to match relevant moments of U.S. economy
- Calibration of parameters related to **bankruptcy procedure** and identify η
- Steady-state **financial distribution** determines share of firms by region

Data sources

- NIPA, Compustat, Federal Judicial Center-IDB, LBD, related papers

Calibration Standard Parameters

Calibration strategy

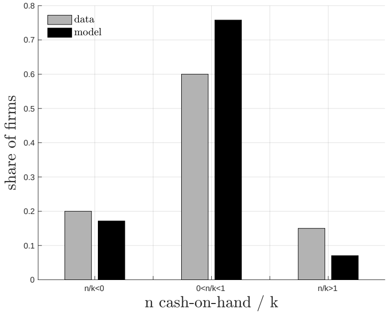
- 9 fixed and 4 fitted parameters not related to runs or bankruptcy
- params: pref, techno, stoch proc, entry/exit
- fit moments: emp, invest, balance sheet, life-cycle

Parameter	Value	Calibration
Fixed		
$\beta = 1/(1+r)$	0.99	fixed to $r = 0.05$ annual
Φ	1.16	fixed to match 58% emp rate
ν	0.64	fixed labor share
α	0.21	fixed capital share
δ	0.025	fixed to match BEA quarterly
ρ_z	0.90	fixed
γ	0.02	fixed to exit rate w/o default
ψ	2	agg AC fixed to lit standard
b_0	0	fixed to no net debt entrants
Fitted		
σ_z	0.032	internally calib
$\underline{\omega}$	−0.33	internally calib
k_0	0.16	internally calib
m	−0.24	internally calib

Relevant Moments

Moment	Data	Model
Aggregates		
K/Y	3.00	2.59
I/Y	0.17	0.15
gross debt: $\mathbb{E}[\,1_{b>0} b]/Y$	1.05	1.79
Credit spreads		
cred spread: $\mathbb{E}[r^Q - r]$	2.2%	0.7%
Investment heterogeneity		
avg invest rate: $\mathbb{E}[i/k]$	0.12	0.17
sd invest rate: $SD[i/k]$	0.34	0.36
Life-cycle		
share exit	0.10	0.11
(L age 1) / L	0.03	0.04
# firms age 1 / # firms	0.10	0.11
# firms age 2 / # firms	0.08	0.09

Moment	Data	Model
Balance sheet		
avg leverage: $\mathbb{E}[1_{b>0} b'/k']$	0.37	0.72
correl (n, k')	0.74	0.23
n distribution		

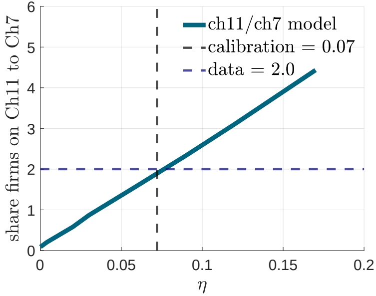


measurement

Identification of η

- $(\alpha_7, \psi_{11}, c_{11})$ match debt haircut under Ch 11 and Ch 7, and leverage in Ch 11

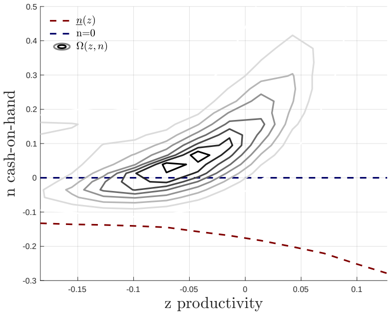
Param.	Value	Moment targeted	Data	Model
α_7	0.38	$\mathbb{E}[R]$	0.27	0.29
ψ_{11}	0.89	$\mathbb{E}[\alpha_{11}]$	0.69	0.82
c_{11}	0.40	$\mathbb{E}[b'/k' \mid \text{Ch } 11]$	0.73	0.67
η	0.07	$\mathbb{E}[\text{Ch11}]/\mathbb{E}[\text{Ch7}]$	2.0	1.9



- Untargeted moments: distribution of leverage in Ch 11 and predictors of Ch 11 validation

Incidence of Rollover Problems

Steady state distribution $\Omega(z, n)$
before bankruptcy choice



$$\underbrace{\int_{(z,n)\in\mathcal{R}} d\Omega(z,n)}_{\substack{\text{share of firms exposed} \\ 0.20}} \times \underbrace{\eta}_{\substack{\text{run likelihood} \\ 0.07}} = 1.5\%$$

Result I: 1.5% of firms are subject to runs

$\Rightarrow$ >60% of bankruptcy events are driven by runs

Outline

- Theoretical Framework
- Identifying Firm Runs
- **Macroeconomic Consequences of Firm Runs**

Macroeconomic Consequences

1. Crises

2. Policies

Crisis Shock

- 3 types of short-lived unexpected aggregate shocks (η fixed):
  1. tfp
  2. cash shock ($n \downarrow$)
  3. credit shock ($\alpha_{11} \downarrow$)

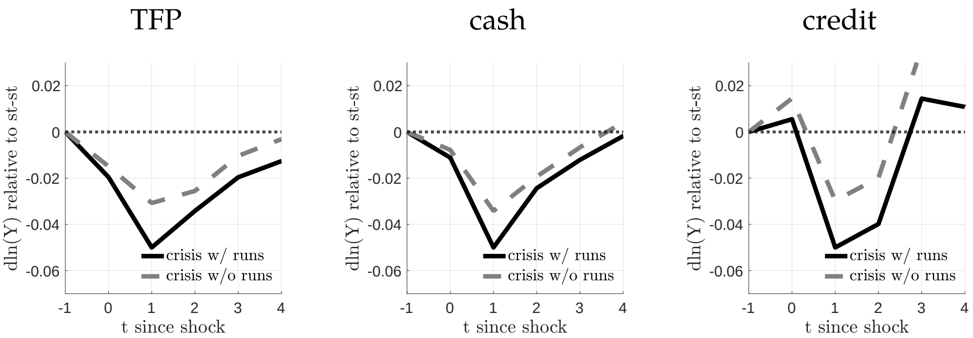
detail shocks

Questions

- contribution of runs to crises impact?
- investment heterogeneity in crises

st-st comparison

Crisis Shock Counterfactuals



detail shocks

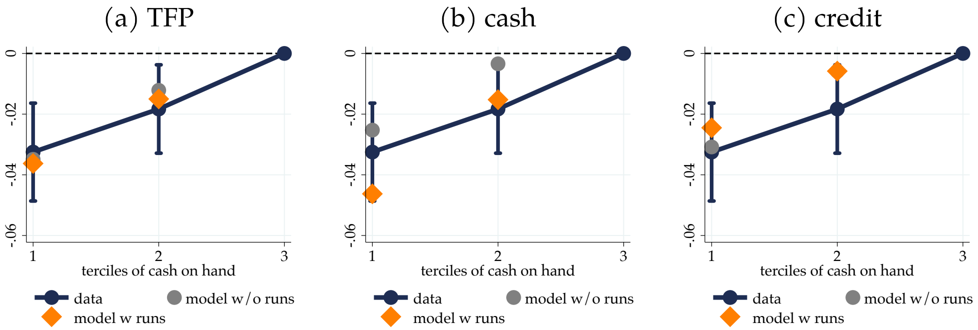
- Firm exit: cash and TFP shocks $\uparrow$; credit shock $\downarrow$ details

Result II: runs amplify significantly the impact of crises

Crisis Shock Heterogeneity

- Estimate heterogeneity in Δk adjustments during crises empirical specification measurement
- Data and model simulation for Great Recession and Covid episodes

Heterogeneity of $\Delta k(\text{crisis}) - \Delta k(\text{no crisis})$ across n/k



note: simple average of both episodes for cash shock individual episode empirical results

Credit Policy Intervention

- Direct lending policy: gov promises an alternative $Q^g(.)$ to a set of elegible firms. Then elegible firms new debt issuance resources are

$$\max\{\underbrace{Q(s, b', k')}_{\text{market}}, \underbrace{Q^g(.)}_{\text{government}}\} \times b'$$

- Policy workings: take elegible firm with $(z, n) \in \mathcal{R}$ under a run
  - faces $Q = 0$ then borrow from government at Q^g
  - If $d = n + \max_{k',b'}\{-k' + Q^g b'\} > 0$ then creditors know the firm could borrow from the gov to rollover the debt $\Rightarrow$ **preclude run**

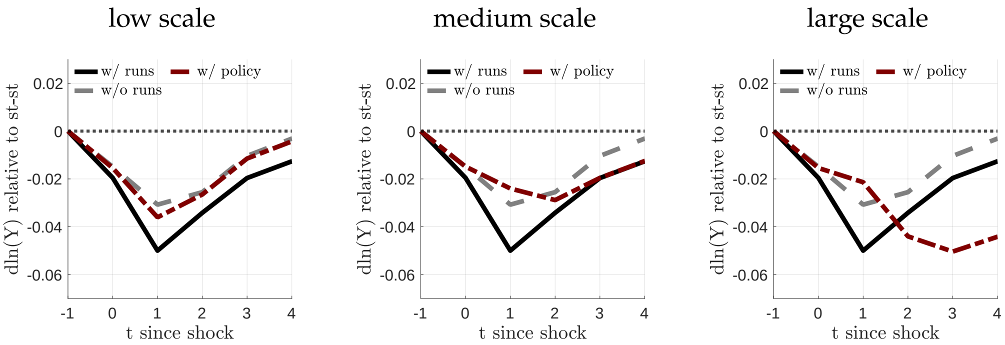
- Imperfect policy faces trade-off between precluding runs and future debt overhang

  parametrization   announcement and implementation   direct lending vs credit guarantees

Question: policy effectiveness during crises

Credit Policy Implications

- Policy active for first two periods and cash shock driven crisis TFP shock results fiscal losses by scale



Result III: imperfect credit policy benefits are ambiguous

(i) low scale policy is very potent

(ii) high scale policy could backfire through future debt overhang

Concluding remarks

Concluding remarks

- Framework where rollover problems can be identified and quantified
- Results
  1. runs are relevant for firms' failure
  2. runs can amplify significantly aggregate impact of crises
  3. role for credit policies to prevent runs, even if imperfect

Future research avenues

- Empirical work
- Extensions: (i) manage liability structure (ii) heterogeneous investors

  liab structure data ex-ante cost of runs
- Other applications: e.g., sovereign debt bankruptcy procedures and self-fullfiling crises

Thank you!

Extra Slides

Related Papers

Brief and non-exhaustive review

- **Quantitative macro models of firms**: heterogeneous firms with corporate finance frictions

  Khan Senga Thomas 2017; Ottonello Winberry 2020 default risk; and Corbae D'Erasmo 2021 bankruptcy

- **Rollover (coord) problems in macro**: creditor's coord problems (run) in banks/firms/countries

  Gertler Kiyotaki 2015 banks; Cole Kehoe 2000; Bocola Dovis 2020 sov debt; Morris Shin 2004 CFin theory; Jackson 1986 CLaw bankruptcy more detail

- **Investment heterogeneity during crises (empirical):** heterogeneity across financial distribution

  Kalemli-Özcan Laeven Moreno 2020; Almeida Campello Laranjeira Weisbenner 2012; Ebsim Faria-e-Castro Kozlowski 2021

- **Credit policy and C-borrowing in crisis:** credit policies implemented to address nonfinancial firms' financing problems in crises

  Crouzet Tourre 2021; Elenev Landvoigt Nieuwerburgh 2021; Ebsim Faria-e-Castro Kozlowski 2021

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Related Papers on Coordination Failures

- **Bank runs**: Diamond Dybvig 1983; Gertler Kiyotaki 2015; Gertler Kiyotaki Prestipino 2020
- **Int'l. macro**: Cole Kehoe 2000; Bocola Dovis 2020; Obstfeld 1994 and 1996
- **Sunspots and business cycles**: Benhabib Wong 2014; Schmitt-Grohe Uribe 2020
- **Corporate finance (theoretical)**: Morris Shin 2004; Acharya Gale Yorulmazer 2011; He Xiong 2012; Halac Kremer Winter 2020; Zhong 2021; Zhong Zhou 2021
- **Corporate law**: Jackson 1986; Baird Jackson 1990; Ayotte Skeel 2013

back related papers   back to paper

US Bankruptcy Code

Bankrupt firms use chapter 11 (11 U.S.C.) or 7 (7 U.S.C.) of US bankruptcy code

- Chapter 7
  - associated with firm's liquidation
  - case impartial trustee appointed to sell the bankrupt firms assets to pay creditors

- Chapter 11
  - associated with firm's restructure (or reorganization)
  - large firms also use to piecemeal liquidate the firm ("363 sale", 11 U.S.C. § 363(a))
  - debtor presents plan, and needs to be approved by judge and, ultimately, negotiated with and voted by creditors
  - provisions to preclude creditor's coordination problem
    1. automatic stay 11 U.S.C. § 362(a): prevents creditors demand payment
    2. debtor-in-possession protection 11 U.S.C. § 1101: allows new financing
    3. creating creditors' committees 11 U.S.C. § 341

back

Bankruptcy Procedure

- Only firms that are insolvent or under a run may restructure their debt
- Recovery rate $\alpha_{11}(.)$ determined by

$$\alpha_{11}(z,k,b,\omega) = \arg\max_{\alpha_{11}} \Bigg[\underbrace{V(z,n^{11}) - 0}_{\text{firm's surplus}}\Bigg]^{1-\Xi} \Bigg[\underbrace{\alpha_{11}b - R(k,b,\omega)\,b}_{\text{creditor's surplus}}\Bigg]^{\Xi}$$

  where $\Xi \in [0,1]$ barg power of creditors, we need that $n_{11} > \underline{n}(z)$ and $\alpha_{11} > R(k,b,\omega) = \min\{1, \alpha_7(1-\delta)\,q\omega k/b\}$

- For computational reasons I approx the barg. Max recov rate $\{\alpha_{11}^{max} : n_{11} = \underline{n}(z)\}$ and min recov rate $\alpha_{11}^{min} = \alpha_7^{min} = R(k,b,\omega)$, then recov rate linear comb of those rates with $\psi_{11} \in (0,1)$ the weight to creditors

back fin frictions   back restructure   back debt price

Entry and Exit

Exogenous exit

- Firms receive exog exit shock with prob γ
- Exiting firms allowed to restructure and liquidate before producing then

$$V^{\text{exit}}(s) = 1_{\{\text{continue}\,|\,\text{exit}\}}(s)\, n + 1_{\{\text{ch11}\,|\,\text{exit}\}}(s)\, n_{11}^{\text{exit}}$$

- Liquidate if $n < 0$ and $n_{11}^{\text{exit}} > 0$ not feasible; restructure if $n < 0$ and $n_{11} > 0$ feasible
- Price of debt conditional on exit is

$$\tilde{Q}_{\text{exit}}\left(z, k', b'\right) = \mathbb{E}_{\left(s'|s\right)}\left[\Lambda\left\{1_{\{\text{continue}\,|\,\text{exit}\}}\left(s'\right) + 1_{\{\text{ch11}\,|\,\text{exit}\}}\left(s'\right)\alpha_{11}^{\text{exit}}\right\}\right] + \mathbb{E}_{\left(s'|s\right)}\left[\Lambda 1_{\{\text{ch7}\,|\,\text{exit}\}}\left(s'\right) R\left(\omega', b', k'\right)\right]$$

Entry

- Mass $\bar{\mu}$ enter each period replacing exiting firms (for all reasons)
- Enter with capital $k = k_0$, $b = 0$ and $z \sim \Omega^e(z)$

back setup back value function back debt price

Liquidation Choice: Characterization

Proposition (Liquidation Choice)

Continuing firms liquidation choice $\tilde{1}_{ch7}\left(s, b', k'\right) \equiv 1_{ch7}(s)$ *where*

$$\tilde{1}_{ch7}(s) = \begin{cases} 1 & \textit{if } n < \underline{n}(z) \\ 1 & \textit{if } n \in \left[\underline{n}(z), 0\right) \quad \textit{and} \quad \phi < \eta \\ 0 & \textit{if } n \geq 0 \quad \textit{or} \quad n \in \left[\underline{n}(z), 0\right) \quad \textit{and} \quad \phi > \eta \end{cases}$$

with $\underline{n}(z) \equiv -\max_{k', b'}\left\{-k' + \tilde{Q}\left(z, b', k'\right) b'\right\}$ *where* $\tilde{Q}$ *debt price if* $[1 - \tilde{1}_{ch7}(s)] = 1$ (*i.e., no liquidation today conjectured*)

- Firms with $n < 0$ are exposed to runs independently of their productivity z
- If $\tilde{Q}$ increasing in z then $\underline{n}(z)$ decreasing in z

back charact   back eq

Liquidation Choice: More General Setup

Long-term debt

- assume portion debt m matures each period (randomly) and nonmatured pays cupon c
- cash-on-hand: $n = \pi + q\omega(1-\delta)k - [m + (1-m)c]b$
- external funds: $Q(.)\left[b' - (1-m)\,b\right] - qk'$
- default threshold: if $n \in [\underline{n}(z, b), 0)$ exposed to runs and $n < \underline{n}(z, b)$ insolvent
- (recursive) debt prices (simplified = no bkrptcy, no exit, no discount, $c = 0$, $\alpha_7 = 0$):

$$\tilde{Q}(z, k', b') = \mathbb{E}_{\mathbf{z}'|\mathbf{z}}\left[\left\{1_{n' \geq 0} + (1-\eta)\,1_{n' \in [\underline{n}(z', b'), 0)}\right\}\left\{(1-m)\,\tilde{Q}(z', k'', b'') + m\right\}\right]$$

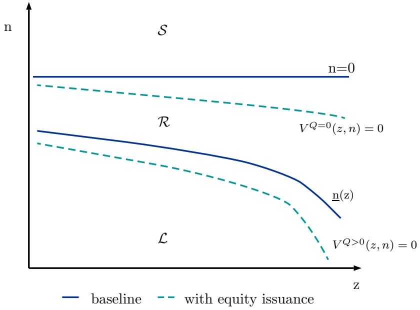
More general (assume $c = 0$ for exposition)

- profits $\pi(\mathbf{z}, k)$ gral $\mathbf{z}$ process, invest $\iota\left(k, k'\right)$ allow for idio k frictions and long-term debt
- dividends (if no run) are $d = \pi(\mathbf{z}, k) - \iota\left(k, k'\right) - bm + \tilde{Q}(.)\left(b' - (1-m)\,b\right)$
- multiple eq if $\max_{k', b'} d \geq 0$ and $n \equiv \pi(\mathbf{z}, k) - bm - \iota(k, 0) < 0$ hold

back liquidation back debt price

Liquidation Choice: Costly Equity Issuance

- Firms can issue equity $e < 0$ at cost $\phi(e)$, which is decreasing in e and unbounded.



- $\mathcal{S} : V^{Q=0}(z,n) \geq 0$
- $\mathcal{L} : V^{Q>0}(z,n) < 0$
- $\mathcal{R} : V^{Q>0}(z,n) \geq 0$ and $V^{Q=0}(z,n) < 0$

- Where $V^{Q=0}$ firm problem with costly equity issuance where $Q = 0$ and $V^{Q>0}$ same but with $Q > 0$

back

HH Problem

HH in equilibrium determines

$$\Lambda' = \beta \frac{u_C(C', L')}{u_C(C, L)}$$

$$1 = E\left[\beta \frac{u_C(C', L')}{u_C(C, L)}(1 + r)\right]$$

$$w = -\frac{u_L(C, L)}{u_C(C, L)}.$$

with utility function $u_C(C, L) = \ln C - \Omega L$

back back eq

Capital Producer

There is a representative aggregate capital producer that maximizes

$$\max_{I} q\Phi\left(\frac{I}{K}\right) - I$$

where I is the amount of final goods used to produce capital, K is the aggregate k stock, and $\Phi(.)$ is the aggregate capital adjustment cost function. FOC:

$$q = \frac{1}{\Phi'\left(\frac{I}{K}\right)}$$

- time-varying q and $\mathcal{R}(.) \rightarrow$ financial accelerator mechanism (Bernanke, Gertler & Gilchrist 1999).

back back eq

Steady-State Equilibrium

Steady-state equilibrium in this economy is Vfunctions of continuing firms $\left\{V, \tilde{V}\right\}$, decision rules $\left\{b', k', l\right\}$, aggregates $\{Y, C, I\}$, price schedule $Q(.)$, interest rate r, prices $\{q, w\}$, default choices $1(.)$, recov rates $\alpha_{11}(.)$ and distribution of firms $\{\Omega(.)\}$

- HHs choices are determined by Euler eq, SDF and labor supply eq detail
- price of capital q determine in K producer problem detail
- debt price satisfy no-profit condition of fin intermediaries detail
- given prices, firm's dec. rules solve the producing firm's problem detail and default choices are consistent with Default Propositions
- recovery rates satisfy bargaining protocol
- markets clear (labor, resources)
- distribution of firms **fixed point** in law of motion detail

back

Law of Motion States

Let Ω be the distribution of firms that produce which they a mass of 1, $\tilde{\Omega}$ the distribution of incumbent firms at the begining of the period, g and $\hat{g}$ the pdf of ω and ϕ respectively, p the conditional pdf of the productivity shocks ϵ_z, and Ω^e the distribution of entrant firms. To define the equilibrium first we need to determine the law of motion of the distribution. Distribution of firms that produce is

$$\begin{aligned}\Omega(z,n) &= (1-\gamma)\int \left[1_{\{ch11\}}(s)\,1_{\{n^{11}(z,k,b,\omega)=n\}} + 1_{\{cont\}}(s)\,1_{\{n(z,k,b,\omega)=n\}}\right] d\tilde{\Omega}(s) \\ &\quad + \bar{\mu}(1-\gamma)\int \left[1_{\{ch11\}}(s)\,1_{\{n^{11}(z,k_0,0,\omega)=n\}} + 1_{\{cont\}}(s)\,1_{\{n(z,k_0,0,\omega)=n\}}\right] \hat{g}(\phi)\,g(\omega)\,d\phi\, d\omega\, d\Omega^e(z) \\ &\quad + \text{lom} \mid \text{exit}\end{aligned}$$

The distribution of incumbent firms at the begining of the period $\tilde{\Omega}(z,\omega,k,b,\phi)$ is

$$\tilde{\Omega}\left(s'\right) = \int 1_{\{k'(z,n)=k'\}}1_{\{b'(z,n)=b'\}}\,\hat{g}\left(\phi'\right) g\left(\omega'\right) p\left(\epsilon_z \mid \rho_z z + \epsilon_z = z'\right) d\epsilon_z d\Omega(z,n)$$

back eq def   back eq paper

Data Sources, Sample and Some Definitions

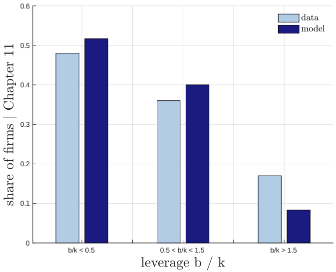
Compustat

- Two samples (accounting changes after 2018, see Ma's online notes)
  - Pre-covid = 1980-2017 (n=179k annual, n=426 k quarterly)
  - Covid = 2019-2020 (n=14k quarterly)
- Sample selection: nonfinancial, $k > 0$, assets > 0, drop outliers and short-spell (<20 q spell)
- Key definitions:
  - n = profits + liq value capital − net liquid liabilities
  - profits = F1.`oiadpq` where F1 = one period ahead in the data
  - net liquid liabilities = `lctq` − `cheq`
  - liq value capital = `invtq` $\times\, \omega_{\text{inv}}$ + `rectq` $\times\, \omega_{\text{rec}}$ + `ppentq` $\times\, \omega_{\text{ppentq}}$ + `acoq` where ω_x is liq value rate (from Kermani Ma 2020) of asset class `x`
- Identify bankrupt firms that operate following Corbae D'Erasmo 2021. Use footnote to total assets and deletion information (`dlrsn` and `dldte`). Bankrupt firms:
  1. report adoption accounting under Ch11, or bankrupt and not deleted
  2. data available next period

back calibration   back invest het   back invest het by episode

Untargeted Moments of Bankruptcy

Distribution of leverage b'/k' firms in Chapter 11



Predictors of Chapter 11

dependent variable: $1^{ch11}_{i,t}$

	(1)		(2)		(3)	
	data	model	data	model	data	model
$n_{i,t-1}/k_{i,t}$	-0.39	-0.05			-0.39	-0.45
	(0.03)				(0.10)	
$b_{i,t}/k_{i,t}$			0.11	0.03	-0.29	-0.41
			(0.04)		(0.09)	
$\log(k_{i,t-1})$	-0.50	-0.06	-0.52	-0.06	-0.49	-0.10
	(0.12)		(0.12)		(0.12)	
$d\log(\text{sales}_{i,t-1})$	-0.04	-0.03	-0.04	-0.02	-0.04	-0.01
	(0.00)		(0.00)		(0.00)	
Sector FE	Y		Y		Y	
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y		Y		Y	
Observations	370,973		373,362		370,973	

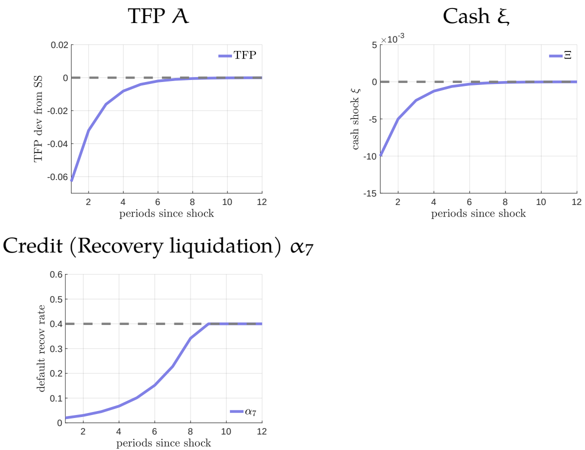
empirical specification: $1^{ch11}_{i,t} = \beta X_{i,t-1} + \alpha_t + \alpha_i + \alpha_s + \epsilon_{i,t}$

back

Crisis Shocks

- Shocks unexpected and perfect foresight of path
- Temporary with persistence 0.5
- Definition of shocks:
  - TFP A: prod funct $Azf(k\omega, l)$
  - Cash ξ: reduction in n by ξk
  - Credit: recov rate if liquidated α_7

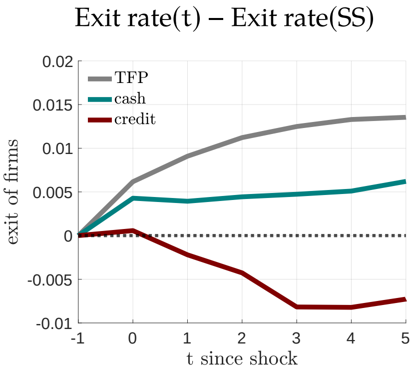
MIT shocks path

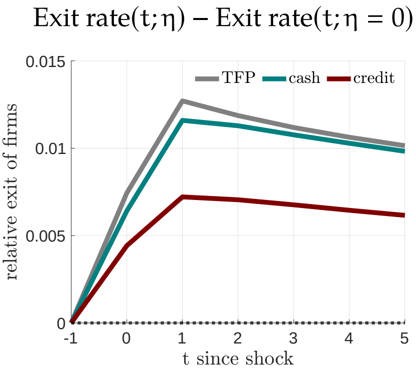


back exercise back results

Steady State Comparison

- Firm exit dynamics during crisis experiments



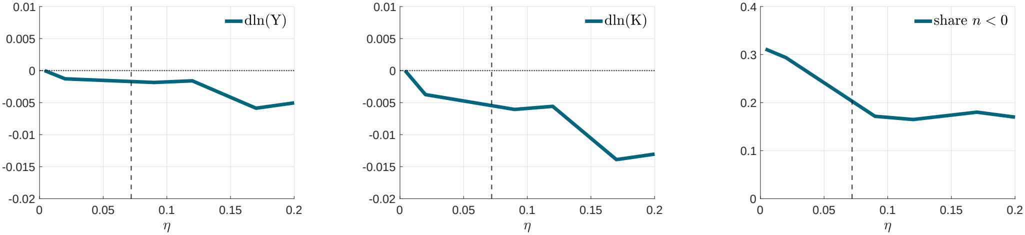


back

Steady State Comparison

- Variables: aggregate income Y, capital, K and share of firms with $n < 0$
- Comparison: steady state for different η

Y | K | share of firms with $n < 0$



notes: log difference relative to st-st with $\eta = 0$ for Y and K and levels for share of firms

back

Heterogeneous Investment Response
Empirical Specification

- Diff-in-diff crisis event estimate
  similar to Kalemli-Özcan Laeven Moreno 2020

$$\Delta \log(k_{it}) = \underbrace{\sum_{j=1}^{J} \beta_j^n \left(Q_{it}^{nj} \times \text{crisis}_t \right)}_{\text{heterogeneity across } n/k} + \underbrace{\sum_{j=1}^{J} \beta_j^b \left(Q_{it}^{bj} \times \text{crisis}_t \right)}_{\text{heterogeneity across } b/k} + \underbrace{\Lambda' Z_{it}}_{\text{controls}} + \varepsilon_{it}$$

let $x_{it} = \{b_{it}, l_{it}\}$ firm i at period t with

- demeaned by sector $\hat{x}_{it} = x_{it} - \mathbb{E}_s[x_{it}]$.
- $\Delta \log(k_{it}) = \log(k_{it+h}) - \log(k_{it})$ with h peak-to-trough length
- crisis_t indicates if a crisis happens during the period considered
- $Z_{i,t}$: sales growth, log firm size, firm FE, sector FE

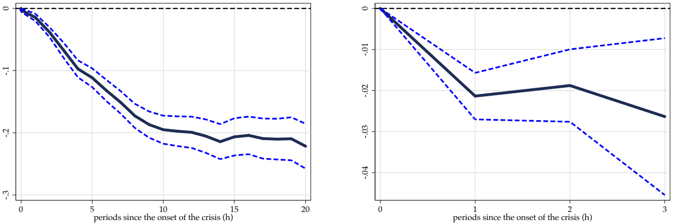
back

Recent Crisis Episodes in U.S.

(a) Great Recession

(b) Covid-19 Crisis

Δ Capital Accumulation



$$\beta_h : \log(k_{it+h}) - \log(k_{it}) = \alpha_i + \beta_h \text{crisis}_t + \varepsilon_{it+h}$$

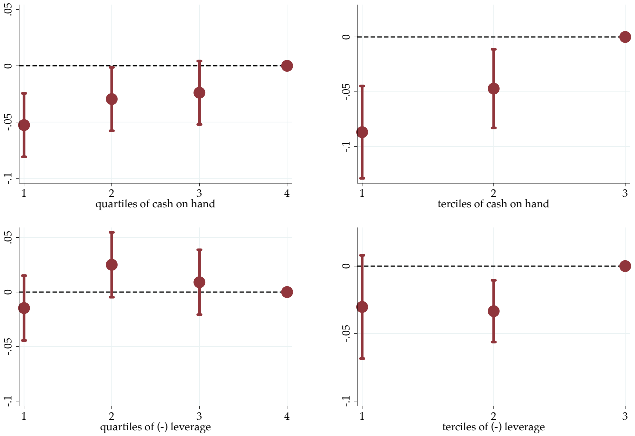
back

Investment Adjustment Heterogeneity
Recent Crisis Episodes in U.S.

Great Recession | Covid-19 Crisis

Across n/k

Across b/k



back

Credit Policy Setup

- Announced unexpectedly at $t = 0$ (same period of shocks) for $T \geq 0$ periods and implemented at $j \in [0, T]$
- Eligible firms $(z, n) \in \mathcal{P}$ offer sequence of $\left\{Q_t^g(.)\right\}$
- Policy $\mathcal{P}$ and labor taxes τ fixed across time
- Budget constraint from $t \geq 1$

$$\tau w_t L_t + B_t + B_{t-1,t}^g = B_t^g + (1 + r_{t-1}) B_{t-1}$$

B_t^g amount lent, $B_{t-1,t}^g$ lent at $t-1$ and recovered at t

back

Credit Insurance Policy: First Best and Trade-off

Proposition (Credit Insurance Policy)

Assume that the government implements the credit insurance policy next period and is predictable today:

1. *First best policy:* $Q^g = \tilde{Q}$ *then no risk of runs and qualified firms indifferent between using public or private credit.*
2. *No screening: fix* z^g *such that* $Q^g = \tilde{Q}\left(z^g, k', b'\right)$ *with firms qualified for credit those with* $0 > n > \underline{n}(z^g)$. *This policy faces a trade-off between lowering firm run risk and greater misallocation.*

- 1st best policy eliminates runs and firms don't use the program's credit
- W/o screening greater z^g will preclude more runs, but firms with $z < z^g$ will draw funds (zombification)

back trade-off

Direct Lending vs. Credit Guarantees

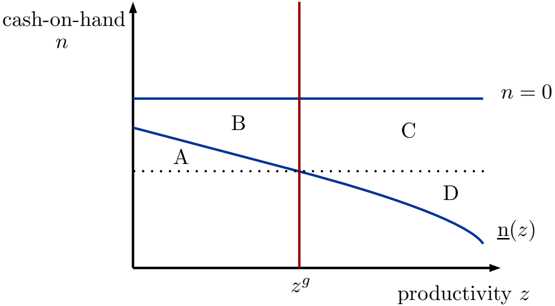
- Examples: direct lending $\approx$ Fed's PMCCF SMCCF and credit guarantee $\approx$ PPP
- In the theory policies are
  - direct lending (DL): alternative $Q^g(.)$ detail theory
  - credit guarantee (CG): repay $\alpha_g^r \geq \alpha^r$ in case of default
- Workings relative to runs
  - DL affects payoffs (outside eq) and could coord creditors in good eq
  - CG relaxes $\underline{n}(z)$ but doesn't *directly* preclude runs

back

Credit Policy Trade-off

Stylized example of 1 period policy in PE and two extreme cases

1. *Perfect screen of z* : $Q^g = \tilde{Q}$ and then remove coord failures for "free"
2. *No screen of z* : gov lends to elegible firms $n \in (0, \underline{n}(z^g)]$ firms at $\tilde{Q}(z^g, k', b')$, with z^g parameterizing policy scope



A ∪ B ∪ C: elegible

D: excluded

A ∪ B: subsidized credit

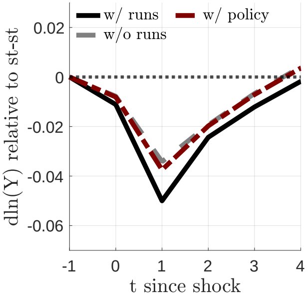
B ∪ C: runs precluded

back proposition

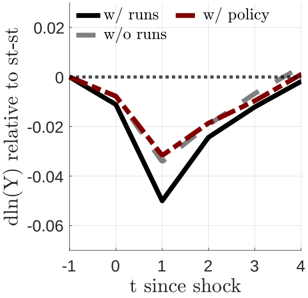
Credit Policy Implications: TFP shock

- Policy active for first two periods and TFP shock driven crisis back cash shock results

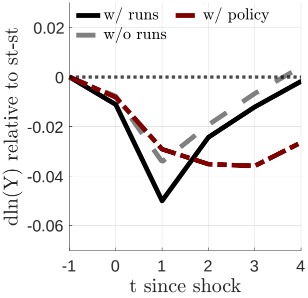
low scale



medium scale



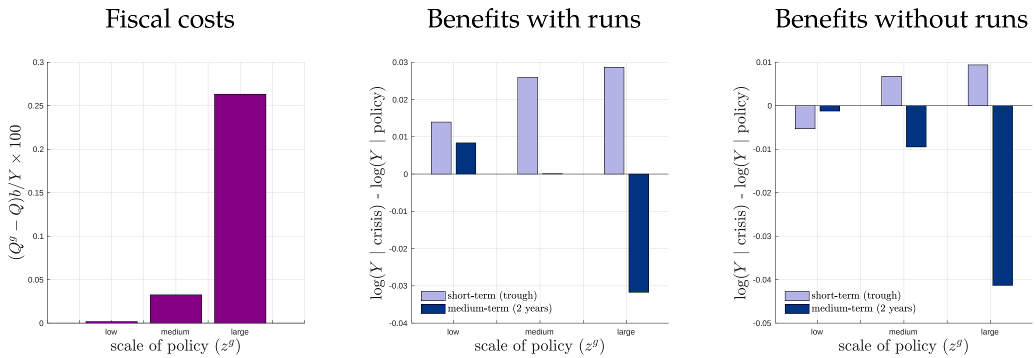
large scale



Credit Policy Implications: TFP shock

- Compute fiscal costs, short and long term benefits back

Costs and benefits



Liability Structure Data

- Debt maturity (Compustat)

	Time to mature (share)		
	< 1 year	1 to 4 years	≥ 5 years
Debt	0.29	0.33	0.38
	< 1 year	> 1 years	
Liabilities	0.61	0.39	

- Number of creditors from bankruptcy filings to Chapter 11 (FJC-IDB)

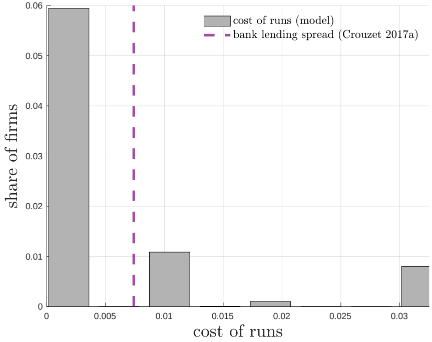
	# Creditors		
	1 to 100	101 to 1,000	>1,000
Medium ($>$ 50 million and $<$ 1 billion assets)	0.16	0.19	0.65
Large ($>$ 1 billion assets)	0.03	0.04	0.93

back

How Costly are Firm Runs?

- (ex-ante) Cost computed as $\tilde{Q}\left(z, k', b'; \eta\right) - \tilde{Q}\left(z, k', b'; 0\right)$
- Only 2.2% of the firms face a cost of runs higher than intermediation spread

Cost of runs (in annual spread terms) distribution



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