Financial Frictions and the Market for Firms*

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Abstract

We study and quantify the aggregate implications of the trade of firms in the presence of financial frictions. In the U.S., one out of four entrepreneurs purchased their business. In the cross-section, younger, smaller, and higher return to capital firms have the highest trading rates. To explain these findings, we propose a general equilibrium model of entrepreneurship with a frictional market for firms where gains from trade arise from credit constraints, incomplete markets, and preference shocks. Using firm-level data from several high-income countries, we document that post-trade firm dynamics are consistent with the trade of firms alleviating financial constraints, as predicted by the model. Our quantitative results for the U.S. suggest that firms' trade significantly improves capital allocation in the economy, accounting for 9.1% of entrepreneurial output and 2.2% of TFP. We argue that the trade of firms can play an even more important role in less financially developed economies.

Keywords: trade of firms, financial frictions, misallocation, search frictions.

JEL classifications: E44, L20, O11, G30.

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1 Introduction

Markets are the predominant allocation mechanism of modern economies. One important market that allocates productive projects and available resources is the market in which firms are bought and sold, i.e., *the market for firms*. In this paper, we argue that the role of this market is particularly relevant in economies where financial constraints are a pervasive feature of entrepreneurial activity. In such environments, who owns the firms matters for allocations, as credit-constrained entrepreneurs will produce at a suboptimal scale, resulting in capital misallocation and lower aggregate output. The market for firms allows financially constrained entrepreneurs to sell their firms to other parties with more financial resources, potentially improving the allocation of capital in the economy.

We study the aggregate implications of the market for firms as an allocation mechanism in multiple steps. First, we use microdata from business owners, households, and firms to document novel facts about firms' trade in the U.S. economy. Second, we develop a macroeconomic model where agents can buy and sell firms in a frictional market. In the model, gains from trading firms arise from financial frictions, namely credit constraints and incomplete financial markets, and preference shocks that capture alternative motives to trade firms. We quantify the model to match salient features of the U.S. market for firms. Third, we provide novel cross-sectional and panel evidence consistent with the main testable predictions from our theory of financial frictions being an important motive to trade firms. Lastly, we use our quantitative framework as a laboratory to study the relevance of this market for aggregate output and productivity.

For our empirical results, we focus on entrepreneurs and study how they acquired their businesses. We define entrepreneurs as self-employed private business owners who actively manage their firms and have at least one employee. Using multiple data sources, we document three main facts about the market for firms. First, one out of four entrepreneurs (around 23% to 26%) in the U.S. acquired their business by purchasing an existing firm, implying an annual trade rate of 3%. This result indicates that private businesses are highly illiquid assets. Compared to housing, for example, Berger and Vavra (2015) reports that 5% of houses are traded annually, higher than the 3% trade rate for private firms. Nonetheless, the trade of firms is more frequent than the trade of intangible assets such as patents. For example, Akcigit, Celik, and Greenwood (2016) documents that 16% of U.S. patents have been traded, smaller than the 26% we find for private businesses.

Second, we document two salient characteristics of business buyers. Our first finding reveals that 66% of buyers have never been entrepreneurs before purchasing their current firm. This finding suggests that buying an existing firm is a relevant way to enter into entrepreneurship, which, to the best of our knowledge, has not been studied before. Besides capturing the illiquidity of private firms, our theoretical framework incorporates this novel feature about households' possible transitions into entrepreneurship through the market for firms. Our second finding shows that the average wealth of firm buyers is about three times that of the average household. This evidence will serve us to test our theory of financial frictions being a relevant driver for firms' trade.

Lastly, we document that younger, smaller, and higher return to capital firms, measured by average revenue product of capital (ARPK), have the highest trading rates. These cross-sectional results about firms' characteristics and trade frequency are informative about the underlying motives behind firms' trade. In this sense, any theory about the trade of firms should be able to accommodate these patterns. Both firms' age and size are associated with financial constraints (Hennessy and Whited, 2007; Hadlock and Pierce, 2010). Furthermore, firms' returns to capital are informative about their access to external finance, as credit-constrained firms may have high returns but cannot increase their investment. By introducing financial frictions as a micro foundation that generates gains from trading firms, our model can account for these cross-sectional facts.

Motivated by these findings, we develop a heterogeneous agent model of entrepreneurship and frictional trade of firms. Our model economy is populated by a continuum of households, which can be firm owners or workers. Firm owners can trade or shut down their firms, while workers can become business owners by buying an existing firm or through an exogenous startup shock. There are credit constraints and incomplete financial markets, so households are subject to uninsurable idiosyncratic risk. On the one hand, firm owners face the risk associated with the quality of their firm, which evolves stochastically. On the other hand, workers are subject to shocks to their labor efficiency.

We characterize firms through the quality of an entrepreneurial project which is indivisible, rival, and excludable. These entrepreneurial projects aim to capture firms' intangible assets.[1] Firms enable their owner to produce the final consumption good with a technology that combines capital, labor, and the firm's quality. Besides the firms owned by a single household, which we call private firms, there is a second production sector with a representative public firm. Both sectors produce the same good, which can be used for consumption or savings in a risk-free asset. There is also a financial intermediary that, each period, takes the savings from the households and rents capital to the firms.

Our empirical findings suggest that private firms are highly illiquid assets, which motivates using a search-theoretic approach to model this market. Specifically, we model the market for firms through a decentralized market subject to search frictions and bilateral random matching. A Nash bargaining protocol between sellers and buyers determines the trading price. One interpretation of these assumptions is that agents can valuate only one firm at a time, which delays trade.[2] This setup is suitable for our quantitative analysis and allows the model to match relevant features about the market for firms.

Exchanges in the market for firms are voluntary. Hence, a necessary condition for trade is that agents have different valuations for the same firm, with the buyer having a higher

[1] Using data from business transactions, Bhandari and McGrattan (2021) documents that when a firm is sold, around 60% of its total value is accounted by intangibles. This evidence supports our characterization of firms by the value of their intangible assets.

[2] BizBuySell, an online marketplace for businesses, surveyed their clients about their major challenges when purchasing a firm. For more than 40%, the major issue was "finding the right business", and for 23% was "valuating the firm". These responses are consistent with our modeling of the market for firms.

valuation than the seller. Heterogeneity in firms' valuations arises from three sources in our theory: credit constraints, incomplete markets, and preference shocks. Credit constraints and incomplete markets generate an endogenous motive to trade. For a given firm, unconstrained agents attain higher profits, grow the firm faster, and bear the risk better than constrained agents. By transferring firms between agents with different wealth levels, the market for firms can improve allocative efficiency. In addition, we assume that potential sellers are subject to idiosyncratic preference shocks that parsimoniously capture other motives to trade firms that we do not explicitly incorporate in our theory.

We calibrate the model to match several features of the U.S. economy. We target moments related to the role of entrepreneurs, the income and wealth distribution across households, the relative importance of the private business sector, and key characteristics of the market for firms. We start by using our calibrated model to quantify the importance of the different motives behind firms' trade. Idiosyncratic preference shocks, which are informed by the frequency of trade of large unconstrained firms, account for 31% of the exchanges. Concerning the two motives related to financial frictions, we find that incomplete markets, which create differences in risk-bearing capacity across agents, explain 16% of the trades. Thus, credit constraints, which limit firms' borrowing to a multiple of the current owner's wealth, account for most of the transactions in our model economy.

We perform three different exercises evaluating testable predictions of our theory about financial frictions being a relevant motive to trade firms. The first two predictions are related to the cross-sectional patterns documented in the empirical part of the paper. First, we compare the model-simulated relations between trade rates and firms' observable attributes with their empirical counterparts. Our model predicts that younger, smaller, and higher ARPK firms have the highest trading rates, consistent with the data. The fact that these groups of firms are associated with binding credit constraints in our model, which generates gains from trading firms, explains this result. Second, we test the prediction of our model regarding business buyers' characteristics and find that, as in the data, buyers are up to three times wealthier than the average household.

Third, we test the predictions of our theory about post-trade firm dynamics. According to our model, if a firm is traded, it was likely financially constrained before the trade. Hence, this firm had lower capital and higher ARPK than its unconstrained level. After a trade, capital expands more than output, reducing the firm's ARPK over time. This is the key prediction of our theory we test in the data. Because of data limitations for the U.S., we use balance sheet and ownership firm-level data from several high-income European countries, which are the most comparable to the U.S., to document firm dynamics after trade. We find that firms' capital increases by 47 log points five years after being traded, while output increases by 23 log points. As a result of capital and output post-trade dynamics, ARPK falls significantly, in 24 log points, five years after trade. This novel empirical evidence about firm dynamics after trade supports our central hypothesis of firms being financially constrained before being traded. We also show that this evidence aligns qualitatively and quantitatively with the post-trade firm dynamics in our model.

After providing evidence consistent with testable predictions from our theory, we quantify the macroeconomic implications of the market for firms as a mechanism that allocates productive projects and available resources in the economy. We perform two counterfactual experiments. In our first experiment, we take our baseline model and analyze a scenario in which the market for firms shuts down. Closing this market implies a fall in aggregate entrepreneurial output and total factor productivity (TFP) of 9.1% and 2.2%, respectively. This result is explained by a lower entrance into entrepreneurship and a poorer capital allocation when this market is absent.

The previous exercise shows that the market for firms alleviates the capital misallocation caused by financial frictions. To better understand the TFP gains from this market, in our second experiment, we consider an alternative economy with no trade of firms. Then, we ask: what credit conditions does the no market economy require to match the TFP level of our baseline economy? The no market model requires looser credit conditions such that private firms' leverage, or the debt-to-capital ratio, increases by 14 percentage points (p.p.), from 0.35 to 0.49. This increase is sizable as, for example, leverage dropped by a total of 5 p.p. in the U.S. during the Great Recession.

Finally, we investigate the interaction between financial development and the trade of firms. In our model, the functioning of both markets for credit and firms determines the allocation of assets and firms. As in the finance and misallocation literature, our model implies that higher levels of financial development lead to a better allocation of capital and higher TFP. Unlike previous work, we show that aggregate TFP can increase through a better-functioning market for firms for any level of financial development. The market for firms can play an even more important role in economies with tighter credit frictions, as the potential gains from trading firms are higher. Consistent with this prediction, we document that post-trade firm dynamics are sizably more pronounced in middle-income and less financially developed countries, with capital and output increasing twofold than in high-income economies. Overall, our results indicate that the market for firms can be a relevant substitute for debt financing in economies with less developed credit markets.

Related Literature Our paper contributes to the following strands of the literature.

Entrepreneurship in Macroeconomics. Our theoretical framework builds on the literature on heterogeneous agents models with entrepreneurship, see, for example, Quadrini (2000), Cagetti and De Nardi (2006), and more recently Bhandari and McGrattan (2021) and Peter (2021). These models can match the observed income and wealth distribution by combining uninsurable income risk and stochastic returns to wealth from entrepreneurial activity. We contribute to this literature by extending the framework to allow for the trade of entrepreneurial projects in a frictional market for firms. Our model can account for the fact that one out of every four entrepreneurs in the U.S. purchased their business, and over two-thirds were not entrepreneurs before acquiring their firm.

Aggregate Implications of the Market for Ideas. From a theoretical perspective, our paper relates to the literature studying the trade of ideas, or patents, and its implications for economic growth (Silveira and Wright, 2010; Akcigit, Celik, and Greenwood, 2016). As in that literature, we use a framework characterized by bilateral meetings subject to search frictions to model the trade of private firms. Hence, the likelihood of trade depends on meeting probability parameters and the endogenous distribution of agents in the economy. Different from papers of non-rival ideas (Lucas and Moll, 2014; Perla and Tonetti, 2014), diffused through imitation, firms are rival and excludable in our environment. Therefore, sellers need to be compensated by buyers. Under credit constraints and incomplete financial markets, the aggregate implications of the market for firms are determined by how this market affects the allocative efficiency of capital in the economy.

Trade of Firms and the (Re)Allocation of Productive Resources. Our work is mainly related to recent, and in some cases contemporaneously developed, literature studying the trade of firms as an allocation mechanism. Caselli and Gennaioli (2013) and Gaillard and Kankanamge (2020) analyze the trade of mature firms, where gains from trade arise from shocks related to business owners' life cycles. David (2021) studies mergers between firms in an environment with complementarities, while Mahone (2021) analyzes firms' trade driven by occupational taste shocks. More recently, Bhandari, Martellini, and McGrattan (2022) studies firms' trade as a mechanism to accumulate intangible capital when it is subject to indivisibilities. Our contribution to this literature is threefold. First, we document several novel facts about the market for firms, particularly about the market participants and the cross-sectional characteristics of traded firms, which are informative about the motives behind firms' trade. Second, different from the other papers in this literature, we study the trade of financially constrained firms. We provide novel empirical evidence of post-trade firm dynamics consistent with our hypothesis of firms being financially constrained before being traded. Finally, we quantify the aggregate implications of the market for firms in an economy with imperfect credit markets.

Financial Frictions and M&A. Our paper also relates to the empirical literature in corporate finance that studies the financial rationale behind mergers and acquisitions (M&A). For example, Liao (2014) and Erel, Jang, and Weisbach (2015) analyze post-M&A firm dynamics and find evidence that acquisitions relieve financial constraints in target firms. We contribute to this literature by providing novel evidence of post-trade firm dynamics for a broader sample of transactions. Using balance sheet and ownership data from several European countries, we identify trades for firms of different sizes and involving households (e.g., workers buying firms in our model) not covered in the M&A data. We document that the post-trade firm dynamics of capital and output are consistent with the alleviation of financial constraints after trade. Our results suggest that financial frictions play an important role in the trade of firms beyond M&A deals.

Finance and Misallocation. Finally, our paper relates to the literature on financial frictions and misallocation as a source behind TFP differences across countries (Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014; Moll, 2014). We contribute to this literature by showing that the market for firms can reduce the capital misallocation caused by financial frictions, especially in less financially developed economies where the gains from trading firms are higher. We provide evidence consistent with this prediction of our model by documenting that post-trade firm dynamics are twice as large in middle-income and less financially developed countries than in high-income countries.

Outline The rest of the paper is organized as follows: Section 2 presents our main empirical results for the U.S. economy; Section 3 presents the model; Section 4 describes our parameterization; Section 5 describes the main properties of our model; Section 6 evaluates several testable predictions of our theory on the trade of firms; Section 7 presents our aggregate results; and finally, Section 8 concludes.

2 Evidence on the U.S. Market for Firms

In this section, we use microdata from business owners, households, and firms, to document relevant facts about the market for firms in the U.S. economy. First, we study how many entrepreneurs purchased their businesses. Next, we present evidence about the previous occupation and wealth of firm buyers. Lastly, we study the characteristics of the traded firms. Appendix A presents robustness checks and additional empirical exercises.

2.1 Data Sources

We use three different surveys related to private firms, their characteristics, and the characteristics of their owners.[3] First, our main data source is the Survey of Business Owners (SBO) Public Use Microdata Sample (PUMS). This survey provides comprehensive information about businesses and business owners. In particular, about how they acquired their business. The PUMS sample is representative of all non-farm private businesses in the U.S. and is available for the year 2007.

Second, we use nine waves of the Survey of Consumer Finances (SCF) covering the period between 1989 and 2016. Importantly, the SCF includes detailed information about households' income and balance sheets, which we will use to discipline our quantitative model's income and wealth distribution. Additionally, this survey asks business owners how they acquired their firms. The information in the different waves of the SCF allows us to study how the ownership of firms has evolved over time.

Finally, we use data from the Kauffman Firm Survey (KFS). The KFS is an eight-year panel of firms that started operations in 2004 and were followed through 2011. Unlike the previous datasets, the KFS contains information about firms' balance sheets, allowing us to compute firm-level capital. However, the KFS sample is not representative of the entire private sector, as it is a survey of startups. We will account for this fact when comparing the KFS evidence with data simulated from our model. Appendix A.1 presents further details about these datasets, variables' definitions, and our sample selection criteria.

[3] In Appendix A, we provide additional evidence using the Annual Survey of Entrepreneurs (ASE). Our main findings are consistent with the results obtained from that complementary data source.

2.2 Entrepreneurs

The empirical analysis in this section focuses on *entrepreneurs* as the observation unit. We follow Cagetti and De Nardi (2006) and define entrepreneurs as self-employed individuals who own a business and have an active management role in it. Given our interest in the trade of firms, we restrict to the entrepreneurs with at least one employee.[4] According to the 2007 SCF, entrepreneurs represent 6% of households. As previous studies have documented, although entrepreneurs represent a small fraction of the population, they earn 20% of total income and hold 33% of total wealth. In our calibration strategy, we will target these key features of the role of entrepreneurs in the economy.

Throughout our analysis, we assume that each entrepreneur owns and manages only one firm. This assumption implies that the number of firms traded every period equals the number of entrepreneurs that trade their firms. Hence, we use both terms interchangeably. Our assumption relies on the fact that, in the SCF, more than 80% of entrepreneurs own only one firm (see Table A.11). Furthermore, according to the SBO, more than 74% of the private firms in the economy have only one entrepreneur and more than 96% of the firms have at most two entrepreneurs (see Table A.12).[5]

2.3 How do Entrepreneurs Acquire Their Firms?

Share of Traded Firms We start our analysis using the SBO and the 2007 SCF to document how entrepreneurs acquire their firms.[6] We focus on three main types of acquisitions: founding a firm, purchasing an existing firm, and inheriting or other kinds of acquisition. Table 1 shows that two-thirds of entrepreneurs acquire their firms by founding their businesses. Also, it shows that 9% to 12% of entrepreneurs acquired it through inheritance or other types of acquisition. The most relevant number for our analysis is that 23% to 26% of the entrepreneurs in the U.S., depending on the survey, acquire their business by purchasing an existing firm.

In Appendix A.2.1, we verify the robustness of these findings. Table A.4 shows that our results are robust to several alternative definitions of entrepreneurs, such as focusing on firms' majority shareholders. Table A.5 shows that the results are almost identical when we compute the share of traded firms at the firm-, instead of the entrepreneur-, level. Table A.6 shows that the presence of franchises does not drive our results, and Figure A.1 documents that the trade of firms is widespread across all sectors of the economy. Finally, using different waves of the SCF, we compute the share of traded firms over time. Figure A.4 shows that the share of entrepreneurs that purchased their business has declined in the last three decades but has been fairly stable since 2007.

[4]We focus on the entrepreneurs with a positive number of employees to exclude the cases of self-employed individuals whose businesses might not be transferable. In Appendix A.2, we present the results considering all entrepreneurs (with employer and non-employer firms).

[5]In this line, Appendix A.3.3 documents that private firms' ownership and management are highly concentrated (Figure A.5), even for the economy's oldest and largest privately held firms (Figure A.6).

[6]Specifically, the SBO asks: "*How did [the owner] initially acquire ownership of this business?*". Similarly, the SCF asks business owners: "*How did you first acquire this business?.*"

Table 1: Share of Entrepreneurs by Business Acquisition

	Founded	Purchased	Inherited/Other
SBO	65.2%	25.5%	9.3%
SCF	65.3%	22.7%	12.0%

Source: SBO and SCF for the year 2007.
Notes: Entrepreneurs are defined as (i) self-employed, (ii) business owners, who (iii) actively manage their firm, and (iv) the firm has at least one employee. Other type of acquisition groups: acquired as a transfer, as a gift or other not specified.

Firms' Trade Rate The previous results refer to the *stock* of firms that have been traded at any point in the past. We are also interested in the annual frequency of trade, i.e., the trade *rate*. We estimate the percentage of firms traded every year using two strategies. The first strategy looks at the percentage of firms purchased in the SBO and SCF data in the same year of the survey. The second strategy, as Appendix A.4 describes, uses the law of motion of the stock of traded firms as a function of firms' entry, exit, and trade rates. Either strategy implies that around 3% of the firms are traded every year. As mentioned above, this suggests that private firms are highly illiquid assets as they trade less frequently than housing but more frequently than, for example, patents.

2.4 Buyers' Characteristics

Buyers' Previous Occupation Using the SBO, we can obtain information regarding entrepreneurs' previous occupations. We found that 66% of the entrepreneurs who purchased their firm have never been self-employed. Hence, most likely, these individuals were in the labor market before acquiring their businesses. This result indicates that buying an existing firm is a relevant channel for entering into entrepreneurship, which, to the best of our knowledge, has not been studied before. Besides capturing the illiquidity of private firms, our theoretical framework will incorporate this novel feature about households' possible transitions into entrepreneurship through the market for firms.[7] Table A.7, in the appendix, shows that this result is robust to alternative samples and definitions, and Table A.8 shows that this number is similar across firms' age and size distributions.

Buyers' Wealth Using the SCF, we can identify the entrepreneurs that recently purchased their businesses and measure their wealth. Table C.1, in the Appendix, reports the wealth of the average buyer relative to the wealth of the average household and the wealth of the average entrepreneur. We consider two definitions of wealth, with and without business wealth. The average buyer is 2.7 times wealthier than the average household and 0.8 times relative to the average entrepreneur, excluding business wealth. Considering total wealth, these numbers are 3.8 and 0.7, respectively. Thus, business buyers are con-

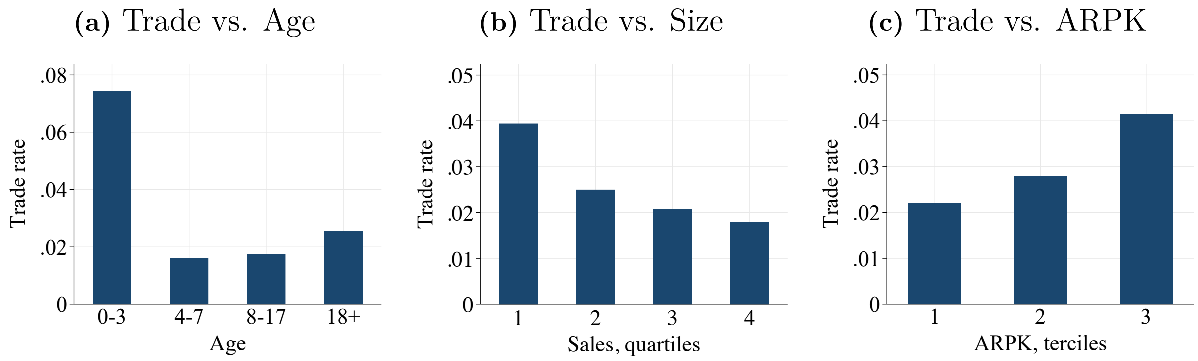
[7] We consider the question in the SBO: "*Prior to acquiring this business, had the owner ever owned a business or been self-employed?*" This number should be interpreted as a lower bound of our non-entrepreneur definition (i.e., the complement of being an entrepreneur).

siderably wealthier than the average household but less wealthy than other entrepreneurs. In Section 6, we compare this evidence with the characteristics of buyers in our model and discuss how the fact that business buyers tend to be wealthy is consistent with our theory of financial frictions being an important driver behind the trade of firms.

2.5 Trade Rate and Firms' Characteristics

In this last section of our motivating empirical results, we document cross-sectional evidence for trade frequency conditional on firms' observable characteristics. We focus on three attributes: firms' age, size, and the average revenue product of capital (ARPK).

Figure 1: Trade Rate by Firms' Characteristics



Source: SBO and KFS.
Notes: Panels (a) and (b) use data from the 2007 SBO, and panel (c) uses data from the KFS. In panel (a), trade is computed using the fraction of owners that acquired their firm through a purchase in 2007. In panel (b), trade is computed using information from firms sold in or after 2007. Size is measured using firms' sales. Panel (c) uses data from KFS. Trade is computed using the firms sold during the years of the sample. We compute this every year and then take the average across time. Average revenue product of capital (ARPK) is measured by sales over capital of the previous year to the sale. Trade rates are normalized to match the aggregate of our baseline calculations.

Firms' Age We measure firms' age using data from the SBO. Specifically, we look at all the businesses purchased in 2007 and compute firms' age as the difference between the year the firm was purchased (2007) and the year when the firm was founded. Panel (a) of Figure 1 presents the trade rate across different age bins. The figure shows that the youngest firms (0-3 years old) have the highest trading rates, with a trading frequency almost three times larger than the other age groups. However, the relation is relatively flat or slightly increasing for the oldest firms. [8]

Firms' Size We use the SBO to study the relation between trade and size, but now we focus on firms that were sold to measure their size before the exchange occurs. For this, we look at the sample of business owners that sold their firm in or after 2007 and measure size using data from the previous year of operation. Thus, we relate the trade probability at t against the firm's size at $t-1$. We measure the firm size using the firms' total sales. Panel (b) of Figure 1 presents the probability of trade for different

[8] Figure A.7, in the appendix, presents the trade rate and the share of transactions by sellers' age.

quintiles of the size distribution. We find that the frequency of trade and firm size are negatively related. Thus, the smaller firms, measure by sales' bottom quartile, have the highest trading probabilities.[9] Figure A.3 shows that the results are very similar when we measure firm size using total payroll.

Firms' ARPK Finally, we document the relation between the trade rate and firms' ARPK. We measure ARPK using data from the KFS, which includes information about firms' balance sheets that allow us to compute a firm-level measure of capital. As the analysis for size, we relate firms' ARPK at period $t-1$ against the probability of trade at t, which we measure as the share of owners that report having sold or merged their business. Panel (c) of Figure 1 shows a positive relation between the frequency of trade and ARPK, with the top terciles ARPK firms having the highest trading rates.

In sum, we document that younger, smaller, and higher ARPK firms have the highest trading rates. These results regarding firms' observable characteristics and trade frequency are informative about the underlying mechanisms behind firms' trade. In this sense, any theory about the trade of firms should be able to accommodate these relations. By introducing financial frictions as a micro foundation that endogenously generates gains from trading firms, the model we now describe can account for these cross-sectional facts.

3 A Model of Entrepreneurship and Trade of Firms

In this section we develop a general equilibrium heterogeneous agent model with four key elements: endogenous occupational choice between entrepreneurship and labor, uninsurable income risk for workers and entrepreneurs, firm-level credit frictions, and a frictional market in which firms can be bought and sold.

3.1 Environment

Our model economy is inhabited by a continuum of households in $[0,1]$. Households can have two possible occupations: *firm owners* or *workers*. Firm owners can buy and sell firms and choose whether to operate their current firm and be *entrepreneurs* or close the firm and become workers. Workers can become firm owners by acquiring a firm or through some exogenous *startup* shock. We explain the transitions between these two occupations in further detail below.

Besides the firms owned and managed by individual households, which we call *private firms*, there is a second sector of production that features a representative *public firm*. Both sectors produce the same good, which can be used for consumption or savings. Capital is produced by a *financial intermediary* which, each period, takes savings from households and rents capital to the firms. All households own the public firm and the financial intermediary in equal shares.[10]

[9] Table A.9 shows that the largest firms in the SBO (e.g., the top 1% of firms) are more likely to have been traded in the past, suggesting that firms tend to be small when traded but grow significantly afterward. This evidence is consistent with the post-trade firm dynamics we document in Section 6.3.

[10] Alternatively, we could have assumed that the intermediary and the public firm issue equity shares,

Time is discrete and infinite, and each time period is divided into two stages. The trade of firms occurs in the first stage, which we call the decentralized market, or *DM*. We assume that, in the market for firms, households meet bilaterally subject to *search frictions*, which may restrain the frequency and the type of the matches. All production, consumption, and saving decisions take place in the second stage, which we call the centralized market, or *CM*.

3.1.1 Households

Households have preferences over consumption c represented by a constant relative risk aversion (CRRA) utility function

$$u(c_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma}$$

where σ is the risk aversion coefficient.

They are heterogeneous in their occupation and their asset holdings a_{it}. Assets are subject to a non-borrowing constraint, $a_{it} \geq 0$, and are deposited with the financial intermediary, which pays a risk-free interest rate of r for the deposits. There is no aggregate uncertainty in this economy. However, households face idiosyncratic uninsurable risks.

Firm owners are endowed with a private firm that enables the owner to produce the final consumption good with a technology that uses capital, labor, and the firm's quality. We describe this technology below. The quality of the firm, denoted by z_{it}, is stochastic and evolves according to the law of motion

$$z_{it+1} = \begin{cases} z_{it} & \text{with pr. } \gamma \\ z' \sim \mathcal{P}(z_{min}, \eta_z) & \text{with pr. } (1-\gamma) \end{cases}$$

where $\mathcal{P}$ denotes a Pareto distribution with scale and a shape parameters z_{min} and η_z, respectively. The $(1-\gamma)$ shock can be interpreted as changes in market conditions that affect the profitability of entrepreneurial projects as in Buera, Kaboski, and Shin (2011).

On the other hand, workers are endowed with one unit of labor, which they supply inelastically, and are heterogeneous in their labor efficiency ε_{it}. We assume that the logarithm of the labor market efficiency evolves according to an AR(1) process with persistence ρ_ε and volatility σ_ε. Specifically,

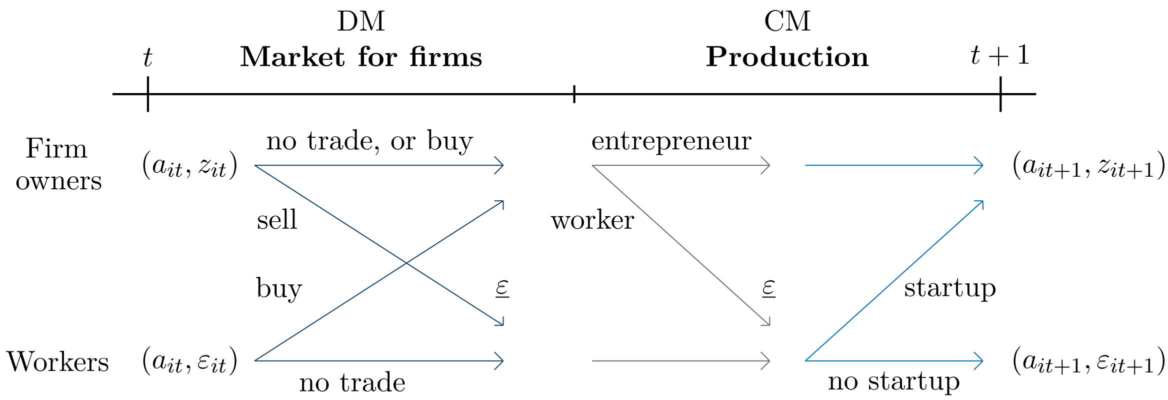
$$\log \varepsilon_{it+1} = \rho_\varepsilon \log \varepsilon_{it} + \sigma_\varepsilon u_{it+1},$$

where u is a standard normal random variable.

Regarding the transitions between occupations, workers can become firm owners by purchasing an existing firm or through an exogenous startup shock at the end of the pe-

which are traded between households in a frictionless centralized market. This setup is analogous, as assets and shares holdings would be indeterminate. Below we assume that the intermediary and the public firm make zero profits. Thus, this modeling choice is not crucial for the analysis.

Figure 2: Transitions Between Occupations



riod. At the beginning of the production stage, firm owners face an occupational choice. They decide whether to operate their firm or shut down the firm and become workers. Upon exit or upon selling, previous firm owners lose the value of their firm and enter the labor market with the lowest labor market efficiency $\underline{\varepsilon}$.[11] We interpret this low entry value as potential costs associated with entrepreneurship, such as lack of experience in the labor market.[12] Figure 2 presents a graphical description of the transitions between occupations.

In this setup, the budget constraint of an entrepreneur, defined as a firm owner that decides to operate, with states (a_{it}, z_{it}) is given by

$$c_{it} = \pi(a_{it}, z_{it}) + (1+r)a_{it} - a_{it+1} + \Pi^p + \Pi^f,$$

and the budget constraint of a worker with states $(a_{it}, \varepsilon_{it})$ is

$$c_{it} = \varepsilon_{it} w + (1+r)a_{it} - a_{it+1} + \Pi^p + \Pi^f,$$

where π are the profits of the entrepreneur's private firm, w is the labor market wage, Π^p and Π^f are the public firm's and the financial intermediary's profits, respectively.

3.1.2 Private Firms

Private firms are endowed with a technology that uses capital k_{it}, labor l_{it}, and the quality of the *entrepreneurial project* z_{it} to produce the final good according to

$$y_{it} = z_{it} k_{it}^{\theta} l_{it}^{\nu}$$

where $\theta + \nu < 1$. The decreasing returns to scale assumption implies that all private firms have an optimal operation scale as in Lucas (1978).

[11] Although the distribution of ε is bounded below by 0, in our numerical solution we take $\underline{\varepsilon}$ to be the lowest value on the ε grid, which is a positive number.

[12] There is also a technical reason behind our assumption that owners that exit go to the labor market with $\underline{\varepsilon}$. If this wasn't the case, and hence suppose they get a value $\tilde{\varepsilon}$, workers with $\varepsilon < \tilde{\varepsilon}$ would have the incentive to buy a low-quality firm and then immediately exit to improve their labor efficiency.

Private firms rent capital and hire workers every period. Hence, they are characterized only by the quality of z_{it}. Private firms are indivisible, rival, and excludable. These features are an important distinction between our model of the trade of firms and the literature that studied the trade of ideas (Silveira and Wright, 2010; Akcigit, Celik, and Greenwood, 2016).[13] Different values of z_{it} aim to capture differences in firms' intangible assets. For example, trademarks, patents, processes, permits, or customer bases.

We assume that entrepreneurs are subject to *financial frictions*, which may prevent the firm from producing at their optimal scale. Specifically, we assume a collateral constraint that limits the firm's debt-borrowing capacity to a multiple of the owner's assets, parameterized by λ. This constraint implies that firms' leverage, or debt to capital ratio, satisfies $(k_{it} - a_{it})/k_{it} \leq (\lambda - 1)/\lambda$.[14]

Given these assumptions, the profit maximization problem of an entrepreneur with assets a_{it} and a firm of quality z_{it} is given by

$$\begin{aligned} \pi(a_{it}, z_{it}) = \max_{k_{it}, l_{it}} \; & y_{it} - Rk_{it} - wl_{it} \\ \text{s.t.} \quad & y_{it} = z_{it} k_{it}^{\theta} l_{it}^{\nu} \\ & k_{it} \leq \lambda a_{it} \end{aligned} \tag{1}$$

where R is the capital rental rate. If the collateral constraint binds ($k_{it} = \lambda a_{it}$), the firm operates at a lower scale compared to the unconstrained profit maximization level.[15]

3.1.3 Public Firm

As in Cagetti and De Nardi (2006), we assume that there is a second sector of production populated by a representative public firm. This aims to capture that, in the U.S. economy, around half of the total output is produced by publicly traded firms.

We assume that the public firm is owned by all households, in equal shares, and faces no financial frictions. The public firm is endowed with a constant return to scale technology

$$Y_{pt} = K_{pt}^{\eta} L_{pt}^{1-\eta}$$

where K_{pt} is the public firm's capital, L_{pt} its labor, and Y_{ct} its total output.

3.1.4 Financial Intermediary

The financial intermediary takes deposits from households and rents capital to the firms at a price equal to the savings interest rate plus the depreciation rate: $R = r + \delta$. We assume that the representative intermediary operates in a perfectly competitive market

[13]By definition, ideas are non-rival. However, ideas might be excludable under certain institutional arrangements such as patents.

[14]This type of constraint can be micro-founded with imperfect enforcement of contracts problem. Consistent with most debt financing contracts, we assume that the firm cannot pledge the quality of the entrepreneurial project as collateral.

[15]Appendix C.1 presents firms' input demand functions that characterize the static solution of (1).

and breaks even (i.e., makes zero profits). The resource constraint of the intermediary is

$$K_{pt} + \int k(a_{it}, z_{it}) \; \mathrm{d}N^e_{cm}(a_{it}, z_{it}) = \int a_{it} \; \mathrm{d}N^e_{cm}(a_{it}, z_{it}) + \int a_{it} \; \mathrm{d}N^w_{cm}(a_{it}, \varepsilon_{it}) \tag{2}$$

where N^e_{cm} and N^w_{cm} are cumulative distribution functions for entrepreneurs and workers, which are normalized such that $\int \mathrm{d}N^e_{cm} + \int \mathrm{d}N^w_{cm} = 1$. These measures correspond to the production stage after firm owners decide whether to be entrepreneurs or workers.

3.2 A Market for Firms

Firms are hard to evaluate and price. This precludes the existence of a centralized market with a complete price schedule for different types of firms. Therefore, we model the market for firms using a search-theoretic approach characterized by bilateral random matching and *quid pro quo* trade. An interpretation of this setup is that agents can evaluate only one firm at a time, which delays trade.

Trade in the market for firms consists of the transfer of both the firm's ownership and management in exchange for assets. Hence, the media of exchange in these transactions are the households' savings a. As we assumed that firms are indivisible, when a buyer and a seller meet they only bargain over the selling price p.

Bilateral Meetings There are two types of meetings in the market for firms: *owner-owner* meetings and *owner-worker* meetings. We allow for different search frictions in each type of meeting. For a firm owner, an owner-owner meeting happens with probability α_o and an owner-worker meeting happens with probability α_w. For a worker, an owner-worker meeting happens with probability α_w.

Note that firm owners are the only potential sellers, while both types of households can be buyers. This implies that in an owner-worker match, the owner is the potential seller, and the worker is the potential buyer. However, in the case of an owner-owner match, who is the buyer and who is the seller depends on the relative quality of the firms.

Let us first consider the owner-owner match and suppose that $z_{it} < z_{jt}$. Then, owner i with states $\mathbf{s}^o_{it} \equiv (a_{it}, z_{it})$ is the potential buyer, and owner j with states $\mathbf{s}^o_{jt} \equiv (a_{jt}, z_{jt})$ is the potential seller. This follows from the assumption that households can own only one firm at a time. Hence, no owner would buy a lower-quality firm. In this case, the total surplus from trading the ownership of firm z_{jt}, in exchange for p assets, is given by

$$\text{Total surplus} \equiv \underbrace{W^o(a_{it} - p, z_{jt}) - W^o(\mathbf{s}^o_{it})}_{\text{Buyer's surplus, } S_b} + \underbrace{W^w(a_{jt} + p, \underline{\varepsilon}) + T_{jt}(p) - W^o(\mathbf{s}^o_{jt})}_{\text{Seller's surplus, } S_s} \tag{3}$$

where W^o and W^w are the value functions at the beginning of the production stage for firm owners and workers, respectively. As described below, T_{jt} is a utility transfer that sellers might receive that captures additional motives to trade firms. Upon selling, the household goes to the labor market with labor efficiency $\underline{\varepsilon}$, as presented in the first term of the seller's surplus.[16] The outside option for both agents (the terms with a minus in the surpluses) is the value of going to the production stage as firm owners with their initial states $\mathbf{s}^o_{it}$ and $\mathbf{s}^o_{jt}$, respectively.

Regarding the owner-worker match, suppose that firm's owner j with states $\mathbf{s}^o_{jt}$ meets with a worker i with states $\mathbf{s}^w_{it} \equiv (a_{it}, \varepsilon_{it})$. Then, the total surplus from trading firm z_{jt} is

$$\text{Total surplus} \equiv \underbrace{W^o(a_{it} - p, z_{jt}) - W^w(\mathbf{s}^w_{it})}_{\text{Buyer's surplus, } S_b} + \underbrace{W^w(a_{jt} + p, \underline{\varepsilon}) + T_{jt}(p) - W^o(\mathbf{s}^o_{jt})}_{\text{Seller's surplus, } S_s} \tag{4}$$

where the only difference relative to the previous match is the buyer's outside option. In this case, if the parties don't trade, the buyer would continue to the production stage as a worker with its initial state $\mathbf{s}^w_{it}$.

Alternative Motives Besides the purely financial reasons to trade firms studied in this paper, related to households' wealth, access to credit, and risk aversion, there could be other motives for why entrepreneurs sell their firms.[17] To account for these alternative motives to trade firms in a parsimonious manner, we assume that potential firms' sellers receive a *preference* shock κ_{jt} that captures additional benefits, or a reduction in the opportunity cost, of selling their firm in the current period. The preference shock follows

$$\kappa_{jt} = \underline{\kappa} + (\overline{\kappa} - \underline{\kappa})\xi_{jt}$$

where $1 \leq \underline{\kappa} < \overline{\kappa}$, and the random variable ξ_{jt} is *iid* across time and firms and drawn from a Beta distribution with $\mathcal{B}(1, \beta_\kappa)$.[18]

The shock κ_{jt}, with domain in $[\underline{\kappa}, \overline{\kappa}]$, determines the additional utility transfer that the seller receives upon selling compared to the trading for a higher price $\kappa_{jt}p \geq p$ but no extra utility. Thus, for each potential seller j with states $\mathbf{s}^o_{jt}$, preference shock κ_{jt}, and price p, the utility transfer $T_{jt}(p) \equiv T(p; \mathbf{s}^o_{jt}, \kappa_{jt})$ is implicitly defined by

$$W^w(a_{jt} + \kappa_{jt}p, \underline{\varepsilon}) = W^w(a_{jt} + p, \underline{\varepsilon}) + T_{jt}(p) \tag{5}$$

which states that the seller is indifferent between selling at a higher price $\kappa_{jt}p$ with no transfer and the case with price p and receiving $T_{jt}(p)$. Hence, this utility transfer is similar in spirit to the classical Hicksian compensation. Intuitively, this transfer implies that the owner is willing to sell the firm at a $1 - \kappa_{jt}^{-1}$ discount, relative to the full price $\kappa_{jt}p$. Thus, all else equal, higher values of κ_{jt} will make sellers willing to sell their firms at larger discounts and lower prices.

[16] If z is very low, some firm owners might even want to pay someone to buy their firm, implying $p < 0$, to transition into the labor market. The free exit assumption, through which firm owners can decide to exit and get the same labor efficiency $\underline{\varepsilon}$, rules out the possibility of negative prices in our model.

[17] Examples of these alternative motives to trade include personal preferences (e.g., the non-monetary value of being self-employed) or related to owners' life cycle (e.g., health shocks or retirement).

[18] We denote the CDF of κ as $\Psi(\kappa)$, which is implicitly defined by the distribution of ξ.

Sufficient Condition for Trade Let $\underline{p}_{jt} \equiv \underline{p}(\mathbf{s}^o_{jt}, \kappa_{jt})$ denote the minimum price at which seller j is willing to sell its firm, i.e., the price at which the seller's surplus is equal to zero. Likewise, let $\overline{p}_{it} \equiv \overline{p}(\mathbf{s}_{it}, z_{jt})$ be the maximum price that buyer i is willing to pay for firm j, i.e., the price at which the buyer's surplus is equal to zero. A sufficient condition for trade, meaning that there are positive gains from trading firm j, is that

$$\underline{p}_{jt} \leq \overline{p}_{it} \tag{6}$$

where the states of buyer i are $\mathbf{s}_{it} \in \{\mathbf{s}^o_{it}, \mathbf{s}^w_{it}\}$, depending on the type of match (owner-owner or owner-worker, respectively). For a given meeting, condition (6) shows that the possibility of trade is a function of the firms' potential sellers' and buyers' characteristics. In Section 5 we characterize, using the quantitative model, the probability of buying and selling the firm across agents' characteristics.

Bargaining If there are positive gains from trade, we assume that the price is determined by a *Nash bargaining* protocol. Thus, the trading price p between buyer i with states $\mathbf{s}_{it} \in \{\mathbf{s}^o_{it}, \mathbf{s}^w_{it}\}$, and seller j with states $\mathbf{s}^o_{jt}$ and preference shock κ_{jt} solves

$$\begin{aligned} p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt}) =& \arg\max_p \Big[S_b(\mathbf{s}_{it}, z_{jt}, p)\Big]^\chi \Big[S_s(\mathbf{s}^o_{jt}, \kappa_{jt}, p)\Big]^{1-\chi} \\ & \text{s.t.} \quad S_b(\mathbf{s}_{it}, z_{jt}, p) \geq 0, \;\; S_s(\mathbf{s}^o_{jt}, \kappa_{jt}, p) \geq 0 \end{aligned} \tag{7}$$

where S_b and S_s are the buyer and seller surpluses, defined in (3) and (4), and $0 \leq \chi \leq 1$ parameterizes buyers' bargaining power. Thus, if χ is near 0, the price will be close to the buyer's maximum price $\overline{p}_{it}$. Conversely, if χ is near 1, the price will be close to the seller's minimum price $\underline{p}_{jt}$. As we will explain in Section 4, information about the ratio of selling prices to firms' profits helps us identify this parameter.

3.3 Timing

The timing of the model can be summarized as follows:

1. The startup shocks, the quality of entrepreneurial projects z, and the labor efficiencies ε are realized.
2. Agents enter the market for firms (DM). Firm owners can buy and sell firms, while workers can only buy. Preference shocks κ are realized for potential sellers.
3. Agents enter the production stage (CM). Given prices and their current z, firm owners decide whether to operate the firm or go to the labor market. Finally, production occurs, and agents choose how much to consume and save.

3.4 Recursive Formulation

We now present the recursive problem of firm owners and workers. First, we describe the value functions at the beginning of the market for firms (the DM subperiod), which we denote by V. Second, we present the value functions at the production stage (the CM subperiod), which we denote by W.

3.4.1 Value at the Market for Firms (DM)

Firm owners have four potential outcomes upon entering the market for firms: (1) don't trade, (2) buy another firm, (3) sell their firm to another owner, and (4) sell their firm to a worker. The no-trade case could arise because the owner did not match with a counterpart or because there was a match, but it did not end with a trade.

The value of a firm owner with states (a_{it}, z_{it}) at the beginning of DM is equal to

$$
\begin{aligned}
V^o(a_{it}, z_{it}) = \mathbb{E}_{\kappa_{it}} \Bigg[& \underbrace{\Pr^o[\text{no trade} \mid a_{it}, z_{it}, \kappa_{it}]\; W^o(a_{it}, z_{it})}_{\text{no trade}} \\
& + \underbrace{\alpha_o \int \int_{z_{it}<z_{jt}, \overline{p}_{it}>\underline{p}_{jt}} W^o(a_{it} - p, z_{jt})\, \mathrm{d}N^o_{dm}(a_{jt}, z_{jt})\, \mathrm{d}\Psi(\kappa_{jt})}_{\text{buy}} \\
& + \underbrace{\alpha_o \int_{z_{it}>z_{jt}, \underline{p}_{it}<\overline{p}_{jt}} [W^w(a_{it} + p, \underline{\varepsilon}) + T_{it}(p)]\, \mathrm{d}N^o_{dm}(a_{jt}, z_{jt})}_{\text{sell to a firm owner}} \\
& + \underbrace{\alpha_w \int_{\underline{p}_{it}<\overline{p}_{jt}} [W^w(a_{it} + p, \underline{\varepsilon}) + T_{it}(p)]\, \mathrm{d}N^w_{dm}(a_{jt}, \varepsilon_{jt})}_{\text{sell to a worker}} \Bigg],
\end{aligned} \tag{8}
$$

where α_o and α_w are exogenous matching probabilities conditional on each match type.[19] These parameters, in $[0, 1]$, govern the degree of search frictions in the market for firms. N^o_{dm} and N^w_{dm} are cumulative distributions for firm owners and workers at the beginning of DM, which satisfy that $\int \mathrm{d}N^o_{dm} + \int \mathrm{d}N^w_{dm} = 1$.

As mentioned in Section 3.2, for the case of owner-owner meetings, who buys and sells depends on the relative firm qualities. Hence, an owner with firm quality z_{it} might buy if it is matched with another owner with a firm of higher quality ($z_{it} < z_{jt}$), as denoted in the integral in the second line of (8). On the contrary, the owner might sell if it is matched with another owner with a firm of lower quality ($z_{it} > z_{jt}$) as denoted in the integral of the third line.[20] Note that the integrals for the buying and selling cases consider only the meetings that result in a trade, which occurs when the seller's minimum price is lower than the buyer's maximum price, as stated in (6). The preference shocks κ, will be relevant in determining these prices.

Workers only have two potential outcomes: (1) don't trade, or (2) buy an existing firm.

[19] In more detail, the probabilities of the bilateral meetings in (8) can be derived as follows. First, note that there is a mass $\int \mathrm{d}N^o_{dm}$ of owners at the beginning of DM. This implies that two owners are matched with probability $\int \mathrm{d}N^o_{dm}$. Due to the search friction, conditional on the match, these owners meet with probability α_o. Thus, the probability of an owner-owner meeting is equal to $\alpha_o \int \mathrm{d}N^o_{dm}$. Similarly, the probability that the owner matches with a worker is equal to $\int \mathrm{d}N^w_{dm} = 1 - \int \mathrm{d}N^o_{dm}$, and conditional on the match they meet with probability α_w. Hence, the probability of an owner-worker meeting is equal to $\alpha_w \int \mathrm{d}N^w_{dm}$. Finally, note that the no-trade probability $\Pr^o[\text{ no trade} \mid a, z\ ]$ sums up the probability of no meetings plus the probability of meetings that do not result in a trade as $\underline{p} < \overline{p}$ is not satisfied.

[20] Here, we assume that meetings in which owners have the same firm quality do not result in a trade.

Hence, the value of a worker with states $(a_{it}, \varepsilon_{it})$ at the beginning of DM is given by

$$V^w(a_{it}, \varepsilon_{it}) = \underbrace{\Pr{}^w[\text{no trade} \mid a_{it}, \varepsilon_{it}]\ W^w(a_{it}, \varepsilon_{it})}_{\text{no trade}}$$
$$+ \underbrace{\alpha_w \int\int_{\overline{p}_{it} > \underline{p}_{jt}} W^o(a_{it} - p, z_{jt})\, \mathrm{d}N^o_{dm}(a_{jt}, z_{jt})\, \mathrm{d}\Psi(\kappa_{jt})}_{\text{buy}}. \tag{9}$$

3.4.2 Value at the Production Stage (CM)

As previously described, firm owners face an occupational choice at the beginning of the production stage. They have to decide whether to operate the firm and be entrepreneurs or shut down and go to the labor market with labor productivity $\underline{\varepsilon}$. Given these assumptions, the value of firm owners at the beginning of CM is

$$W^o(a_{it}, z_{it}) = \max_e \{W^e(a_{it}, z_{it}), W^w(a_{it}, \underline{\varepsilon})\} \tag{10}$$

where e denotes the owners' occupational choice.

The value function of entrepreneurs is given by

$$\begin{aligned} W^e(a_{it}, z_{it}) = \max_{a_{it+1}, c_{it}}\ & u(c_{it})\ +\ \beta\left\{\gamma V^o(a_{it+1}, z_{it}) + (1-\gamma)\mathbb{E}_{z_{it+1}}[V^o(a_{it+1}, z_{it+1})]\right\} \\ \text{s.t.}\quad & c_{it} = \pi(a_{it}, z_{it}) + (1+r)a_{it} - a_{it+1} \\ & c_{it} \geq 0,\ a_{it+1} \geq 0 \end{aligned} \tag{11}$$

and the value function of workers by

$$\begin{aligned} W^w(a_{it}, \varepsilon_{it}) = \max_{a_{it+1}, c_{it}}\ & u(c_{it}) + \beta\left\{\zeta \mathbb{E}_{\varepsilon_{it+1}|\varepsilon_{it}}[V^w(a_{it+1}, \varepsilon_{it+1})] + (1-\zeta)\mathbb{E}_{z_{it+1}}[V^o(a_{it+1}, z_{it+1})]\right\} \\ \text{s.t.}\quad & c_{it} = \varepsilon_{it} w + (1+r)a_{it} - a_{it+1} \\ & c_{it} \geq 0,\ a_{it+1} \geq 0 \end{aligned} \tag{12}$$

where $(1-\zeta)$ is the probability of the exogenous startup shock through which a worker can become a firm owner.[21]

3.5 Competitive Equilibrium

A *competitive stationary equilibrium* in this economy consists of: (i) aggregate prices $\{r, R, w\}$; (ii) terms of trade in the market for firms given by the price functions of seller j and buyer-owner i meetings $\{p\left(\mathbf{s}^o_{it}, \mathbf{s}^o_{jt}, \kappa_{jt}\right), \underline{p}\left(\mathbf{s}^o_j, \kappa_j\right), \bar{p}\left(\mathbf{s}^o_{it}, z_{jt}\right)\}$, and the price functions of seller j and buyer-worker i meetings $\{p\left(\mathbf{s}^w_i, \mathbf{s}^o_j, \kappa_j\right), \underline{p}\left(\mathbf{s}^o_j, \kappa_j\right), \bar{p}\left(\mathbf{s}^w_{it}, z_{jt}\right)\}$; (iii) firm owners' occupational choice decisions $e(a_{it}, z_{it})$; (iv) consumption and savings decisions for entrepreneurs $\{c(a_{it}, z_{it}), a'(a_{it}, z_{it})\}$ and for workers $\{c(a_{it}, \varepsilon_{it}), a'(a_{it}, \varepsilon_{it})\}$; (v) capital

[21] In (11) and (12) we omit the profits of the public firm and the financial intermediary (Π^p and Π^f terms) in the households' budget constraints as both terms are equal to zero, in equilibrium.

and labor demand functions for private and public firms, $\{k(a_{it}, z_{it}), l(a_{it}, z_{it}), K_{pt}, L_{pt}\}$; and (vi) measures of agents over occupations and idiosyncratic states at DM and CM subperiods characterized by $\{N^o_{dm}(a_{it}, z_{it}), N^w_{dm}(a_{it}, \varepsilon_{it})\}$ and $\{N^e_{cm}(a_{it}, z_{it}), N^w_{cm}(a_{it}, \varepsilon_{it})\}$, respectively, such that:

1. In DM, the terms of trade in bilateral meetings solve the Nash bargaining problem.

2. In CM, given prices, households, private, and public firms solve their corresponding optimization problems.

3. Goods market clears, period by period:

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t \tag{13}$$

   where

$$\begin{aligned} Y_t &\equiv Y_{pt} + \int z_{it} k(a_{it}, z_{it})^\theta l(a_{it}, z_{it})^\nu \; \mathrm{d}N^e_{cm}(a_{it}, z_{it}) \\ C_t &\equiv \int c(a_{it}, z_{it}) \; \mathrm{d}N^e_{cm}(a_{it}, z_{it}) + \int c(a, \varepsilon) \mathrm{d}N^w_{cm}(a_{it}, \varepsilon_{it}) \\ K_t &\equiv K_{pt} + \int k(a_{it}, z_{it}) \; \mathrm{d}N^e_{cm}(a_{it}, z_{it}). \end{aligned}$$

4. Labor market clears, period by period:

$$L_{pt} + \int l(a_{it}, z_{it}) \; \mathrm{d}N^e_{cm}(a_{it}, z_{it}) = \int \varepsilon_{it} \; \mathrm{d}N^w_{cm}(a_{it}, \varepsilon_{it}). \tag{14}$$

5. The budget constraint of the financial intermediary, specified in (2), is satisfied period by period.

6. The measures over types and states satisfy

$$\begin{aligned} \int \mathrm{d}N^o_{dm}(a_{it}, z_{it}) + \int \mathrm{d}N^w_{dm}(a_{it}, \varepsilon_{it}) = 1 \\ \int \mathrm{d}N^e_{cm}(a_{it}, z_{it}) + \int \mathrm{d}N^w_{cm}(a_{it}, \varepsilon_{it}) = 1 \end{aligned}$$

   and are consistent with a recursive equilibrium mapping dictated by prices and trades in the market for firms, households' optimal choices, and the stochastic processes for firms' qualities, workers' labor efficiencies, and sellers' preferences shocks. The stationary equilibrium implies that fixed distribution over time (fixed point).

We solve for the stationary equilibrium of this model by approximating the value functions using projection methods on a finite state space for which we solve all the possible matches and trading prices, as well as agents' and firms' optimal choices. See Appendix C.2 for a detailed description of our numerical solution.

4 Parameterization

This section describes our calibration strategy. We calibrate the model, at an annual frequency, to the year 2007. We focus on 2007 as that is the year we have both the SBO and SCF data available.

4.1 Assigned Parameters

We set the relative risk aversion parameter to $\sigma = 1.5$, the capital depreciation to $\delta = 0.06$, and the public's firm capital elasticity to $\eta = 1/3$. All three are common values in the literature. Regarding the preference shock κ, we set its domain to $[1, 3]$, which implies that sellers' have a maximum possible discount of 66% $(1 - 1/\overline{\kappa})$ coming from the preference shocks. Panel (a) of Table 2 summarizes these assigned parameters.

Table 2: Parameterization

Parameter	Value	Description
		(a) *Assigned Parameters*
σ	1.5	CRRA
δ	0.06	Capital depreciation rate
η	1/3	Capital elasticity
$\underline{\kappa}$	1	Preference shock, lower bound
$\overline{\kappa}$	3	Preference shock, upper bound
		(b) *Calibrated Parameters*
β	0.898	Discount factor
Υ	0.724	Curvature private firms technology
$(\lambda - 1)/\lambda$	0.397	Collateral constraint, maximum leverage
γ	0.930	Persistence private firm value
ζ	0.939	1− Startup shock
z_{min}	1.118	Scale, z distribution
η_z	2.419	Shape, z distribution
ρ_ε	0.953	AR(1) parameter, ε distribution
σ_ε	0.240	Std. Deviation, ε distribution
$\mathbb{E}[\kappa]$	1.354	Preference shock, mean
α_o	0.803	Owner-owner | meeting probability
α_w	0.459	Owner-worker | meeting probability
χ	0.436	Buyers' bargaining power

4.2 Calibrated Parameters and Targeted Moments

We calibrate the remaining parameters such that the model replicates several key features of the U.S. economy, focusing on the trade of private firms. To reduce the parameter space dimension, we assume private firms' technology has the same relative elasticity between capital and labor as public firms. In such a way, a single parameter $\Upsilon < 1$ captures the degree of decreasing returns to scale in private firms' technology by setting $\theta = \eta\Upsilon$ and $\nu = (1 - \eta)\Upsilon$. After this, we have thirteen parameters, which we calibrate to match seventeen moments. Panel (b) of Table 2 presents these parameters with their calibrated values. We find those values by minimizing the distance between moments in the data and the model. Table 3 presents the seventeen moments we target in our calibration exercise. For an easier exposition, we divide these moments into five groups, which we now describe.

First, we target moments capturing the role of entrepreneurs in the economy. As reported in the 2007 SCF, we target that 6% of households are entrepreneurs, and they earn 20% of total income and hold 33% of the economy's wealth. Our second set of moments characterizes the distribution of income and wealth across all households and within workers and entrepreneurs. We target six different Gini indexes, which we also compute from the 2007 SCF. The table shows that our model matches the dispersion of income and wealth in the data very well. However, it slightly overpredicts the level of inequality among entrepreneurs. It is worth mentioning that different from the previous literature, which has abstracted from firm prices, our definition of wealth in the model includes the value of private firms $(a + p)$, consistent with the data. The parameters most informative about the income and wealth distribution in the model are the ones characterizing the distribution of firms' quality, z_{min} and η_z, and workers' labor efficiency, ρ_ε and ε.

The third and fourth sets of moments capture relevant characteristics of firms in the US economy. First, we target a capital-output ratio of 3, which disciplines the discount factor β. Second, we target that private firms account for 50% of total output, which is consistent with the estimates in Dinlersoz et al. (2019), and lower than Asker, Farre-Mensa, and Ljungqvist (2014) which calculates that private firms account for 57% of total sales. Regarding private firms' leverage, we target our model's weighted average debt-to-capital ratio to be 0.35, consistent with private firms' leverage in the Flow of Funds Accounts. This moment pins down the collateral constraint parameter λ. We also target a firm-level exit rate of 0.09, which we computed from the Census Business Dynamics Statistics (BDS) for 2007. These moments are especially important to discipline the decreasing return to scale Υ and the parameters γ and ζ.

Our fifth and final set of moments captures relevant features of the trade of private firms documented in Section 2. We target an annual trade rate of 3% and that workers purchase 66% of the firms. These moments are relevant for the search frictions parameters, α_o and α_w. Additionally, to identify the relevance of preference shocks in firms' trade, we target the 1.7% trade rate of the largest firms, defined by firms in the top output quartile. As explained below, preference shocks are particularly relevant for the trade of large and financially unconstrained firms. From our calibration, we get that $\mathbb{E}[\kappa] = 1.354$, which implies an average discount of $\mathbb{E}[1-\kappa^{-1}]$=0.23.[22] Finally, we target a median price-to-profit ratio equal to 3.5, which we obtained from Dealstat.[23] This ratio is most informative for the buyers' Nash bargaining parameter χ. We obtain a value of $\chi = 0.442$, implying that sellers have a slightly higher bargaining power than buyers. Overall, Table 3 shows that our model does an excellent job matching the targeted moments. Especially the ones related to entrepreneurs, private firms, and the market for firms.

Table 3: Targeted Moments

	Source	Data	Model
Entrepreneurs			
Fraction of entrepreneurs	SCF	0.06	0.06
Income share of entrepreneurs	SCF	0.20	0.21
Wealth share of entrepreneurs	SCF	0.33	0.38
Income and Wealth Distribution			
Gini income, all households	SCF	0.62	0.61
Gini wealth, all households	SCF	0.82	0.83
Gini income, entrepreneurs	SCF	0.67	0.77
Gini wealth, entrepreneurs	SCF	0.74	0.81
Gini income, workers	SCF	0.58	0.56
Gini wealth, workers	SCF	0.78	0.79
Private and Public Firms			
Capital to output ratio	See text	3.0	3.0
Private Firms			
Output share	See text	0.50	0.45
Leverage	FoF	0.35	0.35
Exit rate	BDS	0.09	0.09
Trade of Private Firms			
Trade rate, all firms	SBO	0.030	0.031
Trade rate, largest firms	SBO	0.017	0.013
Share purchased by workers	SBO	0.66	0.67
Median price/profits	DealStats	3.5	3.3

Notes: Data moments correspond to the year 2007. Wealth in the model is defined as the sum of the risk-free asset and the value of the firm $a+p$. Trade rate, largest firms is the trading frequency of firms in the top quartile of the output distribution.

[22]We directly target the mean of κ, which implicitly defines the parameter β_κ. In detail, note that $\mathbb{E}[\kappa] = \underline{\kappa} + (\overline{\kappa} - \underline{\kappa})\mathbb{E}[\xi]$ and $\mathbb{E}[\xi] = \frac{1}{1+\beta_\kappa}$, which defines β_κ given $\underline{\kappa}$, $\overline{\kappa}$ and $\mathbb{E}[\kappa]$.

[23]Dealstat (formerly Pratt's Stats) is a database of business transactions. We use their publicly available reports to compute an average median selling price to EBITDA ratio of 3.5 from 2010 to 2018 in the US.

4.3 Other Untargeted Moments

A relevant feature of heterogeneous agents models with entrepreneurship is that they can replicate the income and wealth distribution observed in the data (Quadrini, 2000; Cagetti and De Nardi, 2006). This is possible thanks to the combination of uninsurable income risk and stochastic returns to wealth from entrepreneurial activity. Table C.2, in the Appendix, shows that this is also true in our model. Although we only targeted a set of Gini coefficients, the model does a good job matching the entire income and wealth distribution observed in the data.

5 Model Properties

This section describes the main properties of our model. First, we discuss and quantify the different motives behind the trade of firms. Second, we characterize who buys and who sells firms in our economy. Finally, we describe the implications of this market for firm dynamics and the allocation of capital.

5.1 Motives for Trading Firms

Exchanges in the market for firms are voluntary. Hence, a necessary condition for gains from trade is that agents have different valuations for the same firm. In particular, the buyer must have a higher valuation than the seller. In our theory, given the agents' outside options, heterogeneous valuations for firms arise from three sources: the preferences shocks, firms' credit constraints, and incomplete markets. We now describe and quantify each of these three motives behind the trade of firms.

Table 4: Trade Rate Decomposition

	All Firms	
	Trade rate	Relative
Baseline	3.1%	1.00
No preference shocks	2.1%	0.69
No collateral constraint	1.0%	0.32
No preference, no collateral	0.4%	0.13

Notes: Steady-state comparisons of the market for firms' trade rate under different parameterizations. Relative is the ratio of each trade rate to the Baseline model. No preference shocks turn off the alternative motives to trade firms by setting $\mathbb{E}[\kappa] = 1$ and $Var[\kappa] = 0$. No collateral constraint assumes $\lambda \to \infty$. No preference, no collateral considers both previous cases simultaneously.

Preference Shocks As described above, we introduce alternative motives to trade firms through sellers' κ shocks at the beginning of the market for firms. These shocks aim to capture, parsimoniously, all the motives to trade firms unrelated to the financial channels we study in this paper. To evaluate the role of these preference shocks, the second row of Table 4 presents the trade rate of firms when we turn off these alternative motives.

This comparative static exercise sets $\mathbb{E}[\kappa] = 1$ and $Var[\kappa] = 0$ while keeping the rest of the parameters fixed. Without preference shocks, the economy's annual firms' trade rate falls from 3.1% to 2.1%. This result indicates that preference shocks explain around 31% of the trades in the market for firms, while most exchanges arise from financial frictions, as we explain below. However, Figure C.1 in the appendix shows that preference shocks play a significant role in the trade of large firms, which are less likely to be financially constrained in our model.

Credit Constraints Regarding the financial motives to trade firms, we first focus on the role of credit constraints. This channel arises from the collateral constraint in the entrepreneurs' problem, presented in (1), that restricts firms' capital to a multiple λ of their owners' wealth. Consequently, whenever an entrepreneur is credit constrained, a wealthier buyer can obtain a higher profit stream out of the same firm as it would be able to operate closer to its optimal scale. Thus, credit constraints generate gains from trade between constrained business owners and wealthier buyers. To quantify the importance of this channel, we set $\lambda \to \infty$, which implies that the firms' profits stream is no longer a function of their owners' wealth. The third row of Table 4 shows that removing credit frictions significantly reduces the frequency of trades in the market for firms to 1.0%, indicating that the bulk of the transactions in our baseline economy, 68%, are driven by credit constraints. This result is in line with Figure 6, where we showed that younger, smaller, and high return to capital firms, which are the ones most likely to be credit constrained, are the ones with the higher trade rates, both in the data and in our model.

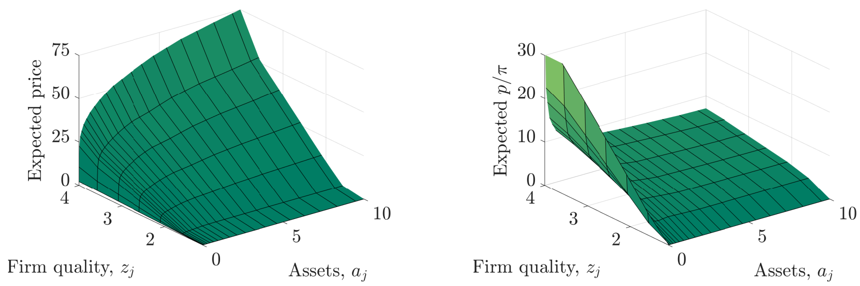
Risk and Incomplete Markets Risk aversion and incomplete financial markets constitute the third motive to trade firms. In our model, owning and operating a firm is associated with uninsurable income risk as the firm's quality z is stochastic, causing agents to have precautionary savings. Thus, even without credit constraints, agents can have different valuations for the same firm as a function of their wealth. For low-wealth owners, selling the firm allows them to front-load consumption and achieve an earlier risk resolution. For high-wealth owners, consumption is less dependent on shocks to the firm's profits. In other words, the covariance between their stochastic discount factor and the realization of profits is small, increasing their ability to bear risk. Therefore, the value of owning a firm will vary across the wealth distribution generating potential gains from trade. To evaluate the importance of this channel, we turn off both the preference shocks and firms' credit constraints. The last row of Table 4 shows that, in this case, the trade rate is 0.4%. This result suggests that risk and incomplete markets account for 13% of the firms' trades in our baseline economy.

5.2 Who Buys and Who Sells Firms?

Now we describe the typical buyers and sellers in the market for firms. We start our characterization by analyzing the prices at which firms trade. Panel (a) of Figure 3 presents the expected price $p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt})$ resulting from the Nash bargaining protocol in the sellers' state space (a_j, z_j), after integrating over the preference shock κ_j and all potential buyers $\mathbf{s}_{it}$. As expected, selling prices are increasing firm quality z_j. However, due to the collateral constraint on firm owners' wealth and incomplete markets, holding the firm's quality fixed, the price is increasing in the owners' assets a_j. Note that firm prices would be unrelated to the current owner's wealth under perfect credit markets. Thus, due to imperfect credit markets, high-quality firm owners with low wealth will be willing to sell their firms at a relatively low price as it will take them a long time, and high saving rates, to grow out of their borrowing constraint through self-financing. Nevertheless, as panel (b) of Figure 3 shows, these transactions have considerably high price-over-profit ratios, which illustrates the small scale of operation of constrained and low-wealth owners with high-quality firms relative to the price at which they can sell their business.[24]

Figure 3: Prices in the Market for Firms

(a) Expected price $\mathbb{E}_{\mathbf{s}_{it},\kappa_{jt}}[p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt})]$ **(b)** Expected price/profits $\mathbb{E}_{\mathbf{s}_{it},\kappa_{jt}}[p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt})/\pi(\mathbf{s}^o_{jt})]$



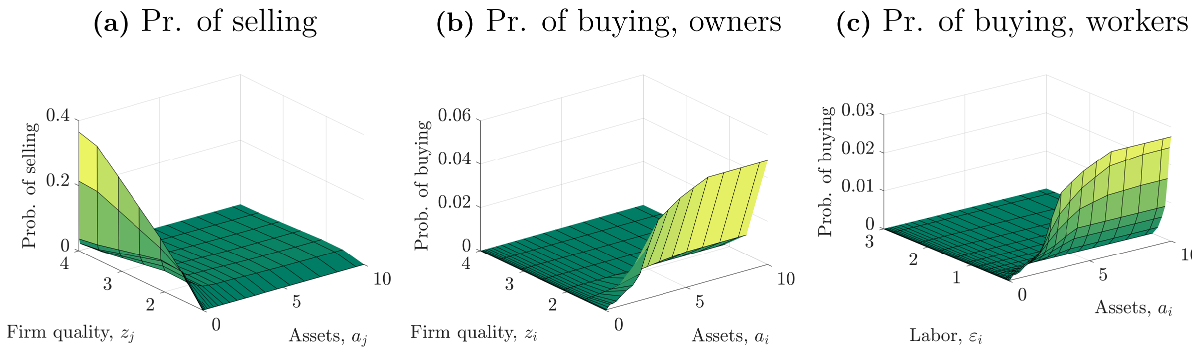
Notes: Expected price, and price over profits, in owners' state space, $\mathbf{s}^o_{jt} = (a_{jt}, z_{jt})$, after integrating over preference shocks, κ_{jt}, and trading counterparts, $\mathbf{s}_{it} \in \{\mathbf{s}^o_{it}, \mathbf{s}^w_{it}\}$.

Considering how trading prices are determined, we characterize who buys and sells firms in our economy. Panel (a) of Figure 4 presents the probability that a firm owner sells its firm in the (a_j, z_j) space. The figure shows that owners with low wealth and high-quality firms have the highest probability of selling. In those cases, there will be high gains from trade as the current owner lacks the assets to operate at the optimal scale. Panels (b) and (c) present the probability of buying a firm for firm owners in the (a_i, z_i) space and for workers in the (a_i, ε_i). These panels show that the probability of buying is the opposite mirror image of the likelihood of selling. Thus, firms' buyers are mostly wealthy households that currently own low-quality firms (low z) or wealthy workers with low labor efficiency (low ε). In Section 6.2, we show that our model's prediction that business buyers are wealthier than the average household is quantitatively consistent with the data.

Overall, the panels in Figure 4 show that the typical seller in our economy will be firm owners with high-quality firms but low wealth, and the typical buyers will be wealthy agents with relatively low-quality firms or low labor efficiency. Thus, as we show and

[24] According to Dealstat, the median price-over-profit ratio in the Information sector equals 9, considerably higher than the economy-wide 3.5 number. This evidence is consistent with our model's large price-over-profit ratios for high-growth potential firms.

Figure 4: Buyers and Sellers in the Market for Firms



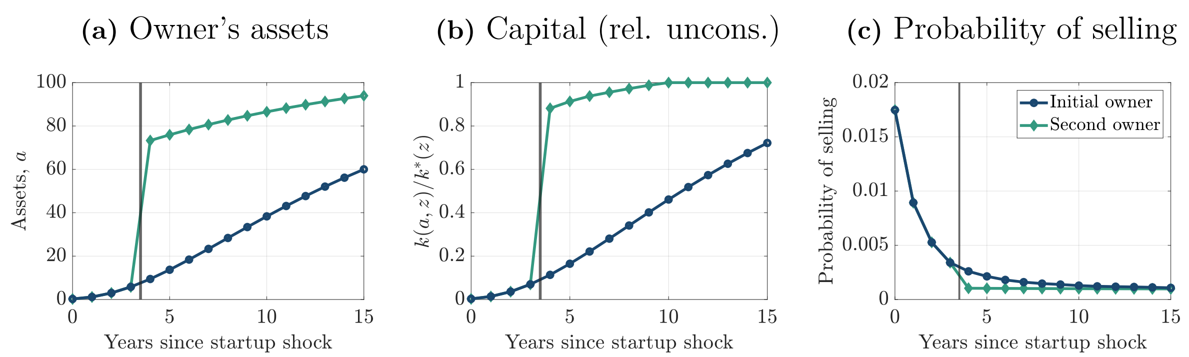
Notes: Probabilities (Pr.) of trade after integrating over preference shocks and trading counterparts.

quantify below, these trades in the market for firms between constrained and potentially unconstrained owners will lead to a better allocation of productive projects and available resources in the economy.

5.3 Implications for Firm Dynamics and Capital Allocation

To provide additional intuition about the implications of the trade of firms, Figure 5 presents the trajectory of a firm in our model. We assume that the initial owner has assets equal to the median wealth among workers and, at period zero, receives a high-quality firm through the startup shock. This entrepreneur will start operating the firm at a low scale because of credit constraints that limit the use of external financing. Panel (a) shows that this business owner will accumulate assets over time to reach the optimal unconstrained size through self-financing. However, panel (b) shows that even after fifteen years, the firm operates at a scale equal to 70% of the optimal unconstrained level.

Figure 5: Firm Dynamics and Trade, An Example



Notes: The vertical line at $t = 3$ indicates when trade takes place. When receiving the startup shock at $t = 0$, the assets of the initial owner are equal to the ones of the median worker. Firm quality z is held constant across time. $k^*(z)$ in panel (b) denotes the unconstrained level of capital for firm z.

Panel (c) of Figure 5 plots the probability of selling the firm over time, which depends on both the owner's idiosyncratic states and the distribution of potential trading counterparts. After receiving the startup shock, this entrepreneur will be willing to sell the firm at a relatively low price as the alternative option of self-financing implies a low-profit stream for several periods. In addition, because of the risk channel previously described, a credit-constrained entrepreneur will be willing to sell the firm because of precautionary motives. As the initial owner accumulates assets, the minimum price at which the owner is willing to sell increases, and the probability of selling falls accordingly.

Suppose now that in period three, a wealthier agent purchases this firm. If the second owner has more resources to invest, this owner can take the firm closer to its optimal operating scale more quickly. In our example, panel (b) shows that with the second owner, the firm reaches its optimal scale ten years after being founded, less than half the time required by the initial owner. In sum, this simple example illustrates how the market for firms reduces the losses from capital misallocation in the economy as it shortens the time that highly productive firms remain financially constrained. In Section 6.3 below, we use microdata from several European countries to provide novel empirical evidence of firm dynamics after trade consistent with our mechanism.

6 Financial Frictions as a Motive to Trade

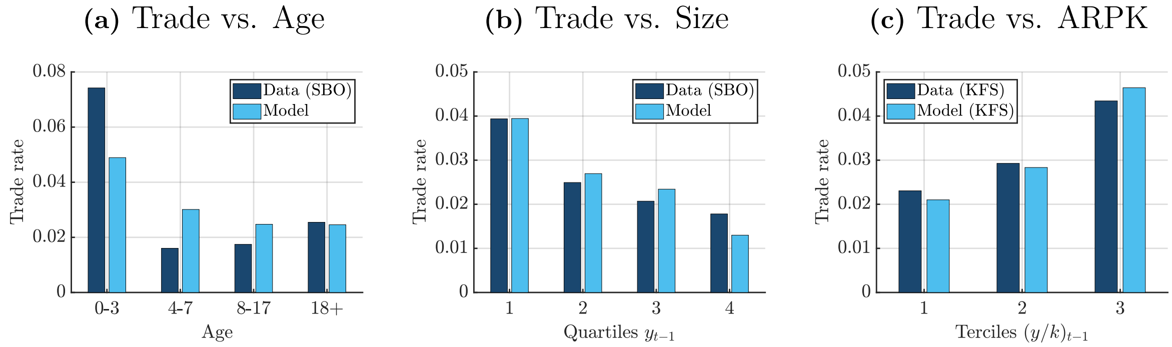
Before moving on to our quantitative results, this section presents three exercises that evaluate some testable predictions of our theory about financial frictions being a relevant motive to trade firms. In the first two exercises, we compare our model predictions about firms' and buyers' characteristics in the cross-section. Our third exercise uses firm-level panel data to show that post-trade firm dynamics are consistent with firms being financially constrained before the trade, as predicted by our theory.

6.1 Trade Rate and Firms' Characteristics in the Model

First, as described throughout the paper, if financial frictions are an important reason for trade, credit-constrained firms should be more likely to be bought and sold. We test this first prediction of the model by analyzing the relation between trade and firms' observable characteristics. As in the empirical section, we consider two commonly used proxies of credit constraints: firms' age and size, as younger and smaller firms are more likely to be financially constrained. In addition, we analyze firms' ARPK, defined as the output to capital ratio, since credit-constrained firms will have high capital returns but cannot increase their investment.

Following the analysis in Section 2.5, we simulate data from our model and compute the trade rate conditional on firms' characteristics. Figure 6 shows that consistent with the data, our model predicts that younger, smaller, and high returns to capital firms exhibit the highest probabilities of trade. It is important to emphasize that these relations were *not* targeted in our calibration exercise. Instead, they result from the key prediction of our theory that credit-constrained firms are the ones more likely to be traded and that these characteristics are strongly correlated with binding credit constraints in our model.

Figure 6: Trade Rate by Firms' Characteristics: Data and Model



Notes: Trade rate by firms' characteristics in the data and data simulated from the model. To be consistent with the data, Model (KFS) restricts to a sample of firms of age less or equal to 7. See the notes in Figure 1 for a description of the data moments.

6.2 Firm Buyers' Wealth

The second prediction that we test relates to the characteristics of business buyers. As we showed in Section 5.2, if financial frictions primarily drive firms' trade, business buyers will be, on average, wealthier than sellers. Although we do not observe the wealth of buyers and sellers in each transaction, we can measure the wealth of the average business buyer in the SCF. In Section 2.4, we documented that the average firm buyer is considerably wealthier than the average household. We compute the analogous moment in our model, average wealth at t for agents that bought a firm in $t-1$, relative to the economy's average household. We also compute this moment relative to the average entrepreneur.

Table C.1, in the Appendix, shows that our model aligns remarkably well with the data despite these moments not being targeted in our calibration. Including business wealth $(a+p)$, our model predicts that buyers are 3.1 times wealthier than the average household, while this number is 3.8 in the data. Excluding business wealth (a), this ratio equals 2.7 in the data and the model. Our model is also consistent with the wealth ratio of firm buyers to the average entrepreneur, which is around 0.69 for total wealth and 0.79 excluding business wealth (0.54 and 0.75 in the model). These results show that although firm buyers are wealthy, they are less so than other entrepreneurs who already own a firm and, hence, have accumulated wealth in the past. Overall, these results about buyers' characteristics suggest that financial frictions are a relevant motive behind firms' trade.

6.3 Firm Dynamics After Trade

Our model predicts that if a firm is traded, it was likely financially constrained before the trade. Hence, this firm was operating with lower capital and higher ARPK relative to their optimal unconstrained level. After a trade, we should see capital increase and, crucially, capital should increase more than output, reducing the firm's ARPK over time. This is the key prediction of our theory we now test in the data.

The Orbis Database Due to data limitations for the U.S., for which we only have cross-sectional data, we use the historical product of Orbis, an extensive firm-level panel database covering millions of companies worldwide, to provide novel empirical evidence about post-trade firm dynamics. We focus on a sample of European private firms, which are the ones with the best coverage in Orbis.[25] For our baseline results, we consider eleven high-income European countries which are the most comparable to the U.S. economy.[26] Appendix B provides a detailed description of this data, discusses our sample selection, presents descriptive statistics, and definitions of the main variables.

Orbis contains income and balance sheet statements from 1996 to 2019, from which we compute firm-level measures of output and capital. From 2007 onward, the data reports annual ownership records with the name and equity shares of firms' shareholders. We use the ownership data to identify trades in the market for firms. We define trade episodes as the years in which we observe a change in the majority shareholder of a firm (equity share above 50%). Therefore, we identify firms' trades by tracking majority shareholders' identities over time. As Appendix B.2 describes, we identify changes in shareholder names using a string similarity algorithm that excludes changes in owners' names that are spurious or could be related to inheritances or family-related transfers.[27]

Empirical Specification After identifying trade episodes, we run a non-parametric regression to analyze the post-trade firm dynamics. Let i denote a firm and t time. We normalize the trading year to $t = 0$. Then, for each variable of interest, x_{it}, we estimate

$$\log x_{it} = \beta_0 + \sum_{h \in \mathcal{T}} \beta_h \mathrm{D}_{it}^h + \gamma \mathbf{c}_{it} + \epsilon_{it}, \tag{15}$$

where $\mathcal{T} = \{-1, 1, \ldots, 5\}$ and D_{it}^h is a indicator variable equal to 1 if time t corresponds to the period h around the trading episode. Thus, as $\mathcal{T}$ indicates, we study firm dynamics from one year before up to five years after trade. For our analysis, we restrict to firms we observe for at least five consecutive years from $t = -1$ to $t = 3$. The variable $\mathbf{c}_{it}$ includes several control variables, such as country, NACE 4-digit sector classifications, and year fixed effects. Given the relevance of firm age for firm dynamics (Haltiwanger, Jarmin, and Miranda, 2013), we also control for firms' age at $t = 0$, the year the firm was traded.

Firm Dynamics After Trade Figure 7 presents the results from estimating (15) for the three main variables of interest: capital, output, and ARPK. Panel (a) shows that, on average, firms' capital significantly grows by 35 log points three years after trade and by 47 log points after five years. These results indicate that capital's five-year growth of traded firms is twice as large as the average firm in our sample, with capital growing at

[25]On average, the historical product of Orbis covers 71% of the gross national output of the countries in our sample. See Kalemli-Özcan et al. (2023) for a thorough analysis of Orbis' coverage in Europe.

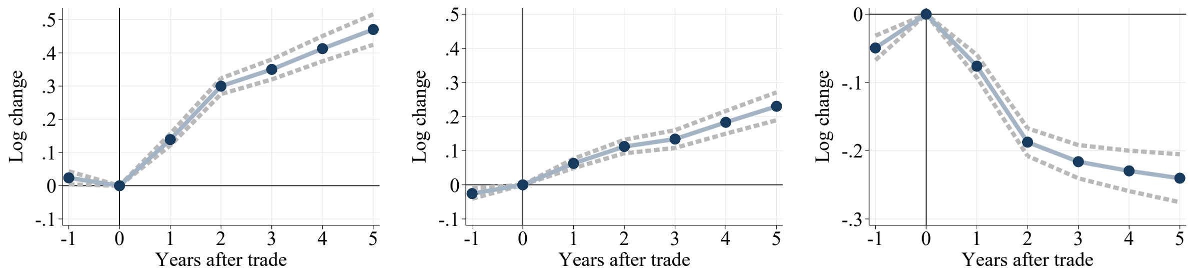
[26]The eleven high-income countries included in our baseline analysis are: Austria, Belgium, Denmark, Finland, France, Germany, Italy, Norway, Spain, Sweden, and the United Kingdom. Appendix B presents additional results for middle-income European countries.

[27]For example, our algorithm excludes changes in owners names such as "Federico Kochen" to "Kochen Federico", or "Federico Guntin" to "Rafael Guntin" as the last one could be related to inheritances.

4.6 log points per year.[28] Further, it is worth noting that there is no clear pre-trade trend as $\hat{\beta}_{-1}$ is close to zero. Panel (b) shows that output also increases, but to a lower extent, in 13 and 23 log points after three and five years, respectively. As a result of capital and output post-trade dynamics, consistent with our theory, ARPK falls by 22 and 24 log points three and five years after trade. These results regarding post-trade firm dynamics support our central hypothesis of financial constraints being alleviated after trade.

Figure 7: Firm Dynamics After Trade in the Orbis Database

(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$



Source: Orbis.
Notes: Estimated coefficients $\hat{\beta}_h$ from (15). The dashed lines correspond to 95% confidence intervals considering firm-level clustered standard errors.

Despite this evidence coming from a different set of high-income countries than the U.S., the economy we primarily study and calibrate our model to, we compare the post-trade dynamics in the European data with the ones implied by our model. We generate a panel of 2.5 million firm-year observations and estimate regression (15) in the model simulated data. Table 5 summarizes the results three and five years after a trade for the variables of interest in Orbis and the model. The table shows that our model is consistent with Orbis's post-trade firm dynamics. Although the dynamics in our model are faster because firms immediately jump closer to their optimal scale as the results for $\hat{\beta}_3$ indicate, the overall effect five years after trade in our model aligns remarkably well with the data. For example, our model predicts that, on average, firms' ARPK falls by 26 log points five years after trading, while this number is 24 log points in the Orbis database.

Additional Results Appendix B presents additional evidence about post-trade firm dynamics in Orbis. First, we present the results of estimating (15) for other variables. Notably, Figure B.1 shows that firms' profitability, measured by the profits to capital ratio, falls by 5 percentage points five years after trade. This pattern also aligns with firms being financially constrained before trade, with our model predicting similar dynamics. Second, we present results for the high- and middle-income European countries studied in Kochen (2023). Consistent with our theory that predicts that the gains for trading firms are larger the more severe are financial frictions, Figure B.2 shows that post-trade firm dynamics are considerably amplified in middle-income and less financially developed countries, with capital and output increasing twofold than in high-income countries. Thus, as discussed

[28] See Table B.1 in the appendix for our sample descriptive statistics.

Table 5: Firm Dynamics After Trade in Orbis and Model

Years After Trade	*Capital*, $\log(k)$ (1) Data	(2) Model	*Output*, $\log(y)$ (3) Data	(4) Model	*ARPK*, $\log(y/k)$ (5) Data	(6) Model
$t=3,\ \hat{\beta}_3$	0.350	0.550	0.134	0.273	-0.216	-0.277
	(0.015)	(0.005)	(0.013)	(0.003)	(0.012)	(0.002)
$t=5,\ \hat{\beta}_5$	0.470	0.527	0.230	0.271	-0.240	-0.256
	(0.023)	(0.005)	(0.021)	(0.003)	(0.018)	(0.003)
Controls	✓		✓		✓	
R^2	0.355	0.165	0.443	0.084	0.375	0.187
N	187,599	147,021	187,599	147,021	187,599	147,021

Notes: Estimated coefficients from (15) after 3 and 5 years from trade in Orbis and in model simulated data. Firm-level clustered standard errors are reported in parentheses. Controls include country, NACE 4-digit sector classifications, year fixed effects, and firms' age when traded.

further below, this evidence is consistent with the trade of firms playing an even more important role in economies with less developed financial markets.

6.4 Alternative Motives to Trade

The empirical evidence presented in this section does not rule out that there could be other motives behind firms' trade. One potential motive, captured in our framework through the exogenous preference shocks, relates to owners' life cycles. Nevertheless, Figure A.7 in the appendix shows the vast majority of firm sellers are young- to middle-aged entrepreneurs, suggesting that sellers' retirement motives explain only a small fraction of the trades in the market for firms. Another motive, absent in our framework, could be the misallocation of heterogeneous managerial abilities (e.g., good managers not managing good quality firms). However, the fact that, on average, firms' profitability falls after a trade, as Figure B.1 shows, poses a challenge to the relevance of this motive.

Taking stock, the empirical results presented in this section are consistent with our hypothesis of financial frictions being an important motive for trading firms. After showing that the main testable predictions of our theory hold true in the data, in the next section, we use our model to quantify the role of the market for firms in the aggregate economy.

7 Macroeconomic Implications

This section presents our main quantitative exercises. First, we perform two counterfactual experiments that quantify the relevance of the market for firms as a mechanism through which entrepreneurial projects and available resources are allocated in the economy. Second, we study the level of TFP predicted by our model across economies with different degrees of financial development and functioning market for firms.

7.1 The Role of the Market for Firms

We consider two counterfactual experiments that quantify the importance of the market for firms. Both experiments consist of steady-state comparisons of our model under different parameterizations. In the first experiment, we take our baseline model and analyze the implications of a partial or total market shutdown. In the second experiment, we compare our baseline economy with an alternative economy with no trade in the ownership of firms. We then analyze the level of external financing that no market economy requires to match the TFP level of our baseline economy.

7.1.1 Closing the Market

Table 6 presents the results of our first counterfactual experiment. As a reference, the first column of the table has some relevant moments of our baseline economy. The second and third columns report the percentage change when the market for firms partially and then completely shut down. In both cases, we only vary the search frictions' parameters in the market for firms, α_o and α_w, while maintaining the rest fixed. For the partial shutdown case, we divide in half both parameters such that their relative values are the same and, hence, the fraction of firms purchased by workers is unchanged. For the complete shutdown case, we set both parameters equal to zero.

Table 6: Closing the Market for Firms

		Δ %	
	Baseline Economy	Partial $(\alpha_o, \alpha_w)/2$	Total $(\alpha_o, \alpha_w) = \mathbf{0}$
Fraction of entrepreneurs	0.06	-2.4%	-4.5%
Private firms output	0.57	-4.8%	-9.1%
Private firms TFP	1.17	-1.2%	-2.2%
Exit rate	0.09	-10.2%	-27.5%
Public firms output	0.71	2.6%	5.1%
Total output	1.29	-0.7%	-1.3%
Interest rate	0.03	2.6%	4.4%
Wage	1.30	-0.4%	-0.7%

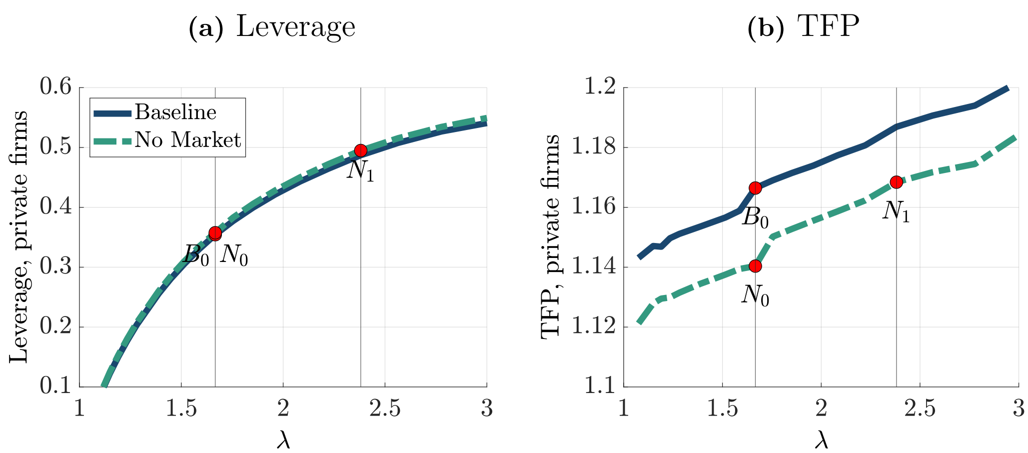
Notes: The Partial column presents the results for the market partial shutdown, obtained dividing by the half the parameters α_o and α_w. The Total column presents the results when both parameters are equal to zero, thus a total market shutdown. TFP is measured as $Y_e/(K_e^{\theta} L_e^{\nu})$, where $(.)_e$ denotes the aggregate variables of the entrepreneurial sector.

In both cases, private firms' output considerably falls by -4.8% and -9.1% for the partial and the complete shutdown case, respectively. For easiness in the exposition, we focus on the total shutdown results. The remaining rows of Table 6 show that both extensive and intensive margins explain the fall in entrepreneurial output. First, regarding the extensive margin, the share of active entrepreneurs falls by 4.5%. Additionally, without the market for firms, the entry and exit rate into entrepreneurship significantly decreases by 27.5%. Regarding the intensive margin, the remaining private firms exhibit a poorer allocation of capital and firms' qualities, as shown by the entrepreneurial TFP, which decreases by 2.2%. Total output in the economy also decreases, but to a lower extent, by 1.3%. General equilibrium effects and the assumption that the production of private and public firms are perfect substitutes explain the smaller aggregate effect. Indeed, an increase in the production of the public firm of 5.1% partially offsets the fall of entrepreneurial output.

7.1.2 Baseline vs. No Market Economy

For our second experiment, we compare the counterfactual economy with $\alpha_o = \alpha_w = 0$, which we call the "No market economy", with our baseline model under alternative credit market frictions. In Figure 8, we present different steady states for the baseline and the no market economy varying firms' credit constraints, which, in the model, is governed by the parameter λ. Higher λ implies easier access to credit as entrepreneurs can borrow more with the same level of assets. From these steady states, we focus on two moments: private firms' leverage (Panel a) and the TFP of the entrepreneurial sector (Panel b).

Figure 8: Baseline vs. No Market Economy



Notes: Steady-state values for the baseline and no market economy varying λ, which parameterizes firms' credit constraints. Panel (a) is private firms' mean leverage, $(k-a)/k$, weighted by capital k. Panel (b) is private firms' TFP. Points B_0 and N_0 denote the allocations in the baseline and no market economies. N_1 is the counterfactual no market economy that attains the same TFP as the baseline model.

Panel (a) of Figure 8 shows that the baseline and the no market economy exhibit almost the same relation between leverage and λ. This finding was expected, as firms' maximum leverage equals $(\lambda-1)/\lambda$. However, as shown in Table 6, this is not the case for the private or entrepreneurial sector TFP. Indeed, panel (b) shows that for the same level of λ, the no market economy achieves a lower TFP than our baseline model. The differences in TFP between these two models are captured by the distance between points B_0 and N_0, which denote the allocations for the baseline and the no market economy, respectively. This result is explained by the higher *misallocation* between entrepreneurial projects and available resources when the market for firms is absent.

With these steady states at hand, we *ask*: what credit conditions does the no market economy require to match the TFP level of our baseline economy? Using Panel (b), we can identify the level of λ such that the no market economy attains the same TFP as the baseline. Graphically, this implies moving from N_0 to N_1 along the no market economy's curve. The allocation N_1 has a higher λ. Thus, it implies *easier credit* conditions than N_0. To better interpret this, we go back to Panel (a) and recover the level of leverage associated with point N_1. These panels show that the no market economy requires an increase in firms' average leverage of 14 p.p., or 40%. This increase is sizable as, for example, firms' leverage fell by around 5 p.p. during the 2008 Great Recession.

Altogether, these two counterfactual exercises show that the market for firms is a quantitatively relevant mechanism through which entrepreneurial projects and available resources can be better allocated in the economy.

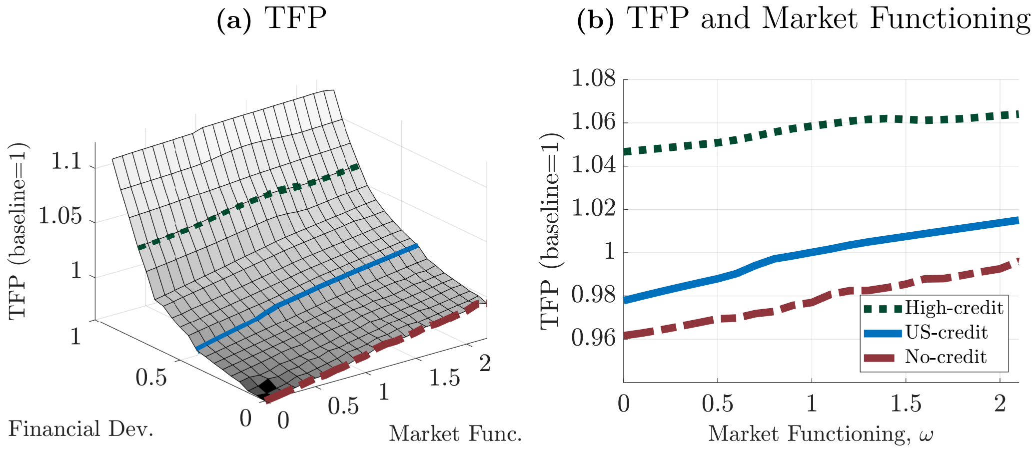
7.2 Financial Development and the Market for Firms

In our model, the functioning of both markets for credit and firms determines the capital allocation in the economy. In this final section, we study the interaction between these two markets and their implications for aggregate productivity. Figure 9 shows private firms' TFP, relative to the baseline, for economies with different degrees of financial development and functioning markets for firms. We parameterize financial development by firms' maximum leverage defined by $(\lambda - 1)/\lambda$. We vary the functioning of the market for firms through different values of a parameter ω that scales the meeting probabilities: $\alpha_o(\omega) = \min\{\omega\alpha_o, 1\}$ and $\alpha_w(\omega) = \min\{\omega\alpha_w, 1\}$ where (α_o, α_w) are the values of the baseline calibration. Thus, $\omega = 0$ corresponds to the total market shutdown, previously analyzed, and $\omega = 1$ to the baseline parameterization. Figure C.2 in the appendix shows similar results for entrepreneurial output and firms' trade rates.

Panel (a) of Figure 9 shows that TFP is increasing in both financial development and the functioning of the market for firms. In particular, note that for the case of $\omega = 0$ (No Market economy), our model implies that higher levels of financial development lead to a better allocation of capital and higher TFP, as in the finance and misallocation literature (Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014; Moll, 2014). However, unlike previous papers in this literature, this figure also shows that for any given level of financial development, aggregate TFP can increase through a better-functioning market for firms.

To better make this point, panel (b) of Figure 9 presents three hyperplanes in the ω-space considering different levels of financial development: High-credit, US-credit, and No-credit (maximum leverage of 0.75, 0.397, and 0, respectively). There are two main takeaways from this panel. First, in a high-credit economy, the TFP gains from a better-functioning market for firms are limited, as shown by the flatter slope in the top line of panel (b). This result is because, in higher-credit environments, firm owners can produce

Figure 9: Financial Development and Functioning of the Market for Firms



Notes: Financial Development is parameterized by firms' maximum leverage, $(\lambda - 1)/\lambda$. Market Functioning is parameterized by ω multiplying the search frictions in the market for firms $\alpha_o(\omega) = \min\{\omega\alpha_o\}, \alpha_w(\omega) = \min\{\omega\alpha_w, 1\}$. Panel (a) plots private firms' TFP in the financial development and functioning of the market for firms' space. Panel (b) plots TFP in the market functioning space for three different levels of financial development: High-credit, US-credit, and No-credit with $(\lambda - 1)/\lambda$ equal to 0.75, 0.397, and 0. The baseline calibration corresponds to the case $(\lambda - 1)/\lambda = 0.397$ and $\omega = 1$.

closer to their optimal scale through debt financing, which reduces the gains from trading firms. Second, economies with less-developed financial markets can achieve TFP levels closer to an economy with US-credit through a better-functioning market for firms. Thus, the market for firms can substitute for debt financing in less developed credit markets. For example, the No-credit economy can attain the same level of TFP as the baseline calibration with twice as large search frictions parameters (ω above 2), with a trade rate in the market for firms of 5% as panel (b) of Figure C.2, in the appendix, shows.

The empirical evidence of firm dynamics after trade documented in Section 6.3 is consistent with these quantitative results. The fact that post-trade firm growth is twice as large in middle-income and less financially developed countries than in high-income economies is in line with the prediction of our model that the gains from trading firms are higher in environments with tighter financial frictions. These results demonstrate that the market for firms can play an even more important role as a capital allocation mechanism in economies with less developed financial markets.

8 Conclusions

We use microdata from business owners, households, and firms to provide novel facts about the trade of firms in the U.S. and post-trade firm dynamics in several European countries. We document that one out of four entrepreneurs purchased their business. In the cross-section, younger, smaller, and higher ARPK firms have the highest trading probabilities. Over time, firms experience significant capital and output growth after trade, while ARPK decreases sharply.

To explain these findings, we develop and quantify a general equilibrium model of entrepreneurship and frictional trade of firms in which gains from trade can arise endogenously from financial frictions, namely credit constraints and incomplete markets, and exogenously from preference shocks, capturing alternative motives to trade. By introducing financial frictions as a micro foundation that generates gains from trade, our model can account for the cross-sectional and panel empirical patterns. It accounts for the fact that younger, smaller, and higher ARPK firms have the highest trading rates, as these firms are more likely to be financially constrained in the model. Furthermore, the model-simulated data quantitatively aligns with the observed post-trade firm dynamics, where capital grows more than output, reducing firms' ARPK over time because firms' trade relieves financial constraints in the model. These empirical and quantitative results support our main hypothesis of financial frictions being an important motive for trading firms.

We then study the aggregate implications of the market for firms implied by our model. We find that the trade of firms significantly improves capital allocation in the economy, as shutting down this market implies losses in private firms' output of 9.1% and TFP of 2.2%. We also investigate the interaction between financial development and firms' trade. We show that aggregate TFP can increase through a better-functioning market for firms for any level of financial development. Finally, we argue that the market for firms can play an even more important role in economies with tighter credit frictions, as the potential gains from trading firms are higher. Our finding about post-trade firm dynamics being twice as large in middle-income and less financially developed countries than in high-income countries is consistent with this prediction.

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Online Appendix
Financial Frictions and the Market for Firms

A U.S. Data Appendix

This appendix describes our primary data sources, presents robustness, and additional exercises about the market for firms in the US economy.

A.1 Data Sources

A.1.1 Survey of Business Owners (SBO) - PUMS

The SBO is a comprehensive survey of firms and firm owners in the U.S. The PUMS sample is representative of non-farm private businesses with receipts of $1,000 or more and is available for the year 2007. The SBO is conducted at the company or firm-level. A company is a business consisting of one or more domestic establishments. The survey is designed to identify the ultimate owners of firms and their characteristics.

Table A.1 reports the total number of owners and firms in the SBO. From those, we first restrict to the owners who report how do they acquire their business. The SBO already restricts to self-employed business owners, thus for our definition of entrepreneurs, we just have to restrict to business owners who actively manage their firm. Our baseline sample consist of almost 700,000 entrepreneurs which own around 500,000 different firms.

Table A.1: 2007 SBO Sample

	#Dropped	#Owners	#Firms
All	-	3,409,393	2,165,680
Report Acquisition	1,244,852	2,164,541	1,291,292
Manage	1,052,287	1,112,254	841,254
Employer firm	413,603	698,651	501,564

From this survey we mainly focus on how the owners acquired their firms. In addition, we use information on the characteristics of the firm (established year, employment, payroll, receipts, sector, location, operation status, number of owners) and of the owners (age, acquisition year, ownership percentage, education level, previous occupation). We use this information to do a thorough characterization of the trade of firms.

Using the SBO we can also obtain information on firms and owners close to the time at which the firm was traded. To study firms' and buyers' characteristics *when purchased* we look at owners that acquired the firm through a purchase in the same year of the survey. Furthermore, the SBO provides information on firms' and owners' characteristics for those owners who report an exit because they sold their firm in the year of the survey. We use this information to characterize firms and their previous owners *when sold*. For all our calculations we use the sample weights provided by the survey.

A.1.2 Survey of Consumer Finances (SCF)

The SCF is a household-level survey that includes extensive information on households' income, balance sheets, and demographic characteristics. The public microdata is available every three years for the period 1989-2016.

Table A.2: 1989-2016 SCF Sample

	#Dropped	#Households
Income and wealth		
All	-	47,769
21 < age < 78	3,528	44,241
Positive income	67	44,174
Firm acquisition		
Manage and own	35,468	8,706
Employer firm	1,379	7,327

In the SCF we identify entrepreneurs as those households whose household head: is self-employed, owns a business, and has an active management role in it. The SCF also provides information of privately held businesses which are actively managed. Business owners can report information for up to three or two firms, depending on the survey year. For our baseline calculations we focus on the characteristics of the main business, defined as the one with higher reported value. Using this information, we can identify the entrepreneurs that own a firm with a positive number of employees.

Table A.2 reports our sample selection criteria and the number of households in our SCF sample. For our calculations of the moments of income and wealth we restrict to a sample of households whose household head is between 22 and 78 years old and have a positive income. For our calculations of the trade of firms trade we focus on entrepreneurs, which considering our baseline definition (with employer firms), are 7,327 households between 1989 and 2016, which is a significantly smaller than the one in our SBO sample.

In addition to the information on entrepreneurs and how do they acquired their firm, we use the SCF to compute relevant moments from the income and wealth distribution in the U.S. economy. Our measure of household wealth is the variable constructed by the Federal Reserve for its Bulletin article which accompanies each wave of the SCF. Wealth is defined as total net worth, which equals assets minus debt. Assets includes both financial and non-financial assets. Financial assets include checking and savings accounts, stocks held directly and indirectly, bonds, etc. Non-financial assets, among others, include the value of houses and other real estate, the value of farm and private businesses owned by the household. Debt includes both housing debt (mortgages), debt from lines of credit and credit cards, and installment loans.

Our measure of income includes all sources of income excluding government transfers (e.g. social security and unemployment benefits) and excluding other (non-classified) sources of income. Thus, we include wage income, income from businesses, income from interests and dividends, from capital gains, rent income and income from pensions and annuities. For all our calculations we use the sample weights provided by the survey.

A.1.3 Kaufman Firm Survey (KFS)

The KFS is a panel survey that tracks almost 5,000 business that start their operations in 2004 through 2011. The initial sample was created by using a list frame sample of start-up businesses from the Dun & Bradstreet Corporation (D&B) database. The KFS collects information from business' and owner's characteristics and, in particular, they provide information about firms' balance sheets.

Table A.3 shows the sample selection. Following the previous literature, we drop firms that at some point refuse to answer and observations with missing values of employment, revenues, sales, assets, cash, and accounts receivable. Our baseline sample remains with 2,841 firms and 13,457 observations (firm×year).

Table A.3: 2004-2011 KFS Sample

	#Dropped	#Owners	#Firms
All	-	39,424	4,928
Answer	13,624	25,800	3,225
Missing	16,684	9,116	2,366

We define capital as total assets without cash holdings and accounts receivable. Total assets is composed by product inventories, land and buildings and structures, vehicles, equipment/machinery, other properties, cash, and other. To approximate the capital returns we consider the average revenue product of capital (ARPK) measured as firms' revenue to capital ratio. In the KFS we identify trades through exits of owners that report having sold or merged their business. For all our calculations we use the sample weights provided by the survey.

A.1.4 Annual Survey of Entrepreneurs (ASE)

The ASE is a representative sample of all non-farm businesses filing Internal Revenue Service (IRS) tax forms as individual proprietorships, partnerships, or any type of corporation, and with receipts of $1,000 or more. The ASE is conducted at the firm-level and gathers information on the firm and owner characteristics. The population represented by the survey focuses on firms with paid employees. This survey is available at an annual frequency starting in 2014.

Similar to the SBO, the ASE collects information regarding owner' and firms' characteristics for a large sample of owners. The difference is that the ASE has an annual frequency and samples only firms with paid employees. One major caveat of the ASE is that we don't have access to the microdata, therefore we use information from the tables provided by the Census Bureau to compare to our baseline estimates and explore the recent evolution in the share of firms traded.

For the table estimates provided by the Census Bureau, a business owner is defined as someone who holds more than 50% of the stake of the firm, where the firm has a positive payroll. This definition is close to our baseline definition of an entrepreneur where firms have at least one employee. Our numbers are retrieved from table `SE1600CSCBO01` where entrepreneurs are classified by the way they acquired their firm.

A.2 Robustness Exercises

A.2.1 How do Entrepreneurs Acquire Their Firms?

Owner-level. Table A.4 report how many entrepreneurs purchased their business for several alternative definitions of entrepreneurship. For example, instead of active management — as in our baseline definition — we restrict to business owners who have more than 50% of the equity of the firm, or to owners who work at least 40 hours a week in the firm. In bold we highlight our baseline definition for entrepreneurs, which implies that firm owners manage an employer firm.

Firm-level. In addition to the business owner-level results, we compute the share of firms that were acquired by their owners through a purchase. We compute the share of firms purchased in two ways: (i) if at least one entrepreneur purchased the firm; (ii) if all the firm's entrepreneurs purchased it. The results are presented in Table A.5. The purchased share computed at the firm- and owner-level are very similar. This is due to the fact that most firms have one entrepreneur, and most entrepreneurs have one firm. As in the business owner-level results, this share is sensitive to the exclusion of firms with no employment. Definitions that consider firms with no employment tend to have lower purchasing ratios as the main input in production is probably the owner human capital, which is hard to transfer.

Franchises. We further analyze whether franchises are driving our results. Table A.6 shows that even excluding all franchises the share of entrepreneurs that purchased their firm is 16.1% and 24.2% for all firms and our baseline definition, respectively. Although is true that, within franchise owners, the share of entrepreneurs that acquired the business is very high, more than 50%, these owners represent a small group in the total number of entrepreneurs: 2.7% and 4.7% for the two definitions used.

Table A.4: Share of Entrepreneurs That Purchased Their Business

Sample	Purchased	N(weighted)	N
All	-	36,856,132	3,409,393
All (Respond acquisition)	16.0%	20,302,192	2,164,541
Manage	17.0%	9,503,681	1,112,254
Employment > 0	25.9%	5,507,460	1,255,134
Receipts > 0	16.9%	17,139,950	1,987,336
Payroll > 0	25.1%	6,045,634	1,338,400
Size (all) > 0	26.1%	5,344,964	1,216,319
Entrepreneur	25.5%	3,167,718	698,651
Share ≥ 50	13.5%	16,274,606	1,479,855
Share ≥ 50 and Employment> 0	23.5%	3,884,071	745,431
Share ≥ 50 and Manage	15.4%	8,064,388	827,286
Entrepreneur and Share ≥ 50	24.0%	2,458,710	469,250
Hours Worked > 40	18.0%	8,928,828	1,164,328
Hours Worked > 40 and Employment > 0	25.6%	3,505,078	802,680
Hours Worked > 40 and Manage	19.6%	5,679,652	806,923
Entrepreneur and Hours Worked > 40	26.0%	2,545,635	582,966
Entrepreneur (Weighted by Employment)	32.2%	3,167,718	698,651

Source: 2007 SBO.
Notes: Purchased refers to the percentage of entrepreneurs that acquire its firm through a purchase. Share refers to the normalized entrepreneur's share of the firm. Hours Worked denotes average number of hours per week the owner spends at the firm.

Table A.5: Share of Firms With Owners That Purchased It

Sample	Owner-level	Firm-level	
		At least one	All
All (Respond acquisition)	16.0%	14.7%	12.0%
Manage	17.0%	16.3%	15.0%
Employment > 0	25.9%	26.8%	20.9%
Entrepreneur	25.5%	25.7%	23.2%
Entrepreneur and Hours Worked > 40	26.0%	26.1%	23.8%

Source: 2007 SBO.
Notes: Hours Worked denotes average number of hours per week the owner spend at the firm.

Table A.6: Share of Firms Purchased: Franchises

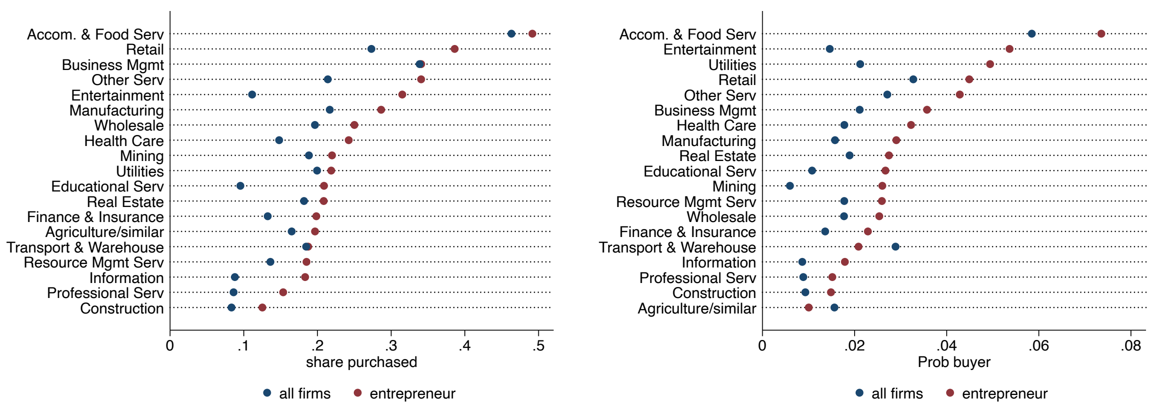
Sample	All firms	Employer firms
Baseline	17.0%	25.5%
W/o franchises	16.1%	24.2%
Franchises only	50.1%	51.8%
Share of Franchises	2.7%	4.7%

Source: 2007 SBO.

Sectors. We also analyze if our results explained by specific sectors of production. The results are presented in Figure A.1. Although there is variability in the stock and rate of trade, we find that the trade of firms is relatively widespread across all sectors.

Figure A.1: Share of Entrepreneurs that Purchased by Sector

(a) Share **(b)** Rate



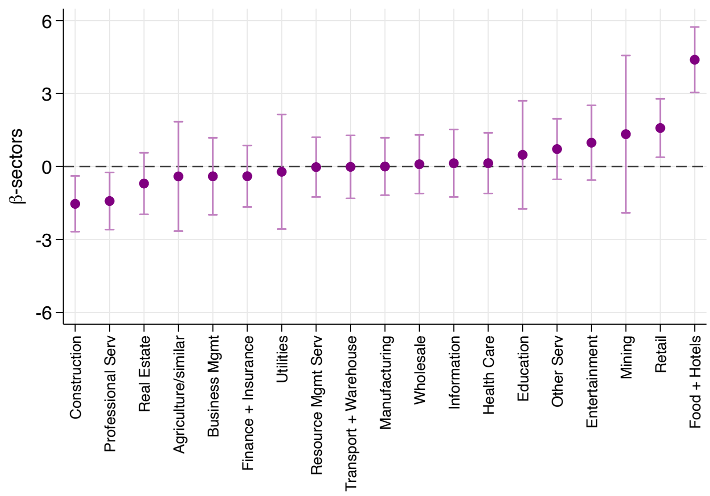
Source: 2007 SBO.
Notes: The rate is constructed as the ratio of firms bought in 2007 to all firms normalized to be 2.0% for all firms and 3.0% for entrepreneurial firms in the aggregate.

To further analyze this, we assess how much of this variability could be related to other observable characteristics correlated to specific sectors, such as firm size. For that we run the following regression

$$\text{Sold}_i = \sum_s \beta_s \times \text{Sector}_{i,s} + \gamma \times X_i + \varepsilon_i \tag{16}$$

where $\text{Sector}_{i,s}$ indicates if the business of entrepreneur i is in sector s, X_i includes several control variables (other observables, such as firm size and age), and Sold_i indicates if the entrepreneur sold its business. Figure A.2 shows the sector specific coefficients. We find that most sectors have a similar propensity to trade. The only sectors with an *unexplained* high propensity to trade are restaurants, hotel and retail sectors, and the ones with low propensities are construction and professional services. These results could be driven by unobservable characteristics such as time-varying demand (restaurants and hotels), fixed costs (construction) and the tradability of the business (professional services).

Figure A.2: Sector Effect on Probability to Sell a Firm



Source: SBO.

Notes: Coefficients are normalized to 0 using median of estimates. Standard errors are clustered by sector and state. Units are in percentage points.

A.2.2 Firm Buyers' Previous Occupation

Alternative Computations. In the main text we documented 66% of current entrepreneurs have never been self-employed (and hence have never been entrepreneurs) prior acquiring its firm. As a robustness we check how many workers, or not self-employed, transition into entrepreneurship by acquiring its firm considering alternative definitions. In Table A.7 we compute the transition rate from worker to entrepreneur conditional on purchasing the firm for: (i) our baseline definition; (ii) when transition to being the main owner of the firm; and (iii) conditional on large firms. Our results are very similar for all these samples.

Table A.7: Firm Buyers' Previous Occupation

Sample	Worker Before Purchasing	
	All firms	Employer firms
Baseline	62.0%	65.9%
Share> 50	61.2%	62.2%
Large Firms	66.9%	69.6%

Source: 2007 SBO.
Notes: Large Firms as those in the top quintile of the employment distribution.

Firms' Characteristics. We also analyze whether workers tend to buy firms with certain characteristics. For example, one could argue that worker-buyers concentrate in small non-growth-oriented type of businesses, compared to firms that are acquired by previous firm owners. Table A.8 shows that there is no stark relation between firm characteristics when purchased and the share of firms purchased by workers and, if something, the share is slightly larger for older and bigger firms.[29]

Table A.8: Share of Firm Buyers Who Were Workers

	Workers	Purchased
By Firm Age		
0-2	50.5%	37.0%
3-7	54.7%	14.0%
8-17	56.9%	16.0%
$\geq$ 18	60.7%	33.0%
By Firm Size		
Q1	54.2%	22.9%
Q2	54.0%	27.7%
Q3	55.3%	16.4%
Q4	56.4%	22.6%
Q5	58.7%	10.4%

Source: 2007 SBO.
Notes: For our calculation we limit to firms purchased in the same year of the survey (2007) and employer firms as in our baseline calculations. The "Workers" column correspond to the ratio of the previously non-self employed entrepreneurs that purchased the firm over the total of firms purchased. The column "Purchased" indicates the amount of firms purchased by characteristic over all firms purchased (i.e., the distribution of purchased firms).

A.2.3 Firm Size and Trade Likelihood

For robustness, we calculate the likelihood of trade across the sales and payroll distributions. As shown in Figure A.3, firms that were in the bottom quartile of the size distribution when traded are the most likely to be traded for both definitions of firm size.

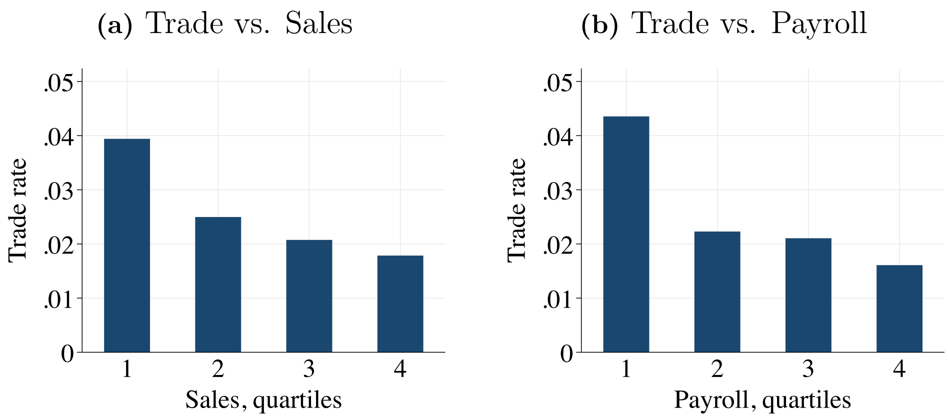
A.3 Additional Evidence on The Market for Firms

A.3.1 Trade share across size and age.

In Section 6, we showed that firms *when purchased* tend to be small and young, consistent with the predictions of our model. In this section, we analyze the share of entrepreneurs that purchased their firm, at any point in the past, conditional on firm observables such as size and age.

[29] The sample is restricted to 2007 such that the characteristics of the firms are approximately to the ones when purchased. For this sample, the share of firm buyers that were workers is slightly lower (less than 60%) than the one of our baseline sample.

Figure A.3: Trade Rate by Firm Size



Source: SBO.
Notes: Panels (a) and (b) use data from the 2007 SBO. The trade rate is computed using information from the firms that were sold in or after 2007. Trade rates are normalized to match the aggregate of our baseline calculations.

Firm Size. Table A.9 presents the share of entrepreneurs that purchased their business across the firm size distribution using three different variables of firm size: receipts, payroll, and employment. We find that the share of traded firms is even higher at the top of the size distribution. For example, in the top 0.1% of receipts, around 39% of entrepreneurs purchased their firm, considerably higher than the unconditional 25.5% share in our baseline calculations.

Table A.9: Firms Purchases, By Firm Size Group

Percentile	Variable	Purchased	Average
Bottom 90	Receipts	24.6%	651
	Payroll	24.6%	153
	Employment	25.2%	8
Top 10\Top 1	Receipts	34.6%	8,624
	Payroll	34.5%	1,773
	Employment	37.9%	83
Top 1\Top 0.1	Receipts	43.8%	57,753
	Payroll	40.0%	9,220
	Employment	37.9%	248
Top 0.1	Receipts	39.0%	381,869
	Payroll	35.3%	49,760
	Employment	32.3%	1,374

Source: 2007 SBO.
Notes: Results are for the baseline definition (employer firms). Average is computed using both purchased and non-purchased firms. Receipts and Payroll are in thousands ('000) of USD.

Firm Age. Next, we study the share of traded firms conditional on the age of the firm. Table A.10 shows that that older firms tend to have larger share of trades. This is consistent either with a higher surviving rate of purchased firms, the declining in trade share we observe in the SCF data, or just a higher probability of being purchased for being around more time. Also, this may reflect some life cycle motives since older entrepreneurs probably manage older firms. Related to this, in Appendix A.3.4 we analyze potential life cycle motives for the trade of firms.

These results suggest that traded firms, *after purchased*, tend to grow bigger and live longer than non-traded firms.

Table A.10: Share of Firms Purchased, By Firm Age

Firm Age	Owner and Manager	Entrepreneur
0-1	8.9%	17.4%
1-2	10.0%	16.3%
2-8	10.9%	16.5%
8-18	13.1%	18.5%
18-28	18.0%	24.9%
+ 28	35.5%	45.2%

Source: 2007 SBO.
Notes: The age of the firm is the age reported at the date of the survey, not when purchased.

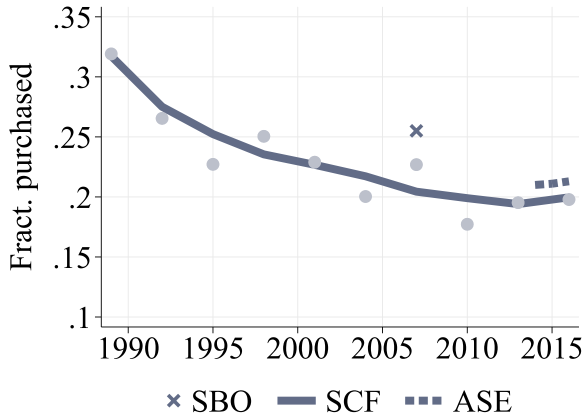
A.3.2 Trade of Firms Across Time

Data sources. As the PUMS version of the SBO is only available for 2007, we use the SCF to document the evolution of the share of entrepreneurs that purchased their firms across different years. Table 1 shows that the SBO and SCF 2007 values are consistent. Additionally, as a robustness check, we also consider data from the ASE available for 2014 to 2016. Overall, the numbers obtained from the SCF align very well with the SBO and ASE for the years in which these surveys overlap.

Results. Figure A.4 shows that between 1989 and 2016, the fraction of entrepreneurs that acquired their firms through a purchase, which proxies for the fraction of traded firms, declined by one-third. More precisely, the fraction of entrepreneurs that purchased their business fell by 12 p.p. going from 32% in 1989 to 20% by 2016. The decreasing trend is robust to alternative definitions of entrepreneurship and changes in the sectoral composition.[30] It is worth mentioning that most of the the share of traded firms is fairly stable since 2007.

[30] These results are available upon request.

Figure A.4: Fraction of Entrepreneurs that Purchased Their Business



Source: SBO, SCF and ASE.
Notes: Entrepreneurs are defined as self-employed, business owners, who actively manage their firm and the firm has at least one employee. The light-colored dots correspond to the time series SCF data points. The solid line trend was estimated using locally weighted smoothing.

A.3.3 Ownership Structure of Private Firms

Number of Firms Owned. Using data from the SCF we document the number of businesses each entrepreneur owns and manages. Table A.11 shows that more than 80% of the entrepreneurs manage one firm at most.

Table A.11: Firms Per Entrepreneur

	# of managed businesses	
	1	≥ 2
Employer firms	83.5%	16.5%
All firms	80.2%	19.8%

DATA SOURCE: SCF 1989-2016.
Notes: Number of employer firms (baseline) and all firms per entrepreneur.

Number of Owners and Entrepreneurs. Table A.12 reports the share of firms in the 2007 SBO conditional on the number of owners and entrepreneurs. The table shows that 74% of the firms have only one entrepreneur, and 96% have at most two. If we include firms with zero employment these numbers are slightly higher (80 and 97%, respectively).

Equity Shares. Figure A.5 shows that, in our SBO sample, more than 60% of the firms have an entrepreneur that holds the 100% of the firm's equity. However, for more than 20% of the firms the entrepreneur shares around 50% of the equity with another non-manager owner. On the other hand, in firms of two entrepreneurs the most common arrangement is 50/50 equity shares. These findings are consistent with what is documented by Espino, Kozlowski, and Sanchez (2016) in other datasets. Finally, we analyze the equity share

Table A.12: Share of Firms by Number of Owners and Entrepreneurs

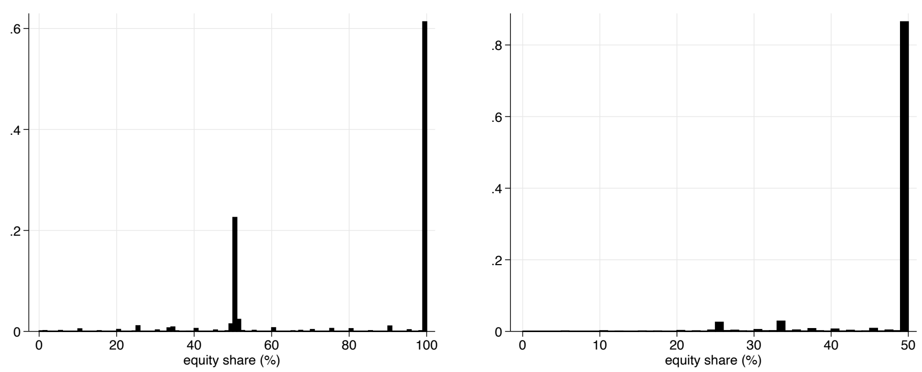
Firms		# of Owners 1	2	3	≥ 4
All	Own	51.4%	39.3%	4.5%	4.8%
	+ Manage	79.8%	18.0%	1.6%	0.6%
Employer firms	Own	43.0%	42.5%	7.1%	7.4%
	+ Manage	73.7%	22.7%	2.7%	0.9%

DATA SOURCE: 2007 SBO.
Notes: Entrepreneurs are defined as (i) self-employed, (ii) business owners, who (iii) actively manage their firm. + Employment > 0 also requires that (iv) the firm has a positive number of employees. Other type of acquisition groups: acquired as a transfer, as a gift or other not specified.

owned by entrepreneurs conditional on firm size and firm age. Figure A.6 reports that the entrepreneurs' equity shares are decreasing with both firm's size and age. Nonetheless this negative relation is relatively weak and even for the firms in the top decile of the size distribution around 75% of the firm equity is held by entrepreneurs. A similar patterns is observed across the firms' age distribution.

Figure A.5: Equity Shares by Number of Entrepreneurs

(a) One Entrepreneur **(b)** Two Entrepreneurs

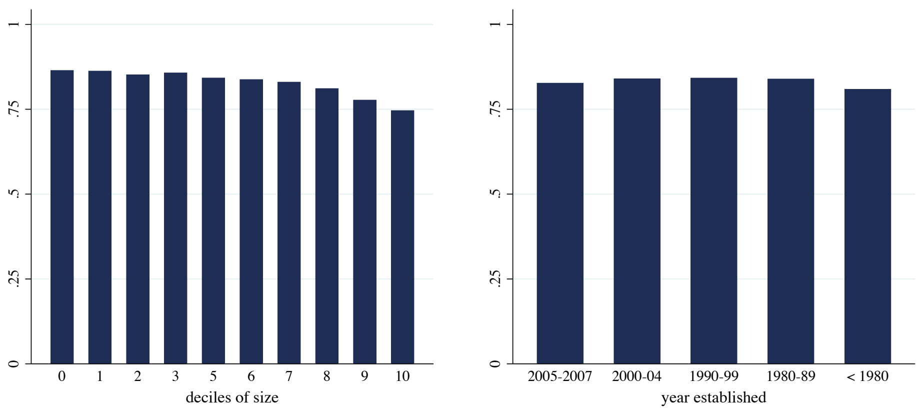


Source: 2007 SBO.
Notes: Use baseline sample of employer firms.

A.3.4 Life Cycle Motives

Another frequently cited motive for the trade of firms are motives related to the entrepreneurs' life cycle. To address this, we study the trade of firms conditional on sellers' age. Panel (a) of Figure A.7 shows that the trade rates are high for young and old entrepreneurs. This is consistent with retirement motives for older entrepreneurs and the lack of access to credit for younger entrepreneurs. However, Panel (b) of Figure A.7 shows that the share of trades is mostly concentrated among middle-aged entrepreneurs, even

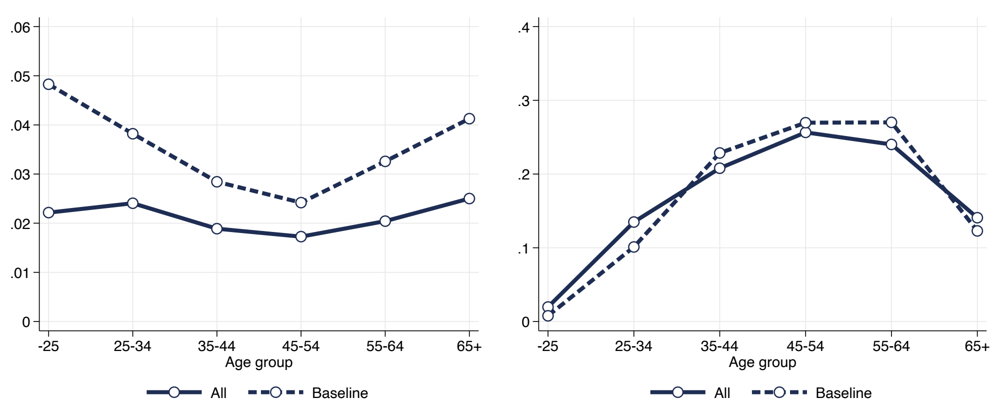
Figure A.6: Equity Shares Across Firm Size and Age



Source: 2007 SBO.
Notes: Deciles of size are constructed using the distribution of firms with positive employment. Decile 0 corresponds to firms with zero employees. Values corresponds to the average value of the sum of entrepreneurial ownership share across the firms' size and age distribution.

though these are the ones that exhibit the lowest trade rates. This result reflects the fact that the age distribution of entrepreneurs has an inverted U-shape. Thus, even though old entrepreneurs' selling rate is relatively high, the fraction of total trades that could be related to retirement, as proxied by share of sells done by entrepreneurs in the 65+ category, is just around 10%.

Figure A.7: Trade of Firms by Sellers' Age Group



Source: 2007 SBO.
Notes: The trade rates in Panel (a) are normalized to match the total trade rate of 2 and 3%.

A.4 Firms' Trade Rate

We indirectly infer the annual trade rate by combining firm dynamics moments, such as the entry and exit rate, and the stock of purchased firms with firms' flow equations.

Define the mass of all firms at t as y_t and the stock of firms purchased at t as x_t. Then, these variables follow the laws of motion

$$
\begin{aligned}
y_{t+1} &= y_t \left[1 - \pi^y_{exit,t} + \pi_{entry,t}\right] \\
x_{t+1} &= x_t \left(1 - \pi^x_{exit,t}\right) + \left[y_{t+1} - x_t \left(1 - \pi^x_{exit,t}\right)\right] \pi_{trade,t_+}
\end{aligned}
$$

where $\pi_{entry,t}$ and $\pi_{exit,t}$ are the annual entry rate and exit rate, respectively, and π_{trade,t_+} is the annual rate of firm trade we want to estimate. Combing the flow equations, we have that the ratio of firms traded evolves as

$$
\left(\frac{x_{t+1}}{y_{t+1}}\right) = \left(\frac{x_t}{y_t}\right) \left\{ \frac{1 - \pi^x_{exit,t} + \frac{y_t}{x_t}\left[1 - \pi^y_{exit,t} + \pi_{entry,t}\right] \pi_{trade,t_+} - \left(1 - \pi^x_{exit,t}\right) \pi_{trade,t_+}}{1 - \pi^y_{exit,t} + \pi_{entry,t}} \right\}
$$

if the exit rate for traded and non-traded firms is equated ($\pi_{exit,t} = \pi^y_{exit,t} = \pi^x_{exit,t}$), and the entry and exit rate coincide ($\pi_{e,t} = \pi_{exit,t} = \pi_{entry,t}$), then the we can calculate the steady state annual trade rate π_{trade} using the observed exit rate of firms π_e and share of traded firms $\frac{x}{y}$ using the following equation

$$
\pi_{trade} = \frac{\pi_e}{\left(\frac{x}{y}\right)^{-1} - 1 + \pi_e}.
$$

B Orbis Database Appendix

This appendix describes the Orbis database. It also presents our algorithm to identify firms' transactions using Orbis' ownership files, and presents some additional results.

B.1 The Orbis Database

To document post-trade firm dynamics, we use the historical product of Orbis, an extensive firm-level database covering millions of companies worldwide. This database is compiled by Moody's Bureau van Dijk (BvD), which aggregates data from various sources, such as national business registries, and harmonizes it into a globally comparable format.

Industry Files From Orbis, we use the industry files reporting annual balance sheets and income statements for private and publicly traded firms. The industry files contain information starting from the early 1990s to 2019. We compute firm-level output, capital, and ARPK measures using these files. This data also includes information about firms' use of inputs, country and industry identifiers, and the year they were founded.

Ownership Files In addition, we use Orbis' ownership files to identify trades in the market for firms. From 2007 onward, this database reports annual snapshots with the list of shareholders for a large number of firms. The data reports the name of the shareholder, with a unique identifier that we can track over time, and the shareholder equity share in the firm. As described below, we identify trades using changes in shareholder names through a string similarity algorithm that excludes spurious changes in owners' names or changes that could be related to inheritances and family-related transfers.

Sample Selection We focus on a sample of European private firms, which are the ones best covered by Orbis. For our baseline results, we study eleven high-income European countries which are the most comparable to the US: Austria, Belgium, Denmark, Finland, France, Germany, Italy, Norway, Spain, Sweden, and the United Kingdom. Below, we also present results for a group of ten middle-income European countries, which include: Bulgaria, Croatia, Czechia, Hungary, Latvia, Lithuania, Poland, Romania, Slovakia, and Slovenia. As documented in Kochen (2023), given the high cross-country correlation between income and finance, the high- and middle-income labels coincide with labels for developed and less financially developed countries. Our analysis focuses on the firm-year observations from 2006 to 2019 with available capital, output, and ownership data. Further, we restrict to the firms we observe for at least five years. Table B.1 presents descriptive statistics of our primary sample. The table shows that while the firms in Orbis with available ownership data are somewhat larger, they are similar in terms of firms' age and capital growth to the complete sample.

Variables' Definitions We follow Kochen (2023) for our definitions of the main variables using the balance sheet and income statements from Orbis. We measure firms' capital as equity plus net financial debt: $k_{it} = e_{it} + b_{it}$, where using Orbis acronyms, we define equity as $e_{it} = \texttt{TOAS}_{it-1} - \texttt{CULI}_{it-1} - \texttt{NCLI}_{it-1}$ and net financial debt as $b_{it} = \texttt{LOAN}_{it-1} + \texttt{LTDB}_{it-1} - \texttt{CASH}_{it-1}$. TOAS denotes total assets, CULI is current liabilities, NCLI is non-current liabilities, LOAN is short-term financial debt (payable within a year), LTDB is long-term financial debt, and CASH denotes the firm's cash and cash equivalents. Balance sheet variables in the data are reported at the end of each year. Hence, we use the one-period lag to measure the beginning of the period variable, as in the model. We measure firms' output using value-added, defined as revenue minus materials: $y_{it} = \texttt{OPRE}_{it} - \texttt{MATE}_{it}$. The wage bill is $wl_{it} = \texttt{STAF}_{it}$. Finally, we measure firms' profits as the sum of profits plus all extraordinary revenues minus extraordinary expenses $\pi_{it} = \texttt{PLAT}_{it} + \texttt{EXTR}_{it}$.[31]

Table B.1: Orbis Database Descriptive Statistics

	High-Income		Middle-Income	
	Mean	SD	Mean	SD
All Firms				
Age	16.0	13.0	12.5	8.2
Output	2.9	54.4	1.5	14.5
$\Delta \log(k)$	0.043	0.58	0.073	0.61
Obs.	16,247,768		4,252,636	
Firms w/ Ownership				
Age	17.1	13.7	12.2	7.7
Output	4.4	72.1	1.5	12.4
$\Delta \log(k)$	0.046	0.59	0.072	0.59
Obs.	8,548,886		2,203,131	

Notes: Descriptive statistics for our sample of firms between 2006-2019, with available output and capital, and observed for at least five years. All Firms are the observations in the Industry Files satisfying these criteria. Firms w/ Ownership are the observations that, in addition, have available data in the Ownership Files. Age is in years, Output is in million 2015 USD at constant exchange rates, and $\Delta \log(k)$ is capital's one-year growth rate measured in log changes.

B.2 Algorithm to Identify Trades in the Market for Firms

Using the Orbis database, we use the following methodology to identify trades in the market for firms. First, to be consistent with our model, we focus on a sample of firms where one shareholder holds at least 50% of the equity. We then identify firms' trades by tracking these majority shareholders' identities over time. After selecting all the years with a change in the name of the top shareholder, we compute four string similarity metrics for all the old and new owners' pairs: Jaro-Winkler distance; Levenshtein distance, normalized by the largest string length among the two names; Soundex; and Token Soundex measures. All these metrics lie in the [0,1] interval. After computing these measures, we

[31] This definition of capital is similar to the one used in the KFS. Appendix A.2 in Kochen (2023) shows that tangible capital and inventories account for the bulk of the balance sheet categories in k. Our definition of output y in Orbis subtract for materials, a variable we don't have available in the other data sets. Our results are almost identical if we define output using only revenue, as in the SBO.

exclude all the pairs that satisfy at least one of the following conditions:

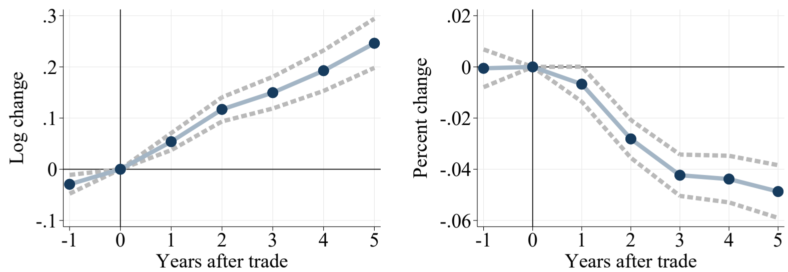
1. The pair is in the top 25 percentile of similarity according to Jaro-Winkler.
2. The pair is in the top 25 percentile of similarity according to Levenshtein.
3. Soundex is equal to 1.
4. Token Soundex is equal to 1.

Conditions 1-4 exclude changes that might be spurious due to mistakes or slight name changes. In addition, this algorithm excludes changes related to inheritances or family transfers as it would identify, for example, the pair of names that share the same last name. Finally, to exclude temporary changes, we also focus on the firms we observe being traded only once in our sample period. Our main results are robust to using alternative similarity metrics or varying the percentile thresholds in conditions 1. and 2.

B.3 Additional Results

Figure B.1: Firm Dynamics After Trade in the Orbis Data

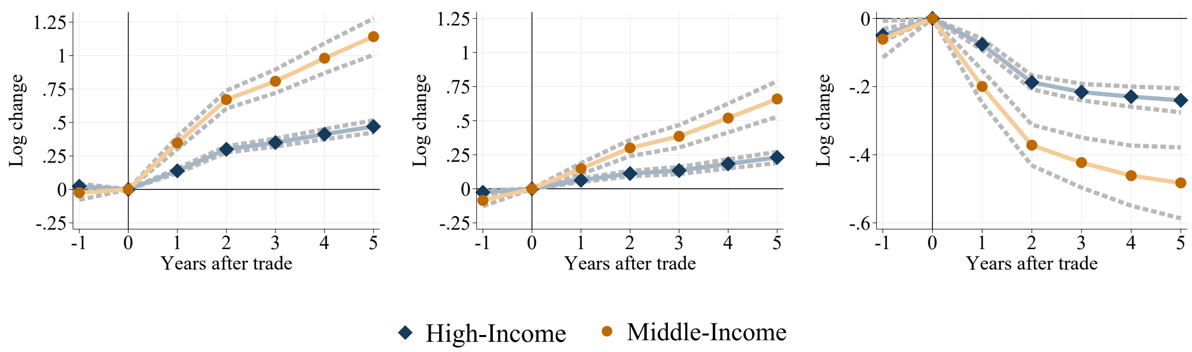
(a) Wage bill, $\log(wl)$ **(b)** Profitability, π/k



Notes: Estimated coefficients $\hat{\beta}_h$ from (15). The dashed lines correspond to 95% confidence intervals considering firm-level clustered standard errors. Profitability is measured by firms' profit to capital ratio.

Figure B.2: Firm Dynamics After Trade in Orbis: High- and Middle-Income Countries

(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$



Notes: Estimated coefficients $\hat{\beta}_h$ from (15), separately for high- and middle-income European countries. The dashed lines correspond to 95% confidence intervals considering firm-level clustered standard errors.

C Model Appendix

This appendix presents additional results and derivations from our model. It also presents a detailed description of our computation solution.

C.1 Additional Derivations

To simplify the notation, we turn to the recursive notation in steady-state.

C.1.1 Private Firms' Optimality Conditions

The solution of entrepreneurs' profit maximization problem, stated in (1), is characterized by the input demand functions

$$k(a,z) = \min\left\{k^*(z),\ \lambda a\right\}, \qquad l(a,z) = \left[\frac{z\nu}{w}\right]^{\frac{1}{1-\nu}} k(a,z)^{\frac{\theta}{1-\nu}},$$

where k^* is the unconstrained optimal level of capital given by

$$k^*(z) = z^{\frac{1}{1-\theta-\nu}} \left[\frac{\theta}{R}\right]^{\frac{1-\nu}{1-\theta-\nu}} \left[\frac{\nu}{w}\right]^{\frac{\nu}{1-\theta-\nu}}$$

which is only a function of the quality of the entrepreneurial project z.

C.1.2 Public Firm's Optimality Conditions.

The FOCs of the public firm's profit maximization problem are

$$\eta \frac{Y_p}{K_p} = R, \qquad (1-\eta)\frac{Y_p}{L_p} = w$$

which imply a relation between the public firm's capital to output and prices.

C.2 Computational Solution

To solve the model we use projection methods to approximate the value functions $\{V^o, W^o, V^w, W^w\}$. Thus, we need to solve for coefficients $\{g_V^o, g_W^o, g_V^w, g_W^w\}$ such that, at the grid points, satisfy: $V^o(a,z) = \Phi^z(a,z)g_V^o$, $W^o(a,z) = \Phi^z(a,z)g_W^o$, $V^w(a,\varepsilon) = \Phi^\varepsilon(a,\varepsilon)g_V^w$, and $W^w(a,\varepsilon) = \Phi^\varepsilon(a,\varepsilon)g_W^w$. Note that the FOCs of the public firm give us a relation between K_p/Y_p, w and r. Both K_p and L_p are determined as residuals from the market clearing conditions of capital and labor, thus we can obtain w as a function of r. This considerably simplifies the solution method of our baseline model as we only need to solve for one equilibrium price: r.

C.2.1 Algorithm

The equilibrium objects we need to solve for are

$$\left\{\underline{p}, \overline{p}, p, g_V^o, g_W^o, g_V^w, g_W^w, n_{dm}^o, n_{dm}^w, n_{cm}^o, n_{cm}^e, P_{dm}^o, P_{dm}^w, P_{cm}^o, P_{cm}^w, \beta\right\}$$

where $\underline{p}$ are sellers' minimum prices, $\overline{p}$ are buyers' maximum prices, p are the Nash bargaining prices, n are the probability densities across states, and P are the transition probability matrices (TPMs).[32] We solve for these objects using the following algorithm:

Iteration on prices

0. Propose an initial guess for r.
1. Given r, solve the model (in partial equilibrium).

Iteration on distributions

1.0. Propose an initial guess for $\{n^o_{dm}, n^w_{dm}\}$.
1.1. Given $\{n^o_{dm}, n^w_{dm}\}$, solve for $\{g^o_W, g^w_W\}$.

Iteration on value functions

1.1.0. Propose an initial guess for $\{g^o_W, g^w_W\}$.
1.1.1. Solve for the prices in the market for firms $\{\underline{p}, \overline{p}, p\}$.
1.1.2. Solve the DM problem: get $\{g^o_V, g^w_V\}$.
1.1.3. Solve the CM problem: obtain e, a' and P_{cm}.
1.1.4. Update $\{g^o_W, g^w_W\}$.
1.1.5. Iterate $\{g^o_W, g^w_W\}$ until convergence.

1.2. Update $\{n^o_{dm}, n^w_{dm}\}$.

1.3. Iterate $\{n^o_{dm}, n^w_{dm}\}$ until convergence.

2. Update r such that the capital market clears.
3. Return to 1. until r converges.

C.2.2 Solving for Prices in the Market for Firms

First, for each potential seller (a, z, κ), we solve for the sellers' minimum price by finding $\underline{p}(a, z, \kappa)$ that implies a sellers surplus, defined in (3) and (4), equal to zero. Using (5), which defines the preference shock utility transfer, the seller's surplus is equal to zero if

$$W^w(a + \kappa\underline{p}, \underline{\varepsilon}) = W^o(a, z)$$

which implicitly defines $\underline{p}(a, z, \kappa)$.

Second, for each potential firm quality z_j, we solve for buyers' maximum price $\overline{p}(\mathbf{s}_i, z_j)$, where $\mathbf{s}_i \in \{\mathbf{s}^o_i, \mathbf{s}^w_i\}$ depending on whether the buyer is a firm owner or a worker. Note that buyers' maximum price is only a function of the seller's firm quality and does not depend on the seller's assets or the preference shock. We compute the buyer's maximum price by solving for $\overline{p}$ that sets the buyer's surplus, defined in (3) and (4), to zero. For the case of current business owners with states $\mathbf{s}^o_i = (a_i, z_i)$, note that they will never buy a lower quality firm $z_j < z_i$. For those cases, we set the buyers' maximum price equal to zero.

Having computed the sellers' minimum prices, $\underline{p}$, and the buyers' maximum prices $\overline{p}$, we can identify the matches with positive gains from trade using the sufficient condition

[32] Where $\int n^o(a, z)\mathrm{d}a\mathrm{d}z = s^o$ and $\int n^w(a, \varepsilon)\mathrm{d}a\mathrm{d}\varepsilon = (1 - s^o)$.

presented in (6). Then, for each potential match of a seller, with states $(\mathbf{s}_j^o, \kappa_j)$, and a buyer, with states $\mathbf{s}_i$, such that there are positive gains from trade, given by $\underline{p}(\mathbf{s}_j^o, \kappa_j) < \overline{p}(\mathbf{s}_i, z_j)$, we approximate the Nash bargaining price, defined in (7), as

$$p(\mathbf{s}_{it}, \mathbf{s}_{jt}^o, \kappa_{jt}) \approx \chi \underline{p}(\mathbf{s}_j^o, \kappa_j) + (1-\chi)\overline{p}(\mathbf{s}_i, z_j) \tag{17}$$

where $\mathbf{s}_i \in \{\mathbf{s}_i^o, \mathbf{s}_i^w\}$ and χ is the buyers' bargaining power. In our numerical simulations, we found that computing the price using (17) is an extremely accurate approximation to the Nash bargaining price obtained from solving the maximization problem (7) while delivering improvements in computational time of several orders of magnitude.

C.2.3 Solving for g_V^o and g_V^w

Given $\{\underline{p}, \overline{p}, p, n_{dm}^o, n_{dm}^w, g_W^o, g_W^w\}$, we can compute the value at DM for firm owners and workers. Then we can solve for g_V^o and g_V^w by inverting the basis functions Φ^z and Φ^ε.

C.2.4 Solving for a', g_W^o and g_W^w

Having solved for the coefficients g_V^o and g_V^w we can solve the households' problems in the production stage (CM). Given r and w, both entrepreneurs and workers problems are a single variable optimization problem in a', which we can solve using golden search.

To obtain g_W^o and g_W^w we use value function iteration. First, by substituting the corresponding optimal policies we obtain two linear systems of equations on g_W^o and g_W^w. Then, we can solve for the coefficients by just inverting the basis functions. For stability reasons we make the update of g_W^o and g_W^w with some dampening.

C.2.5 Transitions and Stationary Distribution

Define the densities across states in DM and CM subperiods as

$$n_{dm} = \begin{bmatrix} n_{dm}^o \\ n_{dm}^w \end{bmatrix} \text{ and } n_{cm} = \begin{bmatrix} n_{cm}^o \\ n_{cm}^w \end{bmatrix}$$

where n_{dm}^o and n_{cm}^o are vectors of size N_o and n_{dm}^w and n_{cm}^w are vectors of size N_w. N_o and N_w are the basis functions grid sizes denoting the number of (a, z) and (a, ε) combinations, respectively. Here $\sum_i n_{dm} = 1$, thus, $\sum_i n_{dm}^o = s_{dm}^o$ and $\sum_i n_{dm}^w = (1 - s_{dm}^o)$.

Then, the TPMs between DM and CM and CM and DM_{+1} solve

$$(n_{cm})^\intercal = (n_{dm})^\intercal P_{dm}, \qquad (n'_{dm})^\intercal = (n_{cm})^\intercal P_{cm}$$

where $(.)^\intercal$ denotes the transpose operator.

We can divide the TPM in blocks differentiating between the two type of agents:

$$P_{dm} = \begin{bmatrix} P_{dm}^{oo} & P_{dm}^{ow} \\ P_{dm}^{wo} & P_{dm}^{ww} \end{bmatrix} \text{ and } P_{cm} = \begin{bmatrix} P_{cm}^{oo} & P_{cm}^{ow} \\ P_{cm}^{wo} & P_{cm}^{ww} \end{bmatrix}$$

where P_{dm}^{oo} captures the transitions of firms' owners that bought another firm or didn't trade, P_{dm}^{ow} is for owners that sold their firm, P_{dm}^{wo} for workers who bought a firm and P_{dm}^{ww} for workers who didn't trade. Regarding CM TPMs, P_{cm}^{oo} is for business owners who operated the firm, P_{cm}^{ow} for owners who didn't operate and went to the labor market, P_{cm}^{wo} for workers who received the $(1-\zeta)$ shock, P_{cm}^{ww} for workers that didn't. Note that besides changes in the exogenous shocks, asset holdings also change due to payments in the market for firms and due to savings in CM.

Stationarity requires that

$$n_{dm}^{\intercal} = n_{dm}^{\intercal} P_{dm} P_{cm}$$

or

$$\left[I - (P_{dm} P_{cm})^{\intercal}\right] n_{dm} = 0$$

which implies that we can solve for n_{dm} by computing the eigenvector of $(P_{dm} P_{cm})^{\intercal}$ associated with the unit eigenvalue, normalized such that $\sum_i n_{dm}(i) = 1$.

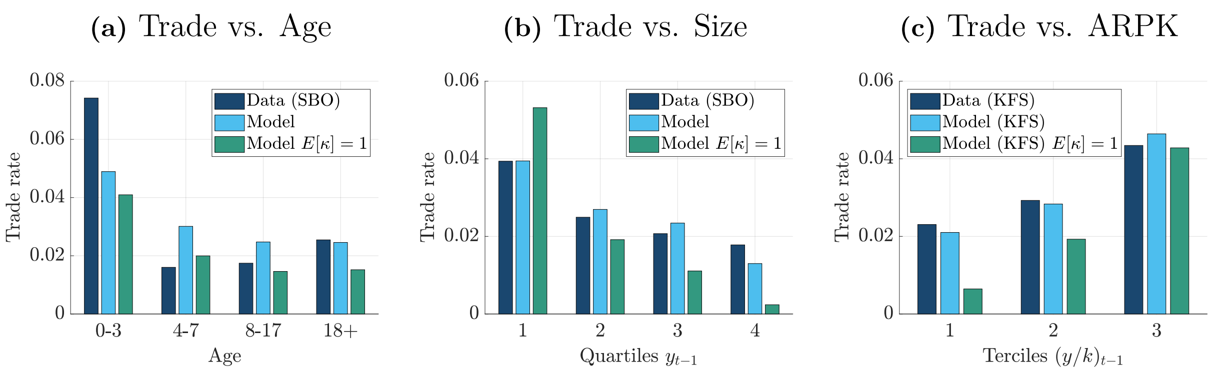
C.3 Additional Results

Table C.1: Wealth Ratio of Firm Buyers' to Households and Entrepreneurs

	Data	Model
Firm Buyers to Average Household		
Wealth $(a+p)$	3.83	3.09
Wealth Excluding Business Wealth (a)	2.71	2.74
Firm Buyers to Average Entrepreneur		
Wealth $(a+p)$	0.69	0.54
Wealth Excluding Business Wealth (a)	0.79	0.75

Source: 1989-2016 SCF.
Notes: We define firm buyers, in the SCF, as those entrepreneurs who purchased their primary business in the year of the survey or the previous one. We compute the ratio as the average wealth of firm buyers divided by the average wealth of all households or entrepreneurs. Because of the small sample of recent business buyers, we take the average across SCF waves. Entrepreneurs are defined as self-employed business owners who manage a business with at least one employee.

Figure C.1: Trade Rate by Firms' Characteristics with and without Preference Shocks



Notes: Trade rate by firms' characteristics in the data and data simulated from the model. To be consistent with the data, Model (KFS) restricts to a sample of firms of age less or equal to 7. See the notes in Figure 1 for a description of the data moments.

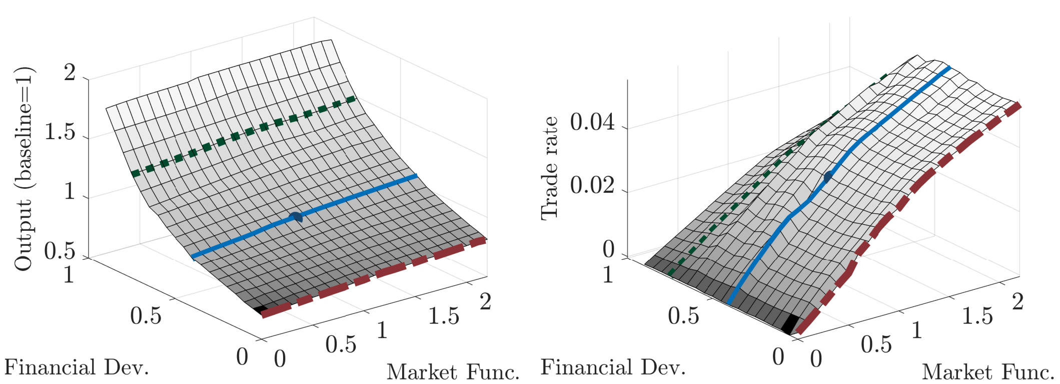
Table C.2: Untargeted Moments

	Data	Model		Data	Model
Income Distribution All Households			*Wealth Distribution All Households*		
Top 1	0.22	0.20	Top 1	0.33	0.40
Top 5	0.39	0.39	Top 5	0.60	0.62
Top 10	0.49	0.54	Top 10	0.72	0.75
Bottom 75	0.31	0.30	Bottom 75	0.13	0.07
Bottom 50	0.12	0.16	Bottom 50	0.02	0.01
Bottom 25	0.02	0.04	Bottom 25	0.00	0.00
Income Distribution Entrepreneurs			*Wealth Distribution Entrepreneurs*		
Top 1	0.23	0.36	Top 1	0.24	0.29
Top 5	0.44	0.67	Top 5	0.45	0.63
Top 10	0.57	0.81	Top 10	0.60	0.80
Bottom 75	0.24	0.15	Bottom 75	0.18	0.11
Bottom 50	0.10	0.11	Bottom 50	0.05	0.06
Bottom 25	0.03	0.07	Bottom 25	0.01	0.04

Source: 2007 SCF.

Figure C.2: Financial Development and Functioning of the Market for Firms

(a) Private Firms' Output **(b)** Trade Rates



Notes: Financial Development is defined by firms' maximum leverage, $(\lambda - 1)/\lambda$. Market Functioning is parameterized by ω multiplying the search frictions in the market for firms $\alpha_o(\omega) = \min\{\omega\alpha_o\}, \alpha_w(\omega) = \min\{\omega\alpha_w, 1\}$.