Financial Frictions and the Market for Firms*

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Abstract

We study and quantify the aggregate implications of the trade of firms in the presence of financial frictions. In the U.S., one out of four entrepreneurs purchased their business. However, this number has decreased in the last three decades. In the cross-section, younger, smaller, and high return to capital firms have the highest trading rates. To explain these findings, we propose a model of entrepreneurship with a frictional market for firms in which gains from trade can arise from credit constraints and incomplete financial markets. Our results suggest that the better allocation of capital due to the trade of firms is significant, accounting for 9.1% of entrepreneurial output and 2.2% of TFP.

Keywords: financial frictions, search frictions, misallocation, entrepreneurship.

JEL classifications: E44, L20, G30.

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1 Introduction

Markets are the predominant allocation mechanism of modern economies. One important market that allocates productive projects and available resources is the market in which firms are bought and sold, i.e., *the market for firms*. In this paper, we argue that the role of this market is particularly relevant in economies where financial constraints are a pervasive feature of entrepreneurial activity. In such environments, who owns the firms matters for allocations, as credit constrained entrepreneurs will produce at a suboptimal scale resulting in capital misallocation and lower aggregate output. The market for firms allows financially constrained entrepreneurs to sell their firms to other parties with more financial resources, potentially improving the allocation of capital in the economy.

We study the aggregate implications of the market for firms as an allocation mechanism in two steps. First, we use micro data from business owners, households, and firms to document novel facts about firms' trade in the U.S. economy. Second, we develop a macroeconomic model where agents can buy and sell firms in a frictional market. In the model, gains from trading firms arise from financial frictions, namely credit constraints and incomplete financial markets, and preference shocks that capture alternative reasons to trade firms. We quantify the model to match salient features of the market for firms we documented from the data. We then use our quantitative framework as a laboratory to analyze the importance of the different motives behind firms' trade and study the relevance of this market for aggregate output and productivity.

For our empirical results, we focus on entrepreneurs and study how they acquired their businesses. We define entrepreneurs as self-employed private business owners who actively manage their firms and have at least one employee. Using multiple data sources, we document four main facts. First, one out of four entrepreneurs (around 23% to 26%) in the U.S. acquired their business by purchasing an existing firm, implying an annual trade rate of 3%. This result indicates that private businesses are highly *illiquid assets*. Compared to housing, for example, Berger and Vavra (2015) reports that 5% of houses are traded every year, higher than the 3% annual trade rate we find for private firms. Nonetheless, the trade of private firms is larger, in terms of volume, than the trade of specific intangible assets such as patents. For example, Akcigit, Celik, and Greenwood (2016) document that 16% of the registered patents in the U.S. have been traded, smaller than the 26% we document for private businesses.

Second, we document that the share of entrepreneurs that purchased their business has declined by roughly one-third in the last 30 years. The fall in firms' trade coincides with a period characterized by declining business dynamism in the U.S. economy, suggesting a possible relation between these trends (see, for example, Akcigit and Ates (2021) for a recent review of that literature). As we explain below, our theory suggests an alternative explanation related to changes in the economy's financial conditions. Intuitively, if

business owners have more access to external financing, the gains from trading firms will be lower. Hence, looser aggregate credit conditions might reduce the number of trades in the economy.

For our third fact, we document two salient characteristics of business buyers. Our first finding reveals that 66% of buyers have never been an entrepreneur before purchasing their current business. This finding suggests that purchasing an existing firm is a relevant way to enter into entrepreneurship, which, to the best of our knowledge, has not been studied before. Besides capturing the illiquidity of private firms, our theoretical framework incorporates this novel feature about households' possible transitions into entrepreneurship through the market for firms. Our second finding shows that the average wealth of firm buyers is about three times that of the average household. This evidence will serve us to test some of the implications of our theory of financial frictions being a relevant driver for firms' trade.

Finally, we document that, in the cross-section, young, small, and high return to capital firms have the highest trading rates. These results regarding firms' observable characteristics and trade frequency are highly informative about the underlying mechanisms behind firms' trade. In this sense, any theory about the trade of firms should be able to accommodate these relations. Both firms' age and size are associated with financial constraints (Hennessy and Whited, 2007; Hadlock and Pierce, 2010). Further, firms' returns to capital are also informative about their access to external finance, as credit constrained firms will have high returns but cannot increase their investment. By introducing financial frictions as a micro foundation that generates gains from trading firms, our model can account for the fact that younger, smaller, and high return to capital firms have the highest probability of trade.

Motivated by these findings, in the second part of the paper, we develop a heterogeneous agent model with entrepreneurship and frictional trade of firms. Our model economy is populated by a continuum of households which can be firm owners or workers. Firm owners can trade or shut down their firm, while workers can become business owners by buying an existing firm or through some exogenous startup shock. There are incomplete financial markets, so households are subject to uninsurable idiosyncratic risk. On the one hand, firm owners are exposed to the risk associated with the quality of their firm, which evolves stochastically. On the other hand, workers are subject to shocks to their labor efficiency.

We characterize firms through the quality of an entrepreneurial project which is indivisible, rival, and excludable. These entrepreneurial projects aim to capture firms' intangible assets.[1] Firms enable their owner to produce the final consumption good with a technol-

[1]Using data from business transactions, Bhandari and McGrattan (2021) document that when a firm is sold, around 60% of its total value is accounted by intangibles. This evidence supports our characterization of firms by the value of their intangible assets.

ogy that combines capital, labor, and the firm's quality. Besides the firms owned by a single household, which we call private firms, there is a second production sector with a representative unconstrained public firm. Both sectors produce the same good, which can be used for consumption or savings in a risk-free asset. There is also a financial intermediary that, each period, takes the savings from the households and rents capital to the firms.

Our empirical findings indicate that private firms are highly illiquid assets, which motivates using a search-theoretic approach to model this market. Specifically, we model the market for firms through a decentralized market subject to *search frictions* and bilateral random matching. One interpretation of our setup is that agents can valuate only one firm at a time, which delays trade.[2] This setup is suitable for our quantitative analysis and allows the model to match relevant features about the trade of firms documented in the empirical section of the paper.

Exchanges in the market for firms are voluntary. Hence, a necessary condition for trade is that agents have different valuations for the same firm. In particular, the buyer must have a higher valuation than the seller. In our theory, heterogeneous valuations for firms arise from three sources: firms' credit constraints, incomplete markets, and preferences shocks. Credit constraints and incomplete markets generate an endogenous motive to trade firms. For a given firm, unconstrained agents attain higher profits, grow the firm faster, and bear the risk better than constrained agents. By transferring firms between agents at different levels of wealth, the market for firms can improve allocative efficiency. In addition, we assume that potential sellers are subject to idiosyncratic preference shocks that capture, in a parsimonious manner, non-pecuniary benefits and other motives to trade firms that we do not explicitly incorporate in our theory.

We calibrate the model's parameters to match several features of the U.S. economy. We target moments related to the role of entrepreneurs, the income and wealth distribution across households, the relative importance of the private business sector, and key characteristics of the market for firms. To validate our theory, we compare the model-simulated relations between trade rates and firms' observable attributes with the ones we document in the micro data. Our model predicts that young, small, and high return to capital firms have the highest trading rates, consistent with the data. This result is explained by the fact that these groups of firms are associated with binding credit constraints in our model, which generates gains from trading firms. We also test the prediction of our model regarding the characteristics of business buyers and find that, as in the data, buyers are up to three times wealthier than the average household in the economy.

Using our calibrated model, we first quantify the importance of the different motives behind the trade of firms. Idiosyncratic preference shocks, which we identify using the

[2] BizBuySell, an online marketplace for businesses, surveyed their clients about their major challenges when purchasing a firm. For more than 40%, the major issue was "finding the right business", and for 23% was "valuating the firm". These responses are consistent with our modeling of the market for firms.

frequency of trade of large unconstrained firms, account for 31% of the exchanges. Concerning the two motives related to financial frictions, we find that incomplete markets, which create differences in risk-bearing capacity across agents, explain 16% of the trades. Thus, collateral constraints, which limit firms' borrowing to a multiple of the current owner's wealth, account for most of the trades in our model economy.

We then quantify the macroeconomic implications of the market for firms as a mechanism that allocates productive projects and available resources in the economy. We perform two counterfactual experiments. In our first experiment, we take our baseline model and analyze a scenario in which the market for firms shuts down. Closing this market implies a fall in aggregate entrepreneurial output and TFP of 9.1% and 2.2%, respectively. This result is explained by the lower entrance into entrepreneurship and the poorer allocation of firms and available resources when this market is absent.

The previous exercise shows that the market for firms alleviates the capital misallocation caused by financial frictions. To better understand the gains in total factor productivity (TFP) from this market, we consider the alternative economy with no trade of firms in our second experiment. Then, we ask: what credit conditions does the no market economy require to match the TFP level of our baseline economy? The no market model requires looser credit conditions such that private firms' leverage, or the debt-to-capital ratio, increases by 14 percentage points (p.p.), from 0.35 to 0.49. This increase is sizable as, for example, during the Great Recession, leverage dropped by a total of 5 p.p.

We close our analysis by studying the relation between aggregate financial conditions and the market for firms. In the empirical section, we document that the share of entrepreneurs that purchased their firm fell by around one-third during the last 30 years. One implication of our theory is that looser credit conditions will decrease the gains from trading firms, resulting in fewer transactions. Consistent with our theory, we show that the time series aggregate data for the U.S. shows a marked negative correlation between firms' leverage and the fraction of traded firms. In the data, however, there could be multiple reasons behind this negative correlation. To isolate the role of credit on firms' trade, we use our model and perform comparative statics for different levels of firms' access to external financing. We find that easier access to credit can explain up to 40% of the fall in the share of traded firms observed in the last three decades in the U.S. economy.

Overall, our paper contributes to the literature in several ways. First, it documents new facts about the trade of privately held firms in the U.S. economy. Second, our paper develops a novel model of entrepreneurship and trade of firms that allows us to study the relation between financial frictions and the market for firms. Third, it presents new evidence indicating that financial frictions are a relevant motive for trading firms. Finally, our paper quantifies the importance of this market as a mechanism that allows for a better allocation of capital across firms in the economy.

Related Literature Our paper is related to the following strands of literature.

Entrepreneurship. Our theoretical framework builds on the literature on heterogeneous agents models with entrepreneurship, see, for example, Quadrini (2000), Cagetti and De Nardi (2006), and more recently Bhandari and McGrattan (2021) and Peter (2021).[3] An important feature of these models is that they can match the observed income and wealth distribution through the combination of uninsurable income risk and stochastic returns to wealth from entrepreneurial activity. We contribute to this literature by allowing entrepreneurial projects to be tradable in a frictional market for firms.

Trade of ideas. Additionally, from a modeling point of view, our paper also relates to the literature that studies the trade of ideas or patents and its implications for economic growth (Silveira and Wright, 2010; Akcigit, Celik, and Greenwood, 2016). Similar to these studies, we use a framework characterized by bilateral meetings subject to search frictions to model the trade of private firms.

Trade of firms. Our work is mostly related to recent literature that studies firms' trade as an allocation mechanism. Caselli and Gennaioli (2013) and Gaillard and Kankanamge (2020) study the trade of mature firms, where gains from trade arise, for example, due to business owners' life cycle considerations. Sevcik (2015) studies the formation of business groups to curtail financial frictions, and David (2021) studies mergers between firms in an environment with complementarities. More recently, Bhandari, Martellini, and McGrattan (2022) studies firms' trade as a mechanism to accumulate intangible capital when it is subject to indivisibilities. Our contribution to this literature is twofold. First, we document novel facts about the market for firms, particularly about the market participants and the characteristics of the traded firms, which are highly informative for the different theories on firms' trade. Second, and different from other papers in the literature, we focus on the trade of financially constrained firms and quantify the importance of this market in an economy with imperfect credit markets.

Finance and misallocation. Our paper also relates to the literature on financial frictions and misallocation as a source of low TFP (Hsieh and Klenow, 2009; Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014). We contribute to this literature by showing that the market for firms can reduce the capital misallocation caused by financial frictions.

Outline The rest of the paper is organized as follows: Section 2 presents our main empirical results; Section 3 presents the model; Section 4 describes our parameterization strategy and validation exercises; Section 5 explains the main properties of our model; Section 6 presents our aggregate results; and finally, Section 7 concludes.

[3]From this literature, the closest to our work is Peter (2021), which allows firms to go public through an initial public offering. Distinctively, our paper focuses on transactions in which ownership and management are fully transferred, which, as we show, is typically the case in the market for private firms.

2 Evidence on the Market for Firms

In this section, we use micro data from business owners, households, and firms, to document relevant facts about the market for firms. First, we study how many entrepreneurs purchased their businesses and how this moment has evolved over the last three decades. Next, we present evidence about the previous occupation and wealth of firm buyers. Lastly, we study the characteristics of the traded firms. Appendix A presents robustness checks and additional empirical exercises.

2.1 Data Sources

We use four different surveys related to private firms, their characteristics, and the characteristics of their owners. First, our main data source is the Survey of Business Owners (SBO) Public Use Microdata Sample (PUMS). This survey provides comprehensive information about businesses and business owners. In particular, about how do they acquired their business. The PUMS sample is representative of all non-farm private businesses in the U.S. and is available for the year 2007.

Second, we use nine waves of the Survey of Consumer Finances (SCF) covering the period between 1989 and 2016. Importantly, the SCF includes detailed information about households' income and balance sheets, which we will use to discipline our quantitative model's income and wealth distribution. Additionally, this survey asks business owners how they acquired their firms. The information in the different waves of the SCF allows us to study how the ownership of firms has evolved over time.

Third, we use the Annual Survey of Entrepreneurs (ASE) as a complementary data source. The ASE is available yearly from 2014 to 2016. As the SBO, this survey is representative of all non-farm private businesses in the U.S. Finally, we use data from the Kauffman Firm Survey (KFS). The KFS is an eight-year panel of firms that started operations in 2004 and were followed through 2011. Compared to the previous datasets, the KFS has the advantage of containing information about firms' balance sheets, allowing us to compute firm-level capital. However, the KFS sample is not representative of the private sector, which we will consider when comparing the KFS evidence with data simulated from our model. See Appendix A.2 for further details about these datasets, definitions, and our sample selection criteria.

2.2 Entrepreneurs

Our empirical analysis focuses on *entrepreneurs* as the observation unit. We follow Cagetti and De Nardi (2006) and define entrepreneurs as self-employed individuals who own a business and have an active management role in it. Given our interest in the trade of firms, for our baseline results, we restrict to the entrepreneurs with at least one

employee.[4] According to the 2007 SCF, entrepreneurs represent 6% of households. As previous studies have documented, although entrepreneurs represent a small fraction of the population, they earn 20% of total income and hold 33% of total wealth. In our calibration strategy, we will target these key features of the role of entrepreneurs in the economy.

Throughout our analysis, we assume that each entrepreneur owns and manages only one firm. This assumption implies that the number of firms traded every period equals the number of entrepreneurs that trade their firms. Hence, we use both terms interchangeably. Our assumption relies on the fact that, according to the SCF, more than 80% of entrepreneurs own only one firm. Additionally, according to the SBO, more than 74% of the private firms in the economy have only one entrepreneur, while more than 96% of the firms have at most two entrepreneurs.[5]

2.3 How do Entrepreneurs Acquire Their Firms?

Share of Traded Firms As a first step, we focus on the SBO and the 2007 SCF and look at how entrepreneurs acquire their firms.[6] We focus on three main types of acquisitions: founded the firm, purchased an existing firm, and inherited or a different kind of acquisition. Table 1 shows that two-thirds of entrepreneurs founded their firms. Founding the firm is, of course, the most common way entrepreneurs acquire their businesses. Also, it shows that between 9 to 12% acquired it through inheritance or other types of acquisition. The most relevant number for our analysis is that 23% to 26% of the entrepreneurs, depending on the survey, acquire their business by purchasing an existing firm.

Table 1: Share of Entrepreneurs by Business Acquisition

	Founded	Purchased	Inherited/Other
SBO	65.2%	25.5%	9.3%
SCF	65.3%	22.7%	12.0%

SOURCE: SBO and SCF for the year 2007.
NOTES: Entrepreneurs are defined as (i) self-employed, (ii) business owners, who (iii) actively manage their firm, and (iv) the firm has at least one employee. Other type of acquisition groups: acquired as a transfer, as a gift or other not specified.

In Appendix A.3.1, we show that our results are robust to several alternative definitions for an entrepreneur. We also show that franchises or some specific production sectors do

[4] We focus on the entrepreneurs with a positive number of employees to exclude the cases of self-employed individuals whose businesses might not be transferable. As a robustness exercise, in Appendix A.3, we present the results considering all entrepreneurs (with employer and non-employer firms).

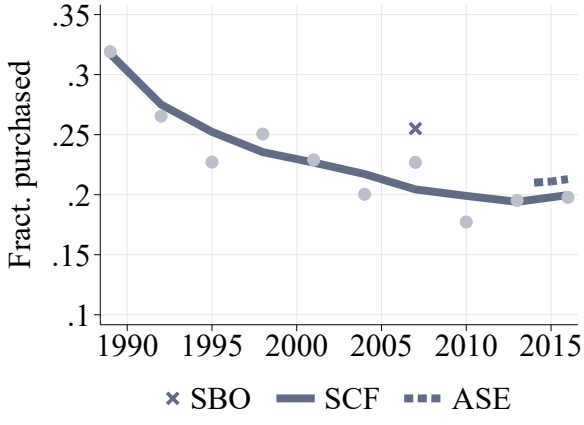
[5] In Appendix A.4.1, we document that the ownership and the management of privately held firms in the U.S. are highly concentrated, even for the economy's oldest and largest private firms.

[6] Specifically, the SBO asks: "*How did [the owner] initially acquire ownership of this business?*". Similarly, the SCF asks business owners: "*How did you first acquire this business?*".

not drive these numbers. Overall, across all our robustness exercises, we find that around one out of four entrepreneurs acquired their firm through a purchase.[7]

Trade of Firms Across Time As the PUMS version of the SBO is only available for 2007, we use the SCF to document the evolution of the share of entrepreneurs that purchased their firms across different years. As the SBO and SCF 2007 results on how entrepreneurs acquire their businesses are consistent, we are confident in using the SCF to measure this moment over time. Additionally, as a robustness check, we also consider data from the ASE available for 2014 to 2016. Overall, the numbers obtained from the SCF align very well with the SBO and ASE for the years in which these surveys overlap.

Figure 1: Fraction of Entrepreneurs that Purchased Their Business



SOURCE: SBO, SCF and ASE.
NOTES: Entrepreneurs are defined as self-employed, business owners, who actively manage their firm and the firm has at least one employee. The light-colored dots correspond to the time series SCF data points. The solid line trend was estimated using locally weighted smoothing.

Figure 1 shows that between 1989 and 2016, the fraction of entrepreneurs that acquired their firms through a purchase, which proxies for the fraction of traded firms, declined by one-third. More precisely, the fraction of entrepreneurs that purchased their business fell by 12 p.p. going from 32% in 1989 to 20% by 2016. In Appendix A.3.3, we show that the decreasing trend is robust to alternative samples and definitions. It is worth mentioning that most of the fall occurred before 2007. Since the Great Recession, this share has been relatively stable. In Section 6, we study, through the lens of our model, how looser credit conditions can explain part of the decline in the share of firms traded.

Firms' Trade Rate The previous results refer to the *stock* of firms that have been traded at any point in the past. We are also interested in the frequency at which firms are traded, i.e., the trade *rate*. We estimate the percentage of firms traded every year using two strategies. The first strategy looks at the percentage of firms purchased in

[7]In Appendix A.4.2, we analyze whether the trade of firms is related to entrepreneurs' life cycle. We find that, at most, 10% of the total trades we observe could be related directly to retirement motives.

the SBO and SCF data in the same year of the survey. The second strategy relies on a back-of-the-envelope calculation using the stock of traded firms and firms' entry and exit rates.[8] Both strategies imply that around 3% of the firms are traded every year. As mentioned above, this trade frequency implies that private firms are highly *illiquid* assets as they trade less frequently than housing but more frequently than, for example, patents.

2.4 Buyers' Characteristics

Buyers' Previous Occupation From the SBO, we can obtain information regarding entrepreneurs' previous occupations. We found that 66% of the entrepreneurs that purchased their firm have never been self-employed. Hence, most likely, these individuals were in the labor market before acquiring their businesses. This result indicates that buying an existing firm is a relevant channel for entering into entrepreneurship, which, to the best of our knowledge, has not been studied before. Besides capturing the illiquidity of private firms, our theoretical framework will incorporate this novel feature about households' possible transitions into entrepreneurship through the market for firms.[9]

Buyers' Wealth Using the SCF, we can identify the entrepreneurs that recently purchased their businesses and measure their wealth. Table B.1, in the Appendix, reports the wealth of the average buyer relative to the wealth of the average household. We consider two definitions of wealth, with and without business wealth. The average firm buyer is 2.7 times wealthier than the average household, excluding business wealth. Considering total wealth, this number is 3.8. In Section 4.3, we compare this evidence with the characteristics of buyers in our model and discuss how the fact that business buyers are wealthier than the average household is consistent with our theory that financial reasons are an important driver behind the trade of firms.

2.5 Trade Rate and Firms' Characteristics

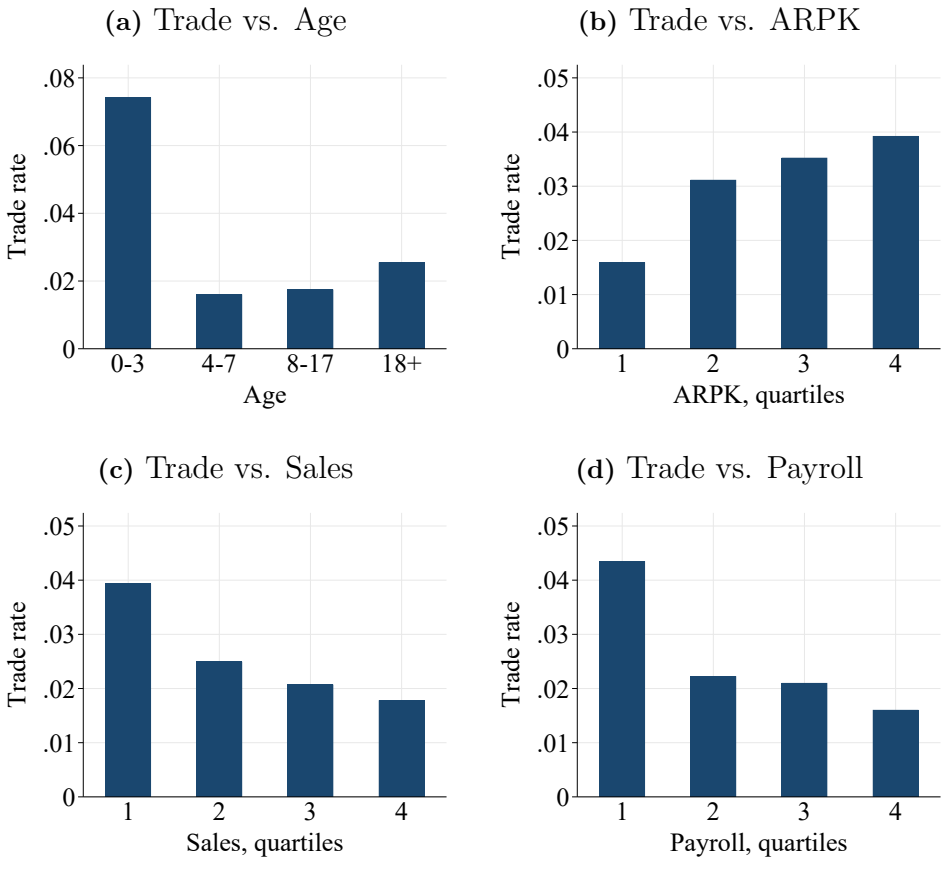
In this last section of the empirical analysis, we document cross-sectional evidence for the frequency of trade conditional on firms' observable characteristics. We focus on three main attributes: firms' age, size, and the average revenue product of capital.

Firm Age We measure firms' age using data from the SBO. Specifically, we look at all the businesses purchased in 2007 and compute firms' age as the difference between the year the firm was purchased (2007) and the year when the firm was founded. Panel (a) of Figure 2 presents the trade rate across different age bins. The figure shows that the youngest firms (0-3 years old) have the highest trading rates, with a trading frequency

[8] See Appendix A.5 for the details of these calculations.

[9] We consider the question in the SBO: "*Prior to acquiring this business, had the owner ever owned a business or been self-employed?*" This number should be interpreted as a lower bound of our non-entrepreneur definition (i.e., the complement of being an entrepreneur). Appendix A.3.2 documents that this result is robust to alternative samples and definitions. Further, we show that the share of workers among firms buyers does not appear to be related to specific sectors or types of firms.

Figure 2: Trade Rate by Firms' Characteristics



SOURCE: SBO and KFS.
NOTES: Panels (a), (c) and (d) use data from the 2007 SBO. In panel (a) trade is computed using the fraction of owners that acquired their firm through a purchase in 2007. The age of the firm is computed as the difference between 2007 and the year when the business was established. Panel (b) uses data from KFS. Trade is computed using information from all the firms sold during the years of the sample. Average revenue product of capital (ARPK) is measured by sales over capital of the previous year to the sale. Capital includes inventories, equipment and machinery, land, buildings, and structures, vehicles and other assets owned by the business. The relation is computed for every year and then averaged across time. In panels (c) and (d) trade is computed using information from the firms that were sold in or after 2007. Trade rates are normalized to match the aggregate of our baseline calculations.

almost three times larger than the other age groups. However, the relation is relatively flat or slightly increasing for the oldest firms.

Firm Size We use the SBO to study the relation between trade and size, but now we focus on firms that were sold to measure their size before the exchange occurs. For this, we look at the sample of business owners that sold their firm in or after 2007 and measure size using data from the previous year of operation. Thus, we relate the trade probability at t against the firm's size at $t-1$. We consider two measures of size given by firms' total sales or total payroll. Panels (c) and (d) of Figure 2 present the probability of trade for different quintiles of the sales and payroll distributions. We find that the frequency of trade and firm size are negatively related. Thus, the smaller firms, measured by either sales or payroll bottom quartile, have the highest trading probabilities.

Returns to Capital Finally, we document the relation between the trade rate and firms' average revenue product of capital (ARPK). We measure ARPK using data from the KFS, as this data includes information about firms' balance sheets that allow us to compute a firm-level measure of capital.[10] As the analysis for size, we relate firms' ARPK at period $t-1$ against the probability of trade at t, which we measure as the share of owners that report having sold or merged their business. Panel (b) of Figure 2 shows a positive relation between the frequency of trade and ARPK, with the top quartiles ARPK firms having the highest trading rates.

In sum, we document that young, small, and high return to capital firms have the highest trading rates. These results regarding firms' observable characteristics and trade frequency are highly informative about the underlying mechanisms behind firms' trade. In this sense, any theory about the trade of firms should be able to accommodate these relations. By introducing financial frictions as a micro foundation that generates gains from trading firms, the model that we describe in the next section can account for these cross-sectional facts.

3 A Model of Entrepreneurship and Trade of Firms

In this section we develop a general equilibrium heterogeneous agent model with four key ingredients: endogenous occupational choice between entrepreneurship and labor, uninsurable income risk for workers and entrepreneurs, firm-level credit frictions, and a frictional market in which firms can be bought and sold.

3.1 Environment

Our model economy is inhabited by a continuum of households in $[0, 1]$. Households can have two possible occupations: *firm owners* or *workers*. Firm owners can buy and sell firms and choose whether to operate their current firm and be *entrepreneurs* or close the firm and become workers. Workers can become firm owners by acquiring a firm or through some exogenous *startup* shock. We explain the transitions between these two occupations in further detail below.

Besides the firms managed by households, which we call *private firms*, there is a second sector of production that features a representative *public firm*. Both sectors produce the same good, which can be used for consumption or savings. Capital is produced by a *financial intermediary* which, each period, takes savings from households and rents capital to the firms. The public firm and the financial intermediary are owned by all households in equal shares.[11]

[10] We measure capital as the sum of the book value of inventories, equipment and machinery, land, buildings, and structures, vehicles, and other types of assets owned by the business.

[11] Alternatively, we could have assumed that the intermediary and the public firm issue equity shares, which are traded between households in a frictionless centralized market. This setup is analogous, as

Time is discrete and infinite, and each time period is divided into two stages. The trade of firms occurs in the first stage, which we call the decentralized market, or *DM*. We assume that, in the market for firms, households meet bilaterally subject to *search frictions*, which may restrain the frequency and the type of the matches. All production, consumption, and saving decisions take place in the second stage, which we call the centralized market, or *CM*.

3.1.1 Households

Households have preferences over consumption c represented by a constant relative risk aversion (CRRA) utility function

$$u(c_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma}$$

where σ is the risk aversion coefficient.

They are heterogeneous in their occupation and their asset holdings a_{it}. Assets are subject to a non-borrowing constraint, $a_{it} \geq 0$, and are deposited with the financial intermediary, which pays a risk-free interest rate of r for the deposits. There is no aggregate uncertainty in this economy. However, households face idiosyncratic uninsurable risks.

Firm owners are endowed with a private firm that enables the owner to produce the final consumption good with a technology that uses capital, labor, and the firm's quality. We describe this technology below. The quality of the firm, denoted by z_{it}, is stochastic and evolves according to the law of motion

$$z_{it+1} = \begin{cases} z_{it} & \text{with pr. } \gamma \\ z' \sim \mathcal{P}(z_{min}, \eta_z) & \text{with pr. } (1-\gamma) \end{cases}$$

where $\mathcal{P}$ denotes a Pareto distribution with scale and a shape parameters z_{min} and η_z, respectively. The $(1-\gamma)$ shock can be interpreted as changes in market conditions that affect the profitability of entrepreneurial projects as in Buera, Kaboski, and Shin (2011).

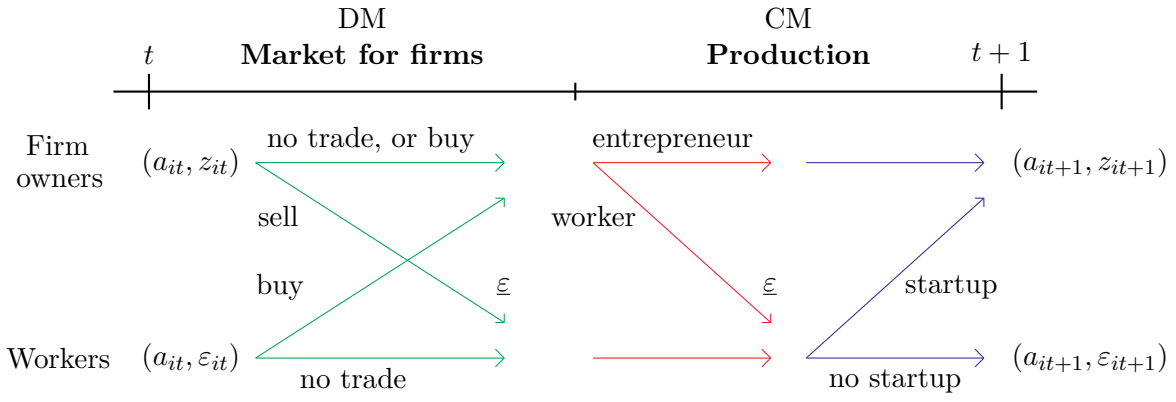
On the other hand, workers are endowed with one unit of labor, which they supply inelastically, and are heterogeneous in their labor efficiency ε_{it}. We assume that the logarithm of the labor market efficiency evolves according to an AR(1) process with persistence ρ_ε and volatility σ_ε. Specifically,

$$\log \varepsilon_{it+1} = \rho_\varepsilon \log \varepsilon_{it} + \sigma_\varepsilon u_{it+1},$$

where u is a standard normal random variable.

assets and shares holdings would be indeterminate. Below we assume that the intermediary and the public firm make zero profits. Thus, this modeling choice is not crucial for the analysis.

Figure 3: Transitions Between Occupations



Regarding the transitions between occupations, workers can become firm owners by purchasing an existing firm or through an exogenous *startup* shock at the end of the period. At the beginning of the production stage, firm owners face an occupational choice. They decide whether to operate their firm or shut down the firm and become workers. Upon exit or upon selling, previous firm owners lose the value of their firm and enter the labor market with the lowest labor market efficiency $\underline{\varepsilon}$.[12] We interpret this low entry value as potential costs associated with entrepreneurship, such as lack of experience in the labor market.[13] Figure 3 presents a graphical description of the transitions between occupations.

In this setup, the budget constraint of an entrepreneur, defined as a firm owner that decides to operate, with states (a_{it}, z_{it}) is given by

$$c_{it} = \pi(a_{it}, z_{it}) + (1+r)a_{it} - a_{it+1} + \Pi^p + \Pi^f,$$

and the budget constraint of a worker with states $(a_{it}, \varepsilon_{it})$ is

$$c_{it} = \varepsilon_{it} w + (1+r)a_{it} - a_{it+1} + \Pi^p + \Pi^f,$$

where π are the profits of the entrepreneur's private firm, w is the labor market wage, Π^p and Π^f are the public firm's and the financial intermediary's profits, respectively.

3.1.2 Private Firms

Private firms are endowed with a technology that uses capital k_{it}, labor l_{it}, and the quality of an *entrepreneurial project* z_{it} to produce the final consumption good according

[12] Although the distribution of ε is bounded below by 0, in our numerical solution we take $\underline{\varepsilon}$ to be the lowest value on the ε grid, which is a positive number.

[13] There is also a technical reason why we assume that firms' owners that exit into the labor market start with $\underline{\varepsilon}$. If this wasn't the case, and hence suppose they get a value $\tilde{\varepsilon}$, workers with $\varepsilon < \tilde{\varepsilon}$ would have the incentive to buy a low-quality firm and then immediately exit to improve their labor efficiency.

to

$$y_{it} = z_{it} k_{it}^{\theta} l_{it}^{\nu}$$

where $\theta + \nu < 1$. The decreasing returns to scale assumption implies that all private firms have an optimal operation scale as in Lucas (1978).

Private firms rent capital and hire workers every period. Hence, they are characterized only by the quality of z_{it}. Private firms are indivisible, rival, and excludable. These features are an important distinction between our model of the trade of firms and the literature that studied the trade of ideas (Silveira and Wright, 2010; Akcigit, Celik, and Greenwood, 2016).[14] Different values of z_{it} aim to capture differences in firms' *intangible assets*. For example, trademarks, patents, processes, permits, or customer bases.

We assume that entrepreneurs are subject to *financial frictions*, which may prevent the firm from producing at their optimal scale. Specifically, we assume a collateral constraint that limits the firm's borrowing capacity to multiple of the owner's assets, parameterized by λ. This constraint implies that firms' leverage, or debt to capital ratio, satisfies $(k_{it} - a_{it})/k_{it} \leq (\lambda - 1)/\lambda$.[15]

Given these assumptions, the profit maximization problem of an entrepreneur with assets a_{it} and a firm of quality z_{it} is given by

$$\begin{aligned} \pi(a_{it}, z_{it}) = \max_{k_{it}, l_{it}} \; & y_{it} - R k_{it} - w l_{it} \\ \text{s.t.} \quad & y_{it} = z_{it} k_{it}^{\theta} l_{it}^{\nu} \\ & k_{it} \leq \lambda a_{it} \end{aligned} \tag{1}$$

where R is the capital rental rate. If the collateral constraint binds ($k_{it} = \lambda a_{it}$), the firm operates at a lower scale compared to the unconstrained profit maximization level.[16]

3.1.3 Public Firm

As in Cagetti and De Nardi (2006), we assume that there is a second sector of production populated by a representative public firm. This aims to capture that, in the U.S. economy, around half of the total output is produced by publicly traded firms.

Specifically, we assume that the public firm is owned by all households, in equal shares, and faces no financial frictions. The public firm is endowed with a constant return to scale technology

$$Y_{pt} = K_{pt}^{\eta} L_{pt}^{1-\eta}$$

[14] By definition, ideas are non-rival. However, ideas might be excludable under certain institutional arrangements such as patents.

[15] This type of constraint can be micro-founded with imperfect enforcement of contracts problem. Consistent with most debt financing contracts, we assume that the firm cannot pledge the quality of the entrepreneurial project as collateral.

[16] Appendix B.2 presents firms' input demand functions that characterize the static solution of (1).

where K_{pt} is the public firm's capital, L_{pt} its labor, and Y_{ct} its total output.

3.1.4 Financial Intermediary

The financial intermediary takes deposits from households and rents capital to the firms at a price equal to the savings interest rate plus the capital depreciation rate: $R = r + \delta$. We assume that the representative intermediary operates in a perfectly competitive market and breaks even (i.e., makes zero profits). The resource constraint of the intermediary is given by

$$K_{pt} + \int k(a_{it}, z_{it}) \, \mathrm{d}N^e_{cm}(a_{it}, z_{it}) = \int a_{it} \, \mathrm{d}N^e_{cm}(a_{it}, z_{it}) + \int a_{it} \, \mathrm{d}N^w_{cm}(a_{it}, \varepsilon_{it}) \tag{2}$$

where N^e_{cm} and N^w_{cm} are cumulative distribution functions for entrepreneurs and workers, which are normalized such that $\int \mathrm{d}N^e_{cm} + \int \mathrm{d}N^w_{cm} = 1$. These measures correspond to the production stage after firm owners decide whether to be entrepreneurs or workers.

3.2 A Market for Firms

Firms are hard to evaluate and price. This precludes the existence of a centralized market with a complete price schedule for different types of firms. Therefore, we model the market for firms using a *search-theoretic* approach characterized by bilateral random matching and *quid pro quo* trade. An interpretation of this setup is that agents can evaluate only one firm at a time, which delays trade.

Trade in the market for firms consists of the transfer of both the firm's ownership and management in exchange for assets. Hence, the media of exchange in these transactions are the households' savings a. As we assumed that firms are indivisible, when a buyer and a seller meet they only bargain over the selling price p.

Bilateral Meetings There are two types of meetings in the market for firms: *owner-owner* meetings and *owner-worker* meetings. We allow for different search frictions in each type of meeting. For a firm owner, an owner-owner meeting happens with probability α_o and an owner-worker meeting happens with probability α_w. For a worker, an owner-worker meeting happens with probability α_w.

Note that firm owners are the only potential sellers, while both types of households can be buyers. This implies that in an owner-worker match, the owner is the potential seller, and the worker is the potential buyer. However, in the case of an owner-owner match, who is the buyer and who is the seller depends on the relative quality of the firms.

Let us first consider the owner-owner match and suppose that $z_{it} < z_{jt}$. Then, owner i with states $\mathbf{s}^o_{it} \equiv (a_{it}, z_{it})$ is the potential buyer, and owner j with states $\mathbf{s}^o_{jt} \equiv (a_{jt}, z_{jt})$ is the potential seller. This follows from the assumption that households can own only one firm at a time. Hence, no owner would buy another firm that has a lower quality. In this

case, the total surplus from trading the ownership of firm z_{jt}, in exchange for p assets, is given by

$$\text{Total surplus} \equiv \underbrace{W^o(a_{it}-p, z_{jt}) - W^o(\mathbf{s}^o_{it})}_{\text{Buyer's surplus, } S_b} + \underbrace{W^w(a_{jt}+p, \underline{\varepsilon}) + T_{jt}(p) - W^o(\mathbf{s}^o_{jt})}_{\text{Seller's surplus, } S_s} \quad (3)$$

where W^o and W^w are the value functions at the beginning of the production stage for firm owners and workers, respectively. As described below, T_{jt} is a utility transfer that sellers might receive that captures additional motives to trade firms. Upon selling, the household goes to the labor market with labor efficiency $\underline{\varepsilon}$, as presented in the first term of the seller's surplus.[17] The outside option for both agents (the terms with a minus in the surpluses) is the value of going to the production stage as firm owners with their initial states $\mathbf{s}^o_{it}$ and $\mathbf{s}^o_{jt}$, respectively.

Regarding the owner-worker match, suppose that firm's owner j with states $\mathbf{s}^o_{jt}$ meets with a worker i with states $\mathbf{s}^w_{it} \equiv (a_{it}, \varepsilon_{it})$. Then, the total surplus from trading firm z_{jt} is now given by

$$\text{Total surplus} \equiv \underbrace{W^o(a_{it}-p, z_{jt}) - W^w(\mathbf{s}^w_{it})}_{\text{Buyer's surplus, } S_b} + \underbrace{W^w(a_{jt}+p, \underline{\varepsilon}) + T_{jt}(p) - W^o(\mathbf{s}^o_{jt})}_{\text{Seller's surplus, } S_s} \quad (4)$$

where the only difference relative to the previous match is the buyer's outside option. In this case, if the parties don't trade, the buyer would continue to the production stage as a worker with its initial state $\mathbf{s}^w_{it}$.

Alternative Motives to Trade Besides the purely financial reasons to trade firms studied in this paper, related to households' wealth, access to credit, and risk aversion, there could be other non-pecuniary motives for why entrepreneurs sell their firms.[18] To account for these alternative motives to trade firms in a parsimonious manner, we assume that potential firms' sellers receive a *preference* shock κ_{jt} that captures additional benefits, or a reduction in the opportunity cost, of selling their firm in the current period. The preference shock follows

$$\kappa_{jt} = \underline{\kappa} + (\overline{\kappa} - \underline{\kappa})\xi_{jt}$$

where $1 \leq \underline{\kappa} < \overline{\kappa}$, and the random variable ξ_{jt} is *iid* across time and firms and drawn from a Beta distribution with $\mathcal{B}(1, \beta_\kappa)$.[19]

The shock κ_{jt}, with domain in $[\underline{\kappa}, \overline{\kappa}]$, determines the additional utility transfer that the

[17] If z is very low, some firm owners might even want to pay someone to buy their firm, implying $p < 0$, to transition into the labor market. The free exit assumption, through which firm owners can decide to exit and get the same labor efficiency $\underline{\varepsilon}$, rules out the possibility of negative prices in our model.

[18] Examples of non-pecuniary reasons to trade firms include personal preferences (e.g., the non-monetary value of being self-employed), owners' life cycle considerations (e.g., health shocks or retirement), or family-related concerns (e.g., spouse's job location).

[19] We denote the CDF of κ as $\Psi(\kappa)$, which is implicitly defined by the distribution of ξ.

seller receives upon selling compared to the trading for a higher price $\kappa_{jt}p \geq p$ but no extra utility. Thus, for each potential seller j with states $\mathbf{s}^o_{jt}$, preference shock κ_{jt}, and price p, the utility transfer $T_{jt}(p) \equiv T(p; \mathbf{s}^o_{jt}, \kappa_{jt})$ is implicitly defined by

$$W^w(a_{jt} + \kappa_{jt}p, \underline{\varepsilon}) = W^w(a_{jt} + p, \underline{\varepsilon}) + T_{jt}(p) \tag{5}$$

which states that the seller is indifferent between selling at a higher price $\kappa_{jt}p$ with no transfer and the case with price p and receiving $T_{jt}(p)$. Hence, this utility transfer is similar in spirit to the classical Hicksian compensation. Intuitively, this transfer implies that the owner is willing to sell the firm at a $1 - \kappa_{jt}^{-1}$ discount, relative to the full price $\kappa_{jt}p$. Thus, all else equal, higher values of κ_{jt} will make sellers willing to sell their firms at larger discounts and lower prices.

Sufficient Condition for Trade Let $\underline{p}_{jt} \equiv \underline{p}(\mathbf{s}^o_{jt}, \kappa_{jt})$ denote the minimum price at which seller j is willing to sell its firm, i.e., the price at which the seller's surplus is equal to zero. Likewise, let $\overline{p}_{it} \equiv \overline{p}(\mathbf{s}_{it}, z_{jt})$ be the maximum price that buyer i is willing to pay for firm j, i.e., the price at which the buyer's surplus is equal to zero. A sufficient condition for trade, meaning that there are positive gains from trading firm j, is that

$$\underline{p}_{jt} \leq \overline{p}_{it} \tag{6}$$

where the states of buyer i are $\mathbf{s}_{it} \in \{\mathbf{s}^o_{it}, \mathbf{s}^w_{it}\}$, depending on the type of match (owner-owner or owner-worker, respectively). For a given meeting, condition (6) shows that the possibility of trade is a function of the firms' potential sellers' and buyers' characteristics. In Section 5 we characterize, using the quantitative model, the probability of buying and selling the firm across agents' characteristics.

Bargaining If there are positive gains from trade, we assume that the price is determined by a *Nash bargaining* protocol. Thus, the trading price p between buyer i with states $\mathbf{s}_{it} \in \{\mathbf{s}^o_{it}, \mathbf{s}^w_{it}\}$, and seller j with states $\mathbf{s}^o_{jt}$ and preference shock κ_{jt} solves

$$\begin{aligned} p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt}) = &\arg\max_p \Big[S_b(\mathbf{s}_{it}, z_{jt}, p)\Big]^{\chi} \Big[S_s(\mathbf{s}^o_{jt}, \kappa_{jt}, p)\Big]^{1-\chi} \\ &\text{s.t.} \quad S_b(\mathbf{s}_{it}, z_{jt}, p) \geq 0, \;\; S_s(\mathbf{s}^o_{jt}, \kappa_{jt}, p) \geq 0 \end{aligned} \tag{7}$$

where S_b and S_s are the buyer and seller surpluses, defined in (3) and (4), and $0 \leq \chi \leq 1$ parameterizes buyers' bargaining power. Thus, if χ is near 0, the price will be close to the buyer's maximum price $\overline{p}_{it}$. Conversely, if χ is near 1, the price will be close to the seller's minimum price $\underline{p}_{jt}$. As we will explain in Section 4, information about the ratio of selling prices to firms' profits helps us identify this parameter.

3.3 Timing

The timing of the model can be summarized as follows:

1. The startup shocks, the quality of entrepreneurial projects z, and the labor efficiencies ε are realized.

2. Agents enter the market for firms (DM). Firm owners can buy and sell firms, while workers can only buy. Preference shock κ is realized for potential sellers.

3. Agents enter the production stage (CM). Given prices and their current z, firm owners decide whether to operate the firm or go to the labor market. Finally, production occurs, and agents choose how much to consume and save.

3.4 Recursive Formulation

We now present the recursive problem of firm owners and workers. First, we describe the value functions at the beginning of the market for firms (the DM subperiod), which we denote by V. Second, we present the value functions at the production stage (the CM subperiod), which we denote by W.

3.4.1 Value at the Market for Firms (DM)

Firm owners have four potential outcomes upon entering the market for firms: (1) don't trade, (2) buy another firm, (3) sell their firm to another owner, and (4) sell their firm to a worker. The no-trade case could arise because the owner did not match with a counterpart or because there was a match, but it did not end with a trade.

The value of a firm owner with states (a_{it}, z_{it}) at the beginning of DM is equal to

$$\begin{aligned} V^o(a_{it}, z_{it}) = \mathbb{E}_{\kappa_{it}}\Bigg[& \underbrace{\Pr{}^o\left[\text{no trade} \mid a_{it}, z_{it}, \kappa_{it}\right] \; W^o(a_{it}, z_{it})}_{\text{no trade}} \\ & + \underbrace{\alpha_o \int\int_{z_{it}<z_{jt}, \overline{p}_{it}>\underline{p}_{jt}} W^o(a_{it}-p, z_{jt}) \, \mathrm{d}N^o_{dm}(a_{jt}, z_{jt}) \, \mathrm{d}\Psi(\kappa_{jt})}_{\text{buy}} \\ & + \underbrace{\alpha_o \int_{z_{it}>z_{jt}, \underline{p}_{it}<\overline{p}_{jt}} \left[W^w(a_{it}+p, \underline{\varepsilon}) + T_{it}(p)\right] \mathrm{d}N^o_{dm}(a_{jt}, z_{jt})}_{\text{sell to a firm owner}} \\ & + \underbrace{\alpha_w \int_{\underline{p}_{it}<\overline{p}_{jt}} \left[W^w(a_{it}+p, \underline{\varepsilon}) + T_{it}(p)\right] \mathrm{d}N^w_{dm}(a_{jt}, \varepsilon_{jt})}_{\text{sell to a worker}} \Bigg], \end{aligned} \tag{8}$$

where α_o and α_w are exogenous matching probabilities conditional on each match type.[20] These parameters, in $[0,1]$, govern the degree of search frictions in the market for firms. N^o_{dm} and N^w_{dm} are cumulative distributions for firm owners and workers at the beginning of DM, which satisfy that $\int \mathrm{d}N^o_{dm} + \int \mathrm{d}N^w_{dm} = 1$.

As mentioned in Section 3.2, for the case of owner-owner meetings, who buys and sells depends on the relative firm qualities. Hence, an owner with firm quality z_{it} might buy if it is matched with another owner with a firm of higher quality ($z_{it} < z_{jt}$), as denoted in the integral in the second line of (8). On the contrary, the owner might sell if it is matched with another owner with a firm of lower quality ($z_{it} > z_{jt}$) as denoted in the integral of the third line.[21] Note that the integrals for the buying and selling cases consider only the meetings that result in a trade, which occurs when the seller's minimum price is lower than the buyer's maximum price, as stated in (6). The preference shocks κ, will be relevant in determining these prices.

There are only two potential outcomes for workers: (1) don't trade, or (2) buy an existing firm. Hence, the value of a worker with states $(a_{it}, \varepsilon_{it})$ at the beginning of DM is given by

$$V^w(a_{it}, \varepsilon_{it}) = \underbrace{\Pr{}^w\left[\text{no trade} \mid a_{it}, \varepsilon_{it}\right] \; W^w(a_{it}, \varepsilon_{it})}_{\text{no trade}} + \underbrace{\alpha_w \int \int_{\overline{p}_{it} > \underline{p}_{jt}} W^o(a_{it} - p, z_{jt}) \, \mathrm{d}N^o_{dm}(a_{jt}, z_{jt}) \, \mathrm{d}\Psi(\kappa_{jt})}_{\text{buy}}. \tag{9}$$

3.4.2 Value at the Production Stage (CM)

As previously described, firm owners face an occupational choice at the beginning of the production stage. They have to decide whether to operate the firm and be entrepreneurs or shut down and go to the labor market with labor productivity $\underline{\varepsilon}$. Given these assumptions, the value of firm owners at the beginning of CM is

$$W^o(a_{it}, z_{it}) = \max_e \left\{ W^e(a_{it}, z_{it}), W^w(a_{it}, \underline{\varepsilon}) \right\} \tag{10}$$

[20] In more detail, the probabilities of the bilateral meetings in (8) can be derived as follows. First, note that there is a mass $\int \mathrm{d}N^o_{dm}$ of owners at the beginning of DM. This implies that two owners are matched with probability $\int \mathrm{d}N^o_{dm}$. Due to the search friction, conditional on the match, these owners meet with probability α_o. Thus, the probability of an owner-owner meeting is equal to $\alpha_o \int \mathrm{d}N^o_{dm}$. Similarly, the probability that the owner matches with a worker is equal to $\int \mathrm{d}N^w_{dm} = 1 - \int \mathrm{d}N^o_{dm}$, and conditional on the match they meet with probability α_w. Hence, the probability of an owner-worker meeting is equal to $\alpha_w \int \mathrm{d}N^w_{dm}$. Finally, note that the no-trade probability $\Pr^o[\text{ no trade} \mid a, z\,]$ sums up the probability of no meetings plus the probability of meetings that do not result in a trade as $\underline{p} < \overline{p}$ is not satisfied.

[21] Here, we assume that meetings in which owners have the same firm quality do not result in a trade.

where e denotes the owners' occupational choice.

The value function of entrepreneurs is given by

$$W^e(a_{it}, z_{it}) = \max_{a_{it+1}, c_{it}} \; u(c_{it}) \; + \; \beta\left\{\gamma V^o(a_{it+1}, z_{it}) + (1-\gamma)\mathbb{E}_{z_{it+1}}\left[V^o(a_{it+1}, z_{it+1})\right]\right\}$$
$$\text{s.t.} \quad c_{it} = \pi(a_{it}, z_{it}) + (1+r)a_{it} - a_{it+1}$$
$$c_{it} \geq 0, \; a_{it+1} \geq 0 \tag{11}$$

and the value function of workers by

$$W^w(a_{it}, \varepsilon_{it}) = \max_{a_{it+1}, c_{it}} \; u(c_{it}) + \beta\left\{\zeta\mathbb{E}_{\varepsilon_{it+1}|\varepsilon_{it}}\left[V^w(a_{it+1}, \varepsilon_{it+1})\right] + (1-\zeta)\mathbb{E}_{z_{it+1}}\left[V^o(a_{it+1}, z_{it+1})\right]\right\}$$
$$\text{s.t.} \quad c_{it} = \varepsilon_{it} w + (1+r)a_{it} - a_{it+1}$$
$$c_{it} \geq 0, \; a_{it+1} \geq 0 \tag{12}$$

where $(1-\zeta)$ represents the exogenous startup shock through which a worker can become a firm owner.[22]

3.5 Competitive Equilibrium

A *competitive stationary equilibrium* in this economy consists of: (i) aggregate prices $\{r, R, w\}$; (ii) terms of trade in the market for firms given by the price functions of seller j and buyer-owner i meetings $\{p\left(\mathbf{s}^o_{it}, \mathbf{s}^o_{jt}, \kappa_{jt}\right), \underline{p}\left(\mathbf{s}^o_j, \kappa_j\right), \bar{p}\left(\mathbf{s}^o_{it}, z_{jt}\right)\}$, and the price functions of seller j and buyer-worker i meetings $\{p\left(\mathbf{s}^w_i, \mathbf{s}^o_j, \kappa_j\right), \underline{p}\left(\mathbf{s}^o_j, \kappa_j\right), \bar{p}\left(\mathbf{s}^w_{it}, z_{jt}\right)\}$; (iii) firm owners' occupational choice decisions $e(a_{it}, z_{it})$; (iv) consumption and savings decisions for entrepreneurs $\{c(a_{it}, z_{it}), a'(a_{it}, z_{it})\}$ and for workers $\{c(a_{it}, \varepsilon_{it}), a'(a_{it}, \varepsilon_{it})\}$; (v) capital and labor demand functions for private and public firms, $\{k(a_{it}, z_{it}), l(a_{it}, z_{it}), K_{pt}, L_{pt}\}$; and (vi) measures of agents over occupations and idiosyncratic states at DM and CM subperiods characterized by $\{N^o_{dm}(a_{it}, z_{it}), N^w_{dm}(a_{it}, \varepsilon_{it})\}$ and $\{N^e_{cm}(a_{it}, z_{it}), N^w_{cm}(a_{it}, \varepsilon_{it})\}$, respectively, such that:

1. In DM, the terms of trade in bilateral meetings solve the Nash bargaining problem.
2. In CM, given prices, households, private and public firms solve their optimization problems.
3. Goods market clears, period by period:

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t \tag{13}$$

[22] In (11) and (12) we omit the profits of the public firm and the financial intermediary (Π^p and Π^f terms) in the households' budget constraints as both terms are equal to zero, in equilibrium.

where

$$Y_t \equiv Y_{pt} + \int z_{it} k(a_{it}, z_{it})^{\theta} l(a_{it}, z_{it})^{\nu} \, \mathrm{d}N^e_{cm}(a_{it}, z_{it})$$
$$C_t \equiv \int c(a_{it}, z_{it}) \, \mathrm{d}N^e_{cm}(a_{it}, z_{it}) + \int c(a, \varepsilon) \mathrm{d}N^w_{cm}(a_{it}, \varepsilon_{it})$$
$$K_t \equiv K_{pt} + \int k(a_{it}, z_{it}) \, \mathrm{d}N^e_{cm}(a_{it}, z_{it}).$$

4. Labor market clears, period by period:

$$L_{pt} + \int l(a_{it}, z_{it}) \, \mathrm{d}N^e_{cm}(a_{it}, z_{it}) = \int \varepsilon_{it} \, \mathrm{d}N^w_{cm}(a_{it}, \varepsilon_{it}). \tag{14}$$

5. The budget constraint of the financial intermediary, specified in (2), is satisfied period by period.

6. The measures over types and states satisfy

$$\int \mathrm{d}N^o_{dm}(a_{it}, z_{it}) + \int \mathrm{d}N^w_{dm}(a_{it}, \varepsilon_{it}) = 1$$
$$\int \mathrm{d}N^e_{cm}(a_{it}, z_{it}) + \int \mathrm{d}N^w_{cm}(a_{it}, \varepsilon_{it}) = 1$$

   and are consistent with a recursive equilibrium mapping dictated by prices and trades in the market for firms, households' optimal choices, and the stochastic processes for firms' qualities, workers' labor efficiencies, and sellers' preferences shocks. The stationary equilibrium implies that fixed distribution over time (fixed point).

We solve for the stationary equilibrium of this model by approximating the value functions using projection methods on a finite state space for which we solve all the possible matches and trading prices, as well as agents' and firms' optimal choices. See Appendix B.3 for a detailed description of our numerical solution.

4 Parameterization

This section describes our calibration strategy and presents our validation exercise. We calibrate the model, at an annual frequency, to the year 2007. We focus on 2007 as that is the year we have both the SBO and SCF data available.

4.1 Assigned Parameters

We set the relative risk aversion parameter to $\sigma = 1.5$, the capital depreciation to $\delta = 0.06$, and the public's firm capital elasticity to $\eta = 1/3$. All three are common values in the literature. Regarding the preference shock κ, we set its domain to $[1, 3]$, which implies that sellers' have a maximum possible discount of 66% $(1 - 1/\overline{\kappa})$ coming from the preference shocks. Panel (a) of Table 2 summarizes these assigned parameters.

Table 2: Parameterization

Parameter	Value	Description
	(a) *Assigned Parameters*	
σ	1.5	CRRA
δ	0.06	Capital depreciation rate
η	1/3	Capital elasticity
$\underline{\kappa}$	1	Preference shock, lower bound
$\overline{\kappa}$	3	Preference shock, upper bound
	(b) *Calibrated Parameters*	
β	0.898	Discount factor
Υ	0.724	Curvature private firms technology
$(\lambda-1)/\lambda$	0.397	Collateral constraint, maximum leverage
γ	0.930	Persistence private firm value
ζ	0.939	1− Startup shock
z_{min}	1.118	Scale, z distribution
η_z	2.419	Shape, z distribution
ρ_ε	0.953	AR(1) parameter, ε distribution
σ_ε	0.240	Std. Deviation, ε distribution
$\mathbb{E}[\kappa]$	1.354	Preference shock, mean
α_o	0.803	Owner-owner | meeting probability
α_w	0.459	Owner-worker | meeting probability
χ	0.436	Buyers' bargaining power

4.2 Calibrated Parameters and Targeted Moments

We calibrate the remaining parameters such that the model replicates several key features of the U.S. economy, focusing on the trade of private firms. To reduce the parameter space dimension, we assume that private firms' technology has the same relative elasticity between capital and labor as the public firm. In such a way, a single parameter $\Upsilon < 1$ captures the degree of decreasing returns to scale in private firms' technology by setting $\theta = \eta\Upsilon$ and $\nu = (1-\eta)\Upsilon$. After this, we have a total of thirteen parameters which we calibrate to match seventeen moments. Panel (b) of Table 2 presents these parameters with their calibrated values. We find those values by minimizing the distance between moments in the data and the model. Table 3 presents the seventeen moments we target in our calibration exercise. For an easier exposition, we divide these moments into five groups which we now describe.

First, we target moments capturing the role of entrepreneurs in the economy. As reported in the 2007 SCF, we target that 6% of households are entrepreneurs, and they

earn 20% of total income and hold 33% of the economy's wealth. Our second set of moments characterizes the distribution of income and wealth across all households and within workers and entrepreneurs. We target six different Gini indexes, which we also compute from the 2007 SCF. The table shows that our model matches the dispersion of income and wealth in the data very well. However, it slightly overpredicts the level of inequality among entrepreneurs. It is worth mentioning that different from the previous literature, which has abstracted from firm prices, our definition of wealth in the model includes the value of private firms $(a + p)$, consistent with the data. The parameters most informative about the income and wealth distribution in the model are the ones characterizing the distribution of firms' quality, z_{min} and η_z, and workers' labor efficiency, ρ_ε and ε.

The third and fourth sets of moments capture relevant characteristics of firms in the US economy. First, we target a capital-output ratio of 3, which disciplines the discount factor β. Second, we target that private firms account for 50% of total output, which is consistent with the estimates in Dinlersoz et al. (2019), and lower than Asker, Farre-Mensa, and Ljungqvist (2014) who calculate that private firms account for 57% of total sales. Regarding private firms' leverage, we target our model's weighted average debt-to-capital ratio to be 0.35, consistent with private firms' leverage in the Flow of Funds Accounts. This moment pins down the collateral constraint parameter λ. We also target a firm-level exit rate of 0.09, which we computed from the Census Business Dynamics Statistics (BDS) for 2007. These moments are especially important to discipline the decreasing return to scale Υ and the parameters γ and ζ.

Our fifth and final set of moments captures relevant features of the trade of private firms that we documented in the empirical section of the paper. Specifically, we target that 3% of the firms are traded every year and, from those, 66% of them are acquired by workers. These moments are relevant for the search frictions parameters, α_o and α_w. Additionally, to identify the relevance of preference shocks in firms' trade, we target the 1.6% trade rate of the largest firms, defined by the top quartile of firms' payroll distribution. As explained below, preference shocks will be particularly relevant for the trade of large and financially unconstrained firms. We get that $\mathbb{E}[\kappa] = 1.354$, which implies an average discount of $\mathbb{E}[1 - \kappa^{-1}]$=0.23.[23] Finally, we target a median price-to-profit ratio equal to 3.5, which we obtained from Dealstat.[24] This ratio is most informative for the buyers' Nash bargaining parameter χ. We obtain a value of $\chi = 0.442$, which implies that sellers have a slightly higher bargaining power than buyers. Overall, Table 3 shows that our model does a very good job matching the targeted moments. Especially the ones related to entrepreneurs, private firms, and the market for firms.

[23] We directly target the mean of κ, which implicitly defines the parameter β_κ. In detail, note that $\mathbb{E}[\kappa] = \underline{\kappa} + (\overline{\kappa} - \underline{\kappa})\mathbb{E}[\xi]$ and $\mathbb{E}[\xi] = \frac{1}{1+\beta_\kappa}$, which defines β_κ given $\underline{\kappa}$, $\overline{\kappa}$ and $\mathbb{E}[\kappa]$.

[24] Dealstat (formerly Pratt's Stats) is a database of business transactions. We use their publicly available reports to compute an average median selling price to EBITDA ratio of 3.5 from 2010 to 2018 in the US.

Table 3: Targeted Moments

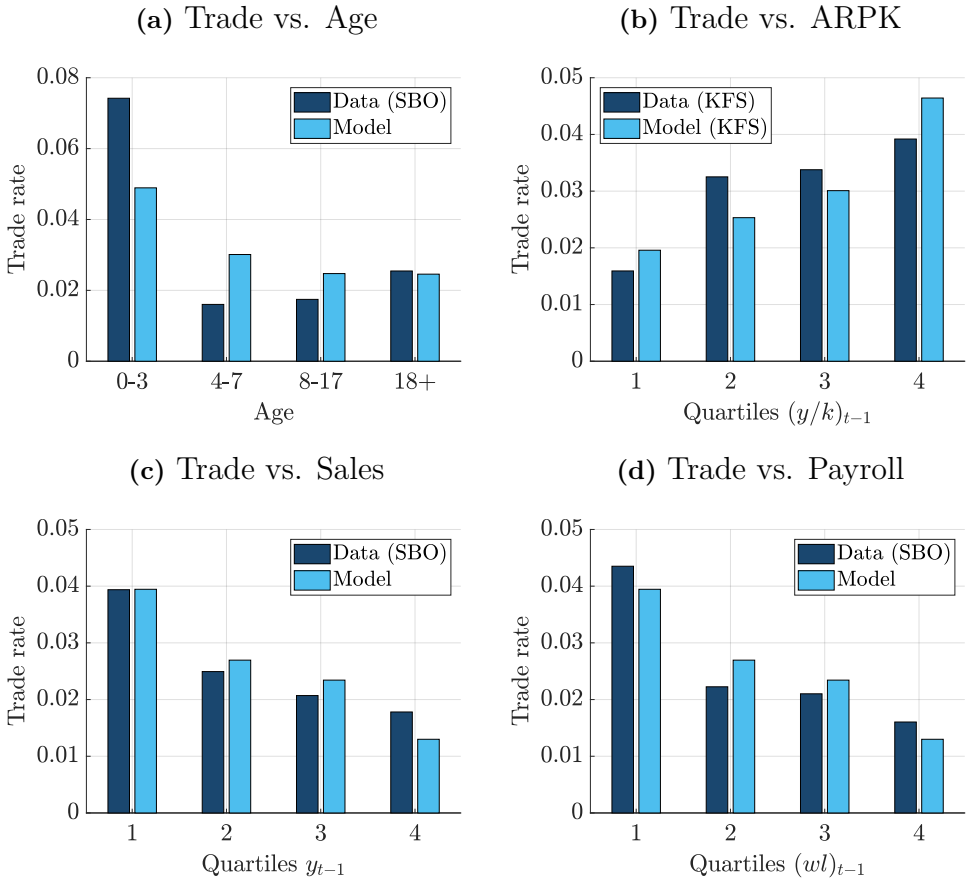
	Source	Data	Model
Entrepreneurs			
Fraction of entrepreneurs	SCF	0.06	0.06
Income share of entrepreneurs	SCF	0.20	0.21
Wealth share of entrepreneurs	SCF	0.33	0.38
Income and Wealth Distribution			
Gini income, all households	SCF	0.62	0.61
Gini wealth, all households	SCF	0.82	0.83
Gini income, entrepreneurs	SCF	0.67	0.77
Gini wealth, entrepreneurs	SCF	0.74	0.81
Gini income, workers	SCF	0.58	0.56
Gini wealth, workers	SCF	0.78	0.79
Private and Public Firms			
Capital to output ratio	See text	3.0	3.0
Private Firms			
Output share	See text	0.50	0.45
Leverage	FoF	0.35	0.35
Exit rate	BDS	0.09	0.09
Trade of Private Firms			
Trade rate, all firms	SBO	0.030	0.031
Trade rate, largest firms	SBO	0.016	0.013
Share purchased by workers	SBO	0.66	0.67
Median price/profits	DealStats	3.5	3.3

NOTES: Data moments correspond to the year 2007. Wealth in the model is defined as the sum of the risk-free asset and the value of the firm $a+p$. Trade rate smallest/biggest denotes the ratio of trade rates for firms in quartile 1 to quartile 4, measured by payroll.

4.3 Validation: Financial Frictions as a Motive for Trade

We now present two exercises to evaluate some testable implications of our theory about financial frictions being an important driver to trade firms. First, as described throughout the paper, if financial frictions are an important reason for trading firms, credit-constrained firms should be more likely to be bought and sold. We test this first prediction of the model by analyzing the relation between trade and firms' observable characteristics. As in the empirical section, we consider two commonly used proxies of credit constraints: firms' age and size, as younger and smaller firms are more likely to be financially constrained. In addition, we analyze firms' ARPK since credit-constrained firms will have high capital returns but cannot increase their investment.

Figure 4: Trade Rate by Firms' Characteristics: Data and Model



NOTES: Trade rate by firms' characteristics in the data and data simulated from the model. To be consistent with the data, Model (KFS) restricts to a sample of firms of age less or equal to 7. See the notes in Figure 2 for a description of the data moments.

Following the analysis presented in Section 2.5, we simulate data from our model and compute the trade rate conditional on these observable characteristics of firms. Figure 4 shows that consistent with the data, our model predicts that younger, smaller, and high returns to capital firms exhibit the highest probabilities of trade. It is important to emphasize that these relations were *not* targeted in our calibration exercise. Instead, they result from the key prediction of our theory that credit-constrained firms are the ones more likely to be traded and that these characteristics are strongly correlated with binding credit constraints in our model.

Now we test a second prediction of our model related to the characteristics of business buyers. As we show in detail in Section 5.2, if financial frictions primarily drive firms' trade, business buyers should be wealthier than business sellers. Although we cannot measure the wealth of buyers and sellers in a given transaction, we do observe the wealth of the average business buyer in the SCF. In Section 2.4, we documented that the average firm buyer is considerably wealthier than the average household. We compute the anal-

ogous moment in our model, average wealth at t for agents that bought a firm in $t-1$, relative to the economy's average wealth. Table B.1, in the Appendix, shows that our model aligns remarkably well with the data despite this moment not being targeted in our calibration. Including business wealth $(a+p)$, our model predicts that buyers are 3.1 times wealthier than the average household, while this number is 3.8 in the data. Excluding business wealth (a), this ratio equals 2.7 in the data and the model. Overall, these results suggest that financial frictions are a relevant motive behind the trade of private firms.

4.4 Other Untargeted Moments

A relevant feature of heterogeneous agents models with entrepreneurship is that they can replicate the income and wealth distribution observed in the data (Quadrini, 2000; Cagetti and De Nardi, 2006). This is possible thanks to the combination of uninsurable income risk and stochastic returns to wealth from entrepreneurial activity. Table B.2, in the Appendix, shows that this is also true in our model. Although we only targeted a set of Gini coefficients, the model does a good job matching the entire income and wealth distribution observed in the data.

5 Model Properties

This section describes the main properties of our model. First, we discuss and quantify the different motives behind the trade of firms. Second, we characterize who buys and who sells firms in our economy. Finally, we describe the implications of this market for firm dynamics and the allocation of capital.

5.1 Motives for Trading Firms

Exchanges in the market for firms are voluntary. Hence, a necessary condition for gains from trade is that agents have different valuations for the same firm. In particular, the buyer must have a higher valuation than the seller. In our theory, given the agents' outside options, heterogeneous valuations for firms arise from three sources: the preferences shocks, firms' credit constraints, and incomplete markets. We now describe and quantify each of these three motives behind the trade of firms.

Preference Shocks As described above, we introduce non-pecuniary motives to trade firms through sellers' κ shocks at the beginning of the market for firms. These shocks aim to capture, parsimoniously, all the motives to trade firms unrelated to the financial channels we study in this paper. To evaluate the role of these preference shocks, the second row of Table 4 presents the trade rate of firms when we turn off these non-pecuniary motives. This comparative static exercise sets $\mathbb{E}[\kappa] = 1$ and $Var[\kappa] = 0$ while keeping the rest of the parameters fixed. Without preference shocks, the economy's annual firms'

Table 4: Trade Rate Decomposition

	All Firms		Largest Firms	
	Trade rate	Relative	Trade rate	Relative
Baseline	3.1%	1.00	1.3%	1.00
No preference shocks	2.1%	0.69	0.2%	0.18
No collateral constraint	1.0%	0.32	0.1%	0.11
No preference, no collateral	0.4%	0.13	0.1%	0.07

NOTES: Steady-state comparisons of the market for firms' trade rate under different parameterizations. Relative is the ratio of each trade rate to the Baseline model. Largest Firms are the top quartile firms, measured by payroll. No preference shocks turn off the alternative motives to trade firms by setting $\mathbb{E}[\kappa] = 1$ and $Var[\kappa] = 0$. No collateral constraint assumes $\lambda \to \infty$. No preference, no collateral considers both previous cases simultaneously.

trade rate falls from 3.1% to 2.1%. This result indicates that preference shocks explain around 31% of the trades in the market for firms, while most exchanges arise from the financial frictions we explain below. However, the last two columns of Table 4 show that preference shocks play a significant role in the trade of large firms, which are less likely to be financially constrained in our model.

Credit Constraints Regarding the financial motives to trade firms, we first focus on the role of credit constraints. This channel arises from the collateral constraint in the entrepreneurs' problem, presented in (1), that restricts firms' capital to a multiple λ of their owners' wealth. Consequently, whenever an entrepreneur is credit constrained, a wealthier buyer can obtain a higher profit stream out of the same firm as it would be able to operate closer to its optimal scale. Thus, credit constraints generate gains from trade between constrained business owners and wealthier buyers. To quantify the importance of this channel, we set $\lambda \to \infty$, which implies that the firms' profits stream is no longer a function of their owners' wealth. The third row of Table 4 shows that removing credit frictions significantly reduces the frequency of trades in the market for firms to 1.0%, indicating that the bulk of the transactions in our baseline economy, 68%, are driven by credit constraints. This result is in line with Figure 4, where we showed that younger, smaller, and high return to capital firms, which are the ones most likely to be credit constrained, are the ones with the higher trade rates, both in the data and in our model.

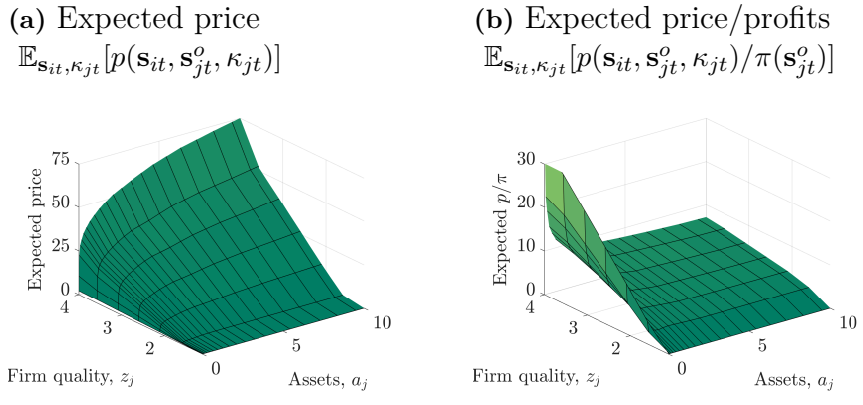
Risk and Incomplete Markets Risk aversion and incomplete financial markets constitute the third motive to trade firms. In our model, owning and operating a firm is associated with uninsurable income risk as the firm's quality z is stochastic, causing agents to have precautionary savings. Thus, even without credit constraints, agents can have different valuations for the same firm as a function of their wealth. For low-wealth owners, selling the firm allows them to front-load consumption and achieve an earlier risk resolution. For high-wealth owners, consumption is less dependent on shocks to the firm's profits. In other words, the covariance between their stochastic discount factor and the

realization of profits is small, increasing their ability to bear risk. Therefore, the value of owning a firm will vary across the wealth distribution generating potential gains from trade. To evaluate the importance of this channel, we turn off both the preference shocks and firms' credit constraints. The last row of Table 4 shows that, in this case, the trade rate is 0.4%. This result suggests that risk and incomplete markets account for 13% of the firms' trades in our baseline economy.

5.2 Who Buys and Who Sells Firms?

Now we describe the typical buyers and sellers in the market for firms. We start our characterization by analyzing the prices at which firms trade. Panel (a) of Figure 5 presents the expected price $p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt})$ resulting from the Nash bargaining protocol in the sellers' state space (a_j, z_j), after integrating over the preference shock κ_j and all potential buyers $\mathbf{s}_{it}$. As expected, selling prices are increasing firm quality z_j. However, due to the collateral constraint on firm owners' wealth and incomplete markets, holding the firm's quality fixed, the price is increasing in the owners' assets a_j. Note that firm prices would be unrelated to the current owner's wealth under perfect credit markets. Thus, due to imperfect credit markets, high-quality firm owners with low wealth will be willing to sell their firms at a relatively low price as it will take them a long time, and high saving rates, to grow out of their borrowing constraint through self-financing. Nevertheless, as panel (b) of Figure 5 shows, these transactions have considerably high price-over-profit ratios, which illustrates the small scale of operation of constrained and low-wealth owners with high-quality firms relative to the price at which they can sell their business.[25]

Figure 5: Prices in the Market for Firms



NOTES: Expected price, and price over profits, in owners' state space, $\mathbf{s}^o_{jt} = (a_{jt}, z_{jt})$, after integrating over preference shocks, κ_{jt}, and trading counterparts, $\mathbf{s}_{it} \in \{\mathbf{s}^o_{it}, \mathbf{s}^w_{it}\}$.

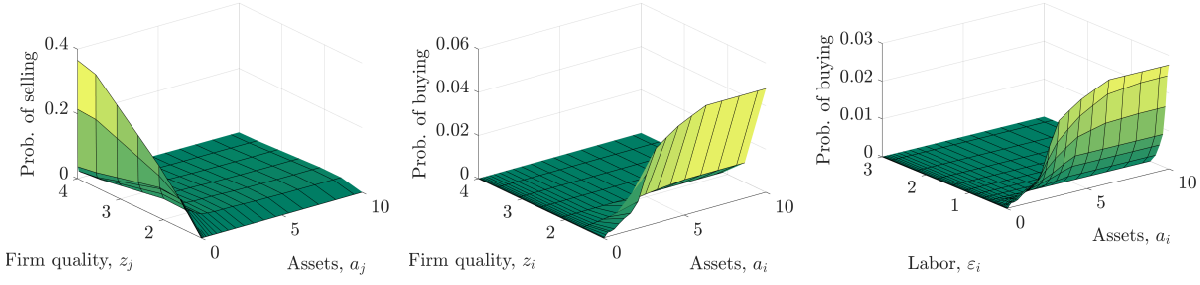
Considering how trading prices are determined, we characterize who buys and sells

[25] According to Dealstat, the median price-over-profit ratio in the Information sector equals 9, considerably higher than the economy-wide 3.5 number. This evidence is consistent with our model's large price-over-profit ratios for high-growth potential firms.

firms in our economy. Panel (a) of Figure 6 presents the probability that a firm owner sells its firm, again, in the state space (a_j, z_j). The figure shows that owners with low wealth and high-quality firms have the highest probability of selling. In those cases, there will be high gains from trade as the current owner lacks the assets to operate at the optimal scale. Panels (b) and (c) present the probability of buying a firm for firm owners in the (a_i, z_i) space and for workers in the (a_i, ε_i). These panels show that the probability of buying is the opposite mirror image of the likelihood of selling. Thus, the firms' buyers will be wealthy households that currently own low-quality firms (low z) or wealthy workers with low labor efficiency (low ε). These trade patterns echo the results in Section 4.3, where we showed that our model's prediction that business buyers are wealthier than the average household is quantitatively consistent with the data.

Figure 6: Buyers and Sellers in the Market for Firms

(a) Pr. of selling **(b)** Pr. of buying, owners **(c)** Pr. of buying, workers



NOTES: Probabilities (Pr.) of trade after integrating over preference shocks and trading counterparts.

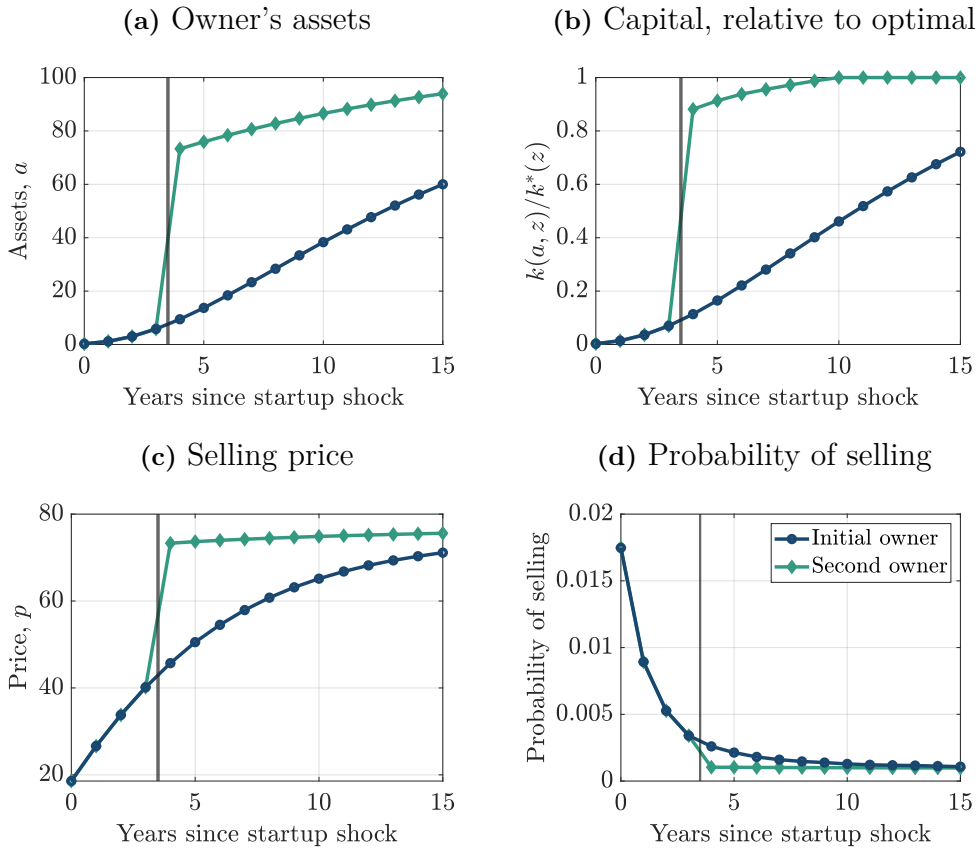
Overall, the panels in Figure 6 show that the typical seller in our economy will be firm owners with high-quality firms but low wealth, and the typical buyers will be wealthy agents with relatively low-quality firms or low labor efficiency. Thus, as we show and quantify below, these trades in the market for firms between constrained and potentially unconstrained owners will lead to a better allocation of productive projects and available resources in the economy.

5.3 Implications for Firm Dynamics and Capital Allocation

To provide further intuition about the implications of the trade of firms, Figure 7 presents a hypothetical trajectory of a firm in our model. We assume that the initial owner of the firm has a level of assets equal to the median wealth among workers who, at period zero, receives a high-quality firm through the exogenous startup shock. Because of credit constraints that limit the use of external funding, this entrepreneur will start operating the firm at a low scale. Panel (a) shows that this business owner will accumulate assets over time to reach the optimal unconstrained size through self-financing. However,

panel (b) shows that even after 15 years the firm operates at a scale equal to 70% of the optimal unconstrained level.

Figure 7: Firm Dynamics and Trade, An Example



NOTES: The vertical line indicates the DM subperiod at $t = 3$, when trade takes place. The initial owner has assets equal to the median worker of the economy upon receiving the startup shock at $t = 0$. In this example firm quality z is held constant across the 15 periods. $k^*(z)$ in panel (b) denotes the unconstrained optimal level of capital for a firm with quality z. The selling price is obtained after integrating over preference shocks and potential buyers.

Now we analyze what happens if owners can sell their firms. Panels (c) and (d) of Figure 7 plots the selling price and the probability of selling the firm, respectively, for the original business owner. After receiving the startup shock, this entrepreneur will be willing to sell the firm at a relatively low price as the alternative option of self-financing implies a low-profit stream for several periods. In addition, because of the risk channel previously described, a credit-constrained entrepreneur will be willing to sell the firm because of precautionary motives. As the initial owner accumulates assets, the trading price increases and the trade probability falls accordingly.

Suppose now that in period three, a wealthier agent purchases this firm. If the second owner has more resources to invest, this owner can take the firm closer to its optimal operating scale more quickly. In our example, panel (b) shows that with the second owner, the firm reaches its optimal scale 10 years after being founded, which is less than

half of the time required by the initial owner. In sum, this simple example illustrates how the market for firms reduces the losses from capital misallocation in the economy as it shortens the time that highly productive firms remain financially constrained. In the next section, we quantify the macroeconomic implications of firms' trade for output and TFP.

6 Macroeconomic Implications

This section presents our main quantitative exercises. First, we present two counterfactual experiments that quantify the relevance of the market for firms as a mechanism through which entrepreneurial projects and available resources are allocated in the economy. Second, we analyze the relation between the economy's aggregate credit conditions and the trade of firms.

6.1 The Role of the Market for Firms

We consider two counterfactual experiments that quantify the importance of the market for firms. Both experiments consist of steady-state comparisons of our model under different parameterizations. In the first experiment, we take our baseline model and analyze the implications of a partial or total market shutdown. In the second experiment, we compare our baseline economy with an alternative economy with no trade in the ownership of firms. We then analyze the level of external financing that no market economy requires to match the TFP level of our baseline economy.

6.1.1 Closing the Market

Table 5 presents the results of our first counterfactual experiment. As a reference, the first column of the table has some relevant moments of our baseline economy. The second and third columns report the percentage change when the market for firms partially and then completely shut down. In both cases, we only vary the search frictions' parameters in the market for firms, α_o and α_w, while maintaining the rest fixed. For the partial shutdown case, we divide in half both parameters such that their relative values are the same and, hence, the fraction of firms purchased by workers is unchanged. For the complete shutdown case, we set both parameters equal to zero.

In both cases, private firms' output considerably falls by -4.8% and -9.1% for the partial and the complete shutdown case, respectively. For easiness in the exposition, we focus on the total shutdown results. The remaining rows of Table 5 show that both extensive and intensive margins explain the fall in entrepreneurial output. First, regarding the extensive margin, the share of active entrepreneurs falls by 4.5%. Additionally, without the market for firms, the entry and exit rate into entrepreneurship significantly decreases by 27.5%. Regarding the intensive margin, the remaining private firms exhibit a poorer allocation of capital and firms' qualities, as shown by the entrepreneurial TFP, which decreases by 2.2%. Total output in the economy also decreases, but to a lower extent, by 1.3%. General

Table 5: Closing the Market for Firms

		Δ %	
	Baseline Economy	Partial $(\alpha_o, \alpha_w)/2$	Total $(\alpha_o, \alpha_w) = \mathbf{0}$
Fraction of entrepreneurs	0.06	-2.4%	-4.5%
Private firms output	0.57	-4.8%	-9.1%
Private firms TFP	1.17	-1.2%	-2.2%
Exit rate	0.09	-10.2%	-27.5%
Public firms output	0.71	2.6%	5.1%
Total output	1.29	-0.7%	-1.3%
Interest rate	0.03	2.6%	4.4%
Wage	1.30	-0.4%	-0.7%

NOTES: The Partial column presents the results for the market partial shutdown, obtained dividing by the half the parameters α_o and α_w. The Total column presents the results when both parameters are equal to zero, thus a total market shutdown. TFP is measured as $Y_e/(K_e^{\theta}L_e^{\nu})$, where $(.)_e$ denotes the aggregate variables of the entrepreneurial sector.

equilibrium effects and the assumption that the production of private and public firms are perfect substitutes explain the smaller aggregate effect. Indeed, an increase in the production of the public firm of 5.1% partially offsets the fall of entrepreneurial output.

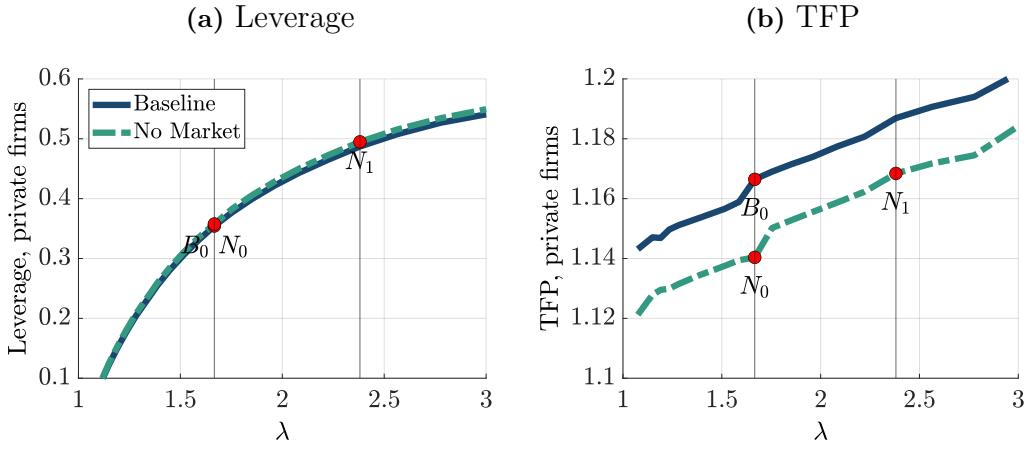
6.1.2 Baseline vs. No Market Economy

For our second experiment, we compare the counterfactual economy with $\alpha_o = \alpha_w = 0$, which we call the "No market economy", with our baseline model under alternative credit market frictions. In Figure 8, we present different steady states for the baseline and the no market economy varying firms' credit constraints, which, in the model, is governed by the parameter λ. Higher λ implies easier access to credit as entrepreneurs can borrow more with the same level of assets. From these steady states, we focus on two moments: private firms' leverage (Panel a) and the TFP of the entrepreneurial sector (Panel b).

Panel (a) of Figure 8 shows that the baseline and the no market economy exhibit almost the same relation between leverage and λ. This finding was expected, as firms' maximum leverage equals $(\lambda-1)/\lambda$. However, as shown in Table 5, this is not the case for the private or entrepreneurial sector TFP. Indeed, panel (b) shows that for the same level of λ, the no market economy achieves a lower TFP than our baseline model. The differences in TFP between these two models are captured by the distance between points B_0 and N_0, which denote the allocations for the baseline and the no market economy, respectively. This result is explained by the higher *misallocation* between entrepreneurial projects and available resources when the market for firms is absent.

With these steady states at hand, we *ask*: what credit conditions does the no market economy require to match the TFP level of our baseline economy? Using Panel (b), we can identify the level of λ such that the no market economy attains the same TFP as the baseline. Graphically, this implies moving from N_0 to N_1 along the no market economy's curve. The allocation N_1 has a higher λ. Thus, it implies *easier credit* conditions than N_0. To better interpret this, we go back to Panel (a) and recover the level of leverage associated with point N_1. These panels show that the no market economy requires an increase in firms' average leverage of 14 p.p., or 40%. This increase is sizable as, for example, firms' leverage fell by around 5 p.p. during the 2008 Great Recession.

Figure 8: Baseline vs. No Market Economy



NOTES: Steady-state values for the baseline and no market economy varying λ, which parameterizes firms' credit constraints. Panel (a) is private firms' mean leverage, $(k-a)/k$, weighted by capital k. Panel (b) is private firms' TFP. Points B_0 and N_0 denote the allocations in the baseline and no market economies. N_1 is the counterfactual no market economy that attains the same TFP as the baseline model.

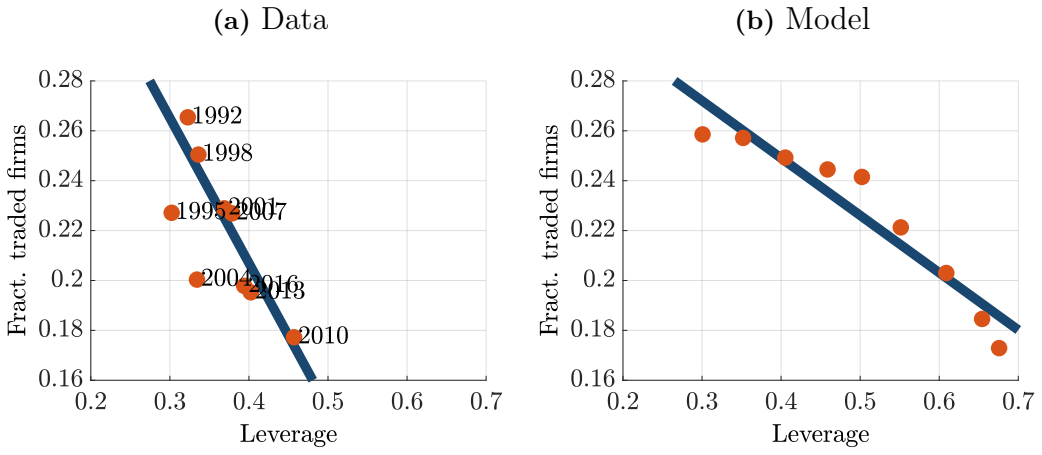
Altogether, these two counterfactual exercises show that the market for firms is a quantitatively relevant mechanism through which entrepreneurial projects and available resources can be better allocated in the economy.

6.2 Aggregate Credit Conditions and the Trade of Firms

In Section 2, we documented that the fraction of entrepreneurs that purchased their firm has decreased in the last decades, going from roughly 30% to 20%. Notably, in addition to the falling share of traded firms, the U.S. economy experienced a secular increase in firms' leverage during the same period. Figure A.1 in the Appendix shows that between the 1980s to 2010s private firms' leverage increased in around 20 p.p. One implication of our theory on firms' trade is that as credit conditions loosen and current firm owners have more access to external finance, the gains from trading firms will be smaller, reducing the total number of trades in the economy. In this section, we analyze the relation between these two variables through the lens of our model and test how much of

the fall in the share of traded firms can be explained by firms' access to external financing.

Figure 9: Credit Conditions and the Trade of Firms



NOTE: Panel (a) reports the empirical relation between leverage and firms' trade over time in the U.S. economy. Panel (b) presents the link between these variables in our model, which we obtain by varying λ keeping the rest of the parameters fixed. The blue line is the linear fit from regressing the share of traded firms on average leverage.

Consistent with our theory, the time series aggregate data for the U.S. economy shows a marked negative correlation between firms' leverage and the fraction of traded firms. Panel (a) of Figure 9 shows this empirical relation with a scatter plot of leverage and trade across years. By fitting a linear regression, we find a slope of -0.58. In the data, however, there could be multiple reasons behind this negative correlation, for example, related to the decline in business dynamism in the U.S. economy. To isolate the role of credit on firms' trade, we use our model and perform comparative statics for different values of λ while maintaining the rest of the parameters fixed at our baseline calibration. Panel (b) shows this causal relation between access to credit and firms' trade in our model economy. Our model implies a linear slope of -0.23 for the relation between leverage and the share of traded firms. Thus, in light of these results, we conclude that easier access to credit can explain up to 39% (-0.23/-0.58) of the fall in the share of traded firms observed in the last three decades. Nevertheless, it is worth noting that our model implies a flatter slope for the levels of leverage observed in the U.S. in the previous thirty years.

7 Conclusions

We study the relation between financial frictions, namely credit constraints and incomplete markets, and the trade of firms. We use micro data from business owners, households, and firms to document novel facts about firms' trade in the U.S. economy. We document that one out of four entrepreneurs purchased their business. In the cross-section, younger, smaller, and high return to capital firms have the highest trading rates.

To explain these findings, we develop a model of entrepreneurship and frictional trade of firms in which gains from trade can arise endogenously from financial frictions and exogenously from preference shocks capturing alternative motives to trade firms. By introducing credit frictions as a micro foundation that generates gains from trading firms, our model can account for the fact that younger, smaller, and high return to capital firms have the highest trading probabilities as these firms are more likely to be financially constrained in the model. We use our model, calibrated to match salient features of the market for firms, to quantify the aggregate importance of firms' trade. We found that shutting down this market implies losses in private firms' output of 9.1% and TFP of 2.2%. Our results show that the market for firms is a quantitatively relevant mechanism through which entrepreneurial projects and available resources can be better allocated in an economy with imperfect credit markets.

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A Data Appendix

A.1 Additional Results

Figure A.1: Credit Conditions and the Trade of Firms in the U.S. Economy

(a) Privately held firms' leverage **(b)** Share of firms purchased

SOURCE: Flow of Funds Accounts and SCF.
NOTE: Panel (a) shows the time-series of privately held firms' leverage (debt/assets). The connected light blue line is the observed series, the dashed blue line is the HP-filter and solid blue line is the linear trend. Panel (b) shows the time-series of the share of purchased firms. The light blue line is the observed share and the blue solid line is the locally weighted smoothing series.

A.2 Data Sources

A.2.1 Survey of Business Owners (SBO) - PUMS

The SBO is a comprehensive survey of firms and firm owners in the U.S. The PUMS sample is representative of non-farm private businesses with receipts of $1,000 or more and is available for the year 2007. The SBO is conducted at the company or firm-level. A company is a business consisting of one or more domestic establishments. The survey is designed to identify the ultimate owners of firms and their characteristics.

Table A.1 reports the total number of owners and firms in the SBO. From those, we first restrict to the owners who report how do they acquire their business. The SBO already restricts to self-employed business owners, thus for our definition of entrepreneurs, we just have to restrict to business owners who actively manage their firm. Our baseline sample consist of almost 700,000 entrepreneurs which own around 500,000 different firms.

From this survey we mainly focus on how the owners acquired their firms. In addition, we use information on the characteristics of the firm (established year, employment, payroll, receipts, sector, location, operation status, number of owners) and of the owners

Table A.1: 2007 SBO Sample

	#Dropped	#Owners	#Firms
All	-	3,409,393	2,165,680
Report Acquisition	1,244,852	2,164,541	1,291,292
Manage and own	1,052,287	1,112,254	841,254
Employer firm	413,603	698,651	501,564

(age, acquisition year, ownership percentage, education level, previous occupation). We use this information to do a thorough characterization of the trade of firms.

Using the SBO we can also obtain information on firms and owners close to the time at which the firm was traded. To study firms' and buyers' characteristics *when purchased* we look at owners that acquired the firm through a purchase in the same year of the survey. Further, the SBO provides information on firms' and owners' characteristics for those owners who report an exit because they sold their firm in the year of the survey. We use this information to characterize firms and their previous owners *when sold*.

For all our calculations we use the sample weights provided by the survey.

A.2.2 Survey of Consumer Finances (SCF)

The SCF is a household-level survey that includes extensive information on households' income, balance sheets, and demographic characteristics. The public microdata is available every three years for the period 1989-2016.

Table A.2: 1989-2016 SCF Sample

	#Dropped	#Households
Income and wealth		
All	-	47,769
$21 <$ age < 78	3,528	44,241
Positive income	67	44,174
Firm acquisition		
Manage and own	35,468	8,706
Employer firm	1,379	7,327

In the SCF we identify entrepreneurs as those households whose household head: is

self-employed, owns a business, and has an active management role in it. The SCF also provides information of privately held businesses which are actively managed. Business owners can report information for up to three or two firms, depending on the survey year. For our baseline calculations we focus on the characteristics of the main business, defined as the one with higher reported value. Using this information, we can identify the entrepreneurs that own a firm with a positive number of employees.

Table A.2 reports our sample selection criteria and the number of households in our SCF sample. For our calculations of the moments of income and wealth we restrict to a sample of households whose household head is between 22 and 78 years old and have a positive income. For our calculations of the trade of firms trade we focus on entrepreneurs, which considering our baseline definition (with employer firms), are 7,327 households between 1989 and 2016, which is a significantly smaller than the one in our SBO sample.

In addition to the information on entrepreneurs and how do they acquired their firm, we use the SCF to compute relevant moments from the income and wealth distribution in the U.S. economy. Our measure of household wealth is the variable constructed by the Federal Reserve for its Bulletin article which accompanies each wave of the SCF. Wealth is defined as total net worth, which equals assets minus debt. Assets includes both financial and non-financial assets. Financial assets include checking and savings accounts, stocks held directly and indirectly, bonds, etc. Non-financial assets, among others, include the value of houses and other real estate, the value of farm and private businesses owned by the household. Debt includes both housing debt (mortgages), debt from lines of credit and credit cards, and installment loans.

Our measure of income includes all sources of income excluding government transfers (e.g. social security and unemployment benefits) and excluding other (non-classified) sources of income. Thus, we include wage income, income from businesses, income from interests and dividends, from capital gains, rent income and income from pensions and annuities.

For all our calculations we use the sample weights provided by the survey.

A.2.3 Annual Survey of Entrepreneurs (ASE)

The ASE is a representative sample of all non-farm businesses filing Internal Revenue Service (IRS) tax forms as individual proprietorships, partnerships, or any type of corporation, and with receipts of $1,000 or more. The ASE is conducted at the firm-level and gathers information on the firm and owner characteristics. The population represented by the survey focuses on firms with paid employees. This survey is available at an annual frequency starting in 2014.

Similar to the SBO, the ASE collects information regarding owner' and firms' characteristics for a large sample of owners. The difference is that the ASE has an annual

frequency and samples only firms with paid employees. One major caveat of the ASE is that we don't have access to the micro data, therefore we use information from the tables provided by the Census Bureau to compare to our baseline estimates and explore the recent evolution in the share of firms traded.

For the table estimates provided by the Census Bureau, a business owner is defined as someone who holds more than 50% of the stake of the firm, where the firm has a positive payroll. This definition is close to our baseline definition of an entrepreneur where firms have at least one employee. Our numbers are retrieved from table `SE1600CSCB001` where entrepreneurs are classified by the way they acquired their firm.

A.2.4 Kaufman Firm Survey (KFS)

The KFS is a panel survey that tracks almost 5,000 business that start their operations in 2004 through 2011. The initial sample was created by using a list frame sample of start-up businesses from the Dun & Bradstreet Corporation (D&B) database. The KFS collects information from business' and owner's characteristics and, in particular, they provide information about firms' balance sheets.

Table A.3 shows the sample selection. Following the previous literature, we drop firms that at some point refuse to answer and observations with missing values of employment, revenues, sales, assets, cash, and accounts receivable. Our baseline sample remains with 2,841 firms and 13,457 observations (firm×year).

Table A.3: 2004-2011 KFS Sample

	#Dropped	#Owners	#Firms
All	-	39,424	4,928
Answer	13,624	25,800	3,225
Missing	15,176	10,624	2,508

We define capital as total assets without cash holdings and accounts receivable. Total assets is composed by product inventories, land and buildings and structures, vehicles, equipment/machinery, other properties, cash, and other. To approximate the capital returns we consider the average revenue product of capital (ARPK) measured as firms' revenue to capital ratio. In the KFS we identify trades as exits due to acquisitions.

For all our calculations we use the sample weights provided by the survey.

A.3 Robustness Exercises

This section presents several robustness exercises for our three main empirical results regarding the trade of firms and firm buyers' previous occupation.

A.3.1 How do Entrepreneurs Acquire Their Firms? - Robustness

Using SBO data we compute the share of business owners that acquired their business through a purchased considering several alternative definitions and restricting to different samples. Our result is robust to these alternative computations. We also compute that share at the firm-level, instead of owner-level as in the baseline computations, and obtain very similar results. Finally, we show that the share of entrepreneurs that purchased their firm is not driven by franchises or by some specific sector of production.

Owner-level. Table A.4 report how many entrepreneurs purchased their business for several alternative definitions of entrepreneurship. For example, instead of active management — as in our baseline definition — we restrict to business owners who have more than 50% of the equity of the firm, or to owners who work at least 40 hours a week in the firm. In bold we highlight our baseline definition for entrepreneurs, which implies that firm owners manage an employer firm.

Firm-level. In addition to the business owner-level results, we study the share of firms which owners acquired them through a purchase at firm-level. We compute the share of firms purchased in two ways: (i) if at least one entrepreneur purchased the firm; (ii) if all the firm's entrepreneurs purchased it. The results are presented in Table A.5. The purchased share computed at the firm- and owner-level are very similar. This is due to the fact that most firms have one entrepreneur, and most entrepreneurs have one firm. As in the business owner results, this share is sensitive to the exclusion of firms with no employment. Definitions that consider firms with no employment tend to have lower purchasing ratios as the main input in production is probably the owner human capital, which is hard to transfer.

Franchises. We further analyze whether franchises are driving our results. Table A.6 shows that even excluding all franchises the share of entrepreneurs that purchased their firm is 16.1% and 24.2% for all firms and our baseline definition, respectively. Although is true that, within franchise owners, the share of entrepreneurs that acquired the business is very high, more than 50%, these owners represent a small group in the total number of entrepreneurs: 2.7% and 4.7% for the two definitions used.

Sectors. We also analyze if our results explained by specific sectors of production. The results are presented in Figure A.2. Although there is variability in the stock and rate of trade, we find that the trade of firms is relatively widespread across all sectors.

To further analyze this, we assess how much of this variability could be related to other observable characteristics correlated to specific sectors, such as firm size. For that we run the following regression

$$\text{Sold}_i = \sum_s \beta_s \times \text{Sector}_{i,s} + \sum_q \beta_q \times \text{Size}_{i,q} + \sum_a \beta_a \times \text{Age}_{i,a}$$

Table A.4: Share of Entrepreneurs That Purchased Their Business

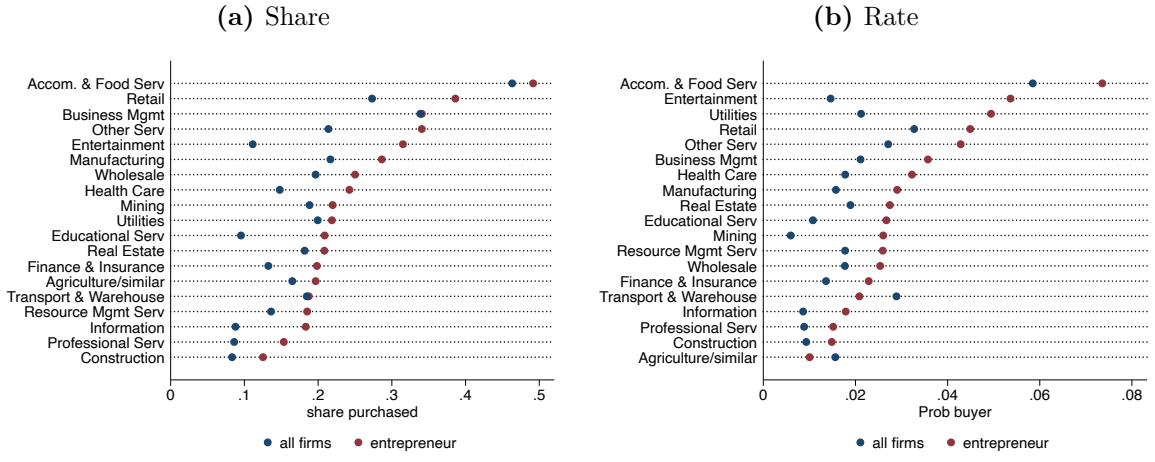
Sample	Purchased	N(weighted)	N
All owners	-	36,856,132	3,409,393
Respond acquisition	16.0%	20,302,192	2,164,541
Manage and own	17.0%	9,503,681	1,112,254
Employment>0	25.9%	5,507,460	1,255,134
Receipts>0	16.9%	17,139,950	1,987,336
Payroll>0	25.1%	6,045,634	1,338,400
All Size>0	26.1%	5,344,964	1,216,319
Entrepreneur	25.5%	3,167,718	698,651
Manage and Payroll >0	24.7%	3,473,610	745,699
Share≥ 50	13.5%	16,274,606	1,479,855
Share≥ 50 and Employment>0	23.5%	3,884,071	745,431
Share≥ 50 + Payroll>0	22.7%	4,320,811	809,769
Share≥ 50 and Manage	15.4%	8,064,388	827,286
Share≥ 50 and Size>0 and Manage	24.2%	2,385,664	455,442
Weighted by Employment	32.2%	3,167,718	698,651
Working Age	17.2%	8,298,522	983,598
Working Age and Employment>0	25.8%	2,838,813	622,336
Hours Worked >40	19.6%	5,679,652	806,923
Hours Worked >40 and Employment>0	26.0%	2,545,635	582,966

SOURCE: 2007 SBO.
NOTES: Purchased refers to the percentage of entrepreneurs that acquire its firm through a purchase. Share refers to the normalized entrepreneur's share of the firm. Hours Worked denotes average number of hours per week the owner spends at the firm. Working age are entrepreneurs which are between 24 and 66 years old.

$$+ \beta_{\text{control}} \times X_{\text{control}} + \varepsilon_i \tag{15}$$

where $\text{Sector}_{i,s}$ indicates if entrepreneur i is in sector s, $\text{Size}_{i,q}$ indicates if size of entrepreneurial firm belongs to quartile q, $\text{Age}_{i,a}$ if entrepreneur belongs to age group a. The dependent variable Sold_i indicates if the entrepreneur sold its business. Figure A.3 exhibit the sector specific effect. We find that, after controlling for these observables, most sectors have a similar propensity. The only sectors with an *unexplained* high propensity to trade are restaurants, hotel and retail sectors, and the ones with low propensities are construction and professional services. These results could be driven by unobservable characteristics such as time-varying demand (restaurants and hotels), fixed costs (construction) and the tradability of the business (professional services).

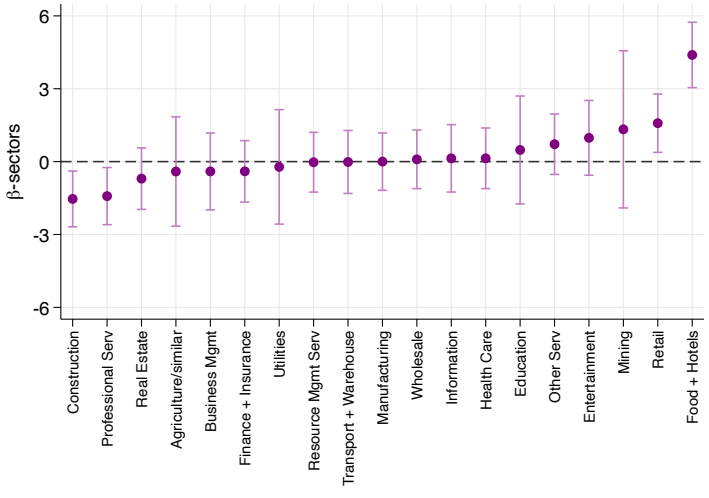
Figure A.2: Share of Entrepreneurs that Purchased by Sector



SOURCE: 2007 SBO.

NOTES: The rate is constructed as the ratio of firms bought in 2007 to all firms normalized to be 2.0% for all firms and 3.0% for entrepreneurial firms in the aggregate.

Figure A.3: Sector Effect on Probability to Sell a Firm



SOURCE: SBO.

NOTES: Coefficients are normalized to 0 using median of estimates. Standard errors are clustered by sector and state. Units are in percentage points.

Table A.5: Share of Firms With Owners That Purchased It

Sample	Owner-level	Firm-level	
		At least one	All
Respond acquisition	16.0%	14.7%	12.0%
All firms	17.0%	26.8%	20.9%
Employment> 0	25.9%	16.3%	15.0%
Entrepreneur	25.5%	25.7%	23.2%
Working Age and Employment> 0	25.8%	25.9%	23.7%
Hours Worked > 40 and Employment> 0	26.0%	26.1%	23.8%

SOURCE: 2007 SBO.
NOTES: Hours Worked denotes average number of hours per week the owner spend at the firm. Working age are entrepreneurs which are between 24 and 66 years old.

Table A.6: Franchises

Sample	All firms	Employer firms
Baseline	17.0%	25.5%
W/o franchises	16.1%	24.2%
Franchises only	50.1%	51.8%
Share of Franchises	2.7%	4.7%

SOURCE: 2007 SBO.

Additional to the previous robustness, we analyze the share of entrepreneurs that purchased their firm conditional on other firm observables such as size and age. In our validation exercise, presented in Section 4, we showed that firms *when purchased* tend to be small and young, consistent with the predictions of our model. Nonetheless, the results of this section show that traded firms, *after purchased*, tend to grow bigger and live longer than non-traded firms.

Firm Size. In table A.7 we present the share of entrepreneurs that purchased their business conditional on the size distribution of three different variables representing firm size: receipts, payroll, and employment. These results show that the trade of firms is even larger, in terms of volume, at the top of the size distribution. For example, in the top 0.1% of receipts, around 39% of entrepreneurs purchased their firm, considerably higher than the unconditional 25.5% share in our baseline calculations.

Firm Age. Finally, we study the share of traded firms conditional on the age of the firm. Table A.8 shows that that older firms tend to have larger share of trades. This is

Table A.7: Firms Purchases, By Firm Size Group

Percentile	Variable	Purchased	Average
Bottom 90	Receipts	24.6%	651
	Payroll	24.6%	153
	Employment	25.2%	8
Top 10\Top 1	Receipts	34.6%	8,624
	Payroll	34.5%	1,773
	Employment	37.9%	83
Top 1\Top 0.1	Receipts	43.8%	57,753
	Payroll	40.0%	9,220
	Employment	37.9%	248
Top 0.1	Receipts	39.0%	381,869
	Payroll	35.3%	49,760
	Employment	32.3%	1,374

SOURCE: 2007 SBO.
NOTES: Results are for the baseline definition (employer firms). Average is computed using both purchased and non-purchased firms. Receipts and Payroll are in thousands ('000) of USD.

consistent either with a higher surviving rate of purchased firms, the declining in trade share we observe in the SCF data, or just a higher probability of being purchased for being around more time. Also, this may reflect some life cycle motives since older entrepreneurs probably manage older firms. Related to this, in Appendix A.4.2 we analyze potential life cycle motives for the trade of firms.

Table A.8: Share of Firms Purchased, By Firm Age

Firm Age	Owner and Manager	Entrepreneur
0-1	8.9%	17.4%
1-2	10.0%	16.3%
2-8	10.9%	16.5%
8-18	13.1%	18.5%
18-28	18.0%	24.9%
+ 28	35.5%	45.2%

SOURCE: 2007 SBO.
NOTES: The age of the firm is the age reported at the date of the survey, not when purchased.

A.3.2 Firm Buyers' Previous Occupation - Robustness

Our second main observation is regarding entrepreneurs' previous occupations. In the main text we documented 66% of current entrepreneurs have never been self-employed (and hence have never been entrepreneurs) prior acquiring its firm. As a robustness we check how many workers, or not self-employed, transition into entrepreneurship by acquiring its firm considering alternative definitions. In Table A.9 we compute the transition rate from worker to entrepreneur conditional on purchasing the firm for: (i) our baseline definition; (ii) when transition to being the main owner of the firm; and (iii) conditional on large firms. Our results are very similar for all these samples.

Table A.9: Firm Buyers' Previous Occupation

Sample	Worker Before Purchasing	
	All firms	Employer firms
Baseline	62.0%	65.9%
Share> 50	61.2%	62.2%
Large Firms	66.9%	69.6%

SOURCE: 2007 SBO.
NOTES: Large Firms as those in the top quintile of the employment distribution.

Firms' Characteristics. We also analyze whether workers tend to buy firms with certain characteristics. For example, one could argue that worker-buyers concentrate in small non-growth-oriented type of businesses, compared to firms that are acquired by previous firm owners. In Table A.10 we show that there is no stark relation between firm characteristics and the share of firms purchased by workers and, if something, the share is slightly larger for older and bigger firms.[26]

A.3.3 Trade of Firms Across Time - Robustness

For our third result regarding the decreasing trend in the trade of firms in the SCF we perform similar robustness exercises. Specifically, we consider: (i) our baseline definition, (ii) our second definition that includes non-employer firms (iii) as entrepreneurs may have more than one firm we can count the number of firms purchased, (iii) entrepreneur as main owner (share $> 50\%$), and (iv) baseline definition computed weighting by value of the firm.

Panel (a) of Figure A.4 presents the results for these alternative definitions and samples. Overall, we find that the decreasing trend in the share of business purchased is

[26]The sample is restricted to 2007 such that the characteristics of the firms are approximately to the ones when purchased. For this sample, the share of firm buyers that were workers is slightly lower (less than 60%) than the one of our baseline sample.

Table A.10: Share of Firm Buyers Who Were Workers

	Workers	Purchased
By Firm Age		
0-2	50.5%	37.0%
3-7	54.7%	14.0%
8-17	56.9%	16.0%
$\geq$ 18	60.7%	33.0%
By Firm Size		
Q1	54.2%	22.9%
Q2	54.0%	27.7%
Q3	55.3%	16.4%
Q4	56.4%	22.6%
Q5	58.7%	10.4%

SOURCE: 2007 SBO.
NOTES: For our calculation we limit to firms purchased in the same year of the survey (2007) and employer firms as in our baseline calculations. The "Workers" column correspond to the ratio of the previously non-self employed entrepreneurs that purchased the firm over the total of firms purchased. The column "Purchased" indicates the amount of firms purchased by characteristic over all firms purchased (i.e., the distribution of purchased firms).

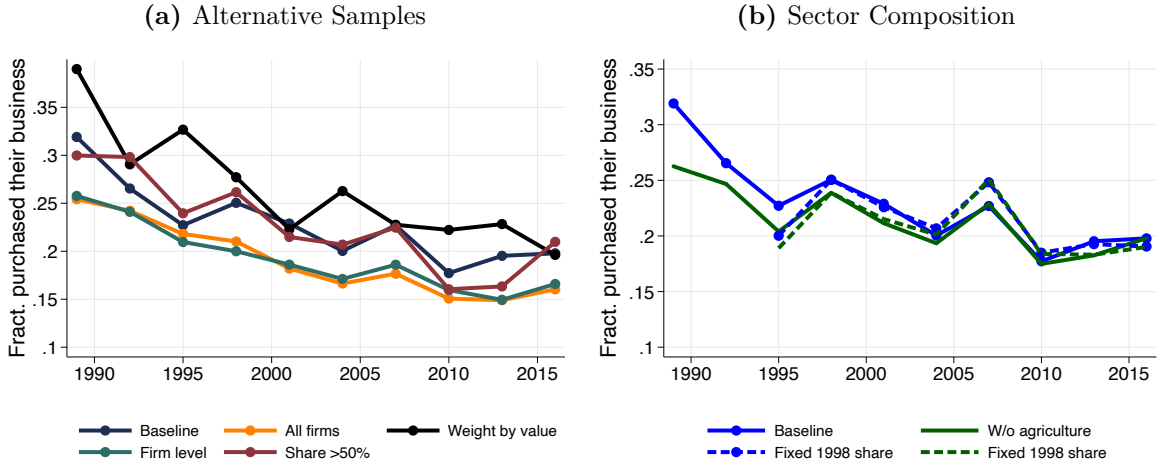
robust across different definitions, both qualitatively and quantitatively. As in the SBO, we find that there is a level difference between weighting the purchase share by size, positive employment or including all firms. This indicates that larger or more valuable firms are more likely to be traded.

Additionally, we explore whether the decreasing trend in the trade of firms is driven by some specific industry. For this, we perform two robustness exercises: (i) we remove the agricultural sector from our estimates, and (ii) we maintain fixed the share of firms by sector in order exclude changes in the composition of sectors across time. The results are presented in Panel (b) of Figure A.4. We find that qualitatively the decreasing trend is robust to these exercises. However, we find that if we exclude agriculture and fix the share of the sectors to the 1998 shares we have that the decrease in the trend remains, but it is less pronounced.

A.4 Additional Evidence on The Market for Firms

In this section we present some additional results regarding the market for firms. First, we analyze the relation between the number of entrepreneurs, owners, and their equity shares. Second, we analyze the relation between the trade of firms and life cycle motives by analyzing the average age of firm buyers and sellers.

Figure A.4: Fraction of Entrepreneurs Who Purchased Their Business - Robustness



SOURCE: 1989-2016 SCF.
NOTES: Fixed 1998 share in panel (b) is created by taking the evolution of purchased firms across time of each sector and aggregate them using their total firm share in 1998.

A.4.1 Entrepreneurs, Owners, and Equity Shares

In the main text we reported that in the 2007 SBO more than 80% of entrepreneurs own only one firm. Further we reported that around 75% of the private firms have only one entrepreneur while more than 96% of the firms have at most two entrepreneurs. These observations support our assumption that each entrepreneur owns only one firm. In this section we further characterize the number of entrepreneurs and owners per firm and study their equity composition.

The main results of this section can be summarized as follows. On the one hand, we find that in the cases in which a firm has only one entrepreneur the equity tends to be concentrated on the single manager-owner. However, there are also several cases in which the entrepreneur share 50/50 the business equity with another non-manager owner. On the other hand, almost all of the firms with two entrepreneurs tend to share the firm equity 50/50.

We also find a decreasing relation between entrepreneurs' equity shares and the size and age of the firm. Nonetheless, entrepreneurs of firms at the top of the size and firm-age distribution still hold large equity shares on their firms of around 50 to 60%. Lastly, we find that at least 82% of the entrepreneurs have one firm that they manage, and all of them (in our sample) have at most 2 employer firms. Taking all these observations into account we conclude that ownership and management of privately held firms in the U.S. is highly concentrated, and usually in a single entrepreneur.

Number of Owners and Entrepreneurs. Table A.11 reports the share of firms in the 2007 SBO conditional on the number of owners and entrepreneurs. The table shows that 74% of the firms have only one entrepreneur, and 96% have at most two. If we include

firms with zero employment these numbers are slightly higher (80 and 97%, respectively).

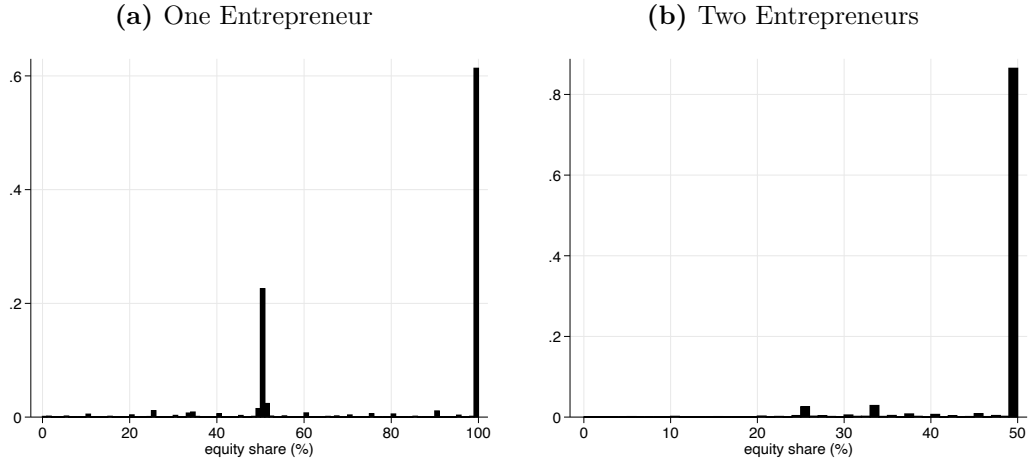
Table A.11: Share of Firms by Number of Owners and Entrepreneurs

Firms		# of Owners			
		1	2	3	≥ 4
All	Own	51.4%	39.3%	4.5%	4.8%
	+ Manage	79.8%	18.0%	1.6%	0.6%
Employer firms	Own	43.0%	42.5%	7.1%	7.4%
	+ Manage	73.7%	22.7%	2.7%	0.9%

DATA SOURCE: 2007 SBO.
NOTES: Entrepreneurs are defined as (i) self-employed, (ii) business owners, who (iii) actively manage their firm. + Employment > 0 also requires that (iv) the firm has a positive number of employees. Other type of acquisition groups: acquired as a transfer, as a gift or other not specified.

Equity Shares. Figure A.5 shows that more than 60% of the firms have an entrepreneur that holds the 100% of the firm's equity. However, for more than 20% of the firms the entrepreneur shares around 50% of the equity with another non-manager owner. On the other hand, in firms of two entrepreneurs the most common arrangement is 50/50 equity shares.

Figure A.5: Equity Shares by Number of Entrepreneurs

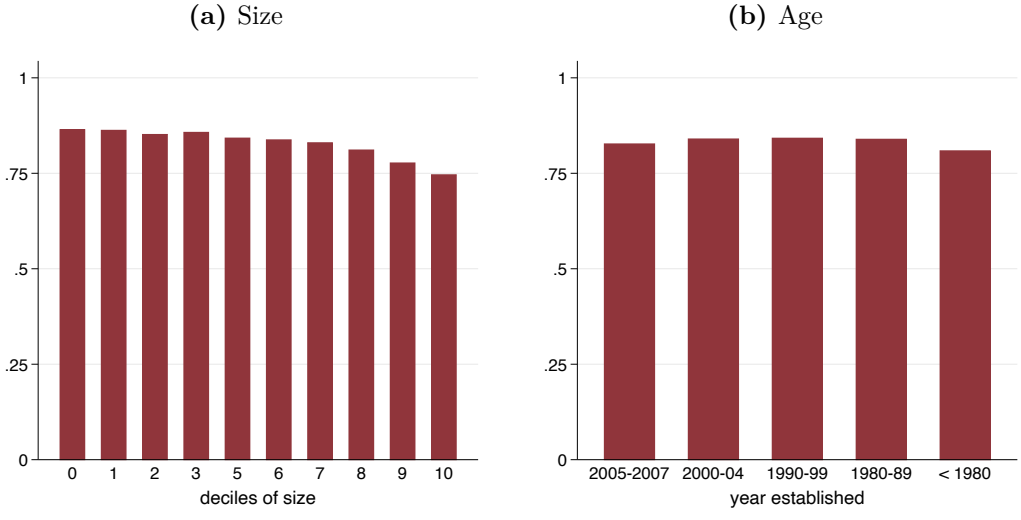


SOURCE: 2007 SBO.
NOTES: Use baseline sample of employer firms.

Next, we analyze the equity share owned by entrepreneurs conditional on firm size and firm age. Figure A.6 reports that the equity share hold by entrepreneurs slightly drops

with firm's size and age. This suggest that larger and older firms do use more equity financing. Nonetheless this negative relation is relatively weak and even for the firms in the top decile of the size distribution around 75% of the firm equity is held by entrepreneurs. Similar patterns are observed across firms' age distribution. Overall figure shows that entrepreneurs own, by a wide margin, the largest share of the equity.

Figure A.6: Equity Shares by Firm Size and Age



SOURCE: 2007 SBO.

NOTES: Deciles of size are constructed using the distribution of firms with positive employment. Decile 0 corresponds to firms with zero employees. Values corresponds to the average value of the sum of entrepreneurial ownership share across the firms' size and age distribution.

Number of Firms Owned. Finally, we use data from the SCF to document the number of businesses each entrepreneur owns and manages. Table A.12 shows that at least 80% of the entrepreneurs manage one firm, and less than 20% manage two firms or more. Both, this and the results in the previous part, suggest that the ownership and management of privately held firms are very concentrated in the U.S. economy.

Table A.12: Firms by Entrepreneurs

	# of managed businesses	
	1	≥ 2
Employer firms	83.5%	16.5%
All firms	80.2%	19.8%

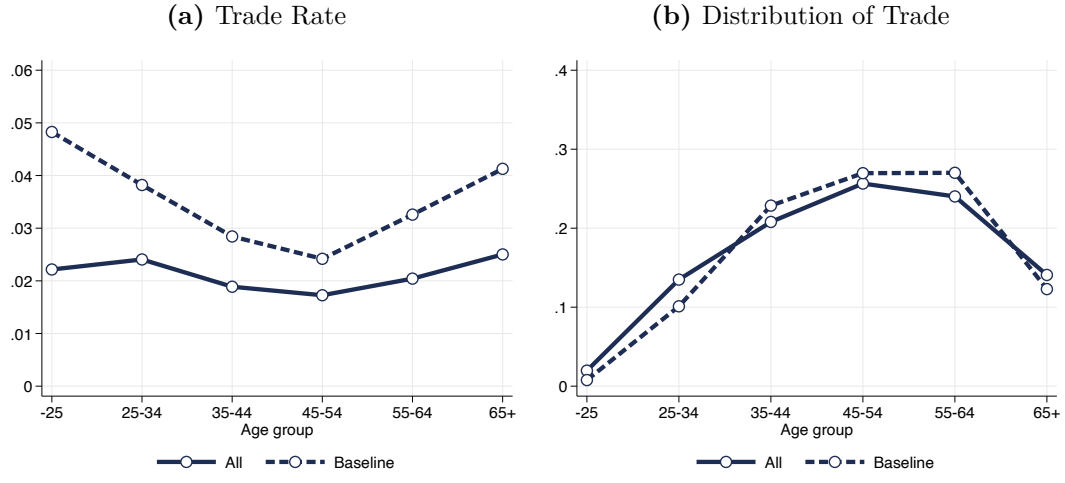
DATA SOURCE: SCF 1989-2016.

NOTES: Number of employer firms (baseline) and all firms per entrepreneur.

A.4.2 Life Cycle Motives

Another important motive for the trade of firms, besides financial frictions, are motives related to the entrepreneurs' life cycle. To address this, we study the trade of firms conditional on sellers' age. Panel (a) of Figure A.7 shows that for our baseline definition, the trade rates across the age of the firms' sellers follows a U-shape. This means that the probability of trade is higher for younger and older entrepreneurs. This evidence is consistent with retirement motives playing a role. Additionally, this could also indicate the presence of financial frictions as younger entrepreneurs are more likely to be more financially constrained, compared to middle-age and older entrepreneurs.

Figure A.7: Trade of Firms by Sellers' Age Group



SOURCE: 2007 SBO.
NOTES: The trade rates in Panel (a) are normalized to match the total trade rate of 2 and 3%.

The previous result looked at the probability of selling. Another question is about the share of total trades conditioning on the age of the seller. Panel (b) shows that, for both definitions, the share of trades is mostly concentrated among middle-aged entrepreneurs, even though these are the ones that exhibit the lowest trade rates. This result reflects the fact that the age distribution of entrepreneurs also follows an inverted U-shape. Thus, even though old entrepreneurs selling rate is relatively high, the fraction of total trades that could be related to retirement, as proxied by share of sells done by entrepreneurs in the 65+ category, is just around 10%.

A.5 Firms' Trade Rate

We indirectly infer the annual trade rate by combining firm dynamics moments, such as the entry and exit rate, and the stock of purchased firms with firms' flow equations. Let us assume the following timing: first, firms exit and enter at some rate, and later the purchase happens. Define the stock of firms purchased as x and the mass of all firms as

y. Then, these variables follow the laws of motion

$$y_{t+1} = y_t \left[1 - \pi^y_{exit,t} + \pi_{entry,t}\right]$$
$$x_{t+1} = x_t \left(1 - \pi^x_{exit,t}\right) + \left[y_{t+1} - x_t \left(1 - \pi^x_{exit,t}\right)\right] \pi_{trade,t_+}$$

where π_{entry} and π_{exit} are the annual entry rate and exit rate, and π_{trade} is the annual rate of firm trade we want to estimate.

Using these flow equations, we have that the ratio evolves as

$$\left(\frac{x_{t+1}}{y_{t+1}}\right) = \left(\frac{x_t}{y_t}\right) \left\{ \frac{1 - \pi^x_{exit,t} + \frac{y_t}{x_t}\left[1 - \pi^y_{exit,t} + \pi_{entry,t}\right] \pi_{trade,t_+} - \left(1 - \pi^x_{exit,t}\right) \pi_{trade,t_+}}{1 - \pi^y_{exit,t} + \pi_{entry,t}} \right\}$$

if no growth of the ratio of firms purchased and same exit rate for x and y then

$$\pi_{entry,t} \left[\frac{y_t}{x_t}\left(1 + \pi_{entry,t} - \pi_{exit,t}\right) - \left(1 - \pi_{exit,t}\right)\right]^{-1} = \pi_{trade,t_+} \tag{16}$$

moreover if we assume that the entry and exit rate coincide we get

$$\pi_{trade,t_+} = \pi_{e,t} \left(\frac{y_t}{x_t} - 1 + \pi_{e,t}\right)^{-1} \tag{17}$$

B Model Appendix

B.1 Additional Results

Table B.1: Ratio of Firms Buyers' Wealth to Average Household's Wealth

	Data	Model
Wealth $(a+p)$	3.83	3.09
Wealth Excluding Business Wealth (a)	2.71	2.74

SOURCE: 1989-2016 SCF.
NOTES: We define firm buyers, in the SCF, as those entrepreneurs who purchased their main business in the year of the survey or the previous one. The ratio is computed as the average wealth of the firm buyers divided the average wealth of the average household. Entrepreneurs are defined as self-employed business owners who manage a business with at least one employee.

Table B.2: Untargeted Moments

	Data	Model		Data	Model
Income Distribution All Households			*Wealth Distribution All Households*		
Top 1	0.22	0.20	Top 1	0.33	0.40
Top 5	0.39	0.39	Top 5	0.60	0.62
Top 10	0.49	0.54	Top 10	0.72	0.75
Bottom 75	0.31	0.30	Bottom 75	0.13	0.07
Bottom 50	0.12	0.16	Bottom 50	0.02	0.01
Bottom 25	0.02	0.04	Bottom 25	0.00	0.00
Income Distribution Entrepreneurs			*Wealth Distribution Entrepreneurs*		
Top 1	0.23	0.36	Top 1	0.24	0.29
Top 5	0.44	0.67	Top 5	0.45	0.63
Top 10	0.57	0.81	Top 10	0.60	0.80
Bottom 75	0.24	0.15	Bottom 75	0.18	0.11
Bottom 50	0.10	0.11	Bottom 50	0.05	0.06
Bottom 25	0.03	0.07	Bottom 25	0.01	0.04

SOURCE: 2007 SCF.

B.2 Additional Derivations

To simplify the notation, in this section, we turn to the recursive notation in steady-state (no time subscripts needed).

B.2.1 Private firms profit maximization

The solution of entrepreneurs' profit maximization problem, stated in (1), is characterized by the input demand functions

$$k(a,z) = \min\left\{k^*(z),\ \lambda a\right\}$$
$$l(a,z) = \left[\frac{z\nu}{w}\right]^{\frac{1}{1-\nu}} k(a,z)^{\frac{\theta}{1-\nu}},$$

where k^* is the unconstrained optimal level of capital given by

$$k^*(z) = z^{\frac{1}{1-\theta-\nu}} \left[\frac{\theta}{R}\right]^{\frac{1-\nu}{1-\theta-\nu}} \left[\frac{\nu}{w}\right]^{\frac{\nu}{1-\theta-\nu}}$$

which is only a function of the quality of the entrepreneurial project z.

B.2.2 Public firm optimality conditions

The FOCs of the public firm profit maximization problem are

$$\eta\frac{Y_p}{K_p} = R$$
$$(1-\eta)\frac{Y_p}{L_p} = w$$

which imply a relation between the public firm's capital to output and prices.

B.3 Computational Solution

To solve the model we use projection methods to approximate the value functions $\{V^o, W^o, V^w, W^w\}$. Thus, we need to solve for coefficients $\{g_V^o, g_W^o, g_V^w, g_W^w\}$ such that, at the grid points, satisfy

$$V^o(a,z) = \Phi^z(a,z)g_V^o$$
$$W^o(a,z) = \Phi^z(a,z)g_W^o$$
$$V^w(a,\varepsilon) = \Phi^\varepsilon(a,\varepsilon)g_V^w$$
$$W^w(a,\varepsilon) = \Phi^\varepsilon(a,\varepsilon)g_W^w.$$

Note that the FOCs of the public firm give us a relation between K_p/Y_p, w and r. Both K_p and L_p are determined as residuals from the market clearing conditions of cap-

ital and labor, thus we can obtain w as a function of r. This considerably simplifies the solution method of our baseline model as we only need to solve for one equilibrium price: r.

B.3.1 Algorithm

The equilibrium objects we need to solve for are

$$\left\{\underline{p}, \overline{p}, p, g_V^o, g_W^o, g_V^w, g_W^w, n_{dm}^o, n_{dm}^w, n_{cm}^o, n_{cm}^e, P_{dm}^o, P_{dm}^w, P_{cm}^o, P_{cm}^w, \beta\right\}$$

where $\underline{p}$ are sellers' minimum prices, $\overline{p}$ are buyers' maximum prices, p are the Nash bargaining prices, n are the probability densities across states, and P are the transition probability matrices (TPMs) across states.[27] We solve for these objects using the algorithm now described. In the remaining subsections, we explain further how we implement some of the steps of the algorithm.

Iteration on prices

0. Propose an initial guess for r.

1. Given r, solve the model (in partial equilibrium).

   Iteration on distributions

   1.0. Propose an initial guess for $\{n_{dm}^o, n_{dm}^w\}$.

   1.1. Given $\{n_{dm}^o, n_{dm}^w\}$, solve for $\{g_W^o, g_W^w\}$.

      Iteration on value functions

      1.1.0. Propose an initial guess for $\{g_W^o, g_W^w\}$.

      1.1.1. Solve for the prices in the market for firms $\{\underline{p}, \overline{p}, p\}$.

      1.1.2. Solve the DM problem: get $\{g_V^o, g_V^w\}$.

      1.1.3. Solve the CM problem: obtain e, a' and P_{cm}.

      1.1.4. Update $\{g_W^o, g_W^w\}$.

      1.1.5. Iterate $\{g_W^o, g_W^w\}$ until convergence.

   1.2. Update $\{n_{dm}^o, n_{dm}^w\}$.

   1.3. Iterate $\{n_{dm}^o, n_{dm}^w\}$ until convergence.

2. Update r such that the capital market clears.[28]

3. Return to 1. until r converges.

[27] Where $\int n^o(a,z)\mathrm{d}a\mathrm{d}z = s^o$ and $\int n^w(a,\varepsilon)\mathrm{d}a\mathrm{d}\varepsilon = (1-s^o)$.

[28] The labor market always as L_p is equal to the residual between the labor supply and the entrepreneurial sector labor demand.

B.3.2 Solving for Prices in the Market for Firms

First, for each potential seller (a, z, κ), we solve for the sellers' minimum price by finding $\underline{p}(a, z, \kappa)$ that implies a sellers surplus, defined in (3) and (4), equal to zero. Using (5), which defines the preference shock utility transfer, the seller's surplus is equal to zero if

$$W^w(a + \kappa \underline{p}, \underline{\varepsilon}) = W^o(a, z)$$

which implicitly defines $\underline{p}(a, z, \kappa)$.

Second, for each potential firm quality z_j, we solve for buyers' maximum price $\overline{p}(\mathbf{s}_i, z_j)$, where $\mathbf{s}_i \in \{\mathbf{s}_i^o, \mathbf{s}_i^w\}$ depending on whether the buyer is a firm owner or a worker. Note that buyers' maximum price is only a function of the seller's firm quality and does not depend on the seller's assets or the preference shock. We compute the buyer's maximum price by solving for $\overline{p}$ that sets the buyer's surplus, defined in (3) and (4), to zero. For the case of current business owners with states $\mathbf{s}_i^o = (a_i, z_i)$, note that they will never buy a lower quality firm $z_j < z_i$. For those cases, we set the buyers' maximum price equal to zero.

Having computed the sellers' minimum prices, $\underline{p}$, and the buyers' maximum prices $\overline{p}$, we can identify the matches with positive gains from trade using the sufficient condition presented in (6). Then, for each potential match of a seller, with states $(\mathbf{s}_j^o, \kappa_j)$, and a buyer, with states $\mathbf{s}_i$, such that there are positive gains from trade, given by $\underline{p}(\mathbf{s}_j^o, \kappa_j) < \overline{p}(\mathbf{s}_i, z_j)$, we approximate the Nash bargaining price, defined in (7), as

$$p(\mathbf{s}_{it}, \mathbf{s}_{jt}^o, \kappa_{jt}) \approx \chi \underline{p}(\mathbf{s}_j^o, \kappa_j) + (1 - \chi)\overline{p}(\mathbf{s}_i, z_j) \tag{18}$$

where $\mathbf{s}_i \in \{\mathbf{s}_i^o, \mathbf{s}_i^w\}$ and χ is the parameter governing buyers' bargaining power. In our numerical simulations, we found that computing the price using (18) is an extremely accurate approximation to the Nash bargaining price obtained from solving the maximization problem presented in (7) while delivering improvements in computational time of several orders of magnitude.

B.3.3 Solving for g_V^o and g_V^w

Given $\{\underline{p}, \overline{p}, p, n_{dm}^o, n_{dm}^w, g_W^o, g_W^w\}$, we can compute the value at DM for firm owners and workers. Then we can solve for g_V^o and g_V^w by inverting the basis functions Φ^z and Φ^ε.

B.3.4 Solving for a', g_W^o and g_W^w

Having solved for the coefficients g_V^o and g_V^w we can solve the households' problems in the production stage (CM). Given r and w, both entrepreneurs and workers problems are a single variable optimization problem in a', which we can solve using golden search.

To obtain g_W^o and g_W^w we use value function iteration. First, by substituting the corresponding optimal policies we obtain two linear systems of equations on g_W^o and g_W^w. Then, we can solve for the coefficients by just inverting the basis functions. For stability reasons we make the update of g_W^o and g_W^w with some dampening.

B.3.5 Transitions and Stationary Distribution

Define the densities across states in DM and CM subperiods as

$$n_{dm} = \begin{bmatrix} n_{dm}^o \\ n_{dm}^w \end{bmatrix} \text{ and } n_{cm} = \begin{bmatrix} n_{cm}^o \\ n_{cm}^w \end{bmatrix}$$

where n_{dm}^o and n_{cm}^o are vectors of size N_o and n_{dm}^w and n_{cm}^w are vectors of size N_w. N_o and N_w are the basis functions grid sizes denoting the number of (a, z) and (a, ε) combinations, respectively. Here $\sum_i n_{dm} = 1$, thus, $\sum_i n_{dm}^o = s_{dm}^o$ and $\sum_i n_{dm}^w = (1 - s_{dm}^o)$.

Then, the TPMs between DM and CM and CM and DM_{+1} solve

$$(n_{cm})^\intercal = (n_{dm})^\intercal P_{dm}$$
$$(n'_{dm})^\intercal = (n_{cm})^\intercal P_{cm}$$

where $(.)^\intercal$ denotes the transpose operator.

We can divide the TPM in blocks differentiating between the two type of agents:

$$P_{dm} = \begin{bmatrix} P_{dm}^{oo} & P_{dm}^{ow} \\ P_{dm}^{wo} & P_{dm}^{ww} \end{bmatrix} \text{ and } P_{cm} = \begin{bmatrix} P_{cm}^{oo} & P_{cm}^{ow} \\ P_{cm}^{wo} & P_{cm}^{ww} \end{bmatrix}$$

where P_{dm}^{oo} captures the transitions of firms' owners that bought another firm or didn't trade, P_{dm}^{ow} is for owners that sold their firm, P_{dm}^{wo} for workers who bought a firm and P_{dm}^{ww} for workers who didn't trade. Regarding CM TPMs, P_{cm}^{oo} is for business owners who operated the firm, P_{cm}^{ow} for owners who didn't operate and went to the labor market, P_{cm}^{wo} for workers who received the $(1 - \zeta)$ shock, P_{cm}^{ww} for workers that didn't. Note that besides changes in the exogenous shocks, asset holdings also change due to payments in the market for firms and due to savings in CM.

Stationarity requires that

$$n_{dm}^\intercal = n_{dm}^\intercal P_{dm} P_{cm}$$

or

$$[I - (P_{dm} P_{cm})^\intercal]\, n_{dm} = 0$$

which implies that we can solve for n_{dm} by computing the eigenvector of $(P_{dm} P_{cm})^\intercal$ associated with the unit eigenvalue, normalized such that $\sum_i n_{dm}(i) = 1$.