QUANTIFYING FIRM RUNS

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October 12, 2021

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Abstract

I study the macroeconomic implications of firms' debt rollover problems. I develop a heterogeneous firm with default risk model where firms can be exposed to rollover and solvency problems. Rollover problems are driven by coordination failures among firms' creditors, akin to runs (*firm runs*), and solvency problems are driven exclusively by firms' weak fundamentals. Further, I allow bankrupt firms to restructure their debt and continue operating, or exit by liquidation. Provisions in the restructuring process mitigate coordination failures among creditors; thus, outcomes of this process jointly with bankrupt firms' characteristics are indicative of the incidence of firm runs. To study the macroeconomic implications of firm runs, I conduct a quantitative analysis of the U.S. economy and find: (i) runs can significantly amplify the impact of crises, explaining around one third of the drop in output; and (ii) credit policy programs — similar to those displayed during the recent Covid crisis — can preclude runs and reduce the impact of the crisis even if participation is low, but can backfire if programs are very ample and subsidize credit to many firms.

*I am highly indebted to Diego Perez, Ricardo Lagos, Jaroslav Borovička and Pablo Ottonello for their continuous guidance and support. I am also grateful to Mark Gertler, Jess Benhabib, Sydney Ludvigson and Federico Kochen for very useful discussions and suggestions. I would like to thank Simon Gilchrist, Gian Luca Clementi, Max Huber, Marina Azzimonti, Michele Cavallo, Dino Palazzo, Juan Martin Morelli, Francisco Vazquez-Grande, Lucia Casal, and participants in seminars at NYU, Central Bank of Uruguay (BCU), and Federal Reserve Board for very helpful comments and suggestions. The paper benefited greatly from several discussions with members of the Division of Monetary Affairs at the Federal Reserve Board. Email: rguntin@nyu.edu

1 Introduction

Economically solvent firms can face liquidity problems — such as debt rollover problems — that lead to their failure. For policy-makers, firms' rollover problems are concerning; therefore, they shape regulations and credit policies, especially in crises.[1] Moreover, entrepreneurs and managers of bankrupt firms frequently cite rollover problems — in contrast to solvency problems — as their main financial problem.[2]

Although firms' debt rollover problems are considered a relevant (and damaging) cause of firms' failure, we know little about their aggregate importance. A major challenge for their quantification is that rollover problems are hard to disentagle from solvency problems. Motivated by these observations, in this paper, I develop a quantitative macroeconomic framework where nonfinancial firms' rollover problems can be identified and quantified. The mechanism behind rollover problems is akin to those of bank runs and self-fullfiling sovereign debt crises; thus, I label them *firm runs*. Using the quantitative setup, combined with data, I answer how relevant are firm runs, and what are their macroeconomic and policy implications.

The quantitative framework builds on general equilibrium models of heterogeneous firms with default risk. In the model, firms maximize their value (i.e., present discounted value of dividends) by making capital investments that can be financed using internal resources (firm's cash-on-hand) or external resources (issuing new debt). Firms have no commitment to repay their debt; thus, default risk limits their borrowing capacity. The rest of the agents in the economy are standard: the corporate debt is purchased and priced by atomistic and perfectly competitive creditors; firms buy capital from a representative capital producer that faces aggregate capital adjustment costs; and households work, consume, save and own all firms in the economy.

I extend the model in two ways. First, using tools from the international macroeco-

[1] See, for example, the discussion described in this column (Financial Times 09/30/2014) around the solvency and liquidity problems (definition akin to a rollover problem) of Lehman Brothers and other financial institutions during the 2008 crisis. More recently, in the Covid crisis, the Federal Reserve intervention in the corporate debt markets was limited to solvent but iliquid nonfinancial firms by Section 13(3) of the Dodd Frank act. Federal Reserve's Chair Jerome Powell, in April 29, 2020, press conference (transcript) explained the limits on the Fed's lending powers as follows: "We can only make loans to solvent entities [...] with the expectation that the loans will be repaid.[..] the Federal Reserve Act [...] it does require that we be secured to our satisfaction and we can't lend it to insolvent companies. [...] We do not make grants. We can't make grants."

[2] This is observed by Ayotte and Skeel (2013) and references within. A notorious example is the case of the Chapter 11 (restructuring) filing of Kodak where its lead bankruptcy lawyer in the first day of the case claimed "We're here for liquidity."

nomics literature, I allow for the possibility of a simple coordination problem among the firm's creditors — firm runs. Second, to introduce a realistic bankruptcy procedure, I assume firms can default their liabilities in two ways: (i) through a liquidation process that allows the firm to exit and creditors recover a fraction of the firm's capital as a payment; and (ii) through a restructuring process that allows the firm and creditors to bargain for a debt haircut, while the firm continues operating. Bankruptcy provisions in the U.S. code — especially those in the restructuring chapter — are designed to preclude coordination failures such as the ones in this paper. Therefore, moments related to the distribution of bankruptcy outcomes — e.g., how many firms liquidate or restructure — and bankrupt firms' characteristics will be central for the identification of firm runs.

Firms decide to liquidate or not *after* issuing new debt, which creates complementarities among creditors and opens the possibility of multiple equilibrium.[3] The contemporaneous lack of repay commitment on new debt issuances implies creditors need to conjecture if the firm is liquidated or not today. Under certain conditions, if creditors (jointly) conjecture the rest of the creditors lend to the firm, such that the firm is not liquidated today, it follows it is optimal for creditors to lend to the firm and the firm is able to continue (repay equilibrium). On the contrary, if creditors (jointly) conjecture the rest of the creditors don't lend to the firm, such that the firm is liquidated today, it follows it is not optimal for firms to lend to the firm and the firm is liquidated (run equilibrium). [4] In case of indeterminacy, to construct the equilibrium, I assume that a stochastic idiosyncratic sunspot variable selects the equilibrium. I show the model has three types of firms according to their current financial position: (i) solvent firms, i.e., repay independently of today's creditors' coordination; (ii) solvent firms exposed to runs, i.e., repay or not depending on the coordination of creditors; and (iii) insolvent firms, i.e., liquidated independently from firms' coordination.

Moreover, firms can decide to restructure. The restructure process has costs and benefits for firms. Restructuring is costly because it is disruptive for firms' operations, has administrative costs, legal fees and reputational costs. To capture the bankruptcy costs, in the model, l simply assume that in the restructure process firms pay an exogenous

[3]This is the well-known Cole and Kehoe (2000) timing in the international macroeconomics literature. Moreover, I focus on the multiple equilibrium that relates to the rollover problem, this is different from the one described by Crouzet (2017b).

[4]Focus on symmetric equilibrium.

cost (proporational to their size). On the other hand, restructuring is beneficial mainly for two reasons. First, firms reduce their debt burden. In the model, I allow firms to bargain with creditors a debt haircut. Second, coordination failures among creditors are mitigated. A fundamental principle in the bankruptcy law literature (see, Jackson (1986) for an early reference) observes that bankruptcy provisions are (and should be) designed to preclude coordination problems among creditors.[5] For example, the Chapter 11 process of the U.S. bankruptcy code — which is the empirical counterpart of the restructure process in the model — has provisions that stops creditors from collecting debts from the firm (11 U.S. Code § 362 - Automatic stay), facilitate the issuance of new debt by the bankrupt firm (Debtor-In-Possesion financing), and create official and ad hoc committees of creditors.[6] These provisions, among others, can stop the run and facilitate coordination among creditors. To incorporate the bankruptcy provisions that coordinate creditors, in the model, I simply assume firms in the restructure process are not subject to coordination failures.

In the model, the incidence of runs is computed as the share of firms exposed to runs times the probability of a run for exposed firms.[7] Both are unobservable; therefore, I inferred them indirectly using the financial distribution of firms and bankruptcy outcomes. The identification follows various steps. First, I calibrate the model parameters unrelated to the bankruptcy process to match salient features of the U.S. economy. The model fits 16 moments using only 4 parameters (the rest are fixed) and performs well. Next, the parameters related to the bankruptcy process are set to match the average debt haircut in the restructure and liquidation procceses — Chapter 11 and Chapter 7, respectively — and the average leverage for firms in the restructure process. The debt haircuts in Chapter 11 are on average low (less than 20%) and the cost of filing is high (required to match the leverage level observed); thus, few insolvent firms will restructure their debt. Therefore, the share of firms that restructure their liabilites — relative to those that are liquidated — identifties very well the conditional probability of firm runs. Combining this probability with the financial distribution of firms, I find that 1.6% of the firms are subject to runs each period, where 21% are exposed and the probability of a run is 7%.

[5]In this literature these problems are frequently called "common pool problem", which are akin to the coordination problems in the model.

[6]Further legal information of the U.S. bankruptcy code in https://www.law.cornell.edu/uscode/text/11.

[7]The probability is implied by the criteria used by the sunspot to coordinate creditors. In the model this probability is a parameter in steady-state.

Using the model fitted to the U.S. economy, I study the role of firm runs during crises. I simulate a prototypical crisis (an unforseen transitory 5% drop in aggregate output from peak-to-trough) and study the transition of aggregate output with and without runs. For various types of shocks driving the crises — total factor productivity (TFP), cash, and credit shocks — I find that runs can significantly amplify the crises, explaining around one third of aggregate output drop. Moreover, runs also increase the persistence of crises, especially when crises are driven by TFP and cash shocks. For crises driven by these shocks, in the presence of runs, a larger fraction of firms exit the economy, which creates further persistence.

Next, I simulate a panel of firms and study the heterogeneous response of investment across firms during the crisis experiments. I contrast the model outcomes with the data. In the data, I focus on two recent crises episodes in U.S.: the Great Recesion and Covid crisis. In both episodes, I find firms with low levels of cash-on-hand adjust their investment significantly more than firms with high levels of cash-on-hand. The model replicates well the heterogeneity (observed in the data) across the cash-on-hand position of firms, but suggests that firms with higher leverage are less sensitive to the crisis (compared to the data). Overall, the data and model suggests that the financial position relative to the cash-on-hand position, instead of the leverage position, is relevant explaining the investment response heterogeneity across firms.

Finally, I study the policy implications of runs. I focus on direct lending policies similar to those displayed by the Fed during the Covid crisis.[8] The goal of these policies was to preclude firms' rollover problems that were exacerbated by the sudden drop in their cashflow due to lockdowns. Although my model does not feature a lockdown shock, I study the effectiveness of these unconventional credit policies in more standard types of crises. Direct lending policies can work "in" and "outside" equilibrium. If firms are subject to runs, the credit facilities from the government could preclude the runs by acting as a form of insurance to creditors and, ultimately, coordinating creditors in the repayment equilibrium. On the other hand, I assume that the government policy has imperfect targeting (the government can't target some relevant characteristics of individual firms); thus, there can be adverse selection and firms receive a subsidized credit.

[8]The Fed provided credit to corporate firms through the Primary Market Corporate Credit Facility (PMCCF) and Secondary Market Corporate Credit Facility (SMCCF).

In my quantitative exercise, I find that policies with few firms that participate and most of the action happening outside equilibrium — credit facilities remain mostly unused — are potent in reducing the effects of runs in crises. On the other hand, credit programs with more participants mitigate more runs, but can subsidize too many firms, which can create future debt overhang problems that extend the crises.

LITERATURE. The paper fits in the broad research agenda described by Brunnermeier and Krishnamurthy (2020). This research agenda aims to incorporate firm-level corporate financing considerations into quantitative macroeconomic models to study aggregate macroeconomic (positive and normative) consequences. My paper's main contributions can be placed in the following strands of literature in macroeconomics and finance:

Financial heterogeneity and business cycles. The framework is built over a flexible price version of Ottonello and Winberry (2020) model, which is very well suited for quantiative studies of firms' responses to aggregate shocks. In their paper, they focus on financial heterogeneity and the investment response of firms to monetary policy shocks. Also, Khan, Senga and Thomas (2020) is closely related. In their paper, they study how default risk can help explaining aggregate dynamics during the Great Recession. I contribute to this literature by extending this framework to incorporate rollover risk and a debt restructure procedure for firms.

Rollover problems and multiple equilibrium in macroeconomics. There is an ample literature that studies rollover problems and multiple equilibrium in macroeconomics. My paper is closely connected to the work on sovereign debt self-fullfiling crises. In my model, the Cole and Kehoe (2000) timing creates rollover risk. Moreover, in the same spirit as Bocola and Dovis (2019), my paper tries to identify and quantify indirectly the incidence of rollover problems. My paper is also connected to the literature on bank runs started by Diamond and Dybvig (1983). Gertler and Kiyotaki (2015) and Gertler, Kiyotaki and Prestipino (2019) use the same timing convention of my paper to introduce a bank run problem in a DSGE model with banks. The contribution of my paper to the literature, on rollover problems and multiple equilibrium in macroeconomics, is to quantifty the rollover problem for nonfinancial firms using an identification strategy that relies on cross-sectional moments of firms.

Firm heterogeneity and bankruptcy. Corbae and D'Erasmo (2021) incorporate a realistic bankruptcy procedure in a general equilibrium model with heterogeneous firms. Their goal is to study how changes in the bankruptcy process could impact aggregate outcomes in the long-run. Different from their paper, inspired by observations in the bankruptcy law literature (for example. Jackson (1986) or more recently Ayotte and Skeel (2013)), I assume the restructure process (Chapter 11 in the U.S. bankruptcy code) works as a coordination device for creditors and could preclude rollover problems.

Rollover (coordination) problems in corporate finance theory. Salient examples are Morris and Shin (2004, 2016) adopt a global game approach to study the rollover problem of firms and the pricing of debt. In related work, He and Xiong (2012a,b); Cheng and Milbradt (2012) study the relation between rollover problems and corporate debt maturity. My paper contribution to this literature is to introduce a simple coordination problem that requires less structure (timing convention) and study the interaction with bankruptcy provisions (which facilitate creditors' coordination).[9]

Corporate credit policy intervention and crises. Sparked by the Covid crisis and the policy response that followed, a set of recent papers study the effectiveness of corporate credit policies during crises using structural models. Ebsim, Faria e Castro and Kozlowski (2021) compare recent crises dynamics and find that the effectiveness of credit policies (akin to those implemented during Covid) depends on the source of the shock. Elenev, Landvoigt and Van Nieuwerburgh (2021) studies similar credit policies and find that they are effective preventing bankruptcies. Crouzet and Tourre (2021) finds that credit policies in crises could backfire through a debt overhang problem. My paper finds similar results, credit policies can potentially prevent bankruptcy and mitigate their risk, but can backfire if they exacerbate future debt overhang problems. The distinctive contribution of my paper, to this strand of literature, is that credit policy works also through a coordination channel, akin to the deposit insurance for bank runs (see, e.g., Diamond and Dybvig (1983)). Thus, credit policy in crises may be potent even if few firms participate in the credit program. Consistent with observations of the Fed's credit programs during Covid (see, e.g., Cox, Greenwald and Ludvigson (2021)).[10]

[9] In my model, the equilibrium is constructed using a sunspot variable, which is a more reduced form approach than in the literature.

[10] Cox *et al.* (2021) observe limited participation in Federal Reserve's Primary and Secondary Market Corporate Credit Facility programs, and argument credit policy (announcements) affected asset prices through nonfundamentals (analogous to the workings of the credit policy in my paper).

Paper's Organization. The paper is organized as follows: Section 2 develops a macroeconomic model where firms can suffer from rollover and solvency problems; Section 3 explains the indentification strategy and calibration of the model; Section 4 quantifies the consequences of runs during crises and provides evidence related to recent crises episodes; Section 5 studies the effectiveness of credit policies during crises and runs; and Section 6 concludes. Further, the Appendix contains further details on the model, data, other exercises and extensions.

2 A Macroeconomic Model of Firm Runs

In this section I describe the theoretical framework used to to identify the incidence of runs and perform the baseline quantitative exercises. I develop a quantitative macroeconomic model of heterogeneous firms with default and runs. The framework is is a flexible price version of Ottonello and Winberry (2020), which I extend in two ways [11] First, firms can be exposed to coordination failures among creditors (runs) á la Cole and Kehoe (2000).[12] Second, bankrupt firms are allowed to liquidate and exit, or restructure their liabilites and continue operating, as in Corbae and D'Erasmo (2021).

To describe the model I follow various steps: Section 2.1 is an overview of the environment; Section 2.2 describes the nonfinancial firms' setup; Section 2.3 shows how creditors determine debt prices given the choices of the firm; Section 2.4 charcaterizes nonfinancial firms' bankruptcy (liquidation and restructure) choices, which depend on debt prices; Section 2.4 shows the nonfinancial firms' recursive problem formulation; Section 2.6 briefly describes the rest of the agents: capital producers and households; and Section 2.7 defines the equilibrium for this economy.

2.1 Environment

The economy has an infinite horizon and is in discrete time, i.e., $t = 0, 1, 2, ...$ It is inhabited by four types of agents: (i) nonfinancial heterogeneous firms that invest, produce and do financial choices in order to maximize their present value of dividends

[11] In particular, I build heavily on Ottonello and Winberry (2020) setup, which is well suited for the quantitative exercises I want to perform in this paper.

[12] Cole and Kehoe (2000) provide a simple mechansim through which the government can be subject to rollover problems in their debt. Further, Gertler and Kiyotaki (2015) and Gertler *et al.* (2019) use the same mechanism for a bank problem. They argue the assumption is weaker than the sequential servicing assumption in Diamond and Dybvig (1983).

(i.e., firm value); (ii) atomistic and perfectly competitive creditors that lend to nonfinancial firms; (iii) representative capital producer that sells capital to the nonfinancial firms; and (iv) a representative household that consumes, saves and works, and owns all the firms in the economy. The price of the final good is normalized to 1, and the price of capital good is q and wages w are determined in general equilibrium. I will assume there is no aggregate risk.

2.2 Nonfinancial Firms Setup

Firm i objective is to maximize its value $V_{it} = \mathbb{E}_t \left[\sum_{t \geq s} \Lambda_s d_{is} \right]$ where Λ_s is the stochastic discount factor of the households and d_{is} is the dividends issued by firm i at period s. The firm has three types of idiosyncratic state variables: (i) exogenous fundamental state variables s^f_{it} ; (ii) exogenous nonfundamental state variable s^n_{it} ; and (iii) endogenous state variables s^e_{it} . Therefore, the idiosyncratic state variable is defined as $s = \left(s^f_{it}, s^n_{it}, s^e_{it} \right)$. Firms are perfectly competitive and there is a continuum of them producing each period with a distribution $\Omega(.)$, which is normalized to $\int d\Omega(.) = 1$.[13]

There is no aggregate risk and the firm's problem can be written recursively (shown later), thus for clarity of exposition I will drop subscripts for firm i and period t, and adopt the recursive timing convention.

Technology and operational profits. Firms combine capital k and labor l to produce a unique final good using a Cobb-Douglas production function

$$y = f(z, \omega, k, l) = z (\omega k)^{\alpha} l^{\nu},$$

where $\alpha \in (0, 1)$ the share of capital and ν the share of labor. I assume the firm operates with decreasing returns to scale $\alpha + \nu < 1$. The firm is subject to two idiosyncratic shocks: (i) a persistent idiosyncratic productivity process where $\ln z' = \rho \ln z + \epsilon_z$ with $\epsilon_z \sim^{\text{iid}} (0, \sigma_z^2)$; and (ii) idiosyncratic capital quality shock ω iid log-normal truncated process where $\ln \omega \in [\underline{\omega}, 0]$.[14]

Fims own capital k, which is inherited from the previous period, and hire labor l at given wage w. The labor choice of firms is static, then I define the operating profits

[13] The distribution of firms may depend on other state variables, thus I don't show them explictly for now.

[14] The purpose of the ω shock is to match quantitatively the default rates observed in the data.

function as

$$\pi\left(z,\omega,k\right)=\max_{l} z\left(\omega k\right)^{\alpha}l^{\nu}-wl \tag{1}$$

with labor demand of firms $l=\left[\frac{\nu z(\omega k)^{\alpha}}{w}\right]^{\frac{1}{1-\nu}}$.

Resources. Firms, each period, can raise external resources by issuing one-period debt b' given price schedule $Q(.)$ offered by creditors. Further, firms can have internal resources n (cash-on-hand), which is the sum of operational profits $\pi\left(z,\omega,k\right)$ and current value of owned capital after depreciation $\left(1-\delta\right)q\omega k$ — where $\delta\in[0,1]$ is the capital depreciation rate and q the price of capital — minus the maturing inherited debt b, i.e.,

$$n=\pi\left(z,\omega,k\right)+\left(1-\delta\right)q\omega k-b. \tag{2}$$

External and own resources are used to issue dividends d and make capital purchases, i.e.,

$$d+qk^{'}=n+Q\left(.\right)b^{'}. \tag{3}$$

As I will show later the financial structure of the firm is going to matter in the presence of financial frictions.

Exit and entry. The mass of entrant firms $\bar{\mu}$ equates the mass of exiting firms in steady-state. I will assume that entrants are endowed with a capital $k=k_0$ and debt $b=0$, and draw their initial productivity level z from an invariant distribution $\Omega^{e}\left(z\right)$ with an average productivity lower than the stationary distribution average by $m\leq 0$ percent.[15] The last assumption is useful to match the firms' life-cycle moments.

Apart from the liquidation choice (explained next), following Khan *et al.* (2020), firms at the begining of each period receive an exogenous exit shock with probability $\gamma\in[0,1]$ which force them to exit after production.[16] This assumption prevents that all firms overcome the financial frictions in steady-state.

Financial frictions. There are two forms of firm-level financial frictions.

[15] This is consistent with evidence that young firms have lower measured productivity as pointed out by Ottonello and Winberry (2020).

[16] I assume exiting firms can still decide to liquidate or restructure their inhertied debt.

First, firms are precluded from issuing equity, i.e.,

$$d \geq 0. \tag{4}$$

This assumption is standard in the literature, is consistent with the scarce issuance of equity by corporate firms in the data and provides greater tractability to the model. Second, firms' debt is defaultable. Each period, firms can decide to repay or not their debt. I assume firms can default their debt in two ways:

1. Liquidate the firm and exit. In this case, all debt is defaulted — inherited b and new issuance $b^{'}$ — and firms exit with value $V = 0$. Upon liquidation, I will assume creditors of b recover fraction $R(b,k,\omega) = \min\left\{1, \alpha_7 \frac{q\omega k}{b}\right\}$ of b, where $\alpha_7 \in [0,1]$ is the parameter that indicates the recovery rate of capital. Futher, I assume that creditors of $b^{'}$ don't recover anything from the current liquidation (on the other hand, they will recover something is the firm is liquidated tomorrow).[17] The empirical counterpart of the liquidation process is the *Chapter 7* of the U.S. Bankruptcy Code.

2. Restructure the firm's debt b and continue operating. In this case, the creditors and the firm bargain on debt recovery rate $\alpha_{11} \in [0,1]$ of inhertied debt b through a Nash Bargaining protocol where the outside option is to continue.[18] Additionally, firms pay $c \in [0,1]$ cost proportional to capital. And most importantly, *runs are precluded* while the firm is in the restructuring process. The resources when restructuring are $n_{11} = \pi(z,\omega,k) + (1-c_{11})(1-\delta) q\omega k - \alpha_{11} b$. In Section 2.4 I provide a thorough discussion of the assumptions related the restructuring process. The empirical counterpart of the restructure process is the *Chapter 11* of the U.S. Bankruptcy Code.

Timing. Figure 1, shows the within period timing of the firm problem for firms that are not subject to the exogenous exit shock. At the begining of the period idiosyncratic states s are realized, i.e., the fundamental and nonfundamental shocks are revealed. After this there is no more within period uncertainty since shocks and states are known,

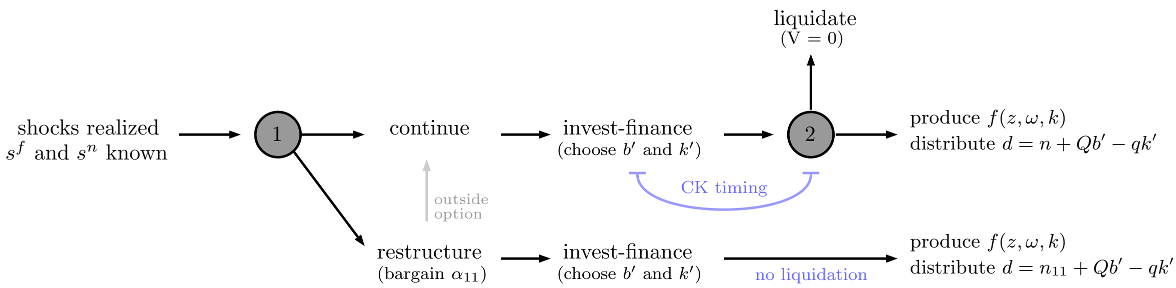
[17] This assumption is done for technical reasons and to capture the fact that inherited creditors tend to have seniority over new creditors.

[18] This assumption can be interpreted as the bankruptcy decision being a joint decision of the creditors and firm or, alternatively, whenever a solvent firms decides to enter the process the judge dismisses the filing. It rules out cases where the firm enters the bankruptcy process as if it is "threatening" creditors.

and common knowledge for all agents. The first gray dot indicates the restructuring choice where firms choose to either continue or restructure.

I the firm decides to contiue, next, it makes the investment and financing choice — i.e., choose (k', b') given pricing schedule $Q(.)$ and price q. After the firm issues the new debt $b^{'}$ the firm makes the liquidation choice (i.e., second gray dot), where it decides to liquidate and exit, or continue and produce. The fact that the liquidation choice takes place after issuing the new debt will be the source of equilibrium multiplicity. This timing is the well-known Cole and Kehoe (2000) timing in the international macroeconomics literature.

Figure 1: Withing Period Timing



Note: timing is conditioned on a firm that doesn't receive an exit shock. In Appendix A.1 I describe and characterize the exiting firm's problem.

If the firm decides to restructure its liabilities then it enters the bargaining for α_{11} process — with outside option to continue (gray arrow up). Under the restructuring process there are no current coordination failures since there is no liquidation choice after issuing new debt in this process, i.e., visually there is no gray dot after the entering the restructure process.

2.3 Creditors and the Debt Pricing Schedule

To characterize the liquidation and restructure choice is relevant to determine the debt pricing schedule before. Creditors to firms borrow from household at the risk free rate and lend to the firms. They are perfectly competitive and atomistic, thus the no-profit condition holds (i.e., equalize expected returns) and prices the debt. All intermediaries are owned by the households then they discount future flow using stochastic discount factor Λ (defined in household problem). Thus, the price of debt is determiend according to

$$Q\left(s,k^{'},b^{'}\right) = \left[1-1_{\{ch7\}}\left(s\right)\right]\mathbb{E}_{\left(s^{'}|s\right)}\left[\Lambda\left\{\left(1-\gamma\right)\mathbf{1}_{\{\text{continue}\}}\left(s^{'}\right)+\gamma\mathbf{1}_{\{\text{continue}|\text{exit}\}}\left(s^{'}\right)\right\}\right]$$
$$+\left[1-1_{\{ch7\}}\left(s\right)\right]\mathbb{E}_{\left(s^{'}|s\right)}\left[\Lambda\left\{\left(1-\gamma\right)\mathbf{1}_{\{\text{Ch11}\}}\left(s^{'}\right)+\gamma\mathbf{1}_{\{\text{Ch11}|\text{exit}\}}\left(s^{'}\right)\right\}\alpha_{11}\left(s^{'}\right)\right]$$
$$+\left[1-1_{\{ch7\}}\left(s\right)\right]\mathbb{E}_{\left(s^{'}|s\right)}\left[\Lambda\left\{\left(1-\gamma\right)\mathbf{1}_{\{\text{Ch7}\}}\left(s^{'}\right)+\gamma\mathbf{1}_{\{\text{Ch7}|\text{exit}\}}\left(s^{'}\right)\right\}R\left(b^{'},k^{'},\omega^{'}\right)\right], \tag{5}$$

where $\mathbf{1}_{\{\text{Ch7}\}}\left(s\right)$ is the indicator of the liquidation choice, $\mathbf{1}_{\{\text{Ch11}\}}\left(s\right)$ is the indicator of the restructure choice (if = 1 then restructure) and the indicator $\mathbf{1}_{\{\text{continue}\}}\left(s\right)$ is defined as $\mathbf{1}_{\{\text{continue}\}}\left(s\right) = 1-\mathbf{1}_{\{\text{Ch11}\}}\left(s\right)-\mathbf{1}_{\{\text{Ch7}\}}\left(s\right)$.[19] Also the indicators are conditioned on the firm receiving the exit shock, i.e. $1_{\{.|\text{exit}\}}$. The liquidation and restructuring choices are characterized in Section 2.4 and Appendix A.1. On the RHS of (5) the first line shows component of the price related to the firms that repay fully next period, the second line the one related to the expectations of restrecturing where they recover $\alpha_{11}\left(s^{'}\right)$ next period, and the last line corresponds to outcomes where the firm is liquidated next period then recover $R\left(b^{'},k^{'},\omega^{'}\right)$. It is useful to define the fundamental price $\tilde{Q}\left(.\right)$ as the price of debt whenever there is no contemporaneous default, i.e.,

$$Q\left(s,k^{'},b^{'}\right) = \left[1-1_{\{ch7\}}\left(s\right)\right]\tilde{Q}\left(z,b^{'},k^{'}\right).$$

The contemporaneous liquidation decision shows up in the debt pricing because of the timing assumption — i.e., the liquidation choice happens after the new debt issuance — and this is the source of potential multiplicity as I will explain next.

2.4 Nonfinancial Firms Liquidation Choice and Restructure Choice

In this subsection I will characterize liquidation and restructure choices. I will start backwards according to Figure 1 timing, since the payoffs of the liquidation choice will affect the decisions in the restructure choice.

I show that the liquidation choice characterization provides a simple way to find out — in the model — which firms (across the state-space) have liquidity (runs) or solvency

[19] As a technical note, the liquidation choice doesn't depend in $\left(k^{'},b^{'}\right)$ because it is a constraint redundant to $d \geq 0$ for the financing-investment choice, and creditors of $b^{'}$ have 0 recovery rate and there is no within period uncertainty after shocks are realized at the begining of the period.

problems.

On the other hand, either insolvent or solvent firms under a run may find beneficial to enter the restructure process. Therefore, the relation between the restructure process and the incidence of runs will depend on if the process works more as a mechanism to solve liquidity problems or a mechanism to resurrect insolvent firms, i.e., how relevant are the costs and benefits of this process.

2.4.1 Liquidation Choice: Liquidity vs Solvency

First I will show how the liquidation choice timing can create the possibility of multiple equilibrium, I briefly discuss some of the assumptions necessary for coordination failures to happen and then I will characterize it across the relevant state-space.

Multiple equilibrium intuition. Lets consider the default decision after new debt issuance for firms that continue. Since firms are subject to the no-equity issuance constraint $d \geq 0$ (feasibility) and their exiting value is 0, then it follows that firms default if they can't issue (weakly) positive dividends.[20] Creditors price the debt according to equation (5). The pricing schedule shows that firms price the debt by calculating the expectations of future default and making conjectures about the firm's liquidation choice today. The price of debt and dividends can be written as

$$Q(.) = \underbrace{\mathbf{1}_{d \geq 0}}_{\text{no liquidation choice}} \quad \underbrace{\tilde{Q}\,(.)}_{\text{debt price if no liquidation}}$$

$$d = n - qk^{'} + Q(.)b^{'},$$

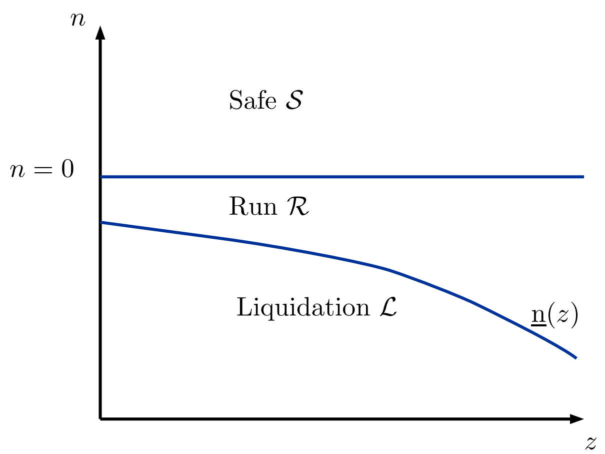
where $\mathbf{1}_{d \geq 0}$ is the no liquidation choice today — i.e., if $\mathbf{1}_{d \geq 0} = 0$ firm cannot satify $d \geq 0$ and is liquidated — and $\tilde{Q}\,(.)$ the pricing schedule without liquidation today. Under some conditions there is a potential feedback between dividends and current debt prices, which could create multiple outcomes. To ilustrate this, assume $n < 0$ then if creditors conjecture $\mathbf{1}_{d \geq 0} = 0$ today then $Q = 0$ and $d = n + \max_{k^{'} \geq 0} \left\{ -qk^{'} \right\} < 0$ then the firm is liquidated. Thus, the outcome is consistent with the conjecture. On the other hand, if creditors conjecture that there is no liquidation today, then $Q = \tilde{Q}$. Moreover, if $\tilde{Q}\,(.)\, b^{'} > n$ for some value of $b^{'}$ then the firm can satisfy $d \geq 0$

[20]This simple result is straightforward from the assumptions.

and doesn't liquidate. Thus, the outcome is consistent with the conjecture. Therefore, under certain conditions — that I will find out next — outcomes could depend on creditor's conjectures of the current liquidation choice and creditors could coordinate in either debt price. I adopt the convention that if creditors coordinate in $Q = 0$ then this means they coordinate in the *run* outcome.

Characterization of the liquidation choice. The *fundamental state-space* (z, n) can be divided in three regions.[21] First, there is a Safe Region $\mathcal{S}$ where firms in this region don't liquidate even if creditors conjecture liquidation today. This means that if $Q = 0$ then they can still satisfy $d = n + \max_{k' \geq 0} \left\{ -qk' \right\} = n \geq 0$. Thus, firms with $n \geq 0$ can always satisfiy $d \geq 0$ and are in $(z, n) \in \mathcal{S}$. Next, there is a Liquidation Region $\mathcal{L}$ where firms are liquidated even if creditors conjecture no liquidation today. This means that even if $Q = \tilde{Q}$ then firms cannot satisfy the $d \geq 0$, i.e., $d = n + \max_{b', k' \geq 0} \left\{ \tilde{Q}(.) b' - qk' \right\} = n < 0$. Since $\tilde{Q}(.) = \tilde{Q}\left(z, b', k'\right)$, then it follows that firms $(z, n) \in \mathcal{L}$ are those with cash-on-hand n bellow a threshold $\underline{n}(z)$ where the threshold is defined by the negative of the maximum amount of external resouces the firm can raise, i.e., $n < \underline{n}(z) = -\max_{b', k' \geq 0} \left\{ \tilde{Q}(.) b' - qk' \right\}$. Finally, there is a Run Region $\mathcal{R}$ where firms can either be liquidated or not depending on creditors' conjecture. This means that if $Q = 0$ then they cannot satisfy $d \geq 0$ so $n < 0$, but if $Q = \tilde{Q} > 0$ then firms satify $d \geq 0$ so $n \geq \underline{n}(z)$. Thus, firms $(z, n) \in \mathcal{R}$ whenever $n \in [\underline{n}(z), 0)$.

Figure 2: Liquidity and solvency regions across (z, n) state-space



Notes: figures shows the state-space (z, n) and the relevant regions for the liquidation choice.

[21] The characterization into three regions is common in models of sovereign that use the Cole and Kehoe (2000) timing convention. See, for example, Bocola and Dovis (2019).

To construct the equilibrium in region $\mathcal{R}$, I define an *idiosyncratic sunspot* variable $\phi \sim^{\text{iid}} U[0,1]$ that is drawn every period at the begining of the period (i.e., the non-fundamental state variable). Given parameter η, if $\phi \leq \eta$ then creditors coordinate in run equilibrium $(Q = 0)$, otherwise creditors coordinate in $Q > 0$ equilibrium.

Formally, the liquidation choice for firms after they decide to continue (i.e., second red dot in Figure 1 from left to right) — $\tilde{\mathbf{1}}_{\{\text{ch7}\}}(.)$ — depends only on states (z, n, ϕ) and is characterized formally as follows

$$\tilde{\mathbf{1}}_{\{\text{ch7}\}}(z, n, \phi) = \begin{cases} 1 & \text{if } \mathrm{n} < \underline{\mathrm{n}}(z) \\ 1 & \text{if } \{\underline{\mathrm{n}}(z) \leq n < 0\} \cap \{\phi \leq \eta\} \\ 0 & \text{if } \{\underline{\mathrm{n}}(z) \leq n < 0\} \cap \{\phi > \eta\} \\ 0 & \text{if } n \geq 0 \end{cases}.$$

This characterization provides a clear distinction between firms that are liquidated because of solvency problems (those in $\mathcal{L}$) and liquidity problems (those in $\mathcal{R}$ with a run). Notice that to determine what firms are liquidated in equilibrium we also need to characterize what firms enter the restructuring process [22]

Discussion of assumption. In the model, I assume the firm borrows from several creditors (i.e., atomistic creditors) and use short-term financing. In Appendix B.3, I show that, in the data, firms tend to borrow from several creditors, especially the large corporate firms. Further, it is well documented that corporate firms' financial debt is mostly composed by corporate debt (instead of bank loans). Thus, corporate firms liabilities tend to have a dispersed ownership. Moreover, in Appendix B.3, I show that the average firm in Compustat has large fractions of their debt maturing in the short-term. Around one third of the financial debt matures in less than 1 year and more than half of the liabilites are due in less than 1 year. For quantiative purposes, I will abstract from long-term debt financing and match moments using liabilites that mature in the short-term in the calibration in Section 3.1. Further, in the model I don't allow firms to manage their liability structure (e.g., extend maturity or concentrate creditors). In Appendix C.2, I show that for the baseline calibration the ex-ante cost of firm runs is negligible for most firms and, when comparing to reasonable costs of

[22] In Appendix A.1, I characterize the liquidation and restructure choice of exiting firms.

managing their liabilites in the literature, most firms wouldn't modify their liability structure (in steady-state) even if allowed. Finally, in Appendix C.3, I characterize the liquidation choice and the conditions for equilibrium multiplicity in various extensions of the model (with long-term debt, costly equity issuance, etc).

2.4.2 Restructure Choice: Bankruptcy Process and Runs

First, I will characterize the Restructure Choice and then I will discuss the assumptions related to the restructuring process.

Characterization. When entering the restructure process firms and creditors will bargain a debt recovery rate $\alpha_{11} \in [0,1]$. I assume that the outside option of the bargaining problem is to continue without restructuring. From the characterization of the liquidation choice we know the payoffs of continuing without restructuring.

A necessary condition for creditors to participate is that they get a higher payoff in the restructuring process relative to continuing without restructuring (outside option). For firms that are solvent and not subject to a run, creditors of b recover fully their debt if the firm continues, so they will not accept a recovery rate lower than 1 on their debt.[23] This rules out cases where solvent firms without a run can restructure their debt, then I can focus only on the cases where firms are liquidated as an outside option.[24]

Let $V(.)$ be the value of the firm when making the financing-investment choice of firms that are solvent without a run, then the Nash bargaining protocol for an insolvent firm or solvent under a run is

$$\alpha_{11}(z,\omega,b,k) = \max_{\alpha_{11}} \left[V(n_{11},z) - 0\right]^{1-\Xi} \left[b\alpha_{11} - bR(b,k,\omega)\right]^{\Xi} \tag{6}$$

where recovery rate α_{11} depends on states (z,ω,b,k) and $\Xi \in (0,1)$ is the bargaining power of creditors. The protocol is subject to the participation constraints

$$n_{11} = \pi(z,\omega,k) + (1-c_{11})(1-\delta)q\omega k - \alpha_{11}b \geq \underline{\mathrm{n}}(z) \tag{7}$$

$$\alpha_{11} \geq R(b,k,\omega). \tag{8}$$

[23]One implicit assumption is that the outside option is costless for the firm and creditors (i.e., same as continuing). This assumption precludes firms from entering the restructure process as if they are "threatening" creditors.

[24]In previous versions of the model although I allow for this possibility the share of solvent firms without a run restructuring their debt was negligible for my baseline calibration.

Notice that, both, insolvent firm or those solvent under a run have $V = 0$ of continuing without restructuring their debt. Equation (7) shows that firms will participate if they are solvent after the restructuring process, equation (8) shows that creditors will participate if they recover more than under liquidation, and (6) shows the objective function of the protocol is a sharing rule of the firm's and creditors' surpluses. For the bargaining process to be feasible we need that the maximum recovery rate the firm is willing to pay is greater than the minimum creditors are willing to accept, i.e.,

$$\frac{\pi(z,\omega,k) + (1-c_{11})(1-\delta)q\omega k}{b} \equiv \alpha_{11}^{\max} > \alpha_{11}^{\min} \equiv R(b,k,\omega).$$

It follows that this is also a sufficient condition for firms to restructure their debt if they are insolvent or solvent under a run. Then firms that go through the restructure process are those that are either insolvent or solvent under a run where their bargaining process is feasible, i.e.,

$$\mathbf{1}_{\{\text{ch11}\}}(z,\omega,\phi,b,k) = \begin{cases} 1 & \text{if } \{(z,n) \in \mathcal{L}\} \cup \{\{(z,n) \in \mathcal{R}\} \cap \{\phi \leq \eta\}\} \cap \{\alpha_{11}^{\max} > \alpha_{11}^{\min}\} \\ 0 & \text{otherwise} \end{cases}. \tag{9}$$

Then the firms that are liquidated are those insolvent or solvent under a run that don't restructure their debt, i.e.,

$$\mathbf{1}_{\{\text{ch7}\}}(z,\omega,\phi,b,k) = \begin{cases} 1 - \mathbf{1}_{\{\text{ch11}\}}(z,\omega,\phi,b,k) & \text{if } \{(z,n) \in \mathcal{L}\} \cup \{\{(z,n) \in \mathcal{R}\} \cap \{\phi \leq \eta\}\} \\ 0 & \text{otherwise} \end{cases}. \tag{10}$$

Finally, the firms that continue without going through the restructuring process are those that are in the safe region or in the run region without a run, i.e.,

$$\begin{aligned} \mathbf{1}_{\{\text{continue}\}}(z,\omega,\phi,b,k) &= 1 - \mathbf{1}_{\{\text{ch11}\}}(z,\omega,\phi,b,k) - \mathbf{1}_{\{\text{ch7}\}}(z,\omega,\phi,b,k) \\ &= \begin{cases} 1 & \text{if } \{(z,n) \in \mathcal{S}\} \cup \{\{(z,n) \in \mathcal{R}\} \cap \{\phi > \eta\}\} \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{11}$$

Further, firms that receive an exit shock can decide to liquidate or restructure before production. The characterization of the restructuring and liquidation choices of exiting firms — i.e., $\mathbf{1}_{\{.|\text{exit}\}}(s)$ indicator functions — is simple and relied to Appendix **A.1**.

Discussion of assumptions. The empirical counterpart of the restructure process is the Chapter 11 of the U.S. Bankruptcy Code. In the model, the cost and benefits for firms from restructuring are three. First, the debt haircut $1 - \alpha_{11}$ benefit captures the formal instance provided by the Chapter 11 to renegotiate the firm's liabilites. Second, the cost of going through the restructure processl $c_{11} \in [0, 1]$ (proportional to capital) captures in a reduced form way legal fees and administrative costs, credit markets reputation penalties, and other costs related to the bankruptcy process — see for example, Corbae and D'Erasmo (2021) and Bris, Welch and Zhu (2006). Third, the restructure process mitigation of coordination failures (i.e., $Q = \tilde{Q}$) benefit captures various provisions of the Chapter 11 process aimed at solving coordination failures among creditors. Ayotte and Skeel (2013) observed that "the dominant normative theory of bankruptcy" (see for example, Jackson (1986)) states that the sole purpose of bankruptcy provisions are to solve "coordination problems caused by multiple creditors". In this spirit, the Automatic Stay provision (11 U.S. Code § 362) that precludes temporarly creditors from individually collecting their debt, Debtor-In-Possesion (DIP) protection that allows firms to issue new debt and continue operating (usually known as DIP financing), and the formation of official and ad hoc committees of creditors during the bankruptcy process, are some of the tools provided by the Chapter 11 to mitigate coordination failures among creditors.

Which firms restructure their debt? Insolvent firms may enter the restructure process if debt haircuts are large and costs are low. On the contrary, firms with liquidity problems (i.e., under a run) may restructure their debt even if there are low haircuts and large costs of bankruptcy. Thus, if the bankruptcy process has low haircuts and is costly, then firms filing for bankruptcy are *mostly* under a run (instead of insolvent). This observation is relevant for the identification strategy described in Section 3.2.

2.5 Nonfinancial Firms' Problem

In this section, I describe the firm's problem recursive formulation. The idiosyncratic states of the firm at the begining of the period are $s = (z, \omega, \phi, b, k)$ where the fundamental exogenous states are $s^f = (z, \omega)$, the nonfundamental exogenous state is $s^n = \phi$, and endogenous states are $s^e = (b, k)$. Let $V(.)$ be the value of the firm that is solvent and not under a run when making the investment-financing decision, and let

$\tilde{V}(.)$ be the value of the firm at the begining of the period (before exit shock). Then the problem of invest-financing for solvent firms without a run today, or recovered from the restructuring process is

$$V(z,n) = \max_{d,k',b'} d + \mathbb{E}_{(z'|z;\omega';\phi')}\left[\Lambda \tilde{V}\left(s'\right)\right] \tag{12}$$

subject to

$$d = n - qk' + \tilde{Q}\left(z,b',k'\right)b' \geq 0$$
$$s' = \left(z',\omega',\phi',b',k'\right)$$

where the continuation value $\tilde{V}(s)$ is defined as

$$\begin{aligned}\tilde{V}(s) = (1-\gamma)&\left[\mathbf{1}_{\{\text{ch11}\}}(s)\,V(z,n_{11}) + \mathbf{1}_{\{\text{continue}\}}(s)\,V(z,n)\right] \\ &+ \gamma\left[\mathbf{1}_{\{\text{ch11|exit}\}}(s)\,n_{11}^{\text{exit}} + \mathbf{1}_{\{\text{continue|exit}\}}(s)\,n\right]\end{aligned} \tag{13}$$

with

$$n = \pi(z,\omega,k) + (1-\delta)\,q\omega k - b$$
$$n_{11} = \pi(z,\omega,k) + (1-c_{11})(1-\delta)\,q\omega k - \alpha_{11}(z,\omega,k,b)\,b$$
$$n_{11}^{\text{exit}} = \pi(z,\omega,k) + (1-c_{11})(1-\delta)\,q\omega k - \alpha_{11}^{\text{exit}}(z,\omega,k,b)\,b,$$

where $\mathbf{1}_{\{.\}}(s)$ indicator functions are described in Section 2.4 and $\mathbf{1}_{\{.|\text{exit}\}}(s)$ are described in Appendix A.1, $\alpha_{11}(z,\omega,k,b)$ solves problem (6) which determines continuing firm's cash-on-hand n_{11} and $\alpha_{11}^{\text{exit}}(z,\omega,k,b)$ solves problem (22) described in Appendix A.1 which determines exiting firm's cash-on-hand n_{11}^{exit}. Investment-financing policy functions $\{b'(z,n), k'(z,n)\}$ solve problem (12).

2.6 Capital Producers and Households

To close the model I describe the problem of the capital producers that sell capital to the firms, and the household that owns all firms, works and consume-save the final good.

2.6.1 Capital Producers.

There is a representative aggregate capital producer that maximizes

$$\max_{I} q\Phi\left(\frac{I}{K}\right) - I$$

where I is the amount of final goods used to produce capital, K is the aggregate capital stock, and $\Phi(.)$ is the aggregate capital adjustment cost function. The first order conditions of the problem is such that

$$q = \frac{1}{\Phi'\left(\frac{I}{K}\right)} \tag{14}$$

where q is the price of capital. The time-varying price of capital q and the incidence of the recovery rate $\mathcal{R}(.)$ on debt prices allow for a channel that maps to the financial accelerator mechanism (Bernanke, Gertler and Gilchrist, 1999) in the transitions. I assume a standard functional form such that $\Phi'\left(\frac{I}{K}\right) = \left[\frac{I/K}{\hat{I}}\right]^{-\psi}$ where $\hat{I}$ is the steady-state investment to capital ratio.

2.6.2 Households.

There is a unit mass of idential households that make the consumption-saving C and labor-leisure L decisions taking wages w, interest rate r and own all the firms in the economy. Then the household determine the stochastic discount factor Λ, the Euler equation holds, and the optimal labor-leisure choice is determined by the marginal rate of subsitution, i.e.,

$$\Lambda' = \beta \frac{U_C\left(C', L'\right)}{U_C(C, L)} \tag{15}$$

$$1 = E\left[\beta \frac{U_C\left(C', L'\right)}{U_C(C, L)}(1+r)\right] \tag{16}$$

$$w = -\frac{U_L(C, L)}{U_C(C, L)} \tag{17}$$

with utility function $U(C, L) = \ln C - \Phi L$.

2.7 Equilibrium

I focus on a steady-state definition of the equilibrium.

Law of montion of the distribution of firms. Let Ω be the distribution of firms that produce which they a mass of 1, $\tilde{\Omega}$ the distribution of incumbent firms at the begining of the period, g and $\hat{g}$ the pdf of ω and ϕ respectively, p the conditional pdf of the productivity shocks ϵ_z, and Ω^e the distribution z of entrant firms. To define the equilibrium first we need to determine the law of motion of the distribution. Distribution of firms that produce is determined by

$$\begin{aligned}\Omega(z,n) = {} & (1-\gamma)\int\left[\mathbf{1}_{\{\text{ch11}\}}(s)\,\mathbf{1}_{\{z,n_{11}(z,\omega,k,b)=n\}}+\mathbf{1}_{\{\text{cont}\}}(s)\,\mathbf{1}_{\{z,n(z,\omega,k,b)=n\}}\right]\mathrm{d}\tilde{\Omega}(s)\\ & +\gamma\int_z\left[\mathbf{1}_{\{\text{ch11}|\text{exit}\}}(s)\,\mathbf{1}_{\left\{z,n_{11}^{\text{exit}}(z,\omega,k,b)=n\right\}}+\mathbf{1}_{\{\text{cont}|\text{exit}\}}(s)\,\mathbf{1}_{\{z,n(z,\omega,k,b)=n\}}\right]\mathrm{d}\tilde{\Omega}(s)\\ & +\bar{\mu}(1-\gamma)\int_z\left[\mathbf{1}_{\{\text{ch11}\}}(s)\,\mathbf{1}_{\{z,n_{11}(z,\omega,k_0,0)=n\}}+\mathbf{1}_{\{\text{cont}\}}(s)\,\mathbf{1}_{\{z,n(z,\omega,k_0,0)=n\}}\right]\hat{g}(\phi)\,g(\omega)\,\mathrm{d}\phi\mathrm{d}\omega\mathrm{d}\Omega^e(z)\\ & +\bar{\mu}\gamma\int_z\left[\mathbf{1}_{\{\text{ch11}|\text{exit}\}}(s)\,\mathbf{1}_{\left\{z,n_{11}^{\text{exit}}(z,\omega,k_0,0)=n\right\}}+\mathbf{1}_{\{\text{cont}|\text{exit}\}}(s)\,\mathbf{1}_{\{z,n(z,\omega,k_0,0)=n\}}\right]\hat{g}(\phi)\,g(\omega)\,\mathrm{d}\phi\mathrm{d}\omega\mathrm{d}\Omega^e(z)\,.\end{aligned} \tag{18}$$

The distribution of incumbent firms $\tilde{\Omega}(z,\omega,k,b,\phi)$ at the begining of the period evolves according to

$$\tilde{\Omega}\left(s'\right)=\int\mathbf{1}_{\left\{k'(z,n)=k'\right\}}\mathbf{1}_{\left\{b'(z,n)=b'\right\}}\hat{g}\left(\phi'\right)g\left(\omega'\right)p\left(\epsilon_z\mid\rho_z z+\epsilon_z=z'\right)\mathrm{d}\epsilon_z\mathrm{d}\Omega(z,n) \tag{19}$$

Equilibrium definition. *Steady-state equilibrium* in this economy is defined as a set of value functions $\{V(z,n),\tilde{V}(s)\}$, firm's decision rules of capital purchases and new debt issuance $\left\{b'(z,n),k'(z,n)\right\}$, bankruptcy decisions $\left\{\mathbf{1}_{\{\text{ch11}\}}(s),\mathbf{1}_{\{\text{ch7}\}}(s),\mathbf{1}_{\{\text{ch11}|\text{exit}\}}(s),\mathbf{1}_{\{\text{ch}\right.$ aggregates $\{Y,C,I\}$, price schedule Q, interest rate r, prices $\{q,w\}$, distributions $\Omega(s)$ and $\tilde{\Omega}(s)$, and debt haircuts $\{\alpha_{11}(z,\omega,b,k)\}$ under Chapter 11.

1. Households choices are determined by (15), (16) and (17).
2. The price of capital is determined by the solution to (14).
3. The debt price satisfy (5) and fundamental price $\tilde{Q}$ is implicit in $Q\left(s,b',k'\right)=\left[1-\mathbf{1}_{\{\text{ch7}\}}(s)\right]\tilde{Q}\left(z,b',k'\right)$.
4. Given prices, firm's decision rules solve the firm problem for firms that produce (12), continuing bankruptcy decisions consistent with (9) (10) (11) and exiting

firms bankruptcy decisions are consistent with equations (21) (23) in Appendix A.1, and the recovery rates are solved by negotation protocols (6) (22).

5. Markets clear: investment is implicitly determined by the law of motion

$$K^{'} = \Phi \left(I/K \right) K + \left(1 - \delta \right) K - \left(k_0 - \left(1 - \delta \right) \mathbb{E} \left[\omega \right] k_0 \right) \bar{\mu}$$

with $K = \int k \mathrm{d}\tilde{\Omega} \left(s \right)$ and aggregate resource constraint is

$$C = Y - I - \mu_{11}$$

where μ_{11} is the aggregate cost of firms filing to Chapter 11.

6. The distribution of firms that produce $\Omega \left(s \right)$ and before bankruptcy $\tilde{\Omega} \left(s \right)$ satisfy (18) and (19).

In steady state the distribution's law of motions is a fixed point, and the households stochastic discount factor is $\Lambda = \beta = \frac{1}{1+r}$ and $q = 1$.

3 Identifying Firm Runs

The incidence of runs depends on how many firms are exposed — i.e., share of firms in region $\mathcal{R}$ — and the likelihood that exposed firms are subject to a rollover problem — i.e., value of η. Both, are not directly observable then I combine data and model to infer them indirectly. In this section I describe the identification strategy and the estimation of firm runs.

To estimate the incidence of runs, first, I fix a set of parameters to standard values in the literature and calibrate the parameters unrelated to the bankruptcy process to fit several moments of the U.S. economy. Next, I calibrate the parameters of the bankruptcy procedure and use the moment that better identifies runs incidence to estimate the value of η. Finally, using the steady-state distribution of firms I determine how many firms are exposed to runs. I find that 1.6% of the firms are subject to runs each period, where 21% of them are exposed and the conditional probability of a run is 7%.

3.1 Standard Calibration

Now, I focus on parameters and moments unrelated to the bankruptcy process. The standard calibration consists on 9 fixed parameters and 4 fitted parameters. To evaluate the empirical fitness of the model I contrast the moments in the model to a wide range of moments (16) in the data. The calibration is done at a quarterly frequency. I use national accounts data from NIPA, firm's balance sheet microdata from Compustat, firms' life-cycle data from the Longitudinal Business Database (LBD), and moments computed in other papers. Further details on the data sources, samples and definitions in Appendix B.

Fixed parameters. Table 1 panel (a) shows the value of the fixed parameters. The subjective discount rate $\beta = 0.99$ is set to fit an annual real interest rate of 5%. The labor disutility parameter $\Phi = 1.16$ is set to match an employment rate of 58%. The parameters of the curvature of the production $\nu = 0.21; \alpha = 0.64$ are set to fit the labor and capital share, respectively, and the capital depreciation rate of $\delta = 0.025$ is set to match estimates from BEA. Following Ottonello and Winberry (2020), I fix the persistence of the idiosyncratic productivity process to $\rho = 0.9$. Further, I fix the initial inhertied debt is $b_0 = 0$ and exogenous exit rate $\gamma = 0.02$ to fit the total the annual exit rate of 10%. Finally, I fix the aggregate capital adjustment cost parameter $\psi = 1/4$ to a standard value in the literature.

Fitted parameters and moments. Table 1 panel (b) shows the value of the fitted parameters unrelated to the bankruptcy process. The volatilty of the idiosyncratic productivity shocks σ_z, the lower bound of the truncated normal process of capital quality shocks (in logs) $\underline{\omega}$, initial capital level k_0, and the relative scale of the initial productivity draw m. Parameters $(\sigma_z, \underline{\omega}, k_0, m)$ are set to fit 16 moments that are related to aggregates, credit spreads and default rates, investment heterogeneity, life-cycle of firms, and balance sheet moments.

Table 1: Standard calibration

Parameter	Value	Calibration
a. fixed		
$\beta = 1/(1+r)$	0.99	fixed to $r = 0.05$ annual
Φ	1.16	fixed to match 58% emp rate
ν	0.64	fixed labor share
α	0.21	fixed capital share
δ	0.025	fixed to match BEA quarterly
ρ_z	0.90	fixed
b_0	0	fixed to no inhertied debt for entrants
γ	0.02	fixed to exit rate 10% annual
ψ	1/4	fixed to standard values in literature
b. fitted		
σ_z	0.032	internally calibrated
$\underline{\omega}$	-0.33	internally calibrated
k_0	0.16	internally calibrated
m	-0.24	internally calibrated

Table 2 shows the moments targeted in the calibration. The model fits fairly well the life-cycle of firms — exit rate, and share of labor and firms at the early stages — and investment rates heterogeneity — average and standard deviation — moments. These moments are from the LBD and Cooper and Haltiwanger (2006). Further, it fits the annual default rate of 3% — from Dun and Bradstreet — which includes defaults by liquidation (Chapter 7) and restructuring (Chapter 11). On the other hand, in the model steady-state equilibrium the average annual credit spreads is 0.7%, which is lower than in the data (2.2%).[25] Also the model fits well the distribution of cash-on-hand n/k' — shares of firms with negative values, between 0 and 1, and greater than 1 — which are particularly relevant moments to estimate the relevance of runs. The cash-on-hand n is measured using data from Compustat, since there is no data on non-Compustat firms I assume they have the same distribution. Details on the measurement are in Appendix B. Finally, the model shows lower correlation between cash-on-hand and capital, and larger average and aggregate leverage than in the data.[26]

[25]This can be explained by the lack of aggregate risk in the model (see, Ottonello and Winberry (2020)). Data excludes recessions.

[26]Leverage is measured as short-term liabilites to capital. Details in Appendix B.

Table 2: Moments standard calibration

Moment	Data	Model	Data source
Aggregates			
K/Y	3.00	2.59	NIPA
I/Y	0.17	0.15	NIPA
gross debt: $\mathbb{E}[\,1_{b>0}b]/Y$	1.05	1.83	NIPA and Flow of Funds
Credit spreads			
default rate: $\mathbb{E}[\mathbf{1}_{\{Ch7\}} + \mathbf{1}_{\{Ch11\}}]$	0.03	0.03	Annual rate from Dun and Bradstreet
cred spread: $\mathbb{E}[r^Q - r]$	2.2%	0.7%	Moody's BAA corporate bonds
Investment heterogeneity			
average investment rate: $\mathbb{E}[i/k]$	0.12	0.20	Cooper and Haltiwanger (2006)
SD investment rate: $\mathrm{SD}[i/k]$	0.34	0.36	Cooper and Haltiwanger (2006)
Life-cycle			
share of firms that exit	0.10	0.11	LBD
share of labor at age 1	0.03	0.04	LBD
share of firms at age 1	0.10	0.11	LBD
share of firms at age 2	0.08	0.09	LBD
Balance sheet			
average leverage: $\mathbb{E}[1_{b>0}b'/k']$	0.37	0.80	Compustat
correlation between n and k'	0.74	0.25	Compustat
fraction of firms with $\frac{n}{k'} < 0$	0.21	0.20	Compustat
fraction of firms with $\frac{n}{k'} \in [0,1]$	0.65	0.75	Compustat
fraction of firms with $\frac{n}{k'} > 1$	0.15	0.05	Compustat

3.2 Identification and Incidence of Runs

In this section, I show how I identify the run likelihood and the share of firms exposed to runs in order to asses the relevance of firm runs. Neither the parameter η nor the threshold function $\underline{n}(z)$ — that defines the regions in the state-space — are directly observable, thus I will infer them indirectly using the firms' bankruptcy choices and financial distribution of firms.[27]

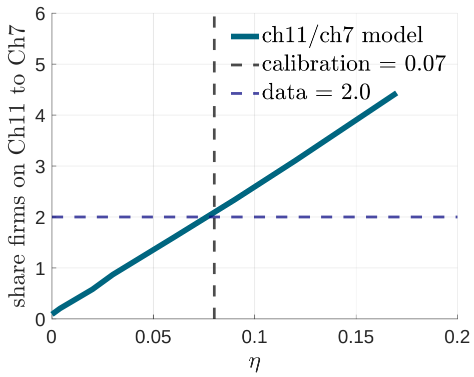
[27] A salient and related example of indirect inference of rollover problems, is Bocola and Dovis (2019) where they infere the rollover risk faced by the government through the time series debt maturity choices. In this paper I use the cross-section of bankruptcy choices to infere the relevance of firm's rollover problems.

Table 3: Identification of η

(a) Parameters and moments of bankruptcy process

Parameters	Value	Moment targeted	Data	Model	Data source
α_7	0.38	$\mathbb{E}[R(b,k,\omega)]$	0.27	0.29	Acharya, Bharath and Srinivasan (2007)
$\tilde{\Xi}$	0.89	$\mathbb{E}[\alpha_{11}]$	0.69	0.82	Acharya *et al.* (2007)
c_{11}	0.40	$\mathbb{E}[b'/k' \mid \text{Ch 11}]$	0.73	0.67	Antill (2021)
η	**0.07**	$\mathbb{E}[1_{\text{Ch11}}]/\mathbb{E}[1_{\text{Ch7}}]$	2.0	1.9	Antill (2021)

(b) Run likelihood and restructure vs liquidation choice



Notes: panel (a) shows the parameters and moments of the bankruptcy process in the baseline calibration. Panel (b) figure shows the relation between η and the share of firms on Chapter 11 relative to Chapter 7, i.e., $\mathbb{E}[1_{\text{Ch11}}]/\mathbb{E}[1_{\text{Ch7}}]$, in the model.

Identification of η. Table 3 panel (a) shows the parameters, and moments in the data and model related to the bankruptcy process. The capital recovery rate of creditors during liquidation $\alpha_7 = 0.29$ is set to match the debt recovery rate $\mathbb{E}[R(b,k,\omega)] = 0.27$ in Chapter 7 reported by Acharya *et al.* (2007). The *approximate* bargaining power of creditors $\tilde{\Xi} = 0.89$ is set to match the debt recovery rate $\mathbb{E}[\alpha_{11}] = 0.69$ in Chapter 11 reported by Acharya *et al.* (2007).[28] The parameter that representes the costs of the Chapter 11 process $c_{11} = 0.40$ is set to fit the leverage choice of firms under Chapter 11 $\mathbb{E}[b'/k' \mid \text{Ch } 11] = 0.73$ reported by Antill (2021).

The data on recovery rates and leverage of Chapter 11 firms, suggest that restructuring the debt is costly and haircuts are relatively low. Thus, *relatively few* insolvent firms will decide to restructure their debt, since gains from Chapter 11 are low.[29] Figure in

[28]For computational efficiency I use a convex-pricing to approximate the bargaining outcome. Details in Appendix A.2.

[29]Consider the extreme case where $\alpha_{11} \to 1$ then only firms under a run restructure their liabilities. In

Table 3 panel (b) shows that a higher level of η in the model shifts the share of firms that restructure relative to those that liquidate.[30]

To approximate better the incidence of liquidation and restructuring in the data, I use the summary statistics provided by Antill (2021). Using Chapter 11 outcomes from the Moody's Ultimate Recovery database Antill (2021) identifies how many Chapter 11 cases end in acquisition or piecemeal liquidations. When considering this, the ratio of firm restructuring to liquidation is around 2. Matching the model to the data — see figure in Table 3 panel (b) — I find a probability $\eta = 0.07$ of a run given firms are exposed, i.e. in region $\mathcal{R}$.[31]

Incidence of runs. Given the value of η, now, it is straightforward to calculate how many firms are subject to runs. First, compute the distribution of firms at the begining of the period across productivity z and cash-on-hand n— i.e., $\Omega^{\text{bop}}(z,n) = (1-\gamma)\int \mathbf{1}_{\{z,n(z,\omega,k,b)=n\}} d\tilde{\Omega}(s)$ — then I estimate the number of firms exposed to runs by computing the share of firms in the run region $\mathcal{R}$ — i.e., $\int_{(z,n)\in\mathcal{R}} d\Omega^{\text{bop}}(z,n)$ — and, finally, I multiply this share by the conditional probability of a run η to estimate the incidence of runs. Figure 3 shows the financial distribution of firms in the model and data. I find that around 20% of the firms are in the run regions and with $\eta = 0.07$ probability they have a run, then 1.6% of the firms are subject to liquidity problems (runs) each period.[32] In the next section, I will asses the macroeconomic importance of runs, focusing on crises episodes, and its policy implications.

the quantitative model less than 10% of the firms in Chapter 11 correspond to insolvent firms.

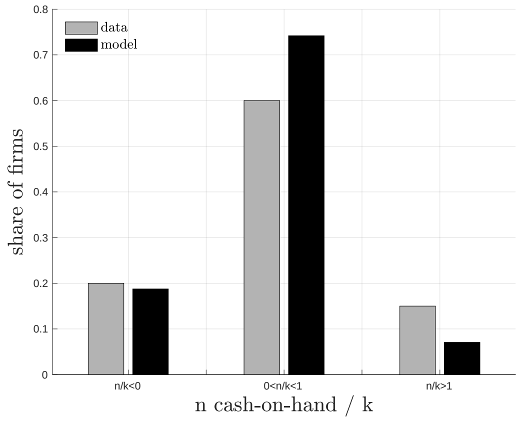
[30]Use the ratio since η shifts the distribution of firms also, thus changes the levels on the opposite direction.

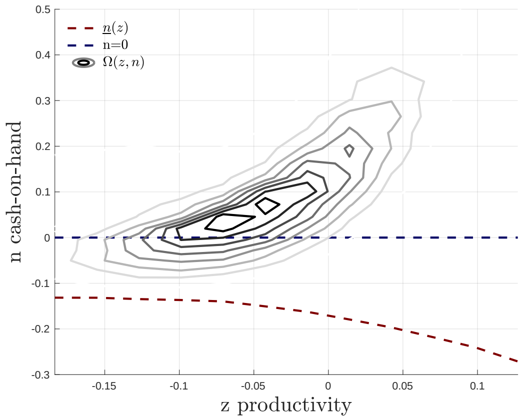
[31]Furthermore, in the calibration only 5% of the firms are restructuring their debt and are insolvent firms, that is consistent with the previous observation that low haircuts and high costs of the restructure process would led to few solvent firms restructuring.

[32]Notice this number includes firms that may not be liquidated because the enter the restructure process.

Figure 3: Financial distribution of firms

(a) Cash-on-hand (n) model and data | (b) Incumbents ($\Omega^{\text{bop}}(z,n)$) model





Notes: Panel (a) compares the distribution of cash-on-hand in the model and the data. Panel (b) shows the contour plot (darker line = higher mass) of the distribution of incumbents firms at the begining of the period (bop) which doesn't receive the exit shock across productivity z (x-axis) and cash-on-hand n (y-axis), i.e., $\Omega^{\text{bop}}(z,n) = (1-\gamma)\int \mathbf{1}_{\{z,n(z,\omega,k,b)=n\}} d\tilde{\Omega}(s)$. The dashed red line is the $\underline{n}(z)$ threshold and the dashed blue line is the $n = 0$ threshold.

Result I:

1.6% of the firms are exposed to runs, with 20% exposed and 7% probability of a run

4 Macroeconomic Consequences of Firm Runs

In this section I study the role of firms runs during crises. First, I simulate a prototypical crisis episode — i.e., crises driven by large aggregate shocks that are standard in the literature — and asses the role of runs by comparing with a counterfactual crisis without runs. Second, I study the heterogeneity in firms investment adjustment during the crises in the model and data. I find that firm runs can significantly amplify the impact of crises, and the heterogeneity observed in recent crises episodes is consistent with rollover problems paying a relevant role in firms investment choices.

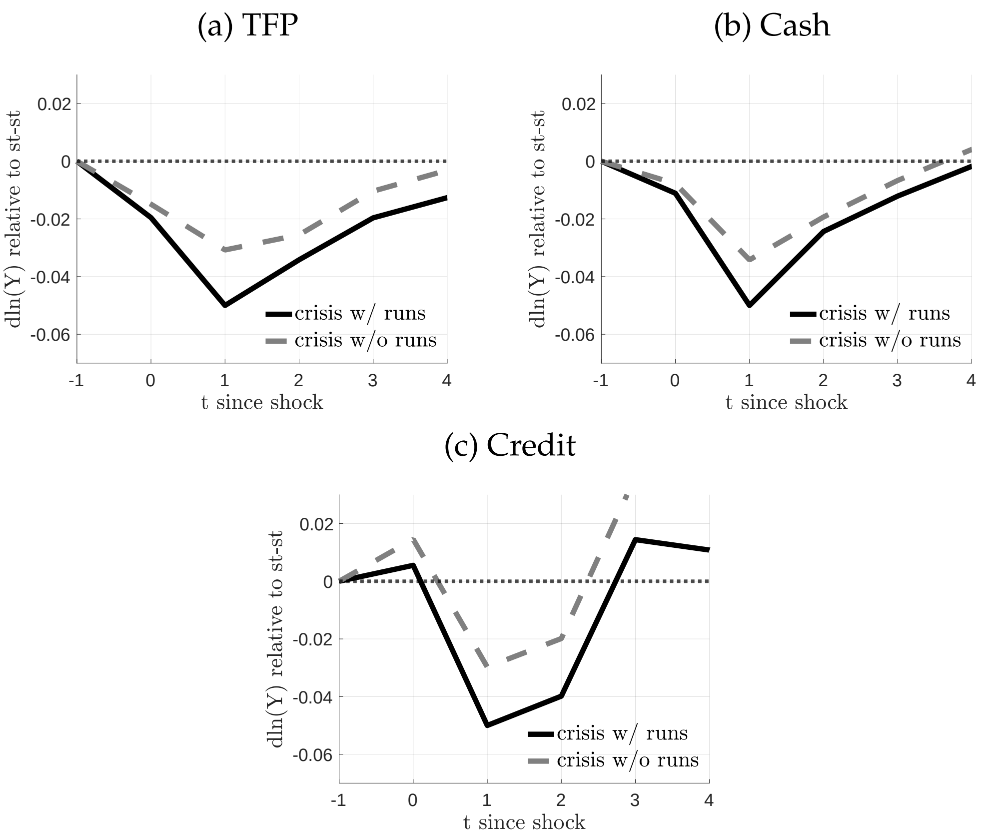
4.1 Crisis and Firm Runs

Now, I study the role of firm runs during crises.[33] Crises, even driven by fundamentals, could deteriorate the firms financial position and suddenly expose them (today

[33]In Appendix C.1, I show that firm runs have negligible consequences over macroeconomic outcomes over the long-run. This happens because firms save away from the run region and in equilibrium they improve their financial position, reducing the overall impact of runs on aggregate outcomes.

and in the future) to rollover problems. To asses the role of runs in crisis I simulate a prototypical crisis — large and unexpected shock — driven by different types of shocks sand perform a counterfactual where runs don't happen during the crisis episode. The three types of crises shocks studied are: (i) aggregate TFP shock; (ii) sudden reduction in firm's cash; (iii) a decrease in liquidation recovery rate α_7, i.e., credit shock. [34] Shocks happen unexpectedly and return to the intial steady state in the long-run. All shocks match a peak-to-trough drop of aggregate output Y of 5%, and have a transitory nature, i.e., persistence of the shocks is set to 0.5. Shocks happen at $t = 0$, where the economy was in steady-state the previous period. Further details of the definition of the shocks and computation of transitions in Appendix **A.3**.

Figure 4: Crisis Shock and Aggregate Output



Notes: panel (a), (b) and (c) show the response of aggregate output Y to an aggregate TFP shock, cash shock and credit shock with runs and without runs (for t=0 and t=1), respectively. The economy at $t = -1$ is in steady-state. The definition of the shocks are in the text and Appendix A.3.

Figure 4 shows the response of aggregate output Y to different types of aggregate

[34]Notice the credit shock in this model is to a reduction in the collateral's value when liquidated, which is different from a credit shock in Khan *et al.* (2020), where the credit shock is closer to the cash shock in this paper.

shocks with runs (black line) and without runs (gray dashed line). In all panels the absence of firm runs —i.e., runs are precluded at $t = 0, 1$ — significantly reduce the depth of the trough in the crises. The counterfactuals indicate that runs happening in the crisis can explain around one third of the drop of output from peak to trough.

Although, on impact, the relevance of runs is similar across shocks, the implications for the dynamics are different. In Appendix A.4, I show the dynamics of net exit and credit spreads during the crises. Crises led by a TFP or cash shock lead to greater firm (net) exit, which makes the crisis more persistent. The persistence is explained by the increase in the number of firms with rollover problems, which are exiting due to a temporarily weaker financial position that exposed them to runs, combined with new firms entering the economy, which are smaller and take time to grow. [35] On the other hand, a crises led by a credit shock will induce firms to deleverage quickly and reduce their investment at $t = 0$, which makes the recovery relatively stronger (compared to other shocks).

Result II:

Firm runs significantly amplify the impact of crises. They explain around one third of the drop in output and make the recovery slower.

4.2 Crisis, Runs and Investment Heterogeneity

Now, I simulate a panel of firms in the model and study the heterogeneity in investment dynamics during the crises. I contrast these results with estimates from the data on recent crises episodes (Great Recession and Covid crisis).

To estimate the *on impact* heterogeneous response — in the model and data — from peak-to-trough of the crisis, I will proceed as follows. First, to account for permanent sectoral heterogeneity I will demean each of the firm-quarter observations of variable x of interest by its sectoral average, i.e. $\hat{x}_{it} = x_{it} - \mathbb{E}_s[x_{it}]$. Next, I will assign each firm-quarter observation of x to different terciles (relative to each period's distribution). Lastly, I run the following panel regression to estimate the heterogeneous responses of

[35] A similar mechanism is in Khan *et al.* (2020).

investment across cash-on-hand n/k and leverage b/k during the crisis:

$$\Delta \log(k_{it}) = \underbrace{\sum_{j=1}^{J} \beta_j^n \left(Q_{it}^{nj} \times \text{crisis}_t\right)}_{\text{heterogeneity across } n/k} + \underbrace{\sum_{j=1}^{J} \beta_j^b \left(Q_{it}^{bj} \times \text{crisis}_t\right)}_{\text{heterogeneity across } b/k} + \underbrace{\Lambda' Z_{it}}_{\text{controls}} + \varepsilon_{it}, \tag{20}$$

where Q_{it}^{xj} indicates if $\hat{x}_{it}$ belongs to tercile j, $\Delta \log(k_{it}) = \log(k_{it+h}) - \log(k_{it})$ is firm's i capital accumulation over a period as long as the crisis studied (i.e., the extension from peak-to-trough of episode studied h), crisis$_t$ indicates if a crisis happens during the period considered (from t to $t+h$) and $Z_{i,t}$ includes the control variables. For the baseline specifications controls, I include firm's fixed effects, sectoral fixed effects, log assets as proxy for size and last quarter sales growth. The coefficients β_j^x are the estimates of interest, and can be interpreted as the diff-in-diff estimates of the crisis impact on capital accumulation for firms in tercile j of $\hat{x}$. Results are shown relative to the third terciles (i.e., the one with highest cash-on-hand or lowest leverage).[36] Further details of the data, estimates and other results are in Appendix B.

Figure 5 shows the results. In all plots, the blue connected line corresponds to the data estimates, which are the average estimates of the Great Recession and Covid crisis. In Appendix B, I show each episode separately. On the other hand, the diamond dots indicate the estimates of the model simulated data for different shocks driving the crisis, and with and without runs.

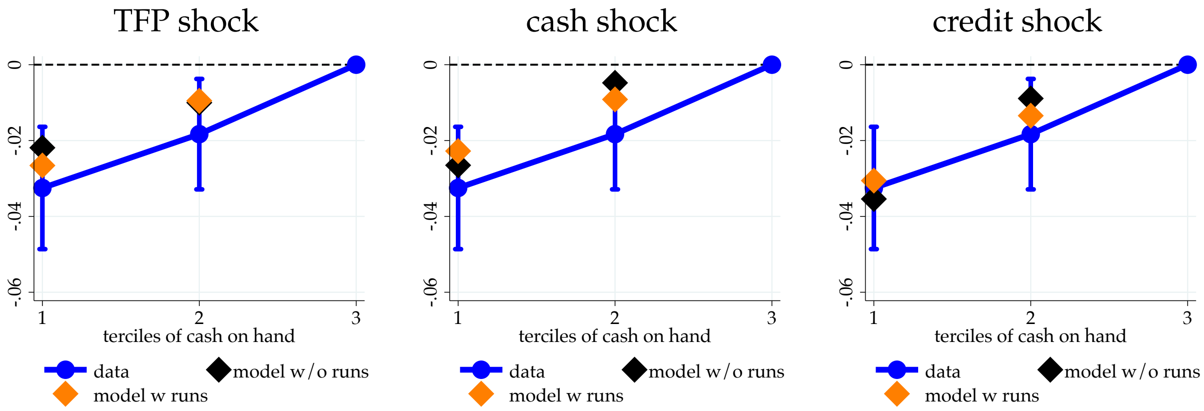
Panel (a) shows that the heterogeneity of investment responses across cash-on-hand is similar in the data and model for different types of shocks. For firms in the lowest tercile of cash-on-hand we observe that the drop in investment (differences in capital accumulation) is 3.5% greater compared to firms with high levels of cash-on-hand. On the contrary, Panel (b) shows that in the data there is no heterogeneity in investment responses across leverage levels. Although the model captures the qualitative difference between heterogeneity in investment responses across leverage and cash-on-hand, it finds that more leveraged firms will adjust less investment (less sensitive to the crises) and this is independent of the shock driving the crisis. This last result is in line with the findings by Ottonello and Winberry (2020) for monetary policy shocks, but inconsistent with my findings in the data of recent crises (where there is little heterogeneity

[36] Notice that for the model estimates I don't include sector fixed effects, also I don't demean by sector the variables of interest.

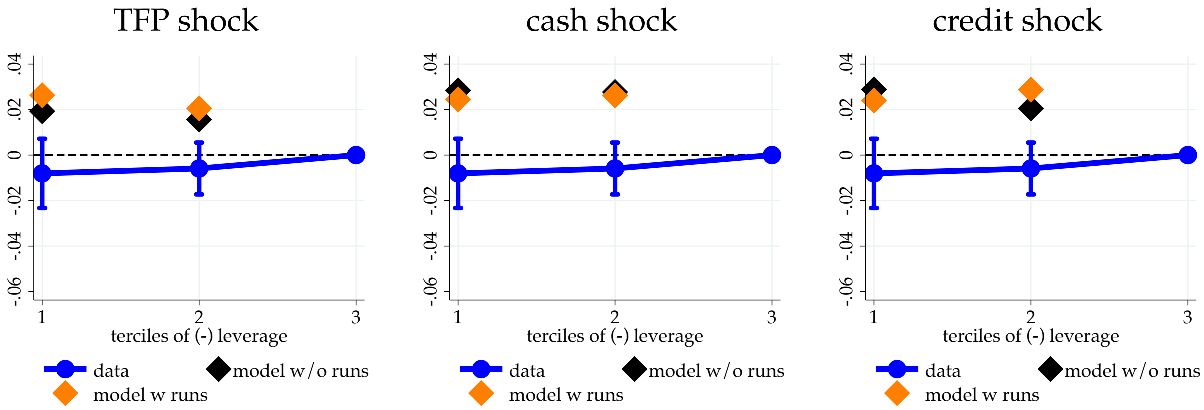
across leverage levels).

Figure 5: Investment Heterogeneity During Crises

(a) Heterogeneity across cash-on-hand: β_j^n



(b) Heterogeneity across (-) leverage: β_j^b



Notes: Panel (a) and (b) show the estimates of β_j^n and β_j^b, respectively, for the data and different aggregate shocks simulated in the model. The blue connected line shows the data and the 90% confidence interval. The diamond dots show the estimates from the model's simulated panel data. In Panel (a), the third tercile corresponds to the firms with the highest cash-on-hand. In Panel (b), the third tercile corresponds to the firms with the lowest leverage. All estimates (in model and data) are from empirical specification (20). The data corresponds to average estimate across the Great Recession and Covid crisis episodes. Estimates are in semester frequency.

5 Policy Response to Crises with Runs

Firm runs spurr the bankruptcy and liquidation of healthy firms, and — as shown in the previous section — they have significant (negative) macroeconomic consequences (i.e., greater depth and slower recovery) during crises. Arguably various of the policies displayed during the recent Covid crises had the motivation of preventing healthy firms from being liquidated.[37] Although the model is not well suited to study some of the particularities of the Covid crisis, it provides a framework where I can explore this new set of unconventional credit policies and how they interact with the firm's rollover problems.

Motivated by this, in this section I study how effective are *imperfect direct lending policies* aimed at reducing the incidence of runs during crises. I find in my quantitative exercises that this policy can be very potent whenever it subsidizes a small number of firms and operates mostly through the insurance channel (precluding runs), otherwise the policy can make the crises and recovery worse.

Credit policy and runs workings. A direct lending policy in the model is promised unexpectedly at $t = 0$ implemented at period J_0 for J periods. When the policy is active, the government offers an alternative pricing schedule $Q_j^g(.)$ for the new debt issuance of the firm at each period $j = J_0, J_0 + 1, .., J$ to a set of elegible firm $\mathcal{P}$ which depends on observable, current or past, characteristics of the firms. The set of elegible firms $\mathcal{P}$ is assumed to be fixed over the time the policy is implemented and I assume creditors know about the policy. Now the external resources from new debt issuance of elegible firms are

$$\max\left\{Q^g(.), Q\left(s, k', b'\right)\right\} b',$$

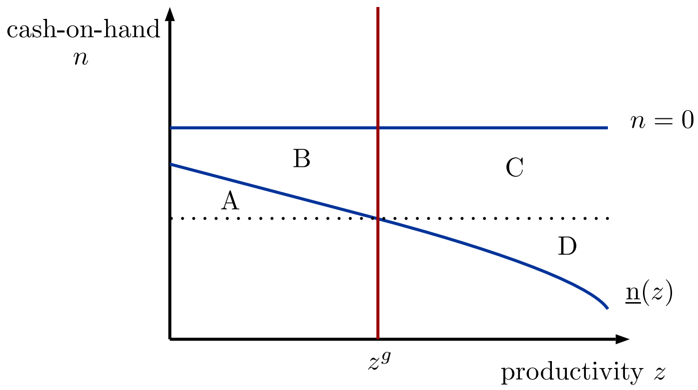
where firms choose the best pricing schedules between the government program and the market.

To understand how this policy works, lets assume that the government sets for 1 period $Q^g(.) = \tilde{Q}\left(z, k', b'\right)$ and a firm with $(z, n) \in \mathcal{R}$ and $\phi < \eta$ — i.e., under a run — is elegible for the government program. Without the policy creditors would offer $Q = 0$ to the firm. With the policy, creditorsknow that since firm are in $\mathcal{R}$ they are solvent under $Q^g = \tilde{Q}$. Thus, if creditors conjecture all other private creditors will not lend to the firm ($Q = 0$) still the firm will be able to satistify $d \geq 0$ at the pricing schedule $Q^g = \tilde{Q}$. The presence of this alternative pricing schedule will coordinate creditors in the $Q = \tilde{Q} > 0$ equilibrium, even if the firm doesn't participate of the policy (i.e., use Q^g) in equilibrium. The intuition is that this policy operates as a form

[37] For example, the Federal Reserve's Primary and Secondary Lending Programs provided direct credit access to a set of seemingly sound corporate firms —i.e., those with a good financial position before the crisis. Providing credit to iliquid but solvent firms is stipulated by the regulation that determines the lending powers of the Fed.

of insurance to creditors, which prevents the $Q = 0$ equilibrium from happening.

Figure 6: Imperfect Credit Policy Elegibility



Notes: figure shows an illustration of the eleigbility and firms participation in the program for a one period example.

Imperfect credit policy quantification. A policy $Q^g = \tilde{Q}$ that precludes all liquidity problems without being used in equilibrium requires the government to observe perfectly each firm's productivity and cash-on-hand (z, n). To make the policy more realistic, I assume that the government observes only n and sets the policy according to a simple rule, i.e., *imperfect credit policy*:

1. The set of elegible firms depend on n only and requires that $n < 0$ (i.e., firms need external resources to satisfy $d \geq 0$). Therefore, the set $\mathcal{P}$ is composed by firms with n such that $n \in [\underline{\mathrm{n}}^g, 0)$, where $\underline{\mathrm{n}}^g < 0$ is a parameter chosen by the government.

2. All elegible firms receive enough funds such that they can satisfy $d \geq 0$, but the government can't discriminate across the n position of elegible firms, i.e., $Q^g\left(z, k^{'}, b^{'}\right)$ is such that $\underline{\mathrm{n}}^g = -\max_{b', k'} Q^g\left(z^g, k^{'}, b^{'}\right) b^{'} - qk^{'} = \underline{\mathrm{n}}\left(z^g\right)$ which implies that n^g determines the choice of $z = z^g$ for the pricing schedule offered by the government.[38]

Figure 6 shows what firms are elegible and the static choice of the participating or not in the program for a 1 period policy.[39] Elegible firms are those in the area $A \cup B \cup C$.

[38]The assumption the government pricing function doesn't depend on the firm's cash-on-hand simplifies greatly the computational problem.

[39]If the policy lasts various periods or is implemented with a lag, then it will affect the solvency thresholds (even in partial equilibrium) since they depend on future prospects of the firm.

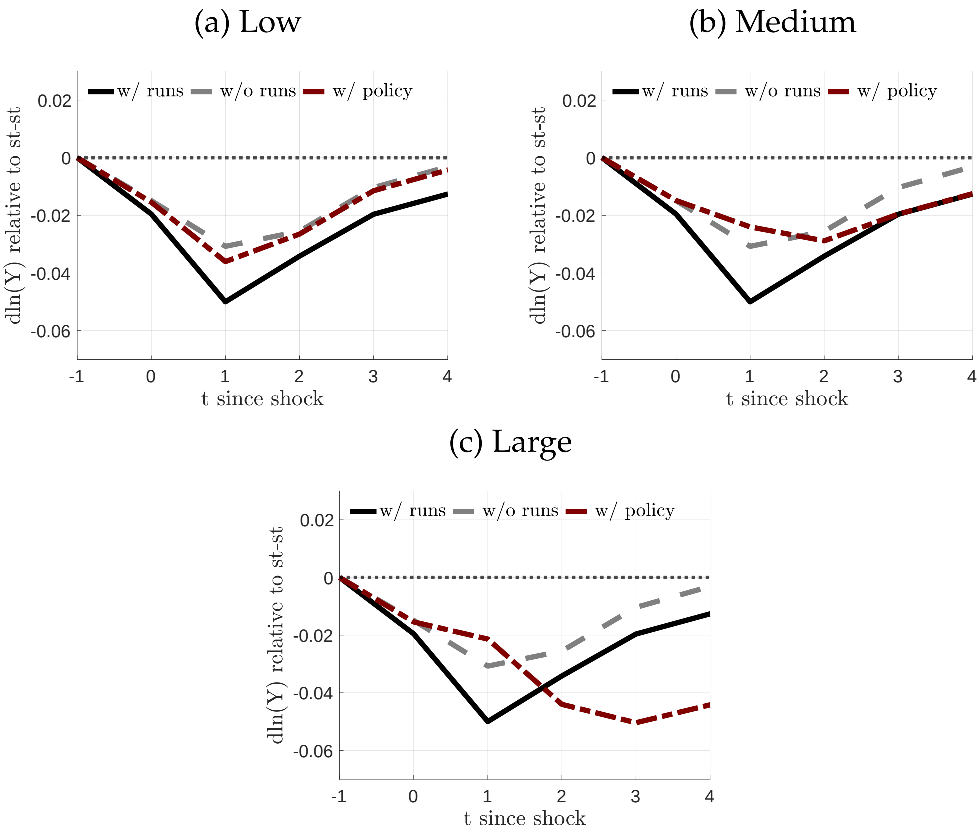
In the case of A in absence of the credit program, the firm would be insolvent, then these firms receive a subsidized credit. On the other hand, firm in B they will find the credit of the program cheaper than the market then they participate so they receive a subsidized credit. On the contrary, firms in region C will have a more expensive credit than the market then they don't participate of the program. Thus, firms in $A \cup B$ receive a subsidized credit and firms in C not. Moreover, the credit program will preclude those firms under a run in $B \cup C$ from being liquidated. Notice that firms under a run in B will participate of the program and in C will not participate of the program but the mere existence of the program will preclude the run. Therefore, if the scope of the policy increases — i.e., lower n^g or, equivalenty, greater z^g — more runs are precluded and more firms are subsidized. The credit subsidizing could exacerbate future debt overhang problems and is costly then the policy faces a potential trade-off when incrementing the scope.

Figure 7 shows the evolution of aggregate Y for different levels of z^g for a policy implemented during $t = 0, 1$ — i.e., peak-to-trough of the crisis without policy. I focus on the cash shock, results for other shocks are in Appendix **X**. I consider a low scope policy where only X% of the firms are subsidized, a medium scope where Y% are subsidized and a large scope where W% are subsidized.[40] At $t = 1$, the larger the scope the lower the impact of the crisis, and even the policy can improve relative to the counterfactual without runs. Thus, larger scope policies not only remove coordination failures, but also provide some extra stimulus. On the contrary, the dynamics indicate that under the low scope policy the economy recovers strongly and similar to the counterfactual without runs, the medium scope policy recovery is weaker and similar to the crisis shock without policy scenario, and the large scope policy makes the crisis longer and deeper. The intuition is that the larger the scope the more subsidized is credit for financially exposed and fundamentally weak firms, which eventually backfire by aggravating the debt overhang problem.[41]

[40]In Appendix X, I show the cost of the different scope.

[41]This mechanism is similar to the one studied by Crouzet and Tourre (2021).

Figure 7: Imperfect Credit Policy Effectiveness by Scope



Notes: panel (a), (b) and (c) show the response of aggregate output Y to an aggregate cash shock with perfectly targeted policy (same as no runs, dashed gray line), imperfectly targeted policy (red dashed line), and without policy intervention (solid black line). The economy at $t = -1$ is in steady-state. The definition of the shocks and crises expermients are in Section 4.1 and Appendix A.3. Further description of the policy in the text.

Result III:

A direct credit policy that is imperfectly targeted is very potent whenever it subsidizes a relatively small number of firms, but backfires whenever many firms receive a subsidized credit.

6 Concluding Remarks

In this paper, I develop a framework where firms' rollover problems (firm runs) can be identified and quantified. The incidence of runs is identified using cross-sectional moments related to the bankruptcy process (outcomes and bankrupt firms' characteristics).

My quantitative results suggest that firm runs, through the failure of healthy firms, have a significant impact during crises. On the other hand, direct credit policies can act as insurance for creditors and prevent runs from happening, but, with imperfect targeting, the government faces a trade-off between preventing runs and future debt overhang problems. Quantiative results suggests that, during crises, the beneifts of a direct credit policy are ambiguous.

In the framework, I focus on the problem of a firm with homogeneous and atomistic creditors, and without active management of its liability structure. Potential extensions could allow for: investors' heterogeneity, as in Halac, Kremer and Winter (2020); endogenous debt maturity structure, as in Bocola and Dovis (2019) (sovereign debt problem); Cheng and Milbradt (2012); Crouzet (2017b) (firm problem); and endogenous number of creditors, as in Bris and Welch (2005); Bolton and Scharfstein (1996). Further, my paper provides insights on the relationship between rollover problems and bankruptcy provisions during crises, which can be applied in other contexts. For example, one potential avenue of future research is to study — in a sovereign debt model with runs — the costs and benefits of including provisions analogous to those of the U.S. bankruptcy code in a supranational sovereign debt bankruptcy process.[42] I leave these extensions and alternative applications for future work.

[42] Guzman and Stiglitz (2016) suggest incorporating U.S. bankruptcy provisions to solve sovereign debt problems. They use arguments related to solvency problems.

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APPENDICES

The Appendix is organized as follows: Appendix A shows additional derivations of the baseline model and further details on the computations; Appendix B includes details of the data sources (sample selection and definitions) and details of the empirical exercises; and Appendix C includes further exercises and extensions of the model.

A Appendix: Model

A.1 Exiting firms problem

Incumbents firms at the begining of the period receive with probability γ a shock that force them to exit after production. I allow for exiting firms to make also the liquidation choice and restructuring choice. Notice that since they exit at the end of the period these firms don't choose $\left(b^{'},k^{'}\right)$ then they are not subject to coordination failures such as the ones described for nonexiting firms. Exiting firms choose to liquidation choice is

$$\mathbf{1}_{\{\text{ch7}|\text{exit}\}}\left(s\right)=\tilde{\mathbf{1}}_{\{\text{ch7}|\text{exit}\}}\left(z,\omega,b,k\right)=\begin{cases}1 & \text{if }\max\left\{n,n_{11}^{\text{exit}}\right\}<0\\ 0 & \text{otherwise}\end{cases}. \tag{21}$$

where n defined as before and $n_{11}^{\text{exit}}=\pi\left(z,\omega,k\right)+\left(1-c_{11}\right)\left(1-\delta\right)q\omega k-\alpha_{11}^{\text{exit}}\left(z,\omega,k,b\right)b$. Since the outside option is to continue then only firms with $n<0$ will restructure their debt and the debt recovery $\alpha_{11}^{\text{exit}}$ is determined by

$$\alpha_{11}^{\text{exit}}\left(z,\omega,b,k\right)=\max_{\alpha_{11}}\left[n_{11}^{\text{exit}}-0\right]^{1-\Xi}\left[b\alpha_{11}^{\text{exit}}-bR\left(b,k,\omega\right)\right]^{\Xi} \tag{22}$$

subject to

$$\begin{aligned}n_{11}^{\text{exit}}&>0\\ \alpha_{11}&\geq R\left(b,k,\omega\right).\end{aligned}$$

The restructure choice is

$$\mathbf{1}_{\{\text{ch11}|\text{exit}\}}\left(s\right)=\tilde{\mathbf{1}}_{\{\text{ch11}|\text{exit}\}}\left(z,\omega,b,k\right)=\begin{cases}1 & \text{if }\{n<0\}\cap\{n_{11}^{\text{exit}}>0\}\cap\left\{\alpha_{11}^{\max|\text{exit}}>\alpha_{11}^{\min}\right\}\\ 0 & \text{otherwise}\end{cases}, \tag{23}$$

where $\alpha_{11}^{\max|\text{exit}}=\frac{\pi(z,\omega,k)+(1-c_{11})(1-\delta)q\omega k}{b}$ and $\alpha_{11}^{\min}=R\left(b,k,\omega\right)$. The firms that continue are defined as $\mathbf{1}_{\{\text{continue}|\text{exit}\}}\left(s\right)=1-\mathbf{1}_{\{\text{ch11}|\text{exit}\}}\left(s\right)-\mathbf{1}_{\{\text{ch7}|\text{exit}\}}\left(s\right)=\mathbf{1}_{\{n\geq0\}}$.

A.2 Computational solution of Bargaining Problem

To solve the bargaining problem I adopt a very simple convex-pricing function to approximate the result from the Nash Bargaining problem.[43] Although this is a reduced form solution to the bargaining problem, it provides better computational speed since we don't need the value function of the firm to compute it. I proceed as follows: I compute the maximum and minimum recovery rates, $\alpha_{11}^{\max}(z,\omega,k,b)$ and $\alpha_{11}^{\min}(\omega,k,b)$, respectively. Using these bounds, for the restructure processes that are feasible I compute the approximate recovery rate $\tilde{\alpha}_{11}(z,\omega,k,b)$ as

$$\tilde{\alpha}_{11}(z,\omega,k,b) = \tilde{\Xi}\alpha_{11}^{\max}(z,\omega,k,b) + \left(1-\tilde{\Xi}\right)\alpha_{11}^{\min}(\omega,k,b)$$

where $\tilde{\Xi} \in [0,1]$ is the approximate bargaining power of the creditors. There is no one-on-one mapping, but to check for robustness I solve for the exact solution and find similar results. Therefore, I adopt this convex-pricing function, which is computationally significantly more efficient than the exact solution.

A.3 Crises shocks and counterfactuals

I work with 3 different types of crisis shocks: a TFP shock, cash shock and credit shock. Shock are unforseen and I study the perfect foresight transitions from $t \geq 0$ where $t = 0$ is the initial impact of the shock (at the begining of the period). The initial impact is calibrated to match a 5% drop in aggregate output from peak-to-trough (large aggregate shock) and the persistence of all shocks is $\rho_{\text{shock}} = 0.5$ (i.e., short lived).[44] I assume the process are

1. TFP shock: firms production function is now $y_{it} = A_t f(z_{it},\omega_{it},k_{it})$ for $t \geq 0$ where $A_t = \exp\left(\rho_{\text{shock}}^t \epsilon_A\right)$ with $\epsilon_A < 0$ the initial shock at $t = 0$.

2. Cash shock: firms cash-on-hand is $n_{it} = \pi_t(z_{it},\omega_{it},k_{it}) + (1-\delta)\, q_t\omega_{it}k_{it} - b_{it} - N_t k_{it}$ for $t \geq 0$ where $N_t = \rho_{\text{shock}}^t \epsilon_N$ with $\epsilon_N > 0$ initial shock to cash proporitional to capital.

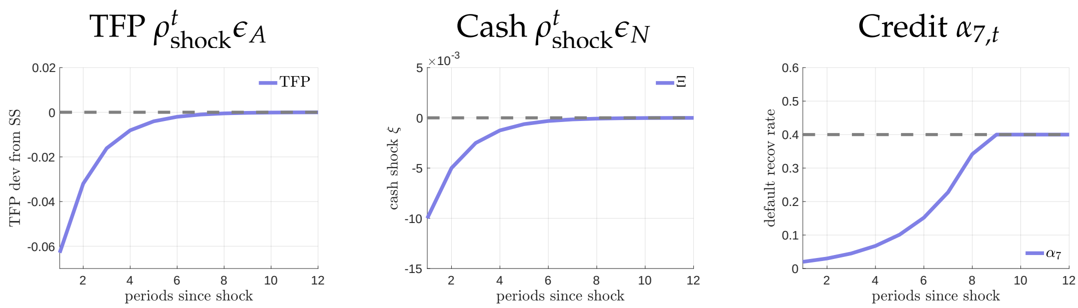
3. Credit shock: recovery rate when liquidated α_{7t} is time-varying for $t \geq 0$ where $\alpha_{7t} = \alpha_7 - \rho_{\text{shock}}^t \epsilon_7$ where $\epsilon_7 > 0$ initial decrease in liquidation recovery rate.

Figure A.1 shows the path for the baseline counterfactuals.

[43] In their robustness exercises Guntin and Kochen (2021) adopt this function to solve computationally for a complex bargaining problem.

[44] More than 95% of the shocks fades away in an year.

Figure A.1: Crises Shocks Path



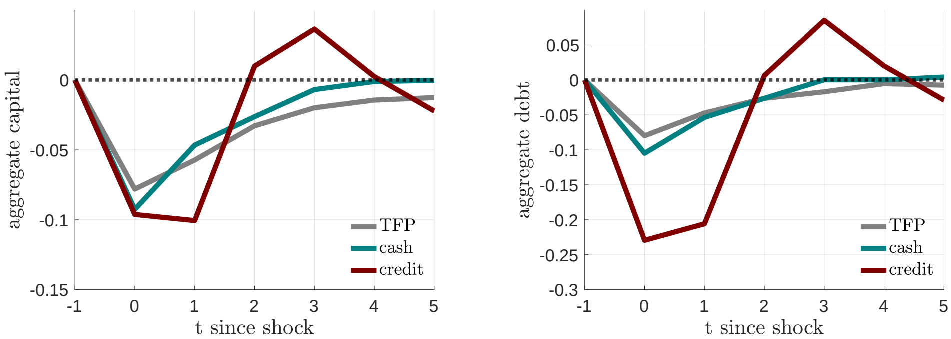
Notes: panel (a), (b) and (c) show the path of the shocks. Shocks happen at $t = 0$. Further description of the shocks in the text.

For computing the aggregates during the transitions I assume that the distribution of firms is no longer a fixed point and allow for *net exit* by fixing the amount of new firms created each period to the one in the steady-state calibration.

A.4 Firm Exit and Spreads during Crises Experiment

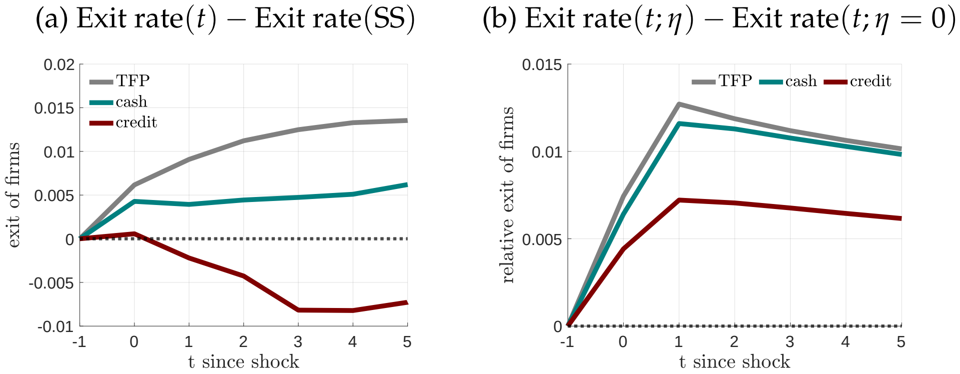
Figure A.2: Capital and Debt during Crises

(a) Capital: $\ln(K_t) - \ln(K_{ss})$ (b) Debt: $\ln(B_t) - \ln(B_{ss})$



Notes: Figures show the dynamics of capital and debt accumulation for the three crisis shocks studied. In both panels, the variables are in terms of log difference relative to steady state — $\ln X_t - \ln X_{SS}$. Panel (a) shows the dynamics of aggregate capital accumulation. Panel (b) shows the dynamics of aggregate debt accumulation.

Figure A.3: Firm Exit during Crises



Notes: Figures show the dynamics of firm exit for the three crisis shocks studied. Panel (a) shows the difference between firm exit rates (exogenous and endogenous) relative to pre-crisis steady-state levels during the crisis episode. Panel (b) shows the difference between firm exit rates with runs relative to the counterfactual without runs during the crisis episode.

A.5 Credit policy program

The baseline credit policy experiment consist of a parameter z^g that determines the pricing schedule $Q_t^g = \tilde{Q}_t\left(z^g, b', k'\right)$ and set of elegible firms $[\underline{n}(z^g), 0)$ and lasts two periods (implemented at $t = 0$ and $t = 1$). The policy is computed backwards, since the presence of the policy at $t+1$ will affect the solvency thresholds $\underline{n}_t(z)$. To estimate the cost of the policy I compute the aggregate credit subsidy as the difference between the price offered by the private sector relative to the government credit program times the amount borrowed for the firms that choose to participate in the program, i.e.,

$$G_t = \int_{(z,n)\in\mathcal{P}} \max\left\{0, \tilde{Q}_t\left(z, k', b'\right) - \tilde{Q}_t\left(z^g, k', b'\right)\right\} b' \mathrm{d}\tilde{\Omega}_t(s).$$

The subsidy is financed through a lump-sum transfer $T_t = G_t$ such that the aggregate output net of government expenditure is $\tilde{Y}_t = Y_t - G_t$.

B Appendix: Data and Empirical Exercises

In this section I outline the data sources, variable definitions, and further empirical exercises and results. I show how the balance sheet moments for the calibration are computed, how it is estimated the heterogeneity in investment dynamics during during the Great Recession and Covid-19 crisis, and study the characteristics of bankrupt firms using microdata.

B.1 Data Sources, Sample Selection and Variable Definitions

In this section I describe the details (definitions and sample construction) of the main data sources used to compute moments related to the balance sheet of firms and empirical exercises in the paper.

B.1.1 Compustat

I use Compustat data to compute moments related to the balance sheet of firms and bankruptcy process, and study the patterns of investment in recent large crises. Compustat is limited to publicly held firms, therefore I assume the balance sheet distribution replicates in the rest of the firms.[45] To construct the sample I follow standard practices in the empirical investment literature.

Balance Sheet Data. Now I explain how I construct the sample and the variables for the balance sheet data used for calibration and empirical exercises. The sample selection criteria follows a firm level filter and firm-date filter. Table B.1 shows the number of observations and those dropped by each filtering step. I drop firms from finance, insurance, and real estate sectors ($\texttt{sic} \in [6000, 6799]$), utilities ($\texttt{sic} \in [4900, 4999]$), non-operating establishments ($\texttt{sic} = 9995$) and industrial conglomerates ($\texttt{sic} = 9997$), and those not incorporated in U.S. and not operate in USD. I drop firm-date observations that with negative capital or total assets, observations with acquisitions of more than 5% of firm's assets, bottom 0.5% and top 99.5% investment rate across the distribution, investment spells of less than 20 quarters, drop if net liquid leverage (net current liquid debt/total assets) is greater than 10 in absolute value, drop if log sales growth is greater than 1 in absolute value, and negative sales or negative liquid assets.

Due to changes in the accounting data of Compustat, I split the sample for the Great Recession (period 1983-2017) and Covid-19 Crisis (period 2019-2020) (see Ma (2020) notes on the accounting changes after 2019).[46] The sample criteria for the 2019-2020 period differs slightly from the 1983-2017 sample. Since the 2019-2020 sample is smaller I exclude filters related to investment outliers and spells, and select firm-quarter observations that register they changed they updated their accounting criteria.[47]

The final sample — pre-Covid — has 426,465 firm-date observations, and the Covid sample has 13,974 firm-date observations.

[45] An alternative approach is to fit the model to a subset of firms that can be defined as the Compustat firms. For simplicity I use the assumption described in the text.

[46] An alternative approach is to use Compustat Snapshot to remove the operation leases from various entries in the balance sheet, but access to this dataset is restricted.

[47] The variable `acctchgq` is "ASU16-02" or "IFRS16" the quarter the firm changes it's accounting criteria.

Table B.1: Sample Selection Compustat - Quarterly Data

	# Drop	# Obs
Baseline		
1983-2017	-	1,484,973
Non-financial sector	474,327	1,010,646
U.S. incorporated and USD currency	212,680	797,966
>20 quarter investment spell	105,503	585,286
No outliers	158,821	426,465
Covid sample		
2019-2020	-	75,712
Change in accounting	38,446	37,266
Other filters	23,292	13,974

The definition of the main variables used for the calibration and regressions are:

1. Capital stock k: is constructed using the perpetual inventory method, following the usual convention in the investment literature.[48] I compute the initial capital level using the level of gross plant, property and equipment `ppegtq`, and using the quarterly change of net plant, property and equipment `ppentq`. The depreciation rates δ are calculated using the BEA accounts to compute investment rates (i.e., change in capital k net of capital depreciation).

2. Net debt stock b: different from other papers in the literature I assume b corresponds to the short-term liabilites. Liabilities include financial debt, debt with suppliers and other firms, accounts and tax payables, and others. `lctq` minus cash holdigns `cheq`. Complementary, the gross debt position I define it as the short-term liabilites `lctq` only.

3. Operating profits π: corresponds to the variable `ibdpq`

4. Liquid value of assets $q\omega k\,(1-\delta)$: to compute this I use the assets of the firm (excluding cash) as follows: for asset category a_{ij} we can compute the liquid value of firms' assets as $\sum_j lr_j \times a_{ij}$ where lr_j is the liquidation rate. The liquidation rates used by asset category are 44% inventories, 63% receivables and 35% physical capital from Kermani and Ma (2021).

5. Cash-on-hand n: is computed as the sum of π and $q\omega k\,(1-\delta)$ minus b. It is assumed that all liabilities can be collected each period.

6. Size: log of total assets `atq`.

[48]See for example, Mongey and Williams (2017); Jeenas (2019); Ottonello and Winberry (2020) for recent references.

7. Sales growth: quarterly growth of sales `saleq`.

Nominal variables are deflated using the BLS implicit price deflator, unless specified. Percentiles of variables used are constructed by year (not quarter). When specified variables are standarized, winzorized and/or demeaned.

Banckurp:tcy Data. To identify when and what firms operate under Chapter 11 in Compustat I use the same strategy as Corbae and D'Erasmo (2021). I use the footnote to total assets (`atq`) and deletion information variables `dlrsn` and `dldte`. A firm is in Chapter 11 if (i) footnote (next period) reports adoption of new accounting under Chapter 11 bankruptcy; (ii) if firm shows as bankrupt but is not deleted; (iii) if the firm shows as bankrupt and deleted but this is not due to liquidation; (iv) and if firm's last observation in the sample is bankrupcy but there is no bankruptcy information.

B.1.2 Federal Judicial Center - Integrated Database (FJC-IDB).

FJC-IDB bankruptcy data includes all petitions filed under the Bankruptcy Code (any of the Chapters) on or after October 1, 2007 and any petitions filed before October 1, 2007 that are still pending. This dataset provides information of the fillings, closures and several firm characteristics.

I will focus on a sample of corporate firms filings to Chapter 11 and Chapter 7. This includes public and privately held firms. Table B.2 shows the sample selection criteria.

Table B.2: Sample Selection FJC-IDB

	# Drop	# Obs
All		32,084,867
Corporations	31,546,855	538,012
Chapter 11 and 7	9,771	528,241
Filings	358,890	169,351
Closures	357,023	171,218

Notes: This table shows the number of observations resulting from the sample selection for the FJC. The first line, *All*, shows the original number of entries from in the dataset. from 2008 to 2020. Two samples are used, one that includes only the filings and other that includes only the closures (closure sample entries include filing characteristics). The following lines detail the set of observations dropped from different filters applied to the sample and the resulting number of observations. More details on these filters can be found in the text.
Data source: FJC-IDB.

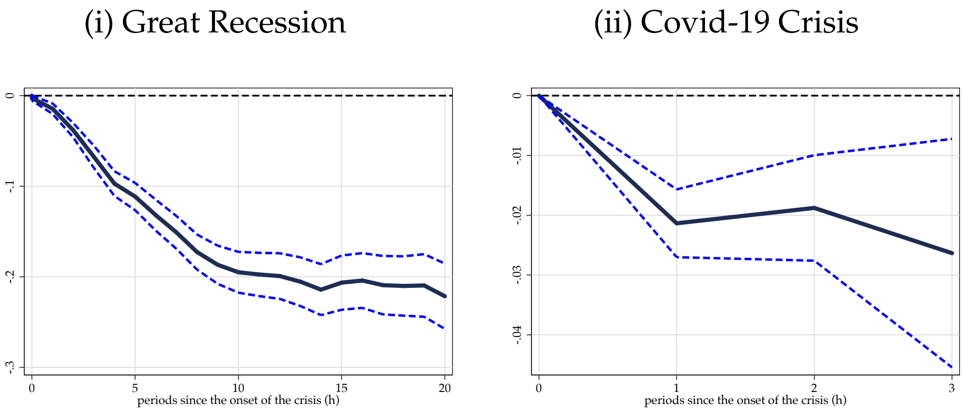
B.2 Heterogeneous Investment Responses During Recent Crises

In this section I will study the heterogeneous investment response of firms during the Great Recession and Covid-19 crisis. First, I will show the aggregate dynamics of the crises. Second, I will show the heterogeneity across the balance sheet positions, focusing on cash-on-hand and leverage positions.

B.2.1 Aggregate Dynamics

Figure B.1 shows that using firm level data of publicly listed firms the capital accumulation rate drop significantly in both episodes — Great Recession and Covid-19 crisis.

Figure B.1: Corporate Investment in Recent Crises Episodes
$\Delta k_{t+h}^{\text{crisis}} - \Delta k_{t+h}^{\text{no crisis}}$



Notes: figures show the dynamics of the capital stock relative accumulation during the Great Recession and Covid crisis. The change in capital accumulation comes from the followin specification using firm-level data: $\log(k_{it+h}) - \log(k_{it}) = \alpha_i + \beta_h \text{crisis}_t + \varepsilon_{it+h}$, where crisis_t indicates the pre-crisis peak and β_h is the h-periods ahead change in the accumulation of capital during the crisis episode relative to no crisis periods. Drop t such that for $\text{crisis}_{t+i} = 1$ for at least one $i \in \{0, .., h\}$, i.e. capital accumulation before the crisis overlaps with the crisis. Panels (a) and (b) show coefficients β_h and their 90% confidence interval. Standard errors are clustered at firm level..
Data sources: Compustat.

B.2.2 Investment Heterogeneous Response Estimation

To estimate the *on impact* heterogeneous response — from peak-to-trough of the crisis — I will proceed as follows. First, to account for permanent sectoral heterogeneity — in my baseline estimations — I will demean each of the firm-quarter observations of variable x of interest by its sectoral average, i.e. $\hat{x}_{it} = x_{it} - \mathbb{E}_s[x_{it}]$. Next, I will assign each firm-quarter observation of x to different quartiles (terciles if Covid sample) relative to the annual distribution. Lastly, I run the following panel regression to estimate the heterogeneous responses of investment across cash-on-hand n/k and leverage b/k during the crisis:

$$\Delta \log(k_{it}) = \underbrace{\sum_{j=1}^{J} \beta_j^n \left(Q_{it}^{nj} \times \text{crisis}_t \right)}_{\text{het across } n/k} + \underbrace{\sum_{j=1}^{J} \beta_j^b \left(Q_{it}^{bj} \times \text{crisis}_t \right)}_{\text{het across } b/k} + \underbrace{\Lambda' Z_{it}}_{\text{controls}} + \varepsilon_{it}, \tag{24}$$

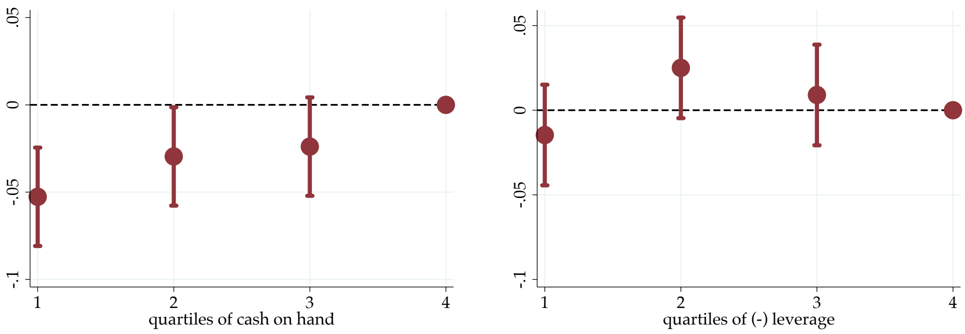
where Q_{it}^{xj} indicates if $\hat{x}_{it}$ belongs to quartile or tercile j, $\Delta \log(k_{it}) = \log(k_{it+h}) - \log(k_{it})$ is the capital accumulation over a period as long as the crisis studied (i.e., the extension from peak-to-trough of episode studied h), crisis$_t$ indicates if a crisis happens during the period considered and $Z_{i,t}$ includes the control variables. For the baseline specifications I include as controls firm's fixed effects, sectoral fixed effects, log assets as proxy for size and last quarter sales growth. The coefficients β_j^x are interpreted as the diff-in-diff estimates of the crisis impact on capital accumulation for firms in quartile or tercile j of $\hat{x}$.

The empirical strategy is close to the one used in other work that studies investment adjustment heterogeneity on recent crises episodes. Salient examles are Almeida, Campello, Laranjeira and Weisbenner (2012) for the Great Recession in U.S. and Kalemli-Özcan, Laeven and Moreno (2020) for the EU crisis.

Figure B.2: Heterogeneous Investment Response during Crises

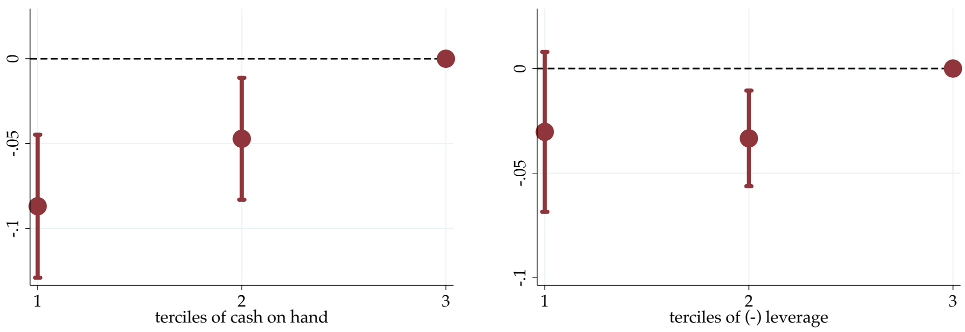
(a) Great Recession

(i) Across cash-on-hand: n/k (ii) Across leverage: b/k



(b) Covid Crisis

(i) Across cash-on-hand: n/k (ii) Across leverage: b/k



Notes: Figures show the change in the capital accumulation from peak to trough in both episodes. For the Great Recession the episode is from 2007q4 to 2009q4, and for the Covid-19 crisis is from 2019q4 to 2020q2. Figures in panel (a) show the coefficient β_j^n and Figures in panels (b) shows β_j^b for the Great Recession and Covid-19 crisis in a joint estimation of specification (24). Coefficients are normalized to 0 with respect to the highest quartile or tercile coefficient. The interval is at 90% confidence level and standard errors are clustered at firm level for the Great Recession and sector level for the Covid-19 crisis. Balance sheet variables are demeaned at sectoral level. Because of data limitations the estimates of the Covid-19 crisis don't include firm's FE. Coefficients are in annual terms.
Data sources: calculations using Compustat data.

Figure B.2 shows the investment response across different levels of cash-on-hand and leverage during the Great Recession and Covid crisis. For both episodes, panel (a) and (b) figure (i) show that firms with low levels of cash-on-hand adjust substantially more their investment, around 5-10 p.p. points in annual terms relative to the firms with the highest levels of cash-on-hand. On the other hand, panel (a) and (b) figure (ii) show that the heterogeneity across leverage is not significant. In Section 4.2, I contrast these results with simulations from the model.

B.3 Other Observations on the Firm's Balance Sheet

In this section I show further facts related to the liability structure (e.g. maturity and number of creditors) of firms that complement the baseline analysis. Table B.3 shows that corporate firms use extensively short-term liabilities to finance their investments and operations, and Table B.4 shows that the great majority of medium to large corporate firms (i.e., with more than 50 million assets) in U.S. borrow from hundreds of creditors. This is complementary to the observation of Crouzet (2017a) that corporate firms financial leverage is mostly in bonds, which they are very likely to have a dispersed ownership. These observations supports the idea that the firms' creditors are likely dispersed and difficult to coordinate, unless the firm wants to incur in costs. Further, in Appendix C.2, I show that the benefits of being able to manage the liability structure in the model are not large (ex-ante), therefore for moderate costs of changing their liabilites most firms will remain inactive.

Table B.3: Firms' Debt Maturity

	Time to mature (share)		
	< 1 year	1 to 4 years	≥ 5 years
Debt	0.29	0.33	0.38
	(0.32)	(0.28)	(0.34)
	< 1 year	> 1 years	
Liabilities	0.61	0.39	
	(0.29)	(0.29)	

Notes: the table shows the share of debt or liabilities maturing at different time horizons. The summary statistic is computed for the average firm, in parenthesis is the standard deviation. Short-term liabilities are `lct` and long-term `lt - lct`. Debt maturing in less than one year is `dlc`, in one to four years is `dd2` + `dd3` + `dd4`, and maturing at 5 or more years is `dltt-dd2-dd3-dd4`. Total debt is `dlc` + `dltt` and total liabilities is `lct`.
Data source: Compustat.

Table B.4: Number of Creditors When Filing to Bankruptcy

	# Creditors		
	1 to 100	101 to 1,000	>1,000
Small ($<$ 50 million assets)	0.88	0.10	0.02
Medium ($>$ 50 million and $<$ 1 billion assets)	0.16	0.19	0.65
Large ($>$ 1 billion assets)	0.03	0.04	0.93
All	0.74	0.10	0.16

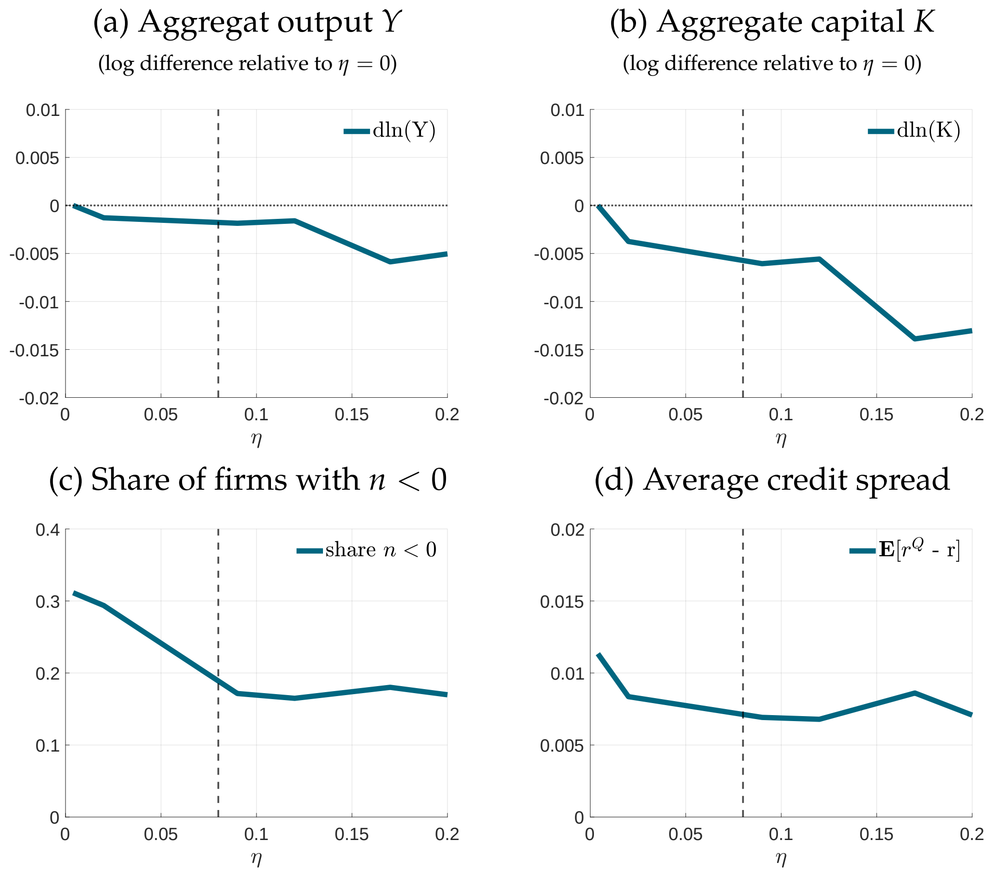
Notes: the table shows the share of firms with by creditor number groups and size when filing to Chapter 11 bankruptcy. Shares are relative to the total filings of each size group. Asset value correspond to the one declared when filing for bankruptcy.
Data source: FJC-IDB.

C Appendix: Further Exercises and Extensions

C.1 Steady-state comparative statics

To study the long-run implications of runs I do some simple comparative statics with η. Figure C.1 shows for different values of η the output and capital level, and the share of firms with negative cash-on-hand and the average spread rate. I find that the incidence of runs in the long-run is relatively low. First, in the long-run, aggregate output Y is 0.2% lower, see panel (a), and aggregate capital K is 0.5% lower, see panel(b), because of runs. Second, the higher is η less firms have a weak balance sheet position in steady-state, see panel (c). Runs shifts (improves) the financial distribution of firms significantly. The increase in the risk of runs, for a given financial position, incentivize firms to save away; thus, accumulating internal resources to preclude runs. The improvement in the financial position is reflected on the little change observerd in credit spreads across η, even if the risk of a run is greater (given the financial position). Overall, the higher incidence of runs η shifts the financial position of firms, but don't impact significanly aggregate outcomes over the long-run.

Figure C.1: Steady-state Comparison

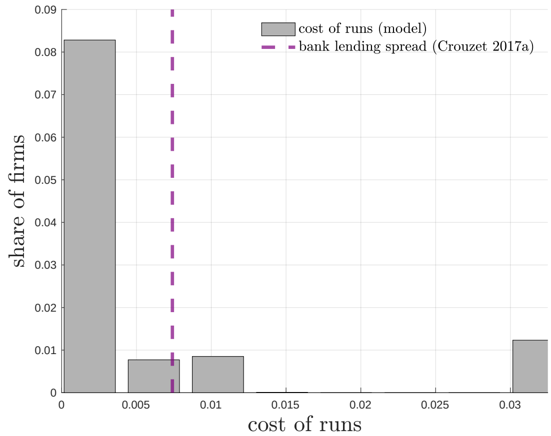


Notes : Panel (a) and (b) show the log difference in aggregate output Y and capital K, respectively, across different values of η in steady-state. Panel (c) and (d) show the share of firms with negative cash-on-hand n and average credit spread rate across, respectively, across different values of η in steady-states. In all the plots, the vertical dashed line indicates the calibrated value of η.

C.2 How costly are firm runs (ex-ante)?

In this section to asses how costly are runs I explore the spread distribution between the pricing schedule with runs and without runs, i.e., $\tilde{Q}\left(z, k^{'}, b^{'}; \eta\right) - \tilde{Q}\left(z, k^{'}, b^{'}; 0\right)$. As a benchmark, I compute how many firms would pay the bank's markup over market borrowing soley to preclude future runs if they could. I use the spread in intermediation costs estimated by Crouzet (2017a) of 0.74% (annual). I find that only 1.25% of the firms face a cost of runs higher than intermediation spread. Figure C.2 shows the distribution of the cost of runs across firms that produce and don't exit at the end of the period. The figure shows that most firms face a cost close to 0 since many firms become exposed tomorrow only in case of an extremely bad shock, therefore the average cost is negligible. The small cost of runs ex-ante in steady state suggests that it can be optimal for firms choose a liability structure they are exposed to rollover problems.

Figure C.2: Cost of Runs (in annual spread terms)



Notes : Figure shows the distribution of the cost of firm runs — i.e., $\tilde{Q}\left(z,k^{'},b^{'};\eta\right)-\tilde{Q}\left(z,k^{'},b^{'};0\right)$ — for producing firms that don't exit at the end of the period. Exclude from plot the ones with 0 cost and truncat distribution at 3% cost. The spread of intermediation between bank and market lending is from the calibration of Crouzet (2017a)

C.3 Model Extensions and Multiple Equilibrium

In this section, I will study two extensions of the model, one that uses more general functional forms for the operational profits function, capital adjustment idiosyncratic frictions and long-term debt, and other that allows firms to issue equity (costly).

Long-term debt and capital adjustment frictions. I assume profits are a function $\pi(\mathbf{z},k)\in\mathbb{R}$ strictly increasing in both arguments, where $\mathbf{z}=(\mathbf{z}^{p},\mathbf{z}^{\text{iid}})$ is a vector of shocks that contain a set of persistent shocks $\mathbf{z}^{p}$ follow a markov process and $\mathbf{z}^{\text{iid}}$ follow an iid process. Both are related to idiosyncratic productivity and cost shocks. Next, I assume that $\iota(\omega k,k')\in\mathbb{R}$ is the investment expenditure function of the firm that is decreasing on k and increasing on k', where $-\iota(\omega k,0)\geq 0$ is the liquidation value of capital.[49] Last, I assume that the firm can issue long-term debt, which fraction $m^{b}\in(0,1]$ matures randomly each period and pays $c^{b}\geq 0$ cupon payments on non-maturing debt. The rest of the model it follows as the baseline model.

I focus on the characterization of the liquidation choice. For the extended setup, firms dividends now can be defined as

$$d=\pi\left(\mathbf{z},k\right)-\iota\left(k,k^{'}\right)-b\left[m^{b}+\left(1-m^{b}\right)c^{b}\right]+Q\left(.\right)\left(b^{'}-\left(1-m^{b}\right)b\right)\geq 0$$

where $Q\left(.\right)\left(b^{'}-\left(1-m^{b}\right)b\right)$ is the amount of new debt issued. Analogous to the

[49] I assume no capital quality shock ω for notational clarity.

baseline model, firms can default after issuing the new debt. The firm never default whenever

$$\max_{k'} \pi\left(\mathbf{z},k\right)-\iota\left(k,k'\right)-b\left[m^{b}+\left(1-m^{b}\right)c^{b}\right]=$$
$$\underbrace{\pi\left(\mathbf{z},k\right)-\iota\left(k,0\right)-b\left[m^{b}+\left(1-m^{b}\right)c^{b}\right]}_{n(\mathbf{z},k,b)}\geq 0. \tag{25}$$

where I can define n as the cash-on-hand of the firm is the sum of operational profits, liquidation value of capital, and maturing debt and cupon payments. On the other hand, we have that the firm will always default whenever

$$\pi\left(\mathbf{z},k\right)-b\left[m^{b}+\left(1-m^{b}\right)c^{b}\right]+\max_{k',b'}\left\{-\iota\left(k,k'\right)+\tilde{Q}\left(\mathbf{z}^{p},k',b'\right)\left(b'-\left(1-m^{b}\right)b\right)\right\}=$$
$$n\left(\mathbf{z},k,b\right)+\underbrace{\max_{k',b'}\left\{-\iota\left(k,k'\right)+\iota\left(k,0\right)+\tilde{Q}\left(\mathbf{z}^{p},k',b'\right)\left(b'-\left(1-m^{b}\right)b\right)\right\}}_{-\underline{\mathrm{n}}\left(\mathbf{z}^{p},k,b\right)}<0 \tag{26}$$

For multiplicity to exists we need that conditions (25) and (26) don't hold, i.e.,

$$0>n\left(\mathbf{z},k,b\right)\geq \underline{\mathrm{n}}\left(\mathbf{z}^{p},k,b\right). \tag{27}$$

Notice $\underline{\mathrm{n}}\left(\mathbf{z}^{p},k,b\right)$ bounded below by 0 (we can always implement $\{k'=0,b'=b\}$). Moreover, there is the possibility of multiple equilibrium whenever the firm can have strictly positive external resources in this region of the state-space. Analogous to the baseline model, the firms default decision is determined by the firm's cash-on-hand and a threshold that depends on the fundamentals of the firm (shocks and financial position).

Further, assume there is no bankruptcy, $c^{b}=0$ and creditors have no recovery for clarity, then the fundamental pricing schedule $\tilde{Q}(.)$ (without coordination problem today) is pinned down by creditors no profit condition and is

$$\tilde{Q}\left(\mathbf{z}^{\mathbf{p}},b',k'\right)=\mathbb{E}\left[\Lambda\left(\mathbf{1}_{\{n\geq\underline{\mathrm{n}}\}}-\eta\mathbf{1}_{\{0>n\geq\underline{\mathrm{n}}\}}\right)\left(m^{b}+\left(1-m^{b}\right)\tilde{Q}'\right)\right]. \tag{28}$$

The pricing schedule with long-term becomes recursive. Also tomorrow's coordination failures show up in the pricing schedule. These two observations suggest, in the firm problem with long-term debt, rollover problems could even be greater than in the baseline model. With long-term debt the pricing schedule is affected by the future stream of expected rollover problems, which can augment their impact.

Equity issuance. In the baseline specification, I don't allow firms to issue equity — $d \geq 0$. This assumption is consistent with the relatively low equity issuance observed in the data, and helps on the tractability of the characterization and computational solution of the model. In this section, I will relax this assumption and show how this affects the characterization of the liquidation problem (equilibrium multiplicity). Moreover, I provide a discussion on the model concepts of liquidity and solvency in the model.

Firms issue equity $e < 0$ at cost $\phi(e)$, which is decreasing in e and unbounded. I assume that equity is raised at the end of the period. Therefore, firms that never default are those when $Q = 0$ they don't default, i.e.,

$$V^{Q=0}(z,n) \geq 0$$

where $V^{Q=0}(z,n)$ is determined by

$$V^{Q=0}(z,n) = d + \mathbb{E}\left[\Lambda \tilde{V}\left(s^{'}\right)\right]$$

subject to

$$d = \begin{cases} e & \text{if } e \geq 0 \\ e - \phi(e) & \text{if } e < 0 \end{cases}$$
$$e = n - qk^{'}$$

where continuation value $\tilde{V}\left(s^{'}\right)$ is analogous to one defined in the baseline firm problem. Thus, we can define safe region

$$\mathcal{S} = \{(z,n) : V^{Q=0}(z,n) \geq 0\}. \tag{29}$$

On the other hand, firms that default are those default even if $Q > 0$, i.e.,

$$V^{Q>0}(z,n) \geq 0$$

where $V^{Q>0}(z,n)$ is determined by

$$V^{Q>0}(z,n) = d + \mathbb{E}\left[\Lambda \tilde{V}\left(s^{'}\right)\right]$$

subject to

$$d = \begin{cases} e & \text{if } e \geq 0 \\ e - \phi(e) & \text{if } e < 0 \end{cases}$$

$$e = n + \tilde{Q}\left(z, b^{'}, k^{'}\right) b^{'} - qk^{'}$$

where $\tilde{Q}$ fundamental pricing schedule (no liquidation today) and continuation value $\tilde{V}\left(s^{'}\right)$ (this analogous as the one in the baseline firm problem). Thus, we can define liquidation region

$$\mathcal{L} = \{(z,n) : V^{Q>0}(z,n) < 0\}. \tag{30}$$

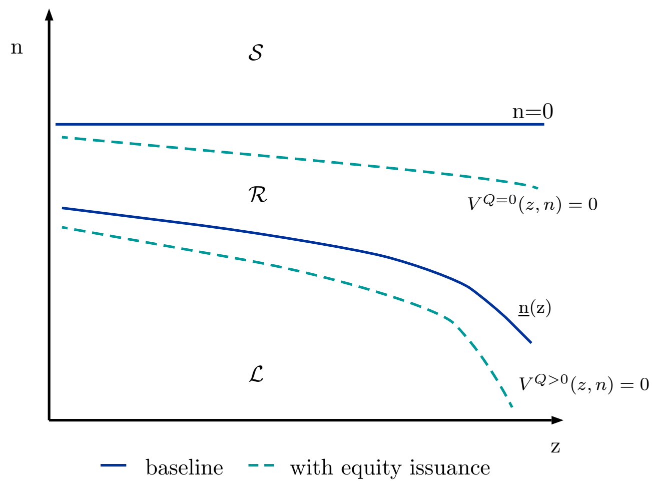
Last, it's straightforward to show that $V^{Q>0}(z,n) \geq V^{Q=0}(z,n)$, then under certain conditions it can be the case that firm is in a region that is undetermined, i.e.,

$$\mathcal{R} = \{(z,n) : V^{Q>0}(z,n) \geq 0 \quad \text{and} \quad V^{Q=0}(z,n) < 0\}. \tag{31}$$

Similar to the baseline mode, we have that firms can be exposed to coordination failure even if they can issue equity. Assume the equity issuance function $\phi(e) = \lambda|e|$ with $\lambda > 0$. In Figure C.3, I illustrate how these affects the characterization of the regions.

Figure C.3: Liquidity and solvency regions across (z,n)

Baseline vs Equity Issuance model



Notes: figures shows the state-space (z,n) and the relevant regions for the liquidation choice for the baseline model (solid blue lines) and the model with equity issuance (dashed cyan lines).

Finally, it's worth noticing that in the model with unbounded equity issuance firms in $\mathcal{L}$ threshold — $V^{Q>0}(z,n) = 0$ — have 0 value, which is the standard notion of economic insolvency. On the other hand, in the baseline model, or with bounded equity issuance, firms in $\mathcal{L}$ threshold ($\underline{\mathrm{n}}(z)$) could have strictly positive value. For my calibration, I find that firms in the insolvency threshold have values close to 0 — $V(z, \underline{\mathrm{n}}(z)) \approx 0$ —; therefore, it approximates well the standard notion of insolvency.