Entrepreneurship, Financial Frictions, and the Market for Firms*

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Abstract

We study how the trade of firms is a mechanism through which productive projects and available resources can be better allocated in an economy in which entrepreneurs are subject to financial frictions. For this we develop a general equilibrium model of entrepreneurship and frictional trade of firms that captures salient features about the trade of private firms in the U.S. economy. Gains from trading firms in our model arise from the presence of financial frictions, with credit constrained firms being the ones most likely to be traded. We present evidence in favor of our theory by documenting that younger, smaller, and high returns to capital firms have the highest probabilities of trade, consistent with the predictions of our model. Finally, we quantify the importance of the market for firms for the aggregate economy. Parameterizations of our model imply that the better allocation of capital due to the trade of firms can account for about 6% of output.

Keywords: entrepreneurship, financial frictions, misallocation, private firms, firm ownership, search frictions.

JEL classifications: D2, D5, L2, E4, G3.

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1 Introduction

We study how the trade of firms is a mechanism through which productive projects and available resources can be better allocated in an economy in which entrepreneurs are subject to financial frictions. In this setup, who are the owners of firms matters for allocations as credit constrained entrepreneurs will produce at a lower, and suboptimal, scale resulting in misallocation of capital and lower aggregate output (Hsieh and Klenow, 2009; Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014). It might be the case, however, that financially constrained entrepreneurs might want to sell their firm to less constrained parties which will be able to take the firm closer to their optimal scale.

Our goal in this paper is to study this type of transactions in which gains from trade arise from the presence of financial frictions. Specifically, we ask: Is there evidence of this type of trades in the data? How important is the market for firms as a mechanism that allocates productive projects and available resources for the aggregate economy? We answer these questions by presenting new evidence about the trade of firms in the U.S. economy and by developing a model of entrepreneurship with a frictional market for firms with which we perform different quantitative experiments.

We start our analysis by documenting three facts about the trade of *privately held firms*. We focus on private firms as these are the ones more likely to be affected by financial frictions that dampen firms' access to external finance and insurance. Additionally, the study of private firms is important by itself as they account for more than 60% of employment and more than 50% of output in the U.S. economy (Asker, Farre-Mensa, and Ljungqvist, 2014). Throughout the analysis we focus on transactions where both the ownership and the management of the firm are traded. The aggregation of all these trades is what we call the *market for firms*.

For our empirical results we focus on *entrepreneurs*, defined as self-employed private business owners who actively manage their firm, and study how do they acquired their business. Using multiple sources of data we document that:

1. Around one out of five (or 20%) of entrepreneurs acquired their business by purchasing an existing firm. This implies an annual trade rate of around 2%.
2. More than 60% of firm buyers have never been entrepreneurs before purchasing their current business.
3. The share of entrepreneurs that purchased their firm has significantly decreased in the last 30 years: from around 30% in 1989 to roughly 20% in 2016 (see Figure 1).

Some comments about these findings are appropriate. First, the trade rate we document indicates that private businesses are highly illiquid assets. For example, compared to housing Berger and Vavra (2015) report that about 5% of houses are traded each year, more than the double of the 2% annual trade rate we estimate. However, the trade of private firms is larger, in terms of volume, than the trade of other intangibles assets used for production such as patents. For example, Akcigit, Celik, and Greenwood (2016) document that 16% of the registered patents in the U.S. have ever been traded, smaller than the 20% we document for private firms.

Our second result, regarding the previous occupation of firm buyers, suggests that purchasing an existing firm is a relevant channel for entering into entrepreneurship which, to best of our knowledge, hasn't been studied before. Our theoretical framework, besides capturing the illiquidity of private firms, will incorporate this novel feature about households' possible transitions into entrepreneurship through the market for firms.

Lastly, is interesting that the fall in the trade of firms coincides with a time period characterized by declining business dynamism and increasing concentration in the U.S. economy, suggesting a possible relation between these trends (see Akcigit and Ates (2019) for a recent

review on this literature). As explained below, however, our model predicts that credit conditions, which ultimately determine entrepreneurs' access to external finance, can also explain the decline in the trade of firms. Intuitively, if business owners have more access to external funding the gains from trading their firms to less constrained parties will be lower. Which, in turn, will reduce the number of trades in the economy.

Motivated by our empirical analysis in the second part of the paper we develop an heterogeneous agent model with entrepreneurship and frictional trade of firms. Our model economy is populated by a continuum of households which can be of two types in function of their occupation: *firm owners* or *workers*. Firm owners can buy, sell or shutdown their firm, while workers can become firm owners by acquiring a firm or through some exogenous *startup* shock.

Firms are characterized by the quality of an entrepreneurial project which is indivisible, rival and excludable. These entrepreneurial projects aim to capture firms' *intangible assets* such as trademarks, patents, customer bases, etc.[1] Firms enable their owner to produce the final consumption good with a technology that uses capital, labor and the quality of the firm.

Both type of households are subject to uninsurable idiosyncratic risk. On the one hand, firm owners are exposed to the risk associated with the quality of their private firm which evolves stochastically. This assumption aims to capture changes in market conditions that affect the profitability of entrepreneurial projects. On the other hand, workers are endowed with one unit of labor supplied inelastically and are subject to shocks to their labor efficiency.

Besides the firms owned by a single household, which we call private firms, there is a second sector of production with a representative public firm. Both sectors produce the same good which can be used for consumption or savings in a risk free asset. There is no aggregate uncertainty in this economy. Finally, there is a financial intermediary that, each period, takes the savings from the households and rent capital to the firms.

Our empirical results about private firms being highly illiquid, together with the fact that firms are complex and unique making them hard to evaluate and price, ask for a *search-theoretic* approach to model this market.[2] Specifically, we model the market for firms through a decentralized market subject to search frictions and bilateral random matching. One interpretation of our setup is that agents can evaluate only one firm at a time, which delays trade. This setup will also be suitable for our quantitative analysis as it give us enough flexibility to match key moments about to the trade of firms that we document in the empirical section of the paper.

Lastly, we assume two *financial frictions* in our model: incomplete markets which leads to uninsurable risk, and a collateral constraint which limits firm owners' use of external funds for production. In our model, financial frictions are the only motive for the trade of firms. Thus, a typical seller in our economy will be a firm owner with low wealth and a high quality firm. The typical buyer is going to be a wealthy firm owner with a low quality firm, or a wealthy worker with low labor efficiency.

We parameterize the model to reproduce salient features of the U.S. economy for the year 2007. Besides matching the income and wealth distribution across households, our model does a good job in replicating salient features of the market for firms. Specifically, the 2% annual trade rate of firms and that, from those, 62% of the purchases are done by workers, capturing the importance of this market as a channel for entering into entrepreneurship.

[1] Using data from business transactions, Bhandari and McGrattan (2018) document that when a firm is sold around 60% of its total value is accounted by intangibles. This evidence supports our modeling decision of characterizing firms' by the value of their intangible assets.

[2] BizBuySell, an online broker that advertises businesses for sale, surveyed some of their clients about the major challenges they faced when purchasing a business. For more than 40% the major issue was "finding the right business" and for 23% the major challenge was "valuating the firm". Both responses are consistent with our search-theoretic approach to model the market for firms.

Our model has strong predictions about which type of firms have the highest probabilities of trade. If financial frictions are an important motive for trade, credit constrained firms should be the ones more likely to be bought and sold. To validate our theory, we go back to the data and look at the trade rate conditional on different observable characteristics of firms. We consider two common proxies of financial frictions: firms' age and firms' size, as younger and smaller firms are the ones more likely to be financially constrained. Additionally, we look at firms' average productivity of capital (APK) as firms with binding credit constraints have high returns but cannot increase their investment.

We find that younger, smaller, and high APK firms are the ones more likely to be traded, in line with the predictions of our model. Although our model predicts starker relations of these variables and the trade rate, it qualitatively aligns well with the data (see Figure 6). Is worth mentioning that these relations were not targeted in our calibration exercise. Rather, these result from the key prediction of our model that credit constrained firms are the ones more likely to be traded. Overall, we consider that these different pieces of evidence present a convincing case for financial frictions being an important motive for the trade of private firms.

With our calibrated model we perform two counterfactual experiments to quantitatively assess the importance of this market for the macroeconomy. In our first experiment we take our baseline economy and analyze what would occur if the market for firms shut downs. We find that shutting down this market would imply a fall in entrepreneurial output of around 6%. This result is explained by the poorer allocation of resources when this market is absent, as the trade in the ownership of firms is a mechanism through which entrepreneurial projects and available resources can be better allocated in the economy.

The previous exercise shows that the trade of firms is a way to alleviate the misallocation caused by financial frictions. To get a better sense of the gains in total factor productivity (TFP) that this market delivers, in our second experiment we consider an alternative economy, which we recalibrate to match the data, with no trade in the ownership of firms. Then, we ask: what are the credit conditions that the no market economy requires such that it matches the TFP level of our baseline economy? We find that the no market economy requires looser credit conditions such that the average debt to capital ratio of entrepreneurs increases in 7 percentage points (p.p.), from from 0.33 to 0.40. This is an sizable amount as, for example, during the Great Recession this number fell by around 5 p.p.

In the last part of our analysis, we study the relation between the credit conditions and the volume of firms traded in the economy. In the empirical section of the paper we document that the share of entrepreneurs that purchased their firm considerably fell during the last 30 years. An implication of our theory is that as credit conditions are looser, and hence firm owners can obtain the required resources by themselves, the gains from trading firms are going to be reduced and hence we will expect to see a reduction in the trade volume.

Interestingly, during the period in which the fraction of firms purchased fell there was also an increase in private firms' leverage, suggesting easier access to credit. To quantitatively asses this relation we feed into the model the change in credit conditions observed between 1989 and 2016 and see its implications for the trade of firms. Our model predicts a fall in the fraction of firms purchased of 3 p.p. This suggest that looser credit conditions can explain 3 out of the 10 p.p. fall in the share of purchased firms observed during the last 30 years. Our results, however, do not rule out that the fall in the volume of traded firms could be related to other aggregate trends, such as the decline in business dynamism in the U.S.

Taking everything into account, we consider that our paper contributes to the literature in several ways. First, by documenting novel facts about the trade of privately held firms in the U.S. economy. Second, by developing a model of entrepreneurship and frictional trade of firms that allow us to study the role of financial frictions for the market for firms. Third, by pre-

senting new evidence that indicates that financial frictions are indeed an important motive for the trade of firms. Fourth, by quantitatively assessing the role of this market in terms of output and TFP. And lastly, by studying the fall in the trade of firms through the lens of our model.

1.1 Related Literature

Entrepreneurship and the Wealth Distribution. Our theoretical framework builds on models of heterogeneous agents with entrepreneurship subject to financial constraints, such as Quadrini (2000) and Cagetti and De Nardi (2006).[3] An important feature of these models is that they are able to match the income and wealth distribution of the data through the combination of uninsurable income risk and stochastic returns to wealth coming from entrepreneurial activity.

From this literature, the closest to our paper is Peter (2019). This paper also builds on this type of models, but, it includes the possibility that private firms go public through an Initial Public Offering (IPO). Then, it analyzes what are the implications of entrepreneurs' difference sources of external financing for wealth inequality, and test these implications using data from a set of European countries. The main difference from our paper is that, in Peter (2019), when entrepreneurs go public they maintain the management of the firm, regardless of how many shares they issue. In contrast, we study transactions in which both the ownership and the management of the firm are traded, which is a pervasive practice in the market for private firms. In a way, our papers are complementary. Peter (2019) studies the implications of IPOs (private firms that go public) while we study the implications of trading private firms (that stay private).

Finance and Misallocation. This paper also relates to the literature of financial frictions and misallocation as a source of low TFP (Hsieh and Klenow, 2009; Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014). We contribute to this literature by highlighting the importance of the market for firms as a mechanism that alleviates the capital misallocation generated by financial frictions. In this sense, how this market function across different countries, due to institutional differences, the enforcement of contracts and the rule of law, may have implications for the severity of financial frictions as a source of capital misallocation.

Market for Ideas. Our paper is also related to the growth literature studying the trade of ideas. For example, Silveira and Wright (2010) study a market where innovators, who are good at coming up with ideas, trade with entrepreneurs, who are better at implementing them. Another example is Akcigit, Celik, and Greenwood (2016) who study the trade of patents and the implications of this market for economic growth. Our paper is similar to those in terms of the theoretical framework, as both papers model the market for ideas through bilateral meetings subject to search frictions.

Market for Firms. Finally, this paper is related to Caselli and Gennaioli (2013) who study the implications of dynastic management, defined as situations in which firm management is passed on between generations of the same family. This could have aggregate implications when, for example, the heirs of firms have no managerial talent. Besides important modeling assumptions, the main difference with our paper is about the motives for trade. In that paper they arise because of life-cycle motives while in ours because of financial frictions.

The reminder of the paper is organized as follows: Section 2 presents our empirical analysis, Section 3 presents the model, Section 4 explains the workings of the model, Section 5 presents the parameterization of the model, Section 6 our quantitative exercises, and finally Section 7 concludes.

[3] In their conclusions, Cagetti and De Nardi (2006) mentioned that an issue left for future research relates to the observation that: "*In the presence of borrowing constraints the entrepreneur might want to sell his idea or project to another, potentially less constrained, party.*" This is exactly the mechanism we study in our paper.

2 Empirical Analysis

2.1 Data

We use four different data sources related to private firms, their characteristics, and the characteristics of their owners. These datasets are now described.

SBO. Our main data source is the Survey of Business Owners (SBO) Public Use Microdata Sample (PUMS). This survey provides comprehensive information about businesses and business owners. In particular, about how do they acquired their business. The PUMS sample is representative of all non-farm private businesses in the U.S. and is available for the year 2007.

SCF. The second dataset we consider is the Survey of Consumer Finances (SCF). This survey is available every three years from 1989 to 2016. The SCF includes detailed information about households' income and balance sheets, information that we'll use to discipline the income and wealth distribution in our quantitative model. Although this survey does not specifically target business owners, as the SBO, it also contains relevant information for our analysis. Specifically, this survey also asks business owners how do they acquired their firm. This information will allow us to study how certain characteristics of firm ownership have evolved over time.

ASE. Third, we use the Annual Survey of Entrepreneurs (ASE) to get complementary information about business owners. The ASE is available on an yearly basis from 2014 to 2016. As the SBO, it's sample is representative of all non-farm private businesses in the U.S.

KFS. Lastly, we use data the Kauffman Firm Survey (KFS). This is an eight year panel of businesses that started operations in 2004 and were followed through 2011. Compared to the other datasets, the KFS contains more detailed information about the firms (e.g. sales, employment, capital, and balance sheets), while also reporting characteristics of the owners. Although this survey is only representative for firms that started operations in the year 2004, it will be useful for our validation exercise.

2.2 Empirical Evidence on the Market for Firms

For all our analysis we focus on *entrepreneurs*. Following Cagetti and De Nardi (2006), we define an entrepreneur as an individual who: (i) is self-employed, (ii) owns a business, and (iii) has an active management role in it. Thus, for example, an investor who doesn't actively manage the business is not an entrepreneur. This definition is particularly suitable for privately held firms in which the ownership and the management is usually concentrated in a single person.

As a robustness, we consider a second definition of an entrepreneur which, additional to (i)-(iii), requires that: (iv) the business has a positive number of employees (i.e. an employer firm). This extra condition aim to exclude cases of self-employed individuals where the main input in the business may not be transferable. We would expect to be less trade in the ownership of that particular type of businesses.

Throughout our analysis we assume that each entrepreneur owns only one firm. Hence, we assume that the amount of firms traded is equivalent to the amount of entrepreneurs that traded their firm. In the SBO data, more than 80% of entrepreneurs have only one firm, and more than 70% of the firms have only one entrepreneur.[4]

[4]See Appendix A for a detailed analysis about entrepreneurs, owners, and their equity shares per firm across U.S. private businesses.

2.2.1 How do Entrepreneurs Acquire Their Firms?

As a first step in our analysis, we focus on data from both the SBO and the SCF for their 2007 waves and look at how do entrepreneurs acquire their firms.[5] Table 1 presents the share of entrepreneurs that founded their firm, that purchased it, and that inherited it or acquired it through any other way. The first set of rows present the results for our baseline definition. The second set of rows present the results for the definition that restricts to employer firms.

The first thing to note is that the numbers obtained from the SBO and the SCF for 2007 align remarkably well. Particularly the ones regarding the share of entrepreneurs that purchased their business. In the light of this result we are more confident about our estimates of type of acquisition across time with the SCF data even though, as mentioned above, that survey is not specifically designed for business owners.

Table 1 shows that more than 70% of entrepreneurs founded their firm. This is, of course, the most common way that entrepreneurs acquire their firm. Also it shows that between 6 to 10% acquired it through inheritance or other type of acquisition. Our main result, is that around 17% of the entrepreneurs in the U.S. acquire their business by purchasing an existing firm. This number is even larger, around 25%, when we restrict to employer firms.

In Appendix A we show that our results are robust to alternative definitions of an entrepreneur. Also, we show that these numbers are not driven by franchises, small firms, or particular sectors of production. Overall, across all our robustness exercises, we find that the fraction of entrepreneurs that acquired their firm through a purchase is, roughly, 20%.

2.2.2 Firm Trade: Stock vs. Rate

The previous result, that around 20% of entrepreneurs bought their firm, is about the current *stock* of firms that have been traded at any point in the past. We are also interested in the frequency with which firms are traded, i.e. the trade *rate*. We estimate the percentage of firms traded every year through two different strategies. The first strategy looks at the percentage of firms purchased in the SBO in the same year of the survey. The second strategy considers a back of the envelope calculation using the stock of traded firms together with firms' entry and exit rates. In Appendix A we present the details of these calculations. Both strategies, imply that around 2% of firms are traded every year. If we restrict to the sample of employer firm we obtain a trade rate of 3%.

2.2.3 Firm Buyers: Previous Occupation

From the SBO we can obtain information regarding entrepreneurs' previous occupation. The survey asks: "Prior acquiring this business, had the owner ever owned a business or been self-employed?". Using this information, we found that between 62 and 66% of entrepreneurs that purchased, for our baseline and secondary definitions, have never been self-employed. Hence, most likely, they were workers before acquiring the firm.[6] This evidence shows that purchasing an existing firm is a relevant channel for entering into entrepreneurship which, to best of our knowledge, hasn't been studied before.

[5] The exact question in the SBO is: "How did [the owner] initially acquire ownership of this business?". The SCF asks business owners: "How did you first acquire this business?".

[6] Because of the design of the question, these numbers should be interpreted as lower bounds for what would be our non-entrepreneur definitions (i.e. the complement of being an entrepreneur).

2.2.4 Trade of Firms Across Time

So far we have focused on cross-sectional features of the trade of firms. Given that the SBO is only available for 2007, we use the SCF to document the evolution of the share of entrepreneurs that purchased their firm during the time period between 1989 to 2016. As a robustness, we also consider data from the ASE which is available for the years 2014 to 2016.[7]

Figure 1 shows a decreasing trend in the share of entrepreneurs that purchased their firm. For both definitions of entrepreneurs, during this time period, the fall in the share of purchased firms is around 10 p.p. In Appendix A we show that the decreasing trend is robust to alternative samples and definitions. It is worth mentioning that most part of the fall occurred before 2007. Since the Great Recession this share has been relatively stable. In Section 6 we study, through the lens of our model, how looser credit conditions may explain the decline in the fraction of firms traded. However, we don't rule out that the fall in the volume of traded firms could be related to other trends such as the decline in business dynamism in the U.S. economy.

3 A Model of Entrepreneurship and Trade of Firms

In this section, we develop an heterogeneous agent model of entrepreneurship with a *frictional* market for firms that accommodates various features documented in the previous section. Our final objective is to use this model to do a quantitative assessment of the macroeconomic implications of the market for firms.

3.1 Environment

The economy is inhabited by a continuum of households in $[0, 1]$. Households can be of two types: *firm owners* or *workers*. Firm owners can buy and sell firms, and choose whether to operate their current firm and be *entrepreneurs*, or close the firm and be workers. Workers can become firm owners by acquiring a firm, or through some exogenous *startup* shock. The transitions between these two types of agents are explained in further detail below.

Besides the firms managed by households, which we call *private firms*, there is a second sector of production with a representative *public firm*. Both sectors produce the same good which can be used for consumption or for savings in the form of capital. Capital is produced by a *financial intermediary* which, each period, takes savings from households and rent capital to the firms. The public firm and the financial intermediary are owned by all households in equal shares.[8]

Time is discrete and infinite and each time period is divided in two stages. In the first stage, which we call the decentralized market, or *DM*, is when the trade in the ownership of private firms takes place. We assume that, in the market for firms, households meet bilaterally subject to *search frictions* which may restrain the frequency and the type of the matches. In the second stage, which we call the centralized market, or *CM*, is when all production, consumption and saving decisions take place.

[7] The definition of an entrepreneur in the ASE is closer to the one of an entrepreneur with positive employment firm of the SBO and the SCF.

[8] Instead, we could have assumed that the intermediary and the public firm issue equity shares which are traded between households in a frictionless centralized market. This setup is analogous, as assets and shares holdings would be indeterminate. Below we assume that the intermediary and the public firm make zero profits, thus, this modeling choice is not crucial for the analysis.

3.1.1 Households

Households have preferences over consumption c represented by a constant relative risk aversion (CRRA) utility function

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

where σ is the risk aversion coefficient.

They are heterogeneous in their type and in their asset holdings a. Assets are subject to a non-borrowing constraint, $a \geq 0$, and are deposited with the financial intermediary which pays a risk-free interest rate r for the deposits. There is no aggregate uncertainty in this economy, however, households face idiosyncratic uninsurable risk.

With respect to households' types, on the one hand, firm owners are endowed with a private firm which enables the owner to produce the final consumption good with a technology that uses capital, labor, and the quality of the firm. This technology is described below. The quality of the firm, denoted by z, is stochastic and evolves according to the law of motion

$$z' = \begin{cases} z & \text{with pr. } \gamma \\ z' \sim \mathcal{P}(z_{min}, \eta_z) & \text{with pr. } (1-\gamma) \end{cases}$$

where $\mathcal{P}$ denotes a Pareto distribution with scale and a shape parameters z_{min} and η_z, respectively. The $(1-\gamma)$ shock can be interpreted as changes in market conditions that affect the profitability of entrepreneurial projects as in Buera, Kaboski, and Shin (2011).

On the other hand, workers are endowed with one unit of labor which they supply inelastically, and are heterogeneous in their labor efficiency ε. We assume that the logarithm of labor market efficiency evolves according to an AR(1) process with persistence ρ_ε and volatility σ_ε. Specifically,

$$\log \varepsilon' = \rho_\varepsilon \log \varepsilon + \sigma_\varepsilon u,$$

where u is a standard normal random variable.

Regarding the transitions between types, workers can become firm owners by purchasing an existing firm or through an exogenous *startup* shock at the end of the period. At the beginning of the production stage, firm owners face an occupational choice. They decide whether to operate their firm and be entrepreneurs, or shut down the firm and be workers. Upon exit or upon selling, previous firm owners lose the value of their firm and enter to the labor market with the lowest labor market efficiency $\underline{\varepsilon}$.[9] We interpret this low value of entry as potential costs associated with entrepreneurship, for example, lack of experience in the labor market.[10] A graphical description of the transitions between occupations is presented in Figure 2.

In this setup, the budget constraint of an entrepreneur, defined as firm owner that decides to operate, with states (a, z) is given by

$$c = \pi(a, z) + (1+r)a - a' + \Pi^c + \Pi^f,$$

and the budget constraint of a worker with states (a, ε) is

$$c = \varepsilon w + (1+r)a - a' + \Pi^c + \Pi^f,$$

where π are the profits of the entrepreneur's private firm, w is the labor market wage, Π^c and Π^f are the public firm and the financial intermediary profits, respectively.

[9] Although the distribution of ε is bounded below by 0, for the numerical solution we take $\underline{\varepsilon}$ to be the lowest value on the ε grid, which is positive number.

[10] There is also a technical reason for which we assume that firm owners that exit obtain the lowest possible value on the workers' labor efficiency grid. If this wasn't the case, and hence suppose they get a value $\tilde{\varepsilon}$, workers with $\varepsilon < \tilde{\varepsilon}$ would have an incentive to buy a low quality firm and then immediately exit just to improve their labor efficiency.

3.1.2 Private Firms

Private firms are endowed with a technology that uses capital k, labor l and the quality of an *entrepreneurial project* z to produce the final consumption good according to

$$y = zk^{\theta}l^{\nu}$$

where $\theta + \nu < 1$. The decreasing returns to scale assumption implies that all private firms have an optimal operation scale as in Lucas (1978).

Private firms rent capital and hire workers every period, thus, they are characterized only by the quality of z. Private firms are indivisible, rival and excludable. This is an important distinction between our model of trade of firms and the literature that studied trade of ideas (Silveira and Wright, 2010; Akcigit, Celik, and Greenwood, 2016).[11] Different valuations of z aim to capture differences in firms' *intangible assets*. For example, trademarks, patents, processes, permits, customer bases, etc.

We assume that entrepreneurs are subject to *financial frictions*, which may prevent the firm to produce at their optimal scale. Specifically, we assume a collateral constraint that limits the borrowing capacity of the firm to a multiple of the owner's assets which is parameterized by λ.[12]

Given these assumptions, the profit maximization problem of an entrepreneur with assets a and a firm of quality z is given by

$$\begin{aligned} \pi(a,z) = \max_{k,l} \; & y - Rk - wl \\ \text{s.t.} \quad & y = zk^{\theta}l^{\nu} \\ & k \leq \lambda a \end{aligned} \tag{1}$$

where R is the capital rental rate.[13] It's worth mentioning that whenever the collateral constraints binds ($k = \lambda a$) the firm operates at a lower scale compared to the unconstrained profit maximization level.

3.1.3 Public Firm

As in Cagetti and De Nardi (2006), we assume that there is a second sector of production populated by a representative public, or corporate, firm. This aim to capture the fact that, in the U.S. economy, around half of the total output is produced by publicly traded firms.

Specifically, we assume that the public firm is owned by all households, in equal shares, and faces no financial frictions. The public firm is endowed with a constant return to scale technology

$$Y_c = K_c^{\eta} L_c^{1-\eta}$$

where K_c is the corporate firm capital, L_c is the corporate firm labor, and Y_c is the total output.

3.1.4 Financial Intermediary

The financial intermediary takes deposits from households and rent capital to the firms at a price equal to the savings interest rate plus the capital depreciation rate: $R = r + \delta$. We assume

[11] By definition ideas are non-rival. However, ideas might be excludable under certain institutional arrangements such as patents.

[12] This type of constraint can be microfounded with an imperfect enforcement of contracts problem. Here, we assume that the quality of the entrepreneurial project cannot be used as a collateral.

[13] In Appendix B we present the entrepreneurs' input demand functions that characterize the static solution of this problem.

that the representative intermediary operates in a perfectly competitive market and breaks even (i.e., makes zero profits). The resource constraint of the intermediary is given by

$$K_c + s^e_{cm} \int k(a,z)\ \mathrm{d}N^e_{cm}(a,z) = s^e_{cm} \int a\ \mathrm{d}N^e_{cm}(a,z) + (1 - s^e_{cm}) \int a\ \mathrm{d}N^w_{cm}(a,\varepsilon) \tag{2}$$

where N^e_{cm} and N^w_{cm} are the cumulative distribution functions of entrepreneurs and workers, respectively, and s^e_{cm} is the share of households that are entrepreneurs.[14] These measures correspond to the production stage (CM) after firm owners' occupational choices took place, i.e. after deciding to be entrepreneurs or workers.

3.2 Timing

The timing of the model can be summarized as follows.

1. Agents' types, the quality of entrepreneurial projects z and labor efficiencies ε are realized.
2. Agents enter to the market for firms (DM). Firm owners can buy and sell firms, while workers can only buy.
3. Agents enter to the production stage (CM). Given prices and their current z, firm owners decide whether to operate the firm or go to the labor market. Finally, production takes place and agents decide how much to consume and how much to save.

3.3 A Market for Firms

Firms are hard to evaluate and price, which prevents the existence of a centralized market with a complete price schedule for different types of firms. Because of this, we model the market for firms using a *search-theoretic* approach characterized by bilateral random matching and *quid pro quo* trade (i.e. no credit). An interpretation of this setup is that agents can evaluate only one firm at a time, delaying trade.

Trade in the market for firms consists in the transfer of both the ownership and the management of the firm in exchange of assets. Thus, the media of exchange in these transactions are the households' savings a. As we assume that firms are indivisible, when a buyer and a seller meet they only bargain over the selling price p.

There are two types of meetings in the market for firms: *owner-owner* meetings and *owner-worker* meetings. We allow for different search frictions in each type of meeting. Note that firm owners are the only potential sellers while both types of households can be buyers. Thus, in an owner-worker match the owner is the potential seller and the worker is the potential buyer. However, in the case of an owner-owner match who is the buyer and who is the seller depends on the relative quality of the firms.

Let us first consider the owner-owner match and suppose that $z < \tilde{z}$. Then, the owner with states $\mathbf{s}^o \equiv (a, z)$ is the potential buyer and the owner with states $\tilde{\mathbf{s}}^o \equiv (\tilde{a}, \tilde{z})$ is the potential seller. This is intuitive as no owner would buy a firm with lower quality. In such a match, the total surplus from trading the ownership of firm $\tilde{z}$, in exchange of p assets, is given by

$$\text{Total surplus} \equiv \underbrace{W^o(a-p,\tilde{z}) - W^o(\mathbf{s}^z)}_{\text{Buyer's surplus, } S_b} + \underbrace{W^w(\tilde{a}+p,\underline{\varepsilon}) - W^o(\tilde{\mathbf{s}}^z)}_{\text{Seller's surplus, } S_s} \tag{3}$$

[14] As the total mass of households is equal to 1, and there are only two types, the mass of workers at CM is given by $(1 - s^e_{cm})$.

where W^o and W^w are the value functions at the beginning of the production stage for firm owners and for workers, respectively. Upon selling the household goes to the labor market with labor efficiency $\underline{\varepsilon}$, as presented in the first term of the seller's surplus. The outside option for both agents (the terms with a minus in the surpluses) is the value of going to the production stage as firm owners with their current states $\mathbf{s}^o$ and $\tilde{\mathbf{s}}^o$, respectively.[15]

For the case of an owner-worker match, suppose that the firm owner with states $\tilde{\mathbf{s}}^z$ meet a worker with states $\mathbf{s}^w \equiv (a, \varepsilon)$. Then, the total surplus from trading firm $\tilde{z}$ is now given by

$$\text{Total surplus} \equiv \underbrace{W^o(a-p,\tilde{z}) - W^w(\mathbf{s}^w)}_{\text{Buyer's surplus, } S_b} + \underbrace{W^w(\tilde{a}+p,\underline{\varepsilon}) - W^o(\tilde{\mathbf{s}}^z)}_{\text{Seller's surplus, } S_s} \tag{4}$$

where the only difference with respect to the previous case is the buyer's outside option. In this case, the buyer would continue to the production stage as a worker with their current $\mathbf{s}^w$.

Let $\underline{p}$ denote the minimum price at which the seller is willing to sell the firm, i.e. the price at which the seller's surplus is equal to zero. Likewise, let $\overline{p}$ be the maximum price that the buyer is willing to pay for the firm, i.e. the price at which the buyer's surplus is equal to zero. A sufficient condition for trade to occur, meaning that there are positive gains from trade, is that

$$\underline{p}\,(\tilde{\mathbf{s}}^o) < \overline{p}\,(\mathbf{s}, \tilde{z}) \tag{5}$$

where $\mathbf{s} \in \{\mathbf{s}^o, \mathbf{s}^w\}$ in function of the type of match (owner-owner or owner-worker, respectively).

In the case that there are positive gains from trade, we assume that the price is determined by *Nash bargaining*. Thus, for $\mathbf{s} \in \{\mathbf{s}^o, \mathbf{s}^w\}$, p solves

$$\begin{aligned} p(\tilde{\mathbf{s}}^o, \mathbf{s}) = &\arg\max_p \Big[S_b(\mathbf{s}, \tilde{z}, p)\Big]^{\chi} \Big[S_s(\tilde{\mathbf{s}}^o, p)\Big]^{1-\chi} \\ &\text{s.t.} \quad S_b(\mathbf{s}, \tilde{z}, p) \geq 0,\ S_s(\tilde{\mathbf{s}}^o, p) \geq 0 \end{aligned} \tag{6}$$

where S_b and S_s are the buyer and seller surpluses, defined in (3) and (4), and χ the parameter determining the buyer's bargaining power. For our quantitative results, we assume that the buyer has all the bargaining power. Hence we study the case where $\chi = 1$.

3.4 Recursive Formulation

We now present the recursive problem of firm owners and workers. First, we describe the value functions at the beginning of the market for firms (the DM subperiod), which we denote by V. Second, we present the value functions at the production stage (the CM subperiod), which we denote by W.

3.4.1 Value at the Market for Firms (DM)

For firm owners, there are four potential outcomes upon entering to the market for firms: (1) they don't trade, (2) they buy a firm, (3) they sell their firm to another owner, and (4) they sell their firm to a worker. The no trade case could arise because the owner did not match with a counterpart, or there was a match that did not end with a trade.

The value at the beginning of DM for firm owners is given by

[15] One concern could be that firm owners would actually like to pay for someone to buy their firm, and hence p would be negative, in the case z is very low. Our model rule out that possibility because of the firm owner occupational choice at the beginning of the production stage. Thus, firm owners can always decide to go the labor market and get the same labor efficiency $\underline{\varepsilon}$.

$$
\begin{aligned}
V^o(a,z) = \Pr^o[\text{ no trade }]\ W^o(a,z) & \qquad \text{(no trade)} \\
+\ \alpha_o\ s^o_{dm} \int_{z<\tilde{z},\ \underline{p}<\overline{p}} W^o(a-p,\tilde{z})\ \mathrm{d}N^o_{dm}(\tilde{a},\tilde{z}) & \qquad \text{(buy)} \\
+\ \alpha_o\ s^o_{dm} \int_{z>\tilde{z},\ \underline{p}<\overline{p}} W^w(a+p,\underline{\varepsilon})\ \mathrm{d}N^o_{dm}(\tilde{a},\tilde{z}) & \qquad \text{(sell to a firm owner)} \\
+\ \alpha_w\ (1-s^o_{dm}) \int_{\underline{p}<\overline{p}} W^w(a+p,\underline{\varepsilon})\ \mathrm{d}N^w_{dm}(\tilde{a},\tilde{\varepsilon}) & \qquad \text{(sell to a worker)}
\end{aligned}
\tag{7}
$$

where α_o and α_w are exogenous matching probabilities for each type of match.[16] These parameters, which lie in $[0,1]$, govern the degree of search frictions in the market for firms. Further, s^o_{dm} is the share of firm owners at the beginning of DM. N^o_{dm} and N^w_{dm} are the cumulative distributions of firm owners and workers, respectively, at the beginning of DM.

As mentioned in Section 3.3, for the case of owner-owner meetings who buys and who sells depend on the relative firm qualities. Hence, an owner with firm quality z, might buy if its matched with another owner with a firm of higher quality (i.e. $z<\tilde{z}$), as denoted in the integral in the second line of (7). On the contrary, the owner might sell if its matched with an owner with a firm of lower quality ($z>\tilde{z}$) as denoted in the integral in the third line.[17] Finally, note that the integrals for the buying and selling cases consider only the meetings that do result in a trade. Which, is captured by the $\underline{p}<\overline{p}$ condition presented in (5).

For workers, there are only two potential outcomes: (1) they don't trade, or (2) they buy an existing firm. Hence, the value for workers at the beginning of DM is given by

$$
\begin{aligned}
V^w(a,\varepsilon) = \Pr^w[\text{ no trade }]\ W^w(a,\varepsilon) & \qquad \text{(no trade)} \\
+\ \alpha_w\ s^o_{dm} \int_{\underline{p}<\overline{p}} W^o(a-p,\tilde{z})\ \mathrm{d}N^o_{dm}(\tilde{a},\tilde{z}) & \qquad \text{(buy)}
\end{aligned}
\tag{8}
$$

3.4.2 Value at the Production Stage (CM)

As previously described, at the beginning of the production stage, firm owners face an occupational choice. They have to decide whether to operate the firm, and be entrepreneurs, or shut down the firm and go to the labor market with labor productivity $\underline{\varepsilon}$. Given these assumptions, the value of firm owners at the beginning of CM is

$$
W^o(a,z) = \max_e \{W^e(a,z), W^w(a,\underline{\varepsilon})\} \tag{9}
$$

where e denotes the owners' occupational choice.

The value function of entrepreneurs is given by

$$
\begin{aligned}
W^e(a,z) = \max_{a',c} \quad & u(c) + \beta\left\{\gamma V^o(a',z) + (1-\gamma)\mathbb{E}_{z'}\left[V^o(a',z')\right]\right\} \\
\text{s.t.} \quad & c = \pi(a,z) + (1+r)a - a' \\
& c \geq 0,\ a' \geq 0
\end{aligned}
\tag{10}
$$

[16] The probabilities of the bilateral meetings in (7) are obtained as follows. On the one hand, with probability s^o_{dm} an owner is matched with another owner, but they actually meet with probability α_o. Hence, the probability of an owner-owner meeting is $\alpha_o s^o_{dm}$. On the other hand, with probability $(1-s^o_{dm})$ an owner is matched with a worker and they meet with probability α_w. Thus, the probability of an owner-worker meeting is $\alpha_w(1-s^o_{dm})$.

[17] Clearly, meetings where the owners have the same firm quality, i.e. $z=\tilde{z}$, do not result on a trade.

and the value function of workers by

$$
\begin{aligned}
W^w(a,\varepsilon) = \max_{a',c} \quad & u(c) + \beta \left\{ \zeta \mathbb{E}_{\varepsilon'|\varepsilon} \left[V^w\left(a',\varepsilon'\right)\right] + (1-\zeta)\mathbb{E}_{z'}\left[V^o\left(a',z'\right)\right] \right\} \\
\text{s.t.} \quad & c = \varepsilon w + (1+r)a - a' \\
& c \geq 0,\ a' \geq 0
\end{aligned} \tag{11}
$$

where $(1-\zeta)$ represents the exogenous startup shock, through which a worker can become a firm owner.[18]

3.5 Equilibrium

A competitive equilibrium in this economy consists of: (i) aggregate prices $\{r, w\}$; (ii) terms of trade in the market for firms given by the prices of owner-owner meetings $(p, \underline{p}, \overline{p})_{\{\tilde{\mathbf{s}}^o, \mathbf{s}^o\}}$ and owner-worker meetings $(p, \underline{p}, \overline{p})_{\{\tilde{\mathbf{s}}^o, \mathbf{s}^w\}}$; (iii) a decision rule for firm owners' occupational choice $e(a,z)$; (iv) consumption and savings decisions for entrepreneurs $\{c(a,z), a'(a,z)\}$ and for workers $\{c(a,\varepsilon), a'(a,\varepsilon)\}$; (v) capital and labor demand functions for private and public firms, $\{k(a,z), l(a,z), K_p, L_p\}$; and (vi) measures of agents over types and idiosyncratic states at DM and CM subperiods characterized by $\{s^o_{dm}, N^o_{dm}(a,z), N^w_{dm}(a,\varepsilon)\}$ and $\{s^e_{cm}, N^e_{cm}(a,z), N^w_{cm}(a,\varepsilon)\}$, respectively, such that:

1. In DM, the terms of trade in bilateral meetings are solved by the bargaining problem.
2. In CM, given prices, households, private and public firms solve their optimization problems.
3. Goods market clears, period by period:

$$Y = C + K' - (1-\delta)K \tag{12}$$

where

$$
\begin{aligned}
Y &\equiv Y_c + s^e_{cm} \int z k(a,z)^\theta l(a,z)^\nu \ \mathrm{d}N^e_{cm}(a,z) \\
C &\equiv s^e_{cm} \int c(a,z) \ \mathrm{d}N^e_{cm}(a,z) + (1-s^e_{cm}) \int c(a,\varepsilon) \mathrm{d}N^w_{cm}(a,\varepsilon) \\
K &\equiv K_c + s^e_{cm} \int k(a,z) \ \mathrm{d}N^e_{cm}(a,z).
\end{aligned}
$$

4. Labor market clears, period by period:

$$L_c + s^e_{cm} \int l(a,z) \ \mathrm{d}N^e_{cm}(a,z) = (1-s^e_{cm}) \int \varepsilon \ \mathrm{d}N^w_{cm}(a,\varepsilon). \tag{13}$$

5. The budget constraint of the financial intermediary, specified in (2), is satisfied period by period.
6. The measures over types and states are consistent with a recursive equilibrium mapping dictated by firms' prices, households' optimal choices, and the stochastic processes for firms' qualities and workers' labor efficiencies.

We solve the stationary equilibrium of this model using projections methods. The details regarding our solution method are presented in Appendix C.

[18] In (10) and (11) we omit the corporate firms and financial intermediary profits (Π^c and Π^f terms) in the households' budget constraints as both terms are equal to zero, in equilibrium.

4 Workings of the Model

In this section we describe the main workings of the model. We first describe how financial frictions generate motives for the trade of firms in our economy. Second, we characterize who buys and who sell firms and then describe the implications of this market for firm dynamics and for the allocation of capital.

4.1 Gains from the Trade of Firms

Given that trade in the market for firms is voluntary, a necessary condition for trade is that agents have different valuations for the same firm. In particular, the buyer needs to have a higher valuation than the seller. In our model, two ingredients generate heterogeneous valuations for firms and, hence, potential gains from trade: *firms' credit constraints* and *incomplete markets*. We explain the workings of these frictions through two different channels: the *credit channel* and the *risk channel*. These channels are now described.

Credit Channel. The first channel arises from the collateral constraint in the entrepreneurs' problem, which may constrain the firm from producing at the optimal scale. As a consequence, whenever an entrepreneur is credit constraint a wealthier buyer would obtain higher profits out of the same firm as it would be able to operate at a level closer to the optimal scale. Thus, the presence of financial frictions in this economy implies that there are potential gains from the trade of firms between constrained business owners and wealthier buyers. It's worth mentioning that as $\lambda \to \infty$, the profits stream of a firm, $\pi(a, z)$, will not depend on the owner's assets and hence there will be no gains from trade through this channel.

Risk Channel. To analyze this channel let us assume that $\lambda = \infty$, hence there are no gains from trade because of heterogeneous profits' streams. Because of incomplete markets, operating a firm comes with an uninsurable risk as its quality z is stochastic. Hence, agents close to the borrowing constraint would like to save today for precautionary motives. An agent can increment its savings either by delaying consumption or, in the case of firm owners, by selling their firm. Further, as wealth increases the precautionary motive is reduced which, in turn, lowers the potential benefits from selling the firm. Because of this risk channel the value of a firm z will vary across the wealth distribution generating potential gains from trade. In our calibrated model firms with zero debt (i.e. firms that are not credit constrained) also have a positive probability of trade. However, their trade rate is around a 1/4 of the probability of trade of the credit constrained firms in the economy.

4.2 Who Buys and Who Sells Firms?

To analyze the workings of the market for firms we start by characterizing the prices at which firms are traded. If we assume that the buyer has all the bargaining power the selling price is equal to the sellers' minimum price which, conveniently, only depend on the seller's idiosyncratic states. Specifically, if $\chi = 1$ the firm selling price that results from Nash bargaining protocol, stated in (6), is equal to

$$p(\tilde{\mathbf{s}}^o, \mathbf{s}) = \underline{p}\,(\tilde{\mathbf{s}}^o)$$

where, as before, $\tilde{\mathbf{s}}^o$ denotes the states of the seller and $\mathbf{s} \in \{\mathbf{s}^o, \mathbf{s}^w\}$ the states of the buyer.

Figure 3 plots the sellers' minimum price in the firm owners' state space (a, z). As expected, sellers' prices are increasing in the firm quality z. But also, due to collateral constraint on firm owners' wealth, the price of a firm of quality z is increasing in a. Note that this relation is particularly strong for high quality firms. Thus, even for the firm with the highest quality in the economy, owners with few assets would be willing to sell the firm for a relatively low price

as it will take them a long time to accumulate enough assets through self-financing.

Now, keeping in mind how trading prices are determined, we characterize who are the sellers of firms in our economy. Panel (a) of Figure 4 presents the probability that a firm owner sells their firm, again, across the state space (a, z). The figure shows that owners with low wealth and high quality firms are going to be the ones more likely to sell. In those cases there will be high gains from trade since the current owner doesn't have enough resources to produce at the optimal scale. These cases are more likely to occur, for example, when a worker receives a high quality firm through the startup shock but it has few assets to operate the firm.

Panels (b) and (c) of Figure 4 present the probability of buying a firm for both type of households: firm owners in the (a, z) space and workers in the (a, ε) space. These panel show that the probability of buying is the opposite mirror image of the probability of selling. Thus, the main buyers in our economy are going to be wealthy households that currently own low quality firms (low z) or wealthy workers with low levels of labor efficiency (low ε).

4.3 Implications for Firm Dynamics and the Allocation of Capital

To provide intuition about the implications of the trade of firms, Figure 5 presents a hypothetical trajectory of a firm in our model. The initial owner of the firm is the median worker in the economy who at period 0 receives a relatively good quality firm through the startup shock. Because of financial frictions that limit the use of external funding, this entrepreneur will start operating the firm at a very low scale. Panel (a) shows that this business owner will accumulate assets over time in order to reach the optimal operating scale through self-financing. Nonetheless, this entrepreneur will take more than ten years to be able to produce at the optimal, unconstrained, scale as shown in panel (b).

Panels (a) and (b) exemplify the basic mechanism through which financial frictions can generate misallocation of capital and, as a consequence, low aggregate output and low TFP. In particular, capital misallocation will be high if this type of situations, where high quality firms or high ability entrepreneurs are credit constrained and hence they produce at a lower scale, are relatively frequent in the economy (Midrigan and Xu, 2014).

Now we examine what would happen if the initial owner has the possibility to sell its firm. Panels (c) and (d) of Figure 5 plots the selling price and the probability of selling the firm for this business owner. Just after receiving the startup shock this entrepreneur will be willing to sell its firm at a relatively low price as the alternative option of self-financing implies a low profit stream for several periods. In addition, because of the risk channel described above, a credit constrained entrepreneur will be willing to sell its firm because of precautionary motives. As the initial owner accumulates assets the seller's minimum price will increase and the probability of trade will fall accordingly.

Suppose that in period 3, a less constrained party purchases this firm. If the second owner has more resources to invest in the firm, this owner will be able to take the firm closer to its optimal operating scale faster. Specifically, panel (b) shows that with the second owner the firm reaches its optimal scale in close to 5 years after being founded. Half of the time required by the initial owner. Overall, this simple example illustrates how this market allow firms to grow faster and allows for a better allocation of available resources and productive projects in an economy in which entrepreneurs are subject to financial frictions. In the following sections we describe the parameterization of our model and then quantitatively evaluate the importance of this mechanism in terms of output and TFP.

5 Parameterization and Validation

This section describes our calibration strategy and present evidence in favor of our theory of financial frictions being an important motive for the trade of firms. The model is calibrated to an annual frequency to the year 2007, the year for which we have the SBO data available. We now describe the parameters we exogenously assign and then describe the parameters we calibrate such that the model matches relevant features of the data.

5.1 Assigned Parameters

As is standard in the literature, we set the relative risk aversion parameter to $\sigma = 1.5$. The depreciation rate of capital is set to $\delta = 0.06$, while the elasticity of capital in the public firms' production function is set to $\eta = 1/3$. Finally we consider a risk free interest rate r such that the capital output ratio of the public firm is equal to $K_c/Y_c = 1/3$ and find a discount factor β such that market clears. For our baseline model we obtain $\beta = 0.895$, $r = .051$ and $w = 1.155$. Finally, we assume that, in the market for firms, the buyers have all the bargaining power and hence set $\chi = 1$. The value of these parameters are summarized in Table 2.

5.2 Calibrated Parameters

For the remaining parameters, first, we assume that the private firms operate with a technology such that the relative elasticity between capital and labor is the same as the one for the public firm. In this way, the decreasing returns to scale are captured by a single parameter $\Upsilon < 1$ such that $\theta = \eta\Upsilon$ and $\nu = (1-\eta)\Upsilon$. After this, we have a total of 10 remaining parameters which we choose in order to minimize the weighted distance between a 14 moments in the data and in the model. The full list of calibrated parameters is presented in Table 3.

The moments we target capture the relative sizes of the public and entrepreneurial sectors, the income and wealth distribution across households, and two moments about the trade of private firms in the U.S. economy. Table 4 presents the list of targeted moments and their values in the data and in the model. For our calibration we assign a higher weight for the moments regarding the fraction of entrepreneurs and their leverage (debt to capital ratio), and for the moments regarding the trade of firms.

Our model matches quite well both the fraction of entrepreneurs in the population as well as their income and wealth shares. Is also important to mention that our model matches very well both the fraction of firms purchased in the economy and, from those, what fraction was purchased by workers, two important features of the trade of firms that we documented in Section 2.

5.3 Untargeted Moments

A relevant feature of heterogeneous agents models with entrepreneurship is that they are able to match the income and wealth distribution observed in the data (Cagetti and De Nardi, 2006). This result relies in the combination of uninsurable income risk and stochastic returns to wealth through entrepreneurial activity. Table 5 shows that, although we only targeted the Gini coefficients, the model replicates the complete income and wealth distribution relatively well.

5.4 Financial Frictions as a Motive for Trade

As has been described through the paper, if financial frictions are an important motive for trade, credit constrained firms should be the ones more likely to be bought and sold. To present evidence in favor of our theory we go back to the data and look at the trade rate conditional on different characteristics of firms that could indicative of the presence of financial frictions. We

consider two commonly used proxies of credit constraints: firms' age and firms' size as younger and smaller firms are more likely to be credit constrained (Gertler and Gilchrist, 1994). In addition, we analyze firms' APK as credit constrained firms will have high returns to capital but are not able to increase their investment.

We analyze firms' trade rate conditional on these variables in the data and in data simulated from our model. We measure firms' age when traded using data from the SBO. Specifically, we look at all the businesses purchased in 2007 and compute firms' age as the difference between the year the firm was purchased (2007) and the year when the firm was established (by the initial owner). Panel (a) of Figure 6 presents the trade rate for different firms' age bins. Both in the SBO data and in our model startups are the ones more likely to be traded, with a probability of trade being more than 5 times larger than the trade rate of older firms.

To measure firm size we also use data from the SBO but focus on the seller as we want to measure firm size before the trade occurred. For this, we look at the sample of business owners that sold their firm in or after 2007 and measure size using data from the year previous to the trade. Hence, in both the data in the model we relate the probability of trade in t against the size of the firm in $t-1$. We measure size by firm's total sales and total payroll. Panels (b) and (c) present the probability of trade for different quintiles of the sales and payroll distributions. In line with the prediction of our model, we find that smaller firms, measured by either sales or payroll, are the ones with the highest probability of trade.

Finally, panel (d) presents the trade rate conditional on firms' APK quintiles. We measure APK using data from the KFS as, different from the SBO, this data includes information about firms balance sheets that allow us to compute a firm-level measure of capital.[19] As the analysis for size, we relate firms' APK in period $t-1$ against the probability of trade in t. Using the panel structure of the KFS, we identify trades through the owners that report to have sold or merged their firm. Consistent with the predictions of our model, high APK firms in the KFS are the ones with the highest probability of trade.

In sum, Figure 6 shows that younger, smaller, and high APK firms are the ones more likely to be traded. Although our model implies starker relations of these variables and the trade rate, it qualitatively aligns well with the data. It is important to mention that these relations were not targeted in our calibration exercise. Rather, these result from the key prediction of our model that credit constrained firms are the ones more likely to be traded. Overall, we consider that these four different pieces of evidence present a convincing case for financial frictions being an important motive for the trade of private firms.

6 Quantitative Analysis

This section presents the quantitative results of the paper. First, we present two counterfactual experiments aimed to quantify the relevance of the market for firms as a mechanism through which entrepreneurial projects and available resources are allocated. Finally, we analyze the relation between the economy credit conditions and the volume of trade in the ownership of firms.

6.1 The Role of the Market for Firms

In this section we present two counteractual experiments aimed to quantify the relevance of the market for firms as an allocation mechanism for the macroeconomy. In the first experiment we take our baseline model and analyze what would occur if the market for firms partially, and then totally, shut downs. In the second experiment we consider an alternative economy with no

[19] We define capital as inventories, equipment and machinery, land, buildings, and structures, vehicles and other assets owned by the business.

trade in the ownership of firms. We then analyze the TFP of the entrepreneurial sector of that economy against our baseline.

In both experiments we make steady state comparisons of our model under different parameterizations. The results presented in this section show that the trade in the ownership of firms is a relevant mechanism through which entrepreneurial projects and resources are allocated.

6.1.1 Closing the Market

Table 6 presents the results of our first counterfactual experiment. As a reference, the second column of the table presents some key moments of our baseline economy. The third and fourth columns present the results for when the market for firms partially and, then, completely shut downs. In this experiment we only vary the parameters governing the trade of firms while maintaining the rest of the parameters fixed.

Remember that, in the model, the parameters determining the search frictions in the market for firms are α_o and α_w, which capture the probability of meeting conditional on the type of match (owner-owner or owner-worker). For the partial shutdown case we divide in half both parameters such that their relative values are the same and, hence, the fraction of firms purchased by workers is mostly unchanged. For the complete shutdown case we set both parameters equal to zero. As a result, for the partial shutdown the fraction of firms purchased drops from 0.19 to 0.11. For the complete shutdown this fraction goes to zero.

In both cases, entrepreneurial output falls considerably: 2.5 and 5.9% for the partial and the complete shutdown case. These results are explained, first, because there are fewer entrepreneurs producing (the fraction of entrepreneurs goes from 0.09 to 0.08 in both cases). And second, because without the market for firms there will be be a poorer allocation of entrepreneurial projects and resources in the economy. As a consequence, the entrepreneurial sector TFP will be 0.6 and 1.5% lower than the baseline for the partial and complete shutdown case, respectively.

Something that stands out from these results is that aggregate output falls very little: 0.1 and 0.2%. This is because of how we model the public sector which, basically, is a residual of the entrepreneurial sector. Hence, as the table shows, the fall in the entrepreneurial output is matched by similar increases in the public firm output. This, keeping in mind that the relative sizes of the two sectors, with respect to total output, are 46 and 54% as shown in Table 4. Thus, in a model without the public sector aggregate output will also fall.

Finally, is worth mentioning what are the implications for the wealth distribution. Consistent with aggregate output not changing much, the wealth distribution considering all households, as well, doesn't change. However, within entrepreneurial households wealth dispersion actually increases. Specifically, Table 6 shows that when the market for firms is shutting down the wealth share of the top 1% of entrepreneurs goes up. This result reflects the fact that, without the market for firms, there would be a poorer allocation of entrepreneurial projects and resources and, hence, a higher dispersion in entrepreneurs' returns to wealth.

6.1.2 Baseline vs. No Market Economy

For our second experiment we consider an alternative economy with $\alpha_o = \alpha_w = 0$ which we recalibrate such that it matches the same moments as our baseline economy (with the exception of the moments regarding the trade of firms). We call this alternative model the "No Market Economy". In Figure 7 we present different steady states for the baseline and for the no market economy under alternative credit conditions which, in the model, are governed by the parameter λ. Higher λ implies looser credit as entrepreneurs can borrow more with the same level of assets. From these steady states, we focus on three moments: the fraction of entrepreneurs (Panel a),

entrepreneurs' debt to capital ratio (Panel b), and the entrepreneurial sector TFP (Panel c).

As the two upper panels of Figure 7 show, the baseline and the no market economy exhibit practically the same relation for both the fraction of entrepreneurs and their debt to capital with respect to the economy's credit conditions. Thus, point A in both panels (a) and (b) shows that for the same level of λ both economies deliver, roughly, the same value regarding these two moments. However, this is not the case for the entrepreneurial sector TFP. Panel (c) shows that for the same level of λ the no market economy will imply a much lower TFP as there will be a higher degree of *misallocation* between entrepreneurial projects and available resources.

Having these results at hand we *ask*: what are the credit conditions that the no market economy requires such that it matches the TFP level of our baseline economy? Using panel (c) we can recover the level of λ such that the no market economy delivers the same TFP level. In the panel, this implies moving from point A to point B. As B is to the right of A this means that the no market economy require *loser credit* conditions. And then, to better interpret this, we go back to panels (a) and (b) and recover the fraction of entrepreneurs and debt to capital implied by point B. We conclude that the no market economy requires a degree of loser credit such that the fraction of entrepreneurs go up by around 2 p.p., while the debt to capital ratio increases by around 7 p.p (20% higher than the observed one).

Overall, these counterfactual exercises show that the market for firms is indeed a relevant mechanism through which entrepreneurial projects, or investment opportunities, and available resources are allocated in the economy.

6.2 Credit Conditions and the Trade of Firms

In this final section we study the relation between aggregate credit conditions and the volume of trade in the market for firms. An implication of the theory presented in this paper is that credit conditions will affect the gains from trade in the ownership of firms which, ultimately, affect the volume of firms traded. Specifically, as credit conditions are looser current firm owners are able to obtain more resources by their own and hence this reduce the gains from trade arising from the credit channel.

In the empirical section of the paper we documented that the share of firms purchased in the U.S. has declined during the last 30 years. Further, using non-corporate firm leverage as a proxy for private firms debt to capital, Panel (a) of Figure 8 shows that during this time period firm leverage increased by around 20 p.p. To obtain this number, we consider a linear trend to control for the boom and bust episodes during this time period.[20] As a reference, Panel (b) plots the time series for the stock of firms purchased in the SCF described in the empirical section.

Now we analyze this relation using our model. Specifically, we compare two steady states that deliver the same levels of debt to capital, as observed in the data, and then see what is the volume of firms traded for both cases. We perform these steps graphically in the lower panels of Figure 8. First, from Panel (c) we obtain points A and B such that the model delivers the same levels of debt to capital as the ones observed in data during that time period. And then for those values of λ we go to Panel (d) and see what is the change in the volume of firms traded between points A and B. We find that the fraction of firms purchased falls by around 3 p.p. This result suggest that around 3 out of the 10 p.p. decrease in the fraction of firms purchased in the U.S. can be explained by the looser credit conditions observed during the same time period.[21]

[20] Note that the figure starts in the year 1980 as the measure we see in the data for the trade of firms is a stock, not a flow. Hence, to allow for a certain lag in this variable, we consider the credit conditions starting 9 years before our first observation for the stock of firms purchased in the SCF, which is 1989.

[21] One caveat of this exercise is that we do it using our baseline calibration for the year 2007. An alternative exercise will be to recalibrate the model to the year 1989 and then perform these computations. However, we would have to take a stand in how other moments not available for previous years changed.

7 Conclusions

In an economy in which entrepreneurs are subject to financial frictions credit constrained entrepreneurs might want to sell their firm to less constrained parties which will be able to produce closer to the firm's optimal scale. We study this motive for the trade of firms both theoretically and empirically. First, by developing a model of entrepreneurship and frictional trade of firms and second by presenting new evidence in favor of this theory. Specifically, we show that younger, smaller and high capital returns firms are the ones more likely to be traded.

Our results present a convincing case in favor of financial frictions being an important motive for the trade of private firms. Still, we acknowledge that other motives for the trade of firms, such as retirement or life-cycle motives, may also be relevant.

We then study the macroeconomic implications of the trade of firms in the context of the U.S. economy. Parameterizations of our model show that the market for firms is a relevant mechanism through which entrepreneurial projects and available resources can be better allocated in the economy. For example, shutting down this market implies a fall in entrepreneurial output of around 6%. Or, alternatively, an economy with no trade of firms requires 7 p.p. (20%) higher debt to capital to match the level of TFP of our baseline economy.

We consider that this paper is a first step towards a better understanding of the role of this market for the aggregate economy. An interesting avenue for future research is to explore the implications of wealth and capital gains taxes for the workings of this market and for efficiency.

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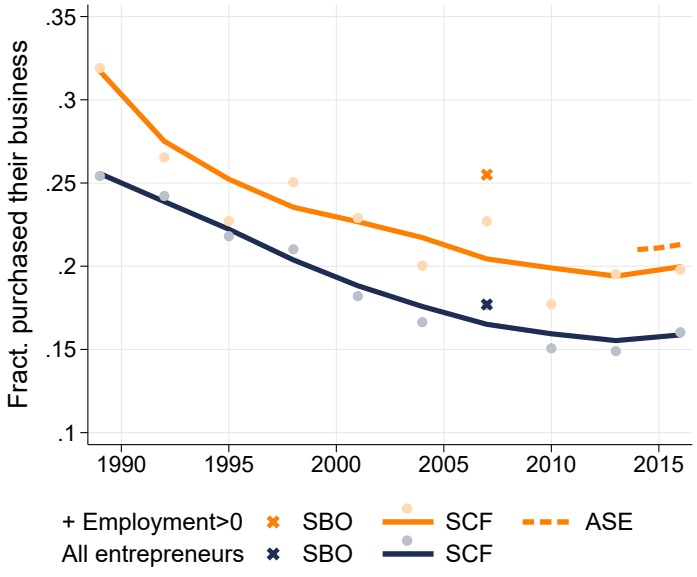
Figures

Figure 1: Fraction of Entrepreneurs that Purchased Their Business



SOURCE: SBO-PUMS, SCF and ASE.

NOTES: Entrepreneurs are defined as (i) self-employed, (ii) business owners, who (iii) actively manage their firm. + Employment > 0 also requires that (iv) the firm has a positive number of employees. The dots correspond to the time series SCF data points, and the solid line to the trend estimated using locally weighted smoothing.

Figure 2: Transitions Between Occupations

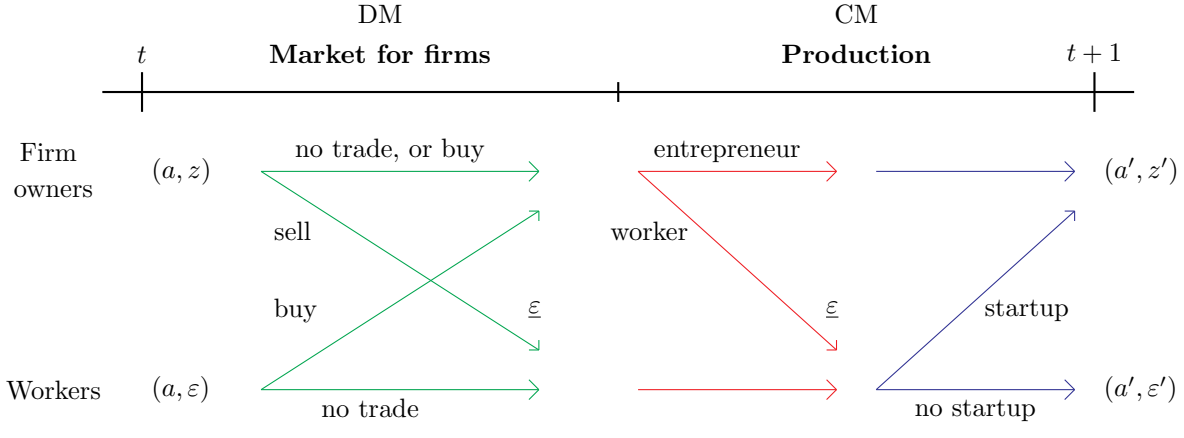


Figure 3: Sellers' Minimum Price $\underline{p}(\tilde{\mathbf{s}}^o)$

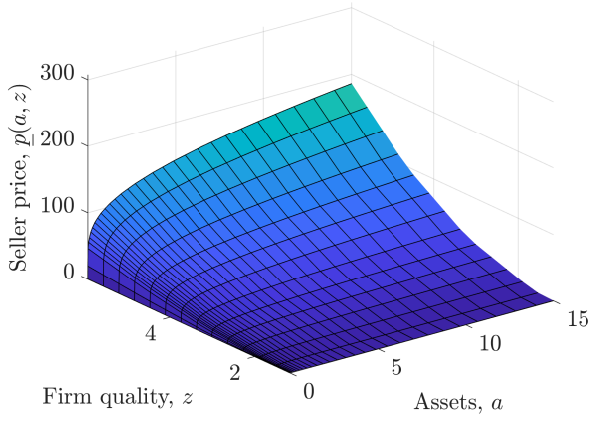
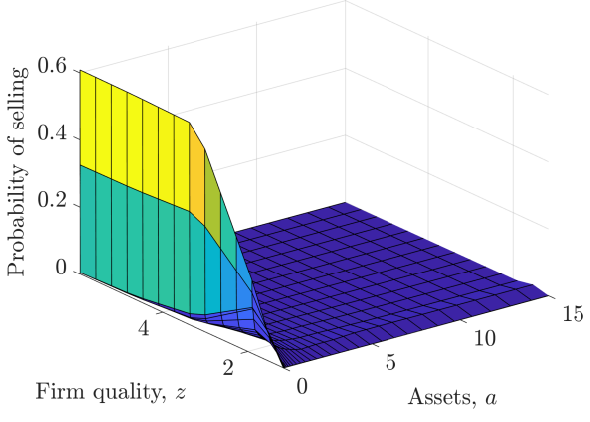
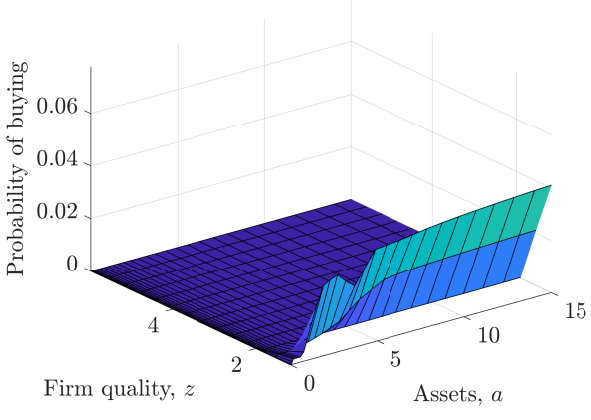


Figure 4: Buyers and Sellers in the Market for Firms

(a) Probability of selling



(b) Probability of buying, owners



(c) Probability of buying, workers

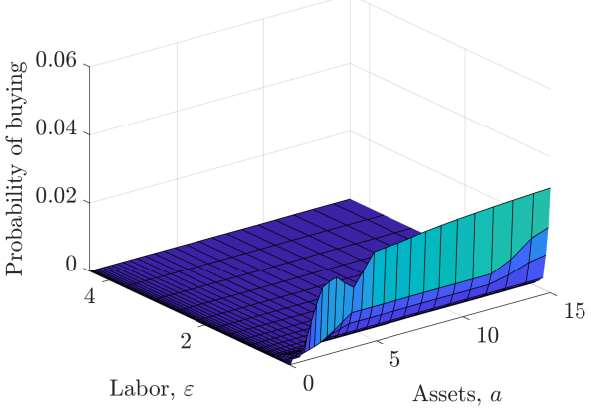
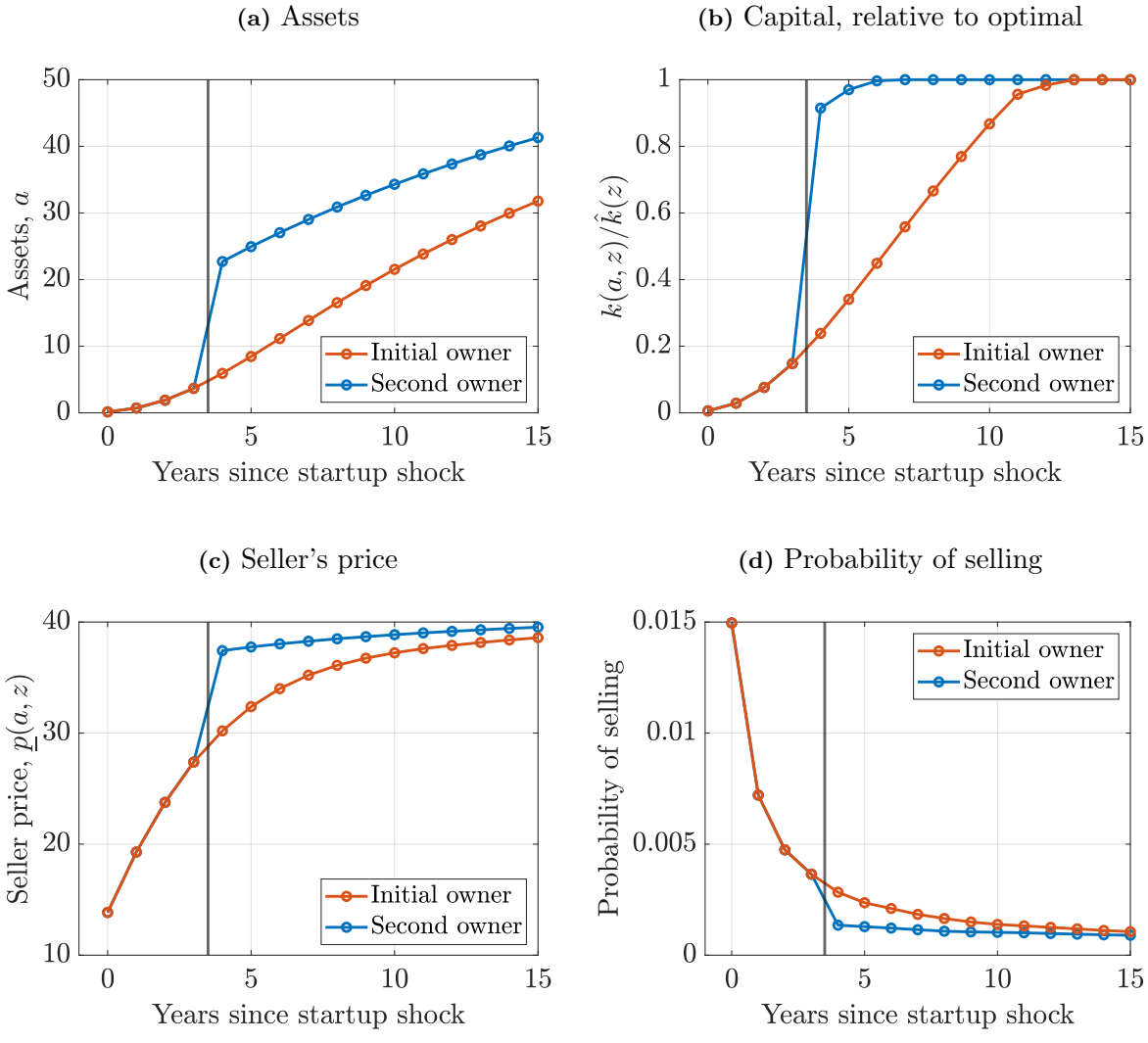


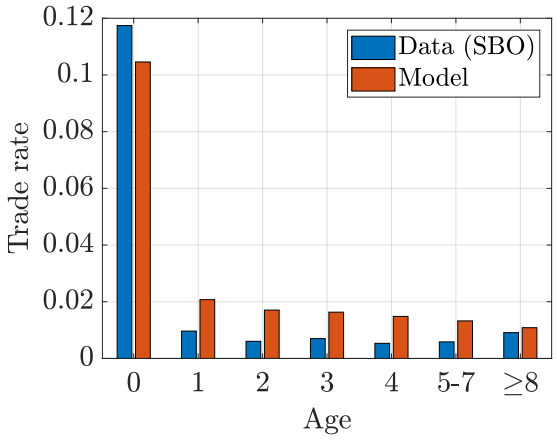
Figure 5: Firm Dynamics and Trade, An Example



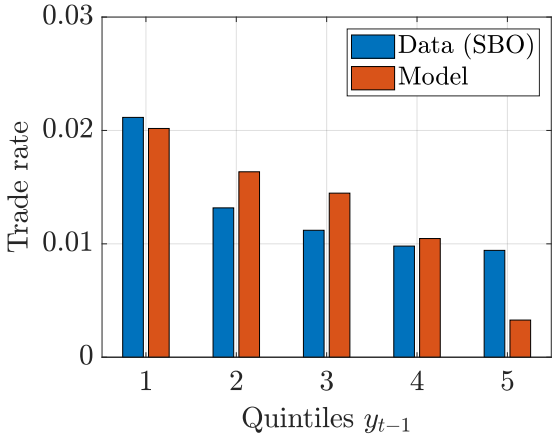
NOTES: The vertical line indicates the DM subperiod at $t = 3$, when trade takes place. The initial owner has assets equal to the median worker of the economy upon receiving the startup shock at $t = 0$. For this example firm quality z is constant across the 15 periods and equal to the 3rd best firm in the z grid. $\tilde{k}(z)$ in panel (b) denotes the unconstrained optimal level of capital for a firm with quality z. $\underline{p}(a,z)$ in panel (c) denotes the seller's minimum price.

Figure 6: Financial Frictions as a Motive for Trade, Data and Theory

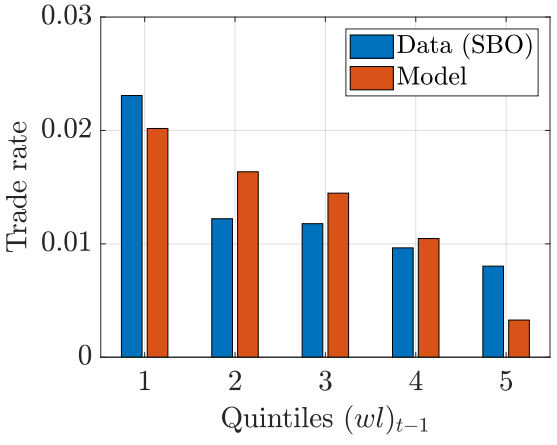
(a) Trade vs. Firm Age



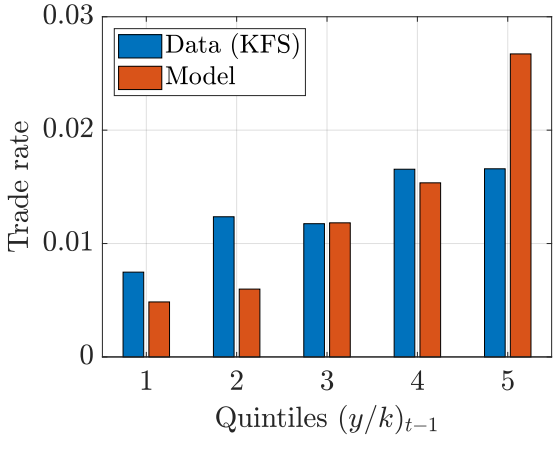
(b) Trade vs. Firm Size (Sales)



(c) Trade vs. Firm Size (Payroll)



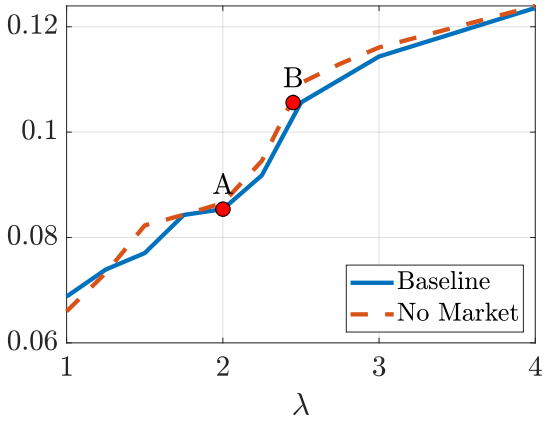
(d) Trade vs. APK



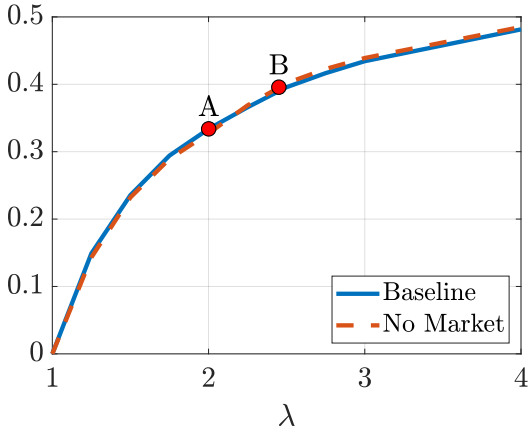
NOTES: Panels (a) to (c) use data from the 2007 SBO. In panel (a) trade is computed using the fraction of owners that acquired their firm through a purchase in 2007. The age of the firm is computed as the difference between 2007 and the year when the business was established. In panels (b) and (c) trade is computed using information from the firms that were sold in or after 2007. Panel (d) uses data from KFS. Trade is computed using information from all the firms sold during the years of the sample. Average productivity of capital (APK) is measured by sales over capital of the previous year to the sale. Capital includes inventories, equipment and machinery, land, buildings, and structures, vehicles and other assets owned by the business. The relation is computed for every year and then averaged across time.

Figure 7: Baseline vs No Market Economy

(a) Fraction of entrepreneurs



(b) Debt to capital, weighted



(c) TFP, entrepreneurial

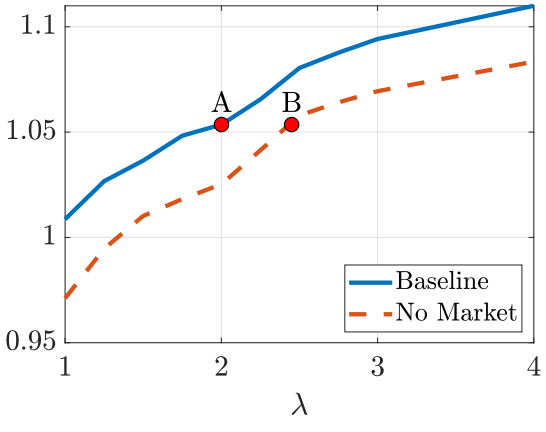
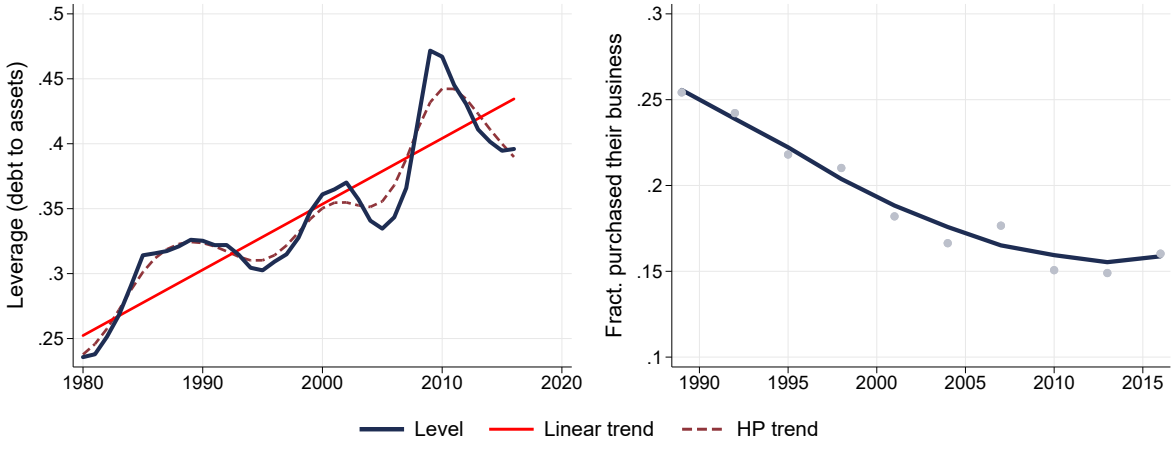


Figure 8: Credit Conditions and the Trade of Firms

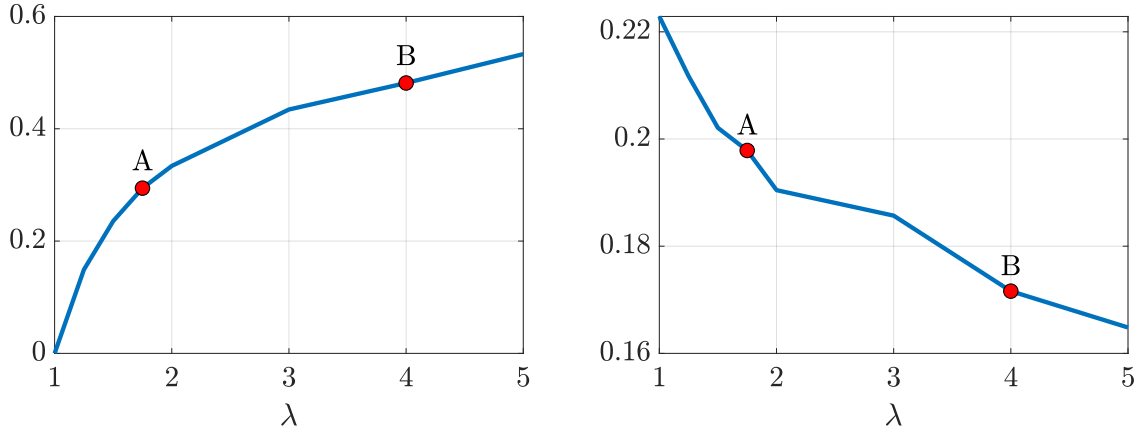
(a) Noncorporate firms leverage

(b) Fract. of entrepreneurs that purchased



(c) Debt to capital, weighted

(d) Fraction of firms purchased



Tables

Table 1: Share of Entrepreneurs, by Business Acquisition

		Founded	Purchased	Inherited/Other
Entrepreneurs	SBO	77.0%	17.0%	6.0%
	SCF	71.9%	17.7%	10.4%
+ Employment > 0	SBO	65.2%	25.5%	9.3%
	SCF	65.3%	22.7%	12.0%

SOURCE: SBO-PUMS and SCF for the year 2007.
NOTES: Entrepreneurs are defined as (i) self-employed, (ii) business owners, who (iii) actively manage their firm. + Employment > 0 also requires that (iv) the firm has a positive number of employees. Other type of acquisition groups: acquired as a transfer, as a gift or other not specified.

Table 2: Assigned Parameters

Parameter	Value	Description
σ	1.5	CRRA
δ	0.06	Capital depreciation rate
η	1/3	Capital elasticity
Y_c/K_c	3	Public firm capital output ratio
χ	1	Buyers' bargaining power

Table 3: Calibrated Parameters

	Value	Description
Υ	0.708	Curvature private firms technology
λ	2.001	Collateral constraint
γ	0.925	Persistence private firm value
ζ	0.933	$1-$ Startup shock
z_{min}	1.166	Scale, z distribution
η_z	2.827	Shape, z distribution
ρ_ε	0.964	AR(1) parameter, ε distribution
σ_ε	0.160	Std. Deviation, ε distribution
α_o	0.802	Owner-owner | meeting probability
α_w	0.603	Owner-worker | meeting probability

Table 4: Targeted Moments

	Data	Model
Fraction of entrepreneurs	0.08	0.09
Debt to capital, weighted	0.35	0.33
Private firms output share	0.48	0.46
Entrepreneurship exit rate	0.08	0.08
Income share, entrepreneurs	0.21	0.21
Wealth share, entrepreneurs	0.35	0.32
Gini income, all households	0.57	0.47
Gini wealth, all households	0.79	0.84
Gini income, entrepreneurs	0.66	0.65
Gini wealth, entrepreneurs	0.75	0.77
Gini income, workers	0.53	0.41
Gini wealth, workers	0.76	0.84
Fraction of firms traded	0.18	0.19
Annual trade rate	0.02	0.02
Share of firms purchased by workers	0.62	0.62

NOTES: The Data numbers come from the SCF and the SBO-PUMS for the year 2007.

Table 5: Untargeted Moments

	Data	Model		Data	Model
Wealth distribution, all households			**Income distribution, all households**		
Top 1	0.33	0.32	Top 1	0.21	0.16
Top 5	0.60	0.59	Top 5	0.37	0.27
Top 10	0.72	0.73	Top 10	0.47	0.38
Bottom 75	0.12	0.08	Bottom 75	0.34	0.45
Bottom 50	0.02	0.01	Bottom 50	0.14	0.18
Bottom 25	0.00	0.00	Bottom 25	0.04	0.08
Wealth distribution, entrepreneurs			**Income distribution, entrepreneurs**		
Top 1	0.24	0.26	Top 1	0.22	0.19
Top 5	0.48	0.58	Top 5	0.44	0.55
Top 10	0.63	0.74	Top 10	0.57	0.68
Bottom 75	0.16	0.15	Bottom 75	0.24	0.24
Bottom 50	0.05	0.08	Bottom 50	0.10	0.12
Bottom 25	0.01	0.03	Bottom 25	0.03	0.06

NOTES: The Data numbers come from the SCF for the year 2007.

Table 6: Closing the Market for Firms

	Baseline economy	Partial $(\alpha_o, \alpha_w)/2$	Complete $(\alpha_o, \alpha_w) = \mathbf{0}$
Fract. firms purchased	0.19	0.11	0.00
Fract. firms purchased by workers	0.64	0.65	-
Fract. entrepreneurs	0.09	0.08	0.08
Δ Output		-0.1%	-0.2%
Δ Output, public		2.0%	4.9%
Δ Output, entrepreneurial		-2.5%	-5.9%
Δ TFP, entrepreneurial		-0.6%	-1.5%
Entrepreneurs' wealth share	0.32	0.29	0.25
Wealth top 1	0.32	0.32	0.32
Wealth top 10	0.73	0.73	0.73
Wealth top 1, entrepreneurs	0.26	0.28	0.31
Wealth top 10, entrepreneurs	0.74	0.74	0.76

NOTES: The Partial column presents the results for when the market for firms is partially shut down, obtained by dividing the parameters α_o and α_w by half. The Complete column presents the results for when both parameters are equal to zero. The rows with Δ report the percentage change of the corresponding variable with respect to its value in the baseline economy. TFP is measured as $Y_e/(K_e^\theta L_e^\nu)$, where $(.)_e$ denotes the aggregate variables of the entrepreneurial sector.

A Data Appendix

A.1 Databases, Samples and Definitions

A.1.1 Survey of Business Owners (SBO) - PUMS

The SBO is a comprehensive survey about firms and firm owners in the U.S. The PUMS sample is representative of the non-farm private businesses and is available for the year 2007. The SBO is conducted at the company or firm-level, rather than at the establishment-level. Where a company, or a firm, is a business consisting of one or more domestic establishments that the reporting firm specified under its ownership or control in 2007. The SBO questionnaire is mailed to a random sample of businesses selected from a list of all firms operating during 2007 with receipts of $1,000 or more.[22]

It's worth noticing that the SBO makes an effort to identify the ultimate owners of the firms and their characteristics. In the survey they are able to identify more than 90% of the privately held firm owners, and in terms of size between 70-80% of the privately held firms.

In the SBO, we identify entrepreneurs simply as those business owners who report to manage their firm. In Appendix A.2, we use alternative ways to identify entrepreneurs. For our baseline definition we have a sample of 1,112,254 entrepreneurs and 841,254 firms as specified in Table A.1.

Table A.1: SBO Sample

	Dropped	Owners	Firms
All	-	8,662,720	2,165,680
Report Acquisition	6,498,179	2,164,541	1,291,292
Owner-Manager	1,052,287	1,112,254	841,254
Employer Firm	413,603	698,651	501,564

SOURCE: SBO-PUMS.

A.1.2 Survey of Consumer Finances (SCF)

Coverage and general information. The SCF is a household-level survey realized by the Federal Reserve Board. The survey data includes information on households' balance sheets, pensions, income, and demographic characteristics. The publicly available microdata is available every three years for the time period between 1989 to 2016.

Variables and sample selection. The SCF reports information regarding firms owned by the members of the household, usually the household head. The exact information we use is

- Acquisition: How the owner acquired their firm?
- Firm value: How much is the value of the owner's stake?
- Ownership: Share hold by the owner?
- Employment status

[22] They exclude some specific small sectors such as "Religious, Grantmaking, Civic, Professional, and Similar Organizations".

- Employment of the firm

For most of our calculations if the entrepreneur has more than one firm we use the characteristics of the largest firm in terms of value. This is not a major issue as most entrepreneurs own only one firm as showed below. In our robustness checks we also consider the rest of the firms owned. The results are very similar.

We also use the SCF to compute moments of the income and wealth distribution. For this we use the standard measures of wealth and income reported by the SCF. From the SCF we also estimate the share of entrepreneurs in the economy and their wealth share. We define them in two ways for our baselines calculations:

1. Self-employed, business owner that actively manage their business
2. Self-employed, business owner that actively manage their business, with a business that employs at least one worker (apart from the owners)

In our robustness checks we consider alternatives definitions.

A.1.3 Annual Survey of Entrepreneurs (ASE)

Coverage and general information. Includes all non-farm businesses filing Internal Revenue Service (IRS) tax forms as individual proprietorships, partnerships, or any type of corporation, and with receipts of $1,000 or more. Represents a similar sample as the SBO, only that the ASE only covers firms with paid employees. The methodology of this survey is different from the SBO (that is based on the Census which is every five years), though the survey provides similar information. We don't have access to the microdata so the results are from the tables presented by the Census Bureau.

Definitions. In the ASE a business owner is defined as someone who holds more than 50% of the stake of the firm, where the firm has a positive payroll. Though this definition is slightly different is close to our second definition of entrepreneur, then we compare the results from this survey with this definition.

A.1.4 Kaufman Firm Survey (KFS)

Coverage and general information. The Kauffman Firm Survey (KFS) tracks close to 5,000 businesses that started operations in 2004 and follows them through 2011. The survey is performed on an yearly basis. The KFS is meant to be representative of start-ups in the US in the year 2004. They collect information from business' and owners' characteristics. In particular they provide information for firms' balance sheets.

Definitions and sample. For the KFS we consider the sample of firms with positive capital and revenues. We define capital as asset without cash holdings and accounts receivable. To approximate the capital returns we consider the average productivity of capital (APK) measured as firms revenue to capital ratio.

A.2 Robustness and Additional Exercises

A.2.1 Definitions and Samples

In this part we want to check if our results are robust to the different definitions of entrepreneurship, samples and computations.

SBO trade of firms. For the SBO we compute several definitions, computations and sample, we do so on a firm and owner level. For the firm level we compute them in two ways: (i) if at least one entrepreneur purchase the firm, we call this *max*; (ii) rate of purchase equally weighted across entrepreneurs of the same firm, we call this *min*.

Results are shown in Table A.2 and Table A.3. First, in Table A.3 we observe that the purchased share computed on a firm or owner level results are very close, this is due to the fact that most firms have one entrepreneur and most entrepreneurs have one firm (i.e. ownership and management very concentrated). Second, in both tables the results are sensitive between definitions that exclude or not firms with no employment. Definitions that consider firms with no employment tend to have lower purchasing ratios. Third, in Table A.2 if we weight by employment the share is much higher (32.2%). This is related to the fact that firms that were purchased some moment in the past on average are larger. Lastly, results are around 20% with minimum values around 15% and maximum around 25%.

SCF trend of traded firms. Other concern is if the decreasing trend is robust to different definitions and samples. To check this we use the following definitions and samples: (i) entrepreneur, (ii) entrepreneur with positive employment, (iii) on a firm level, (iii) main owners (share $> 50\%$), and (iv) weighted by value of the firm.

Figure A.1 shows that the decreasing of trend is robust across different definitions, both qualitatively and quantitatively. Again we find that there is a level difference between weighting the purchase share by size (largest), positive employment or all firms (smallest).

A.2.2 Other Robustness and Exercises

Size and Trade of Firms. Firms purchased are significant in the aggregate economy. Even though they are around 20% of the privately held firms, they are around one third of the total sales, employment and payroll. Next, we show that not only they are relevant also they are a significant fraction of the firms in right tail of the distribution.

In Table A.4 we find that trade of firms is even more common among large and very large private firms. This, jointly with the observation in Section 5 that firms are small at the moment of being purchased, may suggest that firms where bought and later grow. On the contrary, if purchases where focused on slow growing firms then it's unlikely to observe that very large firms where purchased.

Owners and Entrepreneurs by firm. One of our key assumptions is that privately held firm's ownership and management are very concentrated. To corroborate if this assumption is a decent approximation in this subsection we will show how concentrated is the ownership and management for the firms in the SBO.

In Table A.5 we compute how many owners and entrepreneurs are by firm. First, we observe that at least two thirds of the firms have at most 1 entrepreneur and more than 90% have at most two. In addition, using the SCF we found that 90% of the entrepreneurs have at most one firm. Both observations are in line with our assumption that ownership and management is very concentrated in privately held firms. Second, we observe in Table A.5 that there are more owners than entrepreneurs and this gap is larger for larger firms. This suggest it is likely that there is some equity financing for privately held firms, specially large firms.

Through the SBO we can know the equity share of each entrepreneur. In Figure A.2 we observe that in firms of one entrepreneur the most common ownership status is that they earn all the equity, but is not rare that they share the ownership half-and-half with a non-entrepreneur owners. Moreover, most of the one entrepreneur firms have at most 2 owners, which makes likely that in firms with entrepreneurs and non-entrepreneurs the arrangement is a partnership. On the other hand, in firms of two entrepreneurs the most common arrangement is that they share

in equal parts the ownership (i.e. partnership).

Franchises. Are the results driven by franchises? Using the SBO we can identify which businesses are operating as a franchise.

In Table A.6 we see that even if we exclude franchises results are almost the same since franchises make no more than 5% of the total firms.

Sectors. Other robustness check is if there is some effect coming from different sectors driving the results. So we compute for each sector the share of entrepreneurs that purchased their firm in 2007 using the SBO.

We compute this for the definition with positive employment[23]. Results in Figure A.3 show that though there is some variability in the stock of trade we have that firm trade is widespread across sector (lowest has at least 13% of the entrepreneurs that purchased their firm). On the other hand, we find that using the SCF there is some sectorial effect driving the trend. The decreasing trend is robust to this, but some estimations we perform show, in Figure A.4, that the decrease in the trend may be lower.

For example if we exclude agriculture and fix the share of the sectors to 1998 share we have that the decrease in trend remains, but is reduced to some number around 5 percentage points.

Propensity to Sell Regressions To estimate the relation of each sector, age group and size of firm with the probability a firm is sold we estimate the following regression:

$$\begin{aligned}\text{Sold}_i = &\sum_s \beta_s \times \text{Sector}_{i,s} + \sum_q \beta_q \times \text{Size}_{i,q} + \sum_a \beta_a \times \text{Age}_{i,a} \\ &+ \beta_{\text{control}} \times X_{\text{control}} + \varepsilon_i \end{aligned} \tag{14}$$

where $\text{Sector}_{i,s}$ indicates if entrepreneur i is in sector s, $\text{Size}_{i,q}$ indicates if size of entrepreneurial firm belongs to quartile q, $\text{Age}_{i,a}$ if entrepreneur belongs to age group a, and we allow for other controls (e.g. states). The dependent variable Sold_i indicates if entrepreneur sold the business.

Figure A.5 and Figure A.6 exhibit the results. First, most sectors have a similar propensity, only the high propensity of restaurants, hotel and retail sectors, and low propensities of construction and professional services can't be explained by this simple estimation. Other unobservables such as heterogeneity in the the demand (restaurants and hotels) for external funds, fix costs (construction) and tradability (professional services) may explain this results. Next, we find that the U-shape relation of age and propensity remains as in our basic descriptive analysis. Lastly, small firms are much likely to be purchased, which again is the same pattern we observe in our baseline analysis.

A.3 Firm Trade: Relating the Stock and the Rate

We have the stock of traded firms and we want to have an estimate of the annual rate at which firms are traded. We estimate this rate in two ways, by using the SBO and approximating it with the year 2006 purchased flow corrected by exit and purchasing rate before the survey in 2007, or we estimate it through firms flow equations we are going to explain in this section.

A.3.1 Constant stock of traded firms

Let's assume the following timing: first firms exit and enter at some rate, and later the purchase happens. Take x stock of firms purchased and y the stock of all firms. Now the flow considering the timing as in the model of purchased firms and total firms are

[23]This is robust also to this, but we focus on one for the sake of brevity.

$$y_{t+1} = y_t \left[1 - \pi^y_{exit,t} + \pi_{entry,t}\right]$$
$$x_{t+1} = x_t \left(1 - \pi^x_{exit,t}\right) + \left[y_{t+1} - x_t \left(1 - \pi^x_{exit,t}\right)\right] \pi_{trade,t_+}$$

where π_{entry} and π_{exit} are the annual entry rate and exit rate, and π_{trade} is the annual rate of firm trade we want to estimate. Using this flow equations we have that the ratio evolves as

$$\left(\frac{x_{t+1}}{y_{t+1}}\right) = \left(\frac{x_t}{y_t}\right) \left\{ \frac{1 - \pi^x_{exit,t} + \frac{y_t}{x_t}\left[1 - \pi^y_{exit,t} + \pi_{entry,t}\right] \pi_{trade,t_+} - \left(1 - \pi^x_{exit,t}\right) \pi_{trade,t_+}}{1 - \pi^y_{exit,t} + \pi_{entry,t}} \right\}$$

if no growth of the ratio of firms purchased and same exit rate for x and y then

$$\pi_{entry,t} \left[\frac{y_t}{x_t}\left(1 + \pi_{entry,t} - \pi_{exit,t}\right) - \left(1 - \pi_{exit,t}\right)\right]^{-1} = \pi_{trade,t_+} \tag{15}$$

moreover if we assume that entry = exit then

$$\pi_{trade,t_+} = \pi_{e,t} \left(\frac{y_t}{x_t} - 1 + \pi_{e,t}\right)^{-1} \tag{16}$$

A.3.2 Decreasing stock of traded firms: the falling entry rate and traded firms ratio

Now we have in the data that the ratio $\frac{y_t}{x_t}$ is increasing and $\pi_{entry,t}$ decreasing then from the decreasing ratio then we need that

$$\pi_{entry,t} \left[\frac{y_t}{x_t}\left(1 + \pi_{entry,t} - \pi_{exit,t}\right) - \left(1 - \pi_{exit,t}\right)\right]^{-1} > \pi_{trade,t_+}$$

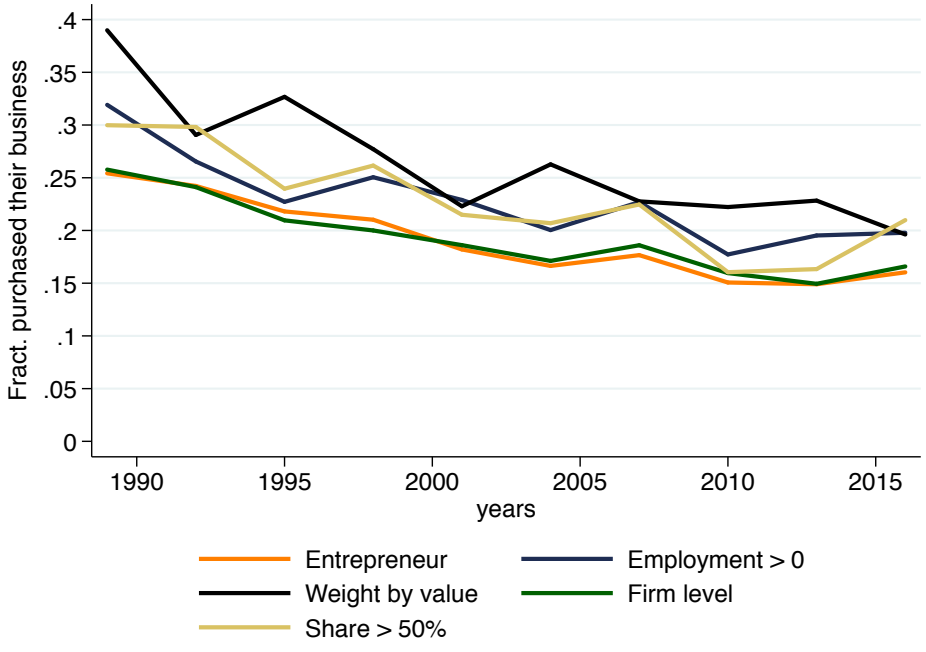
Notice that for the ratio to decrease this condition has to hold, and moreover the RHS is decreasing overtime, e.g.

$$\pi_{trade,80s} < 0.13 \left[\frac{1}{0.3}\left(1 + 0.05\right) - 1 + 0.08\right]^{-1} \approx 0.050 \quad t = 1980s$$
$$\pi_{trade,00s} < 0.08 \left[\frac{1}{0.2} - 1 + 0.08\right]^{-1} \approx 0.020 \quad t = \text{late } 2000s$$

indicating a fall of the rate from 5.0% to 2.0%. In this sense the fall on the upper-bound of the rate doesn't necessarily imply that the rate is falling, but the magnitudes and evolution of the ratio of traded firms suggest that is very likely that this rate dropped (o/w the fall in the ratio would be very abrupt in the first years).

Data Appendix Figures

Figure A.1: Purchased share SCF (1989-2016)

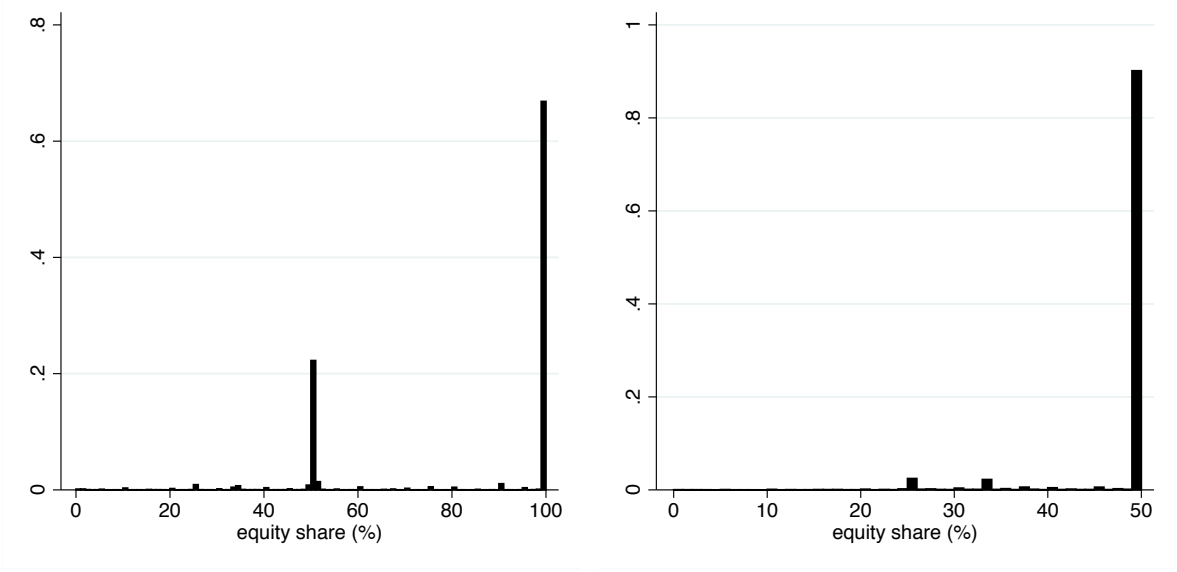


SOURCE: SCF.

Figure A.2: Equity Share of Entrepreneurs

(a) One entrepreneur firms

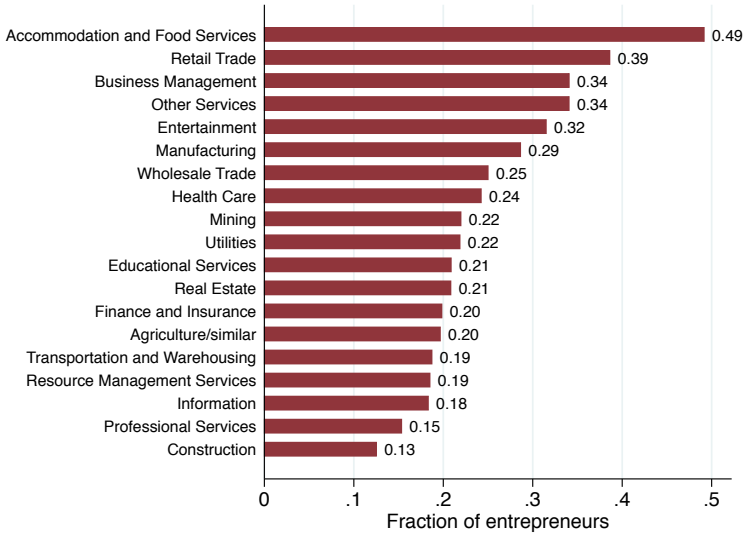
(b) Two entrepreneurs firms



SOURCE: SBO-PUMS.

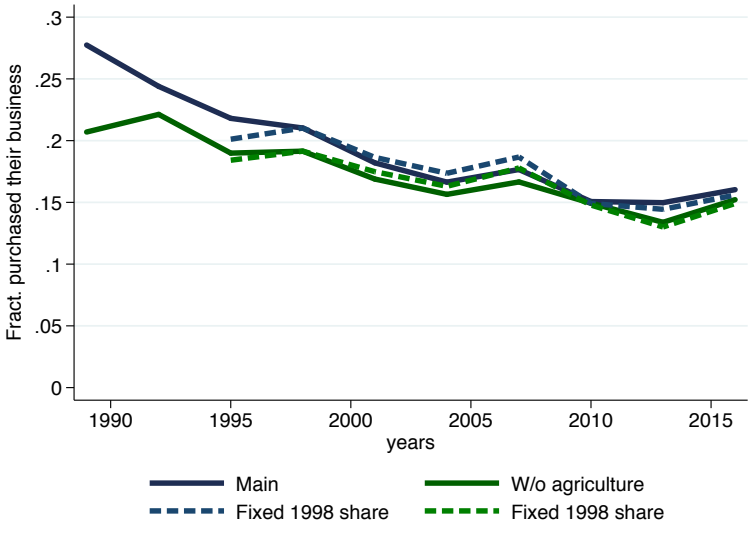
NOTES: firm owners are conditioned to be at most 4 since the equity share is normalized to 4 owners at most. Firms with at most two entrepreneurs make up to 97-98% of all firms.

Figure A.3: Share purchased by Sector (SBO, 2007)



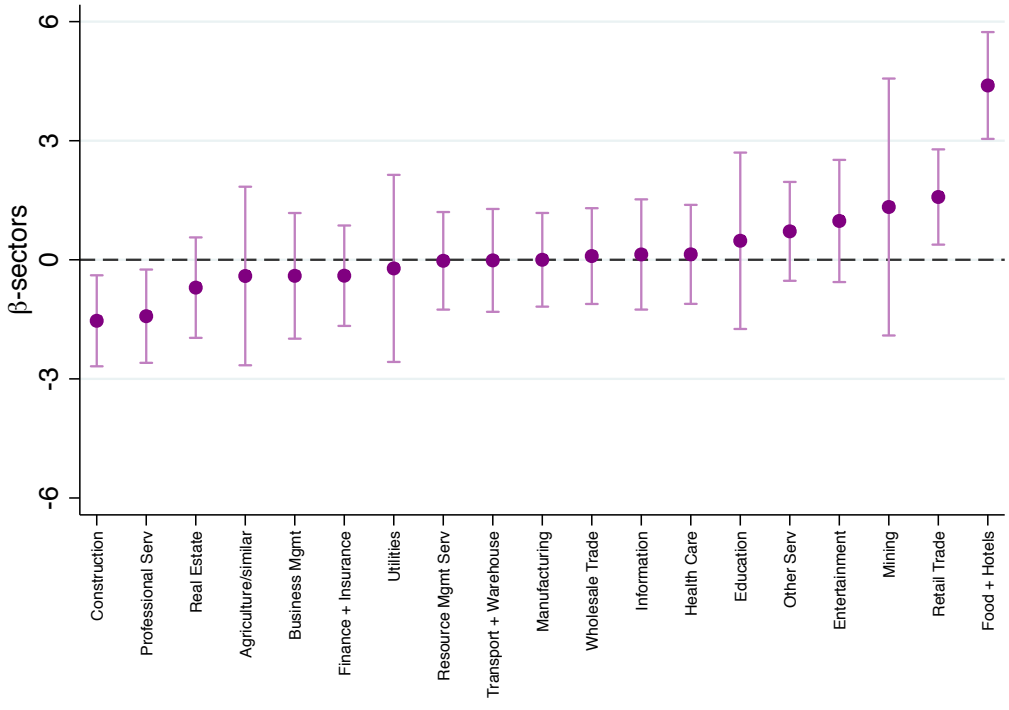
SOURCE: SBO-PUMS.

Figure A.4: Sectorial Effect on Purchased Firms (SCF, 1989-2016)



SOURCE: SCF.

Figure A.5: Sector Effect on Propensity to Sell Firm

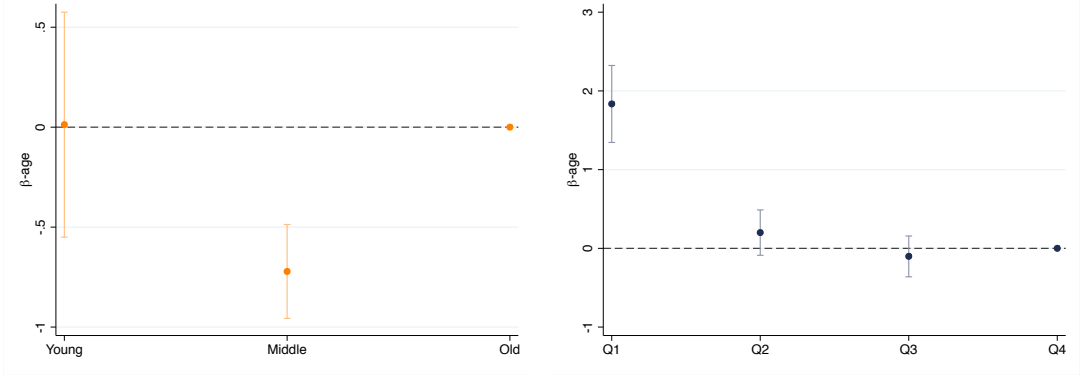


SOURCE: SBO-PUMS.

NOTES: Coefficients are normalized to 0 using median of estimates. Standard errors are clustered by sector and state.

Figure A.6: Age and Size Effect on Propensity to Sell Firm

(a) Age Group **(b)** Size Quartile



SOURCE: SBO-PUMS.

NOTES: Coefficients are normalized to 0 with respect to oldest age group and largest quartile of size. Standard errors are clustered by sector and state. Sales of the firm is used as the size variable.

Data Appendix Tables

Table A.2: Robustness: Share of entrepreneurs that purchased their firm

Sample	purchased	N(weighted)	N
All owners	-	105,568,952	8,662,720
Respond acquisition	16.0%	20,302,192	2,164,541
entrepreneurs (manage)	17.0%	9,503,681	1,112,254
>0 employment	25.9%	5,507,460	1,255,134
>0 receipts	16.9%	17,139,950	1,987,336
>0 payroll	25.1%	6,045,634	1,338,400
>0 all size	26.1%	5,344,965	1,216,319
entrepreneurs + employment>0	25.5%	3,167,718	698,651
manage + payroll >0	24.7%	3,473,610	745,699
share>=50	13.5%	16,274,606	1,479,855
share>=50 + employment>0	23.5%	3,884,071	745,431
share>=50 + payroll>0	22.7%	4,320,811	809,769
share>=50 + manage	15.4%	8,064,388	827,286
share>=50 + size>0 + manage	24.2%	2,385,664	455,442
weight by employment	32.2%	9,503,681	1,112,254
working age	17.2%	8,298,522	983,598
working age + employment>0	25.8%	2,838,812	622,336
> 40 hours	19.6%	5,679,652	806,923
> 40 hours + employment>0	26.0%	2,545,635	582,966

SOURCE: SBO-PUMS.
NOTES: "share" refers to the normalized entrepreneur's share of the firm. ">40 hours" entrepreneurs that report at least 40 hours of labor in the firm. "working age" are entrepreneurs with more than 24 years and less than 66 years.

Table A.3: Robustness: Share of entrepreneurs vs firms purchased

Sample	owner level	firm level	
		at least one	all
Respond acquisition	16.0%	14.7%	12.0%
entrepreneurs (manage)	17.0%	16.3%	15.0%
>0 employment	25.9%	26.8%	20.9%
entrepreneurs + employment>0	25.5%	25.7%	23.2%
working age + employment>0	25.8%	25.9%	23.7%
> 40 hours + employment>0	26.0%	26.1%	23.8%

SOURCE: SBO-PUMS.
NOTES: on a firm level we compute the share of firms where at least one of the entrepreneurs purchased her share of the firm, and if all entrepreneurs purchased their share of the firm. ">40 hours" entrepreneurs that report at least 40 hours of labor in the firm. "working age" are entrepreneurs with more than 24 years and less than 66 years.

Table A.4: Firms purchases, by size group

Percentile	Variable	Purchased	Average
P0-P90	receipts	24.6%	651
	payroll	24.6%	153
	employment	25.2%	8
P10-P1	receipts	34.6%	8,624
	payroll	34.4%	1,773
	employment	37.9%	83
P1-P0.1	receipts	43.8%	57,753
	payroll	40.0%	9,220
	employment	37.9%	248
P0.1-P0	receipts	39.0%	381,869
	payroll	35.3%	49,760
	employment	32.2%	1,374

SOURCE: SBO-PUMS.
NOTES: given that many firms have employment zero in the baseline sample we use employer firms for these estimates. Averages are computed for purchased and non-purchased firms. Payroll and receipts are in thousands of US$.

Table A.5: Owners and Entrepreneurs by Firms

		# of owners or entrepreneurs			
		1	2	3	≥ 4
All Firms	Owners	51.4%	39.3%	4.5%	4.8%
	Entrepreneurs	79.8%	18.0%	1.6%	0.1%
Employer Firms	Owners	43.0%	42.5%	7.1%	7.3%
	Entrepreneurs	73.7%	22.7%	2.7%	0.1%
Wgt by Size	Owners	28.8%	37.3%	11.6%	22.3%
	Entrepreneurs	63.7%	22.0%	6.5%	2.7%

SOURCE: SBO-PUMS 2007.
NOTES: Entrepreneurs are defined as (i) self-employed, (ii) business owners, who (iii) actively manage their firm. + Employment > 0 also requires that (iv) the firm has a positive number of employees. Other type of acquisition groups: acquired as a transfer, as a gift or other not specified. We use employment as the measure of size.

Table A.6: Franchises

	Entrepreneurs	+ Employment > 0
Baseline	17.0%	25.5%
W/o franchises	16.1%	24.1%
Franchises only	50.1%	51.8%
Franchises % of total	2.8%	4.8%

SOURCE: SBO-PUMS.

B Additional Derivations

B.1 Private firms profit maximization

The solution of entrepreneurs' profit maximization problem, stated in (1), is characterized by the input demand functions

$$k(a,z) = \min\left\{\hat{k}(z),\ \lambda a\right\}$$
$$l(a,z) = \left[\frac{pz\nu}{w}\right]^{\frac{1}{1-\nu}} k(a,z)^{\frac{\theta}{1-\nu}},$$

where $\hat{k}$ is the unconstrained optimal level of capital given by

$$\hat{k}(z) = (pz)^{\frac{1}{1-\theta-\nu}} \left[\frac{\theta}{R}\right]^{\frac{1-\nu}{1-\theta-\nu}} \left[\frac{\nu}{w}\right]^{\frac{\nu}{1-\theta-\nu}}$$

which is only a function of the quality of the entrepreneurial project z.

B.2 Public firm optimality conditions

The FOCs of the public firm profit maximization problem are

$$\eta\frac{Y_c}{K_c} = R$$
$$(1-\eta)\frac{Y_c}{L_c} = w$$

which imply a relation between the public firm capital to output and the equilibrium prices.

C Computational Appendix

To solve the model we use projection methods to approximate the value functions $\{V^o, W^o, V^w, W^w\}$. Thus, we need to solve for coefficients $\{g_V^o, g_W^o, g_V^w, g_W^w\}$ such that, at the grid points, satisfy

$$\begin{aligned} V^o(a,z) &= \Phi^z(a,z) g_V^o \\ W^o(a,z) &= \Phi^z(a,z) g_W^o \\ V^w(a,\varepsilon) &= \Phi^\varepsilon(a,\varepsilon) g_V^w \\ W^w(a,\varepsilon) &= \Phi^\varepsilon(a,\varepsilon) g_W^w. \end{aligned}$$

Note that the FOCs of the public firm give us a relation between K_c/Y_c, w and r. Both K_c and L_c are determined as residuals from the market clearing conditions of capital and labor, thus we can obtain w as a function of r. This simplifies the solution method as we don't need to solve for r. For the baseline model we set K_c/Y_c equal to 3, which pin downs r, and vary β such that markets clear.

C.1 Algorithm

The equilibrium objects we need to solve for are

$$\{g_V^o, g_W^o, g_V^w, g_W^w, n_{dm}^o, n_{dm}^w, n_{cm}^o, n_{cm}^e, P_{dm}^o, P_{dm}^w, P_{cm}^o, P_{cm}^w, \beta\}$$

where n are the probability densities across states and P are the transition probability matrices (TPMs) across states.[24] We solve for these objects using the algorithm now described. In the remaining sections we explain in further detail how some steps of the algorithm are implemented.

Iteration on prices

0. Propose an initial guess for β.

1. Given β, solve the model (in partial equilibrium).

   Iteration on distributions

   1.0. Propose an initial guess for $\{n_{dm}^o, n_{dm}^w\}$.

   1.1. Given $\{n_{dm}^o, n_{dm}^w\}$, solve for $\{g_W^o, g_W^w\}$.

      Iteration on value functions

      1.1.0. Propose an initial guess for $\{g_W^o, g_W^w\}$.

      1.1.1. Solve the DM problem: get $\{g_V^o, g_V^w\}$.

      1.1.2. Solve the CM problem: obtain e, a' and P_{cm}.

      1.1.3. Update $\{g_W^o, g_W^w\}$.

      1.1.4. Iterate $\{g_W^o, g_W^w\}$ until convergence.

   1.2. Update $\{n_{dm}^o, n_{dm}^w\}$.

   1.3. Iterate $\{n_{dm}^o, n_{dm}^w\}$ until convergence.

2. Update β using the bisection method on the labor market clearing condition.[25]

3. Iterate β until markets clear.

[24] Where $\int n^o(a,z)\mathrm{d}a\mathrm{d}z = s^o$ and $\int n^w(a,\varepsilon)\mathrm{d}a\mathrm{d}\varepsilon = (1-s^o)$.

[25] K_c is determined as a residual of capital market clearing. Thus β adjusts such that the labor market clears.

C.2 Computing expectations

Besides the above approximations, it is convenient to approximate the expectations over V^o and V^w such that these are functions of the coefficients g_V^o and g_V^w. Note that when the $(1-\gamma)$, or the $(1-\zeta)$, shock hit the expectation over the value of being a firm owner is not a function of previous period z, if any. Thus, we can approximate

$$\mathbb{E}_{z'}\left[V^o\left(a,z'\right)\right] = \Phi^a(a) g_V^{o,E}$$

Then, note that

$$\begin{aligned}\Phi^a(a) g_V^{o,E} &= \mathbb{E}_{z'}\left[V^o(a,z')\right] \\ &= \sum_i \omega_i^z V^o(a,z_i) \\ &= \left[\sum_i \omega_i^z \Phi^z\left(a,z_i\right)\right] g_V^o\end{aligned}$$

where $\{\omega_i\}_i$ are weights that discretize the exogenous process for z'.

This implies that, given g_V^o, the coefficient for the expectation is just

$$\begin{aligned}g_V^{o,E} &= \Phi^a(a)^{-1}\left[\sum_i \omega_i^z \Phi^z\left(a,z_i\right)\right) g_V^o \\ &\equiv \Phi^{z,E} g_V^o\end{aligned}$$

and hence

$$\mathbb{E}_{z'}\left[V^o(a,z')\right] = \Phi^a(a)\Phi^{z,E} g_V^z,$$

where $\Phi^{z,E}$ is computed only once.

For the expectation over the value of being a worker at DM we can do similar steps but now accounting for the persistence in ε:

$$\mathbb{E}_{\varepsilon'|\varepsilon}\left[V^w\left(a,\varepsilon'\right)\right] = \Phi^\varepsilon(a,\varepsilon) g_V^{w,E}$$

where a is the policy chosen at CM (i.e. a'), and ε is the current state at CM.

As before

$$\begin{aligned}\Phi^\varepsilon(a,\varepsilon) g_V^{w,E} &= \mathbb{E}_{\varepsilon'|\varepsilon}\left[V^w(a,\varepsilon')\right] \\ &= \sum_i \omega_i^\varepsilon V^w(a, f(\varepsilon,u_i)) \\ &= \left[\sum_i \omega_i^\varepsilon \Phi^\varepsilon\left(a, f(\varepsilon,u_i)\right)\right] g_V^w\end{aligned}$$

where $\{\omega_i^\varepsilon, u_i\}_i$ are weights and nodes that discretize the exogenous process for ε'. Thus, $\varepsilon_i' = f(\varepsilon,u_i)$, given current period ε.

This implies that, given g_V^w, the coefficient for the expectation is just

$$\begin{aligned}g_V^{w,E} &= \Phi^\varepsilon(a,\varepsilon)^{-1}\left[\sum_i \omega_i^\varepsilon \Phi^\varepsilon\left(a, f(\varepsilon,u_i)\right)\right] g_V^w \\ &\equiv \Phi^{\varepsilon,E} g_V^w\end{aligned}$$

and hence

$$\mathbb{E}_{\varepsilon'|\varepsilon}\left[V^w(a,\varepsilon')\right] = \Phi^\varepsilon(a,\varepsilon)\Phi^{\varepsilon,E} g_V^w,$$

where $\Phi^{\varepsilon,E}$ is computed only once.

C.3 Solving for g_V^o and g_V^w

Given a $\{n_{dm}^o, n_{dm}^w, g_W^o, g_W^w\}$, we can compute the value at DM for both firm owners and workers. Then we can solve for g_V^o and g_V^w by inverting the basis functions Φ^z and Φ^ε.

C.4 Solving for a', g_W^o and g_W^w

Having solved for the coefficients g_V^o and g_V^w we can solve the households' problems in the production stage (CM). Given prices, both entrepreneurs and workers problems are a single variable optimization problem in a', which we can solve using golden search or Brent's method.

To obtain g_W^o and g_W^w we use value function iteration. First, by substituting the corresponding optimal policies we obtain two linear systems of equations on g_W^o and g_W^w. Then, we can solve for the coefficients by just inverting the basis functions. For stability reasons we make the update of g_W^o and g_W^w with some dampening.

C.5 Transitions and Stationary Distribution

Define the densities across states in DM and CM subperiods as

$$n_{dm} = \begin{bmatrix} n_{dm}^o \\ n_{dm}^w \end{bmatrix} \text{ and } n_{cm} = \begin{bmatrix} n_{cm}^o \\ n_{cm}^w \end{bmatrix}$$

where n_{dm}^o and n_{cm}^o are vectors of size N_o and n_{dm}^w and n_{cm}^w are vectors of size N_w. N_o and N_w are the basis functions grid sizes denoting the number of (a, z) and (a, ε) combinations, respectively. Here $\sum_i n_{dm} = 1$, thus, $\sum_i n_{dm}^o = s_{dm}^o$ and $\sum_i n_{dm}^w = (1 - s_{dm}^z)$.

Then, the TPMs between DM and CM and CM and DM_{+1} solve

$$(n_{cm})^\intercal = (n_{dm})^\intercal P_{dm}$$
$$(n'_{dm})^\intercal = (n_{cm})^\intercal P_{cm}$$

where $(.)^\intercal$ denotes the transpose operator.

We can divide the TPM in blocks differentiating between the two type of agents:

$$P_{dm} = \begin{bmatrix} P_{dm}^{oo} & P_{dm}^{ow} \\ P_{dm}^{wo} & P_{dm}^{ww} \end{bmatrix} \text{ and } P_{cm} = \begin{bmatrix} P_{cm}^{oo} & P_{cm}^{ow} \\ P_{cm}^{wo} & P_{cm}^{ww} \end{bmatrix}$$

where P_{dm}^{oo} captures the transitions of firms' owners that bought another firm or didn't trade, P_{dm}^{ow} is for owners that sold their firm, P_{dm}^{wo} for workers who bought a firm and P_{dm}^{ww} for workers who didn't trade. Regarding CM TPMs, P_{cm}^{oo} is for business owners who operated the firm, P_{cm}^{ow} for owners who didn't operate and went to the labor market, P_{cm}^{wo} for workers who received the $(1 - \zeta)$ shock, P_{cm}^{ww} for workers that didn't. Note that besides changes in the exogenous shocks, asset holdings also change due to payments in the market for firms and due to savings in CM.

Stationarity requires that

$$n_{dm}^\intercal = n_{dm}^\intercal P_{dm} P_{cm}$$

or

$$\left[I - (P_{dm} P_{cm})^\intercal\right] n_{dm} = 0$$

which implies that we can solve for n_{dm} by computing the eigenvector of $(P_{dm} P_{cm})^\intercal$ associated with the unit eigenvalue, normalized such that $\sum_i n_{dm}(i) = 1$.