

FIRMS' ROLLOVER RISK AND MACROECONOMIC DYNAMICS

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Abstract

I analyze the aggregate implications of firms' rollover risk. I develop a heterogeneous firm macroeconomic model with rollover risk. Debt rollover problems arise when firms cannot repay outstanding debt because of coordination failures among their creditors. I assess the quantitative relevance of rollover problems by employing a model-based identification strategy which argues that the incidence of rollover problems is informed by the observed distribution of firms' bankruptcy outcomes. I find that around two thirds of bankruptcy events (liquidations and reorganizations) are due to rollover problems. I then use the model to assess the macroeconomic implications of rollover risk for the U.S. economy. I find that the presence of rollover problems can significantly amplify the impact of crises. Lastly, I show that imperfectly targeted credit policies deployed to mitigate rollover problems have a trade-off between less creditors' coordination failures in the short term and greater debt overhang in the future.

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1 INTRODUCTION

In this article, I study the incidence of firm's rollover problems on macroeconomic dynamics. Firms with positive net present value (or economically solvent) can be subject to debt rollover problems that lead to their failure. For policy-makers, firms' rollover problems are a concern, especially in crises; therefore, they shape various regulations and credit policies.¹ Moreover, entrepreneurs and managers of bankrupt firms frequently cite rollover problems as their main financial problem.² In addition, empirical evidence suggests that, during recent crises episodes, nonfinancial firms with higher rollover risk reduced their investment significantly more.³ Although firms' debt rollover problems are considered a relevant (and damaging) cause of firms' failure, we know little about their importance for aggregate outcomes. Especially because firm rollover problems are hard to disentangle from solvency problems.

In this paper, I develop a quantitative macroeconomic framework where nonfinancial firms' rollover problems can be identified and quantified. Debt rollover problems arise when a firm cannot repay outstanding debt because of *coordination failures* among its creditors. Using the framework, I conduct a quantitative analysis of the U.S. economy to answer how relevant are firms' rollover problems, and what are their macroeconomic and policy implications.

In the model, firms maximize their value — i.e., present discounted value of dividends — by making capital investments that can be financed using internal resources (firm's cash-on-hand) or external resources (issuing new debt). The firms' debt is purchased and priced by atomistic and perfectly competitive creditors, and the firms buy capital from a representative capital producer that faces aggregate capital adjustment

¹See, for example, the article [Financial Times \(09/30/2014\)](#) describes the discussion around the solvency and debt rollover problems of Lehman Brothers and other financial institutions during the 2008 crisis. More recently, in the Covid crisis, the Federal Reserve intervention in the corporate debt markets was limited to solvent but "iliquid" nonfinancial firms by Section 13(3) of the Dodd Frank act. Federal Reserve's Chair Jerome Powell, in April 29, 2020, press conference ([link to transcript](#)) explained the limits on the Fed's lending powers as follows: "We can only make loans to solvent entities [...] with the expectation that the loans will be repaid.[...] we can't lend it to insolvent companies. [...] We do not make grants. We can't make grants."

²[Ayotte and Skeel \(2013\)](#), and references within, observe that in the bankruptcy process is usual that the bankrupt firm and/or the judge of the case allude to financial problems akin to rollover problems. For example, when Kodak filed for bankruptcy, its lead bankruptcy lawyer in the first day of the case claimed "[We're here for liquidity](#)". In other example, commentators observe that, during the Covid crisis, Hertz seeked bankruptcy protection due to a combination of transitorily low cashflows (due to the pandemic) and the inability to rollover its maturing debt.

³This is documented by [Almeida, Campello, Laranjeira and Weisbenner \(2012\)](#) for the Great Recession in U.S. and [Kalemli-Özcan, Laeven and Moreno \(2020\)](#) for the European crisis in 2011.

costs. All firms in the economy are owned by households, who work, consume and save.

There are three key ingredients. First, firms have no commitment to repay their debt; thus, default risk limits their borrowing capacity. Second, using tools from the literature that studies self-fulfilling sovereign debt crises [see, for example, [Cole and Kehoe \(2000\)](#)], I incorporate potential coordination problems among firms' creditors — rollover problems. Third, emulating the U.S. bankruptcy code, firms can decide to liquidate and exit, or restructure its liabilities and continue operating.

Firms can decide to be liquidated *after* issuing new debt, which creates complementarities among creditors and opens the possibility of multiple equilibrium. It follows, there are three types of firms according to their current financial position. First, a firm is solvent and not exposed to rollover problems if it is not liquidated independently from creditors' joint conjectures (coordination). Second, a firm is insolvent if it is liquidated independently from creditors' coordination. Third, a firm is solvent and exposed to rollover problems if it is liquidated whenever its creditors' jointly conjecture the firm is liquidated today, but it is not liquidated otherwise. To solve for multiplicity, I assume that (randomly) each period a given share of exposed firms have a rollover problem — idiosyncratic sunspot equilibrium selection.

Moreover, firms can decide to restructure (frequently called "reorganize"). The restructure process has costs and benefits for firms. I assume firms pay an exogenous bankruptcy cost, which captures typical costs observed in the restructure process such as administrative costs, legal fees and reputational deterioration. On the other hand, in the model, firms benefit from lower debt burden and absence of creditors' coordination failures in the restructure process. The lack of coordination failures in the restructure process captures a fundamental principle in the bankruptcy law literature [see, for example, [Jackson \(1986\)](#) for an early reference], which observes that bankruptcy provisions are (and should be) designed to preclude coordination problems among creditors.⁴ For example, to preclude coordination failures for bankrupt firms, provisions in the Chapter 11 of the U.S. bankruptcy code — empirical counterpart of the restructure process in the model — stop creditors from collecting debts (automatic stay), facilitate the issuance of new debt (Debtor-In-Possession financing), and create official and ad

⁴In this literature these problems are frequently called "common pool problem", which are akin to the coordination problems in the model.

hoc committees to facilitate creditors' coordination.

I indirectly infer the incidence of rollover problems using the financial distribution of firms and the distribution of bankruptcy outcomes. The identification strategy follows several steps. First, I calibrate the model parameters unrelated to the bankruptcy process to match salient features of the U.S. economy. Next, I set the parameters related to the bankruptcy process to match the average debt haircut and leverage in the restructure and liquidation processes. The debt haircut in the restructure process is on average low (less than 30%) and the (implicit) bankruptcy costs are high; thus, few insolvent firms choose to restructure (these are less than 10% of the firms that restructure in the baseline calibration). Therefore, I find that the share of firms that restructure their liabilities — relative to those that are liquidated — identifies very well the probability of coordination failures for exposed firms. Lastly, from the stationary distribution of the model I infer how many firms are exposed to rollover problems.

I find that 1.6% of the firms are subject to rollover problems, where 21% of the firms are exposed and the likelihood of a coordination failure is 7%. Therefore, around two thirds of bankruptcy events are driven by rollover problems. Moreover, I find that the model matches how well firm's characteristics predict a bankruptcy event and matches the bankrupt firms' leverage distribution, which are not targeted.

Using the quantitative model calibrated to the U.S. economy, I study the role of firms' rollover problems during crises. I simulate a prototypical crisis — i.e., an unforeseen transitory shock that decreases 5% aggregate output from peak-to-trough — and study the transition of aggregate output with and without coordination failures. For different types of shocks driving the crises — total factor productivity (TFP), cash and credit shocks — I find that rollover risk can significantly amplify the impact of crises, explaining around one fifth of output losses during the episode.⁵ Moreover, rollover risk makes crises more persistent, especially when crises are driven by aggregate TFP or cash shocks. Weaker fundamentals — a sudden reduction in cash-flows — expose (temporarily) more firms to rollover problems, leading to more failures (bankruptcy and liquidation) of healthy firms, which creates the extra persistence.

Next, I simulate a panel of firms and estimate — doing a difference-in-difference regression analysis — the heterogeneity of investment responses across firms during

⁵In my experiments a credit shock is a reduction in the recovery rate of creditors during a firm's liquidation event.

crises. I contrast the model's panel regression estimates with empirical estimates for the Great Recession and Covid crisis in US. The empirical and model simulated data, for both episodes, show that firms with lower levels of cash-on-hand adjust their investment significantly more.

Finally, I study the policy implications of rollover problems. I focus on direct lending policies, which resemble those deployed by the Fed during the Covid crisis.⁶ Although my model does not feature a lockdown shock, I study the effectiveness of these unconventional credit policies in more standard types of crises. I show that direct lending policies can work "in" and "outside" equilibrium. If a firm is exposed to a rollover problem, the credit facilities from the government could preclude the rollover failure by acting as a form of insurance to creditors and, ultimately, coordinating creditors in the repayment equilibrium. On the other hand, I assume that the government policy has imperfect targeting (i.e., the government can't target some relevant characteristics of individual firms); thus, there is adverse selection and some firms receive a subsidized credit.

In my quantitative exercise, I simulate the credit policies in the crisis experiments, and find that policies where most of the action happens outside equilibrium and credit facilities remain mostly unused in equilibrium are the most potent in reducing the short term impact of rollover problems and providing a swift recovery. On the contrary, more ample credit programs, while they mitigate more coordination failures and have greater short term benefits, they subsidize credit to many firms and create future debt overhang problems that extend the crises.

LITERATURE. The paper fits in the broad research agenda described by [Brunnermeier and Krishnamurthy \(2020\)](#). This research agenda aims to incorporate firm-level corporate financing considerations in quantitative macroeconomic models to study their aggregate (positive and normative) consequences. Furthermore, my work identifies the incidence of the financial friction studied (debt rollover problems) using distinctive features of the U.S. bankruptcy code and firms' bankruptcy choices. My paper's main contributions can be placed in the following strands of literature in macroeconomics and finance:

⁶The Fed provided credit to corporate firms through the Primary Market Corporate Credit Facility (PMCCF) and Secondary Market Corporate Credit Facility (SMCCF).

Financial heterogeneity, default risk and business cycles. The framework, in this paper, is built over a flexible price version of [Ottonello and Winberry \(2020\)](#) model, which is well suited for quantitative studies of firms' responses to aggregate shocks. In their paper, they focus on financial heterogeneity and the investment response of firms to monetary policy shocks. Moreover, my paper, is related to [Cooley, Marimon and Quadrini \(2004\)](#); [Arellano, Bai and Kehoe \(2019\)](#); [Khan, Seng and Thomas \(2020\)](#), where they study the implications of aggregate shocks in models of heterogeneous firms with default risk. I contribute to this literature by extending this framework to incorporate rollover risk — driven by creditors' coordination failures — and a debt restructure procedure for firms (i.e., default without liquidation).

Rollover problems and multiple equilibrium in macroeconomics. There is an ample literature that studies rollover problems and multiple equilibrium in macroeconomics. My paper is closely connected to the work on sovereign debt self-fulfilling crises. In my model, the [Cole and Kehoe \(2000\)](#) timing creates rollover risk through creditors' coordination failures. Moreover, in the same spirit as [Bocola and Dovis \(2019\)](#), my paper tries to identify and quantify indirectly the incidence of rollover problems. My paper is also connected to the literature on bank runs, which seminal work is by [Diamond and Dybvig \(1983\)](#). I use a similar timing convention to [Gertler and Kiyotaki \(2015\)](#) and [Gertler, Kiyotaki and Prestipino \(2019\)](#), which introduce a bank run problem in a DSGE model with banks. The contribution of my paper to the literature, on rollover problems and multiple equilibrium in macroeconomics, is to do a quantitative assessment of nonfinancial firms' firm-level rollover problems using an identification strategy that hinges on cross-sectional moments of firms and salient features of the U.S. bankruptcy code.

Firm heterogeneity and bankruptcy in macroeconomics. [Corbae and D'Erasmus \(2021\)](#) incorporate a realistic bankruptcy procedure in a general equilibrium model with heterogeneous firms. Their goal is to study how changes in the bankruptcy process could impact aggregate outcomes in the long run. Different from their paper — inspired by observations in the bankruptcy law literature (for example, [Jackson \(1986\)](#) or more recently [Ayotte and Skeel \(2013\)](#)) — in my model the restructure process, Chapter 11 in the U.S. bankruptcy code, works also as a coordination device for creditors and precludes rollover problems once firms are in the bankruptcy procedure.

Rollover (coordination) problems in corporate finance theory. Salient examples are [Morris and Shin \(2004, 2016\)](#) that adopt a global games approach to study the relation be-

tween firm's rollover problems (coordination) and corporate debt pricing. In related work, [He and Xiong \(2012a,b\)](#); [Cheng and Milbradt \(2012\)](#) study the relation between rollover problems and corporate debt maturity, and [Zhong \(2021\)](#) studies the relation of coordination failures with creditor's concentration. I contribute to this literature by introducing a simple coordination problem that requires less structure and study the interaction of rollover problems with bankruptcy provisions which could facilitate creditors' coordination.

Corporate credit policy intervention and crises. Sparked by the Covid crisis — and the policy response that followed — some papers have studied the effectiveness of corporate credit policies during crises using structural models. [Ebsim, Faria e Castro and Kozłowski \(2021\)](#) find that the effectiveness of credit policies (akin to those implemented during Covid) depends on the source of the shock. [Elenev, Landvoigt and Van Nieuwerburgh \(2021\)](#) study similar credit policies and find that they are effective at preventing firm bankruptcies. [Crouzet and Tourre \(2021\)](#) find that credit policies in crises could backfire because of debt overhang problems in the future. My paper finds similar results, credit policies prevent bankruptcy events and mitigate their risk, but backfire if they exacerbate future debt overhang problems. The distinctive contribution of my paper, to this strand of literature, is that credit policy works also through a coordination channel, akin to the deposit insurance for bank runs [see, for example, [Diamond and Dybvig \(1983\)](#)]. Thus, credit policies in crises may be potent even if few firms participate in the credit program. This last result is related to observations made by [Cox, Greenwald and Ludvigson \(2021\)](#) regarding the workings of the Fed's corporate credit facilities during Covid.⁷

Investment response heterogeneity during crises (empirical). A recent literature studies empirically the heterogeneity of investment responses across firm's financial positions during recent crises episodes. [Almeida et al. \(2012\)](#) explore the relevance of long term debt that matured in the short term during the Great Recession in U.S. and find that firms with more long term debt maturing in the short term reduced more their investment. Moreover, [Kalemli-Özcan et al. \(2020\)](#) find evidence of debt overhang problems and rollover problems being relevant during the EU crisis. These patterns were greater in peripheral europe countries, which were hardly hit by the crisis. In addition, [Eb-](#)

⁷[Cox et al. \(2021\)](#) observe limited participation in Federal Reserve's Primary and Secondary Market Corporate Credit Facility programs, and argue that credit policy (announcements) affected asset prices through nonfundamentals (analogous to the workings of the credit policy in my paper).

[sim et al. \(2021\)](#) show that cash holdings were relevant explaining the heterogeneity of credit spreads dynamics during the Covid crisis, but they weren't relevant during the Great Recession. My paper contribute to this literature by showing that firms with lower internal resources (cash-on-hand) adjusted their investment more during the Great Recession and Covid crisis, and, on the contrary, I find no evidence of significant heterogeneity across leverage position in both episodes.

PAPER'S ORGANIZATION. The paper is organized as follows: Section 2 develops a macroeconomic model where firms can be subject to rollover problems; Section 3 explains the identification strategy and calibration of the model; Section 4 quantifies the consequences of rollover risk during crises and provides evidence of investment heterogeneity during recent crises episodes; Section 5 studies the effectiveness of credit policies, during crises, in the presence of rollover risk; and Section 6 concludes. Lastly, the Appendix contains further details on the theory, data, other exercises and extensions.

2 A MACRO MODEL OF FIRMS' ROLLOVER RISK

In this section I describe the theoretical framework used to to identify the incidence of firms' debt rollover problems and conduct the baseline quantitative exercises. I develop a quantitative macroeconomic model of heterogeneous firms with default and rollover risk. The framework has three key ingredients. First, firms lack repay commitment; therefore, they are subject to default risk. Second, firms can be exposed to coordination failures among creditors á la [Cole and Kehoe \(2000\)](#); therefore, they can be subject to rollover risk. Third, bankrupt firms are allowed to liquidate and exit, or restructure their liabilities and continue operating, as in [Corbae and D'Erasmus \(2021\)](#).

To describe the model, I follow several steps: Section 2.1 is an overview of the environment; Section 2.2 describes the nonfinancial firms' setup; Section 2.3 shows how creditors determine debt prices given the choices of the firm; Section 2.4 characterizes nonfinancial firms' bankruptcy (liquidation and restructure) choices, which depend on debt prices; Section 2.5 shows the nonfinancial firms' recursive problem formulation; Section 2.6 briefly describes the rest of the agents: capital producers and households; and Section 2.7 defines the equilibrium for this economy.

2.1 Environment

The economy has an infinite horizon and is in discrete time, i.e., $t = 0, 1, 2, \dots$. It is inhabited by four types of agents: (i) nonfinancial heterogeneous firms that invest, produce and do financial choices in order to maximize the present value of their dividends (i.e., firm value); (ii) atomistic and perfectly competitive creditors that lend to nonfinancial firms; (iii) a representative capital producer that sells capital to nonfinancial firms; and (iv) a representative household that consumes, saves and works, and owns all the firms in the economy. The price of the final good is normalized to 1, and the price of capital good is q and wages w are determined in general equilibrium. I will assume there is no aggregate risk.

2.2 Nonfinancial Firms Setup

Firm i objective is to maximize its value $V_{it} = \mathbb{E}_t [\sum_{s \geq t} \Lambda_s d_{is}]$ where Λ_s is the stochastic discount factor of the households and d_{is} is the dividends issued by firm i at period s . The firm has three types of idiosyncratic state variables: (i) exogenous fundamental state variables s_{it}^f ; (ii) exogenous nonfundamental state variable s_{it}^n ; and (iii) endogenous state variables s_{it}^e . Therefore, the idiosyncratic state variable is defined as $s = (s_{it}^f, s_{it}^n, s_{it}^e)$. Firms are perfectly competitive and there is a continuum of them producing each period with a distribution $\Omega(\cdot)$, which is normalized to $\int d\Omega(\cdot) = 1$.

There is no aggregate risk and the firm's problem can be written recursively (shown later), thus for clarity of exposition I will drop subscripts for firm i and period t , and adopt the recursive timing convention.

Technology and operational profits. Firms combine capital k and labor l to produce a unique final good using a Cobb-Douglas production function

$$y = f(z, \omega, k, l) = z(\omega k)^\alpha l^\nu,$$

where $\alpha \in (0, 1)$ is the share of capital and ν is the share of labor. I assume the firm operates with decreasing returns to scale $\alpha + \nu < 1$. The firm is subject to two idiosyncratic shocks: (i) a persistent idiosyncratic productivity process $\ln z' = \rho \ln z + \epsilon_z$ with $\epsilon_z \sim^{\text{iid}} (0, \sigma_z^2)$; and (ii) idiosyncratic iid capital quality shock ω , which is drawn from a log-normal truncated distribution where $\ln \omega \in [\underline{\omega}, 0]$. The sole purpose of including

the ω shock is to match quantitatively the default rates observed in the data.

Firms own capital k , which is inherited from the previous period, and hire labor l at given wage w . The labor choice of firms is static, then I define the operating profits function as

$$\pi(z, \omega, k) = \max_l z(\omega k)^\alpha l^\nu - wl \quad (1)$$

with labor demand of firms $l = \left[\frac{\nu z(\omega k)^\alpha}{w} \right]^{\frac{1}{1-\nu}}$.

Resources. Each period firms can raise external resources by issuing one-period debt b' given price schedule $Q(\cdot)$ offered by creditors. Further, firms internal resources are cash-on-hand n , which is the sum of operational profits $\pi(z, \omega, k)$ and current value of owned capital after depreciation $(1 - \delta)q\omega k$ — where $\delta \in [0, 1]$ is the capital depreciation rate and q the price of capital — minus the maturing inherited debt b , i.e.,

$$n = \pi(z, \omega, k) + (1 - \delta)q\omega k - b. \quad (2)$$

External and own resources are used to issue dividends d and make capital purchases, i.e.,

$$d + qk' = n + Q(\cdot)b'. \quad (3)$$

As I will show later the financial structure of the firm is going to matter in the presence of financial frictions.

Exit and entry. The mass of entrant firms $\bar{\mu}$ equates the mass of exiting firms in steady-state. I will assume that entrants are endowed with a capital $k = k_0$ and debt $b = 0$, and draw their initial productivity level z from an invariant distribution $\Omega^e(z)$ with an average productivity lower than the stationary distribution average by $m \leq 0$ percent. This assumption is consistent with evidence that young firms have lower measured productivity, as pointed out by [Ottonello and Winberry \(2020\)](#), and is useful to match the firms' life-cycle moments.

Apart from the liquidation choice (explained next), following [Khan et al. \(2020\)](#), firms at the beginning of each period receive an exogenous exit shock with probability $\gamma \in [0, 1]$ which force them to exit after production. This assumption prevents that all firms overcome the financial frictions in steady-state. Moreover, exiting firms can still decide

to liquidate or restructure their inherited debt.

Financial frictions. There are two forms of firm-level financial frictions.

First, firms are precluded from issuing equity, i.e.,

$$d \geq 0. \tag{4}$$

This assumption is standard in the literature, is consistent with the scarce issuance of equity by corporate firms in the data and provides greater tractability to the model. In Appendix C.3, I extend the model to allow for costly equity issuance and show how the characterization of the firm's liquidation choice changes.

Second, firms' debt is defaultable. Each period, firms decide to repay or not their debt. I assume firms can default their debt in two ways:

1. *Liquidate* the firm and exit. In this case, all debt is defaulted — inherited b and new issuance b' — and firms exit with value $V = 0$. Upon liquidation, I will assume creditors of b recover fraction $R(b, k, \omega) = \min \left\{ 1, \alpha_7 \frac{q\omega k}{b} \right\}$ of b , where $\alpha_7 \in [0, 1]$ is the parameter that indicates the recovery rate of capital. Further, I assume that creditors of b' don't recover anything from the current liquidation. On the other hand, they will recover something if the firm is liquidated tomorrow. This last assumption is done for technical reasons and to capture the fact that inherited creditors tend to have seniority over new creditors. The empirical counterpart of the liquidation process is the *Chapter 7* liquidation and *Chapter 11* piecemeal liquidations of the U.S. Bankruptcy Code. For notational simplicity, I define piecemeal liquidations in Chapter 11 as an equivalent process to the Chapter 7 liquidation.
2. *Restructure* the firm's debt b and continue operating. In this case, the creditors and the firm bargain on debt recovery rate $\alpha_{11} \in [0, 1]$ of inherited debt b through a Nash Bargaining protocol where the outside option is to continue. The assumption, regarding the outside option, rules out cases where the firm enters the bankruptcy process as if it is "threatening" creditors, we can interpret this as cases where "the judge" dismisses the bankruptcy filing. Moreover, we need that the firm and its creditors are willing to participate of the negotiations; thus, the bankruptcy decision can be interpreted as a joint decision. Additionally, firms

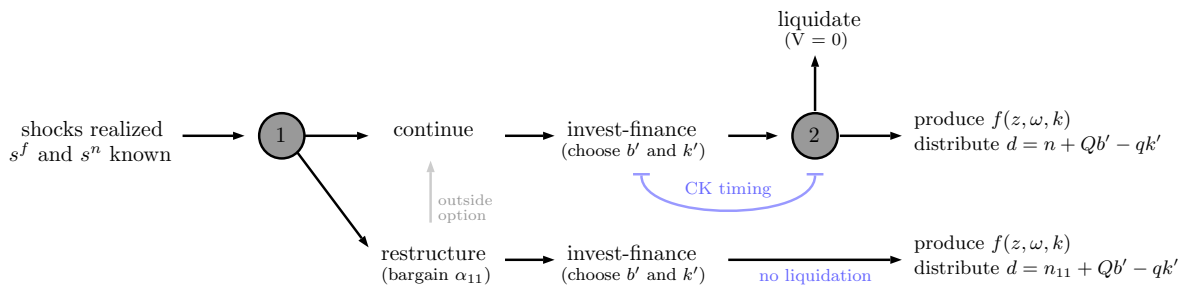
pay $c \in [0, 1]$ costs proportional to capital that captures the usual bankruptcy losses (e.g., legal fees, administrative costs, reputational deterioration). Lastly, in the restructure process *coordination failures are precluded*. The resources when restructuring are $n_{11} = \pi(z, \omega, k) + (1 - c_{11})(1 - \delta)q\omega k - \alpha_{11}b$. In Section 2.4, I provide a thorough discussion of the assumptions related the restructure process. The empirical counterpart of the restructure process is the *Chapter 11* restructure outcomes (excludes Chapter 11 "liquidations") of the U.S. Bankruptcy Code.

In Appendix D, I provide further institutional details of the Chapter 7 and Chapter 11 of the U.S. bankruptcy code.

Timing. Figure 1, shows the within period timing of the firm problem for firms that are not subject to the exogenous exit shock. At the beginning of the period idiosyncratic states s are realized, i.e., the fundamental and nonfundamental shocks are revealed. After uncertainty is resolved, there is no more within period uncertainty since shocks and states (also nonfundamental) are known, and they are common knowledge for all agents. The first gray dot indicates the restructuring choice where firms choose to either continue or restructure.

If the firm decides to continue, next, it makes the investment and financing choice — i.e., choose (k', b') given pricing schedule $Q(\cdot)$ and price q . After the firm issues the new debt b' the firm makes the liquidation choice (i.e., second gray dot), where it decides to liquidate and exit, or continue and produce. The fact that the liquidation choice takes place after issuing the new debt will be the source of equilibrium multiplicity. This timing is the well-known [Cole and Kehoe \(2000\)](#) timing in the international macroeconomics literature.

Figure 1: Within period timing



Note: timing is conditioned on a firm that doesn't receive an exit shock. In Appendix A.1 I describe and characterize the exiting firm's problem.

If the firm decides to restructure its liabilities then it enters the bargaining process for α_{11} — with outside option to continue (gray arrow up). Under the restructuring process there are no current coordination failures since there is no liquidation choice after issuing new debt, i.e., visually there is no gray dot after the entering the restructure process.

2.3 Creditors and the Debt Pricing Schedule

To characterize the liquidation and restructure choice is important to determine the debt pricing schedule. Creditors borrow from household at the risk free rate and lend to the firms. They are perfectly competitive and atomistic; thus, the no-profit condition holds (i.e., equalize expected returns) and prices the debt. All intermediaries are owned by the household, then they discount future flows using stochastic discount factor Λ (defined in household problem in Section 2.6). Thus, the price of debt is determined according to

$$\begin{aligned} Q(s, k', b') = & \left[1 - \mathbf{1}_{\{\text{ch7}\}}(s)\right] \mathbb{E}_{(s'|s)} \left[\Lambda (1 - \gamma) \mathbf{1}_{\{\text{continue}\}}(s') \right] \\ & + \left[1 - \mathbf{1}_{\{\text{ch7}\}}(s)\right] \mathbb{E}_{(s'|s)} \left[\Lambda (1 - \gamma) \mathbf{1}_{\{\text{Ch11}\}}(s') \alpha_{11}(s') \right] \\ & + \left[1 - \mathbf{1}_{\{\text{ch7}\}}(s)\right] \mathbb{E}_{(s'|s)} \left[\Lambda (1 - \gamma) \mathbf{1}_{\{\text{Ch7}\}}(s') R(b', k', \omega') \right] \\ & + \gamma \left[1 - \mathbf{1}_{\{\text{ch7}\}}(s)\right] \tilde{Q}_{\text{exit}}(z, k', b'), \end{aligned} \quad (5)$$

with

$$\begin{aligned} \tilde{Q}_{\text{exit}}(z, k', b') = & \mathbb{E}_{(s'|s)} \left[\Lambda \left\{ \mathbf{1}_{\{\text{continue} | \text{exit}\}}(s') + \mathbf{1}_{\{\text{ch11} | \text{exit}\}}(s') \alpha_{11}^{\text{exit}} \right\} \right] \\ & + \mathbb{E}_{(s'|s)} \left[\Lambda \mathbf{1}_{\{\text{ch7} | \text{exit}\}}(s') R(\omega', b', k') \right], \end{aligned} \quad (6)$$

where $\mathbf{1}_{\{\text{Ch7}\}}(s)$ is the indicator of the liquidation choice (=1 if liquidate), $\mathbf{1}_{\{\text{Ch11}\}}(s)$ is the indicator of the restructure choice (= 1 if restructure) and the indicator $\mathbf{1}_{\{\text{continue}\}}(s)$ that indicates which firms continue is defined as $\mathbf{1}_{\{\text{continue}\}}(s) = 1 - \mathbf{1}_{\{\text{Ch11}\}}(s) - \mathbf{1}_{\{\text{Ch7}\}}(s)$ for firms that don't receive the exit shock.⁸ Further, the indicators condi-

⁸As a technical note, the liquidation choice doesn't depend in (k', b') because the policy function imposes a constraint redundant to $d \geq 0$ for the financing-investment choice, and creditors of b' have 0 recovery rate and there is no within period uncertainty after shocks are realized at the beginning of the period.

tioned on the firm receiving the exit shock, i.e. $1_{\{.\mid \text{exit}\}}$, are defined analogously. The liquidation and restructuring choices are characterized in Section 2.4 for firms that don't receive exit shock and in Appendix A.1 for firms that receive the exit shock. On the RHS of (5) the first line shows component of the price related to the firms that repay fully next period, the second line to the one related to the expectations of restructuring happening next period where they recover $\alpha_{11}(s')$, and the third corresponds to outcomes where the firm is liquidated next period then recover $R(b', k', \omega')$. The last line shows the pricing component related to exiting firms, and (6) shows the determinants of the debt price given no liquidation today and firm receiving the exit shock tomorrow, it is analogous to the pricing function for non exiting firms.

It is useful to define the fundamental price $\tilde{Q}(\cdot)$ as the price of debt whenever there is no contemporaneous liquidation, i.e.,

$$Q(s, k', b') = [1 - 1_{\{\text{ch7}\}}(s)] \tilde{Q}(z, b', k').$$

The contemporaneous liquidation decision shows up in the debt pricing because of the CK timing assumption — i.e., the liquidation choice happens after the new debt issuance — and this is the source of potential multiplicity as I explain next.

2.4 Nonfinancial Firms Liquidation Choice and Restructure Choice

In this subsection, I will characterize liquidation and restructure choices. I will start backwards according to Figure 1 timing, since the payoffs of the liquidation choice will affect the decisions in the restructure choice.

I show that the liquidation choice characterization provides a simple way to find out — in the model — which firms (across the state-space) have rollover or solvency problems.

On the other hand, either insolvent or solvent firms with rollover problems may find beneficial to enter the restructure process. Therefore, the relation between the restructure process and the incidence of rollover risk will depend on if the process works more as a mechanism to solve coordination failures or a mechanism to resurrect insolvent firms, i.e., how relevant are the costs and benefits of this process.

2.4.1 Liquidation Choice: Rollover vs Solvency

First, I show how the liquidation choice timing can create the possibility of multiple equilibrium. Next, I briefly discuss some of the assumptions necessary for coordination failures to happen and, lastly, I characterize the liquidation choice across the relevant state-space.

Multiple equilibrium intuition. Firms are subject to the no-equity issuance constraint $d \geq 0$ (feasibility) and their exiting value is 0, then it follows that firms liquidate if they can't issue (weakly) positive dividends.⁹ Creditors price the debt according to equation (5). The pricing schedule shows that firms price the debt by calculating the expectations of future default and making conjectures about the firm's liquidation choice today. The price of debt and dividends can be written as

$$Q(.) = \underbrace{\mathbf{1}_{d \geq 0}}_{\text{no liquidation choice}} \underbrace{\tilde{Q}(.)}_{\text{debt price if no liquidation}}$$

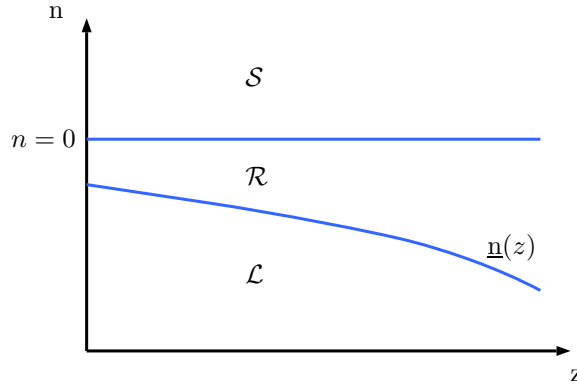
$$d = n - qk' + Q(.)b',$$

where $\mathbf{1}_{d \geq 0}$ is the today's no liquidation indicator — i.e., if $\mathbf{1}_{d \geq 0} = 0$ firm cannot satisfy $d \geq 0$ and is liquidated — and $\tilde{Q}(.)$ the pricing schedule without liquidation today. Under certain conditions there is a potential feedback between dividends and current debt prices, which could create multiple outcomes. To illustrate this, assume $n < 0$ then if creditors conjecture $\mathbf{1}_{d \geq 0} = 0$ today then $Q = 0$ and $d = n + \max_{k' \geq 0} \{-qk'\} < 0$ then the firm is liquidated. Thus, the outcome is consistent with the conjecture. On the other hand, if creditors conjecture that there is no liquidation today, then $Q = \tilde{Q}$. Moreover, if $\tilde{Q}(.)b' > n$ for some value of b' then the firm can satisfy $d \geq 0$ and doesn't liquidate. Thus, the outcome is consistent with the conjecture. Therefore, under certain conditions — that I show next — outcomes could depend on creditor's conjectures of the current liquidation choice and creditors could coordinate in either debt price. I adopt the convention that if creditors coordinate in $Q = 0$, then they coordinate in the liquidation outcome.

⁹This simple result is straightforward from the assumptions.

Characterization of the liquidation choice. Figure 2 shows that the *fundamental state-space* (z, n) can be divided in three regions.¹⁰ First, there is a Safe region \mathcal{S} where firms in this region don't liquidate even if creditors conjecture liquidation today. This means that if $Q = 0$ then they can still satisfy $d = n + \max_{k' \geq 0} \{-qk'\} = n \geq 0$. Thus, firms with $n \geq 0$ can always satisfy $d \geq 0$ and are in $(z, n) \in \mathcal{S}$. Next, there is a Liquidation region \mathcal{L} where firms are liquidated even if creditors conjecture no liquidation today. This means that even if $Q = \tilde{Q}$ then firms cannot satisfy the $d \geq 0$, i.e., $d = n + \max_{b', k' \geq 0} \{\tilde{Q}(\cdot) b' - qk'\} = n < 0$. Since $\tilde{Q}(\cdot) = \tilde{Q}(z, b', k')$, then it follows that firms $(z, n) \in \mathcal{L}$ are those with cash-on-hand n below a threshold $\underline{n}(z)$ where the threshold is defined by the negative of the maximum amount of external resources the firm can raise, i.e., $n < \underline{n}(z) = -\max_{b', k' \geq 0} \{\tilde{Q}(\cdot) b' - qk'\}$. Finally, there is a Risky region \mathcal{R} where firms can either be liquidated or not depending on creditors' conjecture. This means that if $Q = 0$ then they cannot satisfy $d \geq 0$ so $n < 0$, but if $Q = \tilde{Q} > 0$ then firms satisfy $d \geq 0$ so $n \geq \underline{n}(z)$. Thus, firms $(z, n) \in \mathcal{R}$ whenever $n \in [\underline{n}(z), 0)$.

Figure 2: Rollover and solvency: regions across (z, n) state-space



Notes: figures shows the state-space (z, n) and the relevant regions for the liquidation choice.

To construct the equilibrium in region \mathcal{R} , I define an *idiosyncratic sunspot* variable $\phi \sim^{\text{iid}} U[0, 1]$ that is drawn every period at the beginning of the period (i.e., the nonfundamental state variable). Given parameter η , if $\phi \leq \eta$ then creditors coordinate in the liquidation equilibrium ($Q = 0$), otherwise creditors coordinate in $Q > 0$ equilibrium.

Formally, the liquidation choice for firms after they decide to continue (i.e., second red dot in Figure 1 from left to right) — $\tilde{\mathbf{1}}_{\{\text{ch7}\}}(\cdot)$ — depends only on states (z, n, ϕ)

¹⁰The characterization into three regions is common in models of sovereign debt self-fulfilling debt crises that use the [Cole and Kehoe \(2000\)](#) timing convention. See, for example, [Bocola and Dovis \(2019\)](#).

and is characterized formally as follows

$$\tilde{\mathbf{1}}_{\{\text{ch7}\}}(z, n, \phi) = \begin{cases} 1 & \text{if } n < \underline{n}(z) \\ 1 & \text{if } \{\underline{n}(z) \leq n < 0\} \cap \{\phi \leq \eta\} \\ 0 & \text{if } \{\underline{n}(z) \leq n < 0\} \cap \{\phi > \eta\} \\ 0 & \text{if } n \geq 0 \end{cases}.$$

This characterization provides a clear distinction between firms that are liquidated because of solvency problems (those in \mathcal{L}) and rollover problems (those in \mathcal{R} and coordination failure). In Appendix A.1, I characterize the liquidation choice of exiting firms. Notice that to determine what firms are liquidated in equilibrium we also need to characterize what firms enter the restructuring process (which choice is before).

Discussion of assumption. In the model, I assume the firm borrows from several creditors (i.e., atomistic creditors) and use short-term financing. In Appendix B.3, I show that, in the data, firms tend to borrow from several creditors, especially large corporate firms. Further, it is well documented that corporate firms' financial debt is mostly composed by corporate debt (instead of bank loans). Thus, corporate firms liabilities tend to have a dispersed ownership, which could preclude simple coordination among them. Moreover, in Appendix B.3, I show that the average firm in Compustat has large fractions of their debt maturing in the short-term. Around one third of the financial debt matures in less than 1 year and more than half of the liabilities are due in less than 1 year. For quantitative purposes, I will abstract from long-term debt financing and match moments using liabilities that mature in the short-term in the calibration in Section 3.1. Further, in the model, I don't allow firms to manage their liability structure (e.g., extend maturity or concentrate creditors). In Appendix C.2, I show that for the baseline calibration the ex-ante costs of firm rollover risk is negligible for most firms and when comparing to reasonable costs of managing their liabilities in the literature most firms wouldn't modify their liability structure (in steady-state) even if allowed. Finally, in Appendix C.3, I characterize the liquidation choice and the conditions for equilibrium multiplicity in various extensions of the model (with long-term debt, costly equity issuance, etc).

2.4.2 Restructure Choice: Bankruptcy Process and Coordination Failures

First, I will characterize the restructure choice of firms. Next, I discuss in detail the costs and benefits of the restructure process, and how they determine what firms restructure their liabilities.

Characterization. When entering the restructure process, firms and creditors will bargain a debt recovery rate $\alpha_{11} \in [0, 1]$. I assume that the outside option of the bargaining problem is to continue without restructuring. From the characterization of the liquidation choice we know the payoffs of continuing without restructuring.

A necessary condition for creditors to participate in the bargaining process is to get a higher payoff in the restructuring process relative to continuing without restructuring (outside option). For firms that are solvent and not subject to a rollover problems (today), creditors of b recover fully their debt if the firm continues, so they will not accept a recovery rate lower than 1 on their debt. This assumption rules out cases where solvent firms without a rollover problems can restructure their debt, then I can focus on the cases where firms are liquidated as an outside option.¹¹

Let $V(\cdot)$ be the value of the firm when making the financing-investment choice of firms that are solvent and without rollover problems, then the Nash bargaining protocol for an insolvent firm or solvent with rollover problems is

$$\alpha_{11}(z, \omega, b, k) = \max_{\alpha_{11}} [V(n_{11}, z) - 0]^{1-\Xi} [b\alpha_{11} - bR(b, k, \omega)]^{\Xi} \quad (7)$$

where recovery rate α_{11} depends on states (z, ω, b, k) and $\Xi \in (0, 1)$ is the bargaining power of creditors. The protocol is subject to the participation constraints

$$n_{11} = \pi(z, \omega, k) + (1 - c_{11})(1 - \delta)q\omega k - \alpha_{11}b \geq \underline{n}(z) \quad (8)$$

$$\alpha_{11} \geq R(b, k, \omega). \quad (9)$$

The insolvent firm or those solvent with rollover problems have an outside option of $V = 0$ (continuing without restructuring their debt). Equation (8) shows that firms will participate if they are solvent after the restructuring process, equation (9) shows that

¹¹In a previous version of the model, I allowed for solvent firms without coordination failures to restructure their debt and I found that the share of firms that do this was negligible for the baseline calibration.

creditors will participate if they recover more than under liquidation, and (7) shows the objective function of the protocol is a sharing rule of the firm's and creditors' surpluses. For the bargaining process to be feasible we need that the maximum recovery rate that the firm is willing to pay is greater than the minimum creditors are willing to accept, i.e.,

$$\frac{\pi(z, \omega, k) + (1 - c_{11})(1 - \delta)q\omega k}{b} \equiv \alpha_{11}^{\max} > \alpha_{11}^{\min} \equiv R(b, k, \omega).$$

It easy to see that this is also a sufficient condition for firms to restructure their debt (if they are insolvent or solvent with rollover problems). Then the firms that restructure their liabilities are those that are either insolvent or solvent with rollover problems where the bargaining process is feasible, i.e.,

$$\mathbf{1}_{\{\text{ch11}\}}(z, \omega, \phi, b, k) = \begin{cases} 1 & \text{if } \{(z, n) \in \mathcal{L}\} \cup \{ \{(z, n) \in \mathcal{R}\} \cap \{\phi \leq \eta\} \} \cap \{ \alpha_{11}^{\max} > \alpha_{11}^{\min} \} \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

Then the firms that are liquidated are those insolvent or solvent with rollover problem that don't restructure their debt, i.e.,

$$\mathbf{1}_{\{\text{ch7}\}}(z, \omega, \phi, b, k) = \begin{cases} 1 - \mathbf{1}_{\{\text{ch11}\}}(z, \omega, \phi, b, k) & \text{if } \{(z, n) \in \mathcal{L}\} \cup \{ \{(z, n) \in \mathcal{R}\} \cap \{\phi \leq \eta\} \} \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

Finally, the firms that continue without going through the bankruptcy process are those that are in the safe region or in the risky region without rollover problems, i.e.,

$$\begin{aligned} \mathbf{1}_{\{\text{continue}\}}(z, \omega, \phi, b, k) &= 1 - \mathbf{1}_{\{\text{ch11}\}}(z, \omega, \phi, b, k) - \mathbf{1}_{\{\text{ch7}\}}(z, \omega, \phi, b, k) \\ &= \begin{cases} 1 & \text{if } \{(z, n) \in \mathcal{S}\} \cup \{ \{(z, n) \in \mathcal{R}\} \cap \{\phi > \eta\} \} \\ 0 & \text{otherwise} \end{cases} \end{aligned} \quad (12)$$

Further, firms that receive an exit shock can decide to liquidate or restructure before production. The characterization of the restructuring and liquidation choices of exiting firms — i.e., $\mathbf{1}_{\{|\text{exit}\}}(s)$ indicator functions — is simple and relied to Appendix A.1.

Costs and benefits of restructure process. The empirical counterpart of the restructure process is the Chapter 11 of the U.S. Bankruptcy Code. To keep the empirical

counterpart closer to the model, I also exclude piecemeal liquidations in Chapter 11 involving "363 sales", which are a form of liquidation typically used by large firms [see Appendix D for institutional details]. In the model, the cost and benefits for firms from restructuring are three. First, firms benefit from a debt haircut $(1 - \alpha_{11})$. This captures the formal instance provided by the Chapter 11 to renegotiate the firm's liabilities. Second, firms pay the cost of the restructure process $c_{11} \in [0, 1]$ (proportional to capital). This captures in a reduced form way legal fees and administrative costs, credit market reputation penalties, and other costs related to the disruptions created by filing for bankruptcy — some examples of these costs can be found in [Corbae and D'Erasmus \(2021\)](#) and [Bris, Welch and Zhu \(2006\)](#). Third, firms benefit from the removal of coordination failures (i.e., $Q = \tilde{Q}$) during the restructure process. This captures various provisions of the Chapter 11 process aimed at resolving coordination problems among creditors. [Ayotte and Skeel \(2013\)](#) observed that "the dominant normative theory of bankruptcy" (see for example, [Jackson \(1986\)](#)) states that the sole purpose of bankruptcy provisions are to solve "coordination problems caused by multiple creditors". In this spirit, the Automatic Stay provision (11 U.S. Code § 362) that precludes temporarily creditors from individually collecting their debt, Debtor-In-Possession (DIP) protection that allows firms to issue new debt and continue operating (usually known as DIP financing), and the formation of official and ad hoc committees of creditors during the bankruptcy process, are some of the tools provided by the Chapter 11 process to mitigate coordination failures among creditors.

Which firms restructure their debt? Insolvent firms may enter the restructure process if debt haircuts are large and costs are low. On the contrary, firms with rollover problems may restructure their debt even if they receive low haircuts and the process is very costly. Therefore, if the bankruptcy process has low haircuts and is costly, it follows that firms restructuring their liabilities are *mostly* subject to a rollover problem (instead of being insolvent). This observation is crucial for the identification strategy described in Section 3.2.

2.5 Nonfinancial Firms' Problem

In this section, I describe the firm's problem recursive formulation. The idiosyncratic states of the firm at the beginning of the period are $s = (z, \omega, \phi, b, k)$ where the fun-

damental exogenous states are $s^f = (z, \omega)$, the nonfundamental exogenous state is $s^n = \phi$, and endogenous states are $s^e = (b, k)$. Let $V(\cdot)$ be the value of the firm that is solvent and not under a rollover problem when making the investment-financing decision¹², and let $\tilde{V}(\cdot)$ be the value of the firm at the beginning of the period (before exit shock). Then the problem of invest-financing for solvent firms without rollover problems today is as follows

$$V(z, n) = \max_{d, k', b'} d + \mathbb{E}_{(z' | z; \omega'; \phi')} \left[\Lambda \tilde{V}(s') \right] \quad (13)$$

subject to

$$\begin{aligned} d &= n - qk' + \tilde{Q}(z, b', k') b' \geq 0 \\ s' &= (z', \omega', \phi', b', k') \end{aligned}$$

where the continuation value $\tilde{V}(s)$ is defined as

$$\begin{aligned} \tilde{V}(s) &= (1 - \gamma) \left[\mathbf{1}_{\{\text{ch11}\}}(s) V(z, n_{11}) + \mathbf{1}_{\{\text{continue}\}}(s) V(z, n) \right] \\ &\quad + \gamma \left[\mathbf{1}_{\{\text{ch11}|\text{exit}\}}(s) n_{11}^{\text{exit}} + \mathbf{1}_{\{\text{continue}|\text{exit}\}}(s) n \right] \end{aligned} \quad (14)$$

with

$$\begin{aligned} n &= \pi(z, \omega, k) + (1 - \delta) q\omega k - b \\ n_{11} &= \pi(z, \omega, k) + (1 - c_{11}) (1 - \delta) q\omega k - \alpha_{11}(z, \omega, k, b) b \\ n_{11}^{\text{exit}} &= \pi(z, \omega, k) + (1 - c_{11}) (1 - \delta) q\omega k - \alpha_{11}^{\text{exit}}(z, \omega, k, b) b, \end{aligned}$$

where $\mathbf{1}_{\{\cdot\}}(s)$ indicator functions are described in Section 2.4 and $\mathbf{1}_{\{\cdot|\text{exit}\}}(s)$ are described in Appendix A.1, $\alpha_{11}(z, \omega, k, b)$ solves problem (7) which determines continuing firm's cash-on-hand n_{11} and recovery rate $\alpha_{11}^{\text{exit}}(z, \omega, k, b)$ solves problem (23) described in Appendix A.1 which determines exiting firm's cash-on-hand n_{11}^{exit} . Investment-financing policy functions $\{b'(z, n), k'(z, n)\}$ solve problem (13).

¹²Notice these firms can be firms that went through the restructure process and are solvent now, or firms that continue, are solvent and are not under rollover problems.

2.6 Capital Producers and Households

To close the model I describe the problem of the representative capital producer that sell capital to the firms, and the representative household that owns all firms, works, consumes the final good and saves.

2.6.1 Capital Producers.

There is a representative aggregate capital producer that solves

$$\max_I q \Phi \left(\frac{I}{K} \right) - I$$

where I is the amount of final goods used to produce capital, K is the aggregate capital stock, and $\Phi(\cdot)$ is the aggregate capital adjustment cost function. The first order condition of the problem is such that

$$q = \frac{1}{\Phi' \left(\frac{I}{K} \right)} \quad (15)$$

where q is the price of capital. The time-varying price of capital q and the incidence of the recovery rate $\mathcal{R}(\cdot)$ on debt prices allow for a channel that maps to the financial accelerator mechanism (Bernanke, Gertler and Gilchrist, 1999) in the transitions after the shock. I assume a standard functional form such that $\Phi' \left(\frac{I}{K} \right) = \left[\frac{I/K}{\hat{I}} \right]^{-\psi}$ where \hat{I} is the steady-state investment to capital ratio.

2.6.2 Households.

There is a unit mass of identical households that make the consumption-saving C and labor-leisure L decisions taking wages w , interest rate r and own all the firms in the economy. Then the household determine the stochastic discount factor Λ , the Euler equation holds, and the optimal labor-leisure choice is determined by the marginal rate of substitution, i.e.,

$$\Lambda' = \beta \frac{U_C(C', L')}{U_C(C, L)} \quad (16)$$

$$1 = E \left[\beta \frac{U_C(C', L')}{U_C(C, L)} (1 + r) \right] \quad (17)$$

$$w = -\frac{U_L(C, L)}{U_C(C, L)} \quad (18)$$

with utility function $U(C, L) = \ln C - \Phi L$.

2.7 Equilibrium

Now I define the equilibrium of the model. I focus on a steady-state definition, the definition of the equilibrium during transitions follows straightforward from this. First, I show the firms' distribution law of motions and then define the steady-state equilibrium.

Law of motion of the distribution of firms. Let Ω be the distribution of firms that produce which they have a mass of 1, $\tilde{\Omega}$ the distribution of incumbent firms at the beginning of the period, g and \hat{g} the pdf of ω and ϕ respectively, p the conditional pdf of the productivity shocks ϵ_z , and Ω^e the distribution z of entrant firms. To define the equilibrium first we need to determine the law of motion of the distribution. Distribution of firms that produce is determined by

$$\begin{aligned} \Omega(z, n) = & (1 - \gamma) \int \left[\mathbf{1}_{\{\text{ch11}\}}(s) \mathbf{1}_{\{z, n_{11}(z, \omega, k, b) = n\}} + \mathbf{1}_{\{\text{cont}\}}(s) \mathbf{1}_{\{z, n(z, \omega, k, b) = n\}} \right] d\tilde{\Omega}(s) \\ & + \gamma \int_z \left[\mathbf{1}_{\{\text{ch11}|\text{exit}\}}(s) \mathbf{1}_{\{z, n_{11}^{\text{exit}}(z, \omega, k, b) = n\}} + \mathbf{1}_{\{\text{cont}|\text{exit}\}}(s) \mathbf{1}_{\{z, n(z, \omega, k, b) = n\}} \right] d\tilde{\Omega}(s) \\ & + \bar{\mu} (1 - \gamma) \int_z \left[\mathbf{1}_{\{\text{ch11}\}}(s) \mathbf{1}_{\{z, n_{11}(z, \omega, k_0, 0) = n\}} \right. \\ & \left. + \mathbf{1}_{\{\text{cont}\}}(s) \mathbf{1}_{\{z, n(z, \omega, k_0, 0) = n\}} \right] \hat{g}(\phi) g(\omega) d\phi d\omega d\Omega^e(z) \\ & + \bar{\mu} \gamma \int_z \left[\mathbf{1}_{\{\text{ch11}|\text{exit}\}}(s) \mathbf{1}_{\{z, n_{11}^{\text{exit}}(z, \omega, k_0, 0) = n\}} \right. \\ & \left. + \mathbf{1}_{\{\text{cont}|\text{exit}\}}(s) \mathbf{1}_{\{z, n(z, \omega, k_0, 0) = n\}} \right] \hat{g}(\phi) g(\omega) d\phi d\omega d\Omega^e(z). \end{aligned} \quad (19)$$

The distribution of incumbent firms $\tilde{\Omega}(z, \omega, k, b, \phi)$ at the beginning of the period evolves according to

$$\tilde{\Omega}(s') = \int \mathbf{1}_{\{k'(z, n) = k'\}} \mathbf{1}_{\{b'(z, n) = b'\}} \hat{g}(\phi') g(\omega') p(\epsilon_z | \rho_z z + \epsilon_z = z') d\epsilon_z d\Omega(z, n). \quad (20)$$

Equilibrium definition. *Steady-state equilibrium* in this economy is defined as a set of value functions $\{V(z, n), \tilde{V}(s)\}$, firm's decision rules of capital purchases and new debt issuance $\{b'(z, n), k'(z, n)\}$, bankruptcy decisions for firms without the exit shock $\{\mathbf{1}_{\{\text{ch11}\}}(s), \mathbf{1}_{\{\text{ch7}\}}(s)\}$ and with the exit shock $\{\mathbf{1}_{\{\text{ch11}|\text{exit}\}}(s), \mathbf{1}_{\{\text{ch7}|\text{exit}\}}(s)\}$, aggregates

$\{Y, C, I\}$, corporate debt price schedule $Q(s, b', k')$, fundamental corporate debt price schedule $\tilde{Q}(z, k', b')$, interest rate r , prices $\{q, w\}$, distributions $\Omega(s)$ and $\tilde{\Omega}(s)$, and debt haircuts $\{\alpha_{11}(z, \omega, b, k)\}$ under restructure process such that:

1. Households choices are determined by (16), (17) and (18).
2. The price of capital is determined by the solution to (15).
3. The debt price satisfy (5) and fundamental price \tilde{Q} is implicit in $Q(s, b', k') = [1 - \mathbf{1}_{\{\text{ch7}\}}(s)] \tilde{Q}(z, b', k')$.
4. Given prices, firm's decision rules solve the firm problem for firms that produce (13), continuing bankruptcy decisions consistent with (10) (11) (12) and exiting firms bankruptcy decisions are consistent with equations (22) (24) in Appendix A.1, and the recovery rates are solved by negotiation protocols (7) (23).
5. Markets clear: investment is implicitly determined by the law of motion

$$K' = \Phi(I/K) K + (1 - \delta) K - (k_0 - (1 - \delta) \mathbb{E}[\omega] k_0) \bar{\mu}$$

with $K = \int k d\tilde{\Omega}(s)$ and aggregate resource constraint is

$$C = Y - I - \mu_{11}$$

where μ_{11} is the aggregate cost of firms filing to Chapter 11.

6. The distribution of firms that produce $\Omega(s)$ and before bankruptcy $\tilde{\Omega}(s)$ satisfy (19) and (20).

In steady state the distribution's law of motions is a fixed point, and the households stochastic discount factor is $\Lambda = \beta = \frac{1}{1+r}$ and capital prices are $q = 1$.

3 IDENTIFYING FIRMS' ROLLOVER PROBLEMS

The incidence of rollover problems depends on how many firms are exposed — i.e., share of firms in region \mathcal{R} — and the likelihood that exposed firms are subject to a rollover problem — i.e., value of η . Both, are not directly observable then I combine

data and model to infer them indirectly. In this section, I describe the identification strategy and the estimation of firms' rollover problems incidence.

To estimate the incidence of rollover problems, first, I fix a set of parameters to standard values in the literature and calibrate the parameters unrelated to the bankruptcy process to fit several moments of the U.S. economy. Next, I calibrate the parameters of the bankruptcy procedure and use the moment that better identifies rollover problems incidence to estimate the value of η . Finally, using the steady-state distribution of firms I determine how many firms are exposed. I find that 1.6% of the firms are subject to coordination failures each period where 21% of them are exposed and the conditional probability is 7%. Around two thirds of bankruptcy events are due to rollover problems.

3.1 Standard Calibration

Now, I focus on parameters and moments unrelated to the bankruptcy process. The standard calibration consists on 9 fixed parameters and 4 fitted parameters. To evaluate the empirical fitness of the model I contrast the moments in the model to a wide range of moments (16) in the data. The calibration is done at a quarterly frequency. I use national accounts data from NIPA, firm's balance sheet microdata from Compustat, firms' life-cycle data from the Longitudinal Business Database (LBD), and moments computed in other papers. Further details on the data sources, samples and definitions in Appendix B.

Fixed parameters. Table 1 panel (a) shows the value of the fixed parameters. The subjective discount rate $\beta = 0.99$ is set to fit an annual real interest rate of 5%. The labor disutility parameter $\Phi = 1.16$ is set to match an employment rate of 58%. The parameters of the curvature of the production $\nu = 0.21; \alpha = 0.64$ are set to fit the labor and capital share, respectively, and the capital depreciation rate of $\delta = 0.025$ is set to match estimates from BEA. Following [Ottonello and Winberry \(2020\)](#), I fix the persistence of the idiosyncratic productivity process to $\rho = 0.9$. Further, I fix the initial inherited debt is $b_0 = 0$ and exogenous exit rate $\gamma = 0.02$ to fit the total the annual exit rate of 10%. Finally, I fix the aggregate capital adjustment cost parameter $\psi = 1/4$ to a standard value in the literature.

Fitted parameters and moments. Table 1 panel (b) shows the value of the fitted parameters unrelated to the bankruptcy process. The volatility of the idiosyncratic productivity shocks σ_z , the lower bound of the truncated normal process of capital quality shocks (in logs) $\underline{\omega}$, initial capital level k_0 , and the relative scale of the initial productivity draw m . Parameters $(\sigma_z, \underline{\omega}, k_0, m)$ are set to fit 16 moments that are related to aggregates, credit spreads and default rates, investment heterogeneity, life-cycle of firms, and balance sheet moments.

Table 1: Standard calibration

Parameter	Value	Calibration
<i>a. fixed</i>		
$\beta = 1/(1+r)$	0.99	fixed to $r = 0.05$ annual
Φ	1.16	fixed to match 58% emp rate
ν	0.64	fixed labor share
α	0.21	fixed capital share
δ	0.025	fixed to match BEA quarterly
ρ_z	0.90	fixed
b_0	0	fixed to no inherited debt for entrants
γ	0.02	fixed to exit rate 10% annual
ψ	1/4	fixed to standard values in literature
<i>b. fitted</i>		
σ_z	0.032	internally calibrated
$\underline{\omega}$	-0.33	internally calibrated
k_0	0.16	internally calibrated
m	-0.24	internally calibrated

Table 2 shows the moments targeted in the calibration. The model fits fairly well the life-cycle of firms — exit rate, and share of labor and firms at the early stages — and investment rates heterogeneity — average and standard deviation — moments. These moments are from the LBD and [Cooper and Haltiwanger \(2006\)](#). Further, it fits the annual default rate of 3% — from Dun and Bradstreet — which includes defaults by liquidation (Chapter 7 liquidation and Chapter 11 piecemeal liquidation) and restructure (Chapter 11 restructure). On the other hand, in the model steady-state equilibrium the average annual credit spreads is 0.7%, which is lower than in the data (2.2%). This discrepancy can be explained by the absence of aggregate risk in the model. Also the model fits well the distribution of cash-on-hand n/k' — shares of firms with negative values, between 0 and 1, and greater than 1 — which are particularly relevant moments

to estimate the relevance of rollover problems. The cash-on-hand n is measured using data from Compustat. There is no data on non-Compustat firms; thus, I extrapolate the rest of the cash-on-hand distribution. Finally, the model shows lower correlation between cash-on-hand and capital, and larger average and aggregate leverage than in the data. Leverage is measured as short-term liabilities to capital. Further details of the measurement in Appendix B.

Table 2: Moments standard calibration

Moment	Data	Model	Data source
<i>Aggregates</i>			
K/Y	3.00	2.59	NIPA
I/Y	0.17	0.15	NIPA
gross debt: $\mathbb{E}[1_{b>0}b]/Y$	1.05	1.83	NIPA and Flow of Funds
<i>Credit spreads</i>			
default rate: $\mathbb{E}[1_{\text{Ch7}} + 1_{\text{Ch11}}]$	0.03	0.03	Annual rate from Dun and Bradstreet
cred spread: $\mathbb{E}[r^Q - r]$	2.2%	0.7%	Moody's BAA corporate bonds
<i>Investment heterogeneity</i>			
average investment rate: $\mathbb{E}[i/k]$	0.12	0.20	Cooper and Haltiwanger (2006)
SD investment rate: $\text{SD}[i/k]$	0.34	0.36	Cooper and Haltiwanger (2006)
<i>Life-cycle</i>			
share of firms that exit	0.10	0.11	LBD
share of labor at age 1	0.03	0.04	LBD
share of firms at age 1	0.10	0.11	LBD
share of firms at age 2	0.08	0.09	LBD
<i>Balance sheet</i>			
average leverage: $\mathbb{E}[1_{b>0}b'/k']$	0.39	0.72	Compustat
correlation between n and k'	0.74	0.23	Compustat
fraction of firms with $\frac{n}{k'} < 0$	0.21	0.18	Compustat
fraction of firms with $\frac{n}{k'} \in [0, 1]$	0.65	0.77	Compustat
fraction of firms with $\frac{n}{k'} > 1$	0.15	0.05	Compustat

3.2 Identification and Incidence of Rollover Problems

In this section, I show how I identify the likelihood of coordination failures that lead to rollover problems and the share of firms exposed to them. Neither the parameter η nor the threshold function $\underline{n}(z)$ — that defines the regions in the state-space — are directly observable, thus I will infer them indirectly using the firms' bankruptcy choices and

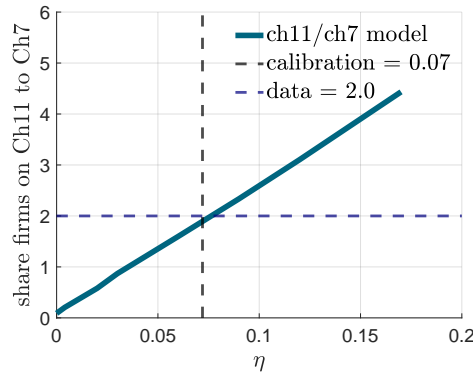
financial distribution of firms.¹³

Table 3: Identifying the likelihood of rollover problems

(a) Parameters and targeted moments of bankruptcy process

Parameters	Value	Moment targeted	Data	Model	Data source
α_7	0.38	$\mathbb{E}[R(b, k, \omega)]$	0.27	0.29	Acharya, Bharath and Srinivasan (2007)
$\tilde{\epsilon}$	0.89	$\mathbb{E}[\alpha_{11}]$	0.69	0.79	Acharya et al. (2007)
c_{11}	0.40	$\mathbb{E}[b'/k' \mid \text{Ch 11}]$	0.73	0.67	Antill (2021)
η	0.07	$\mathbb{E}[1_{\text{Ch11}}]/\mathbb{E}[1_{\text{Ch7}}]$	2.0	1.9	Antill (2021)

(b) Rollover problems likelihood and restructure vs liquidation choice



Notes: panel (a) shows the parameters and moments of the bankruptcy process in the baseline calibration. Panel (b) figure shows the relation between η and the share of firms on Chapter 11 relative to Chapter 7, i.e., $\mathbb{E}[1_{\text{Ch11}}]/\mathbb{E}[1_{\text{Ch7}}]$, in the model.

Identification of the probability η of rollover problems. Table 3 panel (a) shows the parameters, and targeted moments in the data and model related to the bankruptcy process. The capital recovery rate of creditors during liquidation $\alpha_7 = 0.29$ is set to match the debt recovery rate $\mathbb{E}[R(b, k, \omega)] = 0.27$ in Chapter 7 liquidations reported by [Acharya et al. \(2007\)](#). The *approximate* bargaining power of creditors $\tilde{\epsilon} = 0.89$ is set to match the debt recovery rate $\mathbb{E}[\alpha_{11}] = 0.69$ in Chapter 11 restructures reported by [Acharya et al. \(2007\)](#).¹⁴ The parameter that represents the costs of the Chapter 11 process $c_{11} = 0.40$ is set to fit the leverage of firms under Chapter 11 $\mathbb{E}[b'/k' \mid \text{Ch 11}] =$

¹³A salient and related example of indirect inference of rollover problems, is [Bocola and Dovis \(2019\)](#) where they infer the rollover risk faced by the government through the time series debt maturity choices. In this paper I use the cross-section of bankruptcy choices to infer the relevance of firm's rollover problems.

¹⁴For computational efficiency I use a convex-pricing to approximate the bargaining outcome. Details in Appendix A.2.

0.73 reported by Antill (2021).

The data on recovery rates and leverage of Chapter 11 firms suggest that restructuring the debt is costly and haircuts are relatively low. Therefore, *relatively few* insolvent firms will decide to restructure their debt, since gains from Chapter 11 are low.¹⁵ The figure in Table 3 panel (b) shows that a higher level of η in the model shifts the share of firms that restructure relative to those that liquidate.¹⁶

To approximate better the incidence of liquidation and restructuring in the data, I use the summary statistics provided by Antill (2021). Using Chapter 11 outcomes from the Moody's Ultimate Recovery database Antill (2021) identifies how many Chapter 11 cases end in acquisition, piecemeal and full liquidations. When considering this, the ratio of firms restructuring to liquidation is 2. Matching the model to the data — see figure in Table 3 panel (b) — I find a probability $\eta = 0.07$ of rollover problems for firms that are exposed, i.e. in region \mathcal{R} . Furthermore, in the calibration, less than 10% of the firms which restructure their debt are insolvent firms, this is consistent with the previous observation that low haircuts and high costs of the restructure process would led to few solvent firms restructuring.

To validate the results, in Appendix C.4, I show that the model fits other (untargeted) moments of bankrupt firms in the data. The model, not only matches the average leverage of bankrupt firms, but also the distribution. Moreover, when I study the predictors of a bankruptcy event — in particular, restructure — I find that in the data and model — with a similar magnitude and same sign — that smaller size, lower sales growth, lower cash-on-hand and lower leverage increase the likelihood of entering the restructure process. I further discuss the results in Appendix C.4.

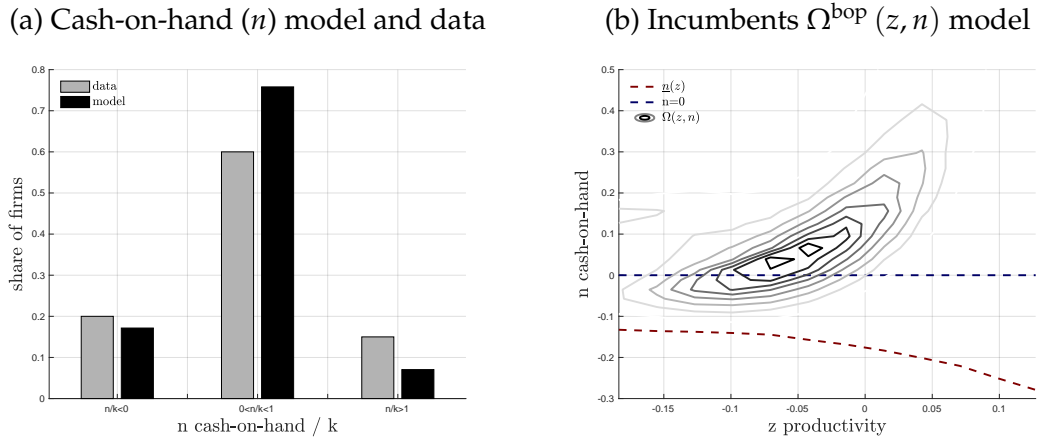
Incidence of rollover problems. Given the value of η , now, it is straightforward to calculate how many firms are subject to rollover problems. First, compute the distribution of firms at the beginning of the period across productivity z and cash-on-hand n — i.e., $\Omega^{\text{bop}}(z, n) = (1 - \gamma) \int \mathbf{1}_{\{z, n(z, \omega, k, b) = n\}} d\tilde{\Omega}(s)$ — then I estimate the number of firms exposed to rollover problems by computing the share of firms in the risky region \mathcal{R} — i.e., $\int_{(z, n) \in \mathcal{R}} d\Omega^{\text{bop}}(z, n)$ — and, finally, I multiply this share by the conditional

¹⁵Consider the extreme case where $\alpha_{11} \rightarrow 1$ then only firms under rollover problems restructure their liabilities.

¹⁶Use the ratio since η shifts the distribution of firms also, thus changes the level on the opposite direction; thus, not useful for identification for some quantitative exercises.

probability of a rollover failure η to estimate the incidence of rollover problems. Figure C.4 shows the financial distribution of firms in the model and data. I find that around 20% of the firms are in the risky region and with $\eta = 0.07$ probability of coordination failure, then 1.6% of the firms are subject to rollover problems each period.¹⁷ To illustrate how relevant is the incidence of rollover problems for firms, I calculate that around two thirds of the bankrupt firms (in restructure or liquidation process) had a rollover problem.

Figure 3: Financial distribution of firms



Notes: Panel (a) compares the distribution of cash-on-hand in the model and the data. Panel (b) shows the contour plot (darker line = higher mass) of the distribution of incumbents firms at the beginning of the period (bop) which doesn't receive the exit shock across productivity z (x-axis) and cash-on-hand n (y-axis), i.e., $\Omega^{\text{bop}}(z, n) = (1 - \gamma) \int \mathbf{1}_{\{z, n(z, \omega, k, b) = n\}} d\tilde{\Omega}(s)$. The dashed red line is the $\underline{n}(z)$ threshold and the dashed blue line is the $n = 0$ threshold.

Result I:

I estimate 1.6% of the firms are exposed to rollover problems, where 20% are exposed and the probability of a rollover problem is 7%. Moreover, around two thirds of bankruptcy events (liquidations and restructures) are driven by rollover problems.

4 MACROECONOMIC IMPLICATIONS OF FIRMS' ROLLOVER RISK

In this section, I study the relation of firms' rollover problems and macroeconomic dynamics during crises. First, I simulate a prototypical crisis episode — i.e., crises

¹⁷Notice this number includes firms that may not be liquidated because they enter the restructure process.

driven by large aggregate shocks that are standard in the literature — and assess the role of rollover problems by comparing with a counterfactual crisis without coordination failures. Second, I study the heterogeneity in firms' investment adjustment during the crises in the model and data. I find that rollover risk can significantly amplify the impact of crises (explain around one fifth of the total impact in output) and the heterogeneity observed of investment responses in recent crises episodes is consistent with the model's crisis simulations.

4.1 Crisis and Rollover Risk

Now, I study the role of firms' rollover risk during crises.¹⁸ Crises, even driven by fundamentals, could deteriorate the firms' financial position and suddenly expose them (today and in the future) to rollover problems. To assess the role of rollover risk in crisis I simulate a prototypical crisis — large and unexpected shock — driven by different types of shocks and perform a counterfactual where coordination failures don't happen during the crisis episode. The three types of crises shocks studied are: (i) aggregate TFP shock; (ii) sudden reduction in firm's cash; (iii) a decrease in liquidation recovery rate α_7 , i.e., credit shock.¹⁹ Shocks happen unexpectedly and return to the initial steady state in the long run. All shocks match a peak-to-trough drop of aggregate output Y of 5%, and have a transitory nature, i.e., persistence of the shocks is set to 0.5. Shocks happen at $t = 0$, where the economy was in steady-state the previous period. Further details of the definition of the shocks and computation of transitions in Appendix A.3.

Figure 4 shows the response of aggregate output Y to different types of aggregate shocks with coordination failures (black line) and without them (gray dashed line). In all panels the absence of firm rollover problems —i.e., no extra rollover problems at $t = 0, 1$ — significantly reduce the depth of the trough in the crises. The counterfactuals indicate that rollover problems happening in the crisis can explain around one fifth of total output losses during the episode.

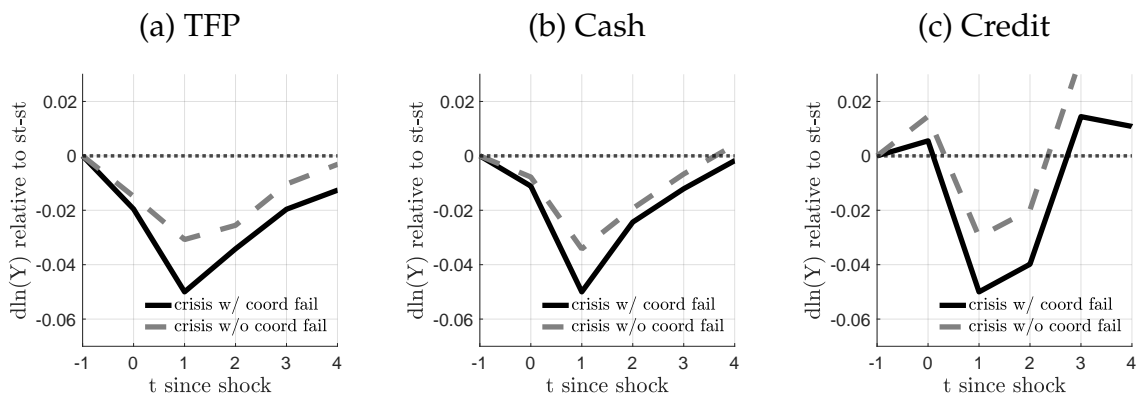
Although, on impact, the relevance of rollover risk is similar across shocks, the dy-

¹⁸In Appendix C.1, I show that firms' rollover risk has negligible consequences over macroeconomic outcomes over the long run. This happens because firms save away from the risky region and in equilibrium they improve their financial position, reducing the overall impact of rollover problems on aggregate outcomes.

¹⁹Notice the credit shock in this model is to a reduction in the collateral's value when liquidated, which is different from a credit shock in Khan *et al.* (2020), where the credit shock is closer to the cash shock in this paper.

namics are different. In Appendix A.4, I show the dynamics of firm (net) exit, debt and capital accumulation during the crises. Weaker fundamentals — due to a TFP or cash shock — expose (temporarily) several solvent firms to rollover problems, which increases firm exit of healthy firms during the crisis. Further, new firms entering the economy are smaller and take time to grow. Therefore, crises driven by a TFP or cash shock — exacerbated by coordination failures — slowdown significantly the recovery. On the other hand, a crisis driven by a credit shock will induce firms to deleverage quickly and reduce their investment initially to preclude liquidation, which makes the recovery relatively stronger (compared to other shocks), although the initial impact is the same.

Figure 4: Crisis Shock and Aggregate Output



Notes: panel (a), (b) and (c) show the response of aggregate output Y to an aggregate TFP shock, cash shock and credit shock with coordination failures and without coordination failures (for $t=0$ and $t=1$), respectively. The economy at $t = -1$ is in steady-state. The definition of the shocks are in the text and Appendix A.3.

Result II:

Firms' rollover risk significantly amplify the impact of crises. It explains around one fifth of total output losses during crises episodes.

4.2 Investment Heterogeneity during Crises

Now, I simulate a panel of firms in the model and study the heterogeneity in investment dynamics during the crises. I contrast these results with estimates from the data from the Great Recession and Covid crisis.

To estimate the *on impact* heterogeneous response — in the model and data — from peak-to-trough of the crisis, I will proceed as follows. First, to account for permanent

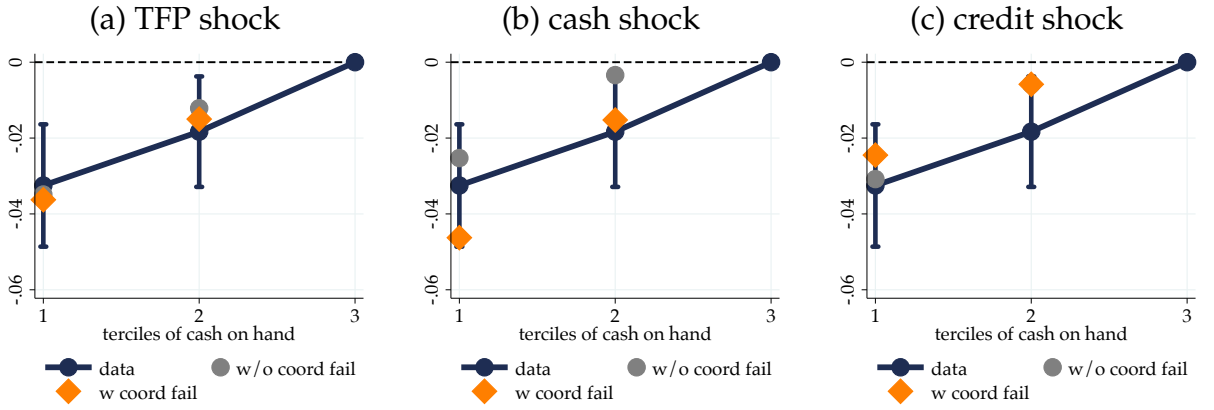
sectoral heterogeneity I will demean each of the firm-quarter observations of cash-on-hand over capital n_{it}/k_{it} for firm i in period t of interest by its sectoral average, i.e. $\hat{n}_{it} = n_{it}/k_{it} - \mathbb{E}_s[n_{it}/k_{it}]$ for firm i in sector s . Next, I will assign each firm-quarter observation of \hat{n} to each tercile (for each period's distribution). Lastly, I run the following panel regression to estimate the heterogeneous responses of investment across cash-on-hand n/k during the crisis:

$$\Delta \log(k_{it}) = \underbrace{\sum_{j=1}^J \beta_j^n \left(Q_{it}^{nj} \times \text{crisis}_t \right)}_{\text{heterogeneity across } n/k} + \underbrace{\Lambda' Z_{it}}_{\text{controls}} + \varepsilon_{it}, \quad (21)$$

where Q_{it}^{nj} indicates if \hat{n}_{it} belongs to tercile j , $\Delta \log(k_{it}) = \log(k_{it+h}) - \log(k_{it})$ is firm's i capital accumulation over a period as long as the crisis studied (i.e., the extension from peak-to-trough of episode studied h), crisis_t indicates if a crisis happens during the period considered (from t to $t+h$) and $Z_{i,t}$ includes the control variables. For the baseline specifications controls, I include firm's fixed effects, sectoral fixed effects, log assets as proxy for size, last quarter sales growth and heterogeneity across firm's leverage. The coefficients β_j^n are the estimates of interest, and can be interpreted as the diff-in-diff estimates of the crisis impact on capital accumulation for firms in tercile j of \hat{n} . Results are shown relative to the highest group (i.e., the one with highest cash-on-hand or lowest leverage). In the empirical application I use data from Compustat (limited to publicly traded firms) and in the model I select firms that approximate this set of firms. Further details of the data, estimates and other results are in Appendix B.

Figure 5 shows the results across cash-on-hand n/k . In all plots, the blue connected line corresponds to the data estimates. On the other hand, the diamond dots indicate the estimates of the model simulated data. For all panels, the heterogeneity of investment responses across cash-on-hand is similar in the data and model for different types of shocks in both episodes. For firms in the lowest group of cash-on-hand we observe that the drop in investment (differences in capital accumulation) is 3.5 percentage points greater compared to firms with high levels of cash-on-hand. Comparing the crisis with and without coordination failures we find that the heterogeneity is similar across models for the TFP shock, slightly lower for the credit shock and much greater for the cash shock. Further, in Appendix B, I show the empirical results across leverage levels and for individual episodes.

Figure 5: Investment Heterogeneity During Crises: β_j^n



Notes: Panel (a), (b) and (c) show the estimates of β_j^n for the Great Recession and Covid Crisis average, for the data and different aggregate shocks simulated in the model. The blue connected line shows the data and the 90% confidence interval. The diamond and regular dots show the estimates from the model's simulated panel data with and without coordination failures, respectively. The coefficients are relative to the tercile of firms with the highest cash-on-hand. All estimates (in model and data) are from empirical specification (21). Estimates are in semester frequency.

5 POLICY RESPONSE TO FIRMS' ROLLOVER RISK

Firms' rollover problems increase the bankruptcy and liquidation of healthy firms, and — as shown in the previous section — they have significant (negative) macroeconomic consequences (i.e., greater depth and slower recovery) during crises. Arguably various of the policies displayed during the recent Covid crises had the motivation of preventing healthy firms from being liquidated. For example, the Federal Reserve's Primary and Secondary Lending Programs provided direct credit access to a set of seemingly sound corporate firms —i.e., those with a good financial position before the crisis. Although the model is not well suited to study some of the particularities of the Covid crisis, it provides a quantitative framework where I can study how these new set of unconventional credit policies interact with firms' rollover problems and macroeconomic dynamics.

In this section, I study how effective are *imperfect direct lending policies* deployed to reduce the incidence of creditors' coordination failures during crises. Policies are imperfect, because I assume the government cannot target the firms using all relevant characteristics. This assumption captures the idea that the government may not observe all the characteristics of individual firms or the policy can't be restricted for other

reasons (e.g., regulations). I find, in my quantitative exercises, that credit policies that operate mostly through the insurance channel (precluding coordination failures) and government credit facilities remain mostly unused (in equilibrium) are very potent. On the contrary, policies that are very ample and subsidize credit for many firms, may preclude several rollover problems in the short-term, but exacerbate debt overhang problems in the future, amplifying the overall impact of the crisis.

Credit policy workings. A direct lending policy in the model is promised unexpectedly at $t = 0$ implemented at period J_0 for J periods. When the policy is active, the government offers an alternative pricing schedule $Q_j^g(\cdot)$ for the new debt issuance of the firm at each period $j = J_0, J_0 + 1, \dots, J$ to a set of eligible firm \mathcal{P} which depends on observable, current or past, characteristics of the firms. The set of eligible firms \mathcal{P} is assumed to be fixed over the time the policy is implemented and I assume creditors know about the policy. Now the external resources from new debt issuance of eligible firms are

$$\max \left\{ Q^g(\cdot), Q(s, k', b') \right\} b',$$

where firms choose the best pricing schedules between the government program and the market.

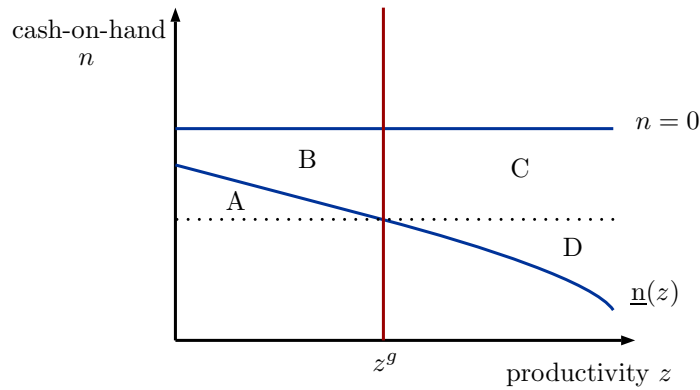
To understand how this policy works, let's assume that the government sets for 1 period $Q^g(\cdot) = \tilde{Q}(z, k', b')$ and a firm with $(z, n) \in \mathcal{R}$ and $\phi < \eta$ — i.e., under a rollover problem — is eligible for the government program. Without the policy creditors would offer $Q = 0$ to the firm. With the policy, creditors know that since firm are in \mathcal{R} they are solvent under $Q^g = \tilde{Q}$. Thus, if creditors conjecture all other private creditors will not lend to the firm ($Q = 0$) still the firm will be able to satisfy $d \geq 0$ at the pricing schedule $Q^g = \tilde{Q}$. The presence of this alternative pricing schedule will coordinate creditors in the $Q = \tilde{Q} > 0$ equilibrium, even if the firm doesn't participate of the policy (i.e., use Q^g) in equilibrium. The intuition is that this policy operates as a form of insurance to creditors, which prevents the $Q = 0$ equilibrium from happening.

Imperfect credit policy quantification. A policy $Q^g = \tilde{Q}$ that precludes all rollover problems without being used in equilibrium requires the government to observe perfectly each firm's productivity and cash-on-hand (z, n) . To make the policy more realistic, I assume that the government observes only n and sets the policy according to a

simple rule, i.e., *imperfect credit policy*:

1. The set of eligible firms depend on n only and requires that $n < 0$ (i.e., firms need external resources to satisfy $d \geq 0$). Therefore, the set \mathcal{P} is composed by firms with n such that $n \in [\underline{n}^g, 0)$, where $\underline{n}^g < 0$ is a parameter chosen by the government.
2. All eligible firms receive enough funds such that they can satisfy $d \geq 0$, but the government can't discriminate across the n position of eligible firms, i.e., $Q^g(z, k', b')$ is such that $\underline{n}^g = -\max_{b', k'} Q^g(z^g, k', b') b' - qk' = \underline{n}(z^g)$ which implies that n^g determines the choice of $z = z^g$ for the pricing schedule offered by the government.²⁰

Figure 6: Imperfect Credit Policy Eligibility



Notes: figure shows an illustration of the eligibility and firms participation in the program for a one period example.

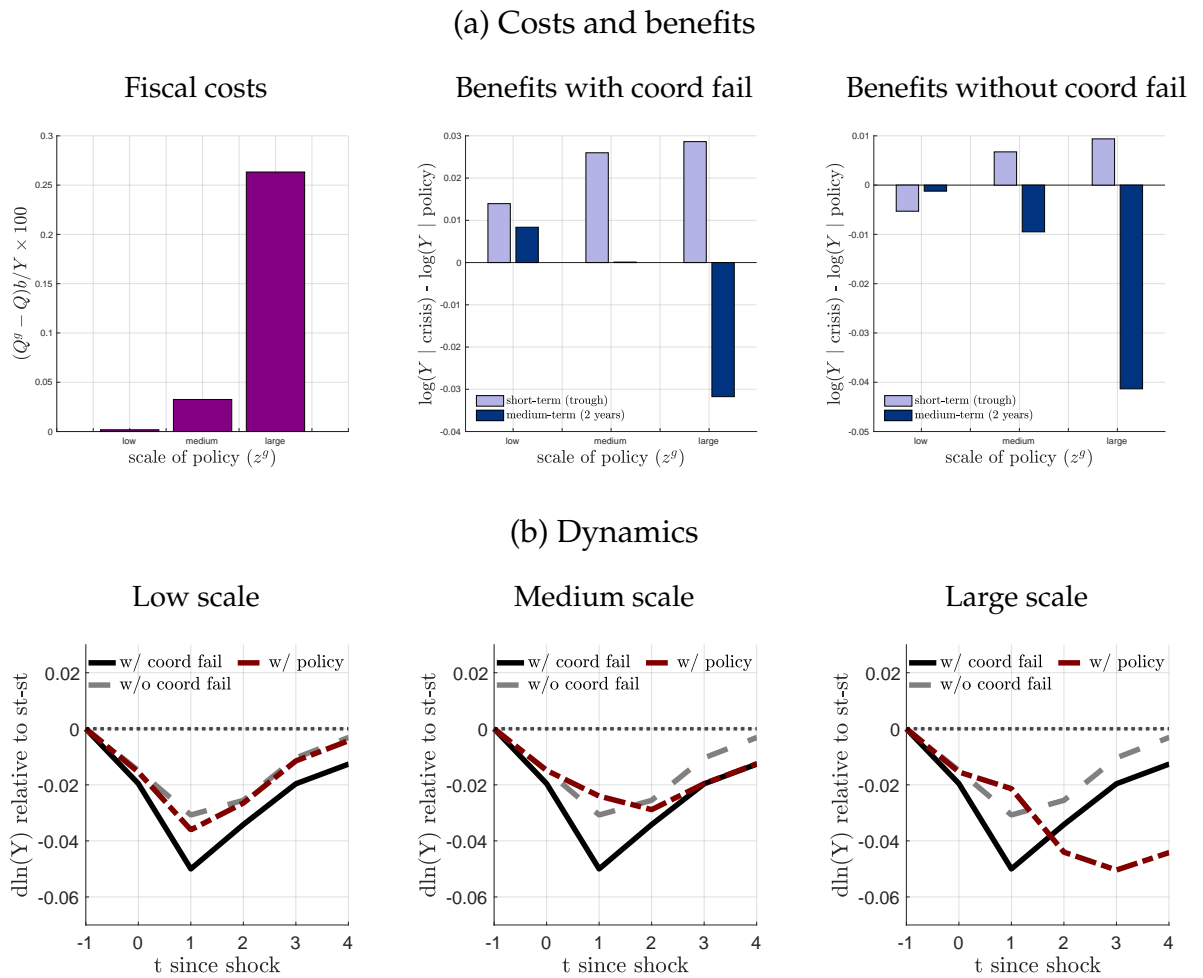
Figure 6 shows what firms are eligible and the static choice of the participating or not in the program for a 1 period policy.²¹ Eligible firms are those in the area $A \cup B \cup C$. In the case of A in absence of the credit program, the firm would be insolvent, then these firms receive a subsidized credit. On the other hand, firm in B they will find the credit of the program cheaper than the market then they participate so they receive a subsidized credit. On the contrary, firms in region C will have a more expensive credit than the market then they don't participate of the program. Thus, firms in $A \cup B$ receive a subsidized credit and firms in C not. Moreover, the credit program will

²⁰The assumption the government pricing function doesn't depend on the firm's cash-on-hand simplifies greatly the computational problem.

²¹If the policy lasts various periods or is implemented with a lag, then it will affect the solvency thresholds (even in partial equilibrium) since they depend on future prospects of the firm.

preclude those firms under a rollover problem in $B \cup C$ from being liquidated. Notice that firms under a rollover problem in B will participate of the program and in C will not participate of the program but the mere existence of the program will preclude coordination failures. Therefore, if the scale of the policy increases — i.e., lower n^g or, equivalently, greater z^g — more coordination failures are precluded and more firms are subsidized. The credit subsidizing could exacerbate future debt overhang problems and is costly then the policy faces a potential trade-off when incrementing the scale of the program.

Figure 7: Imperfect Credit Policy by Scale



Notes: the Figure shows the policy costs and benefits for different policies during a crises driven by a cash shock. Figure (b) shows the fiscal cost of the policy in GDP terms, the short (in the crisis trough) and medium (2 years) term benefits of the policy with and without coordination failures. Figures in Panel (b) show the response of aggregate output Y to an aggregate cash shock with perfectly targeted policy (same as without coordination failures, dashed gray line), imperfectly targeted policy (red dashed line), and without policy intervention (solid black line) for different policy scale. The economy at $t = -1$ is in steady-state. The definition of the shocks and crises experiments are in Section 4.1 and Appendix A.3. Further description of the policy in the text.

Figure 7 shows the evolution of aggregate Y for different levels of z^g for a policy implemented during $t = 0, 1$ — i.e., peak-to-trough of the crisis without policy. I focus on the TFP shock, results for cash shock are very similar and relied to Appendix A.5 for expositional clarity. I consider policies with different scale z^g . Panel (a) shows that the low scale policy that (in equilibrium) the cost is almost 0, a medium scale policy that has a low cost (0.05% of output) and a large scale policy which costs around 0.3% of output. At $t = 1$ (trough of crisis), the larger the scale the lower the impact of the crisis, and even the policy can improve relative to the counterfactual without coordination failures. Thus, larger scale policies not only remove coordination failures, but also provide some extra stimulus. On the contrary, the dynamics (medium term benefits) indicate that under the low scale policy the economy recovers strongly and similar to the counterfactual without coordination failures, the medium scale policy recovery is weaker and similar to the crisis shock without policy scenario, and the large scale policy makes the crisis longer and deeper. The intuition is that the larger the scale the more subsidized is credit for financially exposed and fundamentally weak firms, which eventually backfire by aggravating the debt overhang problem in the future.²²

Result III:

A direct credit policy that is imperfectly targeted is very potent whenever it subsidizes a relatively small number of firms, but backfires whenever many firms receive a subsidized credit.

6 CONCLUDING REMARKS

In this paper, I develop a framework where firms' rollover problems can be identified and quantified. Salient features of the U.S. bankruptcy code allow me to quantify the incidence of rollover problems by using cross-sectional moments related to the bankruptcy outcomes and bankrupt firms' characteristics.

My quantitative results suggest that firms' rollover risk, through the failure of healthy firms, have a significant impact during crises. On the other hand, direct credit policies can act as insurance for creditors and prevent coordination failures from happening, but, with imperfect targeting, the government faces a trade-off between short run mitigation of coordination failures and future debt overhang problems. Quantitative results

²²This mechanism is similar to the one studied by Crouzet and Tourre (2021).

suggests that, during crises, the benefits of direct credit policies are ambiguous.

In the model, I focus on the problem of firms which have homogeneous and atomistic creditors, and without active management of its liability structure. Potential extensions could allow for: investors' heterogeneity, as in [Halac, Kremer and Winter \(2020\)](#); endogenous debt maturity structure, as in [Bocola and Dovis \(2019\)](#) for a sovereign debt problem or [Cheng and Milbradt \(2012\)](#); [Crouzet \(2017b\)](#) for firms; and endogenous number of creditors, as in [Bris and Welch \(2005\)](#); [Bolton and Scharfstein \(1996\)](#).

The evidence presented in this paper suggests that firm-level financial frictions can play a relevant role during crises, which complements the view that creditor-level financial frictions can be important during crises [see, for example, [Gertler and Gilchrist \(2018\)](#)]. One potential avenue of future research is to explore the relation between these two type of credit frictions and their identification. Further, my paper provides insights on the relationship between rollover problems and bankruptcy provisions during crises, which can be applied in other contexts. For example, one potential avenue of future research is to study — in a sovereign debt model with rollover risk — the costs and benefits of including provisions analogous to those of the U.S. bankruptcy code in a supranational sovereign debt bankruptcy process. I leave these extensions and alternative applications for future work.

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APPENDICES

The Appendix is organized as follows: Appendix A shows additional derivations of the baseline model and further details on the computations; Appendix B includes details of the data sources (sample selection and definitions) and details of the empirical exercises; and Appendix C includes further exercises and extensions of the model.

A Appendix: Model

A.1 Exiting firms problem

Incumbents firms at the beginning of the period receive with probability γ a shock that force them to exit after production. I allow for exiting firms to make also the liquidation choice and restructuring choice. Notice that since they exit at the end of the period these firms don't choose (b', k') then they are not subject to coordination failures such as the ones described for nonexiting firms. Exiting firms choose to liquidation choice is

$$\mathbf{1}_{\{\text{ch7|exit}\}}(s) = \tilde{\mathbf{1}}_{\{\text{ch7|exit}\}}(z, \omega, b, k) = \begin{cases} 1 & \text{if } \max\{n, n_{11}^{\text{exit}}\} < 0 \\ 0 & \text{otherwise} \end{cases}. \quad (22)$$

where n defined as before and $n_{11}^{\text{exit}} = \pi(z, \omega, k) + (1 - c_{11})(1 - \delta)q\omega k - \alpha_{11}^{\text{exit}}(z, \omega, k, b)b$. Since the outside option is to continue then only firms with $n < 0$ will restructure their debt and the debt recovery $\alpha_{11}^{\text{exit}}$ is determined by

$$\alpha_{11}^{\text{exit}}(z, \omega, b, k) = \max_{\alpha_{11}} \left[n_{11}^{\text{exit}} - 0 \right]^{1-\Xi} \left[b\alpha_{11}^{\text{exit}} - bR(b, k, \omega) \right]^{\Xi} \quad (23)$$

subject to

$$\begin{aligned} n_{11}^{\text{exit}} &> 0 \\ \alpha_{11} &\geq R(b, k, \omega). \end{aligned}$$

The restructure choice is

$$\mathbf{1}_{\{\text{ch11|exit}\}}(s) = \tilde{\mathbf{1}}_{\{\text{ch11|exit}\}}(z, \omega, b, k) = \begin{cases} 1 & \text{if } \{n < 0\} \cap \{n_{11}^{\text{exit}} > 0\} \cap \left\{ \alpha_{11}^{\text{max|exit}} > \alpha_{11}^{\text{min}} \right\} \\ 0 & \text{otherwise} \end{cases}, \quad (24)$$

where $\alpha_{11}^{\text{max|exit}} = \frac{\pi(z, \omega, k) + (1 - c_{11})(1 - \delta)q\omega k}{b}$ and $\alpha_{11}^{\text{min}} = R(b, k, \omega)$. The firms that continue are defined as $\mathbf{1}_{\{\text{continue|exit}\}}(s) = 1 - \mathbf{1}_{\{\text{ch11|exit}\}}(s) - \mathbf{1}_{\{\text{ch7|exit}\}}(s) = \mathbf{1}_{\{n \geq 0\}}$.

A.2 Computational solution of Bargaining Problem

To solve the bargaining problem I adopt a very simple convex-pricing function to approximate the result from the Nash Bargaining problem.²³ Although this is a reduced form solution to the bargaining problem, it provides better computational speed since we don't need the value function of the firm to compute it. I proceed as follows: I compute the maximum and minimum recovery rates, $\alpha_{11}^{\max}(z, \omega, k, b)$ and $\alpha_{11}^{\min}(\omega, k, b)$, respectively. Using these bounds, for the restructure processes that are feasible I compute the approximate recovery rate $\tilde{\alpha}_{11}(z, \omega, k, b)$ as

$$\tilde{\alpha}_{11}(z, \omega, k, b) = \tilde{\Xi} \alpha_{11}^{\max}(z, \omega, k, b) + (1 - \tilde{\Xi}) \alpha_{11}^{\min}(\omega, k, b)$$

where $\tilde{\Xi} \in [0, 1]$ is the approximate bargaining power of the creditors. There is no one-on-one mapping, but to check for robustness I solve for the exact solution and find similar results. Therefore, I adopt this convex-pricing function, which is computationally significantly more efficient than the exact solution.

A.3 Crises shocks and counterfactuals

I work with 3 different types of crisis shocks: a TFP shock, cash shock and credit shock. Shocks are unforeseen and I study the perfect foresight transitions from $t \geq 0$ where $t = 0$ is the initial impact of the shock (at the beginning of the period). The initial impact is calibrated to match a 5% drop in aggregate output from peak-to-trough (large aggregate shock) and the persistence of all shocks is $\rho_{\text{shock}} = 0.5$ (i.e., short lived).²⁴ I assume the process are

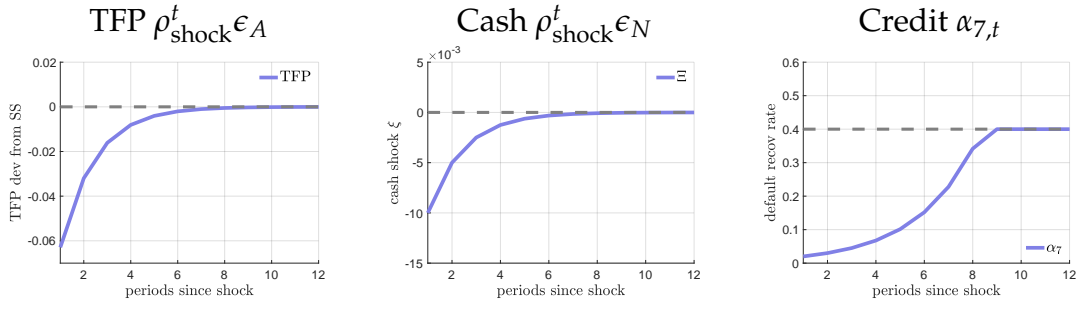
1. TFP shock: firms production function is now $y_{it} = A_t f(z_{it}, \omega_{it}, k_{it})$ for $t \geq 0$ where $A_t = \exp(\rho_{\text{shock}}^t \epsilon_A)$ with $\epsilon_A < 0$ the initial shock at $t = 0$.
2. Cash shock: firms cash-on-hand is $n_{it} = \pi_t(z_{it}, \omega_{it}, k_{it}) + (1 - \delta) q_t \omega_{it} k_{it} - b_{it} - N_t k_{it}$ for $t \geq 0$ where $N_t = \rho_{\text{shock}}^t \epsilon_N$ with $\epsilon_N > 0$ initial shock to cash proportional to capital.
3. Credit shock: recovery rate when liquidated α_{7t} is time-varying for $t \geq 0$ where $\alpha_{7t} = \alpha_7 - \rho_{\text{shock}}^t \epsilon_7$ where $\epsilon_7 > 0$ initial decrease in liquidation recovery rate.

Figure A.1 shows the path for the baseline counterfactuals.

²³In their robustness exercises Guntin and Kochen (2021) adopt this function to solve computationally for a complex bargaining problem.

²⁴More than 95% of the shocks fades away in an year.

Figure A.1: Crises Shocks Path

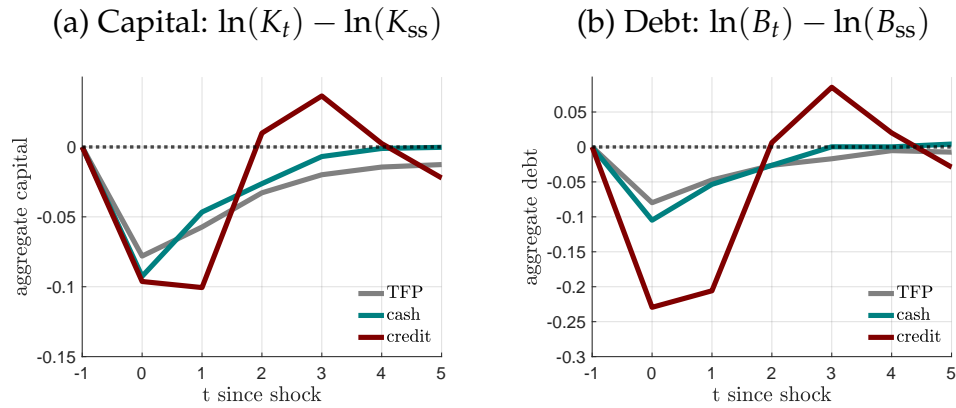


Notes: panel (a), (b) and (c) show the path of the shocks. Shocks happen at $t = 0$. Further description of the shocks in the text.

For computing the aggregates during the transitions I assume that the distribution of firms is no longer a fixed point and allow for *net exit* by fixing the amount of new firms created each period to the initial steady-state calibration, i.e., $\bar{\mu}_t = \bar{\mu}$.

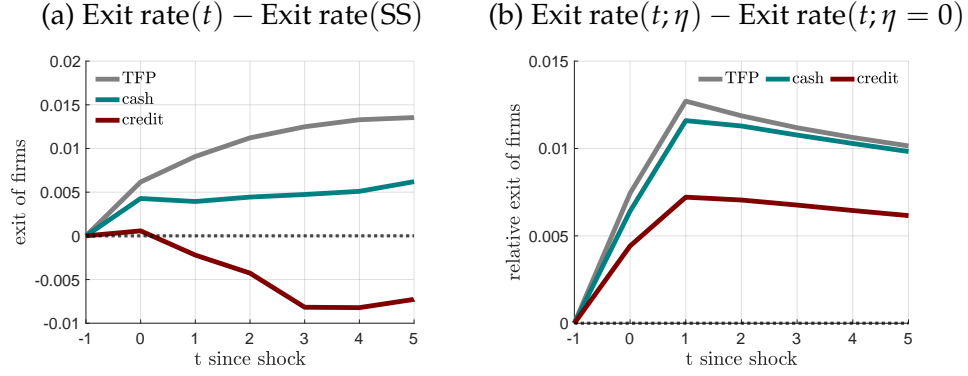
A.4 Firm Exit and Spreads during Crises Experiment

Figure A.2: Capital and Debt during Crises



Notes: Figures show the dynamics of capital and debt accumulation for the three crisis shocks studied. In both panels, the variables are in terms of log difference relative to steady state — $\ln X_t - \ln X_{SS}$. Panel (a) shows the dynamics of aggregate capital accumulation. Panel (b) shows the dynamics of aggregate debt accumulation.

Figure A.3: Firm Exit during Crises



Notes: Figures show the dynamics of firm exit for the three crisis shocks studied. Panel (a) shows the difference between firm exit rates (exogenous and endogenous) relative to pre-crisis steady-state levels during the crisis episode. Panel (b) shows the difference between firm exit rates with coordination failures relative to the counterfactual without coordination failures during the crisis episode.

A.5 Credit policy program

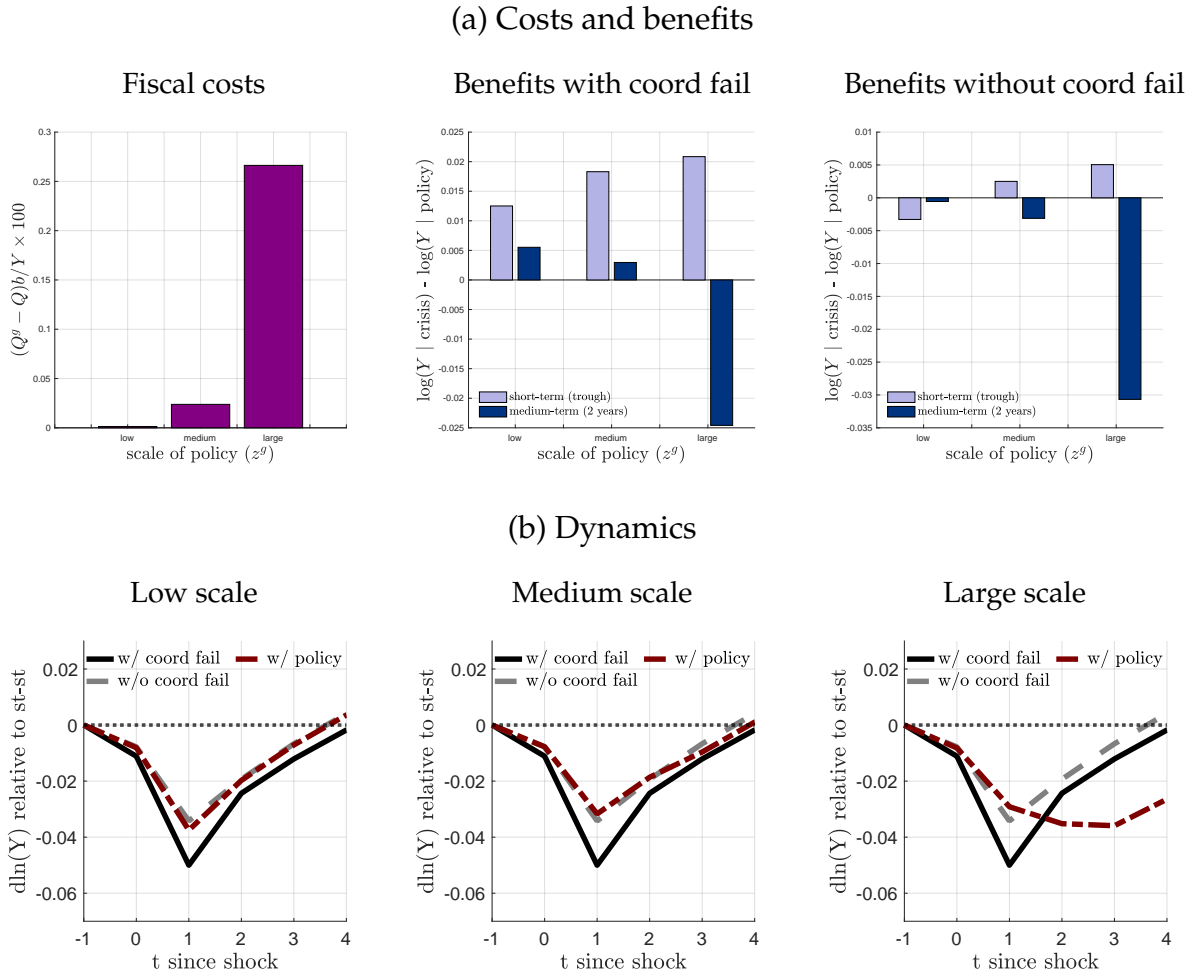
Further details quantitative setup. The baseline credit policy experiment consist of a parameter z^g that determines the pricing schedule $Q_t^g = \tilde{Q}_t(z^g, b', k')$ and set of eligible firms $[\underline{n}(z^g), 0)$ and lasts two periods (implemented at $t = 0$ and $t = 1$). The policy is computed backwards, since the presence of the policy at $t + 1$ will affect the solvency thresholds $\underline{n}_t(z)$. To estimate the cost of the policy I compute the aggregate credit subsidy as the difference between the price offered by the private sector relative to the government credit program times the amount borrowed for the firms that choose to participate in the program, i.e.,

$$G_t = \int_{(z,n) \in \mathcal{P}} \max \left\{ 0, \tilde{Q}_t(z, k', b') - \tilde{Q}_t(z^g, k', b') \right\} b' d\tilde{\Omega}_t(s).$$

The subsidy is financed through a lump-sum transfer $T_t = G_t$ such that the aggregate output net of government expenditure is $\tilde{Y}_t = Y_t - G_t$.

Further quantitative results. Figure A.4 shows the results for the policy experiments when the driving shock is a cash shock.

Figure A.4: Imperfect Credit Policy by Scale (cash shock)



Notes: the Figure shows the policy costs and benefits for different policies during a crises driven by a TFP shock. Figure (b) shows the fiscal cost of the policy in GDP terms, the short (in the crisis trough) and medium (2 years) term benefits of the policy with and without coordination failures. Figures in Panel (b) show the response of aggregate output Y to an aggregate cash shock with perfectly targeted policy (same as without coordination failures, dashed gray line), imperfectly targeted policy (red dashed line), and without policy intervention (solid black line) for different policy scale. The economy at $t = -1$ is in steady-state. The definition of the shocks and crises experiments are in Section 4.1 and Appendix A.3. Further description of the policy in the text.

B Appendix: Data and Empirical Exercises

In this section I outline the data sources, variable definitions, and further empirical exercises and results. I show how the balance sheet moments for the calibration are computed, how it is estimated the heterogeneity in investment dynamics during during the Great Recession and Covid-19 crisis, and study the characteristics of bankrupt firms using microdata.

B.1 Data Sources, Sample Selection and Variable Definitions

In this section I describe the details (definitions and sample construction) of the main data sources used to compute moments related to the balance sheet of firms and empirical exercises in the paper.

B.1.1 Compustat

I use Compustat data to compute moments related to the balance sheet of firms and bankruptcy process, and study the patterns of investment in recent large crises. Compustat is limited to publicly held firms, therefore I assume the balance sheet distribution replicates in the rest of the firms.²⁵ To construct the sample I follow standard practices in the empirical investment literature.

Balance Sheet Data. Now I explain how I construct the sample and the variables for the balance sheet data used for calibration and empirical exercises. The sample selection criteria follows a firm level filter and firm-date filter. Table B.1 shows the number of observations and those dropped by each filtering step. I drop firms from finance, insurance, and real estate sectors ($\text{sic} \in [6000, 6799]$), utilities ($\text{sic} \in [4900, 4999]$), non-operating establishments ($\text{sic} = 9995$) and industrial conglomerates ($\text{sic} = 9997$), and those not incorporated in U.S. and not operate in USD. I drop firm-date observations that with negative capital or total assets, observations with acquisitions of more than 5% of firm's assets, bottom 0.5% and top 99.5% investment rate across the distribution, investment spells of less than 20 quarters, drop if net liquid leverage (net current liquid debt/total assets) is greater than 10 in absolute value, drop if log sales growth is greater than 1 in absolute value, and negative sales or negative liquid assets.

Due to changes in the accounting data of Compustat, I split the sample for the Great Recession (period 1983-2017) and Covid-19 Crisis (period 2019-2020) (see Ma (2020) notes on the accounting changes after 2019).²⁶ The sample criteria for the 2019-2020 period differs slightly from the 1983-2017 sample. Since the 2019-2020 sample is smaller I exclude filters related to investment outliers and spells, and select firm-quarter observations that register they changed they updated their accounting criteria.²⁷

The final sample — pre-Covid — has 426,465 firm-date observations, and the Covid sample has 13,974 firm-date observations.

²⁵An alternative approach is to fit the model to a subset of firms that can be defined as the Compustat firms. For simplicity I use the assumption described in the text.

²⁶An alternative approach is to use Compustat Snapshot to remove the operation leases from various entries in the balance sheet, but access to this dataset is restricted.

²⁷The variable `acctchgq` is "ASU16-02" or "IFRS16" the quarter the firm changes it's accounting criteria.

Table B.1: Sample Selection Compustat - Quarterly Data

	# Drop	# Obs
<i>Baseline</i>		
1983-2017	-	1,484,973
Non-financial sector	474,327	1,010,646
U.S. incorporated and USD currency	212,680	797,966
>20 quarter investment spell	105,503	585,286
No outliers	158,821	426,465
<i>Covid sample</i>		
2019-2020	-	75,712
Change in accounting	38,446	37,266
Other filters	23,292	13,974

The definition of the main variables used for the calibration and regressions are:

1. Capital stock k : is constructed using the perpetual inventory method, following the usual convention in the investment literature.²⁸ I compute the initial capital level using the level of gross plant, property and equipment $ppeg_tq$, and using the quarterly change of net plant, property and equipment $ppentq$. The depreciation rates δ are calculated using the BEA accounts to compute investment rates (i.e., change in capital k net of capital depreciation).
2. Net debt stock b : different from other papers in the literature I assume b corresponds to the short-term liabilities. Liabilities include financial debt, debt with suppliers and other firms, accounts and tax payables, and others. $1ctq$ minus cash holdings $cheq$. Complementary, the gross debt position I define it as the short-term liabilities $1ctq$ only.
3. Operating profits π : corresponds to the variable $ibdpq$
4. Liquid value of assets $q\omega k(1 - \delta)$: to compute this I use the assets of the firm (excluding cash) as follows: for asset category a_{ij} we can compute the liquid value of firms' assets as $\sum_j lr_j \times a_{ij}$ where lr_j is the liquidation rate. The liquidation rates used by asset category are 44% inventories, 63% receivables and 35% physical capital from [Kermani and Ma \(2021\)](#).
5. Cash-on-hand n : is computed as the sum of π and $q\omega k(1 - \delta)$ minus b . It is assumed that all liabilities can be collected each period.
6. Size: log of total assets atq .

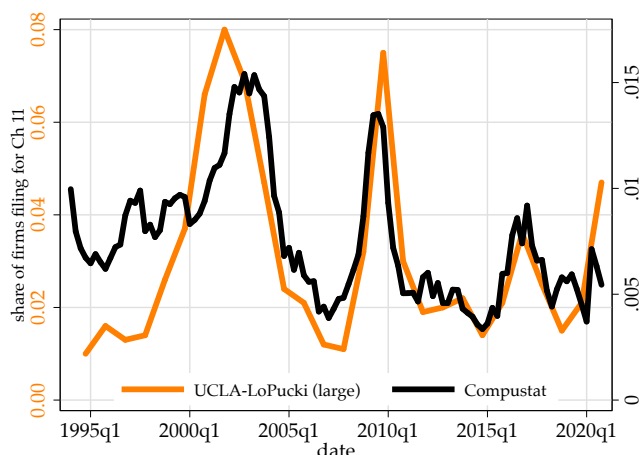
²⁸See for example, Mongey and Williams (2017); Jeena (2019); [Otonello and Winberry \(2020\)](#) for recent references.

7. Sales growth: quarterly growth of sales saleq .

Nominal variables are deflated using the BLS implicit price deflator, unless specified. Percentiles of variables used are constructed by year (not quarter). When specified variables are standardized, winzorized and/or demeaned.

Bankruptcy Data. To identify when and what firms operate under Chapter 11 in Compustat I use the same strategy as [Corbae and D’Erasmus \(2021\)](#). I use the footnote to total assets (atq) and deletion information variables dlrsn and dldte . A firm is in Chapter 11 if (i) footnote (next period) reports adoption of new accounting under Chapter 11 bankruptcy; (ii) if firm shows as bankrupt but is not deleted; (iii) if the firm shows as bankrupt and deleted but this is not due to liquidation; (iv) and if firm’s last observation in the sample is bankruptcy but there is no bankruptcy information.

Figure B.1: Filings to Chapter 11



Notes : Figures shows the flow of filings to Chapter 11 in the last 12 months. Filings to Chapter 11 are identified through the steps detailed in the text. Data sources: Compustat-Quarterly and UCLA-LoPucki.

Figure [B.1](#) shows that the Compustat data evolution is consistent with UCLA-LoPucki estimates which are for large firms in US.

B.1.2 Federal Judicial Center - Integrated Database (FJC-IDB).

FJC-IDB bankruptcy data includes all petitions filed under the Bankruptcy Code (any of the Chapters) on or after October 1, 2007 and any petitions filed before October 1, 2007 that are still pending. This dataset provides information of the fillings, closures and several firm characteristics.

I will focus on a sample of corporate firms filings to Chapter 11 and Chapter 7. This includes public and privately held firms. Table [B.2](#) shows the sample selection criteria.

Table B.2: Sample Selection FJC-IDB

	# Drop	# Obs
All		32,084,867
Corporations	31,546,855	538,012
Chapter 11 and 7	9,771	528,241
Filings	358,890	169,351
Closures	357,023	171,218

Notes: This table shows the number of observations resulting from the sample selection for the FJC. The first line, *All*, shows the original number of entries from in the dataset. from 2008 to 2020. Two samples are used, one that includes only the filings and other that includes only the closures (closure sample entries include filing characteristics). The following lines detail the set of observations dropped from different filters applied to the sample and the resulting number of observations. More details on these filters can be found in the text.

Data source: FJC-IDB.

B.2 Heterogeneous Investment Responses During Recent Crises

In this section I will study the heterogeneous investment response of firms during the Great Recession and Covid-19 crisis. First, I will show the aggregate dynamics of the crises. Second, I will show the heterogeneity across the balance sheet positions, focusing on cash-on-hand and leverage positions.

B.2.1 Aggregate Dynamics

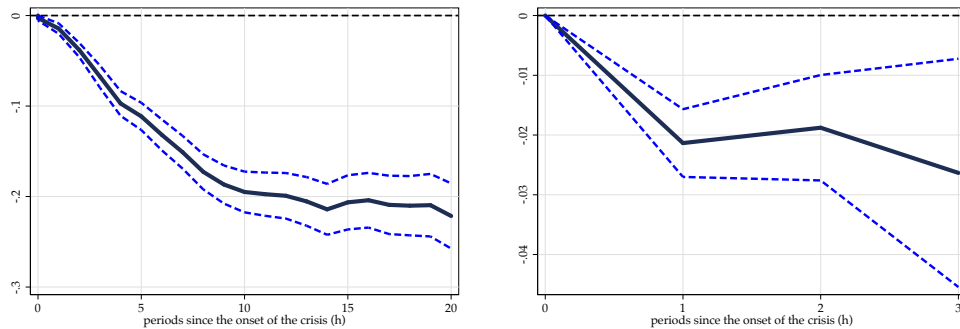
Figure B.2 shows that using firm level data of publicly listed firms the capital accumulation rate drop significantly in both episodes — Great Recession and Covid-19 crisis.

Figure B.2: Corporate Investment in Recent Crises Episodes

$$\Delta k_{t+h}^{\text{crisis}} - \Delta k_{t+h}^{\text{no crisis}}$$

(i) Great Recession

(ii) Covid-19 Crisis



Notes: figures show the dynamics of the capital stock relative accumulation during the Great Recession and Covid crisis. The change in capital accumulation comes from the followin specification using firm-level data: $\log(k_{it+h}) - \log(k_{it}) = \alpha_i + \beta_h \text{crisis}_t + \varepsilon_{it+h}$, where crisis_t indicates the pre-crisis peak and β_h is the h-periods ahead change in the accumulation of capital during the crisis episode relative to no crisis periods. Drop t such that for $\text{crisis}_{t+i} = 1$ for at least one $i \in \{0, \dots, h\}$, i.e. capital accumulation before the crisis overlaps with the crisis. Panels (a) and (b) show coefficients β_h and their 90% confidence interval. Standard errors are clustered at firm level..

Data sources: Compustat.

B.2.2 Investment Heterogeneous Response Estimation

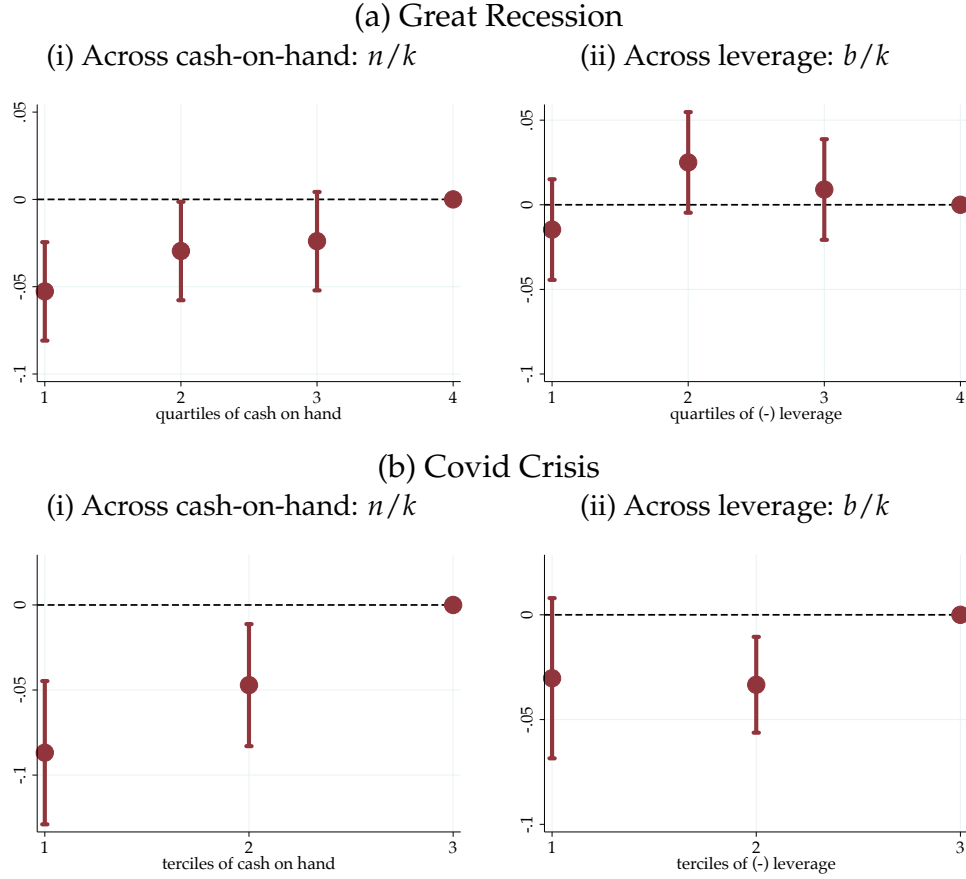
To estimate the *on impact* heterogeneous response — from peak-to-trough of the crisis — I will proceed as follows. First, to account for permanent sectoral heterogeneity — in my baseline estimations — I will demean each of the firm-quarter observations of variable x of interest by its sectoral average, i.e. $\hat{x}_{it} = x_{it} - \mathbb{E}_s[x_{it}]$. Next, I will assign each firm-quarter observation of x to different quartiles (terciles if Covid sample) relative to the annual distribution. Lastly, I run the following panel regression to estimate the heterogeneous responses of investment across cash-on-hand n/k and leverage b/k during the crisis:

$$\Delta \log(k_{it}) = \underbrace{\sum_{j=1}^J \beta_j^n \left(Q_{it}^{nj} \times \text{crisis}_t \right)}_{\text{het across } n/k} + \underbrace{\sum_{j=1}^J \beta_j^b \left(Q_{it}^{bj} \times \text{crisis}_t \right)}_{\text{het across } b/k} + \underbrace{\Lambda' Z_{it}}_{\text{controls}} + \varepsilon_{it}, \quad (25)$$

where Q_{it}^{xj} indicates if \hat{x}_{it} belongs to quartile or tercile j , $\Delta \log(k_{it}) = \log(k_{it+h}) - \log(k_{it})$ is the capital accumulation over a period as long as the crisis studied (i.e., the extension from peak-to-trough of episode studied h), crisis_t indicates if a crisis happens during the period considered and $Z_{i,t}$ includes the control variables. For the baseline specifications I include as controls firm's fixed effects, sectoral fixed effects, log assets as proxy for size and last quarter sales growth. The coefficients β_j^x are interpreted as the diff-in-diff estimates of the crisis impact on capital accumulation for firms in quartile or tercile j of \hat{x} .

The empirical strategy is close to the one used in other work that studies investment adjustment heterogeneity on recent crises episodes. Salient examples are [Almeida *et al.* \(2012\)](#) for the Great Recession in U.S. and [Kalemli-Özcan *et al.* \(2020\)](#) for the EU crisis.

Figure B.3: Heterogeneous Investment Response during Crises



Notes: Figures show the change in the capital accumulation from peak to trough in both episodes. For the Great Recession the episode is from 2007q4 to 2009q4, and for the Covid-19 crisis is from 2019q4 to 2020q2. Figures in panel (a) show the coefficient β_j^n and Figures in panels (b) shows β_j^b for the Great Recession and Covid-19 crisis in a joint estimation of specification (25). Coefficients are normalized to 0 with respect to the highest quartile or tercile coefficient. The interval is at 90% confidence level and standard errors are clustered at firm level for the Great Recession and sector level for the Covid-19 crisis. Balance sheet variables are demeaned at sectoral level. Because of data limitations the estimates of the Covid-19 crisis don't include firm's FE. Coefficients are in annual terms. Data sources: calculations using Compustat data.

Figure B.3 shows the investment response across different levels of cash-on-hand and leverage during the Great Recession and Covid crisis. For both episodes, panel (a) and (b) figure (i) show that firms with low levels of cash-on-hand adjust substantially more their investment, around 5-10 p.p. points in annual terms relative to the firms with the highest levels of cash-on-hand. On the other hand, panel (a) and (b) figure (ii) show that the heterogeneity across leverage is not significant. In Section 4.2, I contrast these results with simulations from the model.

B.3 Other Observations on the Firm's Balance Sheet

In this section I show further facts related to the liability structure (e.g. maturity and number of creditors) of firms that complement the baseline analysis. Table B.3 shows that corporate firms use extensively short-term liabilities to finance their investments and operations, and Table B.4 shows that the great majority of medium to large cor-

porate firms (i.e., with more than 50 million assets) in U.S. borrow from hundreds of creditors. This is complementary to the observation of [Crouzet \(2017a\)](#) that corporate firms financial leverage is mostly in bonds, which they are very likely to have a dispersed ownership. These observations supports the idea that the firms' creditors are likely dispersed and difficult to coordinate, unless the firm wants to incur in costs. Further, in Appendix [C.2](#), I show that the benefits of being able to manage the liability structure in the model are not large (ex-ante), therefore for moderate costs of changing their liabilities most firms will remain inactive.

Table B.3: Firms' Debt Maturity

	Time to mature (share)		
	< 1 year	1 to 4 years	≥ 5 years
Debt	0.29 (0.32)	0.33 (0.28)	0.38 (0.34)
<hr/>			
	< 1 year	> 1 years	
	0.61 (0.29)	0.39 (0.29)	

Notes: the table shows the share of debt or liabilities maturing at different time horizons. The summary statistic is computed for the average firm, in parenthesis is the standard deviation. Short-term liabilities are 1ct and long-term 1t - 1ct. Debt maturing in less than one year is d1c, in one to four years is dd2 + dd3 + dd4, and maturing at 5 or more years is d1tt-dd2-dd3-dd4. Total debt is d1c + d1tt and total liabilities is 1ct.

Data source: Compustat.

Table B.4: Number of Creditors When Filing to Bankruptcy

	# Creditors		
	1 to 100	101 to 1,000	>1,000
Small (< 50 million assets)	0.88	0.10	0.02
Medium (> 50 million and < 1 billion assets)	0.16	0.19	0.65
Large (> 1 billion assets)	0.03	0.04	0.93
All	0.74	0.10	0.16

Notes: the table shows the share of firms with by creditor number groups and size when filing to Chapter 11 bankruptcy. Shares are relative to the total filings of each size group. Asset value correspond to the one declared when filing for bankruptcy.

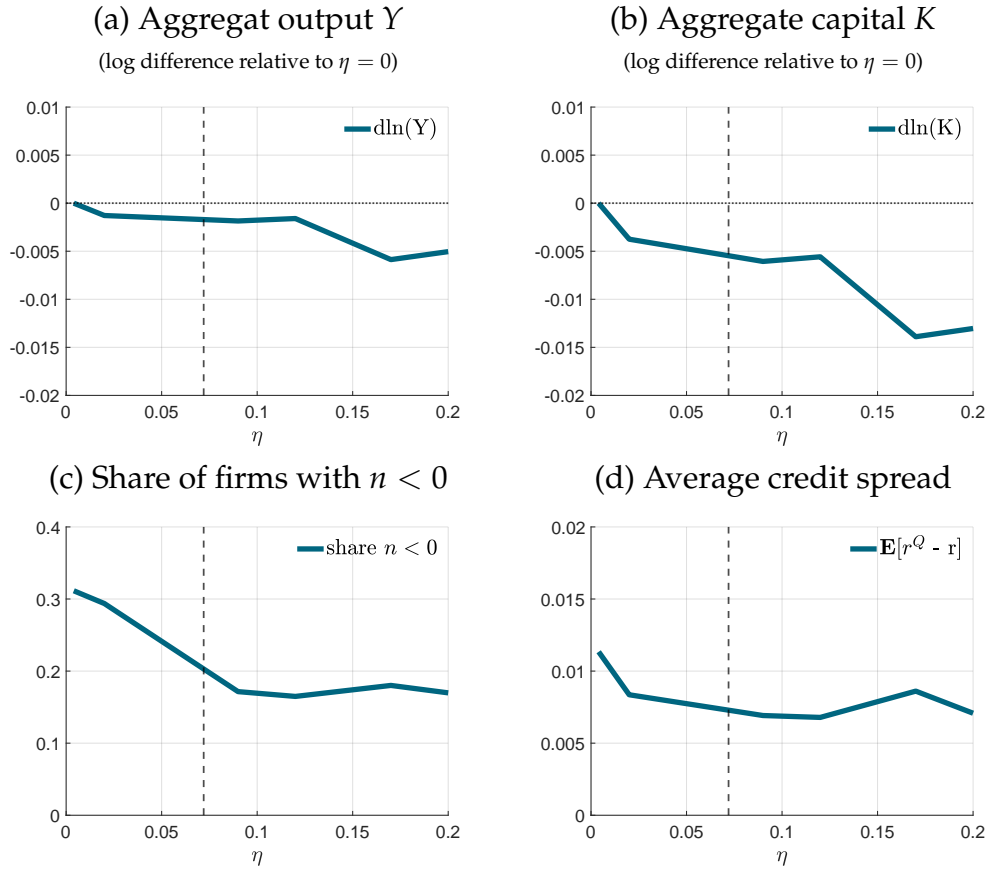
Data source: FJC-IDB.

C Appendix: Further Exercises and Extensions

C.1 Steady-state comparative statics

To study the long run implications of firms' rollover risk I do some simple comparative statics with η . Figure C.1 shows for different values of η the output and capital level, and the share of firms with negative cash-on-hand and the average spread rate. I find that the incidence of rollover problems in the long run is relatively low. First, in the long run, aggregate output Y is 0.2% lower, see panel (a), and aggregate capital K is 0.5% lower, see panel(b), because of creditors' coordination failures. Second, the higher is η less firms have a weak balance sheet position in steady-state, see panel (c). Rollover risk shifts (improves) the financial distribution of firms significantly. The increase in the risk of rollover failure, for a given financial position, incentivize firms to save away; thus, accumulating internal resources to preclude the coordination failure. The improvement in the financial position is reflected on the little change observed in credit spreads across η , even if the risk of rollover problems is greater (given the financial position). Overall, the likelihood of coordination failures for exposed firms η shifts the financial position of firms, but don't impact significantly aggregate outcomes over the long run.

Figure C.1: Steady-state Comparison



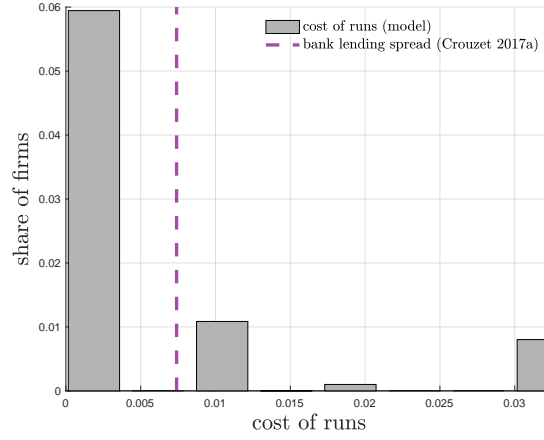
Notes : Panel (a) and (b) show the log difference in aggregate output Y and capital K , respectively, across different values of η in steady-state. Panel (c) and (d) show the share of firms with negative cash-on-hand n and average credit spread rate across, respectively, across different values of η in steady-states. In all the plots, the vertical dashed line indicates the calibrated value of η .

C.2 How costly is firm's rollover risk (ex-ante)?

In this section to asses how costly is rollover risk. For this, I explore the spread distribution between the pricing schedule with and without coordination failures, i.e., $\tilde{Q}(z, k', b'; \eta) - \tilde{Q}(z, k', b'; 0)$. As a benchmark, I compute how many firms would pay the bank's markup over market borrowing solely to preclude future creditors' coordination failures if they could. I use the spread in intermediation costs estimated by Crouzet (2017a) of 0.74% (annual). I find that only 2.2% of the firms face a cost of rollover risk higher than intermediation spread. Figure C.2 shows the distribution of the cost of rollover risk across firms that produce and don't exit at the end of the period. The figure shows that most firms face a cost close to 0 since many firms become exposed tomorrow only in case of an extremely bad shock, therefore the average cost is negligible. The small cost of rollover risk ex-ante in steady state suggests that it can be optimal for firms choose a liability structure where they are exposed to rollover

problems.

Figure C.2: Cost of Rollover Risk (in annual spread terms)



Notes : Figure shows the distribution of the cost of firms' rollover risk — i.e., $\tilde{Q}(z, k', b'; \eta) - \tilde{Q}(z, k', b'; 0)$ — for producing firms that don't exit at the end of the period. Exclude from plot the ones with 0 cost and truncated distribution at 3% cost. The spread of intermediation between bank and market lending is from the calibration of [Crouzet \(2017a\)](#).

C.3 Model Extensions and Multiple Equilibrium

In this section, I will study two extensions of the model, one that uses more general functional forms for the operational profits function, capital adjustment idiosyncratic frictions and long-term debt, and other that allows firms to issue equity (costly).

Long-term debt and capital adjustment frictions. I assume profits are a function $\pi(\mathbf{z}, k) \in \mathbb{R}$ strictly increasing in both arguments, where $\mathbf{z} = (\mathbf{z}^p, \mathbf{z}^{\text{iid}})$ is a vector of shocks that contain a set of persistent shocks \mathbf{z}^p follow a markov process and \mathbf{z}^{iid} follow an iid process. Both are related to idiosyncratic productivity and cost shocks. Next, I assume that $\iota(\omega k, k') \in \mathbb{R}$ is the investment expenditure function of the firm that is decreasing on k and increasing on k' , where $-\iota(\omega k, 0) \geq 0$ is the liquidation value of capital.²⁹ Last, I assume that the firm can issue long-term debt, which fraction $m^b \in (0, 1]$ matures randomly each period and pays $c^b \geq 0$ coupon payments on non-maturing debt. The rest of the model it follows as the baseline model.

I focus on the characterization of the liquidation choice. For the extended setup, firms dividends now can be defined as

$$d = \pi(\mathbf{z}, k) - \iota(k, k') - b \left[m^b + (1 - m^b) c^b \right] + Q(.) \left(b' - (1 - m^b) b \right) \geq 0$$

²⁹I assume no capital quality shock ω for notational clarity.

where $Q(\cdot) \left(b' - (1 - m^b) b \right)$ is the amount of new debt issued. Analogous to the baseline model, firms can default after issuing the new debt. The firm never default whenever

$$\begin{aligned} \max_{k'} \pi(\mathbf{z}, k) - \iota(k, k') - b \left[m^b + (1 - m^b) c^b \right] = \\ \underbrace{\pi(\mathbf{z}, k) - \iota(k, 0) - b \left[m^b + (1 - m^b) c^b \right]}_{n(\mathbf{z}, k, b)} \geq 0. \end{aligned} \quad (26)$$

where I can define n as the cash-on-hand of the firm is the sum of operational profits, liquidation value of capital, and maturing debt and coupon payments. On the other hand, we have that the firm will always default whenever

$$\begin{aligned} \pi(\mathbf{z}, k) - b \left[m^b + (1 - m^b) c^b \right] + \max_{k', b'} \left\{ -\iota(k, k') + \tilde{Q}(\mathbf{z}^p, k', b') \left(b' - (1 - m^b) b \right) \right\} = \\ n(\mathbf{z}, k, b) + \underbrace{\max_{k', b'} \left\{ -\iota(k, k') + \iota(k, 0) + \tilde{Q}(\mathbf{z}^p, k', b') \left(b' - (1 - m^b) b \right) \right\}}_{-\underline{n}(\mathbf{z}^p, k, b)} < 0 \end{aligned} \quad (27)$$

For multiplicity to exists we need that conditions (26) and (27) don't hold, i.e.,

$$0 > n(\mathbf{z}, k, b) \geq \underline{n}(\mathbf{z}^p, k, b). \quad (28)$$

Notice $\underline{n}(\mathbf{z}^p, k, b)$ bounded below by 0 (we can always implement $\{k' = 0, b' = b\}$). Moreover, there is the possibility of multiple equilibrium whenever the firm can have strictly positive external resources in this region of the state-space. Analogous to the baseline model, the firms default decision is determined by the firm's cash-on-hand and a threshold that depends on the fundamentals of the firm (shocks and financial position).

Further, assume there is no bankruptcy, $c^b = 0$ and creditors have no recovery for clarity, then the fundamental pricing schedule $\tilde{Q}(\cdot)$ (without coordination problem today) is pinned down by creditors no profit condition and is

$$\tilde{Q}(\mathbf{z}^p, b', k') = \mathbb{E} \left[\Lambda \left(\mathbf{1}_{\{n \geq \underline{n}\}} - \eta \mathbf{1}_{\{0 > n \geq \underline{n}\}} \right) \left(m^b + (1 - m^b) \tilde{Q}' \right) \right]. \quad (29)$$

The pricing schedule with long-term becomes recursive. Also tomorrow's coordination failures show up in the pricing schedule. These two observations suggest, in the firm problem with long-term debt, rollover problems could even be greater than in the baseline model. With long-term debt the pricing schedule is affected by the future stream of expected rollover problems, which can augment their impact.

Equity issuance. In the baseline specification, I don't allow firms to issue equity — $d \geq 0$. This assumption is consistent with the relatively low equity issuance observed in the data, and helps on the tractability of the characterization and computational solution of the model. In this section, I will relax this assumption and show how this affects the characterization of the liquidation problem (equilibrium multiplicity). Moreover, I provide a discussion on the model concepts of rollover and solvency in the model.

Firms issue equity $e < 0$ at cost $\phi(e)$, which is decreasing in e and unbounded. I assume that equity is raised at the end of the period. Therefore, firms that never default are those when $Q = 0$ they don't default, i.e.,

$$V^{Q=0}(z, n) \geq 0$$

where $V^{Q=0}(z, n)$ is determined by

$$V^{Q=0}(z, n) = d + \mathbb{E} \left[\Lambda \tilde{V}(s') \right]$$

subject to

$$\begin{aligned} d &= \begin{cases} e & \text{if } e \geq 0 \\ e - \phi(e) & \text{if } e < 0 \end{cases} \\ e &= n - qk' \end{aligned}$$

where continuation value $\tilde{V}(s')$ is analogous to one defined in the baseline firm problem. Thus, we can define safe region

$$\mathcal{S} = \{(z, n) : V^{Q=0}(z, n) \geq 0\}. \quad (30)$$

On the other hand, firms that default are those default even if $Q > 0$, i.e.,

$$V^{Q>0}(z, n) < 0$$

where $V^{Q>0}(z, n)$ is determined by

$$V^{Q>0}(z, n) = d + \mathbb{E} \left[\Lambda \tilde{V}(s') \right]$$

subject to

$$d = \begin{cases} e & \text{if } e \geq 0 \\ e - \phi(e) & \text{if } e < 0 \end{cases}$$

$$e = n + \tilde{Q}(z, b', k') b' - qk'$$

where \tilde{Q} fundamental pricing schedule (no liquidation today) and continuation value $\tilde{V}(s')$ (this analogous as the one in the baseline firm problem). Thus, we can define liquidation region

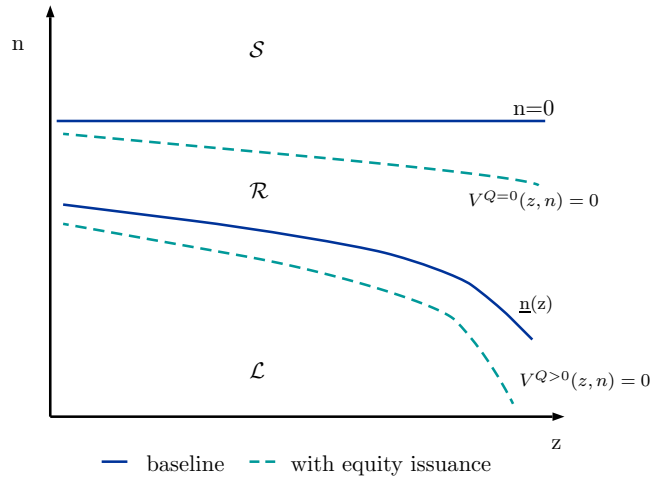
$$\mathcal{L} = \{(z, n) : V^{Q>0}(z, n) < 0\}. \quad (31)$$

Last, it's straightforward to show that $V^{Q>0}(z, n) \geq V^{Q=0}(z, n)$, then under certain conditions it can be the case that firm is in a region that is undetermined, i.e.,

$$\mathcal{R} = \{(z, n) : V^{Q>0}(z, n) \geq 0 \text{ and } V^{Q=0}(z, n) < 0\}. \quad (32)$$

Similar to the baseline mode, we have that firms can be exposed to coordination failure even if they can issue equity. Assume the equity issuance function $\phi(e) = \lambda|e|$ with $\lambda > 0$. In Figure C.3, I illustrate how these affects the characterization of the regions.

Figure C.3: Rollover and solvency regions across (z, n)
Baseline vs Equity Issuance model



Notes: figures shows the state-space (z, n) and the relevant regions for the liquidation choice for the baseline model (solid blue lines) and the model with equity issuance (dashed cyan lines).

Finally, it's worth noticing that in the model with unbounded equity issuance firms in \mathcal{L} threshold — $V^{Q>0}(z, n) = 0$ — have 0 value, which is the standard notion of economic insolvency. On the other hand, in the baseline model, or with bounded

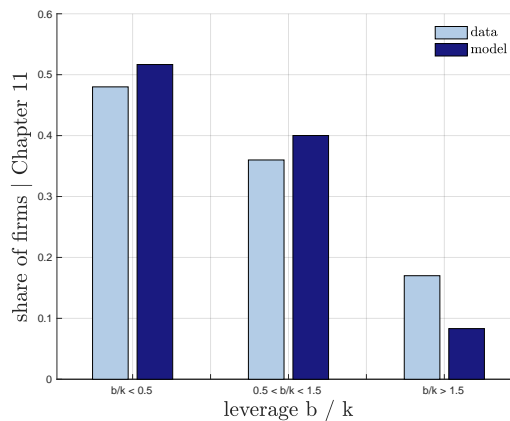
equity issuance, firms in \mathcal{L} threshold ($\underline{n}(z)$) could have strictly positive value. For my calibration, I find that firms in the insolvency threshold have values close to 0 — $V(z, \underline{n}(z)) \approx 0$ —; therefore, it approximates well the standard notion of insolvency.

C.4 Bankruptcy Analysis: Data and Model

In this section, I compute moments related to bankrupt firms, which are not targeted in the baseline calibration. I use Compustat data, and construct the data and identify bankrupt firms following the steps described in Appendix B.1.1. For this analysis I will focus on Chapter 11 events where the firm is operating next period.³⁰ First, I will compare in the data and model the leverage position distribution of bankrupt firms and, next, I will compare how the firm's financial characteristics — in the data and model — predict a bankruptcy event.

Leverage (b'/k') distribution. Although I target the average leverage ratio of firms in Chapter 11, I don't target its distribution. I compare the share of bankrupt firms according to their leverage (liabilities over capital) position. I split in three groups, those bankrupt firms that have choose low leverage (less than 0.5), medium to high leverage (0.5 to 1.5) and extremely high leverage (more than 1.5). Figure C.4 shows that the model fits well the distribution of leverage for firms in Chapter 11 that continue operating.

Figure C.4: Leverage distribution of bankrupt (restructure) firms



Notes: Figure shows the distribution of leverage for firms in the restructure process in the model and the data (in Chapter 11 and operating the next period). Data source: Compustat.

Bankruptcy predictors. Now I study how the firm's lagged financial variables predict the firm's bankruptcy. For this I ran the following regression (in the data and

³⁰There are few events of Chapter 7 for publicly traded firms and Chapter 11 events that lead to liquidation are difficult to identify.

model)

$$\mathbf{1}_{i,t}^{\text{ch11}} = \beta X_{i,t-1} + \alpha_t + \alpha_i + \alpha_s + \epsilon_{i,t}$$

where $\mathbf{1}_{i,t}^{\text{ch11}}$ indicates if the firm i in period t is in Chapter 11 and operating (restructure instead of liquidation), α_i are firm FE, α_s sector FE, α_t time fixed effects and $X_{i,t-1}$ is a vector of characteristics (predictors) of interest lagged one period. In my baseline specification I include in $X_{i,t}$ the size of the firm (assets in logs), real quarterly growth of sales, the cash-on-hand and leverage positions. I standardize all variables in $X_{i,t-1}$ and winzorized the cash-on-hand n/k' and leverage b'/k' at level 0.5% and 99.5%.

Table C.1 shows the results. For all the specifications the coefficients in the model and data have the same sign. In the full specification (3) I find that lower sales growth, smaller size, low cash-on-hand and (surprisingly) low leverage predicts a higher restructure likelihood next period. Further, the magnitudes are relatively similar between the model and data. The relation between leverage and bankruptcy could be explained is reverted if we exclude cash-on-hand from the specification (in (2)). One potential explanation for this counter-intuitive relation is that firms suffering from rollover problems given their financial position — summarized by n — are more likely to restructure if they are in better shape given the costs of bankruptcy.

Table C.1: Predictors of Chapter 11 - Data vs Model

	dependent variable: $\mathbf{1}_{i,t}^{\text{ch11}}$					
	(1)		(2)		(3)	
	data	model	data	model	data	model
$n_{i,t-1}/k_{i,t}$	-0.39 (0.03)	-0.05			-0.39 (0.10)	-0.45
$b_{i,t}/k_{i,t}$			0.11 (0.04)	0.03	-0.29 (0.09)	-0.41
$\log(k_{i,t-1})$	-0.50 (0.12)	-0.06	-0.52 (0.12)	-0.06	-0.49 (0.12)	-0.10
$d \log(\text{sales}_{i,t-1})$	-0.04 (0.00)	-0.03	-0.04 (0.00)	-0.02	-0.04 (0.00)	-0.01
Sector FE	Y		Y		Y	
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y		Y		Y	
Observations	370,973		373,362		370,973	

Notes: This table shows the baseline results of the regression using bankruptcy outcomes in the data and the model simulations. All variables are standarized, and leverage and cash-on-hand are also winsorized at level 0.5% and 99.5% and demeaned relative to the sectoral average. Standard errors (in parenthesis) are clustered by firm. Coefficients are times 100.

Data source: Compustat quarterly.

D Appendix: U.S. Bankruptcy Code Institutional Details

In this section, I provide a brief review of some insitutional details of the bankruptcy process for firms in the U.S. bankruptcy code. Chapter 7 and 11 are the typically used to liquidate or restructure the firm's liabilities. Chapter 7 is associated with firm liquidations, and Chapter 11 with restructures (or sometimes called "reorganizations") and liquidations through piecemeal sales of firms.

Chapter 7 bankruptcy. Firms can enter a Chapter 7 liquidation process by filing directly to this chapter or being redirected by court ruling from other chapters (e.g., a judge may rule that a Chapter 11 case is switched to a Chapter 7). In this process, a case impartial trustee is appointed by the court to gather and sell the bankrupt firms assets to pay the firm's creditors.

Chapter 11 bankruptcy. Cases begin usually with the voluntary filling of the debtor (firm). Involuntary petitions (done by creditors) are very rare. When filling the firm au-

tomatically assumes an additional identity as the "debtor in possession." by 11 U.S.C. § 1101. The DIP provisions can provide access to new credit for the firm (DIP financing) and the automatic stay of firm's debt payments by 11 U.S.C. § 362(a) preclude (most) creditors from collecting the firm's debt. Further, when filing automatically a creditor's committee is appointed, which typically consists of the unsecured creditors who hold the seven largest unsecured claims against the debtor. Further, is common that creditors form ad hoc committees to coordinate their actions and have further surveillance over the debtor-in-possession's management of the firm.

The firm usually files a written disclosure statement and a reorganization plan. The disclosure statement contains information of the firms' assets, liabilities and other business affairs. Typically, the disclosure statements contains a counterfactual analysis of the credit recovery rates under liquidation (liquidation analysis) and other information relevant for the judge to decide if the reorganization chapter is appropriate. Lastly, the plan presented by the creditors needs to be approved by the creditors for the restructure to be executed.

Moreover, Chapter 11 process are sometimes used by large firms to do a piecemeal liquidation of the firm. The provisions provided by 11 U.S.C. § 363(b), "363 sales", allow firms to liquidate part of the firm's assets without the creditors' consent. This process is closer to a Chapter 7 "piecemeal" liquidation of the firm, instead of a restructure or reorganization.