Firms' Rollover Risk and Macroeconomic Dynamics

Rafael Guntin

NYU

Job Market Paper

Click here for most recent version

November 23, 2021

Motivation

- Financial factors are relevant drivers of large economic downturns
  - e.g., through the failure of firms with good fundamentals

This paper

- Focuses on events where firms fail because of **rollover crises**
  - economically solvent firm defaults because its creditors fail to roll over the debt
  - alluded to by policymakers during recessions and top managers of bankrupt firms
- **Quantitative framework** where firms' rollover crises can be identified and quantified

Implications of **firms' rollover risk** in macro dynamics and policy?

What I Find

1. How **relevant** are firms' rollover crises?
   - + **1.6%** firms rollover crises = 21% exposed $\times$ 7% probability
     (roughlt half of bankruptcy events are driven by rollover crises)
   - ▶ indirect inference using bankruptcy outcomes and financial distribution of firms
   - ▶ identification: restructure process "forces" creditors to rollover
2. What are the **macroeconomic** implications of rollover risk?
   - + rollover risk **amplify** significantly impact of recessions
     (more output drop and persistence)
   - ▶ explain between 10% to 30% of aggregate output losses
3. What are the **policy** implications?
   - + credit policy can **undo rollover crises** amplification of crises, but can **backfire**
   - ▶ focus on imperfectly-targeted credit policy

How I Do It

- Build on GE models of heterogeneous firms with **default**
  building block Khan Senga Thomas 2016 and Ottonello Winberry 2020, among others

- Borrow from international macro literature to allow for **rollover crises**
  mechanism in Cole Kehoe 2000 and Bocola Dovis 2020

- Identify incidence of rollover crises using model-based strategy **informed** by distribution of bankruptcy outcomes and bankrupt firm's characteristics
  insights from Corp Law literature, e.g., Jackson 1986; framework Corbae D'Erasmo 2021

- Conduct **quantitative** analysis of U.S. economy to assess aggregate relevance of firms' rollover risk and study policy implications
  novel results on rollover crises amplification channel and imperfectly-targeted policies workings

related papers   related papers on rollover crises

Outline

- Macro Model of Firms' Rollover Risk
- Identifying Rollover Crises
- Macroeconomic Consequences

Outline

- **Macro Model of Firms' Rollover Risk**
- Identifying Rollover Crises
- Macroeconomic Consequences

Overview of the Model

- Quantitative GE models of heterogeneous firms with default

  Khan Senga Thomas 2017; Ottonello Winberry 2020

- Extend model to

  1. possibility of coordination failures among creditors á la Cole Kehoe 2000
  2. allow debt restructuring similar to Corbae D'Erasmo 2021

- Study unforseen crises and policy shocks (MIT shocks)

Environment

- Infinite horizon and discrete time
- Four types of agents
  1. **nonfinancial firms**: invest and produce to maximize their value
  2. **creditors**: lend to nonfinancial firms, and are perfectly competitive and atomistic
  3. **capital producer**: sell capital to nonfinancial firms
  4. **representative HH**: consumes, saves and works. Owns all firms in the economy

Nonfinancial Firms' Environment

- Firm i objective is to max

$$\sum_{t\geq 0} \mathbb{E}_0[\Lambda_t d_{it}]$$

  with Λ_t HH's SDF and d_t firm's dividends

- Idiosyncratic state variables:
  1. s_{it}^f exogenous
  2. s_{it}^{nf} exogenous non-fundamental
  3. s_{it}^e endogenous

  where $s_{it} = (s_{it}^f, s_{it}^{nf}, s_{it}^e)$

- For clarity, drop i and t subscripts

Overview of Nonfinancial Firms' Timing

Within period timing is as follows (firms with no exit shock)

1. **All** uncertainty about fundamentals and nonfundamentals is realized
2. **Restructure** choice
3. Investment and **new debt** issuance choice
4. **Liquidation** choice (if continue in 2)
5. Firms produce and distribute dividends (if don't liquidated in 4)

Nonfinancial Firms' Production

- Operate with technology

$$f(z, \omega, k, l) = z(\omega k)^{\alpha} l^{\nu}$$

  - decreasing returns to scale $\nu + \alpha < 1$
  - idiosyncratic persisent productivity shocks $\ln z' = \rho_z \ln z + \epsilon_z$ with $\epsilon_z \sim N\left(0, \sigma_z^2\right)$
  - idiosyncratic capital quality shock ω iid log-normal trunc. where $\ln \omega \in \left[\underline{\omega}, 0\right]$
    (fit quantitative default rate)

- Own capital k and hire labor l at wage w, then operating profits are

$$\pi(z, \omega, k) = \max_{l} f(z, \omega, k, l) - wl$$

Nonfinancial Firms' Financial Resources

- **Internal** resources (cash-on-hand)
  - inherit k at price q which depreciates at $\delta \in [0,1]$ and maturing b, and has $\pi(z,\omega,k)$

$$n = \underbrace{\pi(z,\omega,k)}_{\text{operational profits}} + \underbrace{(1-\delta)q\omega k}_{\text{selling value of capital}} - \underbrace{b}_{\text{maturing debt}}$$

- (net) **External** resources
  - issues one-period debt b' at price schedule $Q(.)$ and buys k' at price q

- **Dividends**
  - distributed at end of period

$$d = \underbrace{n}_{\text{cash-on-hand}} + \underbrace{Q(.)b'}_{\text{new debt issuance resources}} - \underbrace{qk'}_{\text{capital purchases}}$$

Nonfinancial Firms' Financial Frictions

1. No-equity issuance constraint $d \geq 0$ (data low eq issuance, standard and simplify)

2. Debt is defaultable in two ways

   (a) liquidation (Chapter 7)

   - non selective default on b and b'
   - firm exits with $V = 0$ and creditors of b recover $\alpha_7 \in [0, 1]$ of the liquidated capital

   (b) restructuring (Chapter 11)

   - (Nash) bargain debt recovery rate $\alpha_{11} \in (0, 1)$ over debt b detail
   - firm pays exogenous cost $c_{11} \in [0, 1]$, which is proportional to capital
   - **no coord failure** (bankruptcy provisions, Corp Law)
   - resources after restructuring are: $n_{11} = \pi(z, \omega, k) + (1 - c_{11})(1 - \delta)q\omega k - \alpha_{11} b$

US bankruptcy code

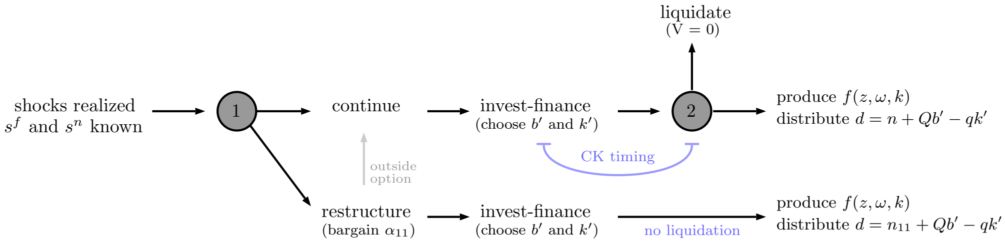
Nonfinancial Firms' Entry/Exit

Technical and quantitative assumptions

- Exogenous exit probability γ (KST 2016, stationary dist)
  - if receive shock the firm exits after production
- Entrants enter on average productivity $m\%$ below ergodic distribution average (OW 2020, life-cycle firms)

details

Within Period Timing Nonfinancial Firms' Problem



- Timing for non-exiting firms
- Cole Kehoe 2000 (CK) timing for liquidation choice

Multiple Equilibrium Intuition

- Liquidate if they can't satisfy $d \geq 0$ (result of assumptions)
- Debt price schedule from no-profit condition of creditors and dividends are

$$Q = \underbrace{\mathbb{I}_{d \geq 0}}_{\text{liquidation choice}} \underbrace{\tilde{Q}}_{\text{price if no liquidation}}$$

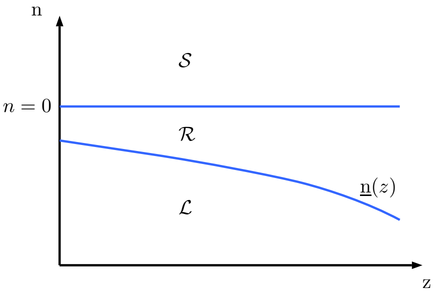
$$d = n - k' + Qb'$$

- Feedback between liquidation choice and prices today could create multiple outcomes

$$Q = 0 \iff d < 0$$
$$Q > 0 \iff d \geq 0$$

Liquidation

- **Fundamental** state-space (z, n) is divided in three regions   liquidation proposition



  - $\mathcal{S}: Q = 0$ then continue if $d = n + \underbrace{\max_{k'}\{-k'\}}_{0} > 0$
  - $\mathcal{L}: Q = \tilde{Q}$ then liquidate if $d = n + \underbrace{\max_{k',b'}\{-k' + \tilde{Q}b'\}}_{-\underline{n}(z)} < 0$
  - $\mathcal{R}$: liquidate if $Q = 0$, continue if $Q = \tilde{Q} > 0$

- Define **idiosyncratic sunspot** shock $\phi \sim^{\text{iid}} U[0, 1]$ draw every period, such that if $(z, n) \in \mathcal{R}$ and $\phi < \eta$ then coord in $Q = 0$ (coord failure)

more general setup   costly equity issuance

Restructure

Bargain outside option is to continue bargain protocol then conditions are

- necessary condition: firms with rollover crises (in $\mathcal{R}$ with $\phi < \eta$) or insolvent (in $\mathcal{L}$)
- sufficient condition: both better-off participating
  1. creditors: $\alpha_{11} > \min\{1, \alpha_7 \frac{(1-\delta)q\omega k}{b}\}$
  2. firm: $n_{11} > \underline{n}(z)$

Costs and **benefits** for firms

1. c_{11} cost (proportional to capital)
2. $1 - \alpha_{11}$ debt haircut
3. no coord failure, i.e., $Q = \tilde{Q}$

observation

if c_{11} large and $(1 - \alpha_{11})$ low $\Rightarrow$
firms with rollover crises **restructure**

Nonfinancial Firm's Recursive Problem

- V value of firm before exit shock and restructure choice with $s = (z, \omega, \phi, k, b)$

$$V(s) = (1-\gamma)\left[1_{\{ch11\}}(s)\,\tilde{V}(z, n_{11}) + 1_{\{cont\}}(s)\,\tilde{V}(z, n) + 1_{\{ch7\}}(s) \times 0\right] + \gamma V_{exit}(s) \tag{1}$$

where indicators follow from previous results, $V_{exit}(s)$ value of exiting firm details and

$$n = \pi(z, \omega, k) + (1-\delta) q \omega k - b$$
$$n_{11} = \pi(z, \omega, k) + (1-c_{11})(1-\delta) q \omega k - \alpha_{11}(s) b$$

- $\tilde{V}(z, n)$ value of the solvent firm and without rollover crisis today is

$$\tilde{V}(z, n) = \max_{d, k', b'} d + \mathbb{E}_{(z' | z; \omega'; \phi')}\left[\Lambda V\left(s'\right)\right] \tag{2}$$

subject to $d = n - qk' + \tilde{Q}\left(z, b', k'\right) b' \geq 0$, where $\tilde{Q}(.)$ debt price without coord failure

Corporate Debt Prices

- $Q = [1 - \mathbf{1}_{ch7}(s)]\tilde{Q}$ from creditor's no profit condition
- $\tilde{Q}$ determined by (discounted) $\mathbb{E}$[prob tomorrow's bankruptcy events]

$$\begin{aligned}\tilde{Q}\left(z,k',b'\right) = &(1-\gamma)\,\mathbb{E}_{(z'|z,\omega',\phi')}\left[\Lambda 1_{\{continue\}}\left(s'\right)\times 1\right]\\&+(1-\gamma)\,\mathbb{E}_{(z'|z,\omega',\phi')}\left[\Lambda 1_{\{Ch11\}}\left(s'\right)\times \alpha_{11}\left(s'\right)\right]\\&+(1-\gamma)\,\mathbb{E}_{(z'|z,\omega',\phi')}\left[\Lambda 1_{\{Ch7\}}\left(s'\right)\times R\left(k',b',\omega'\right)\right]\\&+\gamma\tilde{Q}_{exit}\left(z,k',b'\right)\end{aligned}$$

where

- $\alpha_{11}(s)$ recovery rate of creditors if restructure bargain protocol
- $R(k,b,\omega) = \min\{1, \alpha_7(1-\delta)q\omega k/b\}$ recovery rate if liquidated
- $\tilde{Q}_{exit}\left(z,k',b'\right)$ debt price conditional on exit shock Q with exogenous exit

long-term debt example

Other Agents and Equilibrium

Agents

1. **HH**'s choices are determined by Euler eq, SDF Λ and labor supply eq detail
2. **K producer** problem sells capital at price q and has a standard aggregate capital adjustment function detail
3. **Creditors** price debt through no-profit condition (SDF Λ) detail

Equilibrium

- Steady-state (law of motion fixed point) full definition law of motion firm distribution

Outline

- Macro Model of Firms' Rollover Risk
- **Identifying Rollover Crises**
- Macroeconomic Consequences

Identification

Questions

1. How many firms are in $\mathcal{R}$?
2. Value of η?

Steps

- Calibration of **standard** parameters to match relevant moments of U.S. economy
- Calibration of parameters related to **bankruptcy procedure** and identify η
- Steady-state **financial distribution** determines share of firms by region

Data sources

- NIPA, Compustat, Federal Judicial Center-IDB, LBD, related papers

Calibration Standard Parameters

Calibration strategy

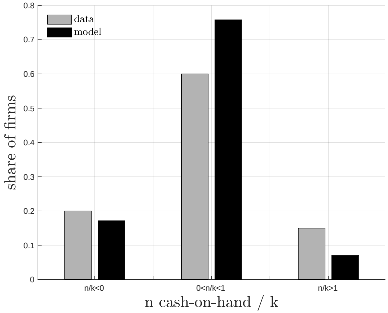
- 9 fixed and 4 fitted parameters not related to bankruptcy process
- params: pref, techno, stoch proc, entry/exit
- fit moments: emp, invest, balance sheet, life-cycle

Parameter	Value	Calibration
Fixed		
$\beta = 1/(1+r)$	0.99	fixed to $r = 0.05$ annual
Φ	1.16	fixed to match 58% emp rate
ν	0.64	fixed labor share
α	0.21	fixed capital share
δ	0.025	fixed to match BEA quarterly
ρ_z	0.90	fixed
γ	0.02	fixed to exit rate w/o default
ψ	2	agg AC fixed to lit standard
b_0	0	fixed to no net debt entrants
Fitted		
σ_z	0.032	internally calib
$\underline{\omega}$	−0.33	internally calib
k_0	0.16	internally calib
m	−0.24	internally calib

Relevant Moments

Moment	Data	Model
Aggregates		
K/Y	3.00	2.59
I/Y	0.17	0.15
gross debt: $\mathbb{E}[\,1_{b>0}b]/Y$	1.05	1.79
Credit spreads		
cred spread: $\mathbb{E}[r^Q - r]$	2.2%	0.7%
Investment heterogeneity		
avg invest rate: $\mathbb{E}[i/k]$	0.12	0.17
sd invest rate: $SD[i/k]$	0.34	0.36
Life-cycle		
share exit	0.10	0.11
(L age 1) / L	0.03	0.04
# firms age 1 / # firms	0.10	0.11
# firms age 2 / # firms	0.08	0.09

Moment	Data	Model
Balance sheet		
avg leverage: $\mathbb{E}[1_{b>0}b'/k']$	0.37	0.72
correl (n, k')	0.74	0.23
n distribution		

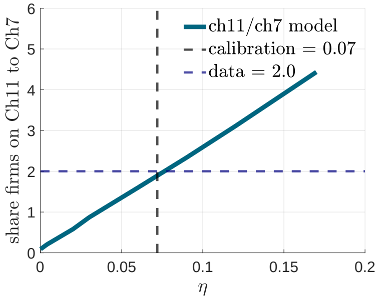


measurement

Identification of η

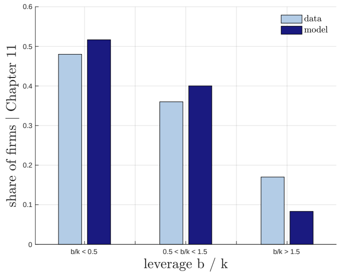
- $(\alpha_7, \psi_{11}, c_{11})$ match debt haircut under Ch 11 and Ch 7, and leverage in Ch 11

Param.	Value	Moment targeted	Data	Model
α_7	0.38	$\mathbb{E}[R]$	0.27	0.29
ψ_{11}	0.89	$\mathbb{E}[\alpha_{11}]$	0.69	0.82
c_{11}	0.40	$\mathbb{E}[b'/k' \mid \text{Ch } 11]$	0.73	0.67
η	0.07	$\mathbb{E}[\text{Ch11}]/\mathbb{E}[\text{Ch7}]$	2.0	1.9



Untargeted Moments of Bankruptcy

Distribution of leverage b'/k'
firms in Chapter 11



Predictors of Chapter 11

dependent variable: $1^{ch11}_{i,t}$

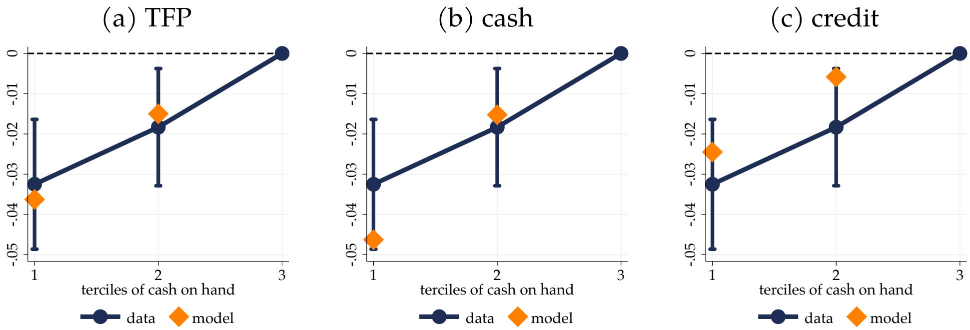
	(1)		(2)		(3)	
	data	model	data	model	data	model
$n_{i,t-1}/k_{i,t}$	-0.39	-0.05			-0.39	-0.45
	(0.03)				(0.10)	
$b_{i,t}/k_{i,t}$			0.11	0.03	-0.29	-0.41
			(0.04)		(0.09)	
$\log(k_{i,t-1})$	-0.50	-0.06	-0.52	-0.06	-0.49	-0.10
	(0.12)		(0.12)		(0.12)	
$d\log(\text{sales}_{i,t-1})$	-0.04	-0.03	-0.04	-0.02	-0.04	-0.01
	(0.00)		(0.00)		(0.00)	
Sector FE	Y		Y		Y	
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y		Y		Y	
Observations	370,973		373,362		370,973	

empirical specification: $1^{ch11}_{i,t} = \beta X_{i,t-1} + \alpha_t + \alpha_i + \alpha_s + \epsilon_{i,t}$

Untargeted Investment Heterogeneity

- Heterogeneity in Δk during last large recessions empirical specification measurement shocks
- Data and model simulation for Great Recession and Covid episodes

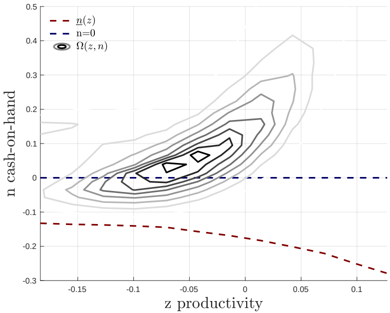
Heterogeneity of $\Delta k(\text{crisis}) - \Delta k(\text{no crisis})$ across n/k



note: simple average of both episodes estimates individual episode empirical results

Incidence of Rollover Crises

Steady state distribution $\Omega(z, n)$
before bankruptcy choice



$$\underbrace{\int_{(z,n)\in\mathcal{R}} d\Omega(z,n)}_{\substack{\text{share of firms exposed} \\ 0.20}} \times \underbrace{\eta}_{\substack{\text{run likelihood} \\ 0.07}} = 1.6\%$$

Result I: 1.6% of firms are subject to rollover crises

$\Rightarrow$ roughly half of bankruptcy events are driven by rollover crises

Outline

- Macro Model of Firms' Rollover Risk
- Identifying Rollover Crises
- **Macroeconomic Consequences**

Aggregate Implications

1. (Large) Recessions

2. Policies

Recession Shock

- 3 types of short-lived unexpected aggregate shocks (η fixed):
  1. tfp
  2. cash shock ($n \downarrow$)
  3. credit shock ($\alpha_{11} \downarrow$)

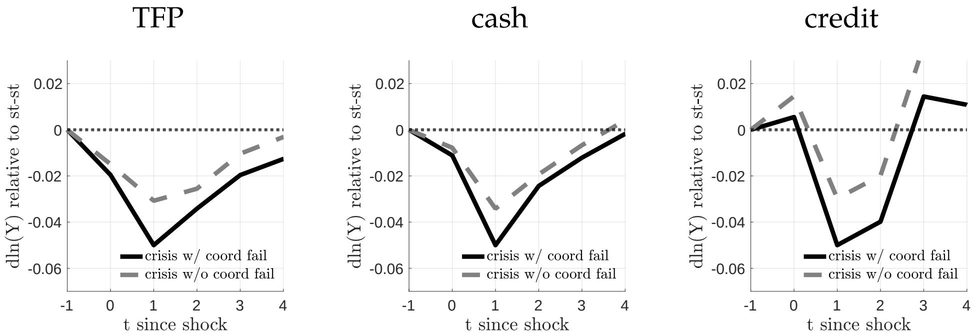
  detail shocks

Questions

- contribution of rollover risk to recession's aggregate impact?
- investment heterogeneity in crises (validation)

st-st comparison

Recession Shock Counterfactuals



detail shocks

- Firm exit: cash and TFP shocks ↑; credit shock ↓ details

Result II: rollover risk amplify significantly the impact of large recessions (explain 10% to 30% of output losses)

Credit Policy Intervention

- Direct lending policy: gov promises an alternative $Q^g(.)$ to a set of elegible firms. Then elegible firms new debt issuance resources are

$$\max\{\underbrace{Q(s, b', k')}_{\text{market}}, \underbrace{Q^g(.)}_{\text{government}}\} \times b'$$

- Policy workings: take elegible firm with $(z, n) \in \mathcal{R}$ under rollover crisis
  - faces $Q = 0$ then borrow from government at Q^g
  - If $d = n + \max_{k',b'}\{-k' + Q^g b'\} > 0$ then creditors know the firm could borrow from the gov to rollover the debt $\Rightarrow$ **preclude rollover crisis**

- Imperfect-targeted policy trade-off between less rollover crises and more debt overhang

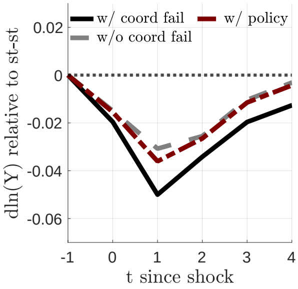
parametrization   announcement and implementation   direct lending vs credit guarantees

Question: policy effectiveness during recessions

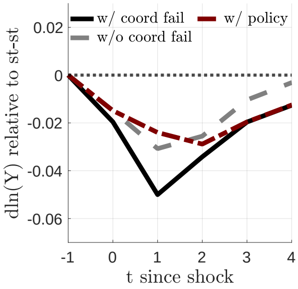
Credit Policy Implications

- Policy active first two periods and cash shock driven recession TFP shock results fiscal losses

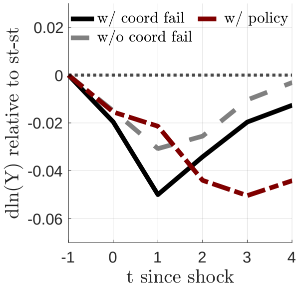
low scale



medium scale



large scale



Result III: imperfectly-targeted credit policy benefits are ambiguous

(i) low scale policy is very potent

(ii) high scale policy could backfire through future debt overhang

Concluding remarks

Concluding remarks

- Framework where rollover crises can be identified and quantified
- Results
  1. rollover crises are relevant for firms' failure
  2. rollover risk can amplify significantly aggregate impact of crises
  3. role for credit policies to prevent coord failures, even if imperfectly-targeted

Future research avenues

- Empirical work
- Extensions: (i) manage liability structure (ii) heterogeneous investors

  liab structure data   ex-ante costs rollover crises
- Other applications: e.g., sovereign debt bankruptcy procedures and self-fullfiling crises

Thank you!

Extra Slides

Related Papers

Brief and non-exhaustive review

- **Quantitative macro models of firms**: heterogeneous firms with corporate finance frictions

  Khan Senga Thomas 2017; Ottonello Winberry 2020 default risk; and Corbae D'Erasmo 2021 bankruptcy

- **Rollover (coord) problems in macro**: creditor's coord problems in banks/firms/countries

  Gertler Kiyotaki 2015 banks; Cole Kehoe 2000; Bocola Dovis 2020 sov debt; Morris Shin 2004 CFin theory; Jackson 1986 CLaw bankruptcy more detail

- **Investment heterogeneity during crises:** heterogeneity across financial distribution

  Kalemli-Özcan Laeven Moreno 2020; Almeida Campello Laranjeira Weisbenner 2012; Ebsim Faria-e-Castro Kozlowski 2021

- **Credit policy and C-borrowing in crisis:** credit policies implemented to address nonfinancial firms' financing problems in crises

  Crouzet Tourre 2021; Elenev Landvoigt Nieuwerburgh 2021; Ebsim Faria-e-Castro Kozlowski 2021

back

Related Papers on Coordination Failures

- **Bank runs**: Diamond Dybvig 1983; Gertler Kiyotaki 2015; Gertler Kiyotaki Prestipino 2020
- **Int'l. macro**: Cole Kehoe 2000; Bocola Dovis 2020; Obstfeld 1994 and 1996
- **Sunspots and business cycles**: Benhabib Wong 2014; Schmitt-Grohe Uribe 2020
- **Corporate finance (theoretical)**: Morris Shin 2004; Acharya Gale Yorulmazer 2011; He Xiong 2012; Halac Kremer Winter 2020; Zhong 2021; Zhong Zhou 2021
- **Corporate law**: Jackson 1986; Baird Jackson 1990; Ayotte Skeel 2013

back related papers back to paper

US Bankruptcy Code

Bankrupt firms use chapter 11 (11 U.S.C.) or 7 (7 U.S.C.) of US bankruptcy code

- Chapter 7
  - associated with firm's liquidation
  - case impartial trustee appointed to sell the bankrupt firms assets to pay creditors
- Chapter 11
  - associated with firm's restructure (or reorganization)
  - large firms also use to piecemeal liquidate the firm ("363 sale", 11 U.S.C. § 363(a))
  - debtor presents plan, and needs to be approved by judge and, ultimately, negotiated with and voted by creditors
  - provisions to preclude creditor's coordination problem
    1. automatic stay 11 U.S.C. § 362(a): prevents creditors demand payment
    2. debtor-in-possession protection 11 U.S.C. § 1101: allows new financing
    3. creating creditors' committees 11 U.S.C. § 341

back

Bankruptcy Procedure

- Only firms that are insolvent or with rollover crises may restructure their debt
- Recovery rate $\alpha_{11}(.)$ determined by

$$\alpha_{11}(z,k,b,\omega) = \arg\max_{\alpha_{11}} \Big[\underbrace{V(z,n^{11}) - 0}_{\text{firm's surplus}}\Big]^{1-\Xi} \Big[\underbrace{\alpha_{11} b - R(k,b,\omega)\, b}_{\text{creditor's surplus}}\Big]^{\Xi}$$

where $\Xi \in [0,1]$ barg power of creditors, we need that $n_{11} > \underline{n}(z)$ and $\alpha_{11} > R(k,b,\omega) = \min\{1, \alpha_7(1-\delta)\, q\omega k/b\}$

- For computational reasons I approx the barg. Max recov rate $\{\alpha_{11}^{max} : n_{11} = \underline{n}(z)\}$ and min recov rate $\alpha_{11}^{min} = \alpha_{7}^{min} = R(k,b,\omega)$, then recov rate linear comb of those rates with $\psi_{11} \in (0,1)$ the weight to creditors

back fin frictions back restructure back debt price

Entry and Exit

Exogenous exit

- Firms receive exog exit shock with prob γ
- Exiting firms allowed to restructure and liquidate before producing then

$$V^{\text{exit}}(s) = 1_{\{\text{continue} \mid \text{exit}\}}(s)\, n + 1_{\{\text{ch11} \mid \text{exit}\}}(s)\, n_{11}^{\text{exit}}$$

- Liquidate if $n < 0$ and $n_{11}^{\text{exit}} > 0$ not feasible; restructure if $n < 0$ and $n_{11} > 0$ feasible
- Price of debt conditional on exit is

$$\tilde{Q}_{\text{exit}}\left(z, k', b'\right) = \mathbb{E}_{\left(s' \mid s\right)}\left[\Lambda \left\{1_{\{\text{continue} \mid \text{exit}\}}\left(s'\right) + 1_{\{\text{ch11} \mid \text{exit}\}}\left(s'\right) \alpha_{11}^{\text{exit}}\right\}\right]$$
$$+ \mathbb{E}_{\left(s' \mid s\right)}\left[\Lambda 1_{\{\text{ch7} \mid \text{exit}\}}\left(s'\right) R\left(\omega', b', k'\right)\right]$$

Entry

- Mass $\bar{\mu}$ enter each period replacing exiting firms (for all reasons)
- Enter with capital $k = k_0$, $b = 0$ and $z \sim \Omega^e(z)$

back setup back value function back debt price

Liquidation Choice: Characterization

Proposition (Liquidation Choice)

Continuing firms liquidation choice $\tilde{1}_{ch7}\left(s, b', k'\right) \equiv 1_{ch7}(s)$ *where*

$$\tilde{1}_{ch7}(s) = \begin{cases} 1 & \text{if } n < \underline{n}(z) \\ 1 & \text{if } n \in \left[\underline{n}(z), 0\right) \quad \text{and} \quad \phi < \eta \\ 0 & \text{if } n \geq 0 \quad \text{or} \quad n \in \left[\underline{n}(z), 0\right) \quad \text{and} \quad \phi > \eta \end{cases}$$

with $\underline{n}(z) \equiv -\max_{k',b'}\left\{-k' + \tilde{Q}\left(z, b', k'\right) b'\right\}$ *where* $\tilde{Q}$ *debt price if* $[1 - \tilde{1}_{ch7}(s)] = 1$ *(i.e., no liquidation today conjectured)*

- Firms with $n < 0$ are exposed to rollover crises independently of z
- If $\tilde{Q}$ increasing in z then $\underline{n}(z)$ decreasing in z

back charact back eq

Liquidation Choice: More General Setup

Long-term debt

- assume portion debt m matures each period (randomly) and nonmatured pays cupon c
- cash-on-hand: $n = \pi + q\omega(1-\delta)k - [m + (1-m)c]b$
- external funds: $Q(.)\left[b' - (1-m)\,b\right] - qk'$
- default threshold: if $n \in [\underline{n}(z,b), 0)$ exposed to rollover crises and $n < \underline{n}(z,b)$ insolvent
- (recursive) debt prices (simplified = no bkrptcy, no exit, no discount, $c = 0$, $\alpha_7 = 0$):

$$\tilde{Q}(z,k',b') = \mathbb{E}_{\mathbf{z}'|\mathbf{z}}\left[\left\{1_{n'\geq 0} + (1-\eta)\,1_{n'\in[\underline{n}(z',b'),0)}\right\}\left\{(1-m)\,\tilde{Q}(z',k'',b'') + m\right\}\right]$$

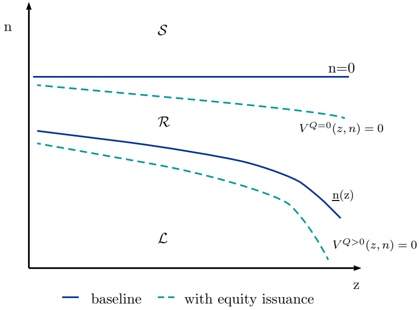
More general (assume $c = 0$ for exposition)

- profits $\pi(\mathbf{z},k)$ gral $\mathbf{z}$ process, invest $\iota\left(k,k'\right)$ allow for idio k frictions and long-term debt
- dividends (if no coord fail) are $d = \pi(\mathbf{z},k) - \iota\left(k,k'\right) - bm + \tilde{Q}(.)\left(b' - (1-m)\,b\right)$
- multiple eq if $\max_{k',b'} d \geq 0$ and $n \equiv \pi(\mathbf{z},k) - bm - \iota(k,0) < 0$ hold

back liquidation back debt price

Liquidation Choice: Costly Equity Issuance

- Firms can issue equity $e < 0$ at cost $\phi(e)$, which is decreasing in e and unbounded.



- $\mathcal{S} : V^{Q=0}(z, n) \geq 0$
- $\mathcal{L} : V^{Q>0}(z, n) < 0$
- $\mathcal{R} : V^{Q>0}(z, n) \geq 0$ and $V^{Q=0}(z, n) < 0$

- Where $V^{Q=0}$ firm problem with costly equity issuance where $Q = 0$ and $V^{Q>0}$ same but with $Q > 0$

back

HH Problem

HH in equilibrium determines

$$\Lambda' = \beta \frac{U_C\left(C', L'\right)}{U_C\left(C, L\right)}$$

$$1 = E\left[\beta \frac{U_C\left(C', L'\right)}{U_C\left(C, L\right)}(1+r)\right]$$

$$w = -\frac{U_L\left(C, L\right)}{U_C\left(C, L\right)}.$$

with utility function $U_C\left(C, L\right) = \ln C - \Omega L$

back back eq

Capital Producer

There is a representative aggregate capital producer that maximizes

$$\max_{I} q\Phi\left(\frac{I}{K}\right) - I$$

where I is the amount of final goods used to produce capital, K is the aggregate k stock, and $\Phi(.)$ is the aggregate capital adjustment cost function. FOC:

$$q = \frac{1}{\Phi'\left(\frac{I}{K}\right)}$$

- time-varying q and $\mathcal{R}(.) \rightarrow$ financial accelerator mechanism (Bernanke, Gertler & Gilchrist 1999).

back back eq

Steady-State Equilibrium

Steady-state equilibrium in this economy is Vfunctions of continuing firms $\left\{V, \tilde{V}\right\}$, decision rules $\left\{b', k', l\right\}$, aggregates $\{Y, C, I\}$, price schedule $Q(.)$, interest rate r, prices $\{q, w\}$, default choices $1(.)$, recov rates $\alpha_{11}(.)$ and distribution of firms $\{\Omega(.)\}$

- HHs choices are determined by Euler eq, SDF and labor supply eq detail
- price of capital q determine in K producer problem detail
- debt price satisfy no-profit condition of fin intermediaries detail
- given prices, firm's dec. rules solve the producing firm's problem detail and default choices are consistent with Default Propositions
- recovery rates satisfy bargaining protocol
- markets clear (labor, resources)
- distribution of firms **fixed point** in law of motion detail

back

Law of Motion States

Let Ω be the distribution of firms that produce which they a mass of 1, $\tilde{\Omega}$ the distribution of incumbent firms at the begining of the period, g and $\hat{g}$ the pdf of ω and ϕ respectively, p the conditional pdf of the productivity shocks ϵ_z, and Ω^e the distribution of entrant firms. To define the equilibrium first we need to determine the law of motion of the distribution. Distribution of firms that produce is

$$\begin{aligned}\Omega(z,n) = {} & (1-\gamma)\int\left[1_{\{ch11\}}(s)1_{\{n^{11}(z,k,b,\omega)=n\}} + 1_{\{cont\}}(s)1_{\{n(z,k,b,\omega)=n\}}\right]d\tilde{\Omega}(s)\\ & + \bar{\mu}(1-\gamma)\int\left[1_{\{ch11\}}(s)1_{\{n^{11}(z,k_0,0,\omega)=n\}} + 1_{\{cont\}}(s)1_{\{n(z,k_0,0,\omega)=n\}}\right]\hat{g}(\phi)\,g(\omega)\,d\phi d\omega d\Omega^e(z)\\ & + \text{lom} \mid \text{exit}\end{aligned}$$

The distribution of incumbent firms at the begining of the period $\tilde{\Omega}(z,\omega,k,b,\phi)$ is

$$\tilde{\Omega}\left(s'\right) = \int 1_{\{k'(z,n)=k'\}}1_{\{b'(z,n)=b'\}}\hat{g}\left(\phi'\right)g\left(\omega'\right)p\left(\epsilon_z \mid \rho_z z + \epsilon_z = z'\right)d\epsilon_z d\Omega(z,n)$$

back eq def   back eq paper

Data Sources, Sample and Some Definitions

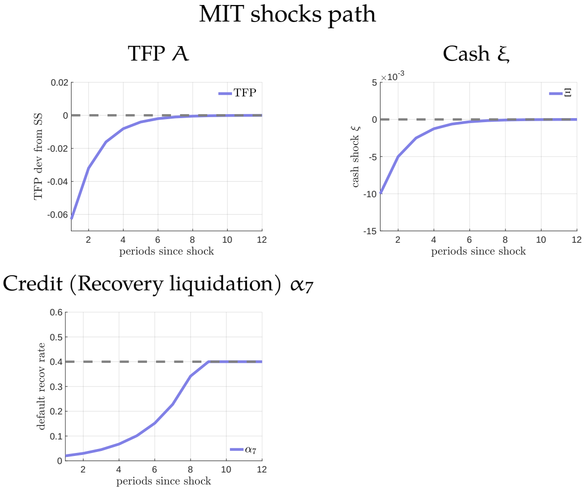
Compustat

- Two samples (accounting changes after 2018, see Ma's online notes)
  - Pre-covid = 1980-2017 (n=179k annual, n=426 k quarterly)
  - Covid = 2019-2020 (n=14k quarterly)
- Sample selection: nonfinancial, $k > 0$, assets > 0, drop outliers and short-spell (<20 q spell)
- Key definitions:
  - n = profits + liq value capital – net liquid liabilities
  - profits = F1.`oiadpq` where F1 = one period ahead in the data
  - net liquid liabilities = `lctq` – `cheq`
  - liq value capital = `invtq` $\times\ \omega_{\text{inv}}$ + `rectq` $\times\ \omega_{\text{rec}}$ + `ppentq` $\times\ \omega_{\text{ppentq}}$ + `acoq` where ω_x is liq value rate (from Kermani Ma 2020) of asset class `x`
- Identify bankrupt firms that operate following Corbae D'Erasmo 2021. Use footnote to total assets and deletion information (`dlrsn` and `dldte`). Bankrupt firms:
  1. report adoption accounting under Ch11, or bankrupt and not deleted
  2. data available next period

back calibration back invest het back invest het by episode

Crisis Shocks

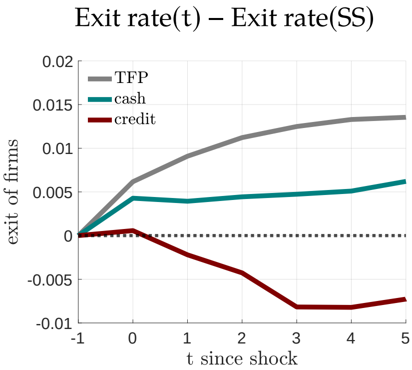
- Shocks unexpected and perfect foresight of path
- Temporary with persistence 0.5
- Definition of shocks:
  - TFP A: prod funct $Azf(k\omega, l)$
  - Cash ξ: reduction in n by ξk
  - Credit: recov rate if liquidated α_7

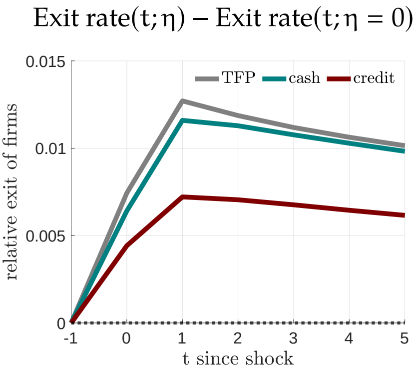


back exercise back results

Steady State Comparison

- Firm exit dynamics during crisis experiments



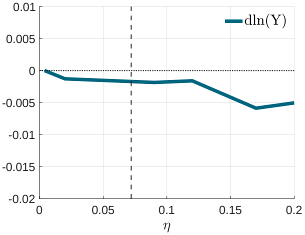


[back](#)

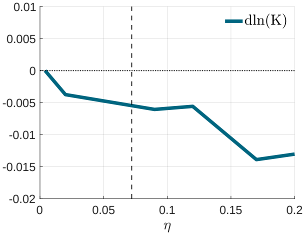
Steady State Comparison

- Variables: aggregate income Y, capital, K and share of firms with $n < 0$
- Comparison: steady state for different η

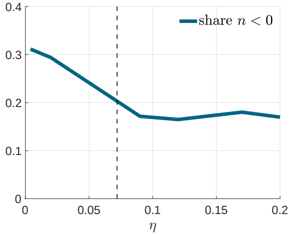
Y



K



share of firms with $n < 0$



notes: log difference relative to st-st with $\eta = 0$ for Y and K and levels for share of firms

back

Heterogeneous Investment Response
Empirical Specification

- Diff-in-diff crisis event estimate
  similar to Kalemli-Özcan Laeven Moreno 2020

$$\Delta \log(k_{it}) = \underbrace{\sum_{j=1}^{J} \beta_j^n \left(Q_{it}^{nj} \times \text{crisis}_t\right)}_{\text{heterogeneity across } n/k} + \underbrace{\sum_{j=1}^{J} \beta_j^b \left(Q_{it}^{bj} \times \text{crisis}_t\right)}_{\text{heterogeneity across } b/k} + \underbrace{\Lambda' Z_{it}}_{\text{controls}} + \varepsilon_{it}$$

let $x_{it} = \{b_{it}, l_{it}\}$ firm i at period t with

- demeaned by sector $\hat{x}_{it} = x_{it} - \mathbb{E}_s[x_{it}]$.
- $\Delta \log(k_{it}) = \log(k_{it+h}) - \log(k_{it})$ with h peak-to-trough length
- crisis_t indicates if a crisis happens during the period considered
- $Z_{i,t}$: sales growth, log firm size, firm FE, sector FE

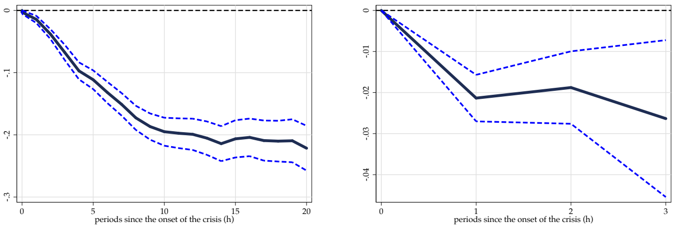
back

Recent Crisis Episodes in U.S.

(a) Great Recession

(b) Covid-19 Crisis

Δ Capital Accumulation



$$\beta_h : \ \log(k_{it+h}) - \log(k_{it}) = \alpha_i + \beta_h \text{crisis}_t + \varepsilon_{it+h}$$

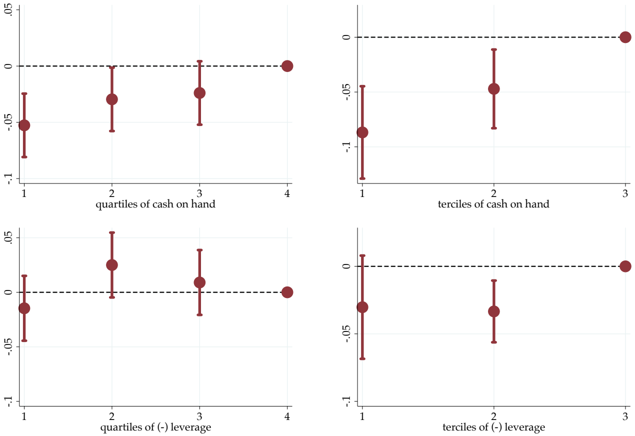
back

Investment Adjustment Heterogeneity
Recent Crisis Episodes in U.S.

Great Recession | Covid-19 Crisis

Across n/k

Across b/k



back

Credit Policy Setup

- Announced unexpectedly at $t = 0$ (same period of shocks) for $T \geq 0$ periods and implemented at $j \in [0, T]$
- Eligible firms $(z, n) \in \mathcal{P}$ offer sequence of $\left\{Q_t^g(.)\right\}$
- Policy $\mathcal{P}$ and labor taxes τ fixed across time
- Budget constraint from $t \geq 1$

$$\tau w_t L_t + B_t + B_{t-1,t}^g = B_t^g + (1 + r_{t-1}) B_{t-1}$$

B_t^g amount lent, $B_{t-1,t}^g$ lent at $t - 1$ and recovered at t

back

Credit Insurance Policy: First Best and Trade-off

Proposition (Credit Insurance Policy)

Assume that the government implements the credit insurance policy next period and is predictable today:

1. *First best policy:* $Q^g = \tilde{Q}$ *then no rollover risk and qualified firms indifferent between using public or private credit.*
2. *No screening: fix* z^g *such that* $Q^g = \tilde{Q}\left(z^g, k', b'\right)$ *with firms qualified for credit those with* $0 > n > \underline{n}(z^g)$. *This policy faces a trade-off between lowering firm rollover risk and greater misallocation.*

- 1st best policy eliminates coord fialures and firms don't use the program's credit
- Imperfect then greater z^g will preclude more coord fail, but firms with $z < z^g$ will draw funds (zombification)

back trade-off

Direct Lending vs. Credit Guarantees

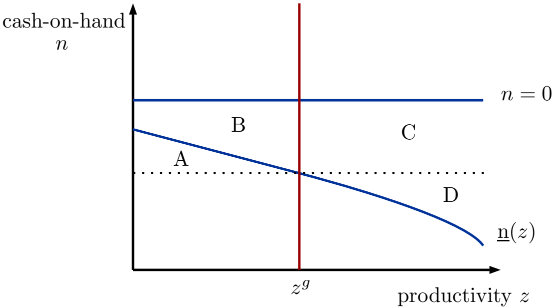
- Examples: direct lending $\approx$ Fed's PMCCF SMCCF and credit guarantee $\approx$ PPP
- In the theory policies are
  - direct lending (DL): alternative $Q^g(.)$ detail theory
  - credit guarantee (CG): repay $\alpha_g^r \geq \alpha^r$ in case of default
- Workings relative to rollover crises
  - DL affects payoffs (outside eq) and could coord creditors in good eq
  - CG relaxes $\underline{n}(z)$ but doesn't *directly* preclude coord failures

back

Credit Policy Trade-off

Stylized example of 1 period policy in PE and two extreme cases

1. *Perfect screen of* z : $Q^g = \tilde{Q}$ and then remove coord failures for "free"
2. *No screen of* z : gov lends to elegible firms $n \in (0, \underline{n}(z^g)]$ firms at $\tilde{Q}(z^g, k', b')$, with z^g parameterizing policy scope



$A \cup B \cup C$: elegible

D: excluded

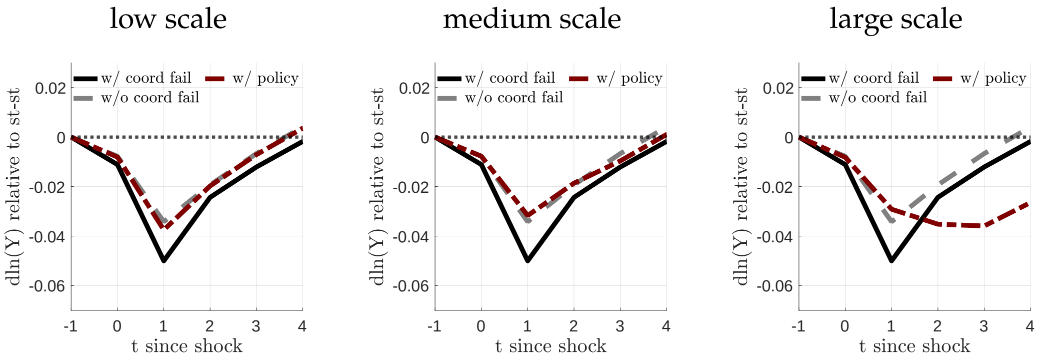
$A \cup B$: subsidized credit

$B \cup C$: coord failures precluded

back proposition

Credit Policy Implications: TFP shock

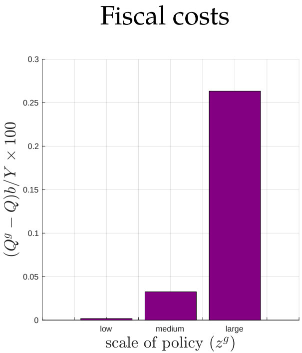
- Policy active for first two periods and TFP shock driven crisis back cash shock results

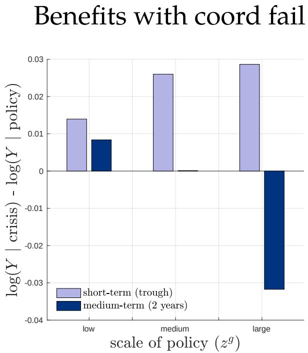


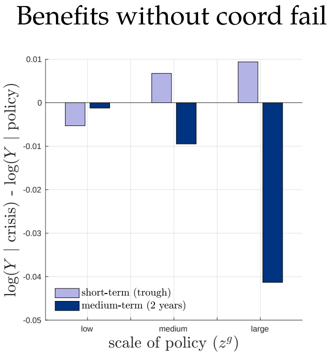
Credit Policy Implications: TFP shock

- Compute fiscal costs, short and long term benefits back

Costs and benefits







Liability Structure Data

- Debt maturity (Compustat)

	Time to mature (share)		
	< 1 year	1 to 4 years	≥ 5 years
Debt	0.29	0.33	0.38
	< 1 year	> 1 years	
Liabilities	0.61	0.39	

- Number of creditors from bankruptcy filings to Chapter 11 (FJC-IDB)

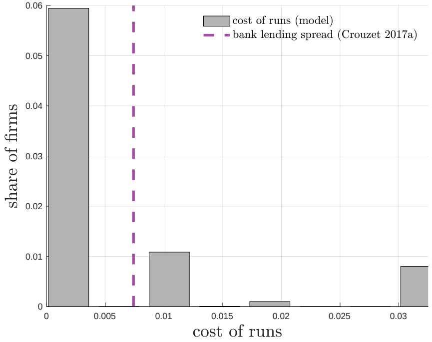
	# Creditors		
	1 to 100	101 to 1,000	>1,000
Medium ($>$ 50 million and $<$ 1 billion assets)	0.16	0.19	0.65
Large ($>$ 1 billion assets)	0.03	0.04	0.93

back

How Costly is Firms' Rollover Risk?

- (ex-ante) Cost computed as $\tilde{Q}\left(z, k', b'; \eta\right) - \tilde{Q}\left(z, k', b'; 0\right)$
- Only 2.2% of the firms face a cost of rollover risk higher than intermediation spread

Cost of runs (in annual spread terms) distribution



back