

# FIRMS' ROLLOVER RISK AND MACROECONOMIC DYNAMICS

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November 5, 2021

# Motivation

- A firm with  $NPV > 0$  can have a **rollover crises** due to coordination failures among its creditors and default
  - ▶ a concern for policy-makers → shape regulations and (credit) policies
  - ▶ frequently cited by owners/managers of bankrupt firms
- Develop **quantitative framework** where firms' rollover crises can be identified and quantified

## Research question

Implications of **firms' rollover risk** in macro dynamics and policy?

# What I Do

- Build on GE models of heterogeneous firms with default  
Khan Sengha Thomas 2016; Ottonello Winberry 2020; and others
- Borrow technics from international macro literature to allow for **rollover crises**  
Cole Kehoe 2000; Bocola Dovis 2020
- Incidence of rollover crises **informed** by distribution of bankruptcy outcomes  
(liquidation vs restructure)  
insights from Corp Law literature, e.g., Jackson 1986; Corbae D'Erasmus 2021
- Conduct **quantitative** analysis of U.S. economy to asses aggregate relevance of firms' rollover risk

# What I Find

## 1. How **relevant** are firms' rollover crises?

- + **1.6%** firms rollover crises = 21% exposed  $\times$  7% probability  
(roughly half of bankruptcy events are driven by rollover crises)
- ▶ indirect inference using bankruptcy outcomes and financial distribution of firms
- ▶ identification: restructure process "forces" creditors to rollover

## 2. What are the **macroeconomic** implications of rollover risk?

- + rollover risk **amplify** significantly impact of recessions  
(more output drop and persistence)
- ▶ explain between 10% to 30% of aggregate output losses

## 3. What are the **policy** implications?

- + credit policy can **undo rollover crises** amplification of crises, but can **backfire**
- ▶ focus on imperfectly-targeted credit policy

# Outline

- Macro Model of Firms' Rollover Risk
- Identifying Rollover Crises
- Macroeconomic Consequences

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- **Macro Model of Firms' Rollover Risk**
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# Overview of the Model

- Quantitative GE models of heterogeneous firms with default  
Khan Sengha Thomas 2017; Ottonello Winberry 2020
- Extend model to
  1. possibility of **coordination failures** among creditors á la Cole Kehoe 2000
  2. allow **debt restructuring** similar to Corbae D'Erasmus 2021
- Study unforeseen crises and policy shocks (MIT shocks)

# Environment

- Infinite horizon and discrete time
- Four types of agents
  1. **nonfinancial firms**: invest and produce to maximize their value
  2. **creditors**: lend to nonfinancial firms, and are perfectly competitive and atomistic
  3. **capital producer**: sell capital to nonfinancial firms
  4. **representative HH**: consumes, saves and works. Owns all firms in the economy



# Nonfinancial Firms' Environment

- Firm  $i$  objective is to max

$$\sum_{t \geq 0} \mathbb{E}_0[\Lambda_t d_{it}]$$

with  $\Lambda_t$  HH's SDF and  $d_t$  firm's dividends

- Idiosyncratic state variables:
  1.  $s_{it}^f$  exogenous
  2.  $s_{it}^{nf}$  exogenous non-fundamental
  3.  $s_{it}^e$  endogenous

where  $s_{it} = (s_{it}^f, s_{it}^{nf}, s_{it}^e)$

- For clarity, drop  $i$  and  $t$  subscripts

# Overview of Nonfinancial Firms' Timing

Within period timing is as follows (firms with no exit shock)

1. **All** uncertainty about fundamentals and nonfundamentals is realized
2. **Restructure** choice
3. Investment and **new debt** issuance choice
4. **Liquidation** choice (if continue in 2)
5. Firms produce and distribute dividends (if don't liquidated in 4)

# Nonfinancial Firms' Production

- Operate with technology

$$f(z, \omega, k, l) = z(\omega k)^\alpha l^\nu$$

- ▶ decreasing returns to scale  $\nu + \alpha < 1$
  - ▶ idiosyncratic persistent productivity shocks  $\ln z' = \rho_z \ln z + \epsilon_z$  with  $\epsilon_z \sim N(0, \sigma_z^2)$
  - ▶ idiosyncratic capital quality shock  $\omega$  iid log-normal trunc. where  $\ln \omega \in [\underline{\omega}, 0]$   
(fit quantitative default rate)
- Own capital  $k$  and hire labor  $l$  at wage  $w$ , then operating profits are

$$\pi(z, \omega, k) = \max_l f(z, \omega, k, l) - wl$$

# Nonfinancial Firms' Financial Resources

- **Internal resources** (cash-on-hand)

- inherit  $k$  at price  $q$  which depreciates at  $\delta \in [0, 1]$  and maturing  $b$ , and has  $\pi(z, \omega, k)$

$$n = \underbrace{\pi(z, \omega, k)}_{\text{operational profits}} + \underbrace{(1 - \delta)q\omega k}_{\text{selling value of capital}} - \underbrace{b}_{\text{maturing debt}}$$

- (net) **External resources**

- issues one-period debt  $b'$  at price schedule  $Q(\cdot)$  and buys  $k'$  at price  $q$

- **Dividends**

- distributed at end of period

$$d = \underbrace{n}_{\text{cash-on-hand}} + \underbrace{Q(\cdot)b'}_{\text{new debt issuance resources}} - \underbrace{qk'}_{\text{capital purchases}}$$

# Nonfinancial Firms' Financial Frictions

1. No-equity issuance constraint  $d \geq 0$  (data low eq issuance, standard and simplify)
2. Debt is defaultable in two ways
  - (a) liquidation (Chapter 7)
    - ▶ non selective default on  $b$  and  $b'$
    - ▶ firm exits with  $V = 0$  and creditors of  $b$  recover  $\alpha_7 \in [0, 1]$  of the liquidated capital
  - (b) restructuring (Chapter 11)
    - ▶ (Nash) bargain debt recovery rate  $\alpha_{11} \in (0, 1)$  over debt  $b$  [detail](#)
    - ▶ firm pays exogenous cost  $c_{11} \in [0, 1]$ , which is proportional to capital
    - ▶ **no coord failure** (bankruptcy provisions, Corp Law)
    - ▶ resources after restructuring are:  $n_{11} = \pi(z, \omega, k) + (1 - c_{11})(1 - \delta)q\omega k - \alpha_{11}b$

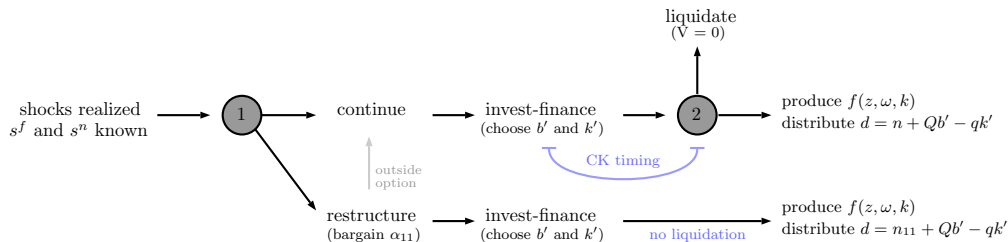
# Nonfinancial Firms' Entry/Exit

## Technical and quantitative assumptions

- Exogenous exit probability  $\gamma$  (KST 2016, stationary dist)
  - ▶ if receive shock the firm exits after production
- Entrants enter on average productivity  $m\%$  below ergodic distribution average (OW 2020, life-cycle firms)

[details](#)

# Within Period Timing Nonfinancial Firms' Problem



- Timing for non-exiting firms
- Cole Kehoe 2000 (CK) timing for liquidation choice

# Multiple Equilibrium Intuition

- Liquidate if they can't satisfy  $d \geq 0$  (result of assumptions)
- Debt price schedule from no-profit condition of creditors and dividends are

$$Q = \underbrace{\mathbb{I}_{d \geq 0}}_{\text{liquidation choice}} \underbrace{\tilde{Q}}_{\text{price if no liquidation}}$$

$$d = n - k' + Qb'$$

- Feedback between liquidation choice and prices today could create multiple outcomes

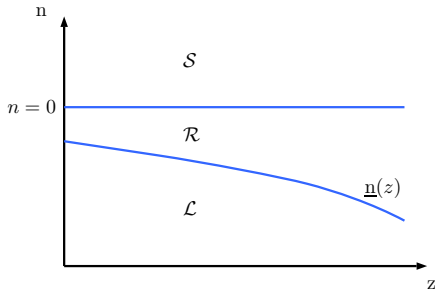
$$Q = 0 \iff d < 0$$

$$Q > 0 \iff d \geq 0$$



# Liquidation

- **Fundamental** state-space  $(z, n)$  is divided in three regions liquidation proposition



- $\mathcal{S}$  :  $Q = 0$  then continue if  $d = n + \underbrace{\max_{k'} \{-k'\}}_0 > 0$
- $\mathcal{L}$  :  $Q = \tilde{Q}$  then liquidate if  $d = n + \underbrace{\max_{k', b'} \{-k' + \tilde{Q} b'\}}_{-\underline{n}(z)} < 0$
- $\mathcal{R}$  : liquidate if  $Q = 0$ , continue if  $Q = \tilde{Q} > 0$

- Define idiosyncratic sunspot shock  $\phi \sim^{\text{iid}} U[0, 1]$  draw every period, such that if  $(z, n) \in \mathcal{R}$  and  $\phi < \eta$  then coord in  $Q = 0$  (coord failure)

more general setup    costly equity issuance

# Restructure

**Bargain** outside option is to continue **bargain protocol** then conditions are

- necessary condition: firms with rollover crises (in  $\mathcal{R}$  with  $\phi < \eta$ ) or insolvent (in  $\mathcal{L}$ )
- sufficient condition: both better-off participating
  1. creditors:  $\alpha_{11} > \min\{1, \alpha_7 \frac{(1-\delta)q\omega k}{b}\}$
  2. firm:  $n_{11} > \underline{n}(z)$

**Costs and benefits for firms**

1.  $c_{11}$  cost (proportional to capital)
2.  $1 - \alpha_{11}$  debt haircut
3. no coord failure, i.e.,  $Q = \tilde{Q}$

**observation**

if  $c_{11}$  large and  $(1 - \alpha_{11})$  low  $\Rightarrow$   
firms with rollover crises **restructure**

# Nonfinancial Firm's Recursive Problem

- $V$  value of firm before exit shock and restructure choice with  $s = (z, \omega, \phi, k, b)$

$$V(s) = (1 - \gamma) \left[ 1_{\{\text{ch11}\}}(s) \tilde{V}(z, n_{11}) + 1_{\{\text{cont}\}}(s) \tilde{V}(z, n) + 1_{\{\text{ch7}\}}(s) \times 0 \right] + \gamma V_{\text{exit}}(s) \quad (1)$$

where indicators follow from previous results,  $V_{\text{exit}}(s)$  value of exiting firm [details](#) and

$$\begin{aligned} n &= \pi(z, \omega, k) + (1 - \delta)q\omega k - b \\ n_{11} &= \pi(z, \omega, k) + (1 - c_{11})(1 - \delta)q\omega k - \alpha_{11}(s)b \end{aligned}$$

- $\tilde{V}(z, n)$  value of the solvent firm and without rollover crisis today is

$$\tilde{V}(z, n) = \max_{d, k', b'} d + \mathbb{E}_{(z'|z; \omega'; \phi')} \left[ \Lambda V(s') \right] \quad (2)$$

subject to  $d = n - qk' + \tilde{Q}(z, b', k')$   $b' \geq 0$ , where  $\tilde{Q}(\cdot)$  debt price without coord failure

# Corporate Debt Prices

- $Q = [1 - \mathbf{1}_{\text{Ch7}}(s)]\tilde{Q}$  from creditor's no profit condition
- $\tilde{Q}$  determined by (discounted)  $\mathbb{E}[\text{prob tomorrow's bankruptcy events}]$

$$\begin{aligned}\tilde{Q}(z, k', b') &= (1 - \gamma) \mathbb{E}_{(z'|z, \omega', \phi')} \left[ \Lambda 1_{\{\text{continue}\}}(s') \times 1 \right] \\ &\quad + (1 - \gamma) \mathbb{E}_{(z'|z, \omega', \phi')} \left[ \Lambda 1_{\{\text{Ch11}\}}(s') \times \alpha_{11}(s') \right] \\ &\quad + (1 - \gamma) \mathbb{E}_{(z'|z, \omega', \phi')} \left[ \Lambda 1_{\{\text{Ch7}\}}(s') \times R(k', b', \omega') \right] \\ &\quad + \gamma \tilde{Q}_{\text{exit}}(z, k', b')\end{aligned}$$

where

- $\alpha_{11}(s)$  recovery rate of creditors if restructure bargain protocol
- $R(k, b, \omega) = \min \{1, \alpha_7 (1 - \delta) q \omega k / b\}$  recovery rate if liquidated
- $\tilde{Q}_{\text{exit}}(z, k', b')$  debt price conditional on exit shock Q with exogenous exit

# Other Agents and Equilibrium

## Agents

1. **HH's** choices are determined by Euler eq, SDF  $\Lambda$  and labor supply eq [detail](#)
2. **K producer** problem sells capital at price  $q$  and has a standard aggregate capital adjustment function [detail](#)
3. **Creditors** price debt through no-profit condition (SDF  $\Lambda$ ) [detail](#)

## Equilibrium

- Steady-state (law of motion fixed point) [full definition](#) [law of motion firm distribution](#)

# Outline

- Macro Model of Firms' Rollover Risk
- **Identifying Rollover Crises**
- Macroeconomic Consequences

# Identification

## Questions

1. How many firms are in  $\mathcal{R}$ ?
2. Value of  $\eta$ ?

## Steps

- Calibration of **standard** parameters to match relevant moments of U.S. economy
- Calibration of parameters related to **bankruptcy procedure** and identify  $\eta$
- Steady-state **financial distribution** determines share of firms by region

## Data sources

- NIPA, Compustat, Federal Judicial Center-IDB, LBD, related papers

## Calibration Standard Parameters

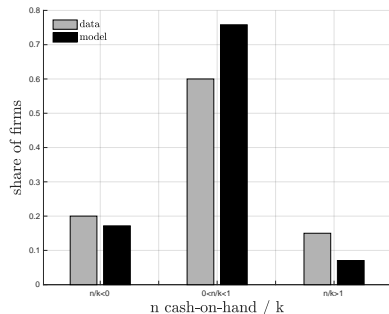
	Parameter	Value	Calibration
Calibration strategy	<i>Fixed</i>		
	$\beta = 1/(1 + r)$	0.99	fixed to $r = 0.05$ annual
	$\Phi$	1.16	fixed to match 58% emp rate
	$\nu$	0.64	fixed labor share
	$\alpha$	0.21	fixed capital share
	$\delta$	0.025	fixed to match BEA quarterly
	$\rho_z$	0.90	fixed
	$\gamma$	0.02	fixed to exit rate w/o default
	$\psi$	2	agg AC fixed to lit standard
	$b_0$	0	fixed to no net debt entrants
	<i>Fitted</i>		
	$\sigma_z$	0.032	internally calib
	$\underline{\omega}$	-0.33	internally calib
	$k_0$	0.16	internally calib
$m$	-0.24	internally calib	



# Relevant Moments

Moment	Data	Model
<i>Aggregates</i>		
K/Y	3.00	2.59
I/Y	0.17	0.15
gross debt: $\mathbb{E}[1_{b>0}b]/Y$	1.05	1.79
<i>Credit spreads</i>		
cred spread: $\mathbb{E}[r^Q - r]$	2.2%	0.7%
<i>Investment heterogeneity</i>		
avg invest rate: $\mathbb{E}[i/k]$	0.12	0.17
sd invest rate: $SD[i/k]$	0.34	0.36
<i>Life-cycle</i>		
share exit	0.10	0.11
(L age 1) / L	0.03	0.04
# firms age 1 / # firms	0.10	0.11
# firms age 2 / # firms	0.08	0.09

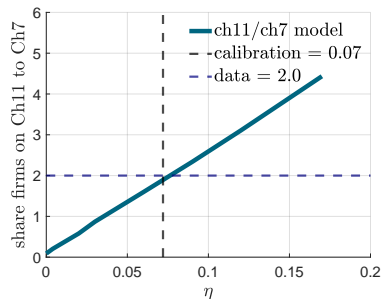
Moment	Data	Model
<i>Balance sheet</i>		
avg leverage: $\mathbb{E}[1_{b>0}b'/k']$	0.37	0.72
correl ( $n, k'$ )	0.74	0.23
n distribution		



# Identification of $\eta$

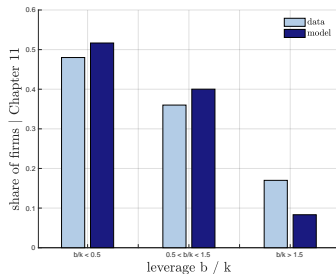
- $(\alpha_7, \psi_{11}, c_{11})$  match debt haircut under Ch 11 and Ch 7, and leverage in Ch 11

Param.	Value	Moment targeted	Data	Model
$\alpha_7$	0.38	$\mathbb{E}[R]$	0.27	0.29
$\psi_{11}$	0.89	$\mathbb{E}[\alpha_{11}]$	0.69	0.82
$c_{11}$	0.40	$\mathbb{E}[b'/k' \mid \text{Ch 11}]$	0.73	0.67
$\eta$	0.07	$\mathbb{E}[\text{Ch11}]/\mathbb{E}[\text{Ch7}]$	2.0	1.9



# Untargeted Moments of Bankruptcy

Distribution of leverage  $b'/k'$   
firms in Chapter 11



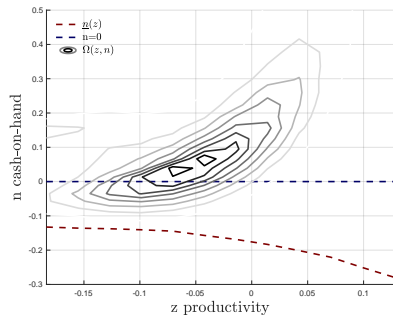
Predictors of Chapter 11

	dependent variable: $1_{i,t}^{\text{ch11}}$					
	(1)		(2)		(3)	
	data	model	data	model	data	model
$n_{i,t-1}/k_{i,t}$	-0.39 (0.03)	-0.05			-0.39 (0.10)	-0.45
$b_{i,t}/k_{i,t}$			0.11 (0.04)	0.03	-0.29 (0.09)	-0.41
$\log(k_{i,t-1})$	-0.50 (0.12)	-0.06	-0.52 (0.12)	-0.06	-0.49 (0.12)	-0.10
$d \log(\text{sales}_{i,t-1})$	-0.04 (0.00)	-0.03	-0.04 (0.00)	-0.02	-0.04 (0.00)	-0.01
Sector FE	Y		Y		Y	
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y		Y		Y	
Observations	370,973		373,362		370,973	

$$\text{empirical specification: } 1_{i,t}^{\text{ch11}} = \beta X_{i,t-1} + \alpha_t + \alpha_i + \alpha_s + \epsilon_{i,t}$$

# Incidence of Rollover Crises

Steady state distribution  $\Omega(z, n)$   
before bankruptcy choice



$$\underbrace{\int_{(z,n) \in \mathcal{R}} d\Omega(z, n)}_{\substack{\text{share of firms exposed} \\ 0.20}} \times \underbrace{\eta}_{\substack{\text{run likelihood} \\ 0.07}} = 1.6\%$$

**Result I:** 1.6% of firms are subject to rollover crises

$\Rightarrow$  roughly half of bankruptcy events are driven by rollover crises

# Outline

- Macro Model of Firms' Rollover Risk
- Identifying Rollover Crises
- **Macroeconomic Consequences**

# Aggregate Implications

1. (Large) Recessions
2. Policies

# Recession Shock

- 3 types of short-lived unexpected aggregate shocks ( $\eta$  fixed):
  1. tfp
  2. cash shock ( $n \downarrow$ )
  3. credit shock ( $\alpha_{11} \downarrow$ )

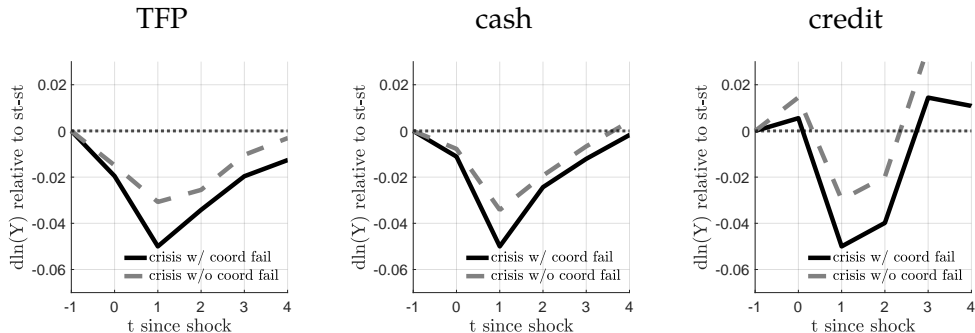
detail shocks

## Questions

- contribution of rollover risk to recession's aggregate impact?
- investment heterogeneity in crises (validation)

st-st comparison

# Recession Shock Counterfactuals



[detail shocks](#)

- Firm exit: cash and TFP shocks  $\uparrow$ ; credit shock  $\downarrow$  [details](#)

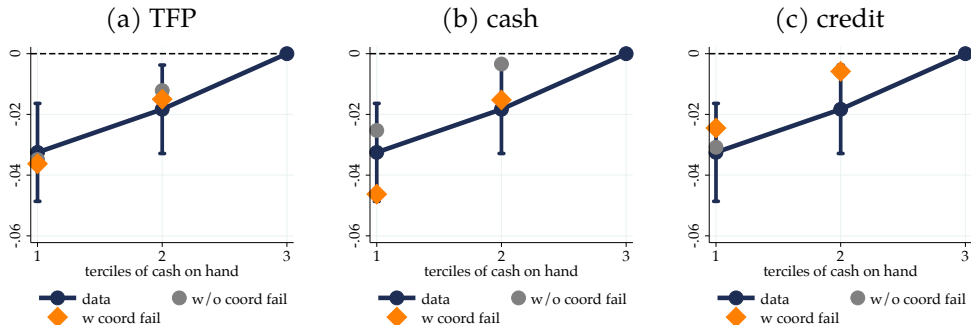
**Result II:** rollover risk amplify significantly the impact of large recessions  
(explain 10% to 30% of output losses)



# Crisis Shock Heterogeneity

- Heterogeneity in  $\Delta k$  during last large recessions [empirical specification](#) [measurement](#)
- Data and model simulation for Great Recession and Covid [episodes](#)

Heterogeneity of  $\Delta k(\text{crisis}) - \Delta k(\text{no crisis})$  across  $n/k$



note: simple average of both episodes for cash shock [individual episode](#) [empirical results](#)

# Credit Policy Intervention

- **Direct lending** policy: gov promises an alternative  $Q^g(.)$  to a set of eligible firms. Then eligible firms new debt issuance resources are

$$\max \left\{ \underset{\text{market}}{Q(s, b', k')}, \underset{\text{government}}{Q^g(.)} \right\} \times b'$$

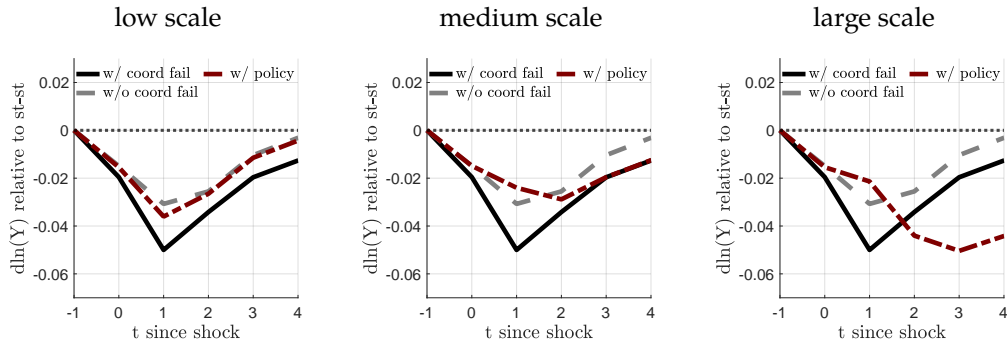
- Policy **workings**: take eligible firm with  $(z, n) \in \mathcal{R}$  under rollover crisis
  - ▶ faces  $Q = 0$  then borrow from government at  $Q^g$
  - ▶ If  $d = n + \max_{k', b'} \{-k' + Q^g b'\} > 0$  then creditors know the firm could borrow from the gov to rollover the debt  $\Rightarrow$  **preclude rollover crisis**
- Imperfect-targeted policy **trade-off** between less rollover crises and more debt overhang
 

parametrization    announcement and implementation    direct lending vs credit guarantees

Question: policy effectiveness during recessions

# Credit Policy Implications

- Policy active first two periods and cash shock driven recession TFP shock results fiscal losses



**Result III:** imperfectly-targeted credit policy benefits are ambiguous

- (i) low scale policy is very potent
- (ii) high scale policy could backfire through future debt overhang

# Concluding remarks

# Concluding remarks

- Framework where rollover crises can be identified and quantified
- Results
  1. rollover crises are relevant for firms' failure
  2. rollover risk can amplify significantly aggregate impact of crises
  3. role for credit policies to prevent coord failures, even if imperfectly-targeted

## Future research avenues

- Empirical work
- Extensions: (i) manage liability structure (ii) heterogeneous investors  
liab structure data      ex-ante costs rollover crises
- Other applications: e.g., sovereign debt bankruptcy procedures and self-fulfilling crises

Thank you!

# Extra Slides

# Related Papers

## Brief and non-exhaustive review

- **Quantitative macro models of firms:** heterogeneous firms with corporate finance frictions  
Khan Sengha Thomas 2017; Ottonello Winberry 2020 default risk; and Corbae D'Erasmus 2021 bankruptcy
- **Rollover (coord) problems in macro:** creditor's coord problems in banks/firms/countries  
Gertler Kiyotaki 2015 banks; Cole Kehoe 2000; Bocola Dovis 2020 sov debt; Morris Shin 2004 CFin theory; Jackson 1986 CLaw bankruptcy [more detail](#)
- **Investment heterogeneity during crises:** heterogeneity across financial distribution  
Kalemli-Özcan Laeven Moreno 2020; Almeida Campello Laranjeira Weisbenner 2012; Ebsim Faria-e-Castro Kozlowski 2021
- **Credit policy and C-borrowing in crisis:** credit policies implemented to address nonfinancial firms' financing problems in crises  
Crouzet Tourre 2021; Elenev Landvoigt Nieuwerburgh 2021; Ebsim Faria-e-Castro Kozlowski 2021



# Related Papers on Coordination Failures

- **Bank runs:** Diamond Dybvig 1983; Gertler Kiyotaki 2015; Gertler Kiyotaki Prestipino 2020
- **Int'l. macro:** Cole Kehoe 2000; Bocola Dovis 2020; Obstfeld 1994 and 1996
- **Sunspots and business cycles:** Benhabib Wong 2014; Schmitt-Grohe Uribe 2020
- **Corporate finance (theoretical):** Morris Shin 2004; Acharya Gale Yorulmazer 2011; He Xiong 2012; Halac Kremer Winter 2020; Zhong 2021; Zhong Zhou 2021
- **Corporate law:** Jackson 1986; Baird Jackson 1990; Ayotte Skeel 2013

[back related papers](#)   [back to paper](#)

# US Bankruptcy Code

Bankrupt firms use chapter 11 (11 U.S.C. ) or 7 (7 U.S.C. ) of US bankruptcy code

- Chapter 7
  - associated with firm's liquidation
  - case impartial trustee appointed to sell the bankrupt firms assets to pay creditors
- Chapter 11
  - associated with firm's restructure (or reorganization)
  - large firms also use to piecemeal liquidate the firm ("363 sale", 11 U.S.C. § 363(a))
  - debtor presents plan, and needs to be approved by judge and, ultimately, negotiated with and voted by creditors
  - provisions to preclude creditor's coordination problem
    1. automatic stay 11 U.S.C. § 362(a): prevents creditors demand payment
    2. debtor-in-possession protection 11 U.S.C. § 1101: allows new financing
    3. creating creditors' committees 11 U.S.C. § 341

# Bankruptcy Procedure

- Only firms that are insolvent or with rollover crises may restructure their debt
- Recovery rate  $\alpha_{11}(\cdot)$  determined by

$$\alpha_{11}(z, k, b, \omega) = \arg \max_{\alpha_{11}} \left[ \underset{\text{firm's surplus}}{V(z, n^{11}) - 0} \right]^{1-\Xi} \left[ \underset{\text{creditor's surplus}}{\alpha_{11}b - R(k, b, \omega)b} \right]^{\Xi}$$

where  $\Xi \in [0, 1]$  barg power of creditors, we need that  $n_{11} > \underline{n}(z)$  and  $\alpha_{11} > R(k, b, \omega) = \min \{1, \alpha_7 (1 - \delta) q \omega k / b\}$

- For computational reasons I approx the barg. Max recov rate  $\{\alpha_{11}^{\max} : n_{11} = \underline{n}(z)\}$  and min recov rate  $\alpha_{11}^{\min} = \alpha_7^{\min} = R(k, b, \omega)$ , then recov rate linear comb of those rates with  $\psi_{11} \in (0, 1)$  the weight to creditors

# Entry and Exit

## Exogenous exit

- Firms receive exog exit shock with prob  $\gamma$
- Exiting firms allowed to restructure and liquidate before producing then

$$V^{\text{exit}}(s) = 1_{\{\text{continue} \mid \text{exit}\}}(s) n + 1_{\{\text{ch11} \mid \text{exit}\}}(s) n_{11}^{\text{exit}}$$

- Liquidate if  $n < 0$  and  $n_{11}^{\text{exit}} > 0$  not feasible; restructure if  $n < 0$  and  $n_{11} > 0$  feasible
- Price of debt conditional on exit is

$$\begin{aligned} \tilde{Q}_{\text{exit}}(z, k', b') &= \mathbb{E}_{(s' \mid s)} \left[ \wedge \left\{ 1_{\{\text{continue} \mid \text{exit}\}}(s') + 1_{\{\text{ch11} \mid \text{exit}\}}(s') \alpha_{11}^{\text{exit}} \right\} \right] \\ &\quad + \mathbb{E}_{(s' \mid s)} \left[ \wedge 1_{\{\text{ch7} \mid \text{exit}\}}(s') R(\omega', b', k') \right] \end{aligned}$$

## Entry

- Mass  $\bar{\mu}$  enter each period replacing exiting firms (for all reasons)
- Enter with capital  $k = k_0$ ,  $b = 0$  and  $z \sim \Omega^e(z)$

# Liquidation Choice: Characterization

## Proposition (Liquidation Choice)

Continuing firms liquidation choice  $\tilde{1}_{ch7}(s, b', k') \equiv 1_{ch7}(s)$  where

$$\tilde{1}_{ch7}(s) = \begin{cases} 1 & \text{if } n < \underline{n}(z) \\ 1 & \text{if } n \in [\underline{n}(z), 0) \text{ and } \phi < \eta \\ 0 & \text{if } n \geq 0 \text{ or } n \in [\underline{n}(z), 0) \text{ and } \phi > \eta \end{cases}$$

with  $\underline{n}(z) \equiv -\max_{k', b'} \{-k' + \tilde{Q}(z, b', k') b'\}$  where  $\tilde{Q}$  debt price if  $[1 - \tilde{1}_{ch7}(s)] = 1$  (i.e., no liquidation today conjectured)

- Firms with  $n < 0$  are exposed to rollover crises independently of  $z$
- If  $\tilde{Q}$  increasing in  $z$  then  $\underline{n}(z)$  decreasing in  $z$

# Liquidation Choice: More General Setup

## Long-term debt

- assume portion debt  $m$  matures each period (randomly) and nonmatured pays coupon  $c$
- cash-on-hand:  $n = \pi + q\omega(1 - \delta)k - [m + (1 - m)c]b$
- external funds:  $Q(.) [b' - (1 - m)b] - qk'$
- default threshold: if  $n \in [\underline{n}(z, b), 0)$  exposed to rollover crises and  $n < \underline{n}(z, b)$  insolvent
- (recursive) debt prices (simplified = no bkruptcy, no exit, no discount,  $c = 0$ ,  $\alpha_7 = 0$ ):

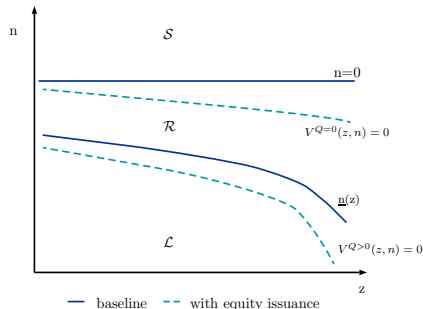
$$\tilde{Q}(z, k', b') = \mathbb{E}_{z'|z} \left[ \left\{ 1_{n' \geq 0} + (1 - \eta) 1_{n' \in [\underline{n}(z', b'), 0)} \right\} \left\{ (1 - m) \tilde{Q}(z', k'', b'') + m \right\} \right]$$

## More general (assume $c = 0$ for exposition)

- profits  $\pi(z, k)$  gral  $z$  process, invest  $\iota(k, k')$  allow for idio  $k$  frictions and long-term debt
- dividends (if no coord fail) are  $d = \pi(z, k) - \iota(k, k') - bm + \tilde{Q}(.) (b' - (1 - m)b)$
- multiple eq if  $\max_{k', b'} d \geq 0$  and  $n \equiv \pi(z, k) - bm - \iota(k, 0) < 0$  hold

# Liquidation Choice: Costly Equity Issuance

- Firms can issue equity  $e < 0$  at cost  $\phi(e)$ , which is decreasing in  $e$  and unbounded.



- $\mathcal{S} : V^{Q=0}(z, n) \geq 0$
- $\mathcal{L} : V^{Q>0}(z, n) < 0$
- $\mathcal{R} : V^{Q>0}(z, n) \geq 0$  and  $V^{Q=0}(z, n) < 0$

- Where  $V^{Q=0}$  firm problem with costly equity issuance where  $Q = 0$  and  $V^{Q>0}$  same but with  $Q > 0$

# HH Problem

HH in equilibrium determines

$$\begin{aligned}\Lambda' &= \beta \frac{u_C(C', L')}{u_C(C, L)} \\ 1 &= E \left[ \beta \frac{u_C(C', L')}{u_C(C, L)} (1 + r) \right] \\ w &= - \frac{u_L(C, L)}{u_C(C, L)}.\end{aligned}$$

with utility function  $u_C(C, L) = \ln C - \Omega L$



# Capital Producer

There is a representative aggregate capital producer that maximizes

$$\max_I q \Phi \left( \frac{I}{K} \right) - I$$

where  $I$  is the amount of final goods used to produce capital,  $K$  is the aggregate  $k$  stock, and  $\Phi(.)$  is the aggregate capital adjustment cost function. FOC:

$$q = \frac{1}{\Phi' \left( \frac{I}{K} \right)}$$

- time-varying  $q$  and  $\mathcal{R}(\cdot) \rightarrow$  financial accelerator mechanism (Bernanke, Gertler & Gilchrist 1999).

# Steady-State Equilibrium

**Steady-state** equilibrium in this economy is Vfunctions of continuing firms  $\{V, \tilde{V}\}$ , decision rules  $\{b', k', l\}$ , aggregates  $\{Y, C, I\}$ , price schedule  $Q(\cdot)$ , interest rate  $r$ , prices  $\{q, w\}$ , default choices  $1(\cdot)$ , recov rates  $\alpha_{11}(\cdot)$  and distribution of firms  $\{\Omega(\cdot)\}$

- HHs choices are determined by Euler eq, SDF and labor supply eq [detail](#)
- price of capital  $q$  determine in K producer problem [detail](#)
- debt price satisfy no-profit condition of fin intermediaries [detail](#)
- given prices, firm's dec. rules solve the producing firm's problem [detail](#) and default choices are consistent with Default Propositions
- recovery rates satisfy bargaining protocol
- markets clear (labor, resources)
- distribution of firms **fixed point** in law of motion [detail](#)

# Law of Motion States

Let  $\Omega$  be the distribution of firms that produce which they a mass of 1,  $\tilde{\Omega}$  the distribution of incumbent firms at the beginning of the period,  $g$  and  $\hat{g}$  the pdf of  $\omega$  and  $\phi$  respectively,  $p$  the conditional pdf of the productivity shocks  $\epsilon_z$ , and  $\Omega^e$  the distribution of entrant firms. To define the equilibrium first we need to determine the law of motion of the distribution. Distribution of firms that produce is

$$\begin{aligned}\Omega(z, n) = & (1 - \gamma) \int \left[ 1_{\{\text{ch11}\}}(s) 1_{\{n^{11}(z, k, b, \omega) = n\}} + 1_{\{\text{cont}\}}(s) 1_{\{n(z, k, b, \omega) = n\}} \right] d\tilde{\Omega}(s) \\ & + \bar{\mu}(1 - \gamma) \int \left[ 1_{\{\text{ch11}\}}(s) 1_{\{n^{11}(z, k_0, 0, \omega) = n\}} + 1_{\{\text{cont}\}}(s) 1_{\{n(z, k_0, 0, \omega) = n\}} \right] \hat{g}(\phi) g(\omega) d\phi d\omega d\Omega^e(z) \\ & + \text{lom} \mid \text{exit}\end{aligned}$$

The distribution of incumbent firms at the beginning of the period  $\tilde{\Omega}(z, \omega, k, b, \phi)$  is

$$\tilde{\Omega}(s') = \int 1_{\{k'(z, n) = k'\}} 1_{\{b'(z, n) = b'\}} \hat{g}(\phi') g(\omega') p(\epsilon_z \mid \rho_z z + \epsilon_z = z') d\epsilon_z d\Omega(z, n)$$

# Data Sources, Sample and Some Definitions

## Compustat

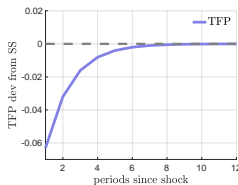
- Two samples (accounting changes after 2018, see Ma's online notes)
  - ▶ Pre-covid = 1980-2017 (n=179k annual, n=426 k quarterly)
  - ▶ Covid = 2019-2020 (n=14k quarterly)
- Sample selection: nonfinancial,  $k > 0$ , assets  $> 0$ , drop outliers and short-spell ( $< 20$  q spell)
- Key definitions:
  - ▶  $n$  = profits + liq value capital – net liquid liabilities
  - ▶ profits =  $F1.oiadpq$  where  $F1$  = one period ahead in the data
  - ▶ net liquid liabilities =  $1ctq - cheq$
  - ▶ liq value capital =  $inv tq \times \omega_{inv} + rectq \times \omega_{rec} + ppentq \times \omega_{ppentq} + acoq$  where  $\omega_x$  is liq value rate (from Kermani Ma 2020) of asset class  $x$
- Identify bankrupt firms that operate following Corbae D'Erasmus 2021. Use footnote to total assets and deletion information ( $d1rsn$  and  $d1dte$ ). Bankrupt firms:
  1. report adoption accounting under Ch11, or bankrupt and not deleted
  2. data available next period

# Crisis Shocks

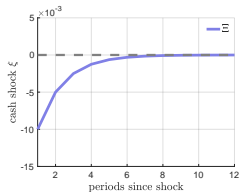
- Shocks unexpected and perfect foresight of path
- Temporary with persistence 0.5
- Definition of shocks:
  - ▶ TFP A: prod funct  $Az f(k, \omega, l)$
  - ▶ Cash  $\xi$ : reduction in  $n$  by  $\xi k$
  - ▶ Credit: recov rate if liquidated  $\alpha_7$

MIT shocks path

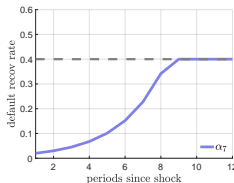
TFP A



Cash  $\xi$

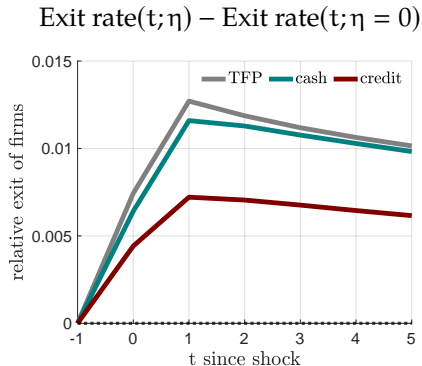
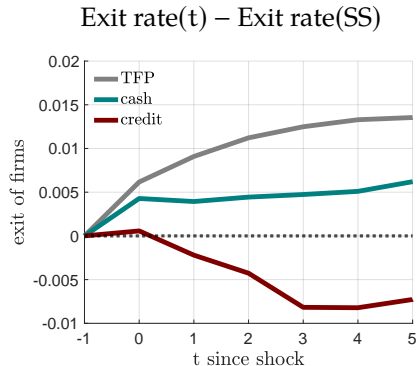


Credit (Recovery liquidation)  $\alpha_7$



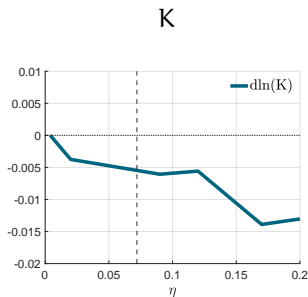
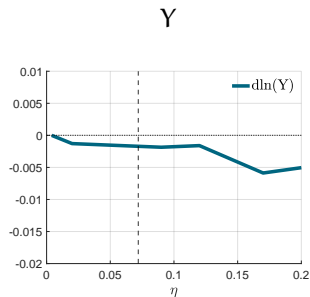
# Steady State Comparison

- Firm exit dynamics during crisis experiments

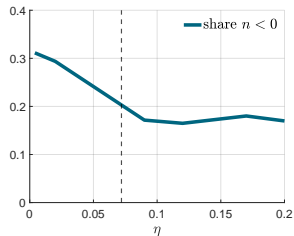


# Steady State Comparison

- Variables: aggregate income  $Y$ , capital,  $K$  and share of firms with  $n < 0$
- Comparison: steady state for different  $\eta$



share of firms with  $n < 0$



notes: log difference relative to st-st with  $\eta = 0$  for  $Y$  and  $K$  and levels for share of firms

[back](#)

# Heterogeneous Investment Response

## Empirical Specification

- Diff-in-diff crisis event estimate  
similar to Kalemli-Özcan Laeven Moreno 2020

$$\Delta \log(k_{it}) = \underbrace{\sum_{j=1}^J \beta_j^n \left( Q_{it}^{nj} \times \text{crisis}_t \right)}_{\text{heterogeneity across } n/k} + \underbrace{\sum_{j=1}^J \beta_j^b \left( Q_{it}^{bj} \times \text{crisis}_t \right)}_{\text{heterogeneity across } b/k} + \underbrace{\Lambda' Z_{it}}_{\text{controls}} + \varepsilon_{it}$$

let  $x_{it} = \{b_{it}, l_{it}\}$  firm  $i$  at period  $t$  with

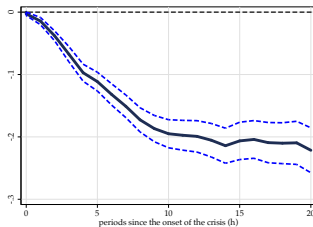
- demeaned by sector  $\hat{x}_{it} = x_{it} - \mathbb{E}_s[x_{it}]$ .
- $\Delta \log(k_{it}) = \log(k_{it+h}) - \log(k_{it})$  with  $h$  peak-to-trough length
- $\text{crisis}_t$  indicates if a crisis happens during the period considered
- $Z_{i,t}$ : sales growth, log firm size, firm FE, sector FE



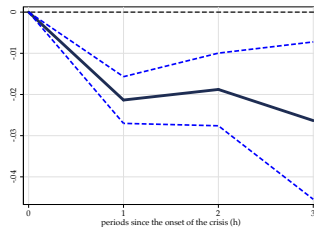
# Recent Crisis Episodes in U.S.

$\Delta$  Capital Accumulation

(a) Great Recession



(b) Covid-19 Crisis



$$\beta_h : \log(k_{it+h}) - \log(k_{it}) = \alpha_i + \beta_h \text{crisis}_t + \varepsilon_{it+h}$$

[back](#)

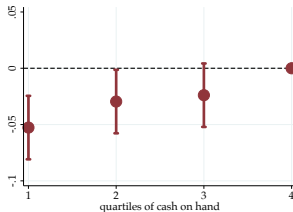
# Investment Adjustment Heterogeneity

## Recent Crisis Episodes in U.S.

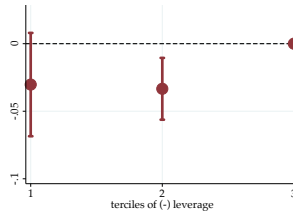
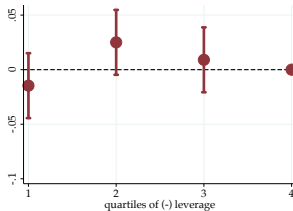
Great Recession

Covid-19 Crisis

Across  $n/k$



Across  $b/k$



# Credit Policy Setup

- Announced unexpectedly at  $t = 0$  (same period of shocks) for  $T \geq 0$  periods and implemented at  $j \in [0, T]$
- Eligible firms  $(z, n) \in \mathcal{P}$  offer sequence of  $\{Q_t^g(.)\}$
- Policy  $\mathcal{P}$  and labor taxes  $\tau$  fixed across time
- Budget constraint from  $t \geq 1$

$$\tau w_t L_t + B_t + B_{t-1,t}^g = B_t^g + (1 + r_{t-1}) B_{t-1}$$

$B_t^g$  amount lent,  $B_{t-1,t}^g$  lent at  $t - 1$  and recovered at  $t$

[back](#)

# Credit Insurance Policy: First Best and Trade-off

## Proposition (Credit Insurance Policy)

*Assume that the government implements the credit insurance policy next period and is predictable today:*

- 1. First best policy:  $Q^g = \tilde{Q}$  then no rollover risk and qualified firms indifferent between using public or private credit.*
- 2. No screening: fix  $z^g$  such that  $Q^g = \tilde{Q}(z^g, k', b')$  with firms qualified for credit those with  $0 > n > \underline{n}(z^g)$ . This policy faces a trade-off between lowering firm rollover risk and greater misallocation.*

- 1st best policy eliminates coord failures and firms don't use the program's credit
- Imperfect then greater  $z^g$  will preclude more coord fail, but firms with  $z < z^g$  will draw funds (zombification)

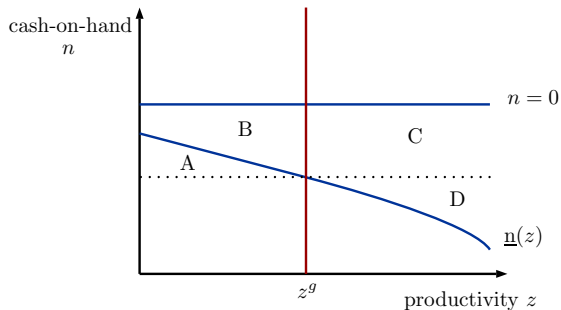
# Direct Lending vs. Credit Guarantees

- Examples: direct lending  $\approx$  Fed's PMCCF SMCCF and credit guarantee  $\approx$  PPP
- In the theory policies are
  - ▶ direct lending (DL): alternative  $Q^g(.)$  [detail theory](#)
  - ▶ credit guarantee (CG): repay  $\alpha_g^r \geq \alpha^r$  in case of default
- Workings relative to rollover crises
  - ▶ DL affects payoffs (outside eq) and could coord creditors in good eq
  - ▶ CG relaxes  $\underline{n}(z)$  but doesn't *directly* preclude coord failures

# Credit Policy Trade-off

Stylized example of 1 period policy in PE and two extreme cases

1. *Perfect screen of  $z$*  :  $Q^g = \tilde{Q}$  and then remove coord failures for "free"
2. *No screen of  $z$*  : gov lends to eligible firms  $n \in (0, \underline{n}(z^g)]$  firms at  $\tilde{Q}(z^g, k', b')$ , with  $z^g$  parameterizing policy scope



$A \cup B \cup C$ : eligible

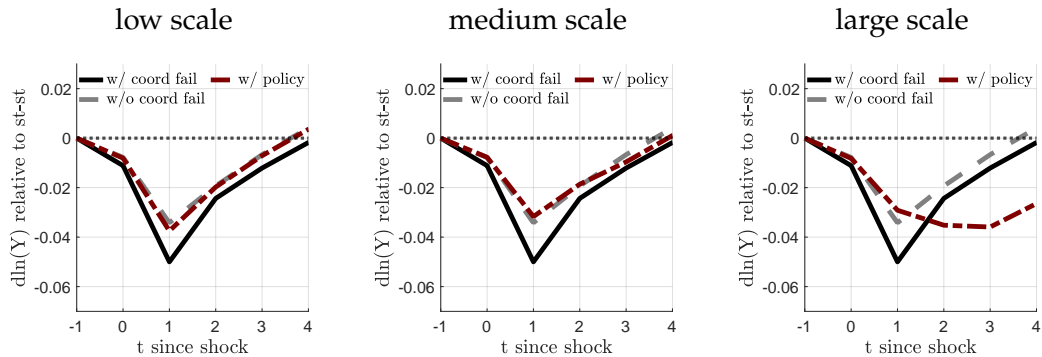
D: excluded

$A \cup B$ : subsidized credit

$B \cup C$ : coord failures precluded

# Credit Policy Implications: TFP shock

- Policy active for first two periods and TFP shock driven crisis [back cash shock results](#)

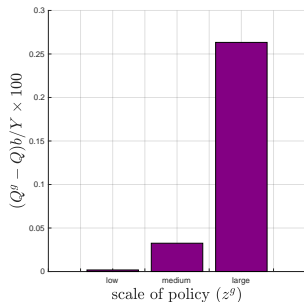


# Credit Policy Implications: TFP shock

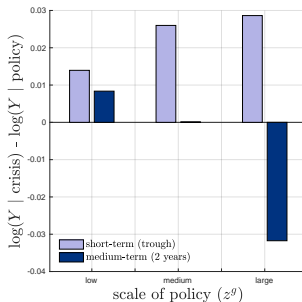
- Compute fiscal costs, short and long term benefits [back](#)

## Costs and benefits

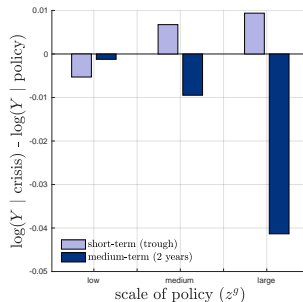
Fiscal costs



Benefits with coord fail



Benefits without coord fail





# Liability Structure Data

- Debt maturity (Compustat)

	Time to mature (share)		
	< 1 year	1 to 4 years	≥ 5 years
Debt	0.29	0.33	0.38

	< 1 year	> 1 years
	0.61	0.39
Liabilities		

- Number of creditors from bankruptcy filings to Chapter 11 (FJC-IDB)

	# Creditors		
	1 to 100	101 to 1,000	>1,000
Medium (> 50 million and < 1 billion assets)	0.16	0.19	0.65
Large (> 1 billion assets)	0.03	0.04	0.93

# How Costly is Firms' Rollover Risk?

- (ex-ante) Cost computed as  $\tilde{Q}(z, k', b'; \eta) - \tilde{Q}(z, k', b'; 0)$
- Only 2.2% of the firms face a cost of rollover risk higher than intermediation spread

Cost of runs (in annual spread terms) distribution

