Financial Frictions and the Market for Firms*

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Abstract

We study and quantify the aggregate implications of the trade of firms in the presence of financial frictions. Empirically, we document that one in four U.S. entrepreneurs purchased their business, with younger, smaller, and higher average revenue product of capital (ARPK) firms having the highest trading rates. After trade, capital outpaces output growth, reducing firms' ARPK over time. To explain these findings, we propose a general equilibrium model of entrepreneurship with a frictional market for firms in which firm trade alleviates financial constraints. We show that the predictions of our theory are consistent with the cross-sectional and longitudinal facts. Quantitatively, we show that taxing transactions in the market for firms generates sizable aggregate losses, reflecting the role of this market in improving allocative efficiency. The productivity gains from this market are potentially larger in less financially developed economies.

Keywords: trade of firms, misallocation, financial frictions, firm dynamics.

JEL classifications: E44, O47, G30.

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1 Introduction

Markets are the predominant allocation mechanism of modern economies. While markets that allocate resources across firms have been well-studied, such as capital and labor markets, less is known about the market where firms themselves can be bought and sold, i.e., *the market for firms*. In this paper, we argue that the role of this market is particularly relevant in economies where financial constraints are a pervasive feature of entrepreneurial activity. In such environments, who owns the firms matters for allocations, as credit-constrained entrepreneurs operate at a suboptimal scale, resulting in capital misallocation and lower output. The market for firms allows constrained entrepreneurs to sell their firms to other parties with more financial resources, potentially improving allocative efficiency in the economy.

Our paper's contribution is to empirically and quantitatively study the market for firms in the presence of financial frictions. We do so in multiple steps. Empirically, we document that one out of four U.S. entrepreneurs purchased their business, with younger, smaller, and higher average revenue product of capital (ARPK) firms more likely to be traded. We also establish novel facts about post-trade firm dynamics. We document that firms' output and capital grow significantly after trade, with capital outpacing output, reducing firms' ARPK over time. Firms' average revenue product of labor (ARPL) remains fairly constant after trade, while profitability and leverage fall. To explain these findings, we develop and quantify a macroeconomic model where agents can buy and sell firms in a frictional market. Gains from trading firms arise from financial frictions, heterogeneity in agents' abilities, and preference shocks that capture alternative motives to trade. We show that both cross-sectional and longitudinal facts are consistent with the trade of firms alleviating financial constraints. We then use our framework as a laboratory to study the macroeconomic relevance of this market. Quantitatively, we show that taxing transactions in the market for firms generates sizable aggregate losses, highlighting the role of this market in improving allocative efficiency. The productivity gains from this market are nonlinear and are larger when financial development is low. We provide cross-country evidence consistent with this prediction.

We start our empirical analysis by using multiple data sources to establish salient cross-sectional features of the market for firms in the U.S. economy. First, we document that one out of four entrepreneurs (around 23% to 26%) in the U.S. acquired their business by purchasing an existing firm, implying an annual trade rate of 3%. Compared to other assets, private firms are traded less frequently than real estate but more so than patents.[1]

Second, we document two salient characteristics of business buyers and sellers. Our first finding reveals that 66% of buyers have never been entrepreneurs before purchasing their

[1] Berger and Vavra (2015) reports that 5% of houses are traded annually, higher than the 3% trade rate for private firms. Akcigit, Celik, and Greenwood (2016) documents that 16% of U.S. patents have been traded, smaller than the 26% we find for private businesses.

current firm. This finding suggests that buying an existing firm is a relevant way to enter entrepreneurship. Besides capturing the trading frequency of private firms, our theoretical framework incorporates these novel possible transitions into entrepreneurship through the market for firms. Our second finding is that the average business buyer is relatively wealthy (about three times the wealth of the average household) and, crucially, wealthier than the average business seller. This evidence will be used to discipline our theory, in which heterogeneity in buyers' and sellers' wealth plays a first-order role in generating gains from trade.

Third, we establish novel cross-sectional facts about the trading frequency and firms' observable characteristics. We document that younger, smaller, and higher ARPK firms have the highest trading rates. These cross-sectional results about firms' characteristics and trade frequency are informative about the underlying motives behind firms' trade. Both firms' age and size are associated with financial constraints (Hennessy and Whited, 2007; Hadlock and Pierce, 2010). Furthermore, firms' ARPK is informative about their access to external finance, as credit-constrained firms may have high returns but cannot increase their investment. By introducing financial frictions as a microfoundation that generates gains from trading firms, our model can account for these cross-sectional facts.

In addition to our cross-sectional analysis, we document novel post-trade firm dynamics to shed light on the different motives driving firms' trade and contrast alternative theories. Due to data limitations, we use data from several high-income European countries, which are the most comparable to the U.S. economy. We document that, on average, firms' capital increases by 109% and output by 37% five years after trade. As a result of these joint dynamics, ARPK sharply falls by 34%. In contrast, firms' ARPL remains almost unchanged after trade, as labor costs grow at nearly the same rate as output. We also document that profitability, measured by the profits-to-capital ratio, decreases as firms' capital grows faster than profits. Finally, we document that firms' leverage falls after trade despite the significant capital increase. All these results are robust across several specifications, including a difference-in-differences framework. As we explain in detail below, the post-trade firm dynamics in the data are consistent with the trade of firms alleviating financial constraints.

Motivated by these findings, we develop a heterogeneous-agent model of entrepreneurship with a frictional market for firms. Households are heterogeneous in assets, abilities, occupations (workers or firm owners), and firm quality if they own a firm. Households' abilities and firms' quality evolve stochastically over time. Firm owners can trade or shut down their firms, while workers can become business owners by buying an existing firm or through a startup shock that allows them to found a new business. There are borrowing constraints and incomplete financial markets, so households face uninsurable idiosyncratic risk.

We characterize firms through the quality of an entrepreneurial project, which is indivisible, rival, and excludable. Firm owners produce the final consumption good using a technology that combines capital, labor, their own ability, and the firm's quality. Because ability is inalienable to the owner, only the firm's quality is tradable in the market for firms. Hence, firms' quality captures all features characterizing a firm beyond the owner's ability and capital and labor inputs, such as organizational capital and intangible assets. Besides the firms owned by a single household, which we call private firms, there is a second sector with a representative public firm. Both sectors produce the same good, which can be used for consumption or savings in a risk-free asset. There is also a financial intermediary that, each period, takes households' savings and rents capital to firms.

Our empirical results show that private firms are traded infrequently, which motivates the use of a search-theoretic approach to model this market. Specifically, we model the market for firms through a decentralized market subject to search frictions and bilateral random matching. A Nash bargaining protocol between sellers and buyers determines the trading price. One interpretation of these assumptions is that agents can valuate only one firm at a time, which slows down the trading process. This setup is suitable for our analysis and allows the model to match key features of the market for firms we previously documented.

Exchanges in the market for firms are voluntary. Hence, a necessary condition for trade is that agents have different valuations for the same firm, with the buyer assigning a higher valuation than the seller. In our theory, heterogeneity in firms' valuations and, hence, gains from trade arises from four sources: borrowing constraints, incomplete markets, heterogeneous ability, and preference shocks. Borrowing constraints and incomplete markets create an endogenous motive to trade: for a given firm, unconstrained agents achieve higher profits, grow the firm faster, and bear risk more effectively than constrained agents. Differences in ability also endogenously generate gains from trade since, all else equal, higher-ability owners will assign a higher value to the same firm as they can operate it more efficiently. By reallocating firms between agents with different abilities and wealth levels, the market for firms can improve allocative efficiency in the economy. Finally, we assume that potential sellers receive idiosyncratic preference shocks that parsimoniously capture other motives for trading firms that we do not explicitly incorporate in our theory.

We calibrate the model to key features of the U.S. economy by targeting a comprehensive set of moments related to entrepreneurship, transitions between employment and business ownership, the distribution of income and wealth across households, firm dynamics, and the market for firms. Importantly, we discipline the trade motives arising from financial frictions by targeting the difference in average wealth between business buyers and sellers. Further, we discipline the role of owners' ability in firms' outcomes, which informs the heterogeneous ability motive, by targeting the relationship between startups' output and owners' pre-entrepreneurship labor income, controlling for wealth. Our calibration indicates that financial frictions, both borrowing constraints and incomplete markets, play a central role in generating gains from trade in the market for firms. Heterogeneous ability and preference shocks also contribute to trade, although to a lesser extent.

After quantifying the model, we perform two exercises to evaluate testable predictions of our theory about financial frictions being a relevant motive for trade. First, we compare the model-simulated relations between trade rates and firms' observable characteristics with their empirical counterparts. Consistent with the data, our model predicts that younger, smaller, and higher ARPK firms have the highest trading rates. These relations arise from the key prediction of our model that constrained firms are more likely to be traded and from the strong correlation between these characteristics and binding borrowing constraints.

In our second exercise, we test the implications of the theory for post-trade dynamics. Our model predicts that firms' trade alleviates financial constraints. Before trade, firms operate with lower capital and higher ARPK than their unconstrained levels. After trade, capital increases more than output, leading to a sizable reduction in ARPK that closely matches the decline observed in the data. Because financial frictions in the model do not distort labor input choices, ARPL is constant, consistent with the nearly flat pattern in the data. We also show that credit-constrained firms have a profit-to-capital ratio above their unconstrained level. Thus, as in the case of ARPK, capital grows more than profits after trade, reducing profitability in line with the empirical evidence. Finally, because buyers are typically wealthier than sellers in the model, the expansion in capital is largely financed by owners' equity, resulting in a decline in leverage comparable to that observed in the data.

Overall, our model is consistent with a broad set of cross-sectional and longitudinal moments we did not target in the calibration. These patterns arise from the key endogenous motive to trade firms we study in this paper, driven by financial frictions and heterogeneity in households' wealth. Indeed, we show that heterogeneous ability and preference shocks, on their own, would lead to counterfactual dynamics after trade. We also discuss alternative motives for trade and compare their predictions with the data, highlighting the usefulness of our novel post-trade firm dynamics for informing theories of the market for firms.

After showing that testable predictions from our theory are consistent with the data, we conduct two counterfactuals to examine the macroeconomic implications of the market for firms as a mechanism that allocates ability, capital, and productive projects in the economy. First, we study the aggregate effects of taxing transactions in this market. Second, we analyze the role of the market for firms across economies with varying levels of financial development.

In the first counterfactual exercise, we show that taxing transactions in the market for firms generates sizable aggregate losses, raises limited fiscal revenue, and delivers modest reductions in income and wealth concentration. As the transaction tax increases, gains from trade decline and the trade rate falls, with the market effectively shutting down under a 100% tax. Tax revenues display a Laffer curve, peaking at a 47.5% tax rate, with maximum revenues of only 0.15% of GDP. By contrast, the aggregate costs are substantial, reflecting the market's role in improving allocative efficiency: shutting down the market reduces private firms' aggregate output by 7.3% and total factor productivity (TFP) by 2.3%. Taxing firms' trade also affects inequality. As the tax rate rises, the income and wealth shares of the top 5% decline, with maximum reductions of 3% and 9%, respectively, when the market shuts down. We show that financial frictions are central for these results. Imposing the same tax in an economy without borrowing constraints leads to aggregate losses that are 60-70% smaller, highlighting the importance of this market in alleviating financial constraints.

We conclude our analysis by investigating the interaction between financial development and firms' trade. In our model, the functioning of both credit markets and the market for firms jointly determines the allocation of productive resources. As in the finance and misallocation literature, higher levels of financial development lead to better capital allocation and higher TFP. Unlike previous work, we show that aggregate TFP can increase through a better-functioning market for firms at any level of financial development. Yet, the TFP gains from improving the market for firms are nonlinear and are substantially larger when financial development is low. Consistent with this prediction, we document that post-trade firm dynamics are more pronounced in middle-income and less financially developed countries, with firms' ARPK decreasing more than in our baseline high-income economies. Altogether, our results indicate that the market for firms can serve as an important substitute for debt financing, particularly in economies with less developed credit markets.

Related Literature Our paper contributes to the following strands of the literature.

Trade of Firms and the (Re)Allocation of Productive Resources. Our work primarily contributes to the literature that studies the trade of private firms as an allocation mechanism. Earlier work includes Holmes and Schmitz (1995) and Caselli and Gennaioli (2013), which study the role of heterogeneous managerial ability in driving trade. We incorporate this motive in our quantitative model and find that it accounts for 23% of transactions. Closest to our work are two contemporaneously developed papers that examine firms' trade in heterogeneous-agent models with entrepreneurship. Gaillard and Kankanamge (2020) studies a model in which only mature firms, which are more productive and have better access to financing, can be bought and sold. Our quantitative model captures that firms become more productive over their life cycles and that more productive firms have more borrowing capacity, while being consistent with the fact that younger firms have the highest trading rates. Mahone (2023) analyzes an economy where firms' trade is entirely driven by exogenous preference shocks, which we find to explain 24% of the transactions in our quantitative model.

Unlike these papers, we study an economy where gains from trade arise endogenously from borrowing constraints and incomplete financial markets. In our setup, the market for firms is multidimensional, with buyers' and sellers' ability and wealth, in addition to firms' quality, playing a first-order role in determining trading surpluses and equilibrium prices. Our paper is the first in this literature to use longitudinal data to document post-trade firm dynamics, which are highly informative about the empirical relevance of different theories on firms' trade. We show that our novel cross-sectional and longitudinal facts are consistent with the main prediction of our theory that the trade of firms alleviates financial constraints.

Entrepreneurship in Macroeconomics. Our theoretical framework builds on the literature on heterogeneous-agent models with entrepreneurship, which started with the seminal work of Quadrini (2000) and Cagetti and De Nardi (2006). We contribute to this literature by extending the framework to allow for the trade of firms in a frictional market, which we show is economically significant and represents an important channel into entrepreneurship.

Finance and Misallocation. Our paper also contributes to the literature on financial frictions and capital misallocation as a source behind TFP differences across countries (Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014; Moll, 2014; Gopinath et al., 2017; David and Venkateswaran, 2019). We show that the market for firms can reduce the capital misallocation caused by financial frictions, especially in economies with tighter credit constraints where the gains from trading firms are higher. Empirically, our novel evidence of post-trade firm dynamics is consistent with Bau and Matray (2023), which documents a reduction in firms' ARPK following a foreign capital liberalization in India.

M&A in Finance and Macroeconomics. Our paper is also related to the literature in finance and macroeconomics on mergers and acquisitions (M&A), in which acquirers are existing firms. This is a crucial distinction relative to our paper, where households can buy and sell firms. Significant contributions to this literature include Jovanovic and Rousseau (2002) and Rhodes-Kropf and Robinson (2008). More recent work studies M&A in firm-dynamics models à la Hopenhayn (1992), including David (2021), that quantifies the implications of mergers under complementarities; Cavenaile, Celik, and Tian (2021) and Celik, Tian, and Wang (2022), which analyze synergistic M&A with endogenous productivity growth under information frictions and oligopolistic competition, respectively; and Bhandari, Martellini, and McGrattan (2025), which studies acquisitions with convex adjustment costs in intangible capital. In addition, our empirical analysis relates to the corporate finance literature on the financial motives behind M&A, showing that acquisitions alleviate financial constraints in target firms (see, e.g., Liao (2014) and Erel, Jang, and Weisbach (2015)).

Aggregate Implications of the Market for Ideas. Finally, as in the literature on the trade of ideas (Silveira and Wright, 2010; Akcigit, Celik, and Greenwood, 2016), we adopt a search-and-matching framework with bilateral meetings, where the likelihood of trade depends on search frictions and the endogenous distribution of agents. Unlike ideas (Lucas and Moll, 2014; Perla and Tonetti, 2014), firms are rival and excludable, requiring compensation from buyers to sellers, and firm trade operates through allocative efficiency rather than diffusion.

Outline The rest of the paper is organized as follows: Section 2 presents our empirical analysis; Section 3 presents the model; Section 4 describes our parameterization; Section 5 describes the model's main properties; Section 6 evaluates predictions of our theory on the trade of firms; Section 7 presents the macroeconomic implications; and Section 8 concludes.

2 Empirical Analysis

In this section, we use microdata on business owners, households, and firms to document several novel cross-sectional and longitudinal facts about the market for firms.

2.1 Cross-Sectional Analysis

This section presents our cross-sectional analysis of the U.S. market for firms. We first describe our data sources and then present our cross-sectional facts. Appendix A provides robustness checks and additional exercises related to our cross-sectional results.

2.1.1 Data Sources

We use four surveys that provide complementary information on private firms and their owners. Our primary source is the 2007 Survey of Business Owners (SBO) Public Use Microdata Sample (PUMS), which is representative of non-farm private businesses and reports how owners acquired their firms. We also use nine waves of the Survey of Consumer Finances (SCF, 1989–2016) to discipline the income and wealth distribution in the model and, as in the SBO, this survey asks business owners how they acquired their firms, allowing us to measure the wealth of recent business buyers. We complement these data with the Panel Study of Income Dynamics (PSID, 1999–2019), which allows us to measure sellers' wealth and study transitions and income dynamics between paid employment and entrepreneurship. Finally, we use the Kauffman Firm Survey (KFS, 2004–2011), an eight-year panel of startups with balance-sheet information that lets us construct firm-level capital. We account for its focus on startups when comparing the KFS evidence with data simulated from our model. See Appendix A.1 for further details on the surveys, variable definitions, and sample selection.

Entrepreneurs Our cross-sectional analysis focuses on *entrepreneurs* as the unit of observation. We follow Cagetti and De Nardi (2006) and define entrepreneurs as self-employed individuals who own a business and have an active management role in it. We focus on entrepreneurs with at least one employee. This sample selection is consistent with the definition of firms used by the US Census and our longitudinal analysis using firm-level data. [2]

Throughout the analysis, we assume that each entrepreneur owns and manages only one firm. This assumption implies that the number of firms traded every period equals the

[2]Nonemployer firms are mainly composed of self-employed individuals. While they account for 58% of the firms in the SBO, they represent only 3.2% of total sales. In Appendix A.2, we report that the share of traded firms considering all entrepreneurs (with employer and nonemployer firms) is lower. This reflects the fact that nonemployer firms are more similar to independent workers whose only input is human capital, which is harder to buy and sell; hence, they are less likely to participate in the market for firms.

number of entrepreneurs that trade their firms. Hence, we use both terms interchangeably. Our assumption relies on the fact that, in the SCF, more than 80% of entrepreneurs own only one firm (see Table A.12). Moreover, in the SBO, more than 74% of the private firms have only one entrepreneur and over 96% have at most two (see Table A.13).[3]

2.1.2 How do Entrepreneurs Acquire Their Firms?

Share of Traded Firms We start our analysis using the SBO and the 2007 SCF to document how entrepreneurs acquire their firms.[4] We focus on three main types of acquisitions: founding a firm, purchasing an existing firm, and inheritance or other types of acquisition. Table 1 shows that two-thirds of entrepreneurs acquire their firms by founding their businesses. Additionally, it shows that 9% to 12% of entrepreneurs acquired it through inheritance or other types of acquisition. From that table, the most relevant number for our analysis of the market for firms is that 23% to 26% of entrepreneurs in the U.S., depending on the survey, acquire their businesses by purchasing an existing firm.[5]

Table 1: Share of Entrepreneurs by Business Acquisition

	Founded	Purchased	Inherited/Other
SBO	65.2%	25.5%	9.3%
SCF	65.3%	22.7%	12.0%

Source: SBO and SCF for the year 2007.
Notes: Entrepreneurs are defined as (i) self-employed, (ii) business owners, who (iii) actively manage their firm, and (iv) the firm has at least one employee. Other types of acquisition groups: acquired as a transfer, as a gift or other not specified.

Appendix A.2.1 verifies the robustness of these findings. Table A.4 shows that our results are robust across alternative definitions of entrepreneurs (e.g., focusing on majority owners). Table A.5 shows that the results are nearly identical when computing the share of traded firms at the firm level (rather than the entrepreneur level). Table A.6 shows that franchises do not drive our results. Table A.7 reports that firms' trade is ubiquitous across industries, although to a lesser extent for construction. Finally, using different SCF waves, we compute the share of traded firms over time. Figure A.2 shows that the share of entrepreneurs who purchased their business has declined in the last decades but has been stable since 2007.

Firms' Trade Rate The previous results refer to the *stock* of firms that have been traded at any point in the past. We are also interested in the annual frequency of trade, i.e., the trade *rate*. We estimate the percentage of firms traded every year using two strategies. The

[3]In this line, Appendix A.3.3 documents that private firms' ownership and management are highly concentrated (Figure A.3), even for the economy's oldest and largest privately held firms (Figure A.4).

[4]Specifically, the SBO asks: "*How did [the owner] initially acquire ownership of this business?*". Similarly, the SCF asks business owners: "*How did you first acquire this business?.*"

[5]Using the same data sources, Gaillard and Kankanamge (2020) and Mahone (2023) contemporaneously documented similar numbers for the stock and frequency of firms' trade in the U.S. economy.

first strategy looks at the percentage of firms purchased in the SBO and SCF data in the same year of the survey. The second strategy, as Appendix A.4 describes, uses the law of motion of the stock of traded firms as a function of firms' entry, exit, and trade rates. Either strategy implies that around 3% of the firms are traded every year.

2.1.3 Buyers' and Sellers' Characteristics

Buyers' Previous Occupation Using the SBO, we can obtain information regarding entrepreneurs' previous occupations. We found that 66% of the entrepreneurs who purchased their firms had never been self-employed. Hence, most likely, these individuals were in the labor market before acquiring their businesses.[6] This result indicates that buying an existing firm is a relevant channel for entering into entrepreneurship, which, to the best of our knowledge, has not been studied before. Our theoretical framework incorporates this novel feature about households' possible transitions into entrepreneurship through the market for firms. Table A.8, in the Appendix, shows that this result is robust to alternative samples, and Table A.9 shows that this number is similar across firms' age and size distributions.

Buyers' and Sellers' Wealth In our theory of the market for firms, differences in the wealth of buyers and sellers play a first-order role in generating gains from trade. We now provide evidence that recent business buyers are, on average, wealthier than sellers. First, using the SCF, we identify entrepreneurs who have recently purchased their businesses (within one year before the survey) and measure their wealth. Panel (a) of Table A.14 in the Appendix reports recent buyers' wealth relative to the average household and to the average entrepreneur under two definitions: non-business wealth (excluding the value of the firm) and total wealth. The average non-business wealth of recent buyers is 2.71 times that of the average household and 0.79 times that of the average entrepreneur. Considering total wealth, the corresponding ratios are 3.83 and 0.69. These results show that recent business buyers are considerably wealthier than the average household but less wealthy than other entrepreneurs.

Next, we measure the wealth of business sellers in the PSID. We identify the households that report having sold their business in the previous two years (the PSID's frequency) and record their wealth in the previous wave. Panel (a) of Table A.14 shows that sellers' average non-business wealth is 1.88 times that of the average household and 0.58 times that of the average entrepreneur. For total wealth, these ratios are 3.04 and 0.54.[7] Hence, like business buyers, sellers are wealthier than the average household but less wealthy than entrepreneurs.

[6] We consider the question in the SBO: "*Prior to acquiring this business, had the owner ever owned a business or been self-employed?*" This number should be interpreted as a lower bound on the share of firms purchased by non-entrepreneurs.

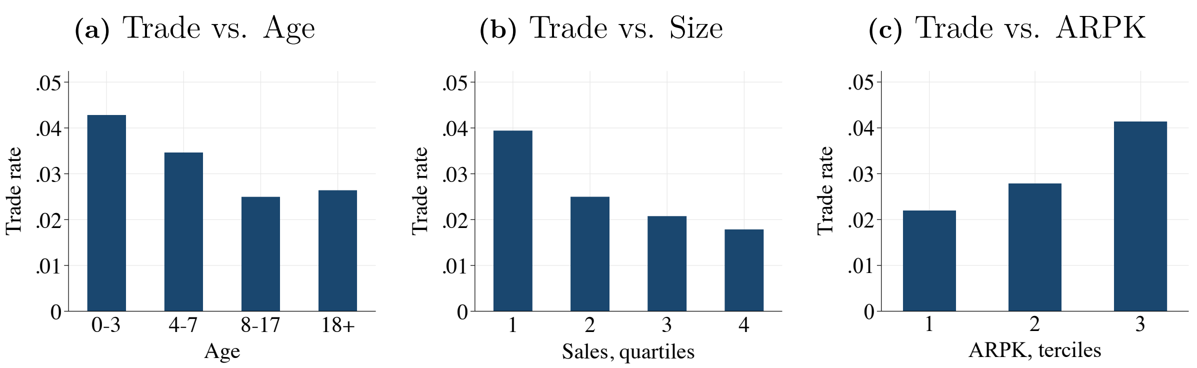
[7] While the PSID lacks the SCF's oversampling of high-wealth households, entrepreneur-to-household wealth ratios are very similar across the two surveys: 3.25 vs. 3.43 for non-business wealth and 5.59 vs. 5.55 for total wealth (see panel (c) of Table A.14). Thus, the buyer-to-entrepreneur and seller-to-entrepreneur wealth ratios are comparable across surveys.

The previous results imply that recent buyers are 1.36 and 1.28 times wealthier than sellers, respectively, for non-business and total wealth, using the ratios relative to entrepreneurs. Note that buyers' wealth is measured after the transaction, whereas sellers' wealth is measured before it. Hence, as we discuss in Section 5.3, these ratios likely understate the differences in wealth at the time of trade. As we argue below, the fact that buyers are, on average, wealthier than sellers is consistent with financial frictions being a significant driver of firm trade. Indeed, to discipline the strength of this motive, our calibration directly targets the relative non-business wealth of recent buyers and sellers documented in this section.[8]

2.1.4 Trade Rate and Firms' Characteristics

We now document novel cross-sectional evidence for trade frequency conditional on firms' observable characteristics. We focus on three attributes: firms' age, size, and ARPK.

Figure 1: Trade Rate by Firms' Characteristics



Source: SBO and KFS.
Notes: Panels (a) and (b) use data from the 2007 SBO, and panel (c) uses data from the KFS. In panels (a) and (b) trade is computed using information from firms sold in or after 2007. Size is measured using firms' sales. Panel (c) uses data from KFS. Trade is computed using the firms sold during the years of the sample. We compute this every year and then take the average across time. Average revenue product of capital (ARPK) is measured by sales over capital of the previous year to the sale. Trade rates are normalized to match the aggregate of our baseline calculations.

Firms' Age We document the trade rate by firm age using the SBO. For this analysis, we focus on the firms sold in or after 2007, the same year as the survey. We measure firms' age as the difference between the survey year and the year the firm was founded.[9] Figure 1a presents the trade rate across different age bins. It shows that the youngest firms (0-3 years

[8] As a robustness check, we also calculate the buyer-to-seller wealth ratio using the National Longitudinal Survey of Youth 1979 cohort (NLSY79) data. This survey allows us to identify the wealth of recent buyers and sellers within the same dataset. However, it is not representative in the cross-section and, hence, we do not use it to discipline our model. Consistent with our baseline results using the SCF and PSID, panel (b) of Table A.14 shows that recent buyers are wealthier than sellers in the NLSY79.

[9] Alternatively, we could measure traded firms' age using information from the buyer side by looking at firms purchased in 2007. However, recently acquired firms incorrectly reported as newly established might contaminate this measure of firms' age, especially for trades at age 0. This issue is unlikely to occur for recently sold firms, as sellers report the firms' year of foundation.

old) have the highest trade rates, with a trading frequency about 50% higher than the older age groups. After age eight, the relationship is relatively flat among the oldest firms.

Firms' Size We also employ the SBO to examine the relation between trade frequency and firm size. For this, we look at a sample of business owners who sold their firms in or after 2007 and measure size using data from the previous year of operation. Thus, we relate the trade probability at t against the firm's size at $t-1$. We measure firm size using sales. Figure 1b presents the trade rate for different quartiles of the size distribution. We find that trade frequency and firm size are negatively related. Thus, the smaller firms, those in the sales' bottom quartile, have the highest trading probabilities.[10] While smaller firms have the highest trading frequencies, Table A.10 in the Appendix shows that the largest firms in the SBO are more likely to have been traded in the past. This evidence suggests that firms tend to be small when traded but grow significantly afterward. These cross-sectional results are consistent with the post-trade firm dynamics we document below in Section 2.2.

Firms' ARPK Finally, we document the relation between the trade rate and firms' ARPK. We measure ARPK using the KFS, which includes balance sheet information, allowing us to measure firms' capital. As in the analysis of firms' size, we relate firms' ARPK at period $t-1$ to the probability of trade at t, which we measure as the share of owners who report having sold or merged their business. Figure 1c shows that trade frequency and ARPK are positively related, with firms in the top ARPK tercile having the highest trading rates.

In sum, we document that younger, smaller, and higher ARPK firms have the highest trade rates. These results provide insights into the mechanisms driving firms' trade. Below, we show that these cross-sectional facts are consistent with our theory's prediction that financially constrained firms are more likely to be bought and sold in the market for firms.

2.2 Post-Trade Firm Dynamics

This section uses longitudinal data from several high-income countries to document novel facts about firm dynamics after trade. First, we describe the data and discuss how we identify firm trade. We then present our empirical specification and describe our results.

2.2.1 The Orbis Database

Due to data limitations for the U.S., for which we only have cross-sectional data, we use Orbis Historical, an extensive firm-level panel database covering millions of companies worldwide, to provide novel evidence on post-trade firm dynamics. We focus on a sample of private firms from eleven high-income European countries that are the most comparable to the U.S. economy.[11] The data contains firms' income and balance sheet statements from

[10] Figure A.1 shows that the results are very similar when we measure size using firms' total payroll.

[11] The eleven high-income countries included in our analysis are: Austria, Belgium, Denmark, Finland, France, Germany, Italy, Norway, Spain, Sweden, and the United Kingdom. Section 7.2 presents additional results for middle-income and less financially developed European countries. Orbis covers 71% of the national output of the countries in our sample and captures well the firm-size distribution (Kalemli-Özcan et al., 2024).

1996 to 2023. From 2007 onward, the data reports annual ownership records, including the names and equity shares of firms' owners. Appendix B.1 describes this data, presents our sample selection, and the variables' definitions. Table B.4 shows that our baseline sample in Orbis includes a substantial share of young, small, and single-owner firms, as in the SBO.

Identifying Trades in the Market for Firms We use the ownership files in Orbis to identify transactions in the market for firms. We define a trade episode as the year in which we observe a change in the majority owner of a firm (equity share above 50%). Having identified these events, we take additional steps to accurately identify firm trades. First, using a string similarity algorithm, we exclude changes in names that are spurious or that are likely related to inheritances or other family-related transfers.[12] Second, for a proper mapping with our theoretical model, we exclude transactions that are closer to corporate M&A by focusing on the events where both the buyer and the seller are individuals, using direct ownership linkages.[13] See Appendix B.2 for further details on how we identify trades in the market for firms using the Orbis data, as well as several robustness checks and additional results. For a detailed description of our string similarity algorithm, see Appendix B.3.

2.2.2 Empirical Specification

After identifying trade episodes, we run a non-parametric regression to analyze post-trade firm dynamics. Let i denote a firm and t time. We normalize the trading year to $t = 0$. Then, for each variable of interest, x_{it}, we estimate

$$\log x_{it} = \beta_0 + \sum_{h \in \mathcal{T}} \beta_h \mathrm{D}_{it}^h + \gamma \mathbf{c}_{it} + \epsilon_{it}, \tag{1}$$

where $\mathcal{T} = \{-1, 1, \ldots, 5\}$ and D_{it}^h is an indicator variable equal to 1 if time t corresponds to the period h around the trading episode. As $\mathcal{T}$ indicates, we study firm dynamics from one year before up to five years after trade. The vector $\mathbf{c}_{it}$ denotes our control variables, including country and NACE 4-digit sector fixed effects. Given the relevance of age effects for firm dynamics (Haltiwanger, Jarmin, and Miranda, 2013; Sterk, Sedláček, and Pugsley, 2021; Kochen, 2025), we also control for firms' age at $t = 0$, the year the firm was traded.

2.2.3 Firm Dynamics After Trade

Capital, Output, and ARPK We now describe the results from estimating (1) for firms' capital, output, and ARPK. Figure 2a shows that, on average, firms' capital increases significantly by 0.22 log points (24%) in the first year after trade and by 0.74 log points (109%) after five years. The observed capital growth of traded firms is much larger than

[12] For example, our algorithm excludes changes in owners' names, such as "Federico Kochen" to "Kochen Federico" or "Luis Guntin" to "Rafael Guntin", as the latter is likely related to inheritances.

[13] As Table B.3 in the Appendix reports, using direct ownership linkages, buyers and sellers are individuals in 34% of the events. This number is higher (60%) when considering ultimate owner information. Our results are consistent whether we include all events or identify individuals using ultimate owners (see Appendix B.2).

the average capital growth of the overall pool of firms in our sample, of 0.06 log points (6%) per year. Notably, most of the growth occurs within the first two years, indicating that firms make large capital investments after being traded in the market for firms. Figure 2b reports that output increases, but to a considerably lesser extent, in 0.08 and 0.32 log points (8% and 37%, respectively) after one and five years. As a result of the joint capital and output post-trade dynamics, Figure 2c shows that ARPK declines sharply by 0.14 and 0.42 log points (13% and 34%, respectively) one and five years after trade.

ARPL, Profitability, and Leverage In addition to our main result on the joint dynamics of capital and output, we also document the post-trade firm dynamics of the ARPL, profitability, and leverage. As we explain in Section 6, the evolution of these ratios and firms' ARPK is informative about the empirical relevance of different motives behind firms' trade. Figure 2d shows the results for firms' ARPL, defined as the ratio of output over labor costs, $\log(y/wl)$. Unlike the ARPK, firms' ARPL falls only 0.04 log points (4%) five years after trade, and in some robustness checks, the change is not statistically different from zero. As Figure B.9 in the Appendix shows, the mostly flat profile of firms' ARPL is because labor costs grow at almost the same rate as output after trade.[14]

Regarding firms' profits, Figure 2e shows that profitability, measured by the profits to capital ratio $\log(\pi/k)$, falls by 0.22 and 0.57 log points (19% and 43%) one and five years after trade, respectively. Figure B.10 in the Appendix shows that the reduction in firms' profitability is not driven by falling profits, which increase after firms are traded. Rather, it is explained by profits growing at a lower rate than capital, especially in the first two years after trade.[15] Finally, regarding firms' debt financing, Figure 2f presents the dynamics of net financial leverage, $\log(b/k)$. This panel shows that leverage falls by 0.12 log points (12%) five years after trade. Figure B.11 in the Appendix shows that while debt, on average, rises after trade, capital grows significantly more.[16] The joint dynamics of capital and debt, captured by firms' leverage, show that businesses traded in the market for firms receive sizable equity injections from their new owners in addition to increasing equity through retained earnings.[17]

Robustness Checks and Additional Analyses Appendix B.5 shows that our key novel findings on the joint post-trade firm dynamics of capital and output are robust across several exercises, such as using a broader sample of trading events without restricting to owners being

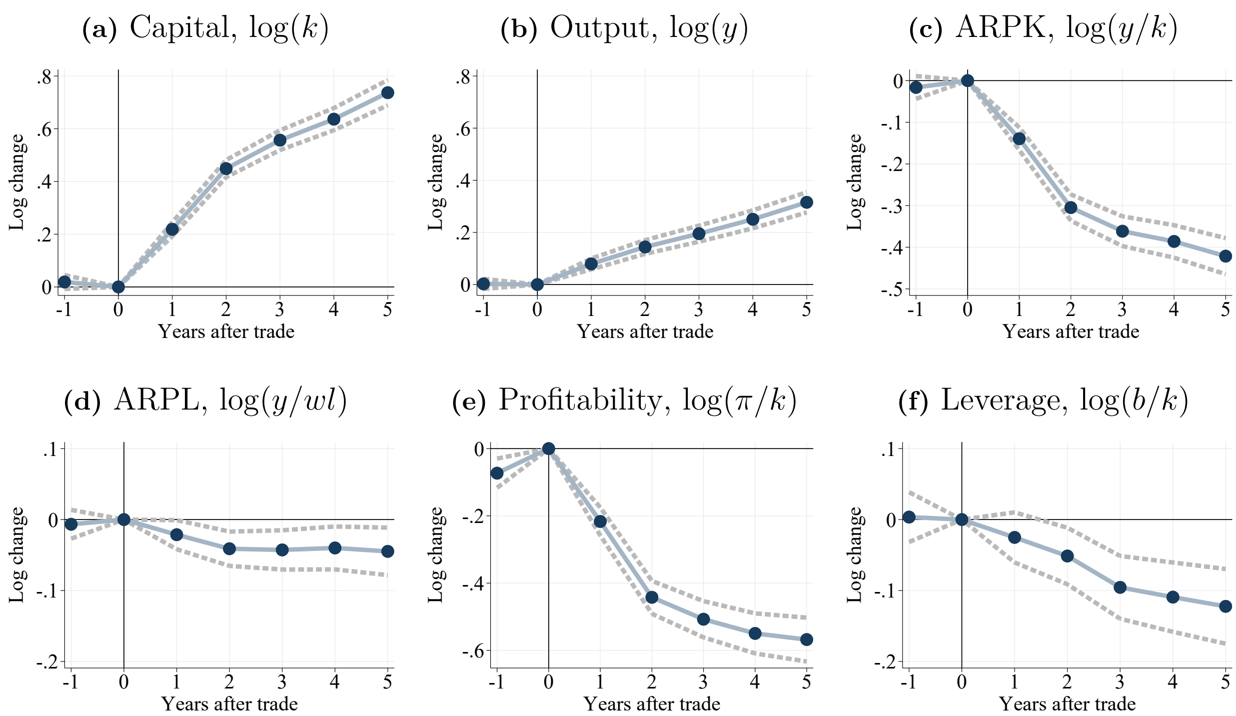
[14]Our findings are consistent with Bau and Matray (2023), which studies firm dynamics after a financial liberalization episode in India. Table 4 of that paper shows that, after the liberalization that reduced financial constraints, firms' ARPK fell sharply by 20%, while ARPL decreased by less than 5%.

[15]The results are very similar if we measure profitability as firms' return on assets (ROA). If firms' profits are negative, they will not be included in our log change measure. Figure B.10b shows that the change in the share of firms with non-positive profits, $\pi \leq 0$, is not statistically different from zero after trade.

[16]Our baseline results are for net financial leverage (Welch, 2011), defining b as financial debt minus cash. Figure B.11c shows that the decline in leverage is slightly larger when using financial debt in the numerator.

[17]Figure B.12 in the Appendix reports the post-trade dynamics of equity and its components. It shows that both external equity, which captures owners' equity injections of funds generated outside the firm, and internal equity, which includes retained earnings or undistributed profits, significantly increase after trade.

Figure 2: Post-Trade Firm Dynamics



Source: Orbis Historical.
Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

individuals (Figure B.4), or identifying ownership linkages using ultimate owner information (Figure B.5). We also present results for additional specifications, including firm-level fixed effects (Figure B.7), or restricting to a balanced panel of firms observed from one year before and five years after trade (Figure B.6). We also extend the event window backward and show that there is no clear pre-trade trend as the estimated coefficients, $\hat{\beta}_h$ for $h < 0$, are close to zero for both capital, output, and ARPK (Figure B.8). Finally, in Appendix B.4, we estimate a difference-in-differences specification in which we compare the trajectories of traded firms with those of a control group of similar non-traded firms matched on observable characteristics. Figure B.2 in the Appendix shows that, following trade, capital and output grow more than in the control group, while ARPK declines by more, in line with our baseline findings.[18]

To summarize, we documented that five years after trade, output and capital increase substantially, with capital outpacing output, leading to a sharp decline in firms' ARPK. Furthermore, unlike the ARPK, the post-trade change in firms' ARPL is much smaller and close to zero. We also documented that firms' profitability decreases while profits rise after trade. Finally, regarding firms' debt financing, we find that leverage falls after trade despite the significant capital increase. As we show below, by introducing financial frictions as a

[18]For our baseline results, we conduct an event study to document post-trade dynamics. We adopt this strategy because the decision to trade is endogenous, so traded firms may differ systematically from the broader pool of firms. Indeed, we are able to identify a non-traded counterpart for only 42% of traded firms.

microfoundation that endogenously generates gains from trade, the model we now describe can account for *all* the cross-sectional and longitudinal facts of the market for firms.

3 A Model of Entrepreneurship and Trade of Firms

In this section, we develop a general equilibrium heterogeneous agent model with four key elements: endogenous occupational choice between entrepreneurship and employment, uninsurable income risk for entrepreneurs and workers, firm-level borrowing constraints, and a frictional market in which firms can be bought and sold.

3.1 Environment

Our model economy is inhabited by a continuum of households in $[0, 1]$. Households can have two possible occupations: *firm owners* or *workers*. Firm owners can buy and sell firms and choose whether to operate their current firm and be *entrepreneurs* or close the firm and become workers. Workers can become firm owners by buying an existing firm or by receiving a startup shock, which allows them to found a new firm. We explain the transitions between these two occupations in further detail below.

Besides the firms owned by individual households, which we call *private firms*, there is a second sector of production that features a representative *public firm*. Both sectors produce the same good, which can be used for consumption or savings. Capital is produced by a *financial intermediary* which takes savings from households and rents capital to the firms.[19]

Time is discrete and infinite. Each time period is divided into two stages. The option to buy or sell firms occurs in the first stage, which we call the decentralized market, or *DM*. We assume that households meet bilaterally in the market for firms subject to *search frictions*, which may restrain the frequency and type of matches. All production, consumption, and saving decisions take place in the second stage, which we call the centralized market, or *CM*.

Households Households have preferences over consumption c represented by a constant relative risk aversion (CRRA) utility function

$$u(c_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma}$$

where σ is the risk aversion coefficient. They are heterogeneous in their asset holdings, abilities, occupations, and the quality of their firms, if they are firm owners. Assets, a_{it}, are subject to a non-borrowing constraint, $a_{it} \geq 0$, and are deposited with the financial intermediary, which pays a risk-free interest rate of r for the deposits. There is no aggregate uncertainty in this economy. However, households face idiosyncratic uninsurable risk.

Households are endowed with a level of *ability*, ε_{it}, and one unit of time which they can supply inelastically in the labor market or put into their firm if they are entrepreneurs. The

[19]Since both the public firm and the financial intermediary earn zero profits in equilibrium, their ownership is irrelevant to the analysis.

ability process evolves stochastically according to

$$\varepsilon_{it} = \rho_\varepsilon \varepsilon_{it-1} + \sigma_\varepsilon v_{it}, \tag{2}$$

where $\rho_\varepsilon \in (0,1)$, $\sigma_\varepsilon > 0$, and v_{it} is an i.i.d. standard normal random variable.

Firm owners are endowed with a private firm that produces the consumption good using a technology that combines capital, hired labor, and the firm's productivity, z_{it}. Productivity depends on the owner's ability, ε_{it}, and the firm's quality. The firm's *quality* consists of a persistent component, θ_{it}, and a transitory shock, e_{it}. Formally, the firm's *productivity* is

$$z_{it} = \underbrace{\omega \varepsilon_{it}}_{\text{owner's ability}} + \underbrace{\theta_{it} + e_{it}}_{\text{firm's quality}} \tag{3}$$

where ω governs the contribution of the owner's ability to the productivity of the firm. Ability is inalienable to the owner and therefore cannot be traded in the market for firms.

The persistent process in firms' quality, θ_{it}, follows an AR(1) process with Gaussian mixture shocks according to the law of motion

$$\theta_{it} = \mu_\theta + \rho_\theta \theta_{it-1} + u_{it}, \tag{4}$$

where $\rho_\theta \in (0,1)$, and u_{it} is drawn from a mixture of two normal distributions with mean zero. In detail, we assume that u_{it} is drawn from a distribution with standard deviation σ_θ with probability p_θ, and from a distribution with standard deviation $s_\theta \sigma_\theta$ with probability $1 - p_\theta$, where $s_\theta > 1$.[20] The transitory component e_{it} is an i.i.d. normal random variable, with mean zero and standard deviation σ_e.

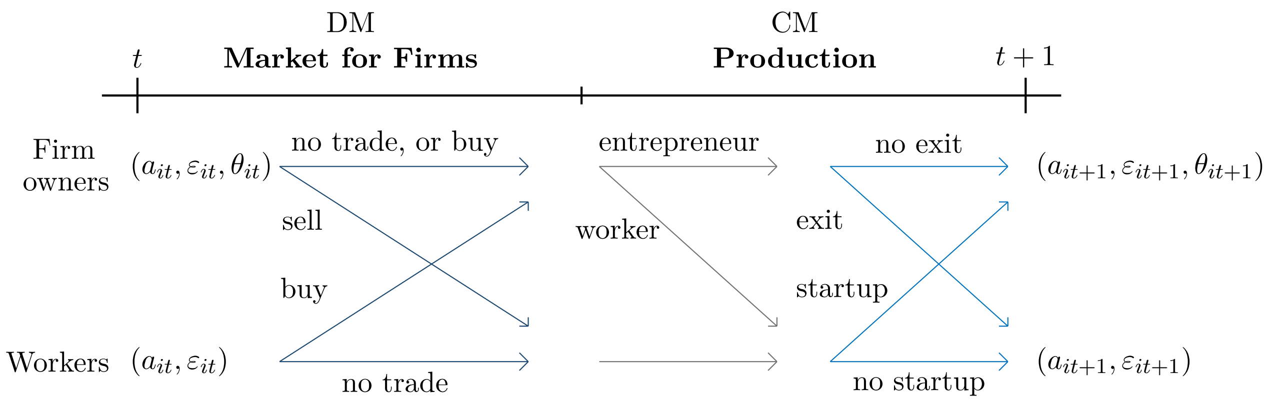
We introduce productivity growth over the life cycle of firms by assuming that when a worker receives a startup shock, they draw heterogeneous quality levels, θ_{it}, from a distribution P_{θ_0} which is the same as incumbents, but with a lower mean, $\underline{\mu_\theta}/(1-\rho_\theta)$. Hence, this assumption captures the role of ex ante heterogeneity in shaping the firm size distribution (Sterk, Sedláček, and Pugsley, 2021), and allows for output growth over firms' life cycles.

Figure 3 illustrates the transitions between occupations. Workers can become firm owners by purchasing an existing firm in the market for firms or by receiving a startup shock at the end of the period, which occurs with probability ζ. On the other hand, firm owners can become workers by selling their firm or by choosing to shut it down at the beginning of the production stage. Besides these transitions, we assume that entrepreneurs receive an exogenous exit shock into the labor market, with probability γ, at the end of the period. It is worth noting that while the startup shock is exogenous, the decision to operate the firm

[20] We use a Gaussian mixture to account for excess kurtosis in firm growth (see, e.g., Boar, Gorea, and Midrigan (2025) and Jaimovich, Terry, and Vincent (2025)), allowing the model to match entrepreneurs' income distribution. We use the method of Farmer and Toda (2017) to discretize the process.

and become an entrepreneur is endogenous and depends on households' ability, wealth, the quality of the firm, and the option value of selling the firm in the market. Indeed, former workers who receive a startup shock can always choose not to start a business and return to the labor market with the same ability that they would have had without the startup shock.

Figure 3: Transitions Between Occupations



In this setup, the budget constraint at CM of an entrepreneur with states $(a_{it}, \varepsilon_{it}, \theta_{it}, e_{it})$, and firm's productivity $z_{it} = \omega\varepsilon_{it} + \theta_{it} + e_{it}$, is given by

$$c_{it} = \pi(a_{it}, z_{it}) + (1+r)a_{it} - a_{it+1},$$

where $\pi(a_{it}, z_{it})$ are the firm's profits defined below. Similarly, the budget constraint at CM of a worker with states $(a_{it}, \varepsilon_{it})$ is

$$c_{it} = w\exp(\varepsilon_{it}) + (1+r)a_{it} - a_{it+1},$$

where w is the labor market equilibrium wage in efficiency ability units.

Private Firms Private firms are endowed with a technology that uses capital k_{it}, labor l_{it}, and the firm's productivity z_{it}, to produce the final good

$$y_{it} = \exp(z_{it})\left[k_{it}^{\alpha} l_{it}^{1-\alpha}\right]^{\eta}$$

where $\alpha \in (0,1)$ is the capital share, and $\eta \in (0,1)$ captures the degree of decreasing returns to scale, implying that private firms have an optimal operation scale as in Lucas (1978).

We assume that entrepreneurs are subject to *financial frictions*, which may prevent the firm from producing at their optimal scale. Specifically, we assume a borrowing constraint that limits the firm's debt-borrowing capacity as a function of the owner's assets, a_{it}, and the firm's productivity, z_{it}. Specifically, we assume that the firm's capital satisfies

$$k_{it} \leq \lambda_a a_{it} + \lambda_\pi \pi^*(z_{it}) \quad (5)$$

where the parameters $\lambda_a \geq 1$ and $\lambda_\pi \geq 0$ jointly govern the tightness of firms' borrowing constraints. The first term captures the standard asset-based constraints, while the latter, which is a function of firms' unconstrained profits π^*, captures earnings-based constraints as those documented in Lian and Ma (2020) and Drechsel (2023).[21]

Under these assumptions, the profit maximization problem of an entrepreneur with assets a_{it} and productivity z_{it} is given by

$$\begin{aligned}\pi(a_{it}, z_{it}) = &\max_{k_{it}, l_{it}} \exp(z_{it}) \left[k_{it}^{\alpha} l_{it}^{1-\alpha}\right]^{\eta} - Rk_{it} - wl_{it} \\ &\text{s.t.} \quad k_{it} \leq \lambda_a a_{it} + \lambda_\pi \pi^*(z_{it})\end{aligned} \tag{6}$$

where R is the capital rental rate. If the borrowing constraint in (5) binds, the firm operates at a lower scale compared to the unconstrained profit maximization level.[22]

Private firms rent capital and hire labor each period.[23] As a result, firms are characterized by their productivity $z_{it} = \omega\varepsilon_{it} + \theta_{it} + e_{it}$. Since ability, ε_{it}, is inalienable to the owner, only the firm's quality is tradable. For tractability, we assume that while the persistent component, θ_{it}, is known upon entering DM, the transitory shock, e_{it}, is realized between the DM and CM subperiods. Consequently, θ_{it} is the component of firms' productivity that households can buy and sell in the market for firms. Different values of firms' persistent quality capture the features beyond labor and capital inputs that characterize a firm. Some examples are firms' organizational capital or intangible assets.[24] Private firms in our model are indivisible, rival, and excludable. These features are an important distinction between our theory of the trade of firms and the literature on the trade of ideas.[25]

Public Firm As in Cagetti and De Nardi (2006), there is a second production sector populated by a representative public firm. This assumption aims to capture that, in the U.S. economy, around half of total output is produced by publicly traded firms. The public firm faces no financial frictions and is endowed with a constant return to scale technology

$$Y_{pt} = A_p K_{pt}^{\alpha} L_{pt}^{1-\alpha}$$

[21]For computational reasons, tractability, and to avoid potential multiplicity issues, we assume that the borrowing constraint depends on the unconstrained profits, π^*, rather than on the level of profits, π, defined in (6). In Appendix C.1.3, we discuss these issues and compare (5) to the constraint with π.

[22]Appendix C.1 presents firms' input demand functions that characterize the static solution of (6).

[23]Without adjustment frictions and if the capital decision at t is measurable with the time t information set, this formulation is equivalent to one in which firms own the capital (see, e.g., Midrigan and Xu (2014)).

[24]Firms' intangible assets could include trademarks, patents, processes, permits, customer bases, business plans, or business knowledge. Consistent with our characterization of firms, Bhandari and McGrattan (2021) document that when a firm is sold, intangible assets account for 58% of the transacted price.

[25]Ideas are non-rival. However, they can be excludable under institutional arrangements such as patents. See Silveira and Wright (2010) and Akcigit, Celik, and Greenwood (2016) for the trade of ideas.

where K_{pt} is the public firm's capital, L_{pt} its labor, and Y_{pt} its total output. The constant productivity term A_p governs the share of total output produced by the public firm.

Financial Intermediary The financial intermediary takes deposits from households and rents capital to the firms at a price equal to the savings interest rate plus the depreciation rate: $R = r + \delta$. We assume that the representative intermediary operates in a perfectly competitive market and breaks even (i.e., makes zero profits). The resource constraint of the intermediary is

$$K_{pt} + \int k_{it} \, \mathrm{d}N^e_{cm} = \int a_{it} \, \mathrm{d}N^e_{cm} + \int a_{it} \, \mathrm{d}N^w_{cm} \tag{7}$$

where N^e_{cm} and N^w_{cm} are cumulative distribution functions for entrepreneurs and workers, which are normalized such that $\int \mathrm{d}N^e_{cm} + \int \mathrm{d}N^w_{cm} = 1$. These measures correspond to the production stage after firm owners decide whether to be entrepreneurs or workers.

3.2 A Market for Firms

Firms are hard to valuate and price. This precludes the existence of a centralized market with a complete price schedule for different types of firms. We therefore model the market for firms using a search-theoretic approach characterized by bilateral random matching and *quid pro quo* trade. An interpretation of this setup is that agents can valuate only one firm at a time, which slows down the trading process. Trade in the market for firms involves transferring firms' ownership and management in exchange for assets, which serve as the media of exchange. As firms are indivisible, when a buyer and a seller meet, they only bargain over the selling price p_{ijt}, where, in what follows, we denote firms' prices first by the buyer i, second by the seller j, and lastly by time t.

Exchanges in the market for firms are voluntary. Hence, a necessary condition for trade is that agents have different valuations for the same firm. Gains from trading firms in our model arise endogenously from heterogeneity in households' wealth and financial frictions, as well as from heterogeneity in ability. We also allow for exogenous preference shocks that parsimoniously capture alternative motives for trade. In Section 5, we discuss in detail the different motives for trading firms and quantify their importance.

Bilateral Meetings There are two types of meetings in the market for firms: owner-owner and owner-worker. Meeting probabilities depend on the *endogenous* distribution of agents in the economy and two exogenous search frictions parameters, which vary by meeting type. Let m^o_{dm} and m^w_{dm} denote the equilibrium shares of owners and workers in the DM sub-period, respectively.[26] Given these shares, an owner meets another owner with probability $\alpha_o m^o_{\mathrm{dm}}$, where $\alpha_o \in [0, 1]$ is the exogenous meeting rate for owner-owner matches. An owner meets a worker with probability $\alpha_w m^w_{\mathrm{dm}}$, where $\alpha_w \in [0, 1]$ is the exogenous meeting rate

[26] Formally, $m^j_{\mathrm{dm}} = \int \mathrm{d}N^j_{\mathrm{dm}}$ for $j \in \{o, w\}$. Note that $\int \mathrm{d}N^o_{\mathrm{dm}} + \int \mathrm{d}N^w_{\mathrm{dm}} = 1$.

for owner-worker matches. Symmetrically, a worker meets an owner with probability $\alpha_w m^o_{\text{dm}}$.

In these meetings, owners may sell their firm to other agents in exchange for assets. Firms are characterized by their productivity. Yet, as noted above, firms' productivity is not fully transferable because ability ε_{it} is inalienable and owner-specific. Moreover, since the shock e_{it} is i.i.d. and realized after the DM, only the persistent component of firm quality, θ_{it}, is traded in the market for firms. Note that only firm owners are potential sellers, while both types of households can be buyers. Therefore, in an owner-worker match, the owner is the potential seller, and the worker is the potential buyer. By contrast, in an owner-owner match, who is the buyer and who is the seller depends on the relative quality of the two firms.

Let us first consider the owner-owner match and suppose that at the DM subperiod $\theta_{it} < \theta_{jt}$. Then, owner i with states $\mathbf{s}^o_{it} \equiv (a_{it}, \varepsilon_{it}, \theta_{it})$ is the potential buyer, and owner j with states $\mathbf{s}^o_{jt} \equiv (a_{jt}, \varepsilon_{jt}, \theta_{jt})$ is the potential seller. This follows from the assumption that households can own only one firm at a time. Hence, no owner would buy a lower-quality firm. In this case, the total surplus from trading the ownership of firm θ_{jt}, in exchange for p_{ijt} assets, is given by

$$\underbrace{W^o(a_{it} - p_{ijt}, \varepsilon_{it}, \theta_{jt}) - W^o(\mathbf{s}^o_{it})}_{\text{Buyer's surplus, } S_b} + \underbrace{W^w(a_{jt} + p_{ijt}, \varepsilon_{jt}) + T_{jt}(p_{ijt}) - W^o(\mathbf{s}^o_{jt})}_{\text{Seller's surplus, } S_s} \tag{8}$$

where W^o and W^w are the value functions for firm owners and workers, respectively, after the market for firms but before owners' occupational choice and the realization of e_{it} shocks. They are defined under the constraints $a_{it} - p_{ijt} \geq 0$ and $a_{jt} + p_{ijt} \geq 0$. As we explain below, T_{jt} is a utility transfer capturing sellers' preference shocks. Upon selling, the household goes to the labor market with its labor efficiency ε_{jt}, as the first term in the seller's surplus shows. The outside option for both agents (the terms with a minus in (8)) is the value of going to the production stage as firm owners with their initial states $\mathbf{s}^o_{it}$ and $\mathbf{s}^o_{jt}$, respectively.[27]

Regarding the owner-worker match, suppose that a firm owner j with states $\mathbf{s}^o_{jt}$ meets with a worker i with states $\mathbf{s}^w_{it} \equiv (a_{it}, \varepsilon_{it})$. Then, the total surplus from trading firm θ_{jt} is

$$\underbrace{W^o(a_{it} - p_{ijt}, \varepsilon_{it}, \theta_{jt}) - W^w(\mathbf{s}^w_{it})}_{\text{Buyer's surplus, } S_b} + \underbrace{W^w(a_{jt} + p_{ijt}, \varepsilon_{jt}) + T_{jt}(p_{ijt}) - W^o(\mathbf{s}^o_{jt})}_{\text{Seller's surplus, } S_s} \tag{9}$$

where the only difference relative to the previous match is the buyer's outside option. In this case, if the parties don't trade, the buyer would continue to the production stage as a worker with its initial state $\mathbf{s}^w_{it}$.

Preference Shocks Beyond the endogenous motives for trading firms studied in this paper, arising from financial frictions and differences in ability, there may be other reasons why owners sell their firms. Examples of these alternative motives include time-varying prefer-

[27]As described below, business owners who do not sell can shut down their firm and enter the labor market with the same labor efficiency ε_{jt}. This free exit assumption rules out negative prices in our model.

ences (e.g., the non-monetary value of being self-employed) or motives related to owners' life cycle (e.g., health shocks or retirement). To parsimoniously account for these motives, which we do not directly model in our theory, we assume that upon matching, potential sellers receive a preference shock that increases the benefit (or lowers the opportunity cost) of selling in period t. We assume that the preference shocks are realized after agents meet and, hence, occur after the identities of the potential buyer and seller are determined.[28]

The preference shock, denoted by $\kappa_{jt} \geq 1$, determines the additional utility transfer that agent j receives when selling in period t at price p_{ijt}. Let $\Psi(\kappa)$ denote the CDF of κ. Then, for a potential seller j with states $\mathbf{s}^o_{jt}$ and preference shock κ_{jt}, the utility transfer $T_{jt}(p_{ijt}) \equiv T(p_{ijt}; \mathbf{s}^o_{jt}, \kappa_{jt})$ associated with selling at price p_{ijt} is implicitly defined by

$$W^w(a_{jt} + \kappa_{jt}p_{ijt}, \varepsilon_{jt}) = W^w(a_{jt} + p_{ijt}, \varepsilon_{jt}) + T_{jt}(p_{ijt}) \tag{10}$$

which states that, because of the utility transfer $T_{jt}(p_{ijt})$, selling at price p_{ijt} yields the same utility as selling at a higher price $\kappa_{jt}p_{ijt}$. Hence, this utility transfer from preference shocks is similar in spirit to the classical Hicksian compensation.

We parameterize the preference shock through an auxiliary i.i.d. random variable $\xi_{jt} \in [0, 1]$ drawn from a Beta distribution with parameters $(1, \beta_\xi)$, and define $\kappa_{jt} = (1 - \xi_{jt})^{-1}$. Figure C.3 in the Appendix exemplifies the distribution of preference shocks under different parameterizations. Since the preference shock, κ_{jt}, is multiplicative on the transaction price, ξ_{jt} can be interpreted as the discount rate that the seller is willing to accept, relative to the no-transfer price $\kappa_{jt}p_{ijt}$.[29] Hence, note that if the discount rate draw is $\xi_{jt} = 0$, the preference shock is $\kappa_{jt} = 1$, and the seller receives no additional utility from selling in period t. In contrast, higher values of ξ_{jt} and κ_{jt} imply larger utility transfers $T_{jt}(p_{ijt})$, making the seller willing to accept lower prices. In our quantitative application, we show that the preference shocks help us fit the relationship between the trade rate and firm size.

Sufficient Condition for Trade Let $\underline{p}_{jt} \equiv \underline{p}(\mathbf{s}^o_{jt}, \kappa_{jt})$ denote the minimum price at which seller j is willing to sell its firm, i.e., the price at which the seller's surplus is equal to zero. Likewise, let $\overline{p}_{it} \equiv \overline{p}(\mathbf{s}_{it}, \theta_{jt})$ be the maximum price that buyer i is willing to pay for firm j, i.e., the price at which the buyer's surplus is equal to zero. A sufficient condition for trade, meaning that there are positive gains from trading firm j, is that

$$\underline{p}_{jt} < \overline{p}_{it} \tag{11}$$

where the states of buyer i are $\mathbf{s}_{it} \in \{\mathbf{s}^o_{it}, \mathbf{s}^w_{it}\}$, depending on the type of match (owner-owner or owner-worker, respectively). For a given meeting, condition (11) shows that whether trade

[28]While we could relax this assumption, doing so would complicate the characterization of the trades in the market for firms without additional insights. Further, note that buyer preference shocks would play an analogous role. For tractability, we assume only sellers receive these shocks.

[29]To see this note that $(1 - \xi_{jt})\kappa_{jt}p_{ijt} = p_{ijt}$, hence p_{ijt} is the price after a $100 \times \xi_{jt}\%$ discount.

occurs depends on the firm's quality and the characteristics of potential sellers and buyers.

Bargaining If there are positive gains from trade, we assume that the price is determined by a *Nash bargaining* protocol. Thus, the trading price p_{ijt} between buyer i with states $\mathbf{s}_{it} \in \{\mathbf{s}^o_{it}, \mathbf{s}^w_{it}\}$, and seller j with states $\mathbf{s}^o_{jt}$ and preference shock κ_{jt} solves

$$
\begin{aligned}
p_{ijt} \equiv p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt}) = \arg\max_p & \Big[S_b(\mathbf{s}_{it}, \theta_{jt}, p)\Big]^{\chi} \Big[S_s(\mathbf{s}^o_{jt}, \kappa_{jt}, p)\Big]^{1-\chi} \\
\text{s.t.} \quad & S_b(\mathbf{s}_{it}, \theta_{jt}, p) \geq 0, \;\; S_s(\mathbf{s}^o_{jt}, \kappa_{jt}, p) \geq 0
\end{aligned} \tag{12}
$$

where S_b and S_s are the buyer and seller surpluses, defined in (8) and (9), and $\chi \in [0, 1]$ parameterizes buyers' bargaining power. Thus, if χ is near 0, the price will be close to the buyer's maximum price $\overline{p}_{it}$. Conversely, if χ is near 1, the price will be close to the seller's minimum price $\underline{p}_{jt}$. As we explain in Section 4 below, information about the ratio of selling prices to firms' EBITDA helps us identify this parameter.

3.3 Timing

The timing of the model can be summarized as follows: (i) Firms' persistent qualities θ, and agents' abilities ε are realized. (ii) Agents enter the market for firms (DM). (iii) Preference shocks κ are realized for potential sellers. (iv) Firm trade occurs. (v) Agents enter the production stage (CM), and firm owners decide whether to operate the firm or enter the labor market. (vi) Transitory shocks e to firm quality are realized for entrepreneurs. (vii) Production takes place, and agents choose how much to consume and save. (viii) Finally, exogenous startup and exit shocks occur.

3.4 Recursive Formulation and Equilibrium

We now present the recursive problem of firm owners and workers and describe the equilibrium definition. First, we describe the value functions at the beginning of the market for firms (the DM subperiod), which we denote by V. Second, we present the value functions at the production stage (the CM subperiod), which we denote by W.

Value at the Market for Firms (DM) Firm owners have four potential outcomes upon entering the market: (i) don't trade, (ii) buy another firm, (iii) sell their firm to another owner, and (iv) sell their firm to a worker. The no-trade case could arise because the owner did not find a counterpart, or because a match occurred but did not result in a trade.

The value of a firm owner with states $(a_{it}, \varepsilon_{it}, \theta_{it})$ at the beginning of DM is equal to

$$
\begin{aligned}
V^o(a_{it}, \varepsilon_{it}, \theta_{it}) = \mathbb{E}_{\kappa_{it}} \Bigg[& \underbrace{\Pr{}^o[\text{no trade} \mid a_{it}, \varepsilon_{it}, \theta_{it}, \kappa_{it}] \;\; W^o(a_{it}, \varepsilon_{it}, \theta_{it})}_{\text{no trade}} \\
& + \underbrace{\alpha_o \int \int_{\theta_{it} < \theta_{jt}, \overline{p}_{it} > \underline{p}_{jt}} W^o(a_{it} - p_{ijt}, \varepsilon_{it}, \theta_{jt}) \, \mathrm{d}N^o_{dm}(a_{jt}, \varepsilon_{jt}, \theta_{jt}) \, \mathrm{d}\Psi(\kappa_{jt})}_{\text{buy}}
\end{aligned}
$$

$$
+ \underbrace{\alpha_o \int_{\theta_{it}>\theta_{jt}, \underline{p}_{it}<\overline{p}_{jt}} \left[W^w\left(a_{it}+p_{jit}, \varepsilon_{it}\right)+T_{it}\left(p_{jit}\right)\right] \mathrm{d} N_{dm}^o\left(a_{jt}, \varepsilon_{jt}, \theta_{jt}\right)}_{\text{sell to a firm owner}}
$$

$$
\left. + \underbrace{\alpha_w \int_{\underline{p}_{it}<\overline{p}_{jt}} \left[W^w\left(a_{it}+p_{jit}, \varepsilon_{it}\right)+T_{it}\left(p_{jit}\right)\right] \mathrm{d} N_{dm}^w\left(a_{jt}, \varepsilon_{jt}\right)}_{\text{sell to a worker}} \right], \tag{13}
$$

where α_o and α_w are parameters for each meeting type, in $[0,1]$, governing the degree of search frictions in the market for firms.[30] N_{dm}^o and N_{dm}^w are cumulative distributions for firm owners and workers at the beginning of DM, which satisfy that $\int \mathrm{d}N_{dm}^o + \int \mathrm{d}N_{dm}^w = 1$.

As described above, in owner-owner meetings, who buys and sells depends on the relative firm qualities. Hence, an owner with firm quality θ_{it} might buy if it is matched with another owner with a firm of higher quality ($\theta_{it} < \theta_{jt}$), as denoted in the integral in the second line of (13). On the contrary, the owner might sell if it is matched with another owner with a firm of lower quality ($\theta_{it} > \theta_{jt}$) as denoted in the integral of the third line. Note that the integrals for the buying and selling cases consider only meetings that result in a trade, i.e., when the seller's minimum price is lower than the buyer's maximum price, as stated in (11).

Workers only have two potential outcomes: (i) don't trade, or (ii) buy an existing firm. Hence, the value of a worker with states $(a_{it}, \varepsilon_{it})$ at the beginning of DM is given by

$$
\begin{aligned}
V^w\left(a_{it}, \varepsilon_{it}\right) = & \underbrace{\operatorname{Pr}^w\left[\text{no trade} \mid a_{it}, \varepsilon_{it}\right] \; W^w\left(a_{it}, \varepsilon_{it}\right)}_{\text{no trade}} \\
& + \underbrace{\alpha_w \int \int_{\overline{p}_{it}>\underline{p}_{jt}} W^o\left(a_{it}-p_{ijt}, \varepsilon_{it}, \theta_{jt}\right) \mathrm{d} N_{dm}^o\left(a_{jt}, \varepsilon_{jt}, \theta_{jt}\right) \mathrm{d} \Psi\left(\kappa_{jt}\right)}_{\text{buy}}.
\end{aligned} \tag{14}
$$

Value at the Production Stage (CM) At the beginning of the production stage, and before drawing e_{it}, firm owners face an occupational choice. They have to decide whether to operate their firm and be entrepreneurs or shut down and enter the labor market. Given these assumptions, the value of firm owners at the beginning of CM is

$$
W^o(a_{it}, \varepsilon_{it}, \theta_{it}) = \max_{h_{it}} \left\{ \int W^e\left(a_{it}, \varepsilon_{it}, \theta_{it}, e_{it}\right) \mathrm{d} P_e\left(e_{it}\right),\; W^w(a_{it}, \varepsilon_{it}) \right\} \tag{15}
$$

where h_{it} denotes the owners' occupational choice and is equal to 1 if the owners go into entrepreneurship. After the realization of e_{it}, the value function of an entrepreneur with

[30] The probabilities of the bilateral meetings in (13) can be derived as follows. First, note that there is a mass $\int \mathrm{d}N_{dm}^o$ of owners at the beginning of DM. This implies that two owners are matched with probability $\int \mathrm{d}N_{dm}^o$. Due to the search friction, conditional on the match, these owners meet with probability α_o. Thus, the probability of an owner-owner meeting is equal to $\alpha_o \int \mathrm{d}N_{dm}^o$. Similarly, the probability that the owner matches with a worker is equal to $\int \mathrm{d}N_{dm}^w = 1 - \int \mathrm{d}N_{dm}^o$, and conditional on the match they meet with probability α_w. Hence, the probability of an owner-worker meeting is equal to $\alpha_w \int \mathrm{d}N_{dm}^w$. Finally, note that the no-trade probability $\operatorname{Pr}^o\left[\text{no trade} \mid a_{it}, \varepsilon_{it}, \theta_{it}, \kappa_{it}\right]$ sums up the probability of no meetings plus the probability of meetings that do not result in a trade as (11) is not satisfied.

states $\mathbf{s}_{it}^{e} = (a_{it}, \varepsilon_{it}, \theta_{it}, e_{it})$ is

$$\begin{aligned}
W^{e}\left(\mathbf{s}_{it}^{e}\right) = \max_{a_{it+1}, c_{it}} \quad & u\left(c_{it}\right) \ + \ \beta(1-\gamma)\mathbb{E}_{\varepsilon_{it+1},\theta_{it+1}|\varepsilon_{it},\theta_{it}}\left[V^{o}\left(a_{it+1}, \varepsilon_{it+1}, \theta_{it+1}\right)\right] \\
& + \beta\gamma\mathbb{E}_{\varepsilon_{it+1}|\varepsilon_{it}}\left[V^{w}\left(a_{it+1}, \varepsilon_{it+1}\right)\right] \\
\text{s.t.} \quad & c_{it} = \pi(a_{it}, z_{it}) + (1+r)a_{it} - a_{it+1} \\
& z_{it} = \omega\varepsilon_{it} + \theta_{it} + e_{it} \\
& c_{it} \geq 0, \ a_{it+1} \geq 0
\end{aligned} \tag{16}$$

where γ is the probability of the exogenous exit shock into the labor market.

The value function at the production stage of a worker with states $\mathbf{s}_{it}^{w} = (a_{it}, \varepsilon_{it})$ is

$$\begin{aligned}
W^{w}\left(\mathbf{s}_{it}^{w}\right) = \max_{a_{it+1}, c_{it}} \quad & u\left(c_{it}\right) + \beta(1-\zeta)\mathbb{E}_{\varepsilon_{it+1}|\varepsilon_{it}}\left[V^{w}\left(\mathbf{s}_{it+1}^{w}\right)\right] \\
& + \beta\zeta\mathbb{E}_{\varepsilon_{it+1}|\varepsilon_{it}}\left[\int V^{o}\left(a_{it+1}, \varepsilon_{it+1}, \theta_{it+1}\right) \mathrm{d}P_{\theta_0}\left(\theta_{it+1}\right)\right] \\
\text{s.t.} \quad & c_{it} = w\exp(\varepsilon_{it}) + (1+r)a_{it} - a_{it+1} \\
& c_{it} \geq 0, \ a_{it+1} \geq 0
\end{aligned} \tag{17}$$

where ζ is the probability of the exogenous startup shock into firm ownership.

Competitive Equilibrium We relegate the definition of the competitive equilibrium to Appendix C.2. See Appendix C.3 for a detailed description of our numerical solution.

3.5 Discussion of Assumptions

We conclude this section by discussing some of our modeling assumptions.

Search and Informational Frictions First, we model the market for firms as a decentralized market with random search. Our modeling is consistent with the accounting literature on firm appraisals (Pratt, 1981) that emphasizes that there is no single method for valuing private businesses due to the absence of centralized markets and the presence of informational frictions. Indeed, public listings are rare. Online platforms, such as *BizBuySell*, account for less than 5% of transactions, suggesting pervasive search frictions that limit targeting. Allowing for directed search or endogenous search effort would increase the gains from trade, so random search makes our quantitative results conservative. Second, our model abstracts from asymmetric information, which, a priori, could have ambiguous effects on the market for firms.[31] While potentially important, incorporating these asymmetries would

[31] On the one hand, informational asymmetries should be most severe for young and small firms, which are more informationally opaque. Standard adverse selection mechanisms would then predict lower trading rates for these firms, which is at odds with the data. Hence, information asymmetries are not sufficiently severe to cause trade in this market segment to disappear. On the other hand, informational asymmetries are cited as one of the reasons young and small firms have limited access to external financing. Hence, directly modeling informational frictions that further restrict access to finance for young and small firms might further strengthen our mechanism on financial frictions driving trade in the market for firms.

substantially complicate the analysis and lies beyond the scope of this paper.

Owner-Owner Meetings We assume that in owner-owner meetings, the buyer operates the acquired firm, and the previous firm is displaced. This assumption simplifies the characterization of trades and reduces the computational burden of the decentralized market problem. One could instead assume that the two firm qualities are aggregated into a new quality level that is higher than the best firm in the match (e.g., if there were complementarities or qualities can be accumulated).[32] Such alternative aggregation rules, which are more suitable for modeling M&A between firms, would significantly complicate the model, and are also ultimately ad hoc. Hence, we see our modeling assumption as conservative and consistent with trade between individuals, which is the primary focus of the paper. Finally, it is worth noting that this assumption applies only to a limited set of transactions, as most trade occurs between workers and owners in our model.[33]

Entrepreneurial and Worker Ability In our model, we assume that entrepreneurial and worker ability are perfectly correlated at the household level. Yet, it is worth noting that labor ($\exp(\varepsilon_{it})w$) and entrepreneurial income ($\pi(a_{it}, z_{it})$) are positively but not perfectly correlated, as in Poschke (2018), as the latter depends on the firm's quality and the owner's assets. We also assume that the probability of receiving a startup shock and the quality of the firm drawn are independent of households' states. However, as the decision to start a firm is *endogenous*, the model predicts selection into entrepreneurship that is consistent with the data. First, higher-ability agents found larger firms, consistent with Queiró (2022), which we directly target below through the relation between startup size and pre-entrepreneurial income. Second, conditional on income, wealthier households are more likely to become entrepreneurs, as in, e.g., Hurst and Lusardi (2004) and Chodorow-Reich et al. (2024).[34]

4 Parameterization

This section describes our calibration strategy. We calibrate the model at an annual frequency to match several key empirical moments of the U.S. economy.

[32]More formally, we could assume that an owner i who buys firm j operates with new quality $\log H(q_i, q_j)$, where $H(q_i, q_j) \geq \max q_i, q_j$, H is a monotone aggregator, and $q_k = \exp(\theta_k)$ for $k \in \{i, j\}$. Our baseline assumption corresponds to $H(q_i, q_j) = \max\{q_i, q_j\}$.

[33]In our calibrated model, 63% of firms are purchased by non-entrepreneurs, defined as households that have not been entrepreneurs in the last 10 years, consistent with the data. As discussed in Section 2.1.3, this number is a lower bound on the share purchased by workers. Indeed, in our model, owner-owner meetings account for only 7% of transactions, implying that owner-worker meetings account for 93%. Hence, these numbers suggest that while a significant share of buyers have been entrepreneurs in the past, they no longer own a firm when they purchase a new one.

[34]In detail, we estimate a probit regression of the decision to enter entrepreneurship (conditional on a startup shock at t) on log income and log wealth at $t-2$. The model implies that a 1% increase in lagged wealth raises entry by 0.034 percentage points. This pattern of selection aligns with a large empirical literature documenting the relationship between wealth and entrepreneurship.

4.1 Assigned Parameters

We set the relative risk aversion parameter to $\sigma = 1.5$, the capital depreciation rate to $\delta = 0.06$, and firms' capital share to $\alpha = 1/3$. All are standard values in the literature.

4.2 Calibrated Parameters and Targeted Moments

We calibrate the remaining 20 parameters to ensure the model replicates several key features of the U.S. economy, focusing on the role of entrepreneurs, inequality, firm dynamics, and the market for private firms. Table 2 presents these parameters with their calibrated values, which we find by minimizing the distance between moments in the data and the model. Table 3 presents the 24 moments we directly target in our calibration exercise. Given our model's characteristics, it is not possible to directly match all parameters to specific moments. Yet, in what follows, we introduce and describe the targeted moments, highlighting how they inform the calibration of the different parameters.

Table 2: Calibrated Parameters

Parameter	Value	Description
β	0.914	Discount factor
A_p	0.886	Public firm's productivity
ρ_ε	0.967	Households' ability, persistence
σ_ε	0.269	Households' ability, std. deviation
ω	0.128	Households' ability in firms' productivity
ζ	0.014	Probability of startup shock
γ	0.079	Probability of exogenous exit shock
$\mu_\theta - \underline{\mu_\theta}$	0.185	Firms' quality, entrants' productivity gap
ρ_θ	0.964	Firms' quality, persistence
σ_θ	0.025	Firms' quality, std. deviation
p_θ	0.887	Firms' quality, mixture probability
s_θ	9.818	Firms' quality, mixture volatility
σ_e	0.113	Firms' transitory quality, std. deviation
η	0.760	Curvature private firms' technology
λ_a	1.547	Borrowing constraint, asset-based
λ_π	0.118	Borrowing constraint, earnings-based
$\mathbb{E}[\xi]$	0.242	Preference shock, mean discount
α_o	0.657	Owner-owner | meeting friction
α_w	0.322	Owner-worker | meeting friction
χ	0.565	Buyers bargaining power

Aggregates and Private Firms We calibrate the discount factor β to match an aggregate capital–output ratio of 3. The productivity of public firms A_p mostly determines the output share of private firms. We target that private firms produce 50% of total output, consistent with Dinlersoz et al. (2024). The entry shock ζ and the exit shock γ are most directly informed by the share of households that are entrepreneurs, measured from the SCF, and firms' exit rate, which we obtained from the Census Business Dynamics Statistics (BDS).

We discipline ω, the elasticity linking the owner's ability to firm productivity, by targeting the relationship between startups' sales and owners' pre-entrepreneurship labor income, controlling for wealth. Specifically, using the PSID, we estimate the following regression for households that become entrepreneurs between $(t-2, t]$:

$$\log y_{it} \;=\; \beta_w \log w_{i,t-2} \;+\; \beta_a \log a_{i,t-2} \;+\; \gamma_x \mathbf{x}_{it} \;+\; \varepsilon_{it},$$

where y_{it} are startups' sales, and $w_{i,t-2}$ and $a_{i,t-2}$ are owners' labor income and wealth, respectively, before entrepreneurship. The control vector $\mathbf{x}_{it}$ includes age and year fixed effects. Appendix A.1.3 provides details on the PSID sample and measurement, and Table C.3 reports the empirical estimates. We run the same regression in the model and target both estimated coefficients β_w and β_a. In the model, a higher value of ω implies a larger β_w, making this moment informative about the role of heterogeneous ability in driving firm trade.

Furthermore, β_a is informative about the strength of financial frictions, as it captures the relationship between households' prior wealth and startups' initial scale. This moment helps discipline the relative importance of asset- and earnings-based borrowing constraints, parameterized by $\{\lambda_a, \lambda_\pi\}$. In particular, note that a higher λ_π reduces the role of owners' wealth in determining firm size, lowering the model-implied β_a. As reported in Table 3, the model-generated estimates of β_w and β_a are close to their empirical counterparts, and the model correctly replicates that β_w is substantially larger than β_a. The borrowing constraint parameters are further disciplined by the aggregate debt-to-capital ratio of private firms, which we target using the 2007 leverage level reported by Bellon et al. (2023).[35] In the model, the debt of a private firm i is $b_{it} = k_{it} - a_{it}$, and leverage is b_{it}/k_{it}.

Firm Dynamics To discipline the stochastic processes related to firms' productivity, we target four moments summarizing firm dynamics in the U.S. economy documented in Sterk, Sedláček, and Pugsley (2021) using the Census Longitudinal Business Database (LBD): average employment growth between ages 0 and 19, the ratio of the standard deviation of log employment at age 19 relative to age 0, and the autocorrelation of log employment between ages 0 and 19 (long run) and between ages 18 and 19 (short run). Figure C.1 presents the full path of these employment dynamics moments by firms' age in the LBD data and in the model. Although the quality process is stylized, the model tracks the data well: it matches average employment growth, slightly overstates the dispersion of log employment at entry,

[35]See Figure 11(b) of Bellon et al. (2023). As Appendix B.1 describes, the definition of capital in that paper (equity plus debt) is consistent with the one we use in Orbis and comparable to that in the KFS.

but then closely follows its evolution with the distribution spanning out with firm age, and delivers an autocorrelation profile that declines more linearly yet remains close to the data.

Table 3: Targeted Moments

	Source	Data	Model
Aggregate			
Capital to output ratio	See text	3.00	2.97
Private Firms			
Output share	See text	0.50	0.50
Fraction of entrepreneurs	SCF	0.06	0.06
Exit rate	BDS	0.09	0.10
Startups' sales at t vs. owners' wage at $t-2$ (β_w)	PSID	0.44	0.58
Startups' sales at t vs. owners' a at $t-2$ (β_a)	PSID	0.21	0.13
Leverage	See text	0.35	0.32
Firm Dynamics			
Employment by age	LBD	See text	
Standard deviation of log employment by age	LBD	See text	
Employment autocorrelation function by age	LBD	See text	
Income and Wealth Distribution			
Gini income, all households	SCF	0.62	0.63
Gini income, entrepreneurs	SCF	0.67	0.62
Gini income, workers	SCF	0.58	0.59
Gini wealth, all households	SCF	0.82	0.81
Gini wealth, entrepreneurs	SCF	0.74	0.62
Gini wealth, workers	SCF	0.78	0.80
Entrepreneurs' income share	SCF	0.20	0.22
Entrepreneurs' wealth share	SCF	0.33	0.30
Trade of Firms			
Trade rate	SBO	0.030	0.030
Trade rate vs. size, slope (%)	SBO	-0.80	-1.26
Share purchased by non-entrepreneurs	SBO	0.66	0.63
Median price/EBITDA	DealStats	2.90	2.72
Mean a_{t+1} buyers $[t, t+1]$/Mean a_{t-2} sellers $[t-2, t]$	SCF, PSID	1.36	1.31

Notes: Most of the data moments correspond to the year 2007. a in the data moments corresponds to non-business wealth. Wealth in the model is defined as the sum of assets and the value of the firm $a+\mathbb{E}[p]$.

These moments mostly inform the parameters of firms' quality processes $\{\theta_{it}, e_{it}\}$ and, to a lesser extent, those of the ability process ε_{it}, since $\omega > 0$. We normalize μ_θ and directly calibrate the difference between incumbents' and entrants' intercepts $\mu_\theta - \underline{\mu_\theta}$. To fit average employment by firms' age, our calibration implies that entrants draw their initial quality from a distribution with lower average quality. The transitory shock volatility σ_e is mainly disciplined by short-run autocorrelations and dispersion. The persistence and volatility parameters $\{\rho_\theta, \sigma_\theta\}$ are informed by the autocorrelation structure of log employment.[36]

Income and Wealth Distribution We also target several moments of the distribution of income and wealth, which we compute from the 2007 SCF. These moments are most informative for the parameters of households' ability process $\{\rho_\varepsilon, \sigma_\varepsilon\}$ and firms' quality, especially those of the Gaussian-mixture component $\{p_\theta, s_\theta\}$. In detail, we target six Gini indices for income and wealth, considering all households and separately for workers and entrepreneurs, as well as the income and wealth shares of entrepreneurs. The model matches the dispersion of income and wealth well but modestly understates inequality among entrepreneurs, mainly because the Gaussian-mixture shock, while generating excess kurtosis, remains thin-tailed. Table C.5 in the Appendix shows that the model also matches income and wealth shares extremely well, except for entrepreneurs at the very top. Lastly, entrepreneurs' income and wealth shares also discipline the curvature of the private firms' technology η, a key determinant of private firms' profitability and, in turn, entrepreneurial income.

Trade of Firms Our final set of moments captures relevant features of the market for private firms documented in Section 2. We target an annual trade rate of 3% and that non-entrepreneurs purchase 66% of the firms.[37] These moments are relevant for the search frictions parameters, α_o and α_w. Additionally, to identify the relevance of preference shocks, we target the relationship between trade and firm size summarized by the slope in panel (b) of Figure 1. Intuitively, preference shocks flatten the size trade relationship by increasing trade among large, unconstrained firms, transactions for which financial frictions' motives are less relevant. Our calibration implies an average discount of $\mathbb{E}[\xi] = 0.242$.[38] We also target a median price-to-EBITDA ratio of 2.9, which we obtained from Dealstat.[39] This ratio is most informative for the buyers' Nash bargaining parameter χ. We obtain a value of $\chi = 0.565$, implying that buyers have a slightly higher bargaining power than sellers.

In our theory, financial frictions and heterogeneity in households' wealth play a first-order role in generating gains from trade. To discipline the strength of this mechanism, we target the ratio of 1.36 between the average non-business wealth of recent business buyers and sellers, obtained using the SCF and PSID. We compute the corresponding moments in the model, using the timing in the data. The model matches well the observed differences

[36]The persistence of θ_{it} is very close to that of the AR(1) component estimated in Sterk, Sedláček, and Pugsley (2021). Persistence in this class of models is crucial to the strength of self-financing and, hence, to capital misallocation losses under financial frictions. In our model, the option to buy and sell firms non-trivially shapes entrepreneurs' saving motives: the option to sell can weaken incentives to grow out of constraints, while the option to buy better firms can strengthen them. In the quantitative results below, the market for firms plays a significant role despite a strong self-financing channel.

[37]In the model, we define non-entrepreneurs as agents that have not operated a firm in the last 10 periods.

[38]We directly calibrate the mean of ξ, which implicitly defines the shape parameter as $\mathbb{E}[\xi] = 1/(1 + \beta_\xi)$. We discretize this process using a four-point quadrature (see Figure C.3 in the Appendix).

[39]Dealstat is a database of U.S. business transactions. We take the median selling price-to-EBITDA ratio from their publicly available reports for the year 2010. In the model, we define EBITDA as $\pi_{it} + Rk_{it}$.

in non-business wealth between recent buyers (measured post-trade) and sellers (measured pre-trade). As we describe in detail below, when we instead measure non-business wealth pre-trade for both buyers and sellers in the model (i.e., at the DM), this ratio is substantially larger, equal to 2.06 (see Table C.6). Hence, our model implies that buyers are, on average, twice as wealthy as sellers. The wealth difference between buyers and sellers helps discipline λ_π. Intuitively, higher values of λ_π relax financial constraints for entrepreneurs with high-quality firms but low wealth, thereby reducing the gains from trade due to financial frictions and, consequently, the difference between buyers' and sellers' wealth.

Overall, Table 3 shows that our model matches the targeted moments well, especially those involving entrepreneurs, private firms, and the market for firms. Next, we analyze additional untargeted moments regarding the role of firm owners and the value of firms' intangible capital, and compare them with estimates in the literature.

4.3 Untargeted Moments

The Importance of Owners in Firms A key feature of our model is that the owner's ability affects firm outcomes and is *inalienable*, implying that there is a component of firms' productivity that cannot be traded in the market for firms. We perform two exercises in our model to quantify the role of owners in firms' outcomes. First, we perform a Shapley variance decomposition on firms' productivity and find that the owner's ability, $\omega\varepsilon_{it}$, explains 30% of the variance, the firm's persistent quality, θ_{it}, accounts for 57%, and the transitory shock, e_{it}, for 13%. Second, motivated by previous empirical work, we estimate the average drop in profits and output following an abrupt 75% reduction in the owner's ability. On average, profits fall by 46% and output by 39%, and when we also reduce the owner's assets, the declines can exceed 50%. These magnitudes are comparable to existing estimates following an owner's premature death or retirement (Smith et al., 2019; Becker and Hvide, 2021; Choi et al., 2025). Taken together, these results show that owners' ability plays a quantitatively important role in firms' outcomes, consistent with the existing evidence, while leaving a substantial transferable component of firm productivity to be traded in the market.

Intangible Assets As firms rent capital and hire labor each period, firms are characterized by the transferable part of their productivity, θ_{jt}. Hence, expected trading prices over potential trading counterparts and the realization of preference shocks capture the value of firms' intangibles in the model. We can then define the intangible capital share of firm j at time t as $\frac{\mathbb{E}_{\mathbf{s}_{it},\kappa_{jt}}[p_{ijt}]}{k_{jt}+\mathbb{E}_{\mathbf{s}_{it},\kappa_{jt}}[p_{ijt}]}$, where $\mathbb{E}_{\mathbf{s}_{it},\kappa_{jt}}[p_{ijt}]$ is the expected transaction price for firm j in the market for firms and k_{jt} is the firm's tangible capital. In our baseline calibration, this share equals 42%. This number is consistent with the 58% share reported by Bhandari and McGrattan (2021), using transaction data.

In Section 6 below, we test the predictions of our model against a battery of additional cross-sectional and longitudinal moments related to financial frictions being an important motive for trade in the market for firms that we did not target in our calibration exercise.

5 Model Properties

This section describes the main workings of our model. First, we discuss and quantify the different motives behind firm trade. Second, we describe the determinants of prices in the market for firms. Third, we characterize who buys and sells firms in our economy.

5.1 Motives for Trading Firms

Because exchanges in the market for firms are voluntary, a necessary condition for trade is that agents assign different values to the same firm, with the buyer valuing the firm more than the seller. In our theory, given agents' outside options, heterogeneous valuations arise from four sources: preferences shocks, heterogeneous abilities, firms' borrowing constraints, and incomplete markets. We now describe and quantify the contribution of these motives to trade in the market for firms. To account for the nonlinear interactions among these mechanisms, we quantify the importance of each motive for the frequency of trade using the Shapley-Owen (S-O) decomposition that averages the marginal effect of shutting down each of the first three motives for trade across all possible permutations. See Appendix C.4 for a detailed description of this decomposition in our setup.

Table 4: Trade Rate Decomposition

		S-O Contribution
Preference shocks	$\xi = 0$	0.24
Heterogeneous ability	$\omega = 0$	0.23
Borrowing constraints	$(\lambda_a, \lambda_\pi) \to \infty$	0.32
Risk and Incomplete Markets		0.21

Notes: Average marginal contribution of shutting down the different motives for trade using the Shapley-Owen (S-O) decomposition. The numbers are reported as shares of the trade rate in the baseline model. Equation (27) in the Appendix presents the formula for the S-O Contribution. Preference shocks turn off the alternative motives to trade firms by setting $\mathbb{E}[\xi] = 0$ and $Var[\xi] = 0$. Heterogeneous abilities assumes the incidence of owners' ability on firms' productivity is zero ($\omega = 0$), and we normalize firm TFP to remove level effects. Borrowing constraints is the case where $(\lambda_a, \lambda_\pi) \to \infty$. Risk and Incomplete Markets is measured as the residual after shutting down the first three motives for trade.

Preference Shocks We introduce alternative motives for trading firms in our model via preference shocks that increase sellers' benefits (or lower their opportunity costs) of trade. These shocks aim to parsimoniously capture all motives for trading firms that are unrelated to ability differences and financial frictions. To quantify their role, we turn off these alternative motives by setting the discount shock to $\xi_{jt} = 0$ ($\kappa_{jt} = 1$) for all potential sellers. Table 4 reports that, using the S-O decomposition, preference shocks account for 24% of the trades in the market for firms.[40] Figure C.4 in the Appendix shows that excluding preference shocks leads to a disproportionately large drop in the trade of large, old, and low ARPK firms. Hence, preference shocks play a significant role in the trade of large, unconstrained firms, for which the gains from trade due to financial frictions are small.

Heterogeneous Ability Given the role of owners in firms' productivity, as stated in (3), agents with different abilities assign heterogeneous valuations to the same firm. In particular, all else equal, higher-ability owners value a firm more because they can operate it more efficiently. This trade motive is related to those first studied in Holmes and Schmitz (1995) and Caselli and Gennaioli (2013). To quantify its importance, we consider the case where $\omega = 0$, so that owners' ability no longer impacts firms' productivity. Table 4 shows that, according to the S-O decomposition, this motive accounts for 23% of transactions.

Borrowing Constraints Regarding the financial motives for trading firms, we first focus on the role of credit constraints. This channel arises from the borrowing constraint in the entrepreneur's problem, given by (6), which makes profits $\pi(a_{it}, z_{it})$ a (weakly) increasing function of the current owner's assets. As a result, when an entrepreneur is constrained, a wealthier buyer, even one with lower ability, can obtain a higher profit stream by relaxing borrowing constraints and operating the firm closer to its optimal scale, generating potential gains from trade. To quantify the importance of this channel, we remove the credit constraint so that firms' profits no longer depend on their owners' wealth. Table 4 reports that borrowing constraints account for 32% of the trades in the market for firms.

Risk and Incomplete Markets Aside from borrowing constraints, there is an additional motive related to financial frictions, driven by risk aversion and imperfect risk-sharing. Owning and operating a firm is risky since the firm's productivity is stochastic. For low-wealth owners, selling the firm allows them to front-load consumption and achieve an earlier risk resolution. For high-wealth owners, consumption is less dependent on shocks to the firm's profits. Hence, the covariance between their stochastic discount factor and the realization of profits is small, increasing their ability to bear risk. Therefore, the value of owning a firm will vary across the wealth distribution, generating potential gains from trade. We evaluate the importance of this channel as a residual after turning off the first three motives for trade. Table C.1 in the Appendix shows that, in this case, the trade rate is 0.6%, suggesting that risk and incomplete markets account for 21% of the trades in our baseline economy.

Our model features strong complementarities across motives for trade, particularly with borrowing constraints. Table C.1 in the Appendix shows that eliminating borrowing constraints alone accounts for 48% of transactions, whereas accounting for their interaction with

[40] Preference shocks capture alternative motives for trade, e.g., owners' health shocks or retirement. Consistent with our results, Figure A.5b shows that the majority of sellers in the SBO are young and middle-aged entrepreneurs, suggesting that retirement-related motives account for a small share of transactions.

other motives (via the S-O decomposition) reduces their contribution to 32%. While there are strong nonlinear interactions among motives, this mechanism is the most relevant driver of trade in the model. Indeed, we show below that firms are predominantly reallocated to wealthier owners in the majority of transactions.

5.2 Equilibrium Prices in the Market for Firms

We now characterize the equilibrium prices at which firms trade. In our model, in addition to firms' quality, the wealth and ability of sellers and buyers play a first-order role in determining gains from trade and equilibrium prices. To illustrate the role of owners' wealth in firm prices, Figure C.2a in the Appendix presents firm prices across sellers' assets a_j and firm quality θ_j. As expected, prices are increasing in firm quality. Notably, holding the firm's quality fixed, the price is increasing in the owner's assets. Firms' prices would be independent of the current owner's wealth under perfect credit markets. Thus, because of financial frictions, high-quality firm owners with low wealth will be willing to sell their firms at a relatively lower price, as it will take time and high saving rates to grow out of their borrowing constraint. However, as Figure C.2b shows, these transactions have considerably high price-to-EBITDA ratios, which reflect the lower scale of operation of low-wealth owners with high-quality firms relative to the price at which they can sell their business.[41]

To analyze the role of sellers' ability in firm pricing, Figure C.2c presents firm prices as a function of sellers' ability ε_j and firm quality θ_j. Whenever $\omega > 0$, the owner's ability contributes to firm productivity, implying that equilibrium prices increase with both firm quality and owner ability. Consequently, holding firm quality fixed, higher-ability owners require a higher compensation to sell their firms, as they can operate them more efficiently.[42] Yet, analogous to the results for owners' wealth, Figure C.2d shows that low-ability owners with high-quality firms exhibit high price-to-EBITDA ratios. This result reflects the fact that such firms are valuable in the market for firms despite operating at a low scale at the time of trade, due to the low ability of their current owners.

5.3 Who Buys and Who Sells Firms?

Next, we study the characteristics of buyers and sellers in the market for firms. We begin by examining the assets and abilities of buyers and sellers at the moment of trade. Table C.6 in the Appendix shows that, at the beginning of the DM subperiod, buyers have 2.06 times higher non-business wealth than sellers. This buyer-to-seller ratio is significantly higher than the one targeted in the calibration, which follows the timing in the SCF and

[41] According to Dealstat, the median price-to-EBITDA ratio in the Information sector equals 6.4, considerably higher than the economy-wide 2.9 number. This evidence is consistent with our model's large price-to-EBITDA ratios for high-growth potential firms.

[42] While high-ability owners also have higher outside options in the labor market, the importance of owners' ability for firms' productivity is quantitatively more important in determining equilibrium prices.

the PSID.[43] Table C.6 also reports that the average buyer is 3.04 times wealthier than the average household, though less wealthy than the average entrepreneur. Regarding ability, the average buyer's ability is 2.28 times that of the average seller. Moreover, the average buyer also has a higher ability than the average household and entrepreneur. In contrast, the average seller in our economy has 1.45 times the assets of the average household and 0.40 times that of the average entrepreneur. The average seller also has a lower ability than the average household and entrepreneur. Overall, these numbers indicate that the typical buyers in our economy are wealthy, high-ability households, while the typical sellers have relatively less wealth and low ability.

Table 5: Taxonomy of Trades by Buyers' and Sellers' Characteristics

	$\varepsilon_{\text{buyer}} > \varepsilon_{\text{seller}}$	$\varepsilon_{\text{buyer}} \leq \varepsilon_{\text{seller}}$	Total
$a_{\text{buyer}} > a_{\text{seller}}$	0.56	0.26	0.82
$a_{\text{buyer}} \leq a_{\text{seller}}$	0.06	0.11	0.17
Total	0.63	0.37	1.00

Notes: Share of trades by buyers' and sellers' assets (a) and ability (ε) at the transaction level.

Finally, to complement the previous analysis, we construct a simple taxonomy of trade types based on the relative assets and abilities of buyers and sellers at the transaction level. Table 5 reports the share of trades in our economy across four groups, defined by the combination of two conditions: whether the buyer is wealthier than the seller ($a_{\text{buyer}} > a_{\text{seller}}$) and whether the buyer has a higher ability than the seller ($\varepsilon_{\text{buyer}} > \varepsilon_{\text{seller}}$). This taxonomy shows that transactions in the market for firms predominantly reallocate businesses from less wealthy to more wealthy households (82%) and from lower- to higher-ability agents (63%). Moreover, 56% of trades involve buyers who are both wealthier and more able, consistent with the descriptive statistics in Table C.6. In 26% of transactions, the buyer is wealthier but less able than the seller, suggesting that financial frictions mainly drive these trades. Roughly 6% of trades occur when the buyer has a higher ability but lower wealth, suggesting motives related only to differences in ability. Lastly, about 11% of transactions involve buyers who are both less wealthy and less able than sellers. These trades are driven primarily by preference shocks. In the next section, we show that post-trade dynamics vary markedly across these four types of transactions, reflecting the distinct motives underlying each case.

Overall, our results show that trade in the market for firms predominantly reallocates businesses from less wealthy to more wealthy households and from less able to more able agents. Thus, as we show below, these trades lead to a better allocation of ability, capital,

[43] In detail, in Table 3 we target Mean a_{t+1} buyers $[t, t+1]$/Mean a_{t-2} sellers $[t-2, t]$, which is equal to 1.31. The ratio of average wealth at DM is equal to Mean a_t buyers at t/Mean a_t sellers at t, which is 2.06. Table C.4 in the Appendix shows that buyers' and sellers' non-business wealth relative to the average household and entrepreneur in the model align well with the data when using the same timing convention.

and productive projects in the economy. In line with these findings, our model predicts that financial frictions, both borrowing constraints and incomplete markets, play a central role in generating gains from trade in the market for firms. Heterogeneous ability and preference shocks also contribute to trade, although to a lesser extent. In the next section, we show that trade motives related to financing frictions are not only consistent with the cross-sectional and longitudinal facts, but are essential for reproducing them in the model.

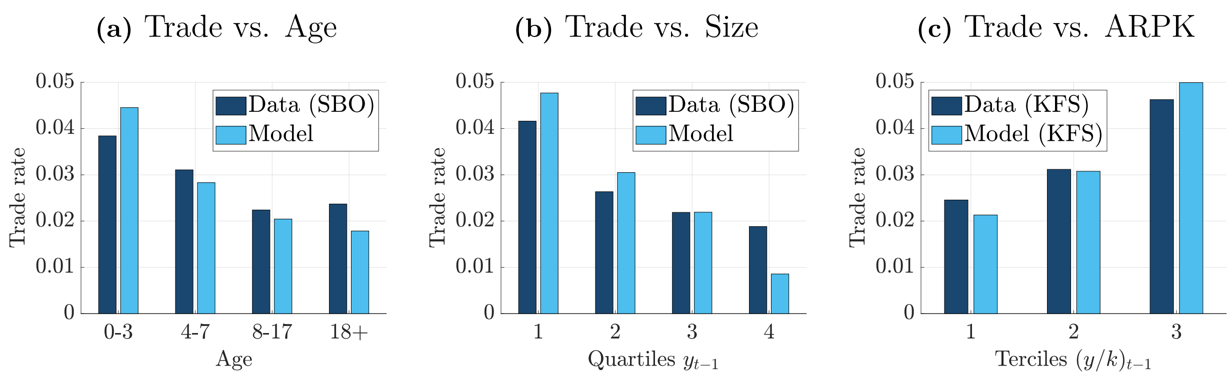
6 Financial Frictions as a Motive to Trade Firms

Before turning to our aggregate results, this section evaluates testable predictions of our theory about financial frictions being a relevant motive to trade firms. First, we compare the characteristics of traded firms in the cross-section to those documented in Section 2.1. Second, we simulate a panel of firms and compare the post-trade firm dynamics implied by our model to those documented in Section 2.2. Finally, we discuss alternative theories of firm trade and compare their predictions to the data, both qualitatively and quantitatively.

6.1 Trade Rate and Firms' Characteristics in Data and Model

If financial frictions are an important reason for trade, credit-constrained firms should be more likely to be bought and sold as gains from trade are the highest for those firms. We test this first prediction of the model by analyzing the relationship between trade and firms' observable characteristics. As in the empirical section, we consider two commonly used proxies of credit constraints: firms' age and size, as younger and smaller firms are more likely to be financially constrained. In addition, we analyze firms' ARPK since credit-constrained firms will have high capital returns but cannot increase their investment.

Figure 4: Trade Rate by Firms' Characteristics in Data and Model



Source: SBO, KFS, and model simulated data.
Notes: Trade rate by firms' characteristics in the data and data simulated from the model. To be consistent with the data, Model (KFS) restricts to a sample of firms of age less or equal to 7. See the notes in Figure 1 for a description of the data moments.

Following the analysis in Section 2.1.4, we simulate data from our model and compute the trade rate conditional on firms' characteristics. Figure 4 shows that, consistent with the data, our model predicts that younger, smaller, and higher-ARPK firms have the highest probabilities of trade. Except for the slope across size (used to calibrate the preference shocks) and the average trade rate, these relations were *not* targeted in our calibration. Instead, they arise from the key prediction of our theory that credit-constrained firms are more likely to be traded and from the strong correlation between these characteristics and binding credit constraints in our model. To show the central role of financing frictions in generating these cross-sectional patterns in the model, Figure C.5 in the Appendix shows that removing borrowing constraints weakens the negative relationship between trade and firm size, flattens the pattern by firm age, and reverses the relationship with firms' ARPK.

6.2 Post-Trade Firm Dynamics in Data and Model

We now compare the dynamics after trade in our model to those documented in Section 2.2.[44] As we show below, post-trade firm dynamics are highly informative about the empirical relevance of the motives driving firms' trade. We first compare average post-trade dynamics in the data and in the model. We then examine how these dynamics differ depending on the motives generating gains from trade. Given that both output and inputs are expected to rise when financial frictions or differences in ability drive trade (see Table C.2), we focus on the joint dynamics of variables, summarized by the key ratios studied in the empirical section, as they are the most informative about the motives driving trade.

Average Post-Trade Dynamics To study the post-trade dynamics implied by our quantitative model, we generate a panel of 2.5 million firm-year observations and estimate regression (1) for each variable of interest using the model simulated data. Figure 5 compares the firm dynamics after trade in our model to those documented in Section 2.2 for firms' ARPK (y/k), profitability (π/k), and leverage (b/k). Despite not being targeted in the calibration, the model reproduces the dynamics of these variables notably well.

We first analyze the joint dynamics of capital and output. Our model predicts that, in most transactions, firms' trade alleviates financial constraints. Hence, before being traded, firms operated with lower capital and higher ARPK relative to their unconstrained level. After a trade, firms' capital increases, and crucially, capital increases more than output, leading to a decline in firms' ARPK over time. Figure C.6 in the Appendix shows that although the dynamics in our model feature a larger initial response and are more pronounced, capital grows significantly more than output, as in the data.[45] As a consequence, as Figure 5a shows,

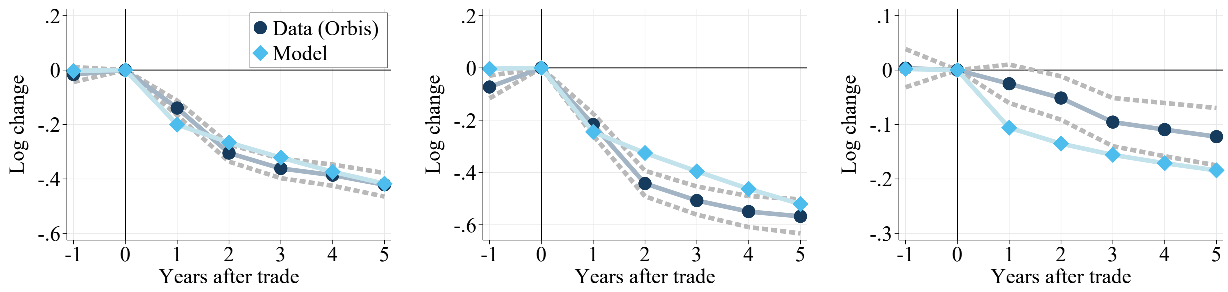
[44] This exercise implicitly assumes that the post-trade firm dynamics in high-income European countries are informative about those in the U.S., the economy for which we calibrate our model. Table B.4 shows that our baseline sample in Orbis includes a substantial share of young, small, and single-owner firms comparable to that in the US SBO. Furthermore, Kochen (2025) shows that key moments related to firm dynamics, including exit and growth rates, over the life cycle of firms in this group of high-income countries are similar to those documented for the U.S. by Haltiwanger, Jarmin, and Miranda (2013).

[45] The absence of other frictions in our quantitative framework, such as capital adjustment costs, can partly explain the faster post-trade dynamics in our model relative to the data.

these joint dynamics result in a sharp reduction in firms' ARPK of 0.42 log points in the model five years after trade, closely matching the decline observed in the data.

Figure 5: ARPK, Profitability, and Leverage Dynamics After Trade in Data and Model

(a) ARPK, $\log(y/k)$ **(b)** Profitability, $\log(\pi/k)$ **(c)** Leverage, $\log(b/k)$



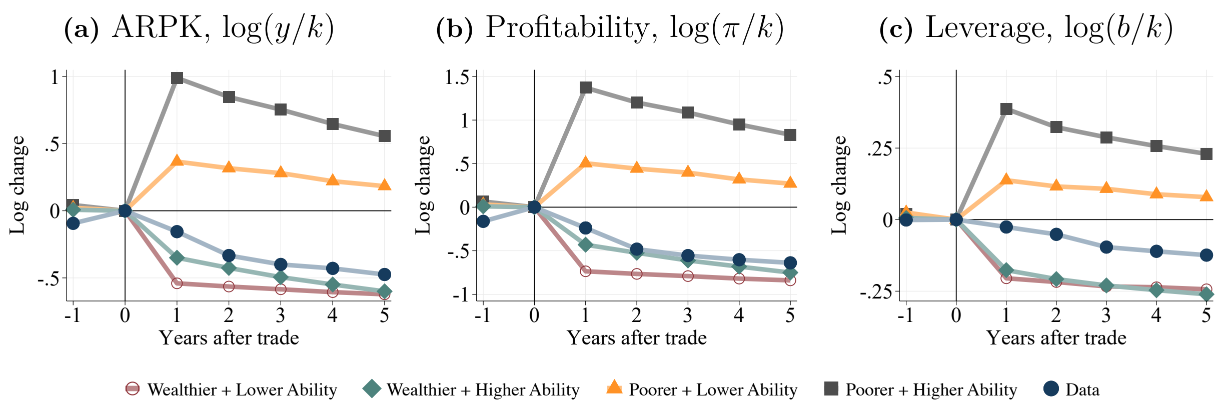
Source: Orbis Historical and model simulated data.
Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Because financial frictions do not distort labor input choices, and differences in ability do not affect the composition of inputs, our model implies that firms' ARPL is constant. The data support these assumptions, as firms' ARPL declines only slightly after trade, by 0.04 log points after five years, and the change is not statistically different from zero in robustness checks. Regarding profitability, Figure 5b shows that profits over capital significantly decrease after trade. In our model, credit-constrained firms have a distorted profit-to-capital ratio that exceeds the unconstrained optimal level. As in the ARPK dynamics, capital grows more than profits after trade, reducing firms' profitability by 0.52 log points after five years, consistent with the 0.57 log points decline in the data. Finally, Figure 5c presents the post-trade dynamics of firms' leverage. Because buyers tend to be wealthier than sellers in the model, most of the additional capital comes from owners' equity, leading to a sizable decline in leverage of about 0.18 log points after five years. This reduction aligns well with the 0.12 log points decline observed in the data.[46] As we formalize below, the post-trade dynamics in Figure 5 are consistent with a substantial easing of financing frictions after trade.

Post-Trade Dynamics by Trade Type To better understand how the underlying motives for buying and selling firms shape post-trade firm dynamics in the model, Figure 6 presents the dynamics of ARPK, profitability, and leverage for the four types of transactions in Table 5, defined by the ability and wealth of buyers and sellers. A clear pattern emerges: transactions in which buyers are wealthier than sellers (where financial frictions play a more relevant role) are associated with declines in ARPK, profitability, and leverage after trade,

[46] The reduction in leverage in the model reflects that, in most transactions, the buyer has higher wealth than the seller, rather than better access to debt financing (e.g., because of higher ability or λ_a or λ_π), which would instead lead to an increase in leverage. This prediction is consistent with the leverage dynamics documented in Section 2.2, which indicate that firms receive sizable equity injections from their new owners.

Figure 6: ARPK, Profitability, and Leverage Dynamics After Trade by Type



Source: Orbis Historical and model simulated data.
Notes: Estimated coefficients $\hat{\beta}_h$ from (1) considering the four types of transactions given buyers' and sellers' characteristics, defined in Table 5. Labels refer to the buyer's assets and ability relative to those of the seller.

consistent with the post-trade dynamics in the data. These patterns hold regardless of buyers' relative ability, with even larger declines in these variables when buyers are less able than sellers, in which case the firm's optimal scale is lower under the new owner. In contrast, trades in which buyers have higher ability but lower wealth (driven mostly by ability heterogeneity) exhibit the opposite pattern: all three variables increase after the trade, as the firm has even higher productivity under the new owner but is more financially constrained. Similarly, trades in which buyers have both lower wealth and lower ability (primarily driven by preference shocks) also display increases in ARPK, profitability, and leverage, although the changes are more moderate. Overall, the results in Figure 6 show that motives related to financial frictions, leading to buyers being wealthier than sellers, are crucial for the model to generate average post-trade firm dynamics consistent with the data.

Implications for Alternative Motives To formalize the usefulness of post-trade firm dynamics in informing theories about the market for firms, Proposition 1 in the Appendix analytically characterizes the dynamics after trade under different motives. Table C.2 summarizes the results. Across the motives considered, both output and capital increase after trade. However, their joint dynamics can help disentangle the motives driving trade.

Under financial frictions motives, trade relaxes borrowing constraints, so capital rises more than output and profits, lowering ARPK and profitability. Leverage also falls as new owners rely relatively more on their own wealth to finance their firms. In contrast, when trade reflects heterogeneous ability and firms move from less to more capable owners, ARPK and profitability are unchanged absent financial frictions (unconstrained), while leverage rises if buyers and sellers have similar wealth. If the buyer has higher ability but lower wealth (constrained), the firm becomes more financially constrained, raising ARPK, profitability, and leverage after trade (consistent with Figure 6). Finally, we consider two motives outside our quantitative model (cost-cutting capabilities and span of control) and find counterfactual predictions. Cost-cutting lowers ARPK but raises profitability and leverage (for constrained firms), while span of control lowers ARPL and raises leverage, both at odds with the data.

To summarize, this section shows that the main testable predictions of our theory about the cross-sectional characteristics of traded firms and regarding post-trade firm dynamics are consistent with the data. While other motives also contribute to trade, financial frictions are essential for the model to reproduce these patterns. In the next section, we use our model to quantify the role of the market for firms in the macroeconomy.

7 Macroeconomic Implications

This section presents our main quantitative exercises. First, we study how taxing the market for firms affects aggregate outcomes.[47] Second, we analyze the importance of the market for firms in economies at different levels of financial development.

7.1 Taxing the Market for Firms

Trades in the market for firms generate a surplus that buyers and sellers divide through bilateral bargaining. Because these gains accrue to a small set of agents, it is natural to consider a tax on these transactions to redistribute part of the surplus to non-participants. Such a tax is analogous to a realized capital gains tax, with the firm's sale price as the tax base. We now examine the efficiency and inequality consequences of such a policy.

Let us consider a tax, $\tau \in [0, 1]$, that is levied on transactions in the market for firms. If agent j sells a firm to buyer i at price p_{ijt}, the seller receives $p_{ijt}(1 - \tau)$, the buyer pays p_{ijt}, and the government collects τp_{ijt}. We assume tax revenues, T_t, are rebated lump-sum to households, so the government budget constraint is

$$T_t = \iint_{(i,j)\in\mathcal{DM}_t} \tau p_{ijt} \,\mathrm{d}i \,\mathrm{d}j,$$

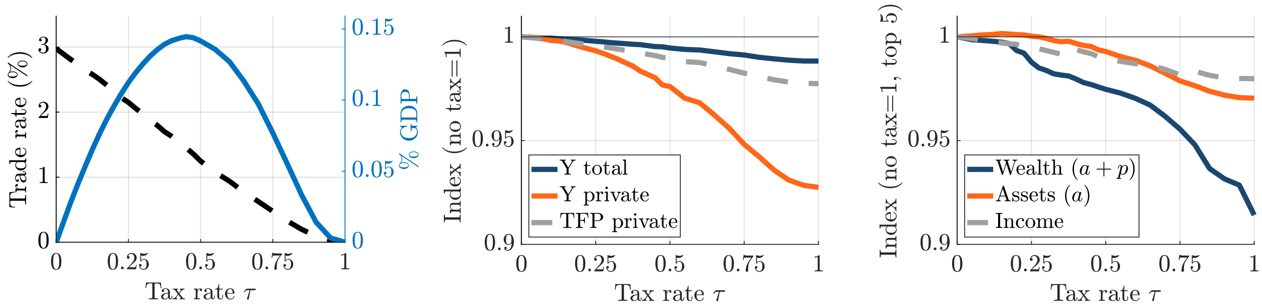
where $\mathcal{DM}_t$ is the set of buyer-seller pairs that trade in the market for firms at time t. The household budget constraint is now $c_{it} = \pi_{it}h_{it} + w \exp(\varepsilon_{it})(1 - h_{it}) + (1 + r)a_{it} - a_{it+1} + T_t$, where $h_{it} = 1$ if the household is an entrepreneur and $h_{it} = 0$ if a worker and a_{it} are the assets remaining after the DM and after paying transaction taxes on firm sales (if they sold a business). Note that the transfer T_t is an equilibrium object that must be determined as part of the competitive equilibrium whenever $\tau > 0$.

Figure 7 shows the aggregate implications of taxing transactions in the market for firms. As taxes increase, gains from trade decline. Hence, as Figure 7a (left y-axis) shows, there is a negative relation between the tax rate and the frequency of trade. In the extreme, if $\tau = 1$, the tax is equivalent to fully shutting down the market. Figure 7a (right y-axis) also plots the

[47] In the following analysis, we focus on aggregate output and productivity, omitting the value of sellers' preference shocks. We do so, as in our baseline economy, the aggregate monetary value of preference shocks, computed as $\mathcal{K}_t = \iint_{(i,j)\in\mathcal{DM}_t}(\kappa_{ijt} - 1)p_{ijt} \,\mathrm{d}i \,\mathrm{d}j$, is small, representing 0.33% of aggregate output ($\mathcal{K}_t/Y_t$).

Figure 7: Taxing the Market For Firms

(a) Trade and Revenues **(b)** Output and TFP **(c)** Top 5% Shares



Notes: Counterfactuals are in general equilibrium. Total wealth ($a+\mathbb{E}[p]$) is measured using after-tax prices.

fiscal revenues from this tax, T_t, as a percentage of aggregate output. Our model implies a Laffer curve that peaks around $\tau = 0.475$, with a relatively small maximum revenue of around 0.15% of GDP. For reference, in the U.S., the top federal capital gains tax rate is around 0.2 and can be as high as 0.3, including state taxes. In Europe it is typically between 0.3 and 0.4.

Next, we examine the implications for aggregate output. Figure 7b shows that total output (including both public and private firms) falls as the tax increases, with a contraction of 1.2% when $\tau = 1$ (i.e., when the market is effectively shut down). We also find that private firms' aggregate output declines sharply, by 7.3% at $\tau = 1$, indicating substantial reallocation from private to public firms, with the latter increasing production. Importantly, private firms' aggregate TFP falls markedly, with a maximum decline of 2.3%, reflecting a reduction in allocative efficiency when this market is absent.

Finally, we explore the implications for income and wealth inequality. Figure 7c shows how the top 5% shares of wealth (including the value of the business), assets (non-business wealth), and income change as τ increases. To measure business wealth, we use the expected after-tax price in the market for firms. As the tax rises, the asset and income shares of the top 5% of households fall by about 3%, while their share of total wealth declines by almost 9% when $\tau = 1$. The after-tax market value of wealth falls sharply because the increase in pre-tax firm prices is smaller than the increase in effective tax rates.

The Role of Financial Frictions We now examine the role of financing frictions in shaping the aggregate effects of taxing transactions in the market for firms. To do so, in Figure C.7 in the Appendix, we compare the previous results with those of an economy without firm-level borrowing constraints. Figure C.7a shows that tax revenues are substantially lower in the economy without credit constraints, reaching at most 0.10% of GDP. This result reflects the lower trade volume when firms are unconstrained, consistent with the trade rate decomposition in Table 4. Turning to aggregate outcomes, Figure C.7b reports that when $\tau = 1$, private firms' output and TFP fall by a maximum of 2.8% and 0.7% in the economy without borrowing constraints. Hence, output and productivity losses are 2.6 and 3.3 times larger, respectively, in the baseline economy with financial frictions. Finally, Figure C.7c shows that eliminating borrowing constraints reduces the impact of the tax on income and wealth concentration by more than half relative to the baseline results.

Taken together, these results indicate that taxing transactions in the market for firms leads to sizable aggregate losses, raises only limited fiscal revenue, and yields a modest reduction in income and wealth concentration at the top. By impeding firm trade, the tax worsens the allocation of ability, capital, and productive projects in the economy. Crucially, it prevents trades in the market for firms from alleviating borrowing constraints, a mechanism that accounts for roughly 60 to 70% of the total output and productivity losses from taxation.

7.2 Financial Development and the Market for Firms

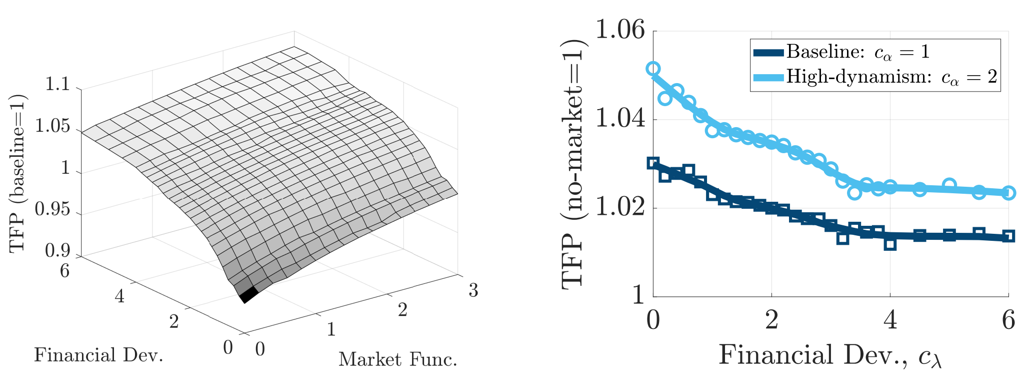
In our model, the functioning of credit markets and the market for firms jointly determines the allocation of capital in the economy. In this final section, we study the interaction between these two markets and their implications for aggregate productivity. Figure 8 reports private firms' TFP across economies with varying degrees of financial development and functioning market for firms. A more financially developed economy has higher λ_a and λ_π (lower borrowing constraints). An economy with a better-functioning market for firms features higher α_o and α_w (lower search frictions). We parameterize these economies through $c_\lambda \geq 0$ and $c_\alpha \geq 0$. Hence, if $c_\lambda = 2$, the economy is twice as financially developed with λ_a and λ_π being twice as large as in the baseline. Analogously, an economy with $c_\alpha = 2$ has a market for firms that functions twice as well, with α_o and α_w twice their baseline values. Figure C.8 in the Appendix shows similar results for private firms' output and trade rates.

Figure 8a shows that TFP is increasing in both financial development and the functioning of the market for firms. In particular, if $c_\alpha = 0$, which corresponds to the no market case (as with $\tau = 1$ above), our model implies that higher levels of financial development lead to a better allocation of capital and higher TFP, as in the finance and misallocation literature (Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014; Moll, 2014). However, unlike previous papers in that literature, this figure also shows that for any given level of financial development, aggregate TFP can increase through a better-functioning market for firms.

The TFP gains from improving the market for firms are nonlinear and are larger when financial development is low. To see this, Figure 8b reports the TFP gains from improving the functioning of the market for firms across different levels of financial development. The figure plots two lines: comparing the no-market case to the baseline ($c_\alpha = 0$ vs. $c_\alpha = 1$), and the no-market economy to a highly dynamic market ($c_\alpha = 0$ vs. $c_\alpha = 2$). There are two main takeaways. First, in high-credit economies ($c_\lambda > 1$), the TFP gains from a better-functioning market for firms are more muted. This result occurs because, in high-credit environments, firm owners operate closer to their optimal scale, thereby reducing the gains from trading

Figure 8: Financial Development and Functioning of the Market for Firms

(a) TFP **(b)** TFP Gains from Market for Firms



Notes: Financial Development is parameterized by c_λ, which scales the borrowing constraint parameters $\lambda_a(c_\lambda) = \max\{c_\lambda \lambda_a, 1\}$ and $\lambda_\pi(c_\lambda) = c_\lambda \lambda_\pi$. Market Functioning is parameterized by c_α multiplying the search frictions $\alpha_o(c_\alpha) = \min\{c_\alpha \alpha_o, 1\}, \alpha_w(c_\alpha) = \min\{c_\alpha \alpha_w, 1\}$. Panel (a) plots private firms' TFP in the financial development and functioning of the market for firms' space. Panel (b) plots the TFP gains from moving from no market ($c_\alpha = 0$) to the baseline ($c_\alpha = 1$) and from no-market to a highly dynamic market for firms ($c_\alpha = 2$) across values of c_λ. The solid lines show locally weighted smoothed values.

firms due to financial frictions. Second, the TFP gains are larger in less-developed financial markets ($c_\lambda < 1$), as trade in the market for firms plays an even more important role in alleviating firms' borrowing constraints and substituting for debt financing.

Post-Trade Firm Dynamics in High- and Middle-Income Countries Motivated by these results, we investigate whether post-trade dynamics vary across countries. We find that, compared to the baseline sample of high-income countries, firms in middle-income and less financially developed economies exhibit higher post-trade output and capital growth, along with a larger decline in ARPK.[48] This evidence is consistent with our theory's prediction that the gains from trading firms are higher in economies with tighter financial constraints.

8 Conclusions

We use microdata from business owners, households, and firms to provide novel cross-sectional and longitudinal facts about the market for firms. We document that one out of four entrepreneurs purchased their business, with younger, smaller, and higher ARPK firms having the highest trading rates. After trade, firms experience substantial capital and output growth, with capital outpacing output, significantly reducing firms' ARPK. Firms' ARPL remains fairly constant after trade, while profitability and leverage decrease.

We explain these empirical findings by developing a rich general equilibrium model of entrepreneurship and frictional trade of firms. By introducing financial frictions as a microfoundation that generates gains from trade, our model can account for the empirical

[48] We use the sample of high- and middle-income European countries in Kochen (2025). See Appendix B for the list of countries included in each group.

patterns. It accounts for the cross-sectional facts, as younger, smaller, and higher ARPK firms are more likely to be financially constrained. Furthermore, it accounts for the longitudinal facts as firms' trade alleviates financial constraints. Other motives for trade, such as preference shocks and heterogeneous ability, cannot account for these patterns on their own.

Our quantitative results show that taxing transactions in the market for firms can generate sizable aggregate losses, reflecting that this market substantially improves the allocation of ability, capital, and productive projects in the economy. Moreover, our model suggests that the market for firms plays an even more important role in economies with tighter credit constraints, where gains from trade due to financial frictions are higher. Thus, a promising avenue for future work is to better understand how policies can improve the functioning of the market for firms, especially in economies with less developed financial markets.

Another exciting avenue for future work is to study the implications for fiscal policy, in particular wealth taxation, in our model. The framework developed in this paper captures important features of private businesses that are typically absent in heterogeneous-agent models (e.g., Guvenen et al. (2023)). In particular, it captures that business wealth is difficult to sell and value, since there is no single price in the market for firms, and that firm values reflect both the firm's quality and the owner's ability. As a result, taxing business wealth could disproportionately affect high-ability entrepreneurs who own high-quality firms.

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Online Appendix for
Financial Frictions and the Market for Firms

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Table of Contents

A U.S. Data Appendix

This appendix describes our primary data sources, presents robustness, and additional exercises about the market for firms in the US economy.

A.1 Data Sources

A.1.1 Survey of Business Owners (SBO) - PUMS

The SBO is a comprehensive survey of firms and firm owners in the U.S. The PUMS sample is representative of non-farm private businesses with receipts of $1,000 or more and is available for the year 2007. The SBO is conducted at the company or firm-level. A company is a business consisting of one or more domestic establishments. The survey is designed to identify the ultimate owners of firms and their characteristics.

Table A.1 reports the total number of owners and firms in the SBO. From those, we first restrict to the owners who report how do they acquire their business. The SBO already restricts to self-employed business owners, thus for our definition of entrepreneurs, we just have to restrict to business owners who actively manage their firm. Our baseline sample consist of almost 700,000 entrepreneurs which own around 500,000 different firms.

Table A.1: 2007 SBO Sample

	#Dropped	#Owners	#Firms
All	-	3,409,393	2,165,680
Report Acquisition	1,244,852	2,164,541	1,291,292
Manage	1,052,287	1,112,254	841,254
Employer firm	413,603	698,651	501,564

From this survey we mainly focus on how the owners acquired their firms. In addition, we use information on the characteristics of the firm (established year, employment, payroll, receipts, sector, location, operation status, number of owners) and of the owners (age, acquisition year, ownership percentage, education level, previous occupation). We use this information to do a thorough characterization of the trade of firms.

Using the SBO we can also obtain information on firms and owners close to the time at which the firm was traded. To study firms' and buyers' characteristics *when purchased* we look at owners that acquired the firm through a purchase in the same year of the survey. Furthermore, the SBO provides information on firms' and owners' characteristics for those owners who report an exit because they sold their firm in the year of the survey. We use this information to characterize firms and their previous owners *when sold*. For all our calculations we use the sample weights provided by the survey.

A.1.2 Survey of Consumer Finances (SCF)

The SCF is a household-level survey that includes extensive information on households' income, balance sheets, and demographic characteristics. The public microdata is available every three years for the period 1989-2016.

Table A.2: 1989-2016 SCF Sample

	#Dropped	#Households
Income and wealth		
All	-	47,769
$22 \leq$ age ≤ 78	3,528	44,241
Positive income	67	44,174
Firm acquisition		
Manage and own	35,468	8,706
Employer firm	1,379	7,327

In the SCF we identify entrepreneurs as those households whose household head: is self-employed, owns a business, and has an active management role in it. The SCF also provides information of privately held businesses which are actively managed. Business owners can report information for up to three or two firms, depending on the survey year. For our baseline calculations we focus on the characteristics of the main business, defined as the one with higher reported value. Using this information, we can identify the entrepreneurs that own a firm with a positive number of employees.

Table A.2 reports our sample selection criteria and the number of households in our SCF sample. For our calculations of the moments of income and wealth we restrict to a sample of households whose household head is between 22 and 78 years old and have a positive income. For our calculations of firm trade, we further focus on entrepreneurs, who, under our baseline definition (employer firms), comprise 7,327 households between 1989 and 2016, a sample that is substantially smaller than our SBO sample.

In addition to the information on entrepreneurs and how did they acquire their firm, we use the SCF to compute relevant moments from the income and wealth distribution in the U.S. economy. Our measure of household wealth is the variable constructed by the Federal Reserve for its Bulletin article which accompanies each wave of the SCF. Wealth is defined as total net worth, which equals assets minus debt. Assets includes both financial and non-financial assets. Financial assets include checking and savings accounts, stocks held directly and indirectly, bonds, etc. Non-financial assets, among others, include the value of houses and other real estate, the value of farm and private businesses owned by the household. Debt includes both housing debt (mortgages), debt from lines of credit and credit cards, and installment loans.

Our measure of income includes all sources of income excluding government transfers (e.g. social security and unemployment benefits) and excluding other (non-classified) sources of income. Thus, we include wage income, income from businesses, income from interests and dividends, from capital gains, rent income and income from pensions and annuities. For all our calculations we use the sample weights provided by the survey.

A.1.3 Panel Study of Income Dynamics (PSID)

The Panel Study of Income Dynamics (PSID) is a nationally representative longitudinal survey of U.S. households that follows the original 1968 sample and their descendants. We use the biennial sample from 1999–2019, restrict to the SRC sample (family IDs ≤ 3000), household heads ages 20–60, and non-missing wealth data. Our baseline sample contains 38,486 cross-section observations and 27,288 panel observations. We compute all statistics using the survey's population weights. We deflate wealth and income variables by the annual PCE price index (FRED).

We define entrepreneurs as business owners (`W10`) who report being self-employed (`B4`). To increase the number of observed firm trades, we use this broader definition and do not additionally require respondents to report managing the firm. Business sellers are those households that report they sold their business in the previous 2 years (`W78`).[49]

Our definition of wealth follows Aguiar, Bils, and Boar (2024) where total wealth ($a_{it}+p_{it}$) includes cash, stocks, home equity, real estate, pensions, bonds, and business wealth, net of debts (mortgage and others). Our measure of non-business wealth (a_{it}) is simply net wealth excluding business wealth. Non-business wealth is total wealth excluding business equity. We define the business seller's wealth as the household's wealth observed in year $t-2$ for those who report selling their business in the next wave at t (i.e., it was sold between years $t-2$ and t). To improve comparability across groups, we compute average household and entrepreneur wealth only for the waves in which sellers' wealth is measured.

To study income dynamics at entry, we define labor income (w_{it}) as the sum of the head's and spouse's labor earnings, including self-employment and farm earnings. We define business net income (π_{it}) as the sum of profits, and business revenue (y_{it}) as the sum of gross receipts, from the household's five main businesses. Lastly, we identify a startup entrepreneur at t as one who was not an entrepreneur at $t-2$.

A.1.4 National Longitudinal Survey of Youth 1979 cohort (NLSY79)

The National Longitudinal Survey of Youth 1979 cohort (NLSY79) is a nationally representative panel survey that follows a cohort of 12,686 individuals who were 14–22 years old when first interviewed in 1979. Respondents have been surveyed annually through 1994 and biennially thereafter. We use observations with non-missing wealth data for the period 1985 to 2020. Our baseline sample includes 131,283 household-wave observations and 14,819 business-module observations (1,084 businesses). All statistics use sample weights `SAMPWEIGHT`. We deflate wealth by the annual PCE price index (FRED).

We identify entrepreneurs as respondents who own a business (`BUSOWN-1`=1 and `BUSOWN-2`<0) and self-identify as entrepreneurs (`BUSOWN-29`=1). We classify firm buyers as those who report purchasing their business (`BUSOWN-7`=3) and firm sellers as those whose ownership ended by sale (`BUSOWN-19`=1). We restrict attention to events involving the main business.

[49] We identify sellers using the item `W78`: "Since January, $t-2$, did you (or anyone in your family) sell part or all of your interest in a business or farm?" Response coded `1 = YES`.

Our measure of total wealth ($a_{it} + p_{it}$) is total family net wealth (`TNFW_TRUNC`) and non-business wealth (a_{it}) excludes net business wealth which we compute as business assets (`Q13-131`) net of business liabilities (`Q13-132`). Wealth is top-coded at the top 2%; accordingly, we winsorize the lower tail at the bottom 2%. In our baseline calculations, to preserve a large sample of trading episodes, we measure buyer and seller wealth in the $[t-3, t+1]$-year window, with the acquisition or sale taking place in year t.

A.1.5 Kaufman Firm Survey (KFS)

The KFS is a panel survey of nearly 5,000 businesses that began operations in 2004 and were tracked through 2011. The initial sample was created by using a list frame sample of start-up businesses from the Dun & Bradstreet Corporation (D&B) database. The KFS collects information from businesses' and owners' characteristics and, in particular, they provide information about firms' balance sheets. Table A.3 shows the sample selection. Following the previous literature, we drop firms that at some point refuse to answer and observations with missing values of employment, revenues, sales, assets, cash, and accounts receivable. Our baseline sample remains with 2,366 firms and 9,116 observations (firm×year).

Table A.3: 2004-2011 KFS Sample

	#Dropped	#Observations	#Firms
All	-	39,424	4,928
Answer	13,624	25,800	3,225
Missing	16,684	9,116	2,366

We define capital as total assets without cash holdings and accounts receivable. Total assets is composed by product inventories, land and buildings and structures, vehicles, equipment/machinery, other properties, cash, and other. In the KFS, we measure firms' ARPK as the revenue to capital ratio. We identify trades through exits of owners that report having sold or merged their business. For all our calculations we use the sample weights provided by the survey.

A.2 Robustness Exercises

A.2.1 How do Entrepreneurs Acquire Their Firms?

Owner-level. Table A.4 report how many entrepreneurs purchased their business for several alternative definitions of entrepreneurship. For example, instead of active management, as in our baseline definition, we restrict to business owners who have more than 50% of the equity of the firm, or to owners who work at least 40 hours a week in the firm. In bold we highlight our baseline definition for entrepreneurs, which implies that firm owners manage an employer firm.

Firm-level. In addition to the business owner-level results, we compute the share of firms that were acquired by their owners through a purchase. We compute the share of firms

Table A.4: Share of Entrepreneurs That Purchased Their Business

Sample	Purchased	N(weighted)	N
All	-	36,856,132	3,409,393
All (Respond acquisition)	16.0%	20,302,192	2,164,541
Manage	17.0%	9,503,681	1,112,254
Employment > 0	25.9%	5,507,460	1,255,134
Receipts > 0	16.9%	17,139,950	1,987,336
Payroll > 0	25.1%	6,045,634	1,338,400
Size (all) > 0	26.1%	5,344,964	1,216,319
Entrepreneur	25.5%	3,167,718	698,651
Share ≥ 50	13.5%	16,274,606	1,479,855
Share ≥ 50 and Employment> 0	23.5%	3,884,071	745,431
Share ≥ 50 and Manage	15.4%	8,064,388	827,286
Entrepreneur and Share ≥ 50	24.0%	2,458,710	469,250
Hours Worked > 40	18.0%	8,928,828	1,164,328
Hours Worked > 40 and Employment > 0	25.6%	3,505,078	802,680
Hours Worked > 40 and Manage	19.6%	5,679,652	806,923
Entrepreneur and Hours Worked > 40	26.0%	2,545,635	582,966
Entrepreneur (Weighted by Employment)	32.2%	3,167,718	698,651

Source: 2007 SBO.
Notes: Purchased refers to the percentage of entrepreneurs that acquire its firm through a purchase. Share refers to entrepreneurs' equity share. Hours Worked denotes average number of hours per week the owner spends at the firm.

purchased in two ways: (i) if at least one entrepreneur purchased the firm; (ii) if all the firm's entrepreneurs purchased it. The results are presented in Table A.5. The purchased share computed at the firm- and owner-level are very similar. This is due to the fact that most firms have one entrepreneur, and most entrepreneurs have one firm. As in the business owner-level results, this share is sensitive to the exclusion of firms with no employment. Definitions that consider firms with no employment tend to have lower purchasing ratios as the main input in production is probably the owner human capital, which is hard to transfer.

Franchises. We further analyze whether franchises are driving our results. Table A.6 shows that even excluding all franchises the share of entrepreneurs that purchased their firm is 16.1% and 24.2% for all firms and our baseline definition, respectively. Although is true that, within franchise owners, the share of entrepreneurs that acquired the business is very high, more than 50%, these owners represent a small group in the total number of entrepreneurs: 2.7% and 4.7% for the two definitions used.

Table A.5: Share of Firms With Owners That Purchased It

Sample	Owner-level	Firm-level	
		At least one	All
All (Respond acquisition)	16.0%	14.7%	12.0%
Manage	17.0%	16.3%	15.0%
Employment > 0	25.9%	26.8%	20.9%
Entrepreneur	25.5%	25.7%	23.2%
Entrepreneur and Hours Worked > 40	26.0%	26.1%	23.8%

Source: 2007 SBO.
Notes: Hours Worked denotes average number of hours per week the owner spends at the firm.

Table A.6: Share of Firms Purchased: Franchises

Sample	All firms	Employer firms
Baseline	17.0%	25.5%
W/o franchises	16.1%	24.2%
Franchises only	50.1%	51.8%
Share of Franchises	2.7%	4.7%

Source: 2007 SBO.

Sectors. We also study whether a particular production sector plays a particularly prominent role in our baseline results regarding the share of traded firms. We consider five main sectors: Manufacturing and Primary; Construction; Wholesale and Retail Trade; Finance, Insurance, Real Estate, and Information Technology; and Services. Table A.7 shows that the share of traded firms is over 20% and the trade rate is over 2.2% in all sectors except the Construction sector, whose trade share is 12.5% and trade rate 1.3%. These results show that firms' trade is widespread across most economic sectors.

A.2.2 Firm Buyers' Previous Occupation

Alternative Computations. In the main text we document that 66% of entrepreneurs have never been self-employed (and hence have never been entrepreneurs) prior acquiring its business. As a robustness, we consider alternative definitions. In Table A.8 we compute the transition rate from worker to entrepreneur conditional on purchasing the firm for: (i) our baseline definition; (ii) when transition to being the main owner of the firm; and (iii) conditional on large firms. Our results are very similar for all these samples.

Firms' Characteristics. We also analyze whether workers tend to buy firms with certain characteristics. For example, one could argue that worker-buyers concentrate in small non-

Table A.7: Share of Firms Purchased and Trade Rates by Sectors

Sector	Share	Share Traded	Trade Rate
Manufacturing and Primary	0.08	27.8%	2.8%
Construction	0.17	11.4%	1.5%
Wholesale and Retail Trade	0.26	32.2%	3.6%
FIRE and ICT	0.11	18.8%	2.4%
Services	0.39	26.5%	3.7%
Total	1.00	25.5%	3.0%

Source: 2007 SBO.
Notes: The first column shows the percentage of firms in each sector, the second column shows the share of firms traded, and the last column shows the trade rate by sector. Manufacturing and Primary = `naics` $\in \{11, 21, 31\}$; Construction = `naics` $\in \{23\}$; Wholesale and Retail Trade = `naics` $\in \{42, 44, 48\}$; FIRE and ICT= `naics` $\in \{51, 52, 53\}$; Services = `naics` $\in \{54, 55, 56, 71, 72, 81\}$. We exclude Utilities, Education Services, and Health Care. Trade rates are computed using businesses purchased in 2007.

Table A.8: Firm Buyers' Previous Occupation

Sample	Worker Before Purchasing	
	All firms	Employer firms
Baseline	62.0%	65.9%
Share> 50	61.2%	62.2%
Large Firms	66.9%	69.6%

Source: 2007 SBO.
Notes: Large Firms as those in the top quintile of the employment distribution.

growth-oriented type of businesses, compared to firms that are acquired by previous firm owners. Table A.9 shows that there is no stark relation between firm characteristics when purchased and the share of firms purchased by workers and, if something, the share is slightly larger for older and bigger firms.[50]

A.2.3 Firm Size and Trade Likelihood

For robustness, we calculate the likelihood of trade across the sales and payroll distributions. As shown in Figure A.1, firms that were in the bottom quartile of the size distribution when traded are the most likely to be traded for both definitions of firm size.

[50]The sample is restricted to 2007 such that the characteristics of the firms are approximately to the ones when purchased. For this sample, the share of firm buyers that were workers is slightly lower (less than 60%) than the one of our baseline sample.

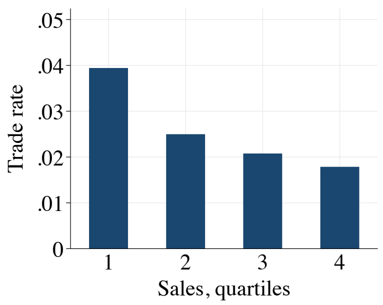
Table A.9: Share of Firm Buyers Who Were Workers

	Workers	Purchased
By Firm Age		
0-2	50.5%	37.0%
3-7	54.7%	14.0%
8-17	56.9%	16.0%
≥ 18	60.7%	33.0%
By Firm Size		
Q1	54.1%	28.9%
Q2	55.8%	25.9%
Q3	55.7%	25.4%
Q4	56.4%	19.8%

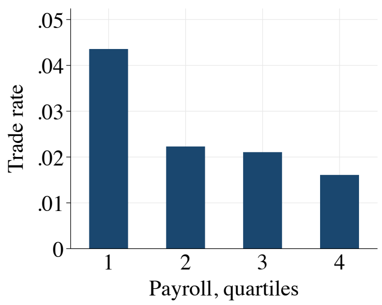
Source: 2007 SBO.
Notes: For our calculation we limit to firms purchased in the same year of the survey (2007) and employer firms as in our baseline calculations. The "Workers" column correspond to the ratio of the previously non-self employed entrepreneurs that purchased the firm over the total of firms purchased. The column "Purchased" indicates the amount of firms purchased by characteristic over all firms purchased (i.e., the distribution of purchased firms). Size is measured by sales.

Figure A.1: Trade Rate by Firm Size

(a) Trade vs. Sales



(b) Trade vs. Payroll



Source: SBO.
Notes: Panels (a) and (b) use data from the 2007 SBO. The trade rate is computed using information from the firms that were sold in or after 2007. Trade rates are normalized to match the aggregate trade rate.

A.3 Additional Evidence on The Market for Firms

A.3.1 Trade share across size and age.

In Section 2.1, we showed that firms *when purchased* tend to be small and young. In this appendix, we analyze the share of entrepreneurs that purchased their firm, at any point in the past, conditional on firm observables such as size and age.

Firm Size. Table A.10 presents the share of entrepreneurs that purchased their business by firm size, considering three definitions: receipts, payroll, and employment. We find that the share of traded firms is even higher at the top of the size distribution. For example, in the top 0.1% of receipts, around 39% of entrepreneurs purchased their firm, considerably higher than the unconditional 25.5% share in our baseline calculations.

Table A.10: Firms Purchases, By Firm Size Group

Percentile	Variable	Purchased	Average
Bottom 90	Receipts	24.6%	651
	Payroll	24.6%	153
	Employment	25.2%	8
Top 10\Top 1	Receipts	34.6%	8,624
	Payroll	34.5%	1,773
	Employment	37.9%	83
Top 1\Top 0.1	Receipts	43.8%	57,753
	Payroll	40.0%	9,220
	Employment	37.9%	248
Top 0.1	Receipts	39.0%	381,869
	Payroll	35.3%	49,760
	Employment	32.3%	1,374

Source: 2007 SBO.
Notes: Results are for the baseline definition (employer firms). Average is computed using both purchased and non-purchased firms. Receipts and Payroll are in thousands ('000) of USD.

Firm Age. Next, we study the share of traded firms conditional on the age of the firm. Table A.11 shows that older firms tend to have larger share of trades. This is consistent either with a higher survival rate of purchased firms, the decline in trade share we observe in the SCF data, or just a higher probability of being purchased for being around more time. Also, this may reflect some life cycle motives since older entrepreneurs probably manage older firms. Related to this, in Appendix A.3.4 we analyze potential life cycle motives for the trade of firms. These results suggest that traded firms, *after purchased*, tend to grow bigger and live longer than non-traded firms.

A.3.2 Trade of Firms Across Time

As the PUMS version of the SBO is only available for 2007, we use the SCF to document the evolution of the share of entrepreneurs that purchased their firms across different years. Table 1 shows that the SBO and SCF 2007 values are consistent. As a robustness check, we also consider data from the Annual Survey of Entrepreneurs (ASE) available from 2014 to 2016. Overall, the numbers obtained from the SCF align very well with the SBO and ASE for the years in which these surveys overlap.[51]

[51] The ASE is representative of all non-farm businesses with annual receipts of $1,000 or more. In this survey, a business owner is defined as someone who holds more than 50% of the firm's stake, provided the firm has a positive payroll. This definition closely aligns with our baseline definition of an entrepreneur,

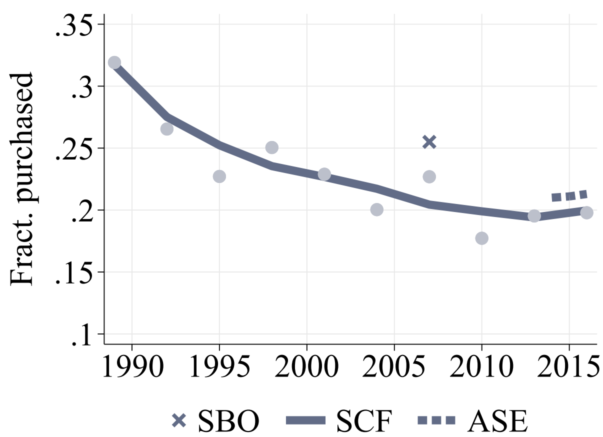
Table A.11: Share of Firms Purchased, By Firm Age

Firm Age	Owner and Manager	Entrepreneur
0-1	8.9%	17.4%
1-2	10.0%	16.3%
2-8	10.9%	16.5%
8-18	13.1%	18.5%
18-28	18.0%	24.9%
+ 28	35.5%	45.2%

Source: 2007 SBO.
Notes: The age of the firm is the age reported at the date of the survey, not when purchased.

Figure A.2 shows that between 1989 and 2016, the fraction of entrepreneurs that acquired their firms through a purchase, which proxies for the fraction of traded firms, declined by one-third. More precisely, the fraction of entrepreneurs that purchased their business fell by 12 p.p. going from 32% in 1989 to 20% by 2016. The decreasing trend is robust to alternative definitions of entrepreneurship and changes in the sectoral composition. The share of traded firms is fairly stable since 2007.

Figure A.2: Fraction of Entrepreneurs that Purchased Their Business



Source: SBO, SCF and ASE.
Notes: Entrepreneurs are defined as self-employed, business owners, who actively manage their firm and the firm has at least one employee. The light-colored dots correspond to the time series SCF data points. The solid line trend was estimated using locally weighted smoothing.

A.3.3 Ownership Structure of Private Firms

Number of Firms Owned. Using data from the SCF we document the number of businesses each entrepreneur owns and manages. Table A.12 shows that more than 80% of the

which requires firms to have at least one employee. The data used in Figure A.2 is retrieved from table `SE1600CSCBO01`, where entrepreneurs are classified based on how they acquired their firms.

entrepreneurs manage one firm at most.

Table A.12: Firms Per Entrepreneur

	# of managed businesses	
	1	≥ 2
Employer firms	83.5%	16.5%
All firms	80.2%	19.8%

Source: SCF 1989-2016.
Notes: Number of employer firms (baseline) and all firms per entrepreneur.

Number of Owners and Entrepreneurs. Table A.13 reports the share of firms in the 2007 SBO conditional on the number of owners and entrepreneurs. The table shows that 74% of the firms have only one entrepreneur, and 96% have at most two. If we include firms with zero employment these numbers are slightly higher (80% and 97%, respectively).

Table A.13: Share of Firms by Number of Owners and Entrepreneurs

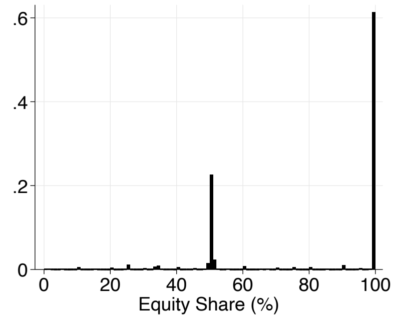
		# of Owners			
Firms		1	2	3	≥ 4
All	Own	51.4%	39.3%	4.5%	4.8%
	+ Manage	79.8%	18.0%	1.6%	0.6%
Employer firms	Own	43.0%	42.5%	7.1%	7.4%
	+ Manage	73.7%	22.7%	2.7%	0.9%

Source: 2007 SBO.
Notes: 'Own' includes owners who do not manage their firm. '+ Manage' includes owners who both own and manage their firm (entrepreneurs).

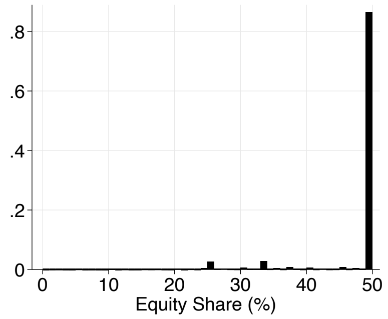
Equity Shares. Figure A.3 shows that, in our SBO sample, more than 60% of the firms have an entrepreneur holding 100% of the firm's equity. However, for more than 20% of firms, the entrepreneur shares around 50% of the equity with another non-manager owner. On the other hand, in firms with two entrepreneurs, the most common arrangement is 50/50 equity shares. These findings are consistent with what is documented by Espino, Kozlowski, and Sanchez (2016) in other datasets. Finally, we analyze the equity share owned by entrepreneurs conditional on firm size and firm age. Figure A.4 reports that the entrepreneurs' equity shares are decreasing with both firms' size and age. Nonetheless, this negative relation is weak, and even for the firms in the top decile of the size distribution, around 75% of the firm equity is held by entrepreneurs. A similar pattern is observed across the age distribution.

Figure A.3: Equity Shares by Number of Entrepreneurs

(a) One Entrepreneur



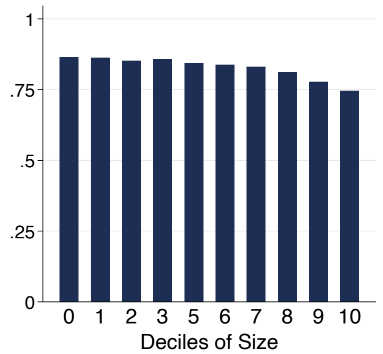
(b) Two Entrepreneurs



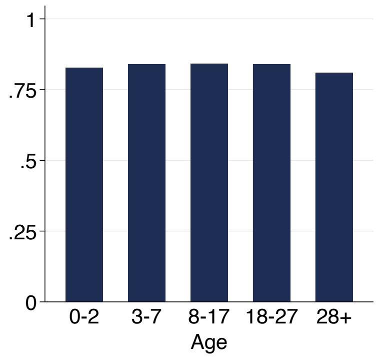
Source: 2007 SBO.
Notes: Use baseline sample of employer firms.

Figure A.4: Equity Shares Across Firm Size and Age

(a) Size



(b) Age

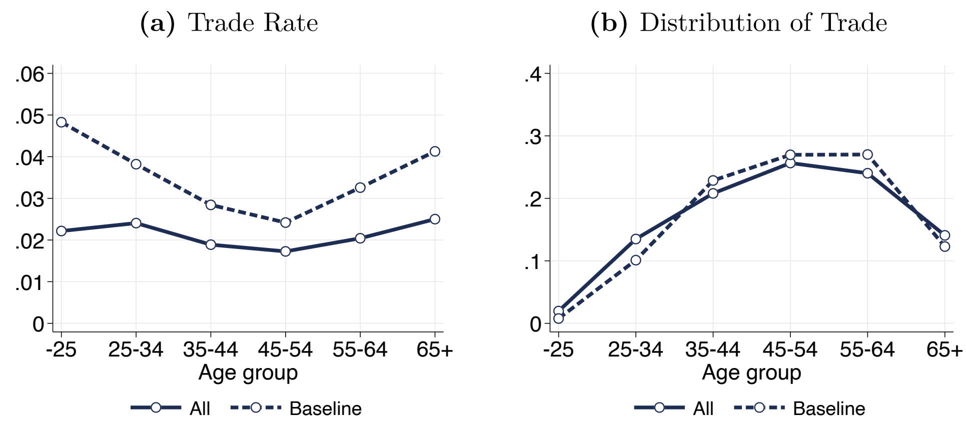


Source: 2007 SBO.
Notes: Deciles of size are constructed using the distribution of firms with positive employment. Decile 0 corresponds to firms with zero employees. Values corresponds to the average value of the sum of entrepreneurial ownership share across the firms' size and age distribution.

A.3.4 Life Cycle Motives

To analyze the role of entrepreneurs' life cycles in driving trade in the market for firms, this appendix examines trade by sellers' age. Figure A.5a shows that the trade rates are higher for young and old entrepreneurs. This is consistent with retirement motives for older entrepreneurs and the lack of access to credit for younger entrepreneurs. However, Figure A.5b shows that the share of trades is primarily concentrated among middle-aged entrepreneurs, even though these are the ones that exhibit the lowest trade rates. This result reflects that the age distribution of entrepreneurs has an inverted U-shape. Thus, even though old entrepreneurs' selling rate is relatively high, the fraction of total trades potentially related to retirement, as proxied by the share of sales done by entrepreneurs in the 65+ category, is just around 10%.

Figure A.5: Trade of Firms by Sellers' Age Group



Source: 2007 SBO.
Notes: The trade rates in Panel (a) are normalized to match the total trade rate of 2 and 3%.

A.3.5 Buyers' and Sellers' Wealth

In our theory of the market for firms, differences in the wealth of buyers and sellers are central for generating gains from trade. In this appendix, we use three household surveys (SCF, PSID, and NLSY79) to measure the wealth of firm buyers and sellers. We find that business buyers are systematically wealthier than sellers. Table A.14 reports results for non-business and total wealth. Panel (a) presents baseline estimates of buyers' and sellers' wealth from the SCF and PSID. Panel (b) reports, as a robustness check, the corresponding estimates from the NLSY79, where we find that wealth differences are even larger. Panel (c) compares the average entrepreneur's wealth to that of the average household across surveys; we find that the PSID and SCF deliver very similar patterns, while the NLSY79 is less comparable.

A.4 Firms' Trade Rate

We indirectly infer the annual trade rate by combining firm dynamics moments, such as the entry and exit rate, and the stock of purchased firms with firms' flow equations. Define the mass of all firms at t as y_t and the stock of firms that have ever been purchased and still active at t as x_t. Then, these variables follow the laws of motion

$$y_{t+1} = y_t \left[1 - \pi^y_{exit,t} + \pi_{entry,t}\right]$$
$$x_{t+1} = x_t \left(1 - \pi^x_{exit,t}\right) + \left[y_{t+1} - x_t \left(1 - \pi^x_{exit,t}\right)\right] \pi_{trade,t_+}$$

where $\pi_{entry,t}$ and $\pi^y_{exit,t}$ are the annual entry and exit rates, respectively, $\pi^x_{exit,t}$ is the annual exit rate conditional on having been traded in the past, and π_{trade,t_+} is the annual firm trade rate that we seek to infer.[52] Combining the flow equations, we have that the ratio of firms

[52] The timing convention is that the trade rate occurs after exit and entry at t and applies to firms that have not been traded. This is innocuous for the steady state calculation.

Table A.14: Firm Buyers' and Sellers' Wealth

	Non-Business	Total
(a) Baseline: SCF and PSID		
Buyer/Avg. HH (SCF)	2.71	3.83
Buyer/Avg. Entrepreneur (SCF)	0.79	0.69
Seller/Avg. HH (PSID)	1.88	3.04
Seller/Avg. Entrepreneur (PSID)	0.58	0.54
Buyer/Seller	1.36	1.28
(b) Robustness: NLSY79		
Buyer/Avg. Entrepreneur	0.81	0.86
Seller/Avg. Entrepreneur	0.57	0.57
Buyer/Seller	1.51	1.42
(c) Avg. Entrepreneurs / Avg. HH		
SCF	3.43	5.55
PSID	3.25	5.59
NLSY79	1.93	2.03

Source: 1989-2016 SCF; 1999-2019 PSID; and 1985-2020 NLSY79.
Panel (a) reports buyers' and sellers' non-business and total wealth relative to entrepreneurs, along with the buyer–seller wealth ratio, using the SCF and PSID. The ratio between buyers and sellers is computed using the ratios relative to entrepreneurs. Panel (b) presents the analogous estimates from the NLSY79. Panel (c) reports entrepreneurs' wealth relative to the average household across all three surveys. In the SCF, we measure buyers' wealth within one year of the purchase; in the PSID, we take sellers' wealth from the wave preceding the reported sale; and in the NLSY79, we measure wealth from three years before through one year after the transaction. All reported wealth ratios are ratios of averages. See the text and Appendix A for details.

traded evolves as

$$\left(\frac{x_{t+1}}{y_{t+1}}\right) = \left(\frac{x_t}{y_t}\right) \left\{ \frac{1 - \pi^x_{exit,t} + \frac{y_t}{x_t}\left[1 - \pi^y_{exit,t} + \pi_{entry,t}\right] \pi_{trade,t_+} - \left(1 - \pi^x_{exit,t}\right) \pi_{trade,t_+}}{1 - \pi^y_{exit,t} + \pi_{entry,t}} \right\}.$$

If the exit rates for traded and non-traded firms are equal ($\pi_{exit,t} = \pi^y_{exit,t} = \pi^x_{exit,t}$) and entry equals exit ($\pi_{e,t} = \pi_{exit,t} = \pi_{entry,t}$), then we can compute the steady state annual firm trade rate π_{trade} from the observed firm exit rate π_e and the share of traded firms $\frac{x}{y}$ (i.e., the stock of traded firms that were traded relative to total firms) using the following equation:

$$\pi_{trade} = \frac{\pi_e}{\left(\frac{x}{y}\right)^{-1} - 1 + \pi_e}.$$

B Orbis Data Appendix

This appendix describes the Orbis database and provides a detailed description of how we identify trades in the market for firms using this data. It also presents additional results and several robustness exercises.

B.1 The Orbis Database

To document post-trade firm dynamics, we use the historical product of Orbis, an extensive firm-level database covering millions of companies worldwide. This database is compiled by Moody's Bureau van Dijk (BvD), which aggregates data from various sources, such as national business registries, and harmonizes it into a globally comparable format.

Industry Files From Orbis, we use the industry files reporting firms' annual balance sheets and income statements. The industry files contain information starting from the early 1990s to 2023. We use these files to compute firm-level output, capital, ARPK, ARPL, profitability, and leverage. This data also includes information about firms' use of inputs, country and industry identifiers, and the year they were founded.

Ownership Files Additionally, we use Orbis' firm-owner linkages to identify trades in the market for firms. From 2007 onward, this database reports annual snapshots with the list of owners for a large number of firms. The data reports owners' names, equity shares, and specifies whether the owner is an individual, another company, a financial institution, or another type of entity. For our baseline results, we focus on firms owned by individuals through direct ownership linkages. This sample of firms is the closest to that in the SBO, and it is the most consistent with our model. We consider robustness exercises that relax our sample selection, both by using ultimate ownership information and including firms owned by other companies, and find consistent results as shown below.[53]

Data Cleaning Starting from Moody's industry financial and ratios data, we first drop firms (`bvdidnumber`) with missing total assets (`toas`) in all the years we observe the firm. Second, we drop the firms with missing NACE sector information or missing incorporation year. We also drop observations with negative firm age (year minus incorporation year). Keeping the tuple defined by firm, consolidation-level, and currency as the unit of observation, we perform some basic cleaning steps. First, if applicable, we correct for any issues arising from changes in the units of reporting over time (e.g., from millions to thousands). Second, following the suggestions in Kalemli-Özcan et al. (2024), we drop partial or entire time spells within a firm that present any of the following issues: (1) balance sheet components do not correctly add up, considering a 0.5% error threshold[54]; (2) if `toas` present large

[53] We construct data for ultimate owners by sequentially matching the ownership files and assigning the respective ownership shares of the parent company. Hence, for example, if 100% of company A is owned by company B, the procedure assigns the ownership information of B to A. If x% of company A is owned by company B, only x% of the ownership of B will be transferred to company A.

[54] In detail, we flag observations where $|\texttt{shfd} - (\texttt{toas} - \texttt{culi} - \texttt{ncli})|/|\texttt{shfd}| > 0.005$. Further, we flag observations where the residual components in total assets and total liabilities are negative and below a 0.5% error threshold. Specifically, if $(\texttt{cuas} - \texttt{cash} - \texttt{stok})/\texttt{toas}$, $((\texttt{toas} - \texttt{cuas}) - \texttt{tfas} - \texttt{ifas})/\texttt{toas}$, $(\texttt{culi} - \texttt{loan} - \texttt{cred})/(\texttt{ncli} + \texttt{culi})$, or $(\texttt{ncli} - \texttt{ltdb})/(\texttt{ncli} + \texttt{culi})$ are < -0.005.

changes above 1000 or below 1/1000 times, or there are changes above 50% that coincide with changes in reporting units; (3) if employment (`empl`) is negative or is larger than that of Walmart (2 million) in any year. If only a subset of a firm's observations have these problems, we keep the last time spell (consecutive observations) of the firm that does not have these issues. Finally, for the variables of interest, we estimate missing values using linear interpolation when a missing observation is located between two periods with non-missing data. If two or more consecutive observations are missing, we do not do any imputation.

Sample We focus on a sample of European private firms, comprising sole proprietorships, partnerships, and private limited companies. Our baseline results focus on eleven high-income European countries that are the most comparable to the US: Austria, Belgium, Denmark, Finland, France, Germany, Italy, Norway, Spain, Sweden, and the United Kingdom. We also present results for a group of ten middle-income European countries: Bulgaria, Croatia, Czechia, Hungary, Latvia, Lithuania, Poland, Romania, Slovakia, and Slovenia. Given the high cross-country correlation between income and finance, the high- and middle-income labels coincide with labels for developed and less financially developed countries (Kochen, 2025) analysis focuses on the firm-year observations from 2006 to 2023 with available capital, output, and ownership data. Table B.4 reports the distribution of age, employment, and number of owners per firm for the high-income European countries in Orbis and the US in SBO. It shows that our baseline sample in Orbis includes a substantial share of young, small, and single-owner firms similar to the SBO. Table B.5 presents additional descriptive statistics for both high- and middle-income countries.

Variables' Definitions We follow Kochen (2025) for our definitions of the main variables using the balance sheet and income statements from Orbis. We measure firms' capital as equity plus net financial debt: $k_{it} = e_{it} + b_{it}$, where using Orbis acronyms, we measure equity as $e_{it} = \texttt{toas}_{it-1} - \texttt{culi}_{it-1} - \texttt{ncli}_{it-1}$ and net financial debt as $b_{it} = \texttt{loan}_{it-1} + \texttt{ltdb}_{it-1} - \texttt{cash}_{it-1}$. The variable `toas` denotes total assets, `culi` is current liabilities, `ncli` is non-current liabilities, `loan` is short-term financial debt (payable within a year), `ltdb` is long-term financial debt, and `cash` denotes the firm's cash and cash equivalents. Balance sheet variables in the data are reported at the end of each year. To be consistent with the model, we use the one-period lag to measure the beginning of the period variable. We measure output using value-added, defined as revenue minus a comprehensive measure of costs, which excludes labor expenses and capital depreciation: $y_{it} = \texttt{ebta}_{it} + \texttt{staf}_{it} = \texttt{opre}_{it} - (\texttt{cost}_{it} - \texttt{staf}_{it} - \texttt{depr}_{it}) - \texttt{oope}_{it}$. Labor costs are $wl_{it} = \texttt{staf}_{it}$. Firms' profits is the sum of profits plus all extraordinary revenues minus extraordinary expenses $\pi_{it} = \texttt{plat}_{it} + \texttt{extr}_{it}$.[55]

[55] The definition of capital we use for Orbis is similar to the one we used for the KFS. Appendix A.2 in Kochen (2025) shows that tangible assets and inventories account for the bulk of the balance sheet categories in k, which also includes intangible assets and a fourth category. Our definition of output y in Orbis subtracts for non-labor and non-capital costs, variables we don't have available in the other data sets. Our results are almost identical if we define output using only revenue, as in the SBO.

B.2 Identifying Trades in the Market for Firms

We identify trades in the market for firms using the Orbis data through changes in firms' majority owners (i.e., equity share above 50%). Considering the sample of firms that appear in the balance sheet and ownership data, we observe a total of 335,558 changes in majority owners, as Table B.1 shows. This table reports the number of ownership changes in the Orbis data under alternative sample definitions. Given the scope of our paper and to ensure a proper measurement, we focus on a subset of these events. First, we employ a string similarity algorithm, described in detail below, to exclude changes in names that are spurious or that are likely related to inheritances or family-related transfers. Second, to adequately capture post-trade firm dynamics, we focus on the sample of firms that change ownership only once in our data sample. Third, we restrict to the sample of firms with available balance sheet data for our main variables of interest: output, capital, wage bill, profits, and debt. After applying these three criteria, we have 71,139 events from the same number of firms. We refer to this as our "broad" sample of trades, which includes all firms for which we can measure post-trade firm dynamics.

Table B.1: Majority Owner Changes in Orbis

	Events	Firms	Observations
Ownership change	335,558	318,508	3,355,346
Drop spurious and inheritances	269,107	257,346	2,686,284
Drop multiple trades	246,736	246,736	2,565,071
Drop missing outcome data (Broad)	71,139	71,139	733,437
Buyer and owner are individuals (Baseline)	22,056	22,056	174,480

Notes: Number of events where the majority owner (equity>50%) of a firm changes in high-income countries under different sample definitions. Firms report the total number of firms for which we observe these events. Observations report the total number of firm-year pairs with available balance sheet data for the sample of firms with at least one majority owner change.

Finally, for a proper mapping with our theoretical model, we focus on a subset of these events where both the buyer (the new majority owner) and the seller (the previous majority owner) are individuals. See Table B.3 below for a taxonomy of the trades in Orbis by buyers and sellers entity type. This last criterion yields a final sample of 22,056 events, which we refer to as our "baseline" sample. The primary rationale behind this sample selection criterion is to focus on transactions between individuals (households in our model) by excluding corporate mergers and acquisitions (M&A). In this same vein, we focus on direct ownership linkages to avoid cases where, although the ultimate majority owner is an individual, this may occur through complex corporate ownership structures.

We consider robustness exercises that relax our sample selection on these two fronts. On the one hand, Figure B.4 shows our main post-trade firm dynamics results for capital, output, and ARPK, using a broader sample of trading events that does not restrict to owners

Table B.2: Traded Firms' Characteristics in Different Samples

	Broad	Baseline	DiD Sample
Age	15.5	11.5	15.2
Output, $\log(y)$	13.15	12.11	12.42
Capital, $\log(k)$	12.82	11.60	12.22
ARPK, $\log(y/k)$	0.33	0.51	0.20
N. of Firms	71,139	22,056	9,353

Notes: Firms' characteristics when traded ($t = 0$). Broad is the sample of trades without restrictions on the entity type of buyers and sellers. Baseline refers to our primary sample, which focuses on transactions where both the buyer and the seller are individuals. DiD Sample denotes the subset of firms in our baseline sample that we can match to a counterfactual non-traded firm and we use in the diff.-in-diff. analysis.

being individuals. Furthermore, the figure shows results for cases in which both buyers and sellers are firms (i.e., M&A transactions). In both cases, the increase in capital and decline in ARPK five years after trade are reduced. Notably, the results for companies suggest that M&A transactions also alleviate target firms' financial constraints, but to a lesser extent. To shed light on the reason behind these differences, Table B.2 reports the average age, firm size (measured by output and capital), and ARPK of firms at the year of trade ($t = 0$). The table shows that firms in our baseline sample are, on average, younger, smaller, and have higher ARPK at the moment of trade relative to the firms in the broad sample. Hence, consistent with our theory's predictions, our baseline sample of firms exhibits more pronounced post-trade firm dynamics than the firms in the broader sample.

On the other hand, we also rerun our analysis using ultimate ownership linkages. Figure B.5 shows that our main findings on post-trade firm dynamics are almost identical using what would be our baseline sample using ultimate owner links, despite having a significantly larger number of trade events. Indeed, as panel (a) of Table B.3 shows, in our broad sample, 59% of sellers are individuals, while in 34% of events, both buyers and sellers are individuals. In 35% of the events, the seller is a (non-financial) company, and 25% of transactions occur between companies. Panel (b) of Table B.3 shows that, considering ultimate owner linkages, the share of events where both buyer and sellers are individuals rises to 60%, while the company-to-company transactions fall to 10%.

B.3 String Similarity Algorithm

As described above, we use a string similarity algorithm in order to exclude changes in majority owner names that are spurious or are likely related to inheritances. In this appendix we describe our algorithm in detail. After identifying all the events where a majority owner changes, we compute four string similarity metrics using the names of old and new owners' pairs: Jaro-Winkler distance; Levenshtein distance, normalized by the largest string length among the two names; Soundex; and Token Soundex measures. All these metrics lie in the [0,1] interval. After computing these measures, we exclude all the pairs that satisfy at least

Table B.3: Buyers and Sellers Taxonomy by Entity Type

(a) *Direct Ownership Linkages*

		Buyer			
		Individual	Company	Financial	Other
Seller	Individual	0.34	0.24	0.02	0.00
	Company	0.07	0.25	0.02	0.00
	Financial	0.01	0.03	0.01	0.00
	Other	0.00	0.00	0.00	0.00

(b) *Ultimate Ownership Linkages*

		Buyer			
		Individual	Company	Financial	Other
Seller	Individual	0.60	0.09	0.01	0.01
	Company	0.11	0.10	0.02	0.01
	Financial	0.02	0.02	0.01	0.00
	Other.	0.00	0.00	0.00	0.01

Notes: Buyers' and sellers' type in the Broad sample considering firms' majority owners. Individual refers to a natural person. Company refers to the cases where the owner is a non-financial firm. Financial refers to entities such as banks, financial and insurance companies, hedge funds, mutual funds, pension funds, private equity firms, and venture capital funds. Other refers to additional or unknown entity types. Panel (a) presents the results using direct ownership linkages. Panel (b) presents the results using ultimate owner linkages after 5 rounds of matching.

one of the following conditions:

1. The pair has a level of similarity above 0.75 according to Jaro-Winkler.
2. The pair has a level of similarity above 0.75 according to Levenshtein.
3. Soundex is equal to 1.
4. Token Soundex is equal to 1.

Conditions 1-4 exclude spurious name changes. In addition, this algorithm excludes changes related to inheritances or family transfers as it identifies, for example, the names that share the same last name. Our results are robust to using alternative similarity metrics or varying the thresholds in conditions 1. and 2.

B.4 Difference-in-Differences Analysis

In our baseline analysis, we employ an event analysis setup to document post-trade firm dynamics along several outcome variables. We consider this to be the most appropriate empirical strategy, as the decision to trade a firm is endogenous and the set of traded firms may therefore differ systematically from the overall pool of firms. To complement this approach, this appendix tests whether our main longitudinal findings remain robust under a difference-in-differences framework, where we compare the trajectory of traded firms with that of a control group of similar firms along several observable characteristics. This analysis yields two key results. First, we successfully identify a non-traded counterpart for 42% of traded firms, echoing our earlier observation of systematic differences between traded and non-traded firms. Second, using the subsample of firms with a non-traded counterpart, we find that capital and output grow more relative to the control group and ARPK falls more five years after trade, consistent with our baseline findings.

Following an empirical strategy similar to that in Smith et al. (2019), we construct a sample comprising traded firms matched to firms that do not experience a change in their majority owner. In detail, for each traded firm i in year t, we look at all "counterfactual" firms at year t, with replacement, that did not change ownership in our sample period and matches the traded firm along five dimensions: (1) the firm operates in the same country; (2) the firm had the same NACE 2-digit industry code; (3) the firm belongs to the same age bin at year t; (4) the firm is in the same output decile at year t; (5) the total number of owners at year t is in the same owner bin as the traded firm.[56] In addition to these five criteria, we discard the matches where the absolute value difference between the traded firm i and counterfactual firms j output is greater than 0.5 log points in any of the three years before trade, i.e., $|\log(y_{it}) - \log(y_{jt})| > 0.5$ for $t \in \{t-2, t-1, t\}$. We considered several robustness exercises relaxing or tightening the criteria described above and find similar results.

We successfully match 9,353 (42%) firms from our baseline sample to at least one non-traded firm. We refer to this subset of firms as the "DiD sample". As the third column of Table B.2 reports, the matched firms are, on average, older, larger, and have lower ARPK than the other firms in the baseline sample. Not surprisingly, given the characteristics of the matched firms, the post-trade firm dynamics using the DiD sample are more muted relative to the baseline results. Figure B.1 shows that, employing our baseline event analysis regression framework in (1), the average increase in capital and decrease in ARPK using the DiD sample is roughly half of the one documented using our baseline sample. These significant differences between the baseline and DiD samples are important to consider when interpreting the difference-in-differences analysis results.

Having constructed the sample of traded firms i and control non-traded firms j, we run the log difference of the main outcome variables on event-time indicators:

$$\log x_{it} - \log x_{jt} = \delta_0 + \sum_{h \in \mathcal{T}} \delta_h \mathrm{D}_{it}^h + \epsilon_{ijt}, \tag{18}$$

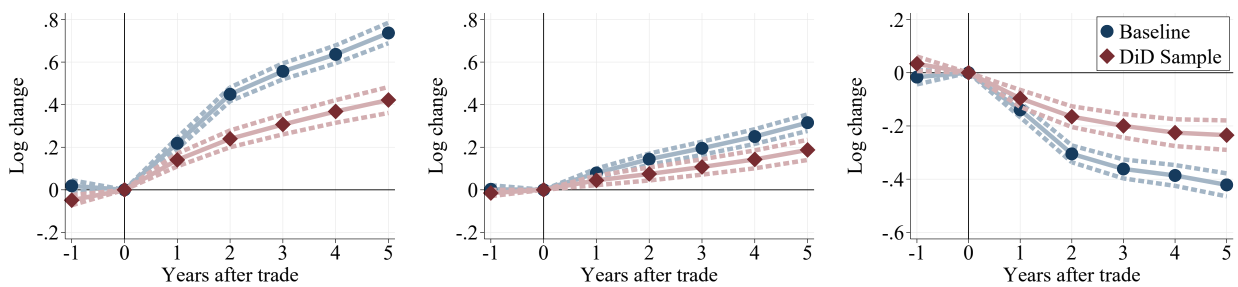
where $\mathcal{T} = \{-2, -1, 1, \ldots, 5\}$ and D_{it}^h is an indicator variable equal to 1 if time t corresponds to the period h around the trading episode. As $\mathcal{T}$ indicates, we study firm dynamics from two years before up to five years after trade. As in Smith et al. (2019), we use the inverse of the number of counterfactual firms j matched to traded firm i as weights in the regression. We cluster standard errors at the level of traded and non-traded firm pairs.

Figure B.2 shows the results from estimating (18) for capital, output, and ARPK. The differences in output, capital, and ARPK between traded and counterfactual firms are not statistically different from zero over the period -2 to 0. Panel (a) shows that in the first

[56] When constructing the matching sample, we consider eight age bins: age 0-2, 3-4, ..., 9-10, 11-15, 16-20, and 21 or higher. For the number of owners, we consider four bins: 1, 2, 3, and 4 or more owners.

Figure B.1: Capital, Output, and ARPK Dynamics After Trade, DiD Sample

(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$

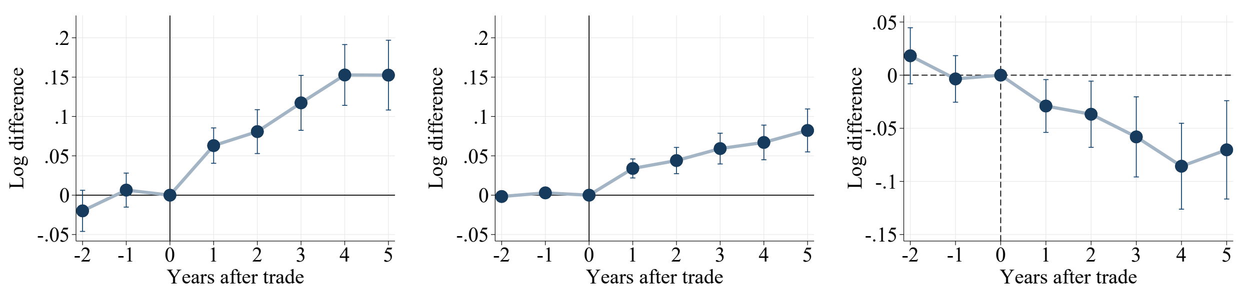


Notes: DiD Sample denotes the estimated coefficients $\hat{\beta}_h$ from (1) considering the sample of traded firms that can be matched to counterfactual non-traded firms. The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

year after trade, the capital of traded firms is 0.063 (6.5%) log points higher than that of comparable non-traded firms, and the difference widens over time, being 0.15 log points (16.5%) higher after five years. Notably, as panel (b) shows, output of traded firms is also higher than that of non-traded firms, but the differences are smaller, being 0.082 log points higher (9%) after five years. Given the differences in the dynamics of capital and output, the ARPK of traded firms is 0.030 and 0.070 log points lower (3% and 7%) one and five years after trade than that of similar non-traded firms. Overall, despite being able to match only a subset of traded firms, which tend to be larger, older, and with higher ARPK, the results of the difference-in-difference analysis are consistent with our baseline results using the full sample of traded firms.

Figure B.2: Capital, Output, and ARPK Dynamics After Trade, DiD Analysis

(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$

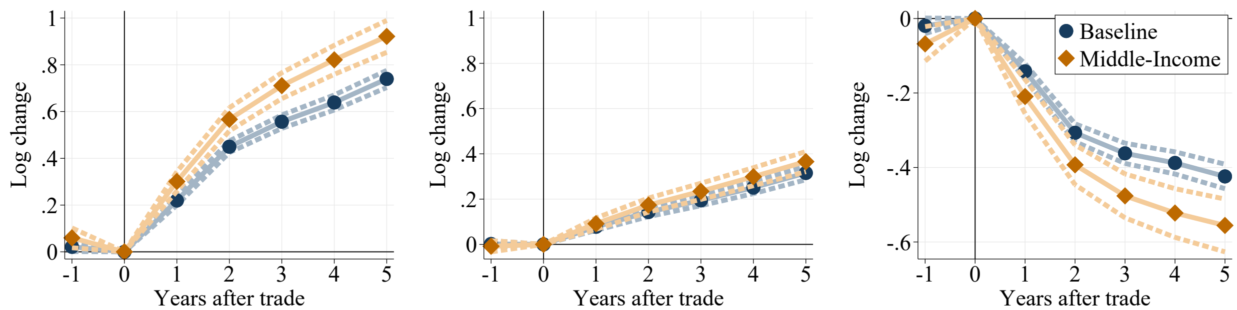


Notes: Estimated coefficients $\hat{\delta}_h$ from (18) using the DiD sample. The vertical lines correspond to 99% confidence intervals considering clustered standard errors at the level of traded and non-traded firm pairs.

B.5 Additional Results

Figure B.3: Firm Dynamics After Trade in High- and Middle-Income Countries

(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$



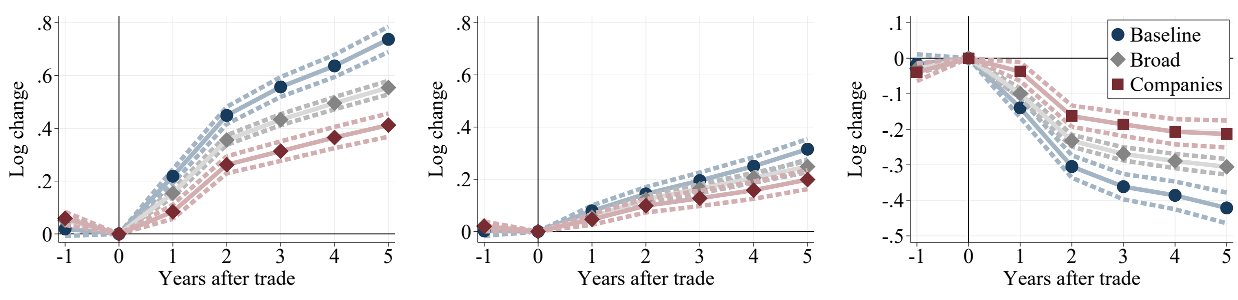
Notes: We estimate a specification similar to (1):

$$\log x_{it} = \beta_0 + \sum_{h \in \mathcal{T}} \Big\{ \beta_h^1 \mathrm{D}_{it}^h + \beta_h^2 \big(\mathrm{D}_{it}^h \times \log y_{it}\big) + \beta_h^3 \big(\mathrm{D}_{it}^h \times \mathrm{MI}_i\big) \Big\} + \gamma \mathbf{c}_{it} + \epsilon_{it},$$

where MI_i equals one if firm i belongs to a middle-income economy and zero otherwise, and $\log y_{it}$ denotes log output residualized by year, sector, and age fixed effects. The average effect for high-income economies is recovered from β_h^1 and β_h^2 evaluated at the mean of $\log y_{it}$ for high-income economies (the same effect estimated in our baseline specification (1) for high-income economies), while β_h^3 captures the difference in the dynamics between middle- and high-income economies. We include the interaction with (residual) firm output to account for differences in coverage across middle- and high-income economies. The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Figure B.4: Capital, Output, and ARPK Dynamics After Trade by Entity Type

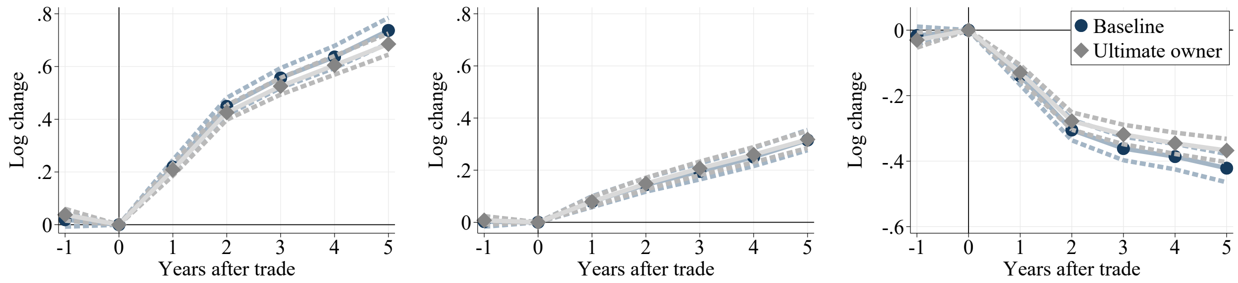
(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$



Notes: Estimated coefficients $\hat{\beta}_h$ from (1) considering different samples by entity type. Broad includes all the transactions. Companies focuses trades where both buyers and sellers are firms. The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Figure B.5: Capital, Output, and ARPK Dynamics After Trade, Ultimate Owner

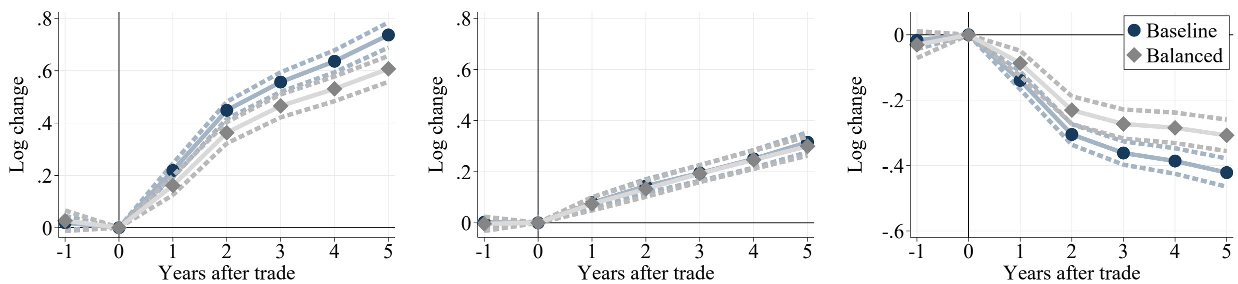
(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$



Notes: Ultimate owner denotes the estimated coefficients $\hat{\beta}_h$ from (1) identifying trades using ultimate owners information after 5 rounds of matching the ownership files. The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Figure B.6: Capital, Output, and ARPK Dynamics After Trade, Balanced Sample

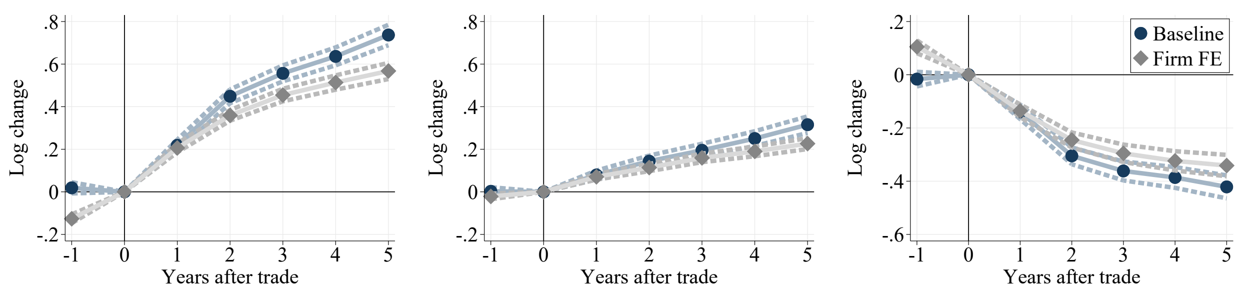
(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$



Notes: Balanced denotes the estimated coefficients $\hat{\beta}_h$ from (1) considering a balanced sample of firms observed from -1 to 5 years after trade. The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Figure B.7: Capital, Output, and ARPK Dynamics After Trade, Firm Fixed Effects

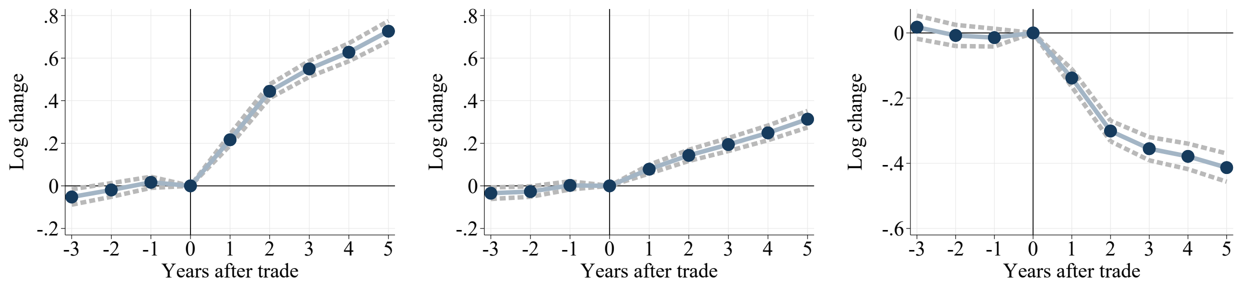
(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$



Notes: Firm FE denotes the estimated coefficients $\hat{\beta}_h$ from (1) considering firm fixed effects. The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Figure B.8: Capital, Output, and ARPK Dynamics After Trade, Extended Window

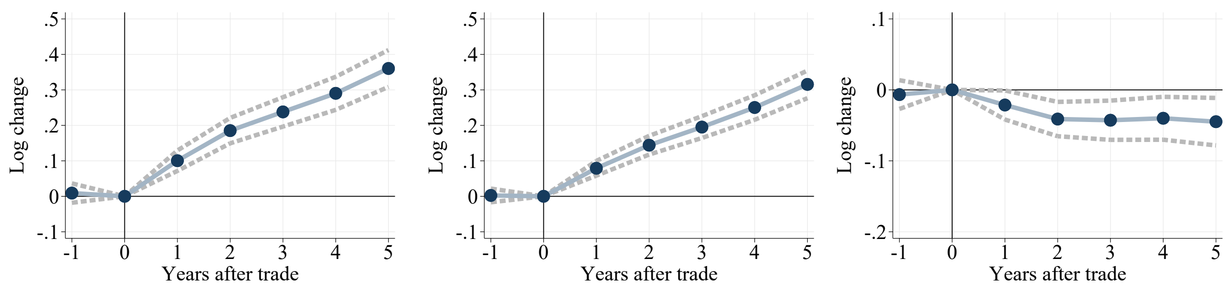
(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$



Notes: Estimated coefficients $\hat{\beta}_h$ from (1) considering an extended window from -3 to 5 years after trade. The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Figure B.9: Labor, Output and ARPL Dynamics After Trade

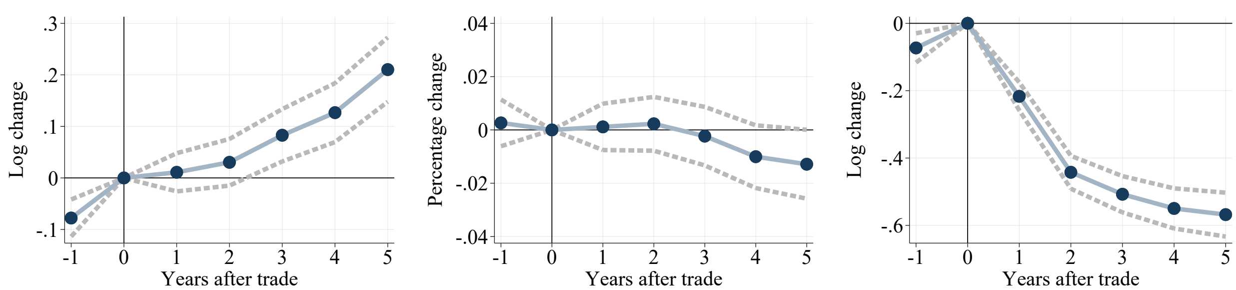
(a) Labor, $\log(wl)$ **(b)** Output, $\log(y)$ **(c)** ARPL, $\log(y/wl)$



Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Figure B.10: Profits Dynamics After Trade in the Orbis Data

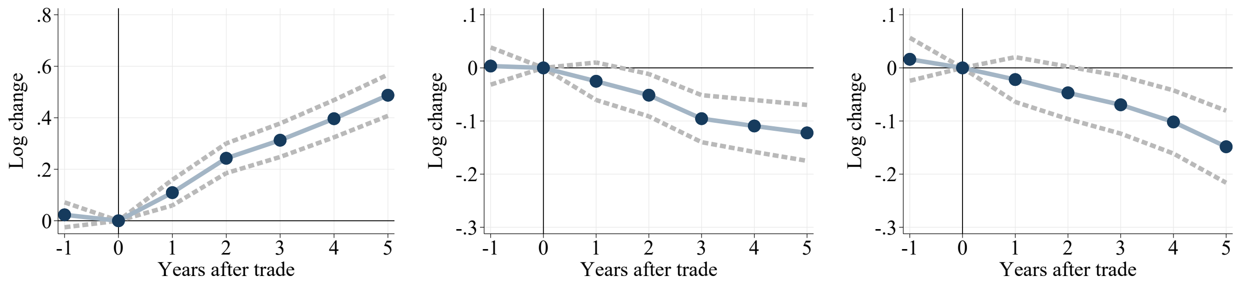
(a) Profits, $\log(\pi)$ **(b)** Neg. Profits, $\mathbb{1}\{\pi \leq 0\}$ **(c)** Profitability, $\log(\pi/k)$



Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Figure B.11: Debt Dynamics After Trade in the Orbis Data

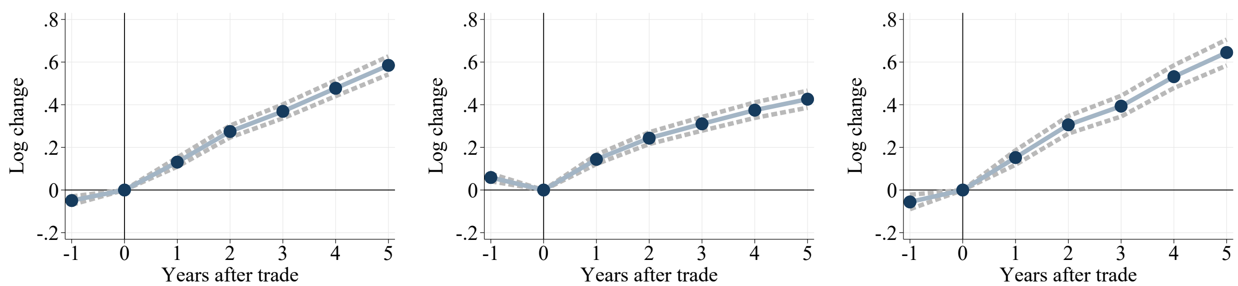
(a) Net Debt, $\log(b)$ **(b)** Leverage, $\log(b/k)$ **(c)** Lev. w/o cash, $\log(d/\mathrm{k})$



Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors. The variable b is net financial debt, $\texttt{loan} + \texttt{ltdb} - \texttt{cash}$, while d is financial debt, $\texttt{loan} + \texttt{ltdb}$.

Figure B.12: Equity Dynamics After Trade in the Orbis Data

(a) Equity **(b)** External Equity **(c)** Internal Equity



Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors. Firms' total equity can be decomposed as the sum of two terms. External equity (variable `capi` in Orbis) are firm owners' external funds that were not generated inside the firm. Internal equity (variable `osfd` in Orbis) captures all the funds internally generated by the firm due to retained earnings or undistributed profits or losses.

Table B.4: Age, Size, and Owner Distribution in SBO and Orbis

Age	0-3	4-7	8-17	18+
SBO	16.9%	15.4%	26.2%	41.6%
Orbis w/ Ownership	10.7%	16.1%	33.5%	39.7%
Orbis w/ Owner Individual	13.7%	19.0%	35.5%	31.8%
Employees	1-9	10-49	50-99	100+
SBO	74.5%	21.7%	2.3%	1.5%
Orbis w/ Ownership	60.9%	28.6%	5.0%	5.6%
Orbis w/ Owner Individual	73.6%	22.3%	2.3%	1.7%
Owners	1	2	3	4+
SBO	43.0%	42.5%	7.1%	7.4%
Orbis w/ Ownership	48.1%	30.2%	11.5%	10.2%
Orbis w/ Owner Individual	56.9%	32.8%	7.1%	3.2%

Source: SBO and Orbis Historical.
Notes: Percentage of firms by age, number of employees, and number of owners. Orbis numbers come from the sample of firms in high-income European countries from 2006-2023. Orbis w/ Ownership are the observations that in addition to the balance sheet data have available ownership linkages. Orbis w/ Owner Individual refers to the sample of firms for which the majority owner is an individual.

Table B.5: Orbis Database Descriptive Statistics

	High-Income		Middle-Income	
	Mean	SD	Mean	SD
Firms w/ Ownership				
Age	16.7	14.3	12.4	8.5
Output	3.0	46.5	2.2	17.8
$\Delta \log(k)$	0.07	0.61	0.09	0.63
Obs.	14,848,005		3,762,476	
Firms w/ Owner Individual				
Age	14.9	12.8	11.2	7.7
Output	0.7	4.8	0.3	1.6
$\Delta \log(k)$	0.08	0.63	0.11	0.66
Obs.	5,382,839		1,948,026	

Notes: Descriptive statistics for our sample of firms between 2006-2023, with available output and capital. All Firms are the observations in the Industry Files satisfying these criteria. Firms w/ Ownership are the observations that in addition have Ownership Files. Age is in years, Output is in million 2015 USD at constant exchange rates, and $\Delta \log(k)$ is capital's one-year growth rate in logs.

C Model Appendix

This appendix includes additional derivations and results from our model. It also presents the equilibrium definition, and a detailed description of our computational solution.

C.1 Additional Derivations

To simplify the notation, we turn to the recursive formulation in the steady state.

C.1.1 Private Firms' Optimality Conditions

The solution of entrepreneurs' profit maximization problem, stated in (6), is characterized by the input demand functions

$$
\begin{aligned}
k(a,z) &= \min\left\{k^*(z),\ \lambda_a a + \lambda_\pi \pi^*(z)\right\} \\
l(a,z) &= \left[\frac{(1-\alpha)\eta\exp(z)}{w}\right]^{\frac{1}{1-(1-\alpha)\eta}} k(a,z)^{\frac{\alpha\eta}{1-(1-\alpha)\eta}}
\end{aligned}
$$

where the unconstrained optimal level of capital and profits given by

$$
\begin{aligned}
k^*(z) &= \left[\frac{\alpha\eta\exp(z)}{R}\right]^{\frac{1}{1-\eta}} \left[\frac{(1-\alpha)}{\alpha}\frac{R}{w}\right]^{\frac{(1-\alpha)\eta}{1-\eta}} \\
l^*(z) &= \left[\frac{(1-\alpha)\eta\exp(z)}{w}\right]^{\frac{1}{1-(1-\alpha)\eta}} k^*(z)^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} \\
\pi^*(z) &= \exp(z)\left[(k^*(z))^{\alpha}\left(l^*(z)\right)^{1-\alpha}\right]^{\eta} - wl^*(z) - Rk^*(z)
\end{aligned}
$$

which is only a function of the firm's productivity z.

C.1.2 Public Firm's Optimality Conditions

The FOCs of the public firm's profit maximization problem are

$$
\alpha\frac{Y_p}{K_p} = R, \qquad (1-\alpha)\frac{Y_p}{L_p} = w
$$

which imply a relation between the public firm's capital to output and prices.

C.1.3 Discussion: Earnings-Based Constraint

We first characterize the earnings-based constraint using equilibrium profits (instead of unconstrained profits, as in (5)). We show that this constraint may admit multiple solutions and usually requires a numerical solution, which raises runtime as model complexity increases (recall that our algorithm iterates over the full agent distribution). We then compare this specification with the baseline (5), highlighting key differences and similarities. The baseline constraint is simpler and substantially faster to compute, though it sacrifices some additional curvature in assets and may understate the role of firm-level financing constraints. Another advantage of the baseline specification is that it is arguably cleaner to calibrate $\{\lambda_a, \lambda_\pi\}$ since it separates the roles of z and a in the constraint.

Characterizing the solution To save notation and make the algebra simpler, we assume the production technology is $y = \exp(z)\, k^{\eta}$, such that profits are $\pi = \exp(z)\, k^{\eta} - Rk$. The borrowing constraint depends on the owner's current assets and equilibrium profits π, i.e.,

$$k \leq \lambda_a a + \lambda_\pi \pi \equiv \underline{k}^{\pi}. \tag{19}$$

The firm profit maximization problem is now

$$\pi(a,z) = \max_k \exp(z)\, k^{\eta} - Rk$$

subject to (19). To characterize the solution we follow these steps:

1. Check if the solution is constrained or unconstrained:
   (a) if $k^* \leq \lambda_a a + \lambda_\pi \pi^* \rightarrow$ unconstrained solution
   (b) if $k^* > \lambda_a a + \lambda_\pi \pi^* \rightarrow$ constrained solution and we proceed to the next step.

2. Next, we impose that the constraint binds

$$k = \lambda_a a + \lambda_\pi \pi.$$

   This implies that profits need to satisfy

$$\pi(a,z): \quad \{\pi = \exp(z)\,(\lambda_a a + \lambda_\pi \pi)^{\eta} - R\,(\lambda_a a + \lambda_\pi \pi)\}.$$

   We can rearrange the terms such that

$$(1 + R\lambda_\pi)\,\pi = \exp(z)\,(\lambda_a a + \lambda_\pi \pi)^{\eta} - R\lambda_a a. \tag{20}$$

   Notice that LHS of (20) is linear in profits and starts from 0 for $\pi \in [0, \pi^*]$ and the LHS is concave, with $\exp(z)\,(\lambda_a a)^{\eta} - R\lambda_a a \geq 0$ at $\pi = 0$ since $k^* > \lambda_a a \geq 0$.

3. The solution to (20) depends on the value of a:
   (a) For $a = 0$ there are **two solutions**: (i) No production, where $\pi = 0$ and $k = 0$ is one solution. (ii) Positive production, where profits are $\pi = \left(\frac{\exp(z)}{1+R\lambda_\pi}\right)^{\frac{1}{1-\eta}} \lambda_\pi^{\frac{\eta}{1-\eta}}$, and the capital is $k = \left(\frac{\exp(z)}{1+R\lambda_\pi}\right)^{\frac{1}{1-\eta}} \lambda_\pi^{\frac{1}{1-\eta}}$ is another solution.
   (b) For $a > 0$ the solution of the problem is **unique** and solves (20) with solution in $\pi \in (0, \pi^*]$, but the solution is **implicitly** defined (this means that we need to solve it numerically).

This earnings-based constraint can admit multiple equilibria at $a = 0$. Even after ruling out the no production equilibrium, we must solve implicitly for constrained profits and capital if $a > 0$, which further increases the computational burden.

Unconstrained solution The unconstrained solution $\{k^*, \pi^*\}$ is

$$k^* = \left(\frac{\eta \exp(z)}{R}\right)^{\frac{1}{1-\eta}}$$

$$\pi^* = \exp(z)^{\frac{1}{1-\eta}} \left(\frac{\eta}{R}\right)^{\frac{\eta}{1-\eta}} (1-\eta)$$

We will use this solution for the next claims.

Comparison between constraints We now compare constraint $\underline{k}^\pi$ to our baseline constraint (5). Aside from the evident point that our baseline constraint is looser, and thus likely understates the role of financing frictions, we make three observations:

- *Observation 1*: Given the distribution of (z, a), the set of constrained and unconstrained firms are the same.

- *Observation 2*: At $a = 0$, productivity affects both constraints in the same way.

- *Observation 3*: Constraint $\underline{k}^\pi$ sensitivity to a is higher and increasing in how constrained are firms, however it is undetermined which constraint is more sensitive relative to z.

The first observation is straightforward. A firm is constrained when $k^* > \lambda_a a + \lambda_\pi \pi^*$ under both constraints. In that sense, they are equally tight. Next, evaluate both constraints at $a = 0$ then the solutions are

$$\lambda_\pi \pi = \lambda_\pi \exp(z)^{\frac{1}{1-\eta}} \left(\frac{\lambda_\pi^\eta}{1 + R\lambda_\pi}\right)^{\frac{1}{1-\eta}}$$

$$\lambda_\pi \pi^* = \lambda_\pi \exp(z)^{\frac{1}{1-\eta}} \left(\frac{\eta}{R}\right)^{\frac{\eta}{1-\eta}} (1-\eta).$$

Notice that productivity affects both constraints in the same way (second observation). Finally, we make a first order perturbation to the profit function implicitly defined by (20). The profits respond to a change in da and dz as

$$\mathrm{d}\pi = \mu\lambda_a \mathrm{d}a + (1 + \mu\lambda_\pi)\, y\mathrm{d}z$$

where μ is the Lagrange multiplier on the borrowing constraint. Then the change in the earnings-based constraint $\underline{k}^\pi$ is

$$\mathrm{d}\underline{k}^\pi = \lambda_a (1 + \lambda_\pi \mu)\, \mathrm{d}a + \lambda_\pi (1 + \mu\lambda_\pi)\, y\mathrm{d}z. \tag{21}$$

On the other hand, the response of the constraint (5), $\underline{k}$, is

$$\mathrm{d}\underline{k} = \lambda_a \mathrm{d}a + \lambda_\pi y^* \mathrm{d}z. \tag{22}$$

On assets, $\underline{k}^\pi$ has greater curvature because an additional term captures how assets also affect profits, which further relaxes the constraint. As firms become unconstrained ($\mu \to 0$), the curvatures coincide. Finally, the relative sensitivity of productivity is ambiguous. Note that $y^* > y$; however, if $\mu > 0$, the second-round effect of productivity on constrained profits raises the sensitivity for $\underline{k}$.

Note that, because the constraint $\underline{k}^\pi$ features feedback between assets a and profits π, it is less clean to calibrate than the baseline $\underline{k}$, where the roles of the two components are separated. For example, under $\underline{k}^\pi$, a higher λ_π also relaxes the constraint for agents with higher a, and a higher λ_a also relaxes the constraint for agents with higher z.

C.2 Competitive Equilibrium

A *competitive stationary equilibrium* in this economy consists of: (i) aggregate prices $\{r, w\}$; (ii) terms of trade in the market for firms given by the price functions of seller j and buyer-owner i meetings $\{p\left(\mathbf{s}^o_{it}, \mathbf{s}^o_{jt}, \kappa_{jt}\right), \underline{p}\left(\mathbf{s}^o_{jt}, \kappa_{jt}\right), \bar{p}\left(\mathbf{s}^o_{it}, \theta_{jt}\right)\}$, and the price functions of seller j and buyer-worker i meetings $\{p\left(\mathbf{s}^w_{it}, \mathbf{s}^o_{jt}, \kappa_{jt}\right), \underline{p}\left(\mathbf{s}^o_{jt}, \kappa_{jt}\right), \bar{p}\left(\mathbf{s}^w_{it}, \theta_{jt}\right)\}$; (iii) firm owners' occupational choice decisions $h(a_{it}, \varepsilon_{it}, \theta_{it})$; (iv) consumption and savings decisions for entrepreneurs $\{c(\mathbf{s}^e_{it}), a'(\mathbf{s}^e_{it})\}$ and for workers $\{c(\mathbf{s}^w_{it}), a'(\mathbf{s}^w_{it})\}$; (v) capital and labor demand functions for private and public firms, $\{k(\mathbf{s}^e_{it}), l(\mathbf{s}^e_{it}), K_{pt}, L_{pt}\}$; and (vi) measures of agents over occupations and idiosyncratic states at DM and CM subperiods characterized by $\{N^o_{dm}(\mathbf{s}^o_{it}), N^w_{dm}(\mathbf{s}^w_{it})\}$ and $\{N^e_{cm}(\mathbf{s}^e_{it}), N^w_{cm}(\mathbf{s}^w_{it})\}$, respectively, such that:

1. In DM, the terms of trade in bilateral meetings solve the Nash bargaining problem.

2. In CM, given prices, households, private, and public firms solve their corresponding optimization problems.

3. Goods market clears, period by period:

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t \tag{23}$$

where

$$\begin{aligned} Y_t &\equiv Y_{pt} + \int y_{it}(\mathbf{s}^e_{it})\ \mathrm{d}N^e_{cm}(\mathbf{s}^e_{it}) \\ C_t &\equiv \int c(\mathbf{s}^e_{it})\ \mathrm{d}N^e_{cm}(\mathbf{s}^e_{it}) + \int c(\mathbf{s}^w_{it})\mathrm{d}N^w_{cm}(\mathbf{s}^w_{it}) \\ K_t &\equiv K_{pt} + \int k(\mathbf{s}^e_{it})\ \mathrm{d}N^e_{cm}(\mathbf{s}^e_{it}). \end{aligned}$$

4. Labor market clears, period by period:

$$L_{pt} + \int l(\mathbf{s}^e_{it})\ \mathrm{d}N^e_{cm}(\mathbf{s}^e_{it}) = \int \exp(\varepsilon_{it})\ \mathrm{d}N^w_{cm}(\mathbf{s}^w_{it}). \tag{24}$$

5. The budget constraint of the financial intermediary, in (7), is satisfied period by period.

6. The measures over types and states satisfy

$$\begin{aligned} \int \mathrm{d}N^o_{dm}(\mathbf{s}^o_{it}) + \int \mathrm{d}N^w_{dm}(\mathbf{s}^w_{it}) &= 1 \\ \int \mathrm{d}N^e_{cm}(\mathbf{s}^e_{it}) + \int \mathrm{d}N^w_{cm}(\mathbf{s}^w_{it}) &= 1 \end{aligned}$$

and are consistent with a recursive equilibrium mapping dictated by prices and trades in the market for firms, households' optimal choices, and the stochastic processes for firms' qualities, workers' labor efficiencies, and sellers' preferences shocks. The stationary equilibrium implies a fixed distribution over time (fixed point).

We solve for the stationary equilibrium of this model by approximating the value functions on a finite state space for which we solve all the possible matches and trading prices, as well as agents' and firms' optimal choices.

C.3 Computational Solution

The model poses two computational challenges. First, we need to solve for the terms of trade in the market for firms. Second, the distribution of agents enters as an explicit state variable, since we use it to compute the value functions before the DM $\{V^o, V^w\}$. In this section, we describe the algorithm, the solution of the market for firms, and the computation of the ergodic distribution.

Note that the FOCs of the public firm give us a relation between K_p/Y_p, w and r. Both K_p and L_p are determined as residuals from the market clearing conditions of capital and labor, thus we can obtain w as a function of r. This considerably simplifies the solution method of our baseline model as we only need to solve for one input price, r, in addition to equilibrium prices in the market for firms.

C.3.1 Algorithm

The equilibrium objects we need to solve for are

$$\left\{r, \underline{p}, \overline{p}, p, V^o, W^o, V^w, W^w, n^o_{dm}, n^w_{dm}, n^o_{cm}, n^e_{cm}, P_{dm}, P_{cm}\right\}$$

where $\underline{p}$ are sellers' minimum prices, $\overline{p}$ are buyers' maximum prices, p are the Nash bargaining prices, n are the probability densities across states, and P are the transition probability matrices (TPMs). We solve for these objects using the following algorithm:

Iteration on prices

0. Propose an initial guess for r.
1. Given r, solve the model (in partial equilibrium).

   Iteration on distributions

   1.0. Propose an initial guess for $\{n^o_{dm}, n^w_{dm}\}$. We use the ergodic distribution of the model without the market for firms.

   1.1. Given $\{n^o_{dm}, n^w_{dm}\}$, We can solve for $\{V^o, W^o, V^w, W^w\}$ using standard value function iteration techniques:

   Iteration on value functions

   1.1.0. Propose an initial guess for $\{W^o, W^w\}$.

   1.1.1. Solve for the prices in the market for firms $\{\underline{p}, \overline{p}, p\}$. For computational efficiency, we implement this step less often than the value function iteration.

   1.1.2. Solve the DM problem: get $\{V^o, V^w\}$

   1.1.3. Solve the CM problem: apply Bellman operator to compute $\{W^o, W^w\}$ and get policy functions $\{a', h\}$.

1.1.4. Iterate $\{W^o, W^w\}$ until convergence.

1.2. Update $\{n^o_{dm}, n^w_{dm}\}$.

1.3. Iterate $\{n^o_{dm}, n^w_{dm}\}$ until convergence.

2. Update r such that the capital market clears.

3. Return to 1. until r converges.

C.3.2 Solving for Prices in the Market for Firms

First, for each potential seller with states $(a, \varepsilon, \theta, \kappa)$, we solve for the sellers' minimum price by finding $\underline{p}(a, \varepsilon, \theta, \kappa)$ that implies that the sellers' surplus, defined in (8) and (9), is equal to zero. Using (10), which defines the preference shock utility transfer, the seller's surplus is equal to zero if

$$W^w(a + \kappa\underline{p}, \varepsilon) = W^o(a, \varepsilon, \theta)$$

which implicitly defines $\underline{p}(a, \varepsilon, \theta, \kappa)$.

Second, for each potential firm quality θ_j, we solve for buyers' maximum price $\overline{p}(\mathbf{s}_i, \theta_j)$, where $\mathbf{s}_i \in \{\mathbf{s}^o_i, \mathbf{s}^w_i\}$ depending on whether the buyer is a firm owner or a worker. Note that the buyer's maximum price does not depend on the seller's assets, ability, or preference shock. We compute the buyer's maximum price by solving for $\overline{p}$ that sets the buyer's surplus, defined in (8) and (9), to zero. For the case of current business owners with states $\mathbf{s}^o_i = (a_i, \varepsilon_i, \theta_i)$, note that they will never buy a lower quality firm $\theta_j < \theta_i$. For those cases, we set the buyers' maximum price equal to zero.

Having computed the sellers' minimum prices, $\underline{p}$, and the buyers' maximum prices $\overline{p}$, we can identify the matches with positive gains from trade using the sufficient condition presented in (11). Then, for each potential match of a seller j, with states $(\mathbf{s}^o_j, \kappa_j)$, and a buyer i, with states $\mathbf{s}_i \in \{\mathbf{s}^o_i, \mathbf{s}^w_i\}$, such that there are positive gains from trade, given by $\underline{p}(\mathbf{s}^o_j, \kappa_j) < \overline{p}(\mathbf{s}_i, \theta_j)$, we approximate the Nash bargaining price, defined in (12), as

$$p_{ijt} \equiv p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt}) \approx \chi\underline{p}(\mathbf{s}^o_j, \kappa_j) + (1 - \chi)\overline{p}(\mathbf{s}_i, \theta_j) \tag{25}$$

where χ is buyers' bargaining power. In our numerical simulations, we found that computing the price using (25) is an extremely accurate approximation to the Nash bargaining price obtained from solving the maximization problem (12) while delivering improvements in computational time of several orders of magnitude.

C.3.3 Transitions and Stationary Distribution

Define the densities across states in DM and CM subperiods as

$$n_{dm} = \begin{bmatrix} n^o_{dm} \\ n^w_{dm} \end{bmatrix} \text{ and } n_{cm} = \begin{bmatrix} n^o_{cm} \\ n^w_{cm} \end{bmatrix}$$

where n^o_{dm} and n^o_{cm} are vectors of size N_o and n^w_{dm} and n^w_{cm} are vectors of size N_w. N_o and N_w are the basis functions grid sizes denoting the number of (a, ε, θ) and (a, ε) combinations, respectively. Here $\sum_i n_{dm} = 1$, thus, $\sum_i n^o_{dm} = m^o_{dm}$ and $\sum_i n^w_{dm} = m^w_{dm} = (1 - m^o_{dm})$.

Then, the TPMs between DM and CM and CM and DM_{+1} solve

$$(n_{cm})^\intercal = (n_{dm})^\intercal P_{dm}, \qquad (n'_{dm})^\intercal = (n_{cm})^\intercal P_{cm}$$

where $(.)^\intercal$ denotes the transpose operator.

We can divide the TPM in blocks differentiating between the two types of agents:

$$P_{dm} = \begin{bmatrix} P_{dm}^{oo} & P_{dm}^{ow} \\ P_{dm}^{wo} & P_{dm}^{ww} \end{bmatrix} \text{ and } P_{cm} = \begin{bmatrix} P_{cm}^{oo} & P_{cm}^{ow} \\ P_{cm}^{wo} & P_{cm}^{ww} \end{bmatrix}$$

where P_{dm}^{oo} captures the transitions of firms' owners that bought another firm or didn't trade, P_{dm}^{ow} is for owners that sold their firm, P_{dm}^{wo} for workers who bought a firm and P_{dm}^{ww} for workers who didn't trade. Regarding CM TPMs, P_{cm}^{oo} is for business owners who operated the firm, P_{cm}^{ow} for owners who didn't operate and went to the labor market or those that did but received the exogenous exit shock γ, P_{cm}^{wo} for workers who received the startup shock ζ, P_{cm}^{ww} for workers that didn't. Note that besides changes in the exogenous shocks, asset holdings also change due to payments in the market for firms and due to savings in CM.

Stationarity requires that

$$n_{dm}^\intercal = n_{dm}^\intercal P_{dm} P_{cm}$$

or

$$\left[I - (P_{dm} P_{cm})^\intercal\right] n_{dm} = 0$$

which implies that we can solve for n_{dm} by computing the eigenvector of $(P_{dm} P_{cm})^\intercal$ associated with the unit eigenvalue, normalized such that $\sum_i n_{dm}(i) = 1$.

C.4 Motives for Trading Firms: Shapley-Owen Decomposition

This appendix describes the Shapley-Owen (S-O) decomposition used in Table 4 to quantify the different moments for trading firms. Given the nonlinear interactions among these mechanisms, measuring their marginal effects by simply shutting down each motive is problematic, as it also captures interactions with the other motives and, hence, the order in which the motives are turned off matters. To account for this, we employ the S-O decomposition for binary variables that averages the marginal effect of shutting down each of the first three motives for trade across all possible permutations.

Shapley-Owen Decomposition for Binary Variables Let $N = \{1, \dots, n\}$ index n binary variables, where each variable $i \in N$ takes values in $\{0, 1\}$. For any subset $S \subseteq N$, the outcome function $f(S)$ is the value of the model outcome when variables in S are set to 1, and all variables in $N \setminus S$ are set to 0. The S-O contribution of variable $i \in N$ is

$$\phi_i = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!\,(n - |S| - 1)!}{n!} \left[f(S \cup \{i\}) - f(S)\right]. \tag{26}$$

where term $f(S \cup \{i\}) - f(S)$ represents the marginal contribution of i when added to S. The weighting term is the fraction of permutations in which the set S precedes variable i.

In our setup, for f being the frequency of trade in the model, we have three binary variables, corresponding to the cases where we shut down each of the first three motives for trade in the following order: no preference shocks ($\xi = 0$), no heterogeneous abilities ($\omega = 0$), and no borrowing constraints ($(\lambda_a, \lambda_\pi) \to \infty$). The second column of Table C.1 reports the eight possible combinations of these three binary variables, where $(0,0,0)$ is our baseline model with all being active, and $(1,1,1)$ is the case where the first three motives are shut down, and hence only risk aversion and incomplete markets are active.

Using (26), the S-O contribution for each of the first three motives for trade is

$$\begin{aligned}
\phi_{\text{No pref. shocks}} &= \frac{1}{3}(f_{100} - f_{000}) + \frac{1}{6}(f_{110} - f_{010}) + \frac{1}{6}(f_{101} - f_{001}) + \frac{1}{3}(f_{111} - f_{011}) \\
\phi_{\text{No het. ability}} &= \frac{1}{3}(f_{010} - f_{000}) + \frac{1}{6}(f_{110} - f_{100}) + \frac{1}{6}(f_{011} - f_{001}) + \frac{1}{3}(f_{111} - f_{101}) \\
\phi_{\text{No borr. const.}} &= \frac{1}{3}(f_{001} - f_{000}) + \frac{1}{6}(f_{101} - f_{100}) + \frac{1}{6}(f_{011} - f_{010}) + \frac{1}{3}(f_{111} - f_{110})
\end{aligned}$$

where the trade rate, f, under all possible combinations are reported in Table C.1.

The efficiency property of the S-O decomposition implies that $\sum_i \phi_i = f_{111} - f_{000}$, which is a negative given that the frequency is higher when all motives are active. Hence, we can decompose the trade rate in the baseline as the sum of four terms

$$\underbrace{-\phi_{\text{No pref. shocks}}/f_{000}}_{\text{Pref. shocks}} \quad \underbrace{-\phi_{\text{No het. ability}}/f_{000}}_{\text{Het. ability}} \quad \underbrace{-\phi_{\text{No borr. const.}}/f_{000}}_{\text{Borr. const.}} \quad \underbrace{+f_{111}/f_{000}}_{\text{Risk and Inc. Markets}} = 1 \quad (27)$$

which correspond to the shares reported in Table 4. Note that we measure the importance of risk and incomplete markets as the residual once the first three motives are shut down.

Table C.1: Trade Rate Decomposition

		Combination	Trade rate	Relative
Baseline		(0,0,0)	3.0%	1.00
No preference shocks	$\xi = 0$	(1,0,0)	2.0%	0.67
No heterogeneous ability	$\omega = 0$	(0,1,0)	1.9%	0.63
No borrowing constraints	$(\lambda_a, \lambda_\pi) \to \infty$	(0,0,1)	1.6%	0.52
No pref. shocks, no het. ability		(1,1,0)	1.2%	0.40
No het. ability, no borr. const.		(0,1,1)	1.1%	0.38
No pref. shocks, no borr. const.		(1,0,1)	1.0%	0.33
No pref. shocks, no borr. const., no het. ability		(1,1,1)	0.6%	0.21

Notes: Steady-state comparisons of the trade rate in the market for firms (annual frequency) under different parameterizations. Relative is the ratio of each trade rate to the Baseline model. No preference shocks turn off the alternative motives to trade firms by setting $\mathbb{E}[\xi] = 0$ and $Var[\xi] = 0$. No heterogeneous abilities assumes the incidence of owners' ability on firms' productivity is zero ($\omega = 0$), and we normalize firm TFP to remove level effects. No borrowing constraint assumes $(\lambda_a, \lambda_\pi) \to \infty$.

C.5 Post-Trade Firm Dynamics: Analytical Results

In this appendix, we derive analytical predictions for the post-trade dynamics of the main variables analyzed in Section 2.2, when firms' trade is driven by: (i) financial frictions and heterogeneity in owners' wealth, (ii) heterogeneity in owners' ability, (iii) heterogeneity in cost-cutting capabilities, and (iii) heterogeneity in owners' span-of-control. First, we demonstrate how profits change with lower financial frictions, improved ability, greater cost-cutting capacity, and increased span of control. Then, we present our main proposition, which characterizes the evolution of ARPK, ARPL, profitability, and leverage under these different motives. Results are summarized in Table C.2.

Table C.2: Post-Trade Firm Dynamics in Data and Proposition 1

	y	k	y/k	y/wl	π/k	b/k
Data	(+)	(+)	(−)	(=)	(−)	(−)
Theory						
Financial Frictions	(+)	(+)	(−)	(=)	(−)	(−)
Heterogeneous Ability: Constrained	(+)	(+)	(+)	(=)	(+)	(+)
Heterogeneous Ability: Unconstrained	(+)	(+)	(=)	(=)	(=)	(+)
Cost-Cutting	(+)	(+)	(−)	(=)	(+)	(+)
Span-of-Control	(+)	(+)	(−)	(−)	(−)	(+)

Source: Orbis Historical and theory results from Proposition 1 in Appendix C.5.
Notes: (+) indicates that the variable increases after trade, (−) decreases, and (=) no change. Heterogeneous Ability: Constrained refers to cases in which the firm is initially constrained or becomes constrained after the trade. Heterogeneous Abilities (Unconstrained) refers to case in which firms are unconstrained before and after trade. In the Cost-Cutting case, we assume the firm is initially constrained and that $\lambda_\pi > 0$ but relatively low. In the Span-of-Control case, we assume the firm is unconstrained.

To study these alternative motives, we modify the private firms' problem in (6) by assuming that firms use a production technology that combines capital, labor, and ability ε such that output is $y = z(\theta, \varepsilon)\left(k^\alpha l^{1-\alpha}\right)^\nu$ where $\nu \in (0, 1)$ captures the production scale over labor and capital, and $z(\theta, \varepsilon)$ is a weakly increasing function on firm quality θ and ability ε. To capture cost-cutting motives to trade, we assume the firm faces a fixed cost $\phi(m) \geq 0$, which is weakly decreasing in cost-cutting capacity m. Agents can vary across $\{a, \varepsilon, m, \nu\}$ and firms across θ.

Firms hire labor at wage w and rent capital at rate R. The amount of capital firms can use for production is limited by $k \leq \lambda_a a + \lambda_\pi \pi^*(\theta, \varepsilon, m, \nu) \equiv \underline{k}(a, \theta, \varepsilon, m, \nu)$ where $\pi^*(\theta, \varepsilon, m, \nu)$ are the unconstrained profits. Thus, in this setup, the firm profit function is then given by

$$\pi(a, \theta, \varepsilon, m, \nu) = \max_{l, k \leq \underline{k}(a, \theta, \varepsilon, m, \nu)} z(\theta, \varepsilon)\left(k^\alpha l^{1-\alpha}\right)^\nu - wl - Rk - \phi(m). \tag{28}$$

From the FOC, we get the capital and labor demand functions

$$k = \left[\frac{\nu\alpha z l^{(1-\alpha)\nu}}{R+\mu}\right]^{\frac{1}{1-\alpha\nu}}$$
$$l = \left[\frac{z\nu(1-\alpha)k^{\alpha\nu}}{w}\right]^{\frac{1}{1-(1-\alpha)\nu}},$$

where μ is the Lagrange multiplier on the capital borrowing constraint $k \leq \underline{k}$. In addition, firms' ARPK, ARPL, and profitability are

$$\frac{y}{k} = \frac{R+\mu}{\nu\alpha} \tag{29}$$
$$\frac{y}{wl} = \frac{1}{\nu(1-\alpha)} \tag{30}$$
$$\frac{\pi}{k} = \left[\frac{1-\nu(1-\alpha)}{\nu\alpha}\right](R+\mu) - R - \frac{\phi(m)}{k} \tag{31}$$

Due to financial frictions, the ARPK and profitability are potentially distorted by the multiplier $\mu > 0$. To simplify the analysis of the span-of-control case in Lemma 1 and Proposition 1, we impose Assumption 1. This restriction implies that profits and output are increasing in ν.

Assumption 1. *We restrict productivity, prices, and parameters to be such that:*

$$\nu > z^{-1}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}.$$

Lemma 1. *Firm profits $\pi(a,\theta,\varepsilon,m)$ are:*

(i) increasing in the owner's wealth a if the firm is initially constrained

(ii) increasing in ability ε and cost-cutting capacity m

(iii) increasing in the span of control ν if Assumption 1 holds and the firm is initially unconstrained

Proof. (Lemma 1) Trivially, if the unconstrained optimal capital choice k^* is lower than $\underline{k}$, then profits don't depend on a. On the other hand, if the firm is constrained, meaning that $k^* > \underline{k}$, then $k = \underline{k}$, and since $\underline{k}$ is increasing in a, then a higher level of a results in strictly higher profits, as profits increase monotonically for values of $k < k^*$. Next, we study how profits change with ε, m, and ν. Denote $\left(\tilde{k},\tilde{l}\right)$ the optimal capital and labor choices for (a,θ,ε,m) states. First, notice that z is increasing in ε, then for $\varepsilon' > \varepsilon$ it holds that

$$\begin{aligned} z(\theta,\varepsilon)\left(\left(\tilde{k}\right)^{\alpha}\left(\tilde{l}\right)^{1-\alpha}\right)^{\nu} - w\tilde{l} - r\tilde{k} &< z\left(\theta,\varepsilon'\right)\left(\left(\tilde{k}\right)^{\alpha}\left(\tilde{l}\right)^{1-\alpha}\right)^{\nu} - w\tilde{l} - r\tilde{k} \\ &\leq \max_{l,k\leq\bar{k}} z\left(\theta,\varepsilon'\right)\left(k^{\alpha}l^{1-\alpha}\right)^{\nu} - wl - rk \end{aligned}$$

showing that greater ability implies greater profits. The analogous argument applies to show that π is increasing in m since cost function $\phi(m)$ is decreasing in m. Finally, consider the case where the firm is unconstrained and Assumption 1 holds, then greater span-of-control $\nu' > \nu$ increases profits π. This stems from Assumption 1 that restricts the parameter space to values of ν where the unconstrained profits are increasing in ν. □

Lemma 1 shows that firms have higher profits when they are transferred to owners with greater wealth a, higher ability ε, greater cost-cutting capacity m, or a larger span of control ν, which suggests that, all else equal, these changes in ownership can generate gains from trade. Next, in our main proposition, we characterize how ARPL, ARPK, profitability, and leverage change after a firm is traded under four motives: (i) financial frictions, with the buyer holding more assets a; (ii) differences in ability, with the buyer having higher ε; (iii) improved cost-cutting capacity, with the buyer having higher m; and (iv) a larger span of control, with the buyer operating with a higher ν.

Proposition 1. *(Firms After Trade)*

Consider the problem of a firm that solves Equation (28):

(1) ***Financial Frictions.*** *If the buyer has more wealth a and the firm is constrained when sold, then the firm's ARPK, profitability, and leverage decrease, while ARPL remains constant.*

(2) ***Heterogeneous Ability.*** *If the buyer has higher ability ε and the firm remains unconstrained after the trade, then ARPK, ARPL, and profitability remain constant, and leverage increases. In addition, if the firm is initially constrained or becomes constrained after the trade, then ARPK, profitability, and leverage all increase.*

(3) ***Cost-Cutting.*** *If the buyer has higher cost-cutting capacity m and the firm is unconstrained or constrained with $\lambda_\pi = 0$, then ARPK, ARPL, and leverage remain constant, while profitability rises. By contrast, if the firm is initially constrained and $\lambda_\pi > 0$, ARPK falls, the change in profitability is ambiguous (declining when k/y is sufficiently low and rising otherwise) and leverage increases.*

(4) ***Span-of-Control.*** *If the buyer has a larger ν, the firm is unconstrained, and Assumption 1 holds, then ARPL, ARPK, and profitability decrease, while leverage increases.*

Proof. (Proposition 1) **Financial Frictions.** Consider the case where firms are constrained, then $k = \lambda_a a + \lambda_\pi \pi^*$ and output is $y = \left[z\left(\frac{\nu(1-\alpha)}{w}\right)^{\nu(1-\alpha)}(\lambda_a a + \lambda_\pi \pi^*)^{\nu\alpha}\right]^{\frac{1}{1-(1-\alpha)\nu}}$, which are both increasing in a. Moreover, the ARPK, ARPL, and profitability $\frac{\pi}{k}$ are

$$\frac{y}{k} = \left[z\left(\frac{\nu(1-\alpha)}{w}\right)^{\nu(1-\alpha)}\right]^{\frac{1}{1-(1-\alpha)\nu}}(\lambda_a a + \lambda_\pi \pi^*)^{-\frac{1-\nu}{1-(1-\alpha)\nu}}$$

$$
\begin{aligned}
\frac{y}{wl} &= \frac{1}{\nu\left(1-\alpha\right)} \\
\frac{\pi}{k} &= \frac{y}{k}\left(1-\nu\left(1-\alpha\right)\right)-R,
\end{aligned}
$$

where ARPK and profitability are decreasing, and ARPL is constant in the owner's wealth a. To make it comparable to the motives in our baseline model, we also assume $\phi(m)=0$, so that $\pi^{*}\geq 0$. Moreover, leverage is $b/k=\frac{k-a}{k}=\frac{(\lambda_a-1)a+\lambda_\pi\pi^{*}}{\lambda_a a+\lambda_\pi\pi^{*}}$, which, given $\pi^{*}\geq 0$ it is also (weakly) decreasing in a and is the upper-bound on unconstrained leverage.

Finally, when the firm is constrained at the time of purchase ($\mu>0$) but becomes unconstrained afterward ($\mu=0$), it follows directly from the baseline solution that ARPK and profitability decline, while ARPL remains unchanged. Leverage is strictly decreasing in this case, since once the firm reaches its optimal scale, the additional assets of the new owner reduce leverage.

Heterogeneous Abilities. The optimal capital and output choices are

$$
\begin{aligned}
k &= \max\left\{\left[z\left(\frac{\alpha\nu}{R}\right)^{1-\nu(1-\alpha)}\left(\frac{\nu\left(1-\alpha\right)}{w}\right)^{\nu(1-\alpha)}\right]^{\frac{1}{1-\nu}},\lambda_a a+\lambda_\pi\pi^{*}\right\} \\
y &= \left[z\left(\frac{\nu\left(1-\alpha\right)}{w}\right)^{\nu(1-\alpha)}k^{\alpha\nu}\right]^{\frac{1}{1-\nu(1-\alpha)}},
\end{aligned}
$$

which are both increasing in z, thus in ε. If the firm remains unconstrained after trade, then

$$
\begin{aligned}
\frac{y}{lw} &= \frac{1}{\nu\left(1-\alpha\right)} \\
\frac{y}{k} &= \frac{R}{\nu\alpha} \\
\frac{\pi}{k} &= \left(\frac{1-\nu}{\nu\alpha}\right)R,
\end{aligned}
$$

are all constant, and leverage $(k-a)/k$ is increasing since k increasing in ε. To make it comparable to the motives in our baseline model, we also assume $\phi(m)=0$, so that $\pi^{*}\geq 0$. On the other hand, if the firm is constrained and $a>0$ then $k=\lambda_a a+\lambda_\pi\pi^{*}$. Using the constrained solution, the elasticity of the ARPK to TFP z can be written as

$$
\frac{\mathrm{d}\ln\frac{y}{k}}{\mathrm{d}\ln z}=\frac{1}{1-\nu\left(1-\alpha\right)}-\frac{1-\nu}{1-\nu\left(1-\alpha\right)}\frac{\mathrm{d}\ln\left(\lambda_a a+\lambda_\pi\pi^{*}\right)}{\mathrm{d}\ln z},
$$

where unconstrained profits are $\pi^{*}=(1-\nu)\left[z\left(\frac{\alpha\nu}{R}\right)^{\alpha\nu}\left(\frac{(1-\alpha)\nu}{w}\right)^{(1-\alpha)\nu}\right]^{\frac{1}{1-\nu}}$. From the unconstrained profits it is easy to see that the elasticity $\mathrm{d}\ln\pi^{*}/\mathrm{d}\ln z=1/(1-\nu)$. Thus, the elasticity of constrained capital to productivity can be written as

$$
\frac{\mathrm{d}\ln\left(\lambda_a a+\lambda_\pi\pi^{*}\right)}{\mathrm{d}\ln z}=\frac{\lambda_\pi\pi^{*}}{\lambda_a a+\lambda_\pi\pi^{*}}\frac{\mathrm{d}\ln\pi^{*}}{\mathrm{d}\ln\mathrm{d}z}=\left(\frac{\lambda_\pi\pi^{*}}{\lambda_a a+\lambda_\pi\pi^{*}}\right)\left(\frac{1}{1-\nu}\right).
$$

Combining these expressions, we can find that

$$\frac{\mathrm{d}\ln\frac{y}{k}}{\mathrm{d}\ln z}=\frac{1}{1-\nu\left(1-\alpha\right)}\left(1-\frac{\lambda_{\pi}\pi^{*}}{\lambda_{a}a+\lambda_{\pi}\pi^{*}}\right)>0,$$

since $a>0$, the elasticity is strictly positive. Notice that for $a=0$ the elasticity is zero. Thus, ARPK is increasing in z, which implies that profitability is also increasing, since $\frac{\pi}{k}=\frac{y}{k}\Big(1-\nu(1-\alpha)\Big)-R$. Lastly, because π^{*} is increasing in z, capital k also rises, so leverage increases.

Finally, it is straightforward to show that ARPK and profitability increase if the firm is initially unconstrained ($\mu=0$) but, with a higher z, becomes constrained ($\mu>0$). In addition, the new unconstrained capital level is higher than the previous one, so even though the firm is now constrained, the amount of capital used is larger than before (i.e., $k_0^{*}<k_1<k_1^{*}$, where k_0^{*} is the initial unconstrained level, and k_1^{*} and k_1 are the new unconstrained and constrained levels, respectively). Thus, leverage also increases.

Cost-Cutting. First, from the unconstrained solution we have that y/k and y/wl remain constant. Since $\phi(m)$ decreases in m and the optimal capital choice does not depend on m, profitability π/k rises and leverage stays constant. Next, we examine the case in which the firm is initially constrained. If the firm is constrained and $\lambda_{\pi}=0$, capital does not respond to a change in m, so ARPK and leverage remain constant and profitability rises as in the unconstrained case. By contrast, when the firm is constrained and $\lambda_{\pi}>0$, the response of capital to an increase in m is

$$\frac{\mathrm{d}k}{\mathrm{d}m}=-\lambda_{\pi}\phi^{'}\left(m\right)>0,$$

where $\phi^{'}\left(m\right)<0$ is the derivative of the cost function to m. This shows that capital increases with m and the ARPK declines as well, since it is a decreasing function of k when firms are constrained. Intuitively, this happens because a higher m reduces the cost ϕ, raises π^{*}, and relaxes financing frictions. Notice that for $\frac{\pi}{k}=\frac{y}{k}\left(1-\nu\left(1-\alpha\right)\right)-R-\frac{\phi(m)}{k}$ we have that since y/k declines but $\frac{\phi(m)}{k}$ also declines ($\phi(m)$ lower and k higher), then the change a priori is ambiguous. Let characterize locally the derivative of $\frac{\mathrm{d}(\pi/k)}{\mathrm{d}m}=\left(1-\nu\left(1-\alpha\right)\right)\frac{\mathrm{d}(y/k)}{\mathrm{d}m}-\frac{\mathrm{d}(\phi(m)/k)}{\mathrm{d}m}$. Using $\frac{\mathrm{d}(y/k)}{\mathrm{d}m}=\left(\frac{1-\nu}{1-\nu(1-\alpha)}\right)\lambda_{\pi}\frac{y}{k}\frac{\phi^{'}(m)}{k}$ and $frac\mathrm{d}\left(\phi\left(m\right)/k\right)\mathrm{d}m=\left[\frac{k+\lambda_{\pi}\phi(m)}{k^{2}}\right]\phi^{'}\left(m\right)$ we find that

$$\frac{\mathrm{d}\left(\pi/k\right)}{\mathrm{d}m}=\left[\left(1-\nu\right)\lambda_{\pi}\frac{y}{k}-\left(1+\frac{\lambda_{\pi}\phi\left(m\right)}{k}\right)\right]\frac{\phi^{'}\left(m\right)}{k},$$

such that the sign of this derivative depends on how constrained the firm is and the importance of λ_{π}. Profitability decreases if $k<\lambda_{\pi}\left[\left(1-\nu\right)y-\phi\left(m\right)\right]$, which implies k is very low relative to y and also λ_{π} is relatively large. Otherwise, profitability will increase as in the unconstrained case. Lastly, since k increases with m, then leverage would increase as well.

Span-of-Control. Finally, we consider the case where span-of-control ν increases. Additionally, we assume that Assumption 1 holds and for simplicity we assume $\phi\left(m\right)=0$. From the unconstrained output and capital equations derived above, an increase in ν results in higher output and capital. Furthermore, ARPL, ARPK, and profitability decrease with ν. Lastly, an increase in the span of control leads to higher leverage, since k is higher. $\square$

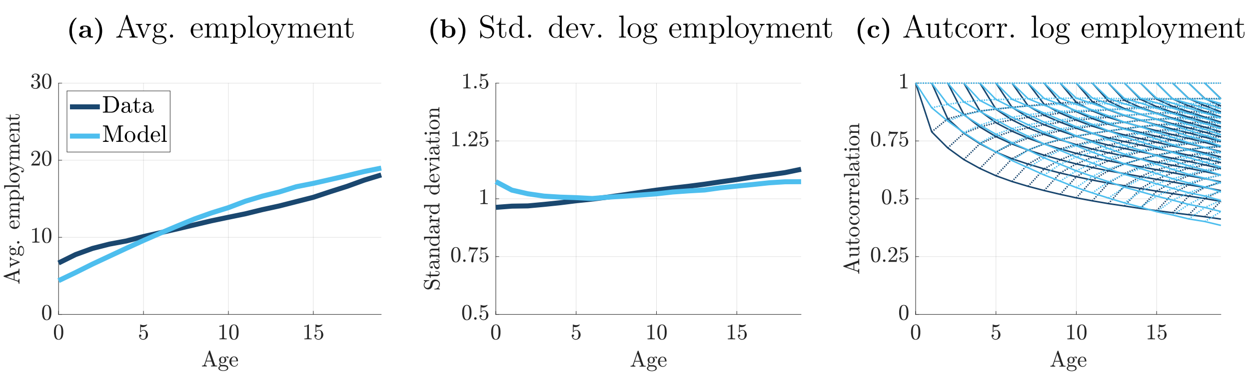
C.6 Additional Results

Table C.3: Startup Income Transition: PSID Data

Dependent variable:	$\log y_{it}$		$\log \pi_{it}$	
	(1)	(2)	(3)	(4)
$\log w_{i,t-2}$	0.53	0.44	0.52	0.47
	(0.41, 0.66)	(0.26,0.62)	(0.40, 0.65)	(0.29,0.65)
$\log a_{i,t-2}$		0.21		0.18
		(0.10,0.33)		(0.07,0.28)
Observations	330	219	355	232
Age FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓

Source: 1999-2019 PSID
The estimates are conditional on those that are entrepreneurs at t, but they are not at $t-2$. 95% confidence intervals in parenthesis. See the text and Appendix A for details.

Figure C.1: Employment Dynamics by Firm's Age



Source: U.S. Census LBD data from Sterk, Sedláček, and Pugsley (2021), and model simulated data.
Notes: Average employment in the model is normalized to match the one in the data.

Table C.4: Buyers' and Sellers' Wealth: Data and Model

Non-Business Wealth (a_{it})	Data	Model
Buyer/Avg. HH	2.71	2.08
Buyer/Avg. Entrepreneur	0.79	0.57
Seller/Avg. HH	1.88	1.59
Seller/Avg. Entrepreneur	0.58	0.44
Recent Buyer/Seller	1.36	1.31

Source: 1989-2016 SCF; 1999-2019 PSID
Notes: Buyers' and sellers' non-business wealth relative to entrepreneurs, along with the buyer-seller wealth ratio, using the SCF, the PSID, and the model-simulated data. The buyer-seller ratio is computed from the respective ratios relative to entrepreneurs. In both the data and the model, buyers' wealth is measured within one year of the purchase (a_{t+1} with trade event between $[t, t+1]$ window), while sellers' wealth is measured before the firm is sold, within a two-year window (a_{t-2} with trade event between $[t-2, t]$ window). See the text and Appendix A for details.

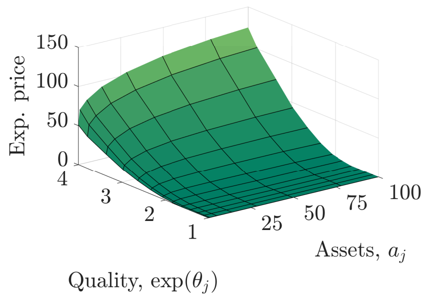
Table C.5: Income and Wealth Shares: Data and Model

	Data	Model		Data	Model
(a) All Households					
Income Distribution			*Wealth Distribution*		
Top 1	0.22	0.14	Top 1	0.33	0.26
Top 5	0.39	0.35	Top 5	0.60	0.53
Top 10	0.49	0.39	Top 10	0.72	0.70
Bottom 75	0.31	0.34	Bottom 75	0.13	0.11
Bottom 50	0.12	0.14	Bottom 50	0.02	0.02
Bottom 25	0.02	0.04	Bottom 25	0.00	0.00
(b) Entrepreneurs					
Income Distribution			*Wealth Distribution*		
Top 1	0.23	0.12	Top 1	0.24	0.12
Top 5	0.44	0.33	Top 5	0.45	0.32
Top 10	0.57	0.48	Top 10	0.60	0.48
Bottom 75	0.24	0.28	Bottom 75	0.18	0.29
Bottom 50	0.10	0.10	Bottom 50	0.05	0.11
Bottom 25	0.03	0.02	Bottom 25	0.01	0.03

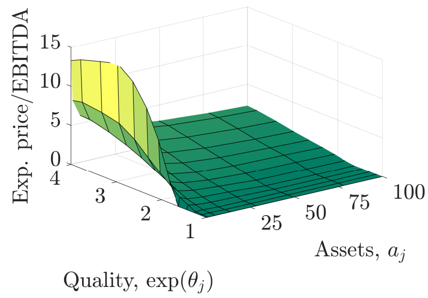
Source: 2007 SCF.
Notes: Wealth ($a + \mathbb{E}[p]$) includes business wealth.

Figure C.2: Expected Price and Price/EBITDA Ratios in the Market for Firms

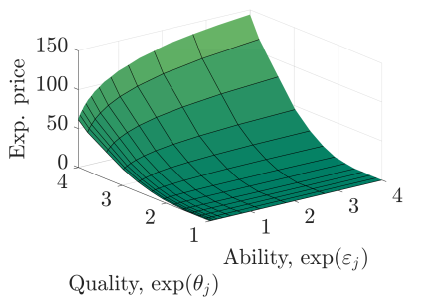
(a) $\mathbb{E}$[price], (a, θ)



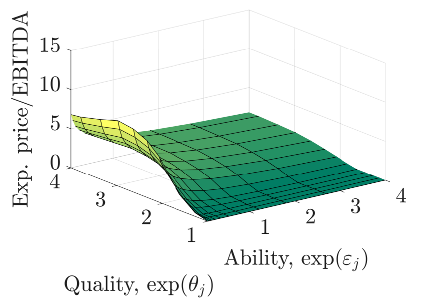
(b) $\mathbb{E}$[price/EBITDA], (a, θ)



(c) $\mathbb{E}$[price], (ε, θ)



(d) $\mathbb{E}$[price/EBITDA], (ε, θ)



Notes: Expected price $\mathbb{E}_{\mathbf{s}_{it},\kappa_{jt}}[p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt})]$, and price-to-EBITDA ratio $\mathbb{E}_{\mathbf{s}_{it},\kappa_{jt},e_{jt}}[p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt})/(\pi(\mathbf{s}^e_{jt}) + Rk(\mathbf{s}^e_{jt}))]$, in sellers' state space $\mathbf{s}^o_{jt} = (a_{jt}, \varepsilon_{jt}, \theta_{jt})$. Expected values are computed by integrating over all potential buyers $\mathbf{s}_{it} \in \{\mathbf{s}^o_{it}, \mathbf{s}^w_{it}\}$ and preference shocks κ_j, over sellers' ability ε_{jt} in panels (a,b), and over sellers' assets a_{jt} in panels (c,d).

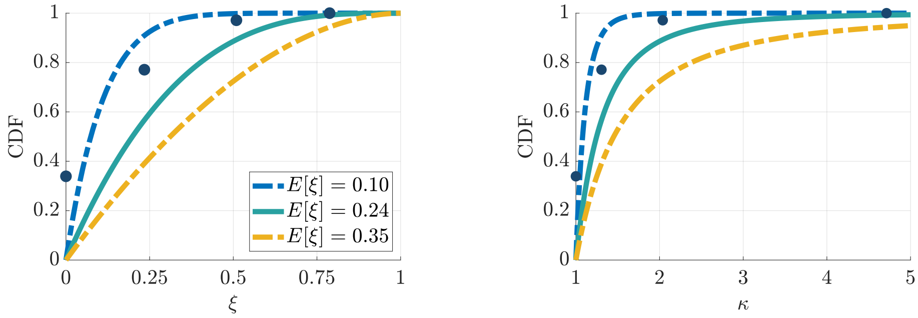
Table C.6: Buyers' and Sellers' Average Assets and Ability

	Assets, a_i	Ability, $\exp(\varepsilon_i)$
Avg. Buyer/Avg. Seller	2.06	2.28
Avg. Buyer/Avg. Household	2.99	1.55
Avg. Buyer/Avg. Entrepreneur	0.82	1.61
Avg. Seller/Avg. Household	1.45	0.68
Avg. Seller/Avg. Entrepreneur	0.40	0.71

Notes: Averages are computed at the beginning of DM.

Figure C.3: Discount and Preference Shocks Distribution

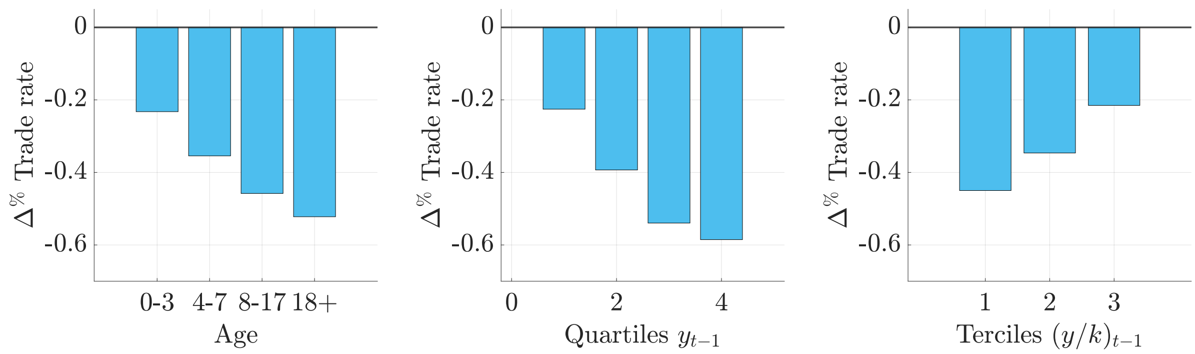
(a) Discount shocks CDF **(b)** Pref. shocks CDF, $\Psi(\kappa)$



Notes: Panel (a) plots the CDF of the discount shocks ξ_{jt} drawn from a Beta distribution with parameters $(1, \beta_\xi)$. Panel (b) plots the resulting preference shocks CDF $(\Psi(\kappa))$, defined by $\kappa_{jt} = (1 - \xi_{jt})^{-1}$. The lines plot to three different parameterizations of $\mathbb{E}[\xi]$. The circles correspond to the four-point quadrature used in the quantitative model.

Figure C.4: Trade Rate and Firms' Characteristics without Preference Shocks

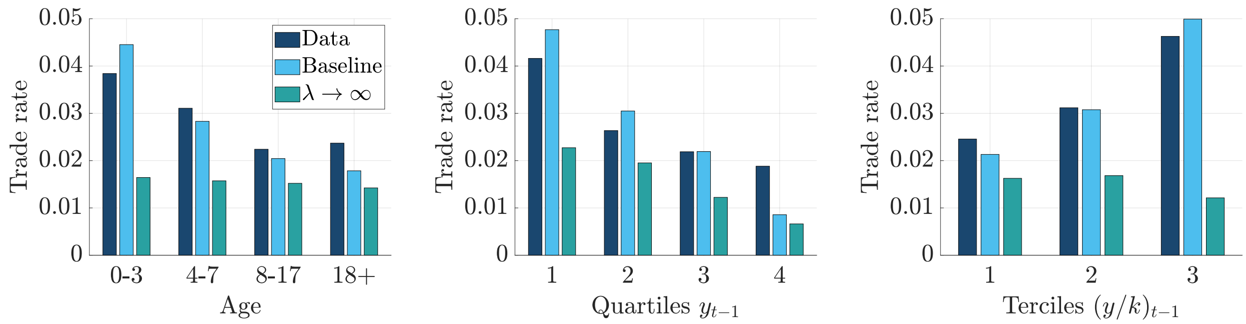
(a) Trade vs. Age **(b)** Trade vs. Size **(c)** Trade vs. ARPK



Notes: $\Delta^{\%}$ in the trade rate between the model without preference shocks ($\xi = 0$) and the baseline.

Figure C.5: Trade Rate by Firms' Characteristics with and without Borrowing Constraint

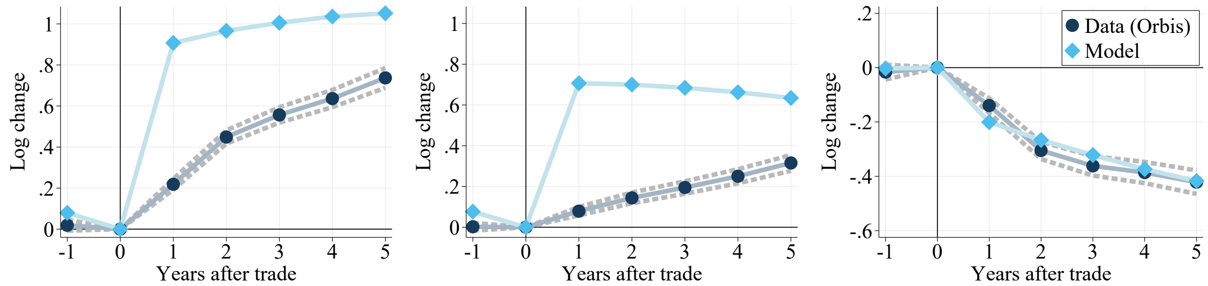
(a) Trade vs. Age **(b)** Trade vs. Size **(c)** Trade vs. ARPK



Source: SBO, KFS, and model simulated data.
Notes: Trade rate by firms' characteristics in the data and data simulated from the model. To be consistent with the data, Model (KFS) restricts to a sample of firms of age less or equal to 7. To remove the borrowing constraint we assume both $\lambda_a, \lambda_\pi \to \infty$. See the notes in Figure 1 for a description of the data moments.

Figure C.6: Capital, Output, and ARPK Dynamics After Trade in Data and Model

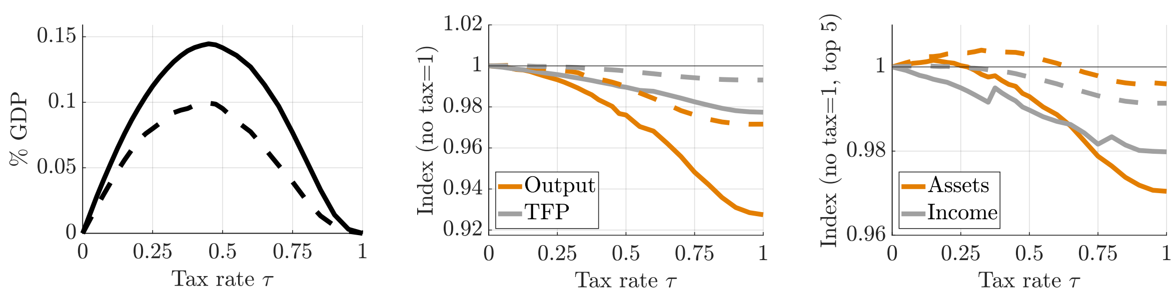
(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$



Source: Orbis Historical and model simulated data.
Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors. For a better comparison, we adjust for differences in the unconditional growth rates of capital and output in the data and model.

Figure C.7: Taxing the Market For Firms Without Borrowing Constraints

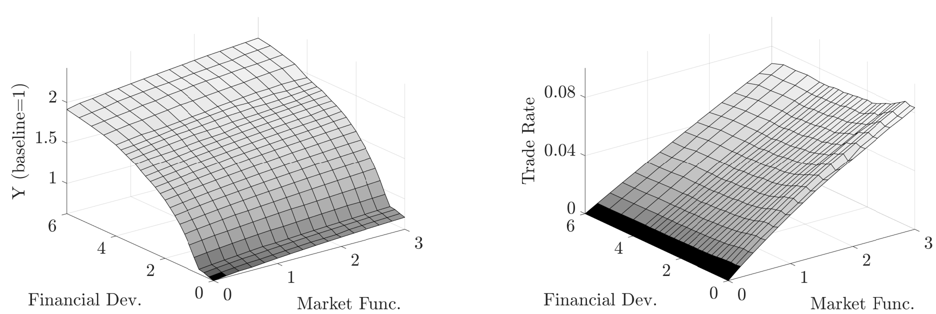
(a) Revenues/Output **(b)** Private Firms **(c)** Top 5% Shares



Notes: Counterfactuals are in general equilibrium. The solid lines are the baseline results. The dashed lines are the results for the economy without borrowing constraints $(\lambda_a, \lambda_\pi) \to \infty$.

Figure C.8: Financial Development and Functioning of the Market for Firms

(a) Private Firms' Output **(b)** Trade Rates



Notes: Financial Development is parameterized by c_λ, which scales the borrowing constraint parameters $\lambda_a(c_\lambda) = \max\{c_\lambda \lambda_a, 1\}$ and $\lambda_\pi(c_\lambda) = c_\lambda \lambda_\pi$. Market Functioning is parameterized by c_α multiplying the search frictions in the market for firms $\alpha_o(c_\alpha) = \min\{c_\alpha \alpha_o, 1\}, \alpha_w(c_\alpha) = \min\{c_\alpha \alpha_w, 1\}$. Panel (a) plots private firms' output in the financial development and functioning of the market for firms' space. Panel (b) plots firms' trade rate in the financial development and functioning of the market for firms' space. The baseline calibration corresponds to the case $c_\lambda = 1$ and $c_\alpha = 1$.