Example 1

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M.F. Atiyah, I.G. MacDonald *Introduction to Commutative Algebra*

Irreducible polynomial in $k[x_1, \ldots, x_n]$ is the same as irreducible element (as there is no difference between primitive and non-primitive elements). Then the factorization of f is $f = f$. In a UFD every irreducible element is also a prime element (J.B. Fraleigh, *A First Course in Abstract Algebra 7Ed*). Then f is a prime element:

$$f|gh \Rightarrow f|g \parallel f|h$$

Now

$$gh \in (f) \Rightarrow g \in (f) \parallel h \in (f)$$

which shows that (f) is a prime ideal.