Przykladowyegzamin

placeholder

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Contents

1 Zad 1

$$\begin{aligned}
\Im\left(\frac{1+3i}{3-2i}+i^3+5\right) &= \Im\left(\frac{1+3i}{3-2i}+\frac{i^3(3-2i)}{3-2i}+\frac{5(3-2i)}{3-2i}\right)\\
&= \Im\left(\frac{1+3i+3i^3-2i^4+15-10i}{3-2i}\right)\\
&= \Im\left(\frac{16-7i+3i^3-2i^4}{3-2i}\right)\\
&= \Im\left(\frac{14-10i}{3-2i}\right)\\
&= \Im\left(\frac{14-10i}{3-2i}\cdot\frac{3+2i}{3+2i}\right)\\
&= \Im\left(\frac{42+28i-30i+20}{9+4}\right)\\
&= \Im\left(\frac{62-2i}{13}\right)\\
&= \frac{-2}{13}
\end{aligned}$$

2 Zad 2

$$\frac{(3-3i)^{14}}{(-1+i\sqrt{3})^{11}} = \frac{z^{14}}{w^{11}}$$

2.1 z

$$\sin(\varphi_z) = \frac{-3}{3\sqrt{2}} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \to \varphi_z = \frac{7}{4}\pi$$

$$\begin{aligned}
z^{14} &= (3-3i)^{14} \\
&= (3-3i)^{14} \\
&= (3\sqrt{2})^{14}(\cos 14\varphi + i\sin 14\varphi) \\
&= (3\sqrt{2})^{14}\left(\cos\left(14\cdot\frac{7}{4}\pi\right) + i\sin\left(14\cdot\frac{7}{4}\pi\right)\right) \\
&= (3\sqrt{2})^{14}\left(\cos\left(\frac{49}{2}\pi\right) + i\sin\left(\frac{49}{2}\pi\right)\right) \\
&= (3\sqrt{2})^{14}\left(\cos\left(\frac{1}{2}\pi\right) + i\sin\left(\frac{1}{2}\pi\right)\right) \\
&= (3\sqrt{2})^{14}(0+i1) \\
&= (3\sqrt{2})^{14} i
\end{aligned}$$

2.2 w

$$\sin(\varphi_w) = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2} \to \varphi_w = \frac{2}{3}\pi$$

$$\begin{aligned}
w^{11} &= 2^{11}\left(\cos\left(11\cdot\frac{2}{3}\pi\right) + i\sin\left(11\cdot\frac{2}{3}\pi\right)\right) \\
&= 2^{11}\left(-\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right) \\
&= 2^{11}\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) \\
&= 2^{10}\left(-1 - i\sqrt{3}\right)
\end{aligned}$$

2.3 Podstawiamy

$$\begin{aligned}
\frac{(3-3i)^{14}}{\left(-1+i\sqrt{3}\right)^{11}} &= \frac{z^{14}}{w^{11}} \\
&= \frac{(3\sqrt{2})^{14}i}{2^{10}(-1-i\sqrt{3})} \\
&= \frac{((3\sqrt{2})^{14}i)(-1+i\sqrt{3})}{2^{10}(-1-i\sqrt{3})(-1+i\sqrt{3})} \\
&= \frac{((3\sqrt{2})^{14}i)(-1+i\sqrt{3})}{2^{10}(-2)} \\
&= \frac{((3\sqrt{2})^{14}i)(-1+i\sqrt{3})}{-2^{11}}
\end{aligned}$$