

Przykładowy egzamin

placeholder

February 5, 2023

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1 DONE Zad 1

$$\begin{aligned}\Im\left(\frac{1+3i}{3-2i} + i^3 + 5\right) &= \Im\left(\frac{1+3i}{3-2i} + \frac{i^3(3-2i)}{3-2i} + \frac{5(3-2i)}{3-2i}\right) \\&= \Im\left(\frac{1+3i+3i^3-2i^4+15-10i}{3-2i}\right) \\&= \Im\left(\frac{16-7i+3i^3-2i^4}{3-2i}\right) \\&= \Im\left(\frac{14-10i}{3-2i}\right) \\&= \Im\left(\frac{14-10i}{3-2i} \cdot \frac{3+2i}{3+2i}\right) \\&= \Im\left(\frac{42+28i-30i+20}{9+4}\right) \\&= \Im\left(\frac{62-2i}{13}\right) \\&= \frac{-2}{13}\end{aligned}$$

2 DONE Zad 2

$$\frac{(3-3i)^{14}}{(-1+i\sqrt{3})^{11}} = \frac{z^{14}}{w^{11}}$$

2.1 z

$$\sin(\varphi_z) = \frac{-3}{3\sqrt{2}} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \rightarrow \varphi_z = \frac{7}{4}\pi$$

$$\begin{aligned}
z^{14} &= (3 - 3i)^{14} \\
&= (3 - 3i)^{14} \\
&= (3\sqrt{2})^{14} (\cos 14\varphi + i \sin 14\varphi) \\
&= (3\sqrt{2})^{14} \left(\cos \left(14 \cdot \frac{7}{4} \pi \right) + i \sin \left(14 \cdot \frac{7}{4} \pi \right) \right) \\
&= (3\sqrt{2})^{14} \left(\cos \left(\frac{49}{2} \pi \right) + i \sin \left(\frac{49}{2} \pi \right) \right) \\
&= (3\sqrt{2})^{14} \left(\cos \left(\frac{1}{2} \pi \right) + i \sin \left(\frac{1}{2} \pi \right) \right) \\
&= (3\sqrt{2})^{14} (0 + i1) \\
&= (3\sqrt{2})^{14} i
\end{aligned}$$

2.2 w

$$\sin(\varphi_w) = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2} \rightarrow \varphi_w = \frac{2}{3}\pi$$

$$\begin{aligned}
w^{11} &= 2^{11} \left(\cos \left(11 \cdot \frac{2}{3} \pi \right) + i \sin \left(11 \cdot \frac{2}{3} \pi \right) \right) \\
&= 2^{11} \left(-\cos \frac{\pi}{3} - i \sin \frac{\pi}{3} \right) \\
&= 2^{11} \left(-\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) \\
&= 2^{10} (-1 - i\sqrt{3})
\end{aligned}$$

2.3 Podstawiamy

$$\begin{aligned}
 \frac{(3-3i)^{14}}{(-1+i\sqrt{3})^{11}} &= \frac{z^{14}}{w^{11}} \\
 &= \frac{(3\sqrt{2})^{14}i}{2^{10}(-1-i\sqrt{3})} \\
 &= \frac{((3\sqrt{2})^{14}i)(-1+i\sqrt{3})}{2^{10}(-1-i\sqrt{3})(-1+i\sqrt{3})} \\
 &= \frac{((3\sqrt{2})^{14}i)(-1+i\sqrt{3})}{2^{10}(-2)} \\
 &= \frac{((3\sqrt{2})^{14}i)(-1+i\sqrt{3})}{-2^{11}}
 \end{aligned}$$

3 PROJ Zad 3

Wyznacznik macierzy głównej = 20.

$$A = \begin{bmatrix} 3 & -2 & 1 & 0 \\ 2 & -1 & 3 & 1 \\ 2 & -1 & 3 & 4 \\ 0 & 1 & 3 & -1 \end{bmatrix}, X = \begin{bmatrix} 4 \\ 1 \\ -2 \\ 3 \end{bmatrix}$$

3.1 DONE A_4

$$\begin{aligned}
 A_4 &= \begin{vmatrix} 3 & -2 & 1 & 4 \\ 2 & -1 & 3 & 1 \\ 2 & -1 & 3 & -2 \\ 0 & 1 & 3 & 3 \end{vmatrix} \xrightarrow[k_3=k_3-k_4]{k_4=k_4-3k_2} \begin{vmatrix} 3 & -2 & -3 & 10 \\ 2 & -1 & 2 & 4 \\ 2 & -1 & 5 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix} \\
 &= 1 \cdot (-1)^6 \cdot \begin{vmatrix} 3 & -3 & 10 \\ 2 & 2 & 4 \\ 2 & 5 & 1 \end{vmatrix} \\
 &= 1 \cdot (6 + 100 - 24) - (40 + 60 - 6) \\
 &= 82 - 94 \\
 &= -12
 \end{aligned}$$

3.2 DONE Podstawianie

$$x_4 = \frac{-12}{20} = \frac{-3}{5}$$