Przykladowyegzamin

placeholder

February 5, 2023

Contents

1 DONE Zad 1

$$\begin{aligned}
\Im\left(\frac{1+3i}{3-2i}+i^3+5\right) &= \Im\left(\frac{1+3i}{3-2i}+\frac{i^3(3-2i)}{3-2i}+\frac{5(3-2i)}{3-2i}\right)\\
&= \Im\left(\frac{1+3i+3i^3-2i^4+15-10i}{3-2i}\right)\\
&= \Im\left(\frac{16-7i+3i^3-2i^4}{3-2i}\right)\\
&= \Im\left(\frac{14-10i}{3-2i}\right)\\
&= \Im\left(\frac{14-10i}{3-2i}\cdot\frac{3+2i}{3+2i}\right)\\
&= \Im\left(\frac{42+28i-30i+20}{9+4}\right)\\
&= \Im\left(\frac{62-2i}{13}\right)\\
&= \frac{-2}{13}
\end{aligned}$$

2 DONE Zad 2

$$\frac{(3-3i)^{14}}{(-1+i\sqrt{3})^{11}} = \frac{z^{14}}{w^{11}}$$

2.1 z

$$\sin(\varphi_z) = \frac{-3}{3\sqrt{2}} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \rightarrow \varphi_z = \frac{7}{4}\pi$$

$$
\begin{aligned}
z^{14} &= (3-3i)^{14} \\
&= (3-3i)^{14} \\
&= (3\sqrt{2})^{14}(\cos 14\varphi + i\sin 14\varphi) \\
&= (3\sqrt{2})^{14}\left(\cos\left(14\cdot\frac{7}{4}\pi\right) + i\sin\left(14\cdot\frac{7}{4}\pi\right)\right) \\
&= (3\sqrt{2})^{14}\left(\cos\left(\frac{49}{2}\pi\right) + i\sin\left(\frac{49}{2}\pi\right)\right) \\
&= (3\sqrt{2})^{14}\left(\cos\left(\frac{1}{2}\pi\right) + i\sin\left(\frac{1}{2}\pi\right)\right) \\
&= (3\sqrt{2})^{14}(0+i1) \\
&= (3\sqrt{2})^{14} i
\end{aligned}
$$

2.2 w

$$
\sin(\varphi_w) = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2} \to \varphi_w = \frac{2}{3}\pi
$$

$$
\begin{aligned}
w^{11} &= 2^{11}\left(\cos\left(11\cdot\frac{2}{3}\pi\right) + i\sin\left(11\cdot\frac{2}{3}\pi\right)\right) \\
&= 2^{11}\left(-\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right) \\
&= 2^{11}\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) \\
&= 2^{10}\left(-1 - i\sqrt{3}\right)
\end{aligned}
$$

2.3 Podstawiamy

$$\begin{aligned}\frac{(3-3i)^{14}}{\left(-1+i\sqrt{3}\right)^{11}} &= \frac{z^{14}}{w^{11}} \\ &= \frac{(3\sqrt{2})^{14}i}{2^{10}(-1-i\sqrt{3})} \\ &= \frac{((3\sqrt{2})^{14}i)(-1+i\sqrt{3})}{2^{10}(-1-i\sqrt{3})(-1+i\sqrt{3})} \\ &= \frac{((3\sqrt{2})^{14}i)(-1+i\sqrt{3})}{2^{10}(-2)} \\ &= \frac{((3\sqrt{2})^{14}i)(-1+i\sqrt{3})}{-2^{11}}\end{aligned}$$

3 DONE Zad 3

Wyznacznik macierzy głownej = 20.

$$A = \begin{bmatrix}3 & -2 & 1 & 0\\ 2 & -1 & 3 & 1\\ 2 & -1 & 3 & 4\\ 0 & 1 & 3 & -1\end{bmatrix},\ X = \begin{bmatrix}4\\ 1\\ -2\\ 3\end{bmatrix}$$

3.1 DONE A_4

$$\begin{aligned}A_4 &= \begin{vmatrix}3 & -2 & 1 & 4\\ 2 & -1 & 3 & 1\\ 2 & -1 & 3 & -2\\ 0 & 1 & 3 & 3\end{vmatrix} \xrightarrow[k_3=k_3-k4]{k_4=k_4-3k_2} \begin{vmatrix}3 & -2 & -3 & 10\\ 2 & -1 & 2 & 4\\ 2 & -1 & 5 & 1\\ 0 & 1 & 0 & 0\end{vmatrix} \\ &= 1\cdot(-1)^6\cdot\begin{vmatrix}3 & -3 & 10\\ 2 & 2 & 4\\ 2 & 5 & 1\end{vmatrix} \\ &= 1\cdot(6+100-24)-(40+60-6) \\ &= 82-94 \\ &= -12\end{aligned}$$

3.2 DONE Podstawianie

$$x_4 = \frac{-12}{20} = \frac{-3}{5}$$

4 DONE Zad 4

$$\left[\begin{array}{cccc|c} 3 & -2 & 1 & 0 & 4 \\ 2 & -1 & 3 & 1 & 1 \\ 2 & -1 & 3 & 4 & -2 \\ x_1 & x_2 & x_3 & x_4 & y \end{array}\right] \xrightarrow[w_1=w_1-w_2]{} \left[\begin{array}{cccc|c} 1 & -1 & -2 & -1 & 3 \\ 2 & -1 & 3 & 1 & 1 \\ 2 & -1 & 3 & 4 & -2 \\ x_1 & x_2 & x_3 & x_4 & y \end{array}\right]$$

$$\xrightarrow[w_2=w_2-2w_1]{w_3=w_3-2w_1} \left[\begin{array}{cccc|c} 1 & -1 & -2 & -1 & 3 \\ 0 & 1 & 7 & 3 & -5 \\ 0 & 1 & 7 & 6 & -8 \\ x_1 & x_2 & x_3 & x_4 & y \end{array}\right]$$

$$\xrightarrow[w_3=w_3-w_2]{w_1=w_1+w_2} \left[\begin{array}{cccc|c} 1 & 0 & 5 & 2 & -2 \\ 0 & 1 & 7 & 3 & -5 \\ 0 & 0 & 0 & 3 & -3 \\ x_1 & x_2 & x_3 & x_4 & y \end{array}\right]$$

$$\xrightarrow[k_4=k_3]{k_3=k_4} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 5 & -2 \\ 0 & 1 & 3 & 7 & -5 \\ 0 & 0 & 3 & 0 & -3 \\ x_1 & x_2 & x_4 & x_3 & y \end{array}\right]$$

$$\xrightarrow[w_3=w_3\cdot\frac{1}{3}]{} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 5 & -2 \\ 0 & 1 & 3 & 7 & -5 \\ 0 & 0 & 1 & 0 & -1 \\ x_1 & x_2 & x_4 & x_3 & y \end{array}\right]$$

$$\xrightarrow[w_2=w_2-3\cdot w_3]{w_1=w_1-2\cdot w_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 5 & 0 \\ 0 & 1 & 0 & 7 & -2 \\ 0 & 0 & 1 & 0 & -1 \\ x_1 & x_2 & x_4 & x_3 & y \end{array}\right]$$

$x_1 = -5x_3$
$x_2 = -7x_3 - 2$
$x_3 \in \mathbb{R}$
$x_4 = -1$

5 PROJ ZAD 5

$$XA = B \xrightarrow[\text{prawostronnie } A^{-1}]{} XAA^{-1} = BA^{-1} \to X = BA^{-1}$$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 0 & -2 \\ -1 & -4 & 5 \end{bmatrix},\ B = \begin{bmatrix} 4 & 2 & -5 \\ 3 & 0 & -2 \end{bmatrix}$$

$\det A = (36+4) - (8+30) = 40 - 38 = 2$

$$
\begin{aligned}
A^{-1} &= \frac{1}{2}\begin{bmatrix}
\begin{vmatrix} 0 & -2 \\ -4 & 5 \end{vmatrix} & -\begin{vmatrix} 3 & -2 \\ -1 & 5 \end{vmatrix} & \begin{vmatrix} 3 & 0 \\ -1 & -4 \end{vmatrix} \\
-\begin{vmatrix} 2 & -3 \\ -4 & 5 \end{vmatrix} & \begin{vmatrix} 1 & -3 \\ -1 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ -1 & -4 \end{vmatrix} \\
\begin{vmatrix} 2 & -3 \\ 0 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & -3 \\ 3 & -2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix}
\end{bmatrix}^T \\
&= \frac{1}{2}\begin{bmatrix} -8 & -13 & -12 \\ 2 & 2 & 2 \\ -4 & -7 & -6 \end{bmatrix}^T \\
&= \frac{1}{2}\begin{bmatrix} -8 & 2 & -4 \\ -13 & 2 & -7 \\ -12 & 2 & -6 \end{bmatrix} \\
&= \begin{bmatrix} -4 & 1 & -2 \\ -\frac{13}{2} & 1 & -\frac{7}{2} \\ -6 & 1 & -3 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
X = BA^{-1} &= \begin{bmatrix} 4 & 2 & -5 \\ 3 & 0 & -2 \end{bmatrix}\begin{bmatrix} -4 & 1 & -2 \\ -\frac{13}{2} & 1 & -\frac{7}{2} \\ -6 & 1 & -3 \end{bmatrix} \\
&= \begin{bmatrix} -16 - 13 + 30 & 4 + 2 - 5 & -8 - 7 + 15 \\ -12 + 0 + 12 & 3 + 0 - 2 & -6 + 0 + 6 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}
\end{aligned}
$$