Przykladowyegzamin

placeholder

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1 DONE Zad 1

$$\Im\left(\frac{1+3i}{3-2i}+i^3+5\right)=\Im\left(\frac{1+3i}{3-2i}+\frac{i^3(3-2i)}{3-2i}+\frac{5(3-2i)}{3-2i}\right)$$

$$=\Im\left(\frac{1+3i+3i^3-2i^4+15-10i}{3-2i}\right)$$

$$=\Im\left(\frac{16-7i+3i^3-2i^4}{3-2i}\right)$$

$$=\Im\left(\frac{14-10i}{3-2i}\right)$$

$$=\Im\left(\frac{14-10i}{3-2i}\cdot\frac{3+2i}{3+2i}\right)$$

$$=\Im\left(\frac{42+28i-30i+20}{9+4}\right)$$

$$=\Im\left(\frac{62-2i}{13}\right)$$

$$=\frac{-2}{13}$$

2 DONE Zad 2

$$\frac{(3-3i)^{14}}{\left(-1+i\sqrt{3}\right)^{11}}=\frac{z^{14}}{w^{11}}$$

2.1 z

$$\sin(\varphi_z)=\frac{-3}{3\sqrt{2}}=\frac{-1}{\sqrt{2}}=\frac{-\sqrt{2}}{2}\rightarrow\varphi_z=\frac{7}{4}\pi$$

$$z^{14} = (3 - 3i)^{14}$$
$$= (3 - 3i)^{14}$$
$$= (3\sqrt{2})^{14}(\cos 14\varphi + i\sin 14\varphi)$$
$$= (3\sqrt{2})^{14}\left(\cos\left(14 \cdot \frac{7}{4}\pi\right) + i\sin\left(14 \cdot \frac{7}{4}\pi\right)\right)$$
$$= (3\sqrt{2})^{14}\left(\cos\left(\frac{49}{2}\pi\right) + i\sin\left(\frac{49}{2}\pi\right)\right)$$
$$= (3\sqrt{2})^{14}\left(\cos\left(\frac{1}{2}\pi\right) + i\sin\left(\frac{1}{2}\pi\right)\right)$$
$$= (3\sqrt{2})^{14}(0 + i1)$$
$$= (3\sqrt{2})^{14}i$$

2.2 w

$$\sin(\varphi_w) = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2} \rightarrow \varphi_w = \frac{2}{3}\pi$$

$$w^{11} = 2^{11}\left(\cos\left(11 \cdot \frac{2}{3}\pi\right) + i\sin\left(11 \cdot \frac{2}{3}\pi\right)\right)$$
$$= 2^{11}\left(-\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$$
$$= 2^{11}\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$
$$= 2^{10}\left(-1 - i\sqrt{3}\right)$$

2.3 Podstawiamy

$$\frac{(3-3i)^{14}}{\left(-1+i\sqrt{3}\right)^{11}} = \frac{z^{14}}{w^{11}}$$

$$= \frac{(3\sqrt{2})^{14}i}{2^{10}(-1-i\sqrt{3})}$$

$$= \frac{((3\sqrt{2})^{14}i)(-1+i\sqrt{3})}{2^{10}(-1-i\sqrt{3})(-1+i\sqrt{3})}$$

$$= \frac{((3\sqrt{2})^{14}i)(-1+i\sqrt{3})}{2^{10}(-2)}$$

$$= \frac{((3\sqrt{2})^{14}i)(-1+i\sqrt{3})}{-2^{11}}$$

3 DONE Zad 3

Wyznacznik macierzy głownej $= 20$.

$$A = \begin{bmatrix} 3 & -2 & 1 & 0 \\ 2 & -1 & 3 & 1 \\ 2 & -1 & 3 & 4 \\ 0 & 1 & 3 & -1 \end{bmatrix}, X = \begin{bmatrix} 4 \\ 1 \\ -2 \\ 3 \end{bmatrix}$$

3.1 DONE A_4

$$A_4 = \begin{vmatrix} 3 & -2 & 1 & 4 \\ 2 & -1 & 3 & 1 \\ 2 & -1 & 3 & -2 \\ 0 & 1 & 3 & 3 \end{vmatrix} \xrightarrow[k_3=k_3-k4]{k_4=k_4-3k_2} \begin{vmatrix} 3 & -2 & -3 & 10 \\ 2 & -1 & 2 & 4 \\ 2 & -1 & 5 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

$$= 1 \cdot (-1)^6 \cdot \begin{vmatrix} 3 & -3 & 10 \\ 2 & 2 & 4 \\ 2 & 5 & 1 \end{vmatrix}$$

$$= 1 \cdot (6 + 100 - 24) - (40 + 60 - 6)$$

$$= 82 - 94$$

$$= -12$$

3.2 DONE Podstawianie

$$x_4 = \frac{-12}{20} = \frac{-3}{5}$$