Applied Mathematics

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Important questions

- Is universe deterministic or probabilistic?
- Can universe be described in mathematically "pretty" way?
- Can we find The Model?

Mechanism of Universe

Simulation Hypothesis

We can't look into the code of the universe. We need models.

Scientific method

Theory is the base of science. It requires falsifiable model described mathematically.

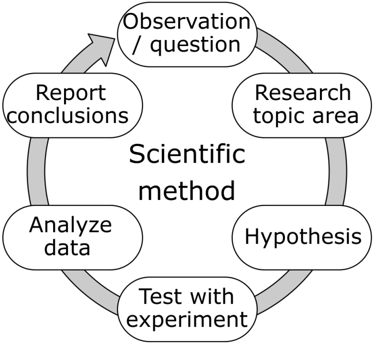

Figure: Source: wikimedia.org, Author: Efbrazil

Important differential equations in physics

Continuity $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathrm{v}) = \sigma$

Diffusion $\frac{\partial \rho}{\partial t} = \nabla \cdot (\mathrm{D}\, \nabla \rho)$

Harmonic oscillator $\frac{\mathrm{d}^2 \phi}{\mathrm{d} t^2} + \omega_0^2 \phi = 0$

Euler-Lagrange $\frac{\mathrm{d}}{\mathrm{d} t} \frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = 0$

Laplace $\nabla^2 V = 0$

Poisson $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$

Helmholtz $\left(\nabla^2 + k^2\right) V = 0$

Wave $\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) V = 0$

Solving differential equation

Gauss's Law in Linear Medium

$$\nabla \cdot \mathbf{E} = \frac{\rho_f}{\varepsilon_r \varepsilon_0} \qquad \oint_{\mathcal{S}} \mathbf{E} \cdot \mathrm{d}\mathbf{S} = \frac{Q_f}{\varepsilon_r \varepsilon_0}$$

Capacitor equation

$$C = \frac{Q_f}{V} = \varepsilon_r \varepsilon_0 \frac{S}{d}$$

Linear Regression

Model

$$y_i = ax_i + b + e_i \quad y_i = C_i \quad a = \varepsilon_r \varepsilon_0 S \quad x_i = \frac{1}{d_i} \quad b = C_0$$

Least Squares Method

$$G(a,b) = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - ax_i - b)^2$$

$$\frac{\partial G(a,b)}{\partial a} = 0 \quad \frac{\partial G(a,b)}{\partial b} = 0$$

Lotka Model

Equations

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = a - axy$$
$$\frac{\mathrm{d}y}{\mathrm{d}\tau} = xy - y$$

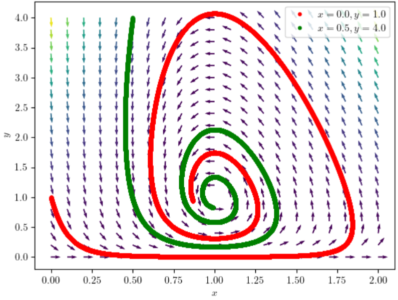

Figure: Lotka model : $a = 0.1$

Lotka-Volterra Model

Equations

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = ax - axy$$
$$\frac{\mathrm{d}y}{\mathrm{d}\tau} = xy - y$$

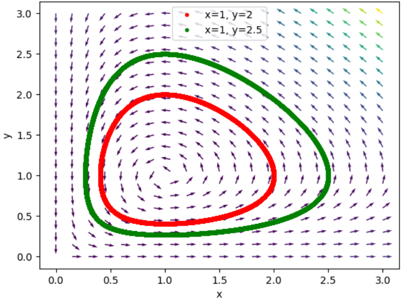

Figure: Lotka-Volterra model : $a = 1$

Brusselator Model

Equations

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = 1 + ax^2y - ax - x$$
$$\frac{\mathrm{d}y}{\mathrm{d}\tau} = -bx^2y + bx$$

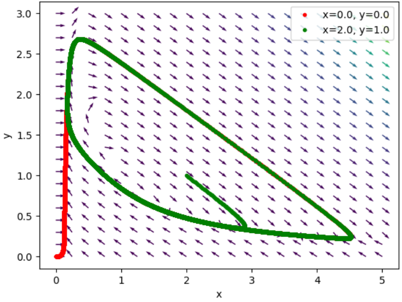

Figure: Brusselator model : $a = 7, \;\; b = 4$

Bibliography

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