

① Row-reduce A - rref(A):

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & -2 & 5 & 1 & 1 \end{pmatrix} \xrightarrow{R_3 + 2R_1} \begin{pmatrix} 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 3 & 1 \end{pmatrix} \xrightarrow{R_3 - R_2} \begin{pmatrix} 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

② Count pivot rows:

There are 2 non-zero (pivots in columns 1 and 3),  
so

$$\boxed{\text{rank}(T) = 2}$$

③ Apply the Rank-Nullity Theorem:

$$\text{rank}(T) + \text{nullity}(T) = 5 \rightarrow \# \text{ of columns}$$

$$\text{nullity}(T) = 5 - 2 = 3$$

Hence:

$$\boxed{\text{nullity}(T) = 3}$$