① **Set up the characteristic equation:**

For a 3x3 matrix A, the characteristic polynomial is:

$$p(\lambda) = \det(A - \lambda I)$$

Here,

$$A = \begin{pmatrix} 5 & 0 & 0 \\ -2 & 2 & -4 \\ 4 & -5 & 3 \end{pmatrix}.$$

② **Form $A - \lambda I$:**

$$A - \lambda I = \begin{pmatrix} 5-\lambda & 0 & 0 \\ -2 & 2-\lambda & -4 \\ 4 & -5 & 3-\lambda \end{pmatrix}$$

③ **Compute the determinant by expansion along the first row:**

Since the first row has 2 zeros, expanding along it gives:

$$\det(A - \lambda I) = (5-\lambda)\det\begin{pmatrix} 2-\lambda & -4 \\ -5 & 3-\lambda \end{pmatrix}$$

④ **Compute 2x2 minor:**

$$\begin{aligned} \det\begin{pmatrix} 2-\lambda & -4 \\ -5 & 3-\lambda \end{pmatrix} &= (2-\lambda)(3-\lambda) - (-4)(-5) \\ &= (2-\lambda)(3-\lambda) - 20 \\ &= (6 - 5\lambda + \lambda^2) - 20 \\ &= \lambda^2 - 5\lambda - 14 \end{aligned}$$

⑤ **Assemble the characteristic polynomial:**

$$p(\lambda) = (5-\lambda)(\lambda^2 - 5\lambda - 14)$$

To match the form $-\lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$,

expand:

$$\begin{aligned} p(\lambda) &= (5-\lambda)(\lambda^2 - 5\lambda - 14) \\ &= 5\lambda^2 - 25\lambda - 70 - \lambda^3 + 5\lambda^2 + 14\lambda \\ &= -\lambda^3 + (5+5)\lambda^2 + (-25+14)\lambda - 70 \\ &= -\lambda^3 + 10\lambda^2 - 11\lambda - 70 \end{aligned}$$

⑥ **Read off the coefficients:**

$$= -\lambda^3 + 10\lambda^2 - 11\lambda - 70$$

$c_1 = 10$, $c_2 = -11$

$c_3 = -70$,

$$\boxed{\text{Answer: } -\lambda^3 + 10\lambda^2 - 11\lambda - 70}$$