

① Compute Cofactor C_{41} :

The cofactor C_{41} is given by:

$$C_{41} = (-1)^{4+1} \det(M_{41}) = -\det(M_{41})$$

Where M_{41} is the 3×3 minor

obtained by deleting row 4 and Column 1 from A .

Thus

$$M_{41} = \begin{bmatrix} 1 & -2 & 3 \\ 3 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

Compute its determinant by expanding along the first row:

$$\begin{aligned} \det(M_{41}) &= 1 \cdot \det \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} - (-2) \cdot \det \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} + 3 \cdot \det \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix} \\ &= 1(1 \cdot 2 - 0 \cdot 1) + 2(3 \cdot 2 - 0 \cdot 0) + 3(3 \cdot 1 - 1 \cdot 0) \\ &= 2 + 2(6) + 3(3) \\ &= 2 + 12 + 9 = 23 \end{aligned}$$

Hence:

$$C_{41} = -23$$

→ don't forget (-) sign

② Compute $\det(A)$ by expansion down column 2:

The determinant can be expanded along column 2:

$$\det(A) = \sum_{i=1}^4 A_{i2} C_{i2} = A_{12} C_{12} + A_{22} C_{22} + A_{32} C_{32} + A_{42} C_{42}$$

From the matrix, $\begin{bmatrix} a & 1 & -2 & 3 \\ b & 3 & 1 & 0 \\ c & 0 & 1 & 2 \\ d & 6 & -2 & 1 \end{bmatrix}$

$$A_{12} = 1 \quad A_{22} = 3 \quad A_{32} = 0 \quad A_{42} = 6$$

We're given:

$$C_{12} = 2 \quad C_{22} = -1 \quad C_{32} = -4 \quad C_{42} = 1$$

Therefore:

$$\begin{aligned} \det(A) &= (1 \cdot 2) + (3 \cdot -1) + (0 \cdot -4) + (6 \cdot 1) \\ &= 2 - 3 + 0 + 6 \end{aligned}$$

$$\boxed{\det(A) = 5}$$