

① Compute determinant and
Set it to zero:

For

$$A = \begin{pmatrix} 3 & k & -1 \\ -3 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix},$$

the determinant is:

$$\begin{aligned} \det(A) &= 3 \det \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} - k \det \begin{pmatrix} -3 & 1 \\ 1 & 1 \end{pmatrix} + (-1) \det \begin{pmatrix} -3 & 2 \\ 1 & 0 \end{pmatrix} \\ &= 3(2 \cdot 1 - 1 \cdot 0) - k(-3 \cdot 1 - 1 \cdot 1) - 1(-3 \cdot 0 - 2 \cdot 1) \\ &= 3(2) - k(-3 - 1) - 1(-2) \\ &= 6 - (-4k) + 2 \\ &= 6 + 4k + 2 \\ &= 4k + 8 \end{aligned}$$

② Solve for invertibility:

A matrix is NOT invertible exactly
when its determinant is zero. Thus,
set:

$$\begin{array}{r} 4k + 8 = 0 \\ \underline{-8 \quad -8} \\ 4k = -8 \\ \boxed{k = -2} \end{array}$$