Conditions:

- $V = \{$all 2x2 real number matrices$\}$
- $H_1 = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \right\}$ – upper triangle matrix
- $H_2 = \{$matrices with at most one non-zero entry$\}$

Statements:

✓ 1.) H_1 is closed under scalar multiplication:

Scaling $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$ by any λ yields $\begin{pmatrix} \lambda a & \lambda b \\ 0 & \lambda c \end{pmatrix}$, which is still upper triangle

✓ 2.) H_1 is a subspace of V:

- It contains the zero matrix ✓
- Closed under addition (sum of 2 upper triangle matrices is upper-triangular) ✓
- Closed under scalar multiplication ✓

✗ 3.) H_2 is closed under matrix addition:

Example:

$$E_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad E_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

H_2 at most must have one non-zero entry

But, $(E_{11} + E_{22})$ yields $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Which has 2 non-zero entries, therefore, false

✓ 4.) H_2 is closed under scalar multiplication:

If a matrix has at most one non-zero entry, multiplying by λ preserves the property of "at most one nonzero entry"

✓ 5.) H_2 is NOT a subspace of V:

- Contains the zero matrix ✓
- Closed under addition ✗
- Closed under scalar mult. ✓

✓ 6.) H_1 is closed under matrix addition:

$$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} + \begin{pmatrix} a' & b' \\ 0 & c' \end{pmatrix} = \begin{pmatrix} a+a' & b+b' \\ 0 & c+c' \end{pmatrix}$$

still upper triangle

$$\boxed{\text{Answer: } 1, 2, 4, 5, 6}$$