$\mathbb{R}^3$ to P_2:

$$T\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}\right) = (9a_1 - 3a_2 + 5a_3)x^2 + (3a_1 + a_2 - 4a_3)x + 8a_1 - 4a_2$$

From the definition:

$$T(e_1) = T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = (9\cdot \overset{a_1}{1} - 3\cdot \overset{a_2}{0} + 5\cdot \overset{a_3}{0})x^2 + (3\cdot 1 + 0 - 4\cdot 0)x + (8\cdot 1 - 4\cdot 0)$$

$$= 9x^2 + 3x + 8$$

Expressed in the basis $\{x^2, x, 1\}$, this is the coordinate vector:

$$[T(e_1)]_{B'} = \begin{pmatrix} 9 \\ 3 \\ 8 \end{pmatrix}$$

Similarly:

$$T(e_2) = T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = (9\cdot \overset{a_1}{0} - 3\cdot \overset{a_2}{1} + 5\cdot \overset{a_3}{0})x^2 + (3\cdot 0 + 1 - 4\cdot 0)x + (8\cdot 0 - 4\cdot 1)$$

$$= -3x^2 + 1x - 4$$

So:

$$[T(e_2)]_{B'} = \begin{pmatrix} -3 \\ 1 \\ -4 \end{pmatrix}$$

Finally:

$$T(e_3) = T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = (9\cdot 0 - 3\cdot 0 + 5\cdot 1)x^2 + (3\cdot 0 + 0 - 4\cdot 1)x + (8\cdot 0 - 4\cdot 0)$$

$$= 5x^2 - 4x + 0$$

So:

$$[T(e_3)]_{B'} = \begin{pmatrix} 5 \\ -4 \\ 0 \end{pmatrix}$$

Answer:

T relative to B, B': $A = \begin{pmatrix} 9 & -3 & 5 \\ 3 & 1 & -4 \\ 8 & -4 & 0 \end{pmatrix}$