① <u>Set up the characteristics polynomial:</u>

For

$$A = \begin{pmatrix} -7 & -3 & 8 \\ 0 & 5 & -3 \\ 0 & -4 & 6 \end{pmatrix},$$

the characteristics polynomial is

$$p(\lambda) = \det(A - \lambda I) = \det\begin{pmatrix} -7-\lambda & -3 & 8 \\ 0 & 5-\lambda & -3 \\ 0 & -4 & 6-\lambda \end{pmatrix}.$$

② <u>Exploit the block-triangular structure:</u>

Since the entries below the first column are zero,

$$p(\lambda) = (-7-\lambda)\det\begin{pmatrix} 5-\lambda & -3 \\ -4 & 6-\lambda \end{pmatrix}$$

③ <u>Compute the 2×2 minor:</u>

$$\det\begin{pmatrix} 5-\lambda & -3 \\ -4 & 6-\lambda \end{pmatrix} = (5-\lambda)(6-\lambda) - (-3)(-4)$$
$$= (30 - 11\lambda + \lambda^2) - 12$$
$$= \lambda^2 - 11\lambda + 18$$

④ <u>Factor and find roots:</u>

- The factor $-7-\lambda = 0$ gives $\lambda_1 = -7$
- Solve $\lambda^2 - 11\lambda + 18 = 0$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \lambda = \frac{11 \pm \sqrt{121 - 72}}{2} = \frac{11 \pm 7}{2} \Longrightarrow \lambda_2 = 2, \ \lambda_3 = 9$$

Answer:

$\lambda_1 = -7 \quad \lambda_2 = 2 \quad \lambda_3 = 9$

$(-7, 2, 9)$