<u>Identify pivot and free columns</u>

$$\begin{bmatrix} \boxed{1} & -2 & 0 & 2 & 0 & | & -3 \\ 0 & 0 & \boxed{1} & 3 & 0 & | & 8 \\ 0 & 0 & 0 & 0 & \boxed{1} & | & 5 \end{bmatrix}$$

• Pivot
• Free

1 2 3 4 5

Pivot Variables: x_1, x_3, x_5

Free Variables: x_2, x_4

① Free variables are x_2 and x_4
with indices $i = 2 < j = 4$

<u>Express each pivot variable in terms
of the free variables:</u>

From the reduced system:

$$\begin{cases} x_1 - 2x_2 + 2x_4 = -3 \\ x_3 + 3x_4 = 8 \\ x_5 = 5 \end{cases}$$

We solve:

$$x_1 = 2x_2 - 4x_4 - 3$$
$$x_3 = 8 - 3x_4$$
$$x_5 = 5$$

② <u>Write the solution in the required
vector form:</u>

Let x_2 and x_4 be the free parameters by setting
them equal to zero

<u>Get vector a (set $x_2 = 0, x_4 = 0$):</u>
From

$$x_1 = 2x_2 - 2x_4 - 3 \quad , \quad x_3 = 8 - 3x_4 \, , \quad x_5 = 5$$

we get

$$x_1 = -3, \quad x_2 = 0 \, , \quad x_3 = 8 \, , \quad x_4 = 0, \quad x_5 = 5$$

$$a = \begin{pmatrix} -3 \\ 0 \\ 8 \\ 0 \\ 5 \end{pmatrix}$$

<u>Get vector b (set $x_2 = 1, x_4 = 0$):</u>

$$x_1 = 2(1) - 2(0) - 3, \quad x_2 = 1$$
$$x_3 = 8 - 3(0) \quad , \quad x_4 = 0 \quad \quad x_5 = 5$$

Subtract result from vector a:

$$b = \begin{pmatrix} -1 \\ 1 \\ 8 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \\ 8 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

<u>Get vector c (set $x_2 = 0, x_4 = 1$):</u>

Subtract result from vector a:

$$c = \begin{pmatrix} -5 \\ 0 \\ 5 \\ 1 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \\ 8 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -3 \\ 1 \\ 0 \end{pmatrix}$$

Answer:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 8 \\ 0 \\ 5 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -2 \\ 0 \\ -3 \\ 1 \\ 0 \end{pmatrix}$$