

Find the rank of $A = \begin{bmatrix} -2 & 2 \\ 1 & 1 \\ 7 & 3 \end{bmatrix}$

Step 1: Write down original Matrix:

$$A = \begin{pmatrix} -2 & 2 \\ 1 & 1 \\ 7 & 3 \end{pmatrix}$$

Step 2: Aim for a leading 1 in the top left corner of the matrix:

- The simplest way to start is to swap a row that has a non-zero entry to the top.
- Here, row 2 has a 1 in the first column, which is convenient.

Swap Row₁ and Row₂

$$\begin{pmatrix} 1 & 1 \\ -2 & 2 \\ 7 & 3 \end{pmatrix}$$

Step 3: Use Row Operations to Clear Out the first Column:

- We want zeros below the leading 1 in the first column:

1. Row₂ \leftarrow Row₂ + 2 · Row₁

$$\begin{pmatrix} 1 & 1 \\ -2 & 2 \end{pmatrix} + 2 \cdot \begin{pmatrix} 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 4 \end{pmatrix}$$

2. Row₃ \leftarrow Row₃ - 7 · Row₁

$$\begin{pmatrix} 7 & 3 \end{pmatrix} - 7 \begin{pmatrix} 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -4 \end{pmatrix}$$

- The matrix becomes:

$$\begin{pmatrix} 1 & 1 \\ 0 & 4 \\ 0 & -4 \end{pmatrix}$$

Step 4: Get a leading 1 in the second Row, second Column

- Next we focus on the second pivot (the entry in row 2 column 2). We want that to be 1.

$$\text{Row}_2 \leftarrow \frac{1}{4} \text{Row}_2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & -4 \end{pmatrix}$$

Step 5: Clear out Above and Below the second Pivot:

- Now eliminate the 1 in row 1, column 2, and the -4 in row 3, column 2:

1. Row₁ \leftarrow Row₁ - (Row₂)

$$\begin{pmatrix} 1 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

2. Row₃ \leftarrow Row₃ + 4 · (Row₂)

$$\begin{pmatrix} 0 & -4 \end{pmatrix} + 4 \cdot \begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

- The matrix is now in reduced echelon form:

$$A' = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

Step 6: Count the Pivot Columns to Determine the Rank:

- In the RREF A' , the pivot positions (leading 1s) are in the first and second columns.
- Therefore, there are 2 pivot columns.

Answer, $\text{rank}(A) = 2$