

Proof by Cases for Problem 3:

CASE 1: $x-6 \geq 0$ aka $x \geq 6$:

1. Simplify absolute value:

Since $x \geq 6$, $x-6 \geq 0$. Hence, $|x-6| = x-6$.

2. Plug this into the expression:

$$|x-6| + x = (x-6) + x = 2x-6$$

3. Check if it is greater than 3:

We need $2x-6 > 3$.

$$\begin{array}{r} 2x-6 > 3 \\ +6 \quad +6 \\ \hline 2x > 9 \\ \hline x > 4.5 \end{array}$$

Since $x \geq 6$ implies $x > 4.5$, it follows immediately that

$$2x-6 > 3$$

CONCLUSION for CASE 1:

Whenever $x \geq 6$, $|x-6| + x > 3$

CASE 2: $x-6 < 0$ aka $x < 6$:

1. Simplify the absolute value:

Since $x < 6$, $x-6 < 0$. Hence, $|x-6| = -(x-6) = 6-x$

2. Plug into expression:

$$|x-6| + x = (6-x) + x = 6$$

3. Check if it is greater than 3:

Clearly, $6 > 3$.

CONCLUSION for CASE 2:

Whenever $x < 6$, $|x-6| + x = 6 > 3$.