

### ① Checking reflexivity:

Let  $A = \{1, 2, 3\}$  and  $R = \{(1, 1), (2, 2), (3, 3)\}$

Step 1- Verify that every element in  $A$  is related to itself:

- $(1, 1) \in R$  ✓
- $(2, 2) \in R$  ✓
- $(3, 3) \in R$  ✓

Answer:  $R$  is reflexive on  $A$

### ② Checking Symmetry and Anti-Symmetry:

Let  $A = \{a, b\}$  and  $R = \{(a, b), (b, a)\}$

Step 1- Check Symmetry:

- $(a, b) \in R$  and  $(b, a) \in R \rightarrow$  Symmetric ✓

Step 2- Check anti-Symmetry:

- Since  $(a, b) \in R$  and  $(b, a) \in R$  but  $a \neq b$ , anti-Symmetry is violated ✗

Answer: The relation is symmetric but not anti-symmetric

### ③ Checking Transitivity:

Let  $A = \{1, 2, 3\}$  and  $R = \{(1, 2), (2, 3), (1, 3)\}$

- Verify that if  $(1, 2) \in R$  and  $(2, 3) \in R$ , then  $(1, 3) \in R$  ✓

Answer: The relation  $R$  is transitive

### ④ A relation with Mixed Properties:

Let  $A = \{x, y, z\}$  and  $R = \{(x, x), (y, y), (z, z), (x, y), (y, x), (y, z)\}$

Reflexivity:

- Each element  $x, y, z$  is related to itself ✓

Symmetry:

- $(x, y) \in R$  and  $(y, x) \in R \rightarrow$  Symmetric for  $x$  and  $y$
- However  $(y, z) \in R$  but  $(z, y) \notin R \rightarrow$  Not Symmetric overall ✗

Anti-Symmetry:

- $(x, y)$  and  $(y, x)$  are both present and  $x \neq y$  so anti-symmetry is violated. ✗

Transitivity:

- Check:  $(x, y) \in R$  and  $(y, z) \in R$  require  $(x, z) \in R$ , which is missing ✗

Answer: The relation  $R$  is reflexive, but neither symmetric, anti-symmetric, nor transitive