Question 14 8 pts

Find the rank and nullity of the matrix $A = \begin{bmatrix} 3 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 2 & 2 & 6 & 3 & 0 \\ 0 & 0 & 0 & -3 & 0 \end{bmatrix}$.

$\text{Rank}(A) =$

$\text{Nullity}(A) =$

B for Col(A):

$$\begin{bmatrix} 3 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 2 & 2 & 6 & 3 & 0 \\ 0 & 0 & 0 & -3 & 0 \end{bmatrix} \qquad \left\{ \begin{bmatrix} 3 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \\ -3 \end{bmatrix} \right\}$$

$-2 \quad -2 \quad -6 \quad -3 \quad 0$

$$\xrightarrow{-R_3+R_1 \to R_1} \begin{bmatrix} 1 & -3 & -5 & -3 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 2 & 2 & 6 & 3 & 0 \\ 0 & 0 & 0 & -3 & 0 \end{bmatrix} \xrightarrow{-2R_1+R_3 \to R_3}$$

$-2 \quad 6 \quad 10 \quad 6 \quad 0$

$$\begin{bmatrix} 1 & -3 & -5 & -3 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 8 & 16 & 9 & 0 \\ 0 & 0 & 0 & -3 & 0 \end{bmatrix} \xrightarrow{-8R_2+R_3 \to R_3} \begin{bmatrix} 1 & -3 & -5 & -3 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 \end{bmatrix}$$

$0 \quad -8 \quad -16 \quad -8 \quad 0$

$R = 3 \quad N = 2 \quad x_1 \quad x_2 \quad x_4$

Question: Basis for Col(A)?

Nul(A)??? Solutions of

$$Ax = 0 \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$x_1 - x_3 + 3x_5 = 0$

$$A \rightsquigarrow \begin{bmatrix} 1 & 0 & -1 & 0 & 3 \\ 0 & 1 & 3 & 0 & 4 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_1 &= x_3 - 3x_5 \\ x_2 &= -3x_3 - 4x_5 \\ x_3 &= 1 \cdot x_3 + 0 \cdot x_5 \\ x_4 &= 0 \cdot x_3 + 5 \cdot x_5 \\ x_5 &= 0 \cdot x_3 + 1 \cdot x_5 \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ -4 \\ 0 \\ 5 \\ 1 \end{bmatrix}$$

Question 15 9 pts

Find the coordinates change matrix when we change from basis $\mathcal{B}$ to basis $\mathcal{C}$, where
$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.

Your answer is $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ where

$a_{11} =$ ___ , $a_{12} =$ ___ , $a_{13} =$ ___

$a_{21} =$ ___ , $a_{22} =$ ___ , $a_{23} =$ ___

$a_{31} =$ ___ , $a_{32} =$ ___ , $a_{33} =$ ___

$$\frac{M_C^{-1} M_B}{M_C}$$

Question 10 6 pts

Let V be the vector space of all 3×3 matrices with real number entries. Let H_1 be the subset of V that contains all 3×3 triangular matrices and H_2 be the subset of V that contains all 3×3 matrices whose traces are integers. Choose all statements that are correct.

- ✓ ☐ H_2 is closed under matrix addition.
- ✓ ☐ H_2 is NOT a subspace of V.
- ✗ ☐ H_1 is closed under matrix addition.
- ✗ ☐ H_2 is closed under scalar multiplication.
- ✗ ☐ H_1 is a subspace of V.
- ✓ ☐ H_1 is closed under scalar multiplication.

$$\frac{1}{5}\begin{bmatrix} 1 & 0 & 1 \\ 0 & \frac{1}{2} & 0 \\ 0 & 5 & 3/2 \end{bmatrix} + \begin{bmatrix} \frac{1}{3} & & * \\ & \frac{2}{3} & \\ * & & 5 \end{bmatrix}$$

#2.

$\underline{B}$

Question 9 6 pts

Determine the values of k so that the vectors $\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} k & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 0 & 3 \end{bmatrix}$ are linearly dependent. If your answer is a fraction, enter it as a fraction.

$k =$ ___

$$\begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix} \quad \begin{bmatrix} k \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 2 \\ -2 \\ 0 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

$$\det \begin{bmatrix} 2 & k & 2 \\ 3 & 1 & -2 \\ -1 & 1 & 3 \end{bmatrix} = 0 \quad \text{Solve for } k.$$

Chapter 5: Problem 7

(1 point) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_6.pg

This set is **visible to students.**

$\{1, x, x^2\}$

The set $B = \{-(1+3x^2),\ -(3+2x+9x^2),\ -(8+4x+27x^2)\}$ is a basis for P_2. Find the coordinates of $p(x) = 9 + 4x + 30x^2$ relative to this basis:

$[p(x)]_B = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$

$$V = \begin{bmatrix} 9 \\ 4 \\ 30 \end{bmatrix}$$

$$B = \left\{ \begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ -9 \end{bmatrix}, \begin{bmatrix} -8 \\ -4 \\ -27 \end{bmatrix} \right\}$$

$$M_B^{-1} \cdot V = \begin{bmatrix} -1 & -3 & -8 \\ 0 & -2 & -4 \\ -3 & -9 & -27 \end{bmatrix}^{-1} \begin{bmatrix} 9 \\ 4 \\ 30 \end{bmatrix}$$

$$\begin{bmatrix} -3 \\ -1 \\ 4 \end{bmatrix} \Rightarrow -3$$