Problem 1. (1 point)

Let $M_{n,n}(\mathbb{R})$ denote the vector space of $n\times n$ matrices with real entries. Let $f : M_{2,2}(\mathbb{R}) \to M_{2,2}(\mathbb{R})$ be the function defined by $f(A) = A^T$ for any $A \in M_{2,2}(\mathbb{R})$. Is f a linear transformation?

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ be any two matrices in $M_{2,2}(\mathbb{R})$ and let $c \in \mathbb{R}$.

(1) $f(A+B) = \begin{bmatrix} \_\_\_ & \_ \\ \_\_\_ & \_ \end{bmatrix}$. (Enter a_{11} as a11, etc.)

$f(A)+f(B) = \begin{bmatrix} \_\_\_ & \_ \\ \_\_\_ & \_ \end{bmatrix} + \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$.

Does $f(A+B) = f(A)+f(B)$ for all $A,B \in M_{2,2}(\mathbb{R})$?

- choose
- Yes, they are equal
- No, they are not equal

(2) $f(cA) = \begin{bmatrix} \_\_\_ & \_ \\ \_\_\_ & \_ \end{bmatrix}$.

$c(f(A)) = \_\_ \left(\begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \right)$.

Does $f(cA) = c(f(A))$ for all $c \in \mathbb{R}$ and all $A \in M_{2,2}(\mathbb{R})$?

- choose
- Yes, they are equal
- No, they are not equal

(3) Is f a linear transformation?

- choose
- f is a linear transformation
- f is not a linear transformation

Problem 2. (1 point)

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that sends the vector $\vec{u} = (5,2)$ into $(2,1)$ and maps $\vec{v} = (1,3)$ into $(-1,3)$. Use properties of a linear transformation to calculate the following. (Enter your answers as ordered pairs, such as (1,2), including the parentheses.)

$T(5\vec{u}) =$ ______,

$T(-7\vec{v}) =$ ______,

$T(5\vec{u} - 7\vec{v}) =$ ______.

Problem 3. (1 point)

Determine which of the following transformations are linear transformations.

- A. The transformation T defined by $T(x_1,x_2,x_3) = (x_1,0,x_3)$
- B. The transformation T defined by $T(x_1,x_2,x_3) = (1,x_2,x_3)$
- C. The transformation T defined by $T(x_1,x_2,x_3) = (x_1,x_2,-x_3)$
- D. The transformation T defined by $T(x_1,x_2) = (4x_1 - 2x_2, 3|x_2|)$.
- E. The transformation T defined by $T(x_1,x_2) = (2x_1 - 3x_2, x_1+4, 5x_2)$.

Problem 4. (1 point)

Which of the following transformations are linear? Select all of the linear transformations. There may be more than one correct answer. Be sure you can justify your answers.

- A. $T(A) = SAS^{-1}$ from $\mathbb{R}^{2\times 2}$ to $\mathbb{R}^{2\times 2}$, where $S = \begin{bmatrix} 7 & 6 \\ -8 & 0 \end{bmatrix}$
- B. $T(A) = A\begin{bmatrix} -3 & 4 \\ 6 & 7 \end{bmatrix}$ from $\mathbb{R}$ to $\mathbb{R}^{2\times 2}$
- C. $T(A) = \det(A)$ from $\mathbb{R}^{3\times 3}$ to $\mathbb{R}$
- D. $T(A) = A\begin{bmatrix} 9 & -1 \\ 2 & 6 \end{bmatrix} - \begin{bmatrix} 9 & 1 \\ 2 & 5 \end{bmatrix}A$ from $\mathbb{R}^{2\times 2}$ to $\mathbb{R}^{2\times 2}$
- E. $T(A) = ASA^{-1}$ from $\mathbb{R}^{2\times 2}$ to $\mathbb{R}^{2\times 2}$, where $S = \begin{bmatrix} -6 & 3 \\ 1 & -2 \end{bmatrix}$
- F. $T(A) = A^T$ from $\mathbb{R}^{5\times 3}$ to $\mathbb{R}^{3\times 5}$

Problem 5. (1 point)

Let V be a vector space, and $T : V \to V$ a linear transformation such that $T(5\vec{v}_1 + 3\vec{v}_2) = -2\vec{v}_1 - 3\vec{v}_2$ and $T(3\vec{v}_1 + 2\vec{v}_2) = 3\vec{v}_1 - 5\vec{v}_2$. Then

$T(\vec{v}_1) =$ ____ $\vec{v}_1 +$ ____ $\vec{v}_2$,

$T(\vec{v}_2) =$ ____ $\vec{v}_1 +$ ____ $\vec{v}_2$,

$T(-4\vec{v}_1 + 2\vec{v}_2) =$ ____ $\vec{v}_1 +$ ____ $\vec{v}_2$.

Problem 6. (1 point)

Let $T : P_3 \to P_3$ be the linear transformation such that

$T(2x^2) = -2x^2 - 4x$, $T(-0.5x+5) = -2x^2+4x-4$, $T(4x^2+1) = -4x+1$.

Find $T(1)$, $T(x)$, $T(x^2)$, and $T(ax^2+bx+c)$, where a, b, and c are arbitrary real numbers.

$T(1) =$ ______________________,
$T(x) =$ ______________________,
$T(x^2) =$ ______________________,
$T(ax^2+bx+c) =$ ______________________.

Problem 7. (1 point)

Determine which of the following functions are one-to-one.

- A. $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x,y) = (x+y, x-y)$.
- B. $f : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $f(x,y,z) = (x+y, y+z, x+z)$.
- C. $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$.
- D. $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x,y) = (x+y, 2x+2y)$.
- E. $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 - x$.

Problem 8. (1 point)

Let T be a linear transformation from $\mathbb{R}^r$ to $\mathbb{R}^s$.

Determine whether or not T is one-to-one in each of the following situations:

___1. $r < s$
___2. $r > s$
___3. $r = s$

A. T is not a one-to-one transformation
B. T is a one-to-one transformation
C. There is not enough information to tell

Problem 9. (1 point)

Let T be an linear transformation from $\mathbb{R}^r$ to $\mathbb{R}^s$. Let A be the matrix associated to T.

Fill in the correct answer for each of the following situations.

___1. Every column in the row-echelon form of A is a pivot column.
___2. The row-echelon form of A has a column corresponding to a free variable.
___3. The row-echelon form of A has no column corresponding to a free variable.
___4. Two columns in the row-echelon form of A are not pivot columns.

A. T is one-to-one
B. T is not one-to-one
C. There is not enough information to tell.

Problem 10. (1 point)

Determine which of the following functions are onto.

- A. $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x,y) = (x+y, x-y)$.
- B. $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$.
- C. $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 + x$.
- D. $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x,y) = (x+y, 2x+2y)$.
- E. $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3$.

Problem 11. (1 point)

Match the following concepts with the correct definitions:

___1. f is an onto function from $\mathbb{R}^3$ to $\mathbb{R}^3$
___2. f is a function from $\mathbb{R}^3$ to $\mathbb{R}^3$
___3. f is a one-to-one function from $\mathbb{R}^3$ to $\mathbb{R}^3$

A. For every $x \in \mathbb{R}^3$, there is a $y \in \mathbb{R}^3$ such that $f(x) = y$.
B. For every $y \in \mathbb{R}^3$, there is a $x \in \mathbb{R}^3$ such that $f(x) = y$.
C. For every $y \in \mathbb{R}^3$, there is a unique $x \in \mathbb{R}^3$ such that $f(x) = y$.
D. For every $y \in \mathbb{R}^3$, there is at most one $x \in \mathbb{R}^3$ such that $f(x) = y$.

Problem 12. (1 point)

The matrix

$$A = \begin{bmatrix} -1 & 1 & -1 & 2 \\ -1.5 & 1.5 & -1.5 & 3 \\ 2 & -2 & 2 & -4 \end{bmatrix}$$

is a matrix of a linear transformation $T : \mathbb{R}^k \to \mathbb{R}^n$.

(1) Find k and n.
$k =$___, $n =$___,

(2) Find the dimension of the kernel (or null space) and range (or image).
dim(Ker(T)) = ___, dim(Range(T)) =___.

(3) Is T onto? [choose/onto/not onto]

(4) Is T one-to-one?
- choose
- one-to-one
- not one-to-one

Problem 13. (1 point)

Let $f:\mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation determined by $f(\vec{x}) = A\vec{x}$ where

$$A = \begin{bmatrix} 7 & 0 \\ -6 & 3 \\ -4 & 2 \end{bmatrix}$$

(1) Find bases for the kernel and image of f. vector

A basis for Kernel(f) is $\left\{ \underline{\hspace{5em}} \right\}$.

A basis for Image(f) is $\left\{ \underline{\hspace{5em}} \right\}$.

(2) The dimension of the kernel of f is ___ and the dimension of the image of f is ___.

(3) The linear transformation f is

- injective
- surjective
- bijective
- none of these

Problem 14. (1 point)

Let $f:\mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $f(x,y) = \langle 4x-2y, 2x+2y \rangle$.

(1) Find the matrix of the linear transformation f.

$$f(x,y) = \begin{bmatrix} \_\_\_ & \_\_ \\ \_\_\_ & \_\_ \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

(2) The linear transformation f is

- injective
- surjective
- bijective
- none of these

(3) If f is bijective, find the matrix of its inverse. If f is not bijective, enter DNE in every answer blank.

$$f^{-1}(x,y) = \begin{bmatrix} \_\_\_ & \_\_ \\ \_\_\_ & \_\_ \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

Problem 15. (1 point)

Let T be the linear transformation defined by

$T(x,y) = (4x+5y, -2y, 9y-x, -6x-7y)$.

Find its associated matrix A.

$$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}.$$

Problem 16. (1 point)

Consider a linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$ for which

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

Find the matrix A of T.

$$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}.$$

Problem 17. (1 point)

Find the determinant of the linear transformation

$$T(M) = \begin{bmatrix} 4 & -5 \\ 0 & 9 \end{bmatrix} M$$

from the space V of upper triangular 2×2 matrices to V.

det = ___

Problem 18. (1 point)

Find the matrix A of the linear transformation $T(f(t)) = f(-6)$ from P_2 to P_2 with respect to the standard basis for P_2, $\{1, t, t^2\}$.

$$A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

Note: You should be viewing the transformation as mapping to constant polynomials rather than real numbers, e.g. $T(2+t-t^2) = -4+0t+0t^2$.

Problem 19. (1 point)

Find the matrix A of the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix} x_1 + \begin{bmatrix} 5 \\ 4 \\ 7 \end{bmatrix} x_2.$$

$$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

Problem 20. (1 point)

Let $A = \begin{bmatrix} 4 & -2 & -2 \\ -2 & 0 & -2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -2 \\ -6 \\ 2 \end{bmatrix}$, and $\mathbf{c} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$. Define $T(\mathbf{x}) = A\mathbf{x}$.

Select true or false for each statement.

[?] 1. The vector $\mathbf{b}$ is in the kernel of T

[?] 2. The vector $\mathbf{c}$ is in the range of T

Problem 21. (1 point)

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by

$$T(x,y) = (8x + 8y, 8x + 8y).$$

Find a vector $\vec{w}$ that is not in the image of T.

$\vec{w} = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$.

Problem 22. (1 point)

Let

$$A = \begin{bmatrix} -4 & 5 & 5 \\ 5 & -2 & -4 \\ 2 & 5 & -6 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} -20 \\ -1 \\ -6 \end{bmatrix}.$$

Define the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T(\vec{x}) = A\vec{x}$. Find a vector $\vec{x}$ whose image under T is $\vec{b}$.

$\vec{x} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Is the vector $\vec{x}$ unique? [choose/unique/not unique]

Problem 23. (1 point)

Let

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 0 & 0 & 0 \\ 0 & -2 & 4 \\ 0 & 2 & -4 \end{bmatrix}.$$

Find dimensions of the kernel and image of $T(\vec{x}) = A\vec{x}$.

$\dim(\mathrm{Ker}(A)) = \_\_$,

$\dim(\mathrm{Im}(A)) = \_\_$.

Problem 24. (1 point)

Let

$$A = \begin{bmatrix} -6 & 2 \\ -9 & 3 \end{bmatrix}.$$

Find bases for the kernel and image of $T(\vec{x}) = A\vec{x}$.

A basis for the kernel of A is $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix} \right\}$.

A basis for the image of A is $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix} \right\}$.

Problem 25. (1 point)

Let $V = \mathbb{R}^{2\times 2}$ be the vector space of 2×2 matrices and let $L : V \to V$ be defined by $L(X) = \begin{bmatrix} -6 & 3 \\ -8 & 4 \end{bmatrix} X$. Hint: The image of a spanning set is a spanning set for the image.

a. Find $L\left(\begin{bmatrix} 5 & 2 \\ -1 & -2 \end{bmatrix}\right) = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

b. Find a basis for $\ker(L)$:

$\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

c. Find a basis for $\mathrm{ran}(L)$:

$\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

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