Lecture 6: Hypotheses Testing (II)

P-value

Definition:

The probability, computed assuming H_0 is true, that the statistic would take a value as extreme as or more extreme than the one actually observed is called the ***P*-value** of the test. The smaller the *P*-value, the stronger the evidence against H_0 provided by the data.

- ✓ It measures whether the test statistic is **likely** or **unlikely**, assuming H_0 is true.
- ✓ Small p-values suggest that the null hypothesis is unlikely to be true. The smaller it is, the more convincing is the rejection of the null hypothesis.
- ✓ It indicates the strength of evidence for rejecting the null hypothesis H_0.

Decision

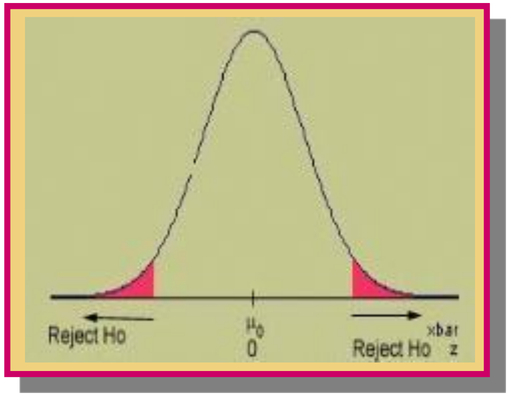
A decision as to whether H_0 should be rejected results from comparing the p-value to the chosen significance level α:

- H_0 should be rejected if p-value $\leq \alpha$.
- H_0 should not be rejected if p-value $> \alpha$.

Large Sample Test for Population Mean

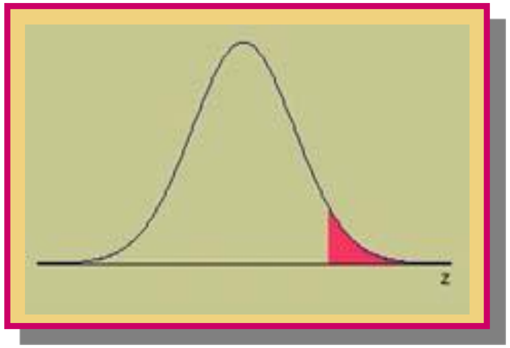
– **H_a: $\mu \neq \mu_0$** (two-tail test)

$$\text{p-value} = P(z < -|z^*|) + P(z > |z^*|) = 2P(z > |z^*|)$$



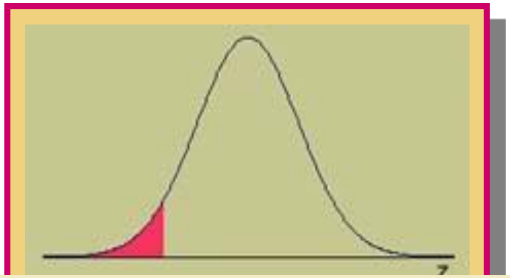
– **H_a: $\mu > \mu_0$** (right-tail test)

$$\text{p-value} = P(z > z^*)$$



– **H_a: $\mu < \mu_0$** (left-tail test)

$$\text{p-value} = P(z < z^*)$$



P(z>|z*|), P(z>z*) and P(z<z*) can be found from the normal table

Example 1

A homeowner wants to estimate the average house price in the neighborhood. She randomly samples 64 homes similar to her own and finds that the average selling price is \$252,000 with a standard deviation of \$15,000. Is this sufficient evidence to conclude that the average selling price is greater than \$250,000? Use $\alpha = .01$.

Example 2

The daily yield for a chemical plant has averaged 880 tons for several years. The quality control manager wants to know if this average has changed. She randomly selects 50 days and records an average yield of 871 tons with a standard deviation of 21 tons. Conduct the test using $\alpha=.05$.

Small Sample Test for Population Mean

Step 1: Specify the null and alternative hypothesis

- **H_0: $\mu = \mu_0$ versus H_a: $\mu \neq \mu_0$** (two-tail test)
- **H_0: $\mu = \mu_0$ versus H_a: $\mu > \mu_0$** (right-tail test)
- **H_0: $\mu = \mu_0$ versus H_a: $\mu < \mu_0$** (left-tail test)

Step 2: Test statistic for small sample

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

where n, $\bar{x}$ and s are sample size, mean and standard deviation

Step 3: When samples are from a normal population, under H_0, the sampling distribution of t has a Student's t distribution with n-1 degrees of freedom.

Small Sample Test for Population Mean

Step 3: Find p-value. Compute sample statistic $t^* = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$

- $\mathbf{H_a: \mu \neq \mu_0}$ (two-tail test) $\quad$ $\text{p-value} = 2P(t > |t^*|)$
- $\mathbf{H_a: \mu > \mu_0}$ (right-tail test) $\quad$ $\text{p-value} = P(t > t^*)$
- $\mathbf{H_a: \mu < \mu_0}$ (left-tail test) $\quad$ $\text{p-value} = P(t < t^*)$

$P(t>|t^*|)$, $P(t>t^*)$ and $P(t<t^*)$ can be found from the t table

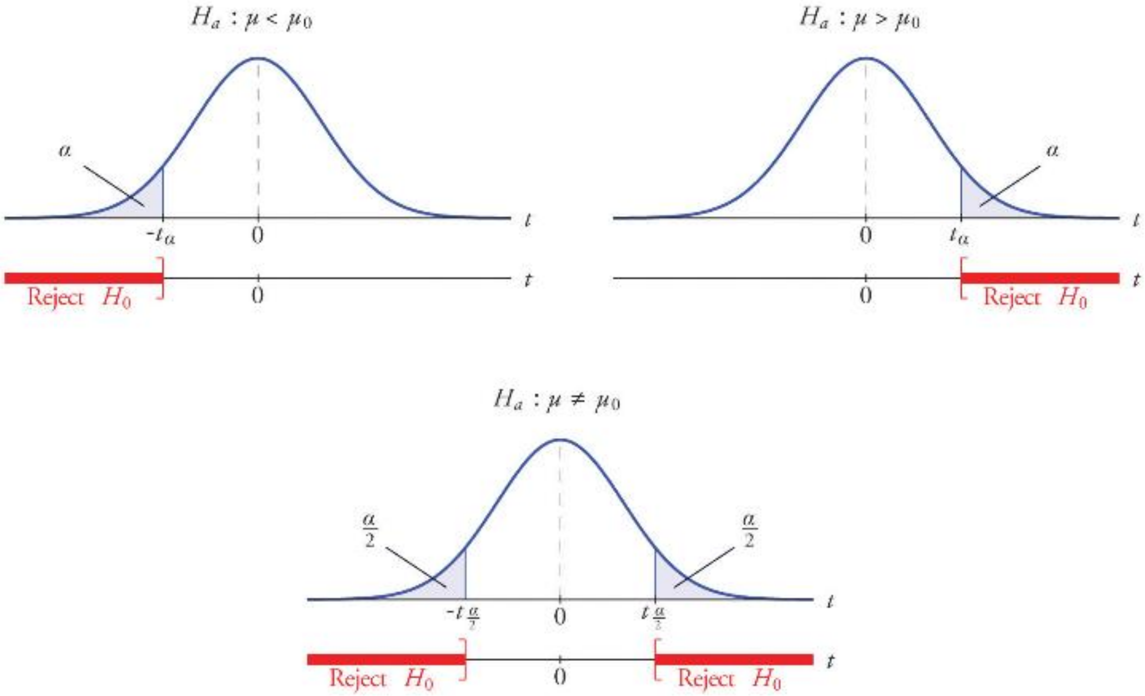
Example

A sprinkler system is designed so that the average time for the sprinklers to activate after being turned on is **no more than** 15 seconds. An agency claims that the system may not work as it proposes. A test of 5 systems gave the following times:

17, 31, 12, 17, 13, 25

Test agency's claim using $\alpha = .05$.

Rejection for small sample case



Example: In the population of Americans who drink coffee, the average daily consumption is 3 cups per day. A university wants to know if their students tend to drink more coffee than the national average. They ask 25 students how many cups of coffee they drink each day and found $\bar{x} = 3.8$ and s=1.5. At $\alpha = .05$, do they have evidence that their students drink more than the national average?