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A generalized Hollander–Proschan type test for NBUE alternatives

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ABSTRACT

In this note we develop a family of test statistics for testing exponentiality against NBUE alternatives. The asymptotic distribution of the test statistics is derived. The test statistics are shown to be asymptotically normal and consistent. This family of test statistics includes the test proposed by Hollander and Proschan (1975) as a special case. Efficiency studies have also been done.

1. Introduction

The assumption of exponentiality is widely used in the theory of reliability and life testing. This essentially implies that a used item is stochastically as good as a new one. The unit in question does not age with time. Hence, there is no reason to replace a unit which is working. However, this is not always a realistic assumption; and age does have an effect on the residual life time. Positive (negative) aging means that age has an adverse (beneficial) effect, in some probabilistic sense, on the residual life. Hence, it is of interest to check possible departure from exponentiality in the data.

When aging is studied using the failure rate function, we get the well-known classes of increasing failure rate (IFR), increasing failure rate average (IFRA), and new better than used (NBU); and their corresponding duals. On the other hand when the mean residual life is the vehicle for the study of aging, we get the classes of decreasing mean residual life (DMRL), new better than used in expectation (NBUE) and harmonic new better than used in expectation (HNBUE). It is well known that

$$\text{IFR} \Rightarrow \text{IFRA} \Rightarrow \text{NBU} \Rightarrow \text{NBUE} \Rightarrow \text{HNBUE}$$

and

$$\text{IFR} \Rightarrow \text{DMRL} \Rightarrow \text{NBUE} \Rightarrow \text{HNUBE}.$$

We shall, for completeness, define the NBUE class here and refer the interested reader to Barlow and Proschan (1975) and Bryson and Siddiqui (1969) for definition and properties of the other classes.

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Definition 1.1. A non-negative random variable X with finite mean μ is said to possess the NBUE (*New Better than Used in Expectation*) property if $E[X - t|X > t] \leq E[X]$; i.e.

$$\int_t^\infty \overline{F}(x)\mathrm{d}x \leq \mu\overline{F}(t).$$

The classification of a life time distribution is important and useful. It helps, for example, to derive bounds for the reliability function as is seen in the works of Korzeniowski and Opawski (1976), Cheng and He (1989) and Cai (1995). Moreover it is a tool to reject some parametric model for the data. For example, if we know that the underlying distribution is gamma, with shape parameter θ, and NBUE, then θ must be greater than unity. Other applications can be found in Fagiouli et al. (1999). Important results for this class can be found in Barlow and Proschan (1975). Specifically the NBUE family of distributions is important in the study of replacement policies. In particular, Marshall and Proschan (1972) have shown that the average waiting time between any two consecutive failures when no planned replacement policies are adopted is smaller than or equal to the similar quantity when an age replacement policy is adopted if and only if the life distribution is NBUE. Moreover, this average waiting time is the same under both policies if the system life is exponential. If the average waiting time between the consecutive failures is an important criterion in deciding whether to adopt an *age replacement policy* over the *failure replacement policy* for a given system, then a reasonable way to decide would be to test whether the life distribution of the given system is exponential. Rejection of the exponentiality hypothesis on the basis of the observed data would suggest as favouring the adoption of age replacement plan.

The problem of testing exponentiality against a particular aging class has received considerable attention in the literature; see Lai (1994) and Lai and Xie (2006) for an overview of such procedures. In particular, the problem of testing exponentiality against NBUE alternatives was first considered by Hollander and Proschan (1975). Subsequently Koul (1978) presented an ingenious test for the same problem. Later de Souza Borges et al. (1984) proposed a test based on the coefficient of variation. Kanjo (1993) proposed an exact test for this problem. Fernandez-Ponce et al. (1996) suggested a test based on a property of the right-spread function to characterize different partial orderings between lifetime distributions. Belzunce et al. (2000) have considered the same problem by means of the dispersion of residual lives. Recently Belzunce et al. (2001) have proposed a test based on the right spread order, while Ahmad (2001) proposed a test using moment inequalities.

While most test procedures available in the literature define a single test statistic for the problem at hand, some also provide a class of test statistics. Mention may be made of Deshpande (1983) and Ahmad (1994) for the IFRA class; Bandyopadhyay and Basu (1990) and Bergman and Klefsjö (1989) for the DMRL class; and Klefsjö (1983) and Klar (2000) for the HNBUE class. To the best of our knowledge no attempt has been made to propose a class of statistics for the NBUE class. It should be noted that the major advantage in having a class of test statistics for a testing problem is that we may be able to pick up members of the class which have maximum efficiency or power. We address this problem in this paper. Essentially, we propose a generalized Hollander–Proschan type of test statistics for testing exponentiality against NBUE alternatives. Our test statistic is an L-statistic; i.e. a linear combination of order statistics. It is interesting to note that our proposed family of test statistics includes the test statistic proposed by Hollander and Proschan (1975) as a special case. This is discussed in Section 2. In Section 3, we find the asymptotic distribution of the proposed family of test statistics. The test is shown to be consistent in Section 4. A simulation study is reported in Section 5, while Pitman efficacy is discussed in Section 6. Examples demonstrating how the test procedure can be carried are presented in Section 7; while Section 8 concludes the paper.

2. Detecting the NBUE (NWUE) property

Let $\mathcal{E}$ be the class of exponential distributions, with the distribution function $F(x) = 1 - \mathrm{e}^{-\lambda x},\ x \geq 0$ where λ is any positive number, typically unknown. Formally our problem is to test

$$H_0 : F \varepsilon \mathcal{E}$$
$$vs. H_1 : F \varepsilon \text{ NBUE} - \mathcal{E}$$

based on a random sample $X_1, X_2, \ldots, X_n$ of size n from an absolutely continuous distribution F with density f and survival function $\overline{F}$.

We propose to measure the deviation of a NBUE distribution from exponentiality using the measure $\gamma_j(F)$ defined below:

$$\gamma_j(F) = \int_0^\infty [\bar{F}(t)]^j [e_F(0) - e_F(t)]\mathrm{d}F(t), \tag{1}$$

where j is a positive real number and $e_F(t) = \int_t^\infty \bar{F}(u)\mathrm{d}u/\bar{F}(t)$. The parameter $\gamma_j(F)$ in (1) may be viewed as follows. Define

$$D(t) = \overline{F}^j(t)\left\{e_F(0) - e_F(t)\right\}.$$

Observe that $D(t) = 0$ if and only if H_0 is true. $D(t)$ is a weighted measure of the deviation from H_0 towards H_1; and $\gamma_j(F)$ is an average value of this deviation. Clearly, $\gamma_j(F) = 0$ when F is exponential. Let us simplify $\gamma_j(F)$. Now,

$$\begin{aligned}\gamma_j(F) &= \int_0^\infty [\bar{F}(t)]^j \left[\mu - \frac{1}{\bar{F}(t)} \int_t^\infty \bar{F}(x)\mathrm{d}x\right] \mathrm{d}F(t) \\ &= \mu \int_0^\infty [\bar{F}(t)]^j \mathrm{d}F(t) - \int_0^\infty [\bar{F}(t)]^{j-1} \left[\int_t^\infty \bar{F}(x)\mathrm{d}x\right] \mathrm{d}F(t) \\ &= \mu I_1 - I_2, \text{ (say)}.\end{aligned} \tag{2}$$

On simplification, we have

$$\begin{aligned} I_1 &= \frac{1}{(j+1)} \\ \text{and} \quad I_2 &= \int_0^\infty \bar{F}(x) \frac{1}{j}\left[1 - \left\{\bar{F}(x)\right\}^j\right] \mathrm{d}x \\ &= \frac{1}{j}\left[\mu - \int_0^\infty \left\{\bar{F}(x)\right\}^{j+1} \mathrm{d}x\right]. \end{aligned}$$

We, therefore, re-write (2) as:

$$\gamma_j(F) = \int_0^\infty \left[a_1 \bar{F}(x) + a_2 \left\{\bar{F}(x)\right\}^{j+1}\right] \mathrm{d}x, \tag{3}$$

where $a_1 = -\frac{1}{j(j+1)}$ and $a_2 = \frac{1}{j}$. In order to make $\gamma_j(F)$ scale invariant, we shall consider

$$\gamma_j^\star(F) = \frac{\gamma_j(F)}{\mu}.$$

We now replace the unknown life distribution function F by the empirical distribution function F_n to obtain

$$\gamma_j(F_n) = \int_0^\infty \left\{a_1 \bar{F}_n(x) + a_2 \left(\bar{F}_n(x)\right)^{j+1}\right\} \mathrm{d}x$$

which on simplification yields

$$\gamma_j(F_n) = \sum_{k=1}^n X_{(k)} \left[\frac{a_1}{n} + a_2 \left\{\left(\frac{n-k+1}{n}\right)^{j+1} - \left(\frac{n-k}{n}\right)^{j+1}\right\}\right], \tag{4}$$

where $X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)}$ denotes the order statistics based on the random sample $X_1, \ldots, X_n$. Hence, the empirical version of $\gamma_j^\star(F_n)$ will analogously be given by

$$\gamma_j^\star(F_n) = \frac{\gamma_j(F_n)}{\bar{X}_n} \tag{5}$$

with $\gamma_j(F_n)$ as described in (4).

Remark 2.1. It is important to note that this statistic reduces to that of Hollander and Proschan (1975) for the special case $j = 1$. In fact we have $\gamma_1(F_n) = \frac{1}{n^2} \sum_{i=1}^n X_{(i)} \left\{\frac{3n}{2} - 2i + 1\right\}$. In their paper, Hollander and Proschan (1975) erroneously gave the expression as $K = \frac{1}{n^2} \sum_{i=1}^n X_{(i)} \left\{\frac{3n}{2} - 2i + \frac{1}{2}\right\}$. However, this does not affect their asymptotic results as the difference is of the order of $1/n^2$.

3. The asymptotic distribution of the test statistic

We shall now derive the asymptotic distribution of our test statistic. First, we state the following useful theorem.

Theorem 3.1. *Let X be a continuous non-negative random variable, such that $E[X^2] < +\infty$ and let*

$$\mu(J_j, F(x)) = \int_0^\infty x J_j(F(x)) \mathrm{d}F(x)$$

and

$$\sigma^2(J_j, F) = \int_0^\infty \int_0^\infty J_j(F(x)) J_j(F(y)) \left\{F(\min(x, y)) - F(x)F(y)\right\} \mathrm{d}x\mathrm{d}y$$

where

$$J_j(u) = a_1 + a_2(j+1)(1-u)^j.$$

Then

$$\frac{\sqrt{n}(\gamma_j(F_n) - \mu(J_j, F))}{\sigma(J_j, F)} \xrightarrow{L} N(0, 1).$$

The above theorem is an easy consequence of Theorems 2 and 3 of Stigler (1974). Using Theorem 3.1 and Slutsky's theorem, we obtain the limiting distribution of $\gamma_j^\star(F_n)$ which is presented in the next theorem.

Theorem 3.2. *Under the conditions of Theorem 3.1 it follows that*

$$\sqrt{n}\left(\gamma_j^\star(F_n) - \frac{\mu(J_j, F)}{E[X]}\right) \xrightarrow{L} N\left(\mu^*, (\sigma^*)^2\right)$$

where $\mu^* = \mu(J_j^*, F)/E(X)$ *and* $(\sigma^*)^2 = \sigma^2(J_j^*, F)/(E(X))^2$ *and* $J_j^*(u) = J_j(u) - \frac{\mu(J_j,F)}{\mu}$.

Let us now obtain the limiting distribution under the null hypothesis of exponentiality. Since the statistic $\gamma_j^\star(F)$ is scale invariant, we may without loss of generality take the scale parameter λ to be equal to 1. Hence, in particular, we get under H_0, i.e. when $F(x) = F_0(x) = 1 - e^x$, $x \geq 0$ that

$$\mu(J_j, F_0) = \gamma_j(F_0) = 0$$

and

$$\sigma^2(J_j, F_0) = \frac{1}{(2j+1)(j+1)^2},$$

after tedious but straight forward algebra. Thus, under H_0 the limiting distribution of $(j+1)\sqrt{n(2j+1)}\gamma_j^\star(F_n)$ is $N(0, 1)$. Hence, for large values of n, we reject the null hypothesis of exponentiality if

$$\frac{\sqrt{n}\gamma_j^\star(F_n)}{\sigma(J_j, F_0)} > z_\alpha$$

where z_α is the upper α-percentile of $N(0, 1)$.

Remark 3.1. Note that for $j = 1$ the variance reduces to $\frac{1}{12}$, which agrees with the result obtained in Hollander and Proschan (1975).

Remark 3.2. If

$$\frac{\sqrt{n}\gamma_j^\star(F_n)}{\sigma(J_j, F_0)} < -z_\alpha$$

we reject the exponentiality assumption in favour of NWUE alternative.

4. Consistency

We shall now establish that the proposed test is consistent against NBUE alternatives. Since $\mu(J, F) = 0$ when F is exponentially distributed, the consistency follows if and only if $\mu(J, F) > 0$, when X is NBUE but it is not exponential. This follows from the equality $\gamma_j(F) = \mu(J_j, F)$ which we prove below.

Theorem 4.1. *Let X be a continuous non-negative random variable and let us suppose that* $\lim_{x\to+\infty} x\overline{F}(x) = 0$. *Then,* $\gamma_j(F) = \mu(J_j, F)$ *where J_j is as given in Section 3.*

Proof. Substituting for $J_j(u)$, we have that

$$\mu\left(J_j, F\right) = \int_0^\infty xJ_j\left(F(x)\right) \mathrm{d}F(x) = \int_0^\infty x\left\{a_1 + a_2\left(j+1\right)\left(\overline{F}(x)\right)^j\right\} \mathrm{d}F(x).$$

It is easy to see that the first integral is equal to

$$a_1 \int_0^\infty \overline{F}(x)\mathrm{d}x.$$

Table 1
Empirical power estimates.

j	Gamma (θ) distribution				Weibull (θ) distribution				LFR (θ) distribution			
	1.4	1.6	1.8	2.0	1.2	1.3	1.4	1.5	0.25	0.50	0.75	1.00
0.50	0.7179	0.9250	0.9875	0.9985	0.7258	0.9489	0.9943	0.9995	0.4217	0.6996	0.8505	0.9273
1.00	0.7783	0.9565	0.9940	0.9996	0.7590	0.9592	0.9963	0.9995	0.4053	0.6774	0.8318	0.9129
1.50	0.8144	0.9695	0.9969	0.9998	0.7740	0.9619	0.9974	0.9995	0.3848	0.6448	0.8026	0.8921
2.00	0.8334	0.9762	0.9983	1.0000	0.7790	0.9627	0.9975	0.9995	0.3692	0.6146	0.7773	0.8699
2.50	0.8468	0.9793	0.9988	1.0000	0.7821	0.9624	0.9977	0.9996	0.3541	0.5893	0.7531	0.8510
3.00	0.8572	0.9815	0.9990	1.0000	0.7826	0.9617	0.9973	0.9995	0.3407	0.5678	0.7308	0.8323
3.50	0.8644	0.9827	0.9990	1.0000	0.7820	0.9601	0.9969	0.9995	0.3293	0.5476	0.7104	0.8118
4.00	0.8696	0.9838	0.9991	1.0000	0.7792	0.9583	0.9963	0.9996	0.3191	0.5303	0.6885	0.7926
4.50	0.8717	0.9846	0.9991	1.0000	0.7784	0.9566	0.9961	0.9995	0.3133	0.5176	0.6717	0.7740
5.00	0.8764	0.9851	0.9993	1.0000	0.7763	0.9542	0.9959	0.9995	0.3058	0.5053	0.6532	0.7578

The second integral on integrating by parts simplifies to

$$\int_0^\infty a_2\overline{F}^{j+1}(x)\mathrm{d}x.$$

Thus,

$$\mu\left(J_j, F\right) = \int_0^\infty \left\{a_1\overline{F}(x) + a_2\overline{F}^{j+1}(x)\right\}\mathrm{d}x = \gamma_j(F). \quad \square$$

5. A simulation study

In this section we report a simulation study done to evaluate the performance of our test against various alternatives. The simulation was done using Matlab 7.5 on PC platform.

We simulate observations from the Weibull and the gamma distributions (for different shape parameter ($\theta > 1$)) and the linear failure rate (LFR) distribution (for different shape parameter ($\theta > 0$)) as they are typical members of the NBUE class. A random sample of size hundred (a large sample) is drawn from each of these distributions and the value of the test statistic $\gamma_j^\star(F)$ is calculated. Since the test is scale invariant, we can take the scale parameter λ to be unity, without loss of generality, while performing the simulations. We check whether this particular realization of the test statistic accepts or rejects the null hypothesis of exponentiality. Next we repeat the whole procedure ten thousand times and observe the proportion of times the proposed test statistic takes the correct decision of rejecting the null hypothesis H_0. This procedure has been repeated for different values of j. It is observed that all the statistics $\left\{\gamma_j^\star(F)\right\}$ perform very well. Thus we estimate the power of the test in different situations.

The empirical power obtained in the three situations based on simulation is reported in Anis (2009) for $j = 0.25(0.25)10$. Here, we reproduce a subset of the table for illustration. It is clear from Table 1 that the proposed test enjoys high power even for small departures from exponentiality for the gamma and Weibull distributions. For the LFR distribution, the proposed test is better than the test proposed by Hollander and Proschan (1975) for $j < 1$. The performance declines sharply as j increases for the LFR distribution.

6. Efficacy values

When comparing consistent tests of a simple hypothesis $\theta = \theta_0$ against an alternative $\theta > \theta_0$ (say) different measures of asymptotic efficiencies can be applied (see e.g. Rao (1973), pp. 464–468). When the test statistic T, based on n observations, is asymptotically normally distributed with mean $\mu_T(\theta)$ and variance $\sigma_T^2(\theta)/n$ the most frequently used measure when testing $H_0 : F \in \mathcal{E}$ against a sequence of alternative distribution functions $\left\{F_{\theta_n}\right\}$, indexed by the parameter $\theta_n = \theta_0 + cn^{-\frac{1}{2}}$, where c is an arbitrary positive constant and θ_0 corresponds to the exponential distribution, is the Pitman efficacy. Specifically the Pitman efficacy of a test statistic T is given by

$$E_F(T) = \frac{\mu_T'(\theta_0)}{\sigma_T(\theta_0)},$$

whenever this expression is well defined; here the prime denotes differentiation.

The efficacy value of a test statistic can be interpreted as a power measure of the corresponding test. When testing against $\theta_n = \theta_0 + cn^{-\frac{1}{2}}$, $c > 0$, the limit of the power when $n \to \infty$ under general assumptions is equal to $\Phi\ (cE_F(T) - \lambda_\alpha)$, where Φ is the cumulative distribution function of $N(0, 1)$; see Lehmann (1975) for details. This means that if $E_F\ (T_1) > E_F\ (T_2)$, then T_1 has better asymptotic power than T_2 in the studied situation.

Table 2
Summary statistics for examples.

j	Example 1		Example 2	
	$(j+1)\sqrt{n(2j+1)}\gamma_j^*(F_n)$	p-value	$(j+1)\sqrt{n(2j+1)}\gamma_j^*(F_n)$	p-value
0.1	1.197881	0.115482	1.954224	0.025337
0.2	1.169198	0.121162	1.963601	0.024788
0.3	1.135999	0.127979	1.963715	0.024782
0.4	1.101044	0.135439	1.958485	0.025087
0.5	1.065988	0.143214	1.950395	0.025565
0.75	0.983284	0.162734	1.926677	0.027010
1.00	0.911283	0.181073	1.905891	0.028332
1.25	0.850236	0.197597	1.891210	0.029298

In our case we get

$$E_F(\gamma_j) = \frac{\mu'(J_j, F)_{\theta=\theta_0}}{\sigma(J_j, F_0)_{\theta=\theta_0}}$$

where θ_0 corresponds to the exponential distribution. The Pitman asymptotic relative efficiency of a test based on a statistic T_1 relative to that based on T_2, when both T_1 and T_2 are asymptotically normally distributed, is defined as $\{E_F(T_1)/E_F(T_2)\}^2$, i.e. the ratio between the squared efficacy values of the two statistics. Roughly the Pitman ARE is the limiting ratio of the sample sizes n_2/n_1 needed to get the same power against alternatives close to H_0.

Three of the most commonly used alternatives are:

(i) the Weibull family: $F_1(x;\theta) = 1 - \exp\left(-x^\theta\right)$ for $\theta > 1,\ x \geq 0$
(ii) the linear failure rate family: $F_2(x;\theta) = 1 - \exp\left(-\left(x + \frac{1}{2}\theta x^2\right)\right)$ for $\theta > 0,\ x \geq 0$
(iii) the Makeham family: $F_3(x;\theta) = 1 - \exp\left(-\left(x + \theta\left(x + e^{-x} - 1\right)\right)\right)$ for $\theta > 0,\ x \geq 0$

For F_1 we get the exponential distribution when $\theta = \theta_0 = 1$ and for F_2 and F_3 when $\theta = \theta_0 = 0$.

Routine but tedious direct calculations of the efficacies of these families give the following results:

(i) the Weibull family:

$$\mathrm{PAE}\left(\gamma_j^\star(F)\right) = \frac{\sqrt{2j+1}}{j}\ln(j+1)$$

(ii) the linear failure rate family:

$$\mathrm{PAE}\left(\gamma_j^\star(F)\right) = \frac{\sqrt{2j+1}}{(j+1)}$$

(iii) the Makeham family:

$$\mathrm{PAE}\left(\gamma_j^\star(F)\right) = \frac{\sqrt{2j+1}}{2(j+2)}.$$

The maximum values are located at approximately at $j = 2.25$, 0.25 and 1.0 for the Weibull, LFR and Makeham families respectively; and they are 1.228; 0.980 and 0.289. The benchmark test for NBUE due to Hollander and Proschan (1975) has PAE values equal to 1.200; 0.866 and 0.289. Hence our test is a valid competitor and we may choose the value of j accordingly if we suspect a particular alternative. Anis (2009) tabulates the efficacy values for these three distributions for $j = 0.25(0.25)10.00$. A suggestion regarding the choice of j is made after the examples in the next section.

7. Examples

One of the reviewers suggested that we use real data set to demonstrate how the test procedure for testing exponentiality against NBUE alternatives can be carried out. Accordingly we have analyzed two data sets which we describe below.

Example 1. This data set consists of $n = 27$ observations of the intervals between successive failures of the air-conditioning systems of 7913 jet airplanes of a fleet of Boeing 720 jet airplanes as reported in Proschan (1963). We have analyzed this data set for different values of j, viz. $j = 0.1(0.1)0.5(0.25)1.25$. The computed value of the statistic together with the associated p-value are reported in Table 2. It is clear that the null hypothesis of exponentiality will be accepted for all values of j. This agrees with the conclusion of Anis and Hoque (2008).

Example 2. This data is reported in Bryson and Siddiqui (1969). It represents the survival times, in days from diagnosis, of patients suffering from chronic granulocytic leukemia. As before we have analyzed this data set for different values of j, viz. $j = 0.1(0.1)0.5(0.25)1.25$. The computed value of the statistic together with the associated p-value are reported in Table 2. We observe that the null hypothesis of exponentiality is rejected for all values of j, and more strongly for lower values of j. This agrees with the conclusion of Hollander and Proschan (1975).

Remark. Observe that $\bar{F}(t) < 1$ for all $t > 0$; hence the effect of the weight $\bar{F}^j$ is to "dampen" the departure from exponentiality. Hence a small value of j seems suitable. We would recommend a value of $j = 0.25$, which seems to be a good trade-off and seems to work well with some of the typical alternatives considered.

8. Conclusion

In this paper we have studied a family of test statistics, intended for testing exponentiality against NBUE alternatives, which includes the test statistic proposed by Hollander and Proschan (1975) as a special case. We have found the asymptotic distribution of the test statistic. We have shown that our test is consistent and we have also studied the efficacy of the proposed procedure.

We have used

$$\gamma_j(F) := \int_0^\infty \overline{F}^j(t) \left\{e_F(0) - e_F(t)\right\} \mathrm{d}F(t)$$

as a measure of NBUE-ness. Of course there are many generalizations of this approach. One of these is to change $\overline{F}^j(t)$ to another weight function $w(t) \geq 0$ and then try to determine $w(t)$ in order to get as good properties as possible; e.g. with respect to power, when we use the empirical counterpart to

$$\gamma_j(F) := \int_0^\infty \overline{F}^j(t) \left\{e_F(0) - e_F(t)\right\} \mathrm{d}F(t)$$

as a test statistic for testing exponentiality against the NBUE-property.

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