Computers & Industrial Engineering 64 (2013) 11–18

Contents lists available at SciVerse ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Change-point detection in failure intensity: A case study with repairable artillery systems ☆

Suk Joo Bae [a,b,*], Byeong Min Mun [b], Kyung Yong Kim [b,c]

[a] *Department of Technology Management, Hanyang University, Seoul, Republic of Korea*
[b] *Department of Industrial Engineering, Hanyang University, Seoul, Republic of Korea*
[c] *RAM Analysis Team, Defense Agency for Technology and Quality, Seoul, Republic of Korea*

A R T I C L E   I N F O

Article history:
Received 9 February 2012
Received in revised form 7 September 2012
Accepted 8 September 2012
Available online 6 October 2012

Keywords:
Change-point
Homogeneous Poisson process
Modified information criterion
Repairable system
Schwarz information criterion
Statistical trend test

A B S T R A C T

Repairable systems can experience unexpected environmental changes over long operational periods. Such changes affect the incidence of failures, causing different system failure patterns before and after the changes. In this article, we propose an informational change-point approach for the pattern of recurrent failures in repairable artillery systems. Unlike other trend tests, this approach provides additional information about the locations of change-points over rates of occurrence of failures (ROCOFs) as well as failure trends. We adopt the modified information criterion (MIC) proposed by Pan and Chen (2006) to detect the locations of the changes and propose sequential procedures for determining the number of change-points in independent exponential sequences. The change-point approach is applied to unscheduled maintenance data from eight artillery system exercises performed by the Republic of Korea Army. The change-point test along with a graphical presentation of estimated ROCOF lines can provide easy interpretation of changes in failure trends/intensities in a homogeneous Poisson process.

1. Introduction

Modern artillery consists of numerous parts working together in a complex system. When the artillery system fails to perform its required functions in the course of operation, some repair action is executed to restore it to an operating condition again without replacement of the entire system (called *repairable system*). Over long operational period, artillery system can experience unexpected changes under the influences of various factors (e.g., operating pattern, working intensity, operating environment, and incomplete repairs (Syamsundar and Naikan, 2007)), affecting the incidence of failures. As a result, the system may exhibit different failure patterns before and after the changes. In applications involving artillery systems, it is often of interest to detect possible changes in the failure patterns over time. Early change identification can help maintenance engineers respond quickly with maintenance actions to reduce unwanted changes. Based on the estimated changes, an optimal maintenance strategy can also be developed to prevent the failures which may be followed by the changes.

The Republic of Korea (ROK) Army collected unscheduled maintenance data during exercise in the field over a fixed period of time. Some of the artillery systems are subject both to early failures due to the presence of defective parts or assembling defects, as well as wear-out failures caused by deteriorating phenomena. This causes a non-monotonic trend in the failure data in which the intensity function initially decreases followed by a long period of constant or near constant intensity until wear-out finally occurs, at which time the intensity function begins to increase. This is called the *bathtub shaped* failure intensity, which is typical for large and complex equipments with different failure modes under various operational environments (Nelson, 1988). First of all, it is crucial to locate the time to early failures to establish burn-in policy so that defective parts can be weeded out of the system completely.

In practice, decisions concerning failure patterns have been made using graphical techniques or statistical trend tests (Rigdon and Basu, 2000). Graphical methods are useful for gaining insights into the data through the illustration of observed failure data. Plotting the cumulative number of failures against respective failure-times is a simple but informative way to identify possible trends between failures. A straight line indicates a constant failure rate over the data collection period. Additionally, if the times between failures are independent, a renewal process (RP) or a homogeneous Poisson process (HPP) for exponentially distributed inter-failure times may be appropriate. The HPP is the simplest statistical model for describing the occurrence of failures in a repairable system. If the Duane plot (Duane, 1964), which graphically fits a line to the log-transformed cumulative failure rate against the log-transformed failure times, has a zero slope, then the HPP may be

☆ This manuscript was processed by Area Editor Min Xie.

* Corresponding author at: Department of Technology Management, Hanyang University, Seoul, Republic of Korea. Tel.: +82 2 2220 0473; fax: +82 2 2296 0471.

E-mail addresses: sjbae@hanyang.ac.kr (S.J. Bae), stat@psm.hanyang.ac.kr (B.M. Mun), tad@dtaq.re.kr (K.Y. Kim).

0360-8352/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.cie.2012.09.010

an appropriate model. When the HPP cannot be applied to the entire life cycle of a repairable system, especially to the early operating period or the final deterioration period, then the nonhomogeneous Poisson process (NHPP) may be used to model the failure history. Statistical trend tests have also been introduced to determine the statistical significance of systematic trends in the patterns of failures. Among them, the most popular methods include the Laplace test (Cox, 1955) and the Military Handbook-189 test (1981), which test the null hypothesis of an HPP against the alternative hypothesis of a monotonic trend, *i.e.,* either a decreasing or increasing failure rate trend.

In this article, we propose an informational change-point approach to analyze the non-monotonic patterns of recurrent failures in repairable artillery systems. The change-point approach enables us to detect the positions of change-points as well as the existence of change-points between failure-times. We also suggest sequential procedures for determining the number of change-points as the dimension of the change-point model. Unlike the other classes of trend tests such as the Laplace test and the Military Handbook-189 test, this approach provides additional information about the locations of change-points, which divide the failure data of a repairable system into several homogeneous groups when heterogeneity is present. The change-point approach has been used in various area, e.g., software reliability (Huang and Lyu, 2011; Li et al., 2010), burn-in (Yuan and Kuo, 2010), nanotechnology (Yang et al., 2012), and preventive maintenance (Li et al., 2010). The change-point problem for a Poisson process has been developed by several researchers. Loader (1991) discussed the likelihood ratio method to construct confidence regions for a hazard rate change-point in an HPP, then Loader (1992) extended the likelihood ratio method to an NHPP. Zhang (2005) proposed a modified Schwarz information criterion for determining the number of change-points in a Poisson process with piecewise constant rate parameter. The applications of the change-point approach to repairable systems has been examined by Syamsundar and Naikan (2007), Syamsundar and Naikan (2008) and Guo et al. (2010). Syamsundar and Naikan (2009) proposed the segmented point process model to identify the change-points in the failure process of a maintained system. They used the maximum likelihood (ML) method to determine the change-points in the Poisson point process.

Unlike common model selection problems, the change-point problem contains the locations of the changes as another set of special parameters. To effectively locate the change-points in the population of recurrent failures, we adopt a modified version of the Schwarz information criterion (Pan and Chen, 2006) to individual failure sequences. We specify the method to the case of multiple change-points at independent exponential sequences, whereas the method was originally developed for general cases with multiple change-points at individual event sequences. We introduce a concept of the critical value associated with a significance level α to produce a statistically convincing decision about the change-points. According to the principle of information criterion, the positions of change-points are estimated such that the criterion is minimized.

The rest of this paper is organized as follows. Section 2 illustrates the proposed informational change-point approach for inter-failure times. Section 3 offers the analysis of actual artillery failure data using the proposed method, along with traditional trend tests. Section 4 presents some concluding remarks and discussion.

2. Change-point approach for inter-failure times

2.1. Notations and definitions

Consider a repairable system observed from time $t = 0$. The system is observed in the time interval $(0, \tau]$ with n independent failures occurring at times $T_1, T_2, \ldots, T_n$. The systems are assumed to be either all time truncated (n is random) or all failure truncated (T_n is random). Inter-failure times in the system are denoted as $X_i = T_i - T_{i-1}$ for $i = 1, \ldots, n$.

The failure process is equivalently represented in terms of the counting process $\{N(t); t \geqslant 0\}$. The counting process $\{N(t); t \geqslant 0\}$ is an NHPP with intensity function $\lambda(t)$ if (1) $N(0) = 0$, (2) non-overlapping increments are independent, (3) $P[N(t+h) - N(h) = 1] = \lambda(t)h + o(h)$, and (4) $P[N(t+h) - N(t) \geqslant 2] = o(h)$ for all t, where $o(h)/h \to 0$ as $h \to 0$. The intensity function $\lambda(t)$ coincides with the rate of occurrence of failures (ROCOF) associated with the repairable system (Ascher and Feingold, 1984). The expected cumulative number of events in $[0,t]$, $\Lambda(t)$ satisfies

$$\Lambda(t) = \int_0^t \lambda(u)du, \tag{1}$$

and the number of failures in an interval $[t, t+h]$, $N(t+h) - N(t)$ has a Poisson distribution with mean $\Lambda(t+h) - \Lambda(t)$.

The NHPP is the most widely used model for repairable systems because it is not only flexible and mathematically tractable, but it can also provide a theoretical justification for minimal repairs in many applications (Ascher and Feingold, 1984). When the intensity function is constant, *i.e.,* $\lambda(t) \equiv \lambda$, the process is an HPP. If the inter-failure times are not all uniformly distributed, then it is said that there is a trend in the failure pattern. When the expected length of the inter-failure time increases or decreases monotonically over time, then there is a monotonic trend. Otherwise, there is a non-monotonic trend.

2.2. Informational change-point test

Maguire et al. (1952) discussed the advantages of using inter-failure times rather than failure counts to test changes in failure occurrence rates over time. Following their recommendation, we express the failure process of a repairable system in terms of inter-failure times. Let inter-failure times $X_1, \ldots, X_n$ be independent exponential random variables with respective parameters $\lambda_1, \ldots, \lambda_n$ that have a density of $f(x_i; \lambda_i) = \lambda_i e^{-\lambda_i x_i}$ for $i = 1, \ldots, n$. Under the null hypothesis of an HPP, $\lambda_1 = \cdots = \lambda_n \equiv \lambda$ represents no change-point among the inter-failure times. Suppose that there are $R (\geqslant 1)$ unknown change-points $\kappa_1, \ldots, \kappa_R$, satisfying $1 \leqslant \kappa_1 < \cdots < \kappa_R < n$ and $\lambda_{\kappa_r} \neq \lambda_{\kappa_r+1}$ for $r = 1, \ldots, R$. The test problem for an HPP against HPP with R change-points can be represented as

$$H_0 : \lambda_1 = \cdots = \lambda_n \equiv \lambda \text{ vs. } H_1 : \lambda_1 = \cdots = \lambda_{\kappa_1} \neq \lambda_{\kappa_1+1} = \cdots \\ = \lambda_{\kappa_R} \neq \lambda_{\kappa_R+1} = \cdots = \lambda_n. \tag{2}$$

Testing H_0 against H_1 is equivalent to selecting the best model among the proposed models. The best model can be selected by comparing the Schwarz information criterion (SIC) in a Bayesian context (Schwarz, 1978), which is expressed as

$$\mathrm{SIC}(\widehat{\Theta}_d) = -2 \log L(\widehat{\Theta}_d) + d \log\ n,$$

where $L(\widehat{\Theta}_d)$ is the maximum likelihood function for a model with d parameters with respect to d-dimensional parameter space Θ_d. Apparently, the SIC is a modified form of the Akaike information criterion (AIC) (Akaike, 1973); the only difference is in the penalty term. In the AIC, the $d\log n$ of the SIC is replaced with $2d$. However, the SIC provides an asymptotically consistent estimate of the order of the true model. The model which minimizes the AIC or SIC is selected as the best model.

Let $\mathcal{L}_0(\hat{\lambda}, n, 0)$ denote the maximum log-likelihood under H_0, then

$$\mathcal{L}_0(\hat{\lambda}, n, 0) = n \log \hat{\lambda} - \hat{\lambda} \sum_{i=1}^{n} x_i,$$

where the MLE of $\lambda, \hat{\lambda} = \left(\sum_{i=1}^{n} x_i/n\right)^{-1}$, and the SIC under H_0 is

$$\mathrm{SIC}(\hat{\lambda}, n, 0) = -2\mathcal{L}_0(\hat{\lambda}, n, 0) + \log n. \tag{3}$$

For the AIC, log$n$in (3) is replaced with 2.

Suppose that there are R change-points. To simplify the notation, let $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_{\kappa_R+1})^T$ and $\boldsymbol{\kappa} = (\kappa_1, \ldots, \kappa_R)^T$. When $\mathcal{L}_1(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, R)$ denotes the maximum log-likelihood under H_1, that is, $\mathcal{L}_1(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, R) \equiv \max_{\lambda,\boldsymbol{\kappa}} \mathcal{L}_1(\boldsymbol{\lambda}, \boldsymbol{\kappa}, R)$, then

$$\mathcal{L}_1(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, R) = \sum_{j=1}^{R+1} \sum_{i=\hat{\kappa}_{j-1}+1}^{\hat{\kappa}_j} \{\log \hat{\lambda}_{\hat{\kappa}_j} - x_i \hat{\lambda}_{\hat{\kappa}_j}\}, \tag{4}$$

with $\hat{\kappa}_0 = 0, \hat{\kappa}_{R+1} = n$. Here, $\hat{\lambda}_{\hat{\kappa}_1}, \ldots, \hat{\lambda}_{\hat{\kappa}_{R+1}}$ are the MLEs of $\lambda_{\kappa_1}, \ldots, \lambda_{\kappa_{R+1}}$, where

$$\hat{\lambda}_{\hat{\kappa}_{j+1}} = \left\{ \frac{\sum_{i=\hat{\kappa}_j+1}^{\hat{\kappa}_{j+1}} x_i}{\hat{\kappa}_{j+1} - \hat{\kappa}_j} \right\}^{-1}.$$

The SIC under H_1 is then

$$\mathrm{SIC}(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, R) = -2\mathcal{L}_1(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, R) + (2R+1)\log n, \tag{5}$$

where $2R + 1$ is the number of unknown parameters: $(R + 1)$ failure rates $(\lambda_1, \ldots, \lambda_{\kappa_R+1})$ and R change-points $(\kappa_1, \ldots, \kappa_R)$ for a given value of R.

In comparison with typical model selection problems, the change-point problem contains additional special parameters: the locations of the changes. As mentioned in Pan and Chen (2006), when the change-points are evenly distributed between 1 and n, all of the parameters $\lambda_1, \ldots, \lambda_{\kappa_R+1}$ are effective, making the change-point model the least complex. However, when one or more change-points are close to 1 or n, or are clustered, some of the parameters $\lambda_1, \ldots, \lambda_{\kappa_R+1}$ become redundant. As a result, some of the change-points $\kappa_1, \ldots, \kappa_R$ become undesirable parameters, unnecessarily complicating the model. To consider the model complexity as well as the dimensionality of the parameter space, we adopt the modified information criterion (MIC) proposed by Pan and Chen (2006) in order to detect the locations of the changes. We specify the method to the case of multiple change-points at independent exponential sequences. The MIC for multiple change-points at exponential sequences is defined as

$$\begin{aligned}\mathrm{MIC}(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, R) &= -2\mathcal{L}_1(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, R) + (R+1)\log n \\ &\quad + C\sum_{j=1}^{R+1}\left(\frac{\hat{\kappa}_j - \hat{\kappa}_{j-1}}{n} - \frac{1}{R+1}\right)^2 \log n \\ &= \mathrm{SIC}(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, R) - R\log n \\ &\quad + C\sum_{j=1}^{R+1}\left(\frac{\hat{\kappa}_j - \hat{\kappa}_{j-1}}{n} - \frac{1}{R+1}\right)^2 \log n,\end{aligned} \tag{6}$$

for a constant $C(>0)$. The MIC under $H_0, \mathrm{MIC}(\hat{\lambda}, n, 0)$, is the same as $\mathrm{SIC}(\hat{\lambda}, n, 0)$. If

$$\min\ \mathrm{MIC} \equiv \min\{\mathrm{MIC}(\hat{\lambda}, n, 0), \mathrm{MIC}(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, R)\}$$

is attained at $\mathrm{MIC}(\hat{\lambda}, n, 0)$ for some R, then we fail to reject to the null hypothesis H_0, and there is no change-point, thus confirming a homogeneous failure process. Otherwise, H_0 is rejected and there are R change-points at $\kappa_1, \ldots, \kappa_R$. It should be noted that the MIC is only justified for the case of independent and identically distributed (*iid*) observations from an exponential family. Pan and Chen (2006) assumed that the number of change-points R as fixed and left room for further research whether $\widehat{R}$ is consistent if R is not fixed. Yao (1988) proved that $\widehat{R}$ is weakly consistent for an independent normal sequence at some regularity conditions.

The MIC procedure presented in this section presents a simple method for estimating the number and locations of change-points. the number of change-points and their locations can be determined simultaneously by comparing the MIC values. The failure trend for a repairable system can be determined using the change-points; for example, $\lambda_{\kappa_1} > \lambda_{\kappa_2} > \cdots > \lambda_{\kappa_R+1}$ indicates an improving system, while $\lambda_{\kappa_1} < \lambda_{\kappa_2} < \cdots < \lambda_{\kappa_R+1}$ indicates a deteriorating system.

However, when the modified information criteria are very close to one another, one may question whether the small differences among them are caused by fluctuation in the data. To produce a statistically convincing decision about the change-points, we introduce the significance level α and its associated critical value $c_\alpha(\geqslant 0)$. Instead of making a decision to fail to reject to H_0 when $\mathrm{MIC}(\hat{\lambda}, n, 0) < \mathrm{MIC}(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, R)$, we fail to reject to the null hypothesis H_0 if $\mathrm{MIC}(\hat{\lambda}, n, 0) < \mathrm{MIC}(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, R) + c_\alpha$, where c_α and α have the relationship

$$1 - \alpha = P[\mathrm{MIC}(\hat{\lambda}, n, 0) < \mathrm{MIC}(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, R) + c_\alpha | H_0]. \tag{7}$$

To obtain the critical value c_α, we must obtain the null distribution of the MIC. To derive this null distribution, we first define the test statistic for the hypothesis of R change-points against the null of no changes as

$$T_{0,R} = \mathrm{MIC}(\hat{\lambda}, n, 0) - \mathrm{MIC}(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, R) + R\log n. \tag{8}$$

The limiting distribution of the test statistic $T_{0,R}$ is Chi-square distribution with R degrees of freedom under H_0 (Pan and Chen, 2006) and we fail to reject to the null hypothesis H_0 at a significance level α if $T_{0,R} < \chi^2_\alpha(R)$.

2.3. Determining the number of change-points

Both the number and positions of change-points are generally unknown; hence, it is necessary to determine both R and $\kappa_1, \ldots, \kappa_R$ in order to execute the testing procedure. According to the principle of information criterion, the change-points $\kappa_1, \ldots, \kappa_R$ are estimated by $\hat{\kappa}_1, \ldots, \hat{\kappa}_R$ such that the MIC is minimized for a given R. To determine the dimension of the model, the number of change-points can be determined in the following steps:

Step I: For a fixed value of change-points r, estimate the unknown failure rates $(\hat{\lambda}_1, \ldots, \hat{\lambda}_{\kappa_r+1})$ and change-points $\hat{\kappa}_1, \ldots, \hat{\kappa}_r$ that maximize (4), that is:

$$M(r) = \max_{\lambda,\boldsymbol{\kappa}} \mathcal{L}_1(\boldsymbol{\lambda}, \boldsymbol{\kappa}, r) \equiv \mathcal{L}_1(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, r). \tag{9}$$

Step II: Evaluate the following modified information criterion

$$\mathrm{MIC}(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, r) = -2M(r) + \left(r + 1 + C\sum_{j=1}^{r+1}\left(\frac{\hat{\kappa}_j - \hat{\kappa}_{j-1}}{n} - \frac{1}{r+1}\right)^2\right)\log n,$$

subject to $r \leqslant R_U$, where R_U is the given upper bound for the change-points. If $T_{r-1,r} \equiv \mathrm{MIC}(\hat{\lambda}, \hat{\boldsymbol{\kappa}}, r-1) - \mathrm{MIC}(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, r) + \log n \geqslant \chi^2_\alpha(1)$, increase r to $r + 1$ and repeat *Step I–II* until $T_{r,r+1} < \chi^2_\alpha(1)$ for $r = 1, 2, \ldots, R_U$. Then, terminate the steps and estimate the number of change-points as $R = r$.

For illustration, consider a simple case in which there is only one change-point at κ. The test problem of (2) becomes

$$H_0 : \lambda_1 = \cdots = \lambda_n \equiv \lambda \text{ vs. } H_1 : \lambda_1 = \cdots = \lambda_\kappa \neq \lambda_{\kappa+1} = \cdots = \lambda_n, \tag{10}$$

for $1 \leqslant \kappa < n$. Let $\mathcal{L}_1(\hat{\lambda}, \hat{\kappa}, 1)$ be the maximum log-likelihood under the above H_1. Then, for MLEs $\hat{\lambda}_1 = \cdots = \hat{\lambda}_{\hat{\kappa}} = \hat{\kappa}/\left(\sum_{i=1}^{\hat{\kappa}} x_i\right)$ and $\hat{\lambda}_{\hat{\kappa}+1} = \cdots = \hat{\lambda}_n = (n - \hat{\kappa})/\left(\sum_{i=\hat{\kappa}+1}^{n} x_i\right)$, the MIC under H_1 is

$$
\begin{aligned}
\mathrm{MIC}(\hat{\boldsymbol{\lambda}},\hat{\kappa},1) &= -2\mathcal{L}_1(\hat{\boldsymbol{\lambda}},\hat{\kappa},1) \\
&\quad + \left(2 + C\sum_{j=1}^{2}\left(\frac{\hat{\kappa}_j-\hat{\kappa}_{j-1}}{n}-\frac{1}{1+1}\right)^2\right)\log n \\
&= 2\hat{\kappa}\log\left(\sum_{i=1}^{\hat{\kappa}}x_i\right) + 2(n-\hat{\kappa})\log\left(\sum_{i=\hat{\kappa}+1}^{n}x_i\right) \\
&\quad - 2\hat{\kappa}\log\hat{\kappa} - 2(n-\hat{\kappa})\log(n-\hat{\kappa}) + 2n \\
&\quad + \left(2+2\left(\frac{\kappa}{n}-\frac{1}{2}\right)^2\right)\log n, \qquad (11)
\end{aligned}
$$

for $C = 1$. If $T_{0,1} \equiv \mathrm{MIC}(\hat{\lambda},n,0) - \mathrm{MIC}(\hat{\boldsymbol{\lambda}},\hat{\kappa},1) + \log n < \chi^2_\alpha(1)$, we fail to reject to the null hypothesis H_0; that is, there is no change-point.

3. Examples

3.1. Submarine diesel engine data

Ascher and Feingold (1984), p. 75 presented arrival times for unscheduled maintenance actions for the U.S.S. Halfbeak No. 3 main propulsion diesel engine. We analyzed 71 failure points excluding scheduled overhaul data. The cumulative number of failures is plotted against the cumulative engine operating times in Fig. 1. The plot shows that inter-failure times tend to decrease and form a concave graph, providing strong evidence of deterioration. First, we applied the Laplace test statistic (Cox, 1955)

$$L = \frac{\sum_{i=1}^{\hat{n}} T_i - \frac{1}{2}\hat{n}\tau}{\sqrt{\frac{1}{12}\hat{n}\tau^2}},$$

where $\hat{n}$ denotes the total number of failures ($\hat{n}$ is equal to n for time truncation and to $n-1$ for failure truncation). According to the central limit theorem, an asymptotic distribution of L is a standard normal distribution under H_0: HPP; hence, the null hypothesis is rejected if $L < -z_{\alpha/2}$ or $L > z_{\alpha/2}$, where z_α is $(1-\alpha)$ th quantile of the standard normal distribution. According to the Laplace test, the test statistic was $L = 7.443$, and its p-value was less than 0.0001. As the other trend test, the Military Handbook-189 test statistic MIL-HDBK-189 (1981)

$$M = 2\sum_{i=1}^{n}\ln\left(\frac{\tau}{T_i}\right)$$

is exactly chi-square distributed with $2n$ degrees of freedom under the null hypothesis. The null hypothesis of an HPP is rejected if $M > \chi^2_{\alpha/2}(2n)$ or $M < \chi^2_{1-\alpha/2}(2n)$. The value of test statistic was $M = 51.443$, and its p-value was also less than 0.0001. The small p-values of the two test statistics consistently support the alternative hypothesis of not HPP, and the positive L value indicates a deteriorating system.

Vaurio (1999) proposed three different test statistics under a time-truncated sampling for testing nonmonotonic trend. In particular, the statistic

$$V = \frac{\sum_{i=1}^{n}|T_i - T_n/2| - nT_n/4}{T_n\sqrt{n/48}} \qquad (12)$$

is approximately a standard normal distribution under the null hypothesis of no trend. We also applied the Vaurio test to determine the existence of a non-monotonic trend, resulting in a Vaurio test statistic of $V = 4.257$ and its p-value less than 0.0001, strongly supporting the alternative hypothesis of a non-monotonic trend. Also, the positive V value indicates bathtub behavior. Note that the traditional trend tests conclude that the failure rates are monotonically increasing, whereas the Vaurio test concludes that they are bathtub-behaviored for the submarine engine data. To investigate the gap between the results, we also applied the monotonic trend test in Vaurio (1999) to the data. The test statistic is

$$J = \frac{\sum_{i=1}^{n}T_i - nT_n/2}{s[n(n+1)(n+2)/12]^{1/2}} \sim t(n),$$

where $s = \sqrt{\sum_{i=1}^{n}(T_i-\overline{T})^2/(n-1)}$ for the average failure-times $\overline{T}$. The test result gives $J = 4.577$, and its p-value was less than 0.0001, concluding monotonic trend in the data. It is noted that the trend tests produce different results according to the alternative hypothesis. Determined from the trend test results and Fig. 1, in summary, the submarine diesel engine data ultimately illustrated a deteriorating system.

The MIC values and the estimated locations of change-points of the change-point test are presented for each change-point in Table 1. Based on the simulation results by Pan and Chen (2006) who examined the effect of constant C over a wide range of C, we decided the value of C as 1. For $R = 1$, $T_{0,1} \equiv$ MIC ($R = 0$) − MIC ($R = 1$) + log (71) = 75.343 is much greater than $\chi^2_{.05}(1) \equiv 3.841$ with a significance level of $\alpha = 0.05$, indicating that there are at least one change-point. For $R = 2$, $T_{1,2} = 2.797$ is less than $\chi^2_{.05}(1)$, hence we determined a single change-point for the submarine diesel engine data. The estimated location of the change-point, $\hat{\kappa} = 18$ (19,067 h), is presented as a vertical line in Fig. 1. The rates of occurrence of failures (ROCOFs) before and after the change-point were $\lambda_1 = \cdots = \lambda_{18} \equiv 9.44 \times 10^{-4}$ and $\lambda_{19} = \cdots = \lambda_{71} \equiv 8.22 \times 10^{-3}$, respectively. The increasing ROCOF after the change-point represents a deteriorating system, coinciding with graphical interpretation and statistical trend test results. For comparison, we employed the segmented HPP model with a single change-point of Syamsun-

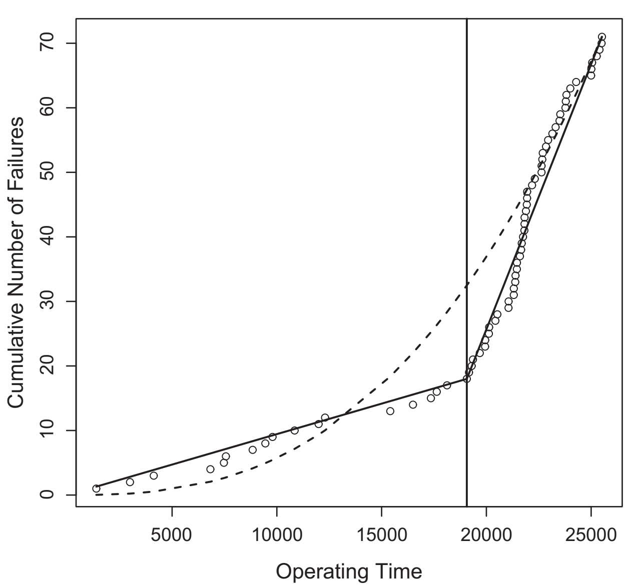

Fig. 1. Cumulative number of failures vs. cumulative operation time for U.S.S. Halfbeak No. 3 main propulsion diesel engine data (dotted line represents the power law process fit and bold line represents separate HPP fits before and after estimated change-point).

Table 1
Change-point test results for submarine diesel engine data.

# Of change-points (R)	MIC (R)	Locations of change-points	$T_{R-1,R}$
0	981.856	–	–
1	910.776	18	75.343
2	912.242	18, 29	*2.797*

dar and Naikan (2007), who proposed a goodness of fit test for a single change-point in a sequence of observed failure points in a maintenance system. In applying their method, the change-point was detected at the 18th failure, same as our method. To test the null hypothesis of HPP against the alternative hypothesis of a segmented HPP with a single change-point, the test statistic $F_0 = \hat{\lambda}_1/\hat{\lambda}_2 \sim F(2n_2, 2n_1)$ suggests rejection of the null hypothesis because $F_0(\equiv 0.0009442/0.008216 = 0.1149) < F_{1-\alpha/2}(2n_2, 2n_1)(\equiv F_{0.975}(106,36) = 1.7821)$ with the significance level $\alpha = 0.05$, where $\hat{\lambda}_1(\hat{\lambda}_2)$ is the estimated ROCOF before (after) the change-point, and $n_1(n_2)$ is the number of failures before (after) the change-point. Though the test has indicated the correct change-point, the use of the *F*-test may be considered appropriate only if the location of the change point is known *a priori* and not if it is to be estimated.

The change-point approach is more informative in that it provides information about the locations of failure rate changes as well as failure trends, while the statistical trend tests determine only whether or not the failure process is an HPP. In general, the power law process has been employed to model the NHPP with a monotonic trend. Meeker and Escobar (1998) analyzed the same data using the power law process and the log-linear process. They also noticed a change in the distribution of times between failures after rather 17th failure than 18th failure. Modeling the diesel engine data using the power law process with the intensity function $\lambda(t) = \theta\beta t^{\beta-1}$, where $\theta(>0)$ is a scale parameter and β is a shape parameter, the estimated scale and shape parameters were $\hat{\theta} = 4.86 \times 10^{-11}$ and $\hat{\beta} = 2.7603$, respectively. A shape parameter greater than one represents a deteriorating system; however, the power law process (dotted line in Fig. 1) does not adequately represent the observed failure data, especially in the middle of the failure sequence. On the other hand, Fig. 1 shows that our informational change-point approach (bold line) successfully captures the observed failures in the submarine engine data.

3.2. Steam exhauster data

Syamsundar and Naikan (2008) analyzed recurrent failure data of a steam exhauster in a continuous casting machine of an integrated steel plant. The data consists of a total of 17 failure times in number of days. Fig. 2 plots the cumulative number of failures against the cumulative system operating times, showing that inter-failure times tend to be non-monotonic during whole observation period of system failures. Syamsundar and Naikan (2008) fitted the data with both an HPP and a nonhomogeneous power law process. The parameter estimate of the HPP is $\hat{\lambda} = 0.0424$, and the parameter estimates of the power law process are $\hat{\theta} = 0.0240$ and $\hat{\beta} = 1.0948$. The shape parameter estimate close to one in the power law process supports an HPP for recurrent failures of a steam exhauster. As a goodness of fit test, they used the AIC likelihood ratio test

$$\mathrm{AIC}_{LR}(\boldsymbol{\Theta}_0, \boldsymbol{\Theta}) \equiv -2\{\log L(\boldsymbol{\Theta}_0) - \log L(\boldsymbol{\Theta})\} + 2k,$$

and it is asymptotically χ^2 distributed with degrees of freedom k, which is equal to the difference in number of parameters in $\boldsymbol{\Theta}_0$ and $\boldsymbol{\Theta}$, where $\boldsymbol{\Theta}_0$ are the set of parameters in the partial model of null hypothesis (e.g., HPP) and $\boldsymbol{\Theta}$ are the set of parameters in the full model of alternative hypothesis (e.g., NHPP). The null hypothesis is rejected if $\mathrm{AIC}_{LR}(\boldsymbol{\Theta}_0, \boldsymbol{\Theta}) < \chi^2_{1-\alpha/2}(k)$ or if $\mathrm{AIC}_{LR}(\boldsymbol{\Theta}_0, \boldsymbol{\Theta}) > \chi^2_{\alpha/2}(k)$. By applying the AIC likelihood ratio in order to test between the HPP and the power law process, the estimated ratio $\widehat{\mathrm{AIC}}_{LR}(\lambda, \{\theta, \beta\}) = -2(-70.7327 + 70.6650) + 2 \times 1 = 2.1354$, lies between $\chi^2_{0.975}(1)(\equiv 9.8207 \times 10^{-4})$ and $\chi^2_{0.025}(1)(\equiv 5.0239)$, hence we fail to reject to the null hypothesis of an HPP. Parameter estimate results in the power law process and the AIC likelihood ratio test results validate no trend for the failure data of a steam exhauster, whereas the plot in Fig. 2 shows a non-monotonic trend during whole observation period of failures. To investigate the gap between the analytical and graphical results, we applied the Vaurio's statistic (12) to test the existence of a non-monotonic trend. The test result gives the Vaurio test statistic of $V = -3.0624$ with corresponding p-value equal to 0.0011, strongly supporting the alternative hypothesis of a non-monotonic trend, and the negative V value indicates inverse-bathtub behavior.

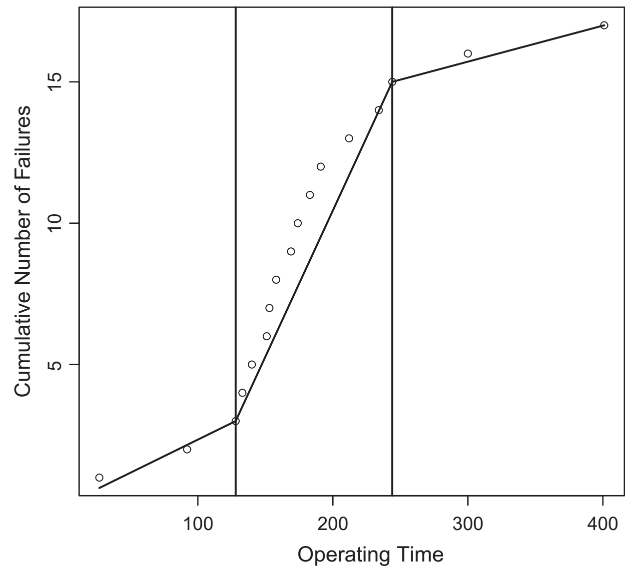

Fig. 2. Cumulative number of failures vs. cumulative operation time for a steam exhauster data (bold line represents separate HPP fits before and after estimated change-points).

Next, the MIC values and the locations of change-points were estimated following the procedure in Section 2. We fixed the value of C as 1. For $R = 1$, $T_{0,1} = 5.511$ is larger than $\chi^2_{.05}(1) \equiv 3.841$ with a significance level of $\alpha = 0.05$, indicating that the null hypothesis of the HPP is rejected. For $R = 2$, $T_{1,2} = 6.938$ is larger than $\chi^2_{.05}(1)$, and $R = 3$, $T_{2,3} = 2.485$ is smaller than $\chi^2_{.05}(1)$ hence we concluded that there are two change-points for the steam exhauster data. The estimated locations of the change-points are $\hat{\kappa}_1 = 3$ (128 days) and $\hat{\kappa}_2 = 15$ (244 days) and they are presented as vertical lines in Fig. 2. The rates of occurrence of failures (ROCOFs) before and after the change-point were $\lambda_1 = \cdots = \lambda_3 \equiv 2.34 \times 10^{-2}$, $\lambda_4 = \cdots = \lambda_{15} \equiv 10.34 \times 10^{-2}$, and $\lambda_{16} = \lambda_{17} \equiv 1.27 \times 10^{-2}$, respectively. The change-point fit (bold line) in Fig. 2 captures the non-monotonic trend of the exhauster failures with inverse-bathtub behavior.

3.3. Artillery repair data

While the Republic of Korea (ROK) Army was performing artillery system exercises in the field, they collected unscheduled maintenance data over a fixed period of time. The unscheduled maintenance actions were caused by system failures or imminent failures. Fig. 3 shows an event plot of eight artillery repair data sets along with their respective observation periods and reported failure times. In general, many failures were observed during the early and final periods of data collection. The test statistics according to the statistical trend tests are summarized in Table 2 along with their p-values. At a significance level of $\alpha = 0.05$, the artillery data exhibited an HPP except for Artillery ID-7 when applying the Laplace test and the Military Handbook-189 test. For Artillery ID-7, a negative value in the Laplace test indicates an improving system. Conversely, the Vaurio test results consistently supported the alternative hypothesis of a non-monotonic trend for all of the artil-

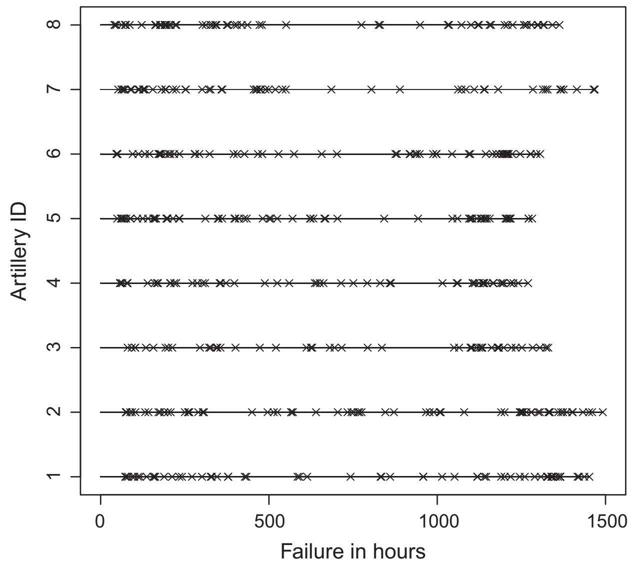

Fig. 3. Event plot showing failure times and the period of observation for the artillery repair data.

Table 2
Statistical trend tests for the K-artillery repair data.

K-artillery	Laplace test		Military handbook-189 test		Vaurio test	
	L	p-Value	M	p-Value	V	p-Value
1	0.7846	0.4326	120.6868	0.9667	3.9790	<0.0001
2	0.6293	0.5291	126.7703	0.7756	1.9863	0.0235
3	1.0438	0.2965	91.9809	0.4972	2.3641	0.0090
4	0.9682	0.3329	102.4246	0.6328	2.5967	0.0047
5	−0.7223	0.4701	149.8833	0.2734	3.3462	0.0004
6	1.0104	0.3123	112.4851	0.7480	3.6172	0.0001
7	−2.8483	0.0043	159.3161	0.0094	3.5558	0.0002
8	−1.4754	0.1401	152.9821	0.0791	4.3812	<0.0001

lery data. The statistical test results are inconsistent in accordance with the alternative hypothesis, thus requiring careful attention in use.

In applying the change-point test, the modified-information criterion (MIC) values and the estimated change-point locations along with the estimated ROCOFs before and after the change-points are presented for the eight artillery repair data points in Table 3. For example, at a significance level of $\alpha = 0.01$, when $R = 1$ for Artillery ID-1, $T_{0,1} = 10.332 > \chi^2_{.01}(1) \equiv 6.635$, indicating that there are multiple change-points. For $R = 2$, $T_{1,2} = 11.406 > \chi^2_{.01}(1)$; thus, we increased the value to $R = 3$ to determine whether there are more than two change-points in the inter-failure times. However, because $T_{2,3} = 6.176 < \chi^2_{.01}(1)$, we concluded that there were two change-points for Artillery ID-1. Significant test values are marked with an asterisk $(*)$ in the last column of Table 3. The ROCOFs before and after the change-points were 5.77×10^{-2},1.75×10^{-2},7.50×10^{-2}, respectively. The estimated locations of the change-points are also presented as vertical lines in Fig. 4. As shown in Table 2, the Laplace and Military Handbook-189 tests result in acceptance of the null hypothesis of an HPP of the inter-failure times for Artillery ID-1. However, the plot in Fig. 4a reveals that a bathtub-shaped intensity function is appropriate for Artillery ID-1, and its Vaurio test indicated a non-monotonic trend, consistent with the results of the informational change-point approach which detected two change-points. In addition, the ROCOF values before and after the two change-points conform to the interpretation in the event plot for ID-1 in Fig. 3. For ID-4, an HPP with a change-point was selected, and the ROCOF increased after the change-point, representing a deteriorating system with respect to the artillery system exercises. Note that the Vaurio test indicated a non-monotonic trend for the sample.

Table 3
Change-point test results for the K-artillery repair data.

K-artillery ID	# Of change-points (R)	MIC (R)	Locations of change-points	ROCOF ($\times 10^{-2}$)	$T_{R-1,R}$
1	0	519.169	–	4.27	–
	1	512.964	48	3.62, 11.13	10.332*
	2	505.685	25,37	5.77, 1.75, 7.50	11.406*
	3	503.636	23,48,58	6.07, 2.64, 25.13, 4.65	6.176
2	0	554.031	–	4.49	–
	1	549.665	46	3.69, 8.50	8.570*
	2	545.856	20,46	6.51, 2.77, 8.50	8.014*
	3	546.807	20,46,49	6.51, 2.77, 30.00, 7.59	3.255
3	0	445.087	–	3.910	–
	1	436.206	32	2.91, 8.66	12.832*
	2	432.398	18,32	5.06, 1.88, 8.66	7.759*
	3	431.519	13,18,32	4.03, 15.00, 1.88, 8.66	4.830
4	0	465.624	–	4.41	–
	1	456.189	38	3.44, 10.91	13.461*
	2	454.107	21,38	5.29, 2.40, 10.91	6.108
	3	455.373	21,38,45	5.29, 2.40, 20.00, 8.46	2.759
5	0	533.704	–	5.23	–
	1	528.112	47	4.29, 10.70	9.796*
	2	516.598	41,47	6.14, 1.40, 10.70	15.718*
	3	514.314	22,42,47	9.39, 4.26, 1.28, 10.70	6.489
6	0	493.740	–	4.59	–
	1	486.686	44	3.78, 11.29	11.148*
	2	483.565	19,30	6.48, 1.89, 6.98	7.216*
	3	482.226	19,30,44	6.48, 1.89, 4.86, 11.29	5.433
7	0	501.344	–	4.02	–
	1	488.084	38	6.91, 2.29	17.337*
	2	480.722	38,42	6.91, 0.78, 4.19	11.439*
	3	480.771	19,38,42	9.84, 5.32, 0.78, 4.19	4.029
8	0	517.525	–	4.62	–
	1	508.765	34	7.80, 3.13	12.903*
	2	495.376	36,42	7.52, 1.08, 6.34	17.532*
	3	496.326	22,36,42	9.78, 5.51, 1.08, 6.34	3.193

4. Conclusions

In practice, decisions concerning failure patterns have been made using graphical techniques or statistical trend tests. Graphical methods are useful for gaining insights into the data through the illustration of observed failure data. Statistical trend tests determine statistical significance of systematic trends in the patterns of failures. In this article, we proposed an informational change-point approach for detecting change-points which may

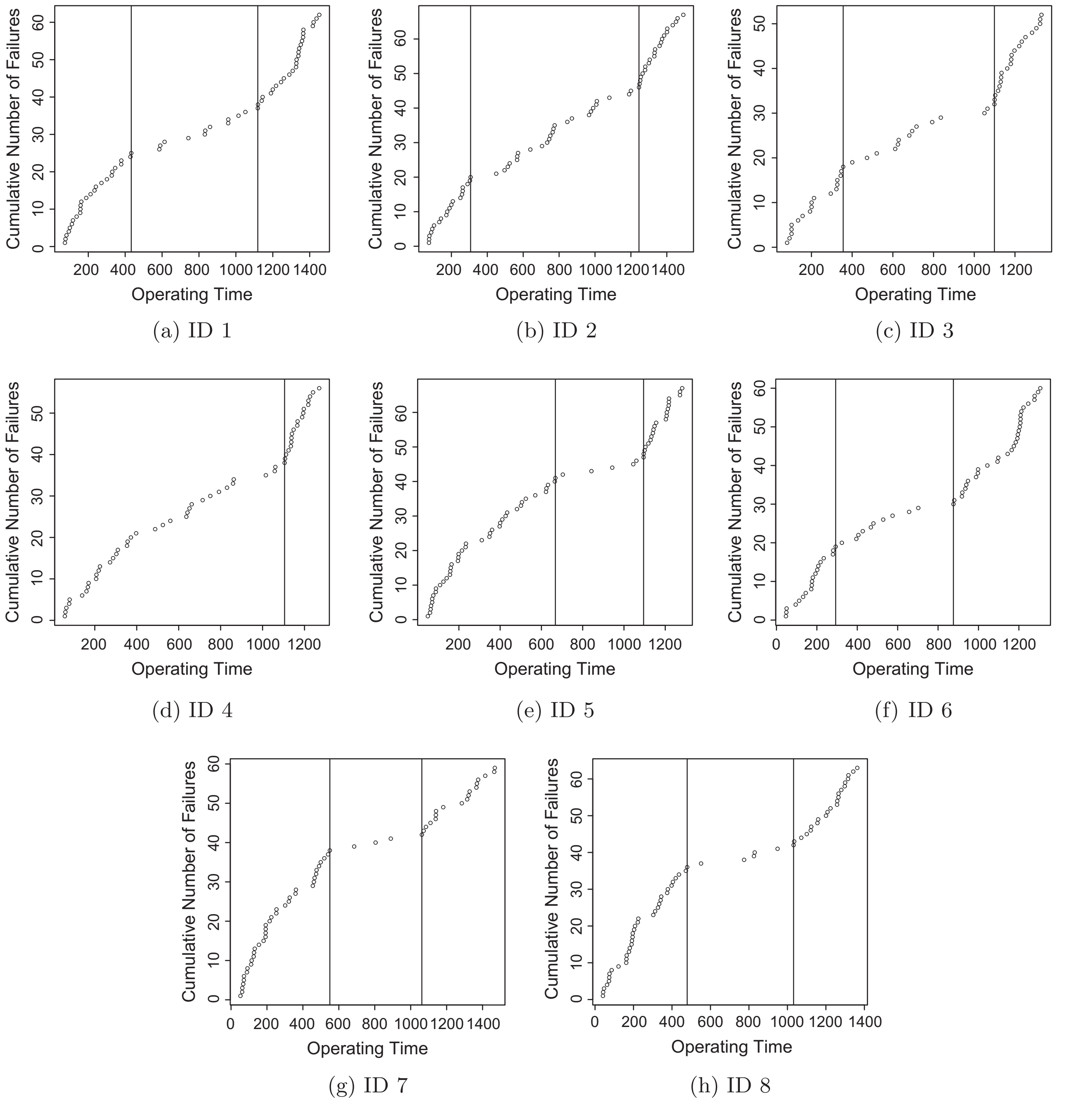

Fig. 4. Cumulative number of failures vs. cumulative operation time for the artillery repair data.

represent unexpected changes in an operating environment. The change-point test determines the number and locations of changes during the failure process in a repairable system. Early detection of such changes can prevent failures by initiating necessary maintenance actions for the system. We applied the proposed method to recurrent failure data collected from artillery system exercises which were performed in the field. Analytical results indicated that more care must be taken when using statistical trend tests because the testing statistics may exhibit conflicting results in accordance with the alternative hypothesis. Thus, we recommend the serial application of several trend tests to produce improved failure trend identification in a reparable system. The change-point test, along with a graphical presentation of the estimated ROCOF lines before and after the change-points, can provide easier interpretation of failure trends than do the statistical trend tests, especially for the systems with non-monotonic failure intensities.

Guo et al. (2010) proposed a piecewise NHPP model and used the maximum likelihood method to estimate the parameters in segmented power law process models. As shown in Fig. 2, the middle data of failure sequence can be modeled via an NHPP model with concave trend. The proposed informational change-point approach in this article will be extended to general NHPP models including the power law process in future work.

Acknowledgements

We thank the referees who helped us gain substantial improvements in the manuscript. This work was supported by Mid-career

Researcher Program through NRF Grant funded by the MEST (2011-0016598).

References

Akaike, H. (1973). Information theory and an extension of the maximum likelihood principle. In B. N. Petrov & E. Csaki (Eds.), *2nd International symposium of information theory* (pp. 267–281). Budapest: Akademiai Kiado.

Ascher, H., & Feingold, H. (1984). *Repairable systems reliability: Modeling, inference, misconceptions and their causes*. New York: Marcel Dekker.

Cox, D. R. (1955). Some statistical methods connected with series of events. *Journal of the Royal Statistical Society B, 17*, 129–164.

Duane, J. T. (1964). Learning curve approach to reliability monitoring. *IEEE Transactions on Aerospace, 2*, 563–566.

Guo, H., Mettas, A., Sarakakis, G., & Niu, P. (2010). Piecewise NHPP models with maximum likelihood estimation for repairable systems. *Proceedings of RAMS*.

Huang, C. Y., & Lyu, M. R. (2011). Estimation and analysis of some generalized multiple change-point software reliability models. *IEEE Transactions on Reliability, 60*, 498–514.

Li, X., Xie, M., & Ng, S. H. (2010). Sensitivity analysis of release time of software reliability models incorporating testing effort with multiple change-points. *Applied Mathematical Modelling, 34*, 3560–3570.

Li, Z. G., Zhou, S. Y., Sievenpiper, C., & Choubey, S. (2010). Change detection in the Cox proportional hazards models from different reliability data. *Quality and Reliability Engineering International, 26*, 677–689.

Loader, C. R. (1991). Inference for a hazard rate change point. *Biometrika, 78*, 749–757.

Loader, C. R. (1992). A log-linear model for a Poisson process change point. *The Annals of Statistics, 20*, 1391–1411.

Maguire, B. A., Pearson, E. S., & Wynn, A. H. A. (1952). The time intervals between industrial accidents. *Biometrika, 39*, 168–180.

Meeker, W. Q., & Escobar, L. A. (1998). *Statistical methods for reliability data*. New York: Wiley.

MIL-HDBK-189, (1981). *Reliability growth management, headquaters*. US Army Communications Research and Development Command, ATTN: DRDCO-PT, Fort Monmouth, NJ 07702.

Nelson, W. (1988). Graphical analysis of system repair data. *Journal of Quality Technology, 20*, 24–35.

Pan, J., & Chen, J. (2006). Application of modified information criterion to multiple change point problems. *Journal of Multivariate Analysis, 97*, 2221–2241.

Rigdon, S. E., & Basu, A. P. (2000). *Statistical methods for the reliability of repairable systems*. New York: Wiley.

Schwarz, G. (1978). Estimating the dimension of a model. *The Annals of Statistics, 6*, 461–464.

Syamsundar, A., & Naikan, V. N. A. (2007). Segmented point process models for maintained systems. *International Journal of Reliability, Quality and Safety Engineering, 14*, 431–458.

Syamsundar, A., & Naikan, V. N. A. (2008). Hierarchical segmented point process models with multiple change points for maintained systems. *International Journal of Reliability, Quality and Safety Engineering, 15*, 261–304.

Syamsundar, A., & Naikan, V. N. A. (2009). Sequential detection of change points for maintained systems using segmented point process models. *Quality and Reliability Engineering International, 25*, 739–757.

Vaurio, J. K. (1999). Identification of process and distribution characteristics by testing monotonic and non-monotonic trends in failure intensities and hazard rates. *Reliability Engineering & System Safety, 64*, 345–357.

Yang, C. H., Yuan, T., Kuo, W., & Kuo, Y. (2012). Non-parametric Bayesian modeling of hazard rate with a change point for nanoelectronic devices. *IIE Transactions, 44*, 496–506.

Yao, Y. C. (1988). Estimating the number of change-points via Schwarz' criterion. *Statistics & Probability Letters, 6*, 181–189.

Yuan, T., & Kuo, Y. (2010). Bayesian analysis of hazard rate, change point, and cost-optimal burn-in time for electronic devices. *IEEE Transactions on Reliability, 59*, 132–138.

Zhang NR. (2005). *Change point detection and sequence alignment: statistical problems of genomics*. Ph.D thesis at Stanford University.