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TTT-SiZer: A graphic tool for aging trends recognition ☆

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Abstract

A new graphic tool is presented to test aging trends based on lifetime data. The graphical test is developed by means of scale and space inference about the Total-Time-on-Test transform and its first and second derivatives. The graphic tool TTT-SiZer is defined considering nonparametric local polynomial kernel estimators and constructing the corresponding (simultaneous as well as punctual) confidence intervals around three different curves. The finite sample properties of the method are evaluated by a simulation study and the comparison with other non-graphical tests shows that the graphical test helps localize discrepancies of empirical data concerning a given hypothesized aging property, thus allowing to solve the problem locally.

Keywords: Total-Time-on-Test Transform, Kernel smoothing, Order statistics, SiZer map
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1. Introduction

In a technical context, aging is usually understood as a gradual decrease in performance over time, so it can be expected that the aging of a component or system increases the probability that it will fail.

Let X be the random variable that represents the lifetime of a mechanism. A natural measure of aging is the probability that X exceeds a time $t+x$, assuming that the mechanism has an age of at least t. This idea leads to

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the concept of residual life. Also, the instantaneous propensity to the failure of a mechanism as a function of its age, that is, its failure rate, is a useful measure of aging.

Aging trends evaluation of units and systems is a challenging subject in reliability and maintainability analysis and has attracted the attention of an increasing number of researchers. As a result, during the last years of the past century, different notions of aging have given rise to a well-known nonparametric classification of life distributions, which can be found in [1], [2], [3], [4] and [5], among others. Since then, the literature in reliability about stochastic aging and its different characterizations have been prolific and new aging criteria have been mathematically formulated, see for example [6], [7], [8], [9]. The monograph [10] provides an excellent revision and summary of the most widely studied aging concepts that are based on reliability characteristics such as the failure rate, the survival function or the mean residual life. Several examples of lifetime distributions together with their aging properties are given, with special attention to the aging behavior of finite mixtures of life distributions.

Se pueden eliminar [1, 3, 4] [7]

The mathematical description of aging is mainly done based on the failure rate, residual life functions, and the survival function so an inspection of the shape of these functions is helpful to evaluate the performance of a system in the future, which is very important for making decisions in many practical applications in reliability and maintainability. In [11] aging criteria based on the failure rate and mean residual life are considered. It is particularly interesting whether a specific property of aging defined in terms of a reliability measure implies the same property, or some other criterion is presented with another reliability measure. Other approaches have contributed to extending the mathematical formulation of aging in different application areas where the concept of failure rate, for example, is not so usual. Recently, in Chapter 4 of [12], the authors review some of the important results of the mathematical treatment of aging and translate the basic definitions to make them amenable for a quantile-based analysis. In [8], [9], a family of generalized aging intensity functions of univariate absolutely continuous lifetime random variables is introduced and studied. They allow the analysis and measurement of the aging tendency from various points of view.

In this paper, the attention is focused on the Total-Time-on-Test (TTT) concept. The shape of the TTT curve is studied to figure out different types of aging of elements and systems. Our proposal is a useful and novel graphical tool that allows recognizing some important types of aging from empirical data. The TTT plot was introduced by [13] as a tool for analyzing failure data and some important classes of life distributions can be characterized in terms of the Total-Time-on-Test curve, see [14].

The concept of TTT transform has been extended and applied to different research fields, such as economics and actuarial sciences or biostatistics, see [15] and references therein for further details. In the field of reliability engineering the TTT curve has revealed as a very useful tool for aging characterization [15], model identification [16] and [17], hypothesis testing [18], [19], [20], stochastic ordering [12], age replacement policies in maintenance [21], structural reliability [22], among others. Recently [15] and [6] have listed some known characterizations of common aging notions in terms of TTT function and have also derived some new ones.

One of the advantages of the TTT curve is that it is confined to the compact interval $[0, 1]$ whereas the failure rate is not bounded in general. Given the relationship between the failure rate and the TTT transform, one can understand the shape of the failure rate function by exploring the TTT curve. In this paper, the scaled TTT plot is used to discover a possible aging tendency of an underlying distribution based on sample information. In particular, increasing failure rate (IFR), bathtub failure rate (BFR) and new better than used in expectation (NBUE) are considered.

Identification of models by testing monotonic and non-monotonic trends has been a matter of research for decades in reliability and maintainability, [23] and [24], where the property of non-aging can also be interpreted as the so-called "as-good-as-new" condition. In particular, the interest has traditionally been put on testing exponentiality against aging alternatives (see [10] and [25] for a detailed discussion) and it continues being a subject of study nowadays, [26], [27], and [28] are some recent contributions. In the late 70's and early 80's several tests based on the TTT transform were proposed to test exponentiality against different aging trends ([29], [30], [16], [18], [19], [20]).

¿Son los del ejemplo?

The contribution of this paper is twofold. On the one hand, a new estimator of the TTT-transform is proposed based on nonparametric statistics, more specifically, based on kernel smoothing techniques. To our knowledge, this is an innovative contribution to this area of Statistics. Although nonparametric smoothing techniques are very popular in related areas such as survival analysis, they are not very usual in reliability engineering where parametric methods have been traditionally preferred by the practitioners

for solving inference problems. One motivation is the following. From available data, the theoretical TTT transform is usually approximated by means of the TTT-plot, the empirical estimator. While the theoretical curve is an absolutely continuous function, the empirical curve is always discontinuous and may be viewed as a poor approximation of the true curve, among other reasons, because it is a step function and the derivative either equals zero or is not defined, which makes difficult to evaluate shape characteristics. Kernel smoothing avoids discontinuities in the empirical function.

The second contribution is a new method for assessing non-aging properties in the underling lifetime. The new proposal is a graphical test able to recognize aging trends from data. The main advantage, compared to existing tests, is that it allows identifying specific areas where discrepancies from non-aging are detected. In contrast, standard testing methods summarize the sampling information in the single p-value to decide on the adequacy of the model specified in the null hypothesis.

The paper is organized as follows. In Section 2 the Total-Time-on-Test transform is formally defined and some relevant properties are reviewed. Section 3 introduces local-polynomial estimators for the TTT-curve and its two first derivatives. Section 4 is devoted to SiZer map methodology. First, the general idea is exposed and then a new graphic tool is presented based on the SiZer map aimed to recognize aging trends in a lifetime by developing scale and space inferences about the TTT curve and some particular transformations of this curve. Finally, a graphical method for hypothesis testing is proposed. The good performance of the new graphical test is evaluated by an extensive simulation study and the method is applied to several real datasets in Section 5. Conclusions and future research interests are summarized in Section 6. Finally, some mathematical developments are included in the Appendix.

Acortar.

2. Preliminary

This section is devoted to some aging classes defined through the failure rate and/or the residual life and that are univocally described in terms of the TTT-transform ([16]).

2.1. Reliability classes based on aging notions

Let X denote a random lifetime with absolutely continuous distribution function, with survival function S, for any fixed $t > 0$ let X_t denote the