Kinetic phase transition in the Ising model with on- and two- spin flip competing dynamics on the small-world network

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Abstract

We have studied the Ising model in a 2D small-world network (SWN) with competing dynamics. The network consists of a $L \times L$ square lattice with random addition of long-range interactions, and the competing dynamics are between the one- and two-spin flip mechanisms that govern the critical phenomena. Employing Monte Carlo simulations and finite-size scaling analysis, we have obtained the stationary state phase transitions and phase diagrams based on thermodynamic quantities, besides the critical exponents for the system. For the competition of the two dynamics, ferromagnetic F to anti-ferromagnetic AF phase transitions are found and vary as the addition of long-range interactions.

Model

For the SWN, initially we have a regular square lattice, with a high clustering coefficient, but with a certain probability p, a long-range random interaction can be added to each site of the network, guaranteeing the small characteristic path length [1,2]. Then the average coordination number on the network is $z = 4 + p$.

The ferromagnetic Ising model Hamiltonian is as follows:

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j,$$

where $J (J > 0)$ is the coupling constant if there is a link between the sites i and j and in which the sum is made.

With probability q we have simulated the system in contact with the heat bath by the one-spin flip mechanism with transition rate $W(\sigma_i \to \sigma_i')$, where the lowest energy state is preferable, and with probability $1 - q$ the system is subject to an external energy flux using the two-spin mechanism flip with transition rate $W(\sigma_i, \sigma_j \to \sigma_i', \sigma_j')$, where the highest energy state is preferred.

Monte Carlo simulation

Let (k, l) and (k', l') be the coordinates of a site in our two-dimensional SWN and one of your closest neighbors respectively, the one-spin flip transition rate is given by Metropolis prescription, and for two-spin flip we have

$$W(\sigma_{kl}\sigma_{k'l'} \to \sigma_{kl}'\sigma_{k'l'}') = \begin{cases} 0 \; if \; \Delta E_{kl,k'l'} \leq 0 \\ 1 \; if \; \Delta E_{kl,k'l'} > 0 \; . \end{cases}$$

The measured thermodynamic quantities in our simulations are magnetization per spin m_F, anti-ferro magnetization m_{AF}, magnetic susceptibility χ and reduced fourth-order Binder cumulant U:

$$m_F = \frac{1}{L^2} \left\langle \sum_{kl} \sigma_{kl} \right\rangle \; ; \; m_{AF} = \frac{1}{L^2} \left\langle \sum_{kl} (-1)^{(k+l)} \sigma_{kl} \right\rangle ,$$

$$\mathcal{X} = \frac{L^2}{k_B T} \left[\langle m^2 \rangle - \langle m \rangle^2 \right],$$

$$U = 1 - \frac{\langle m^4 \rangle}{3 \langle m^2 \rangle^2},$$

where m can be m_F or m_{AF}. These above defined quantities obey the following finite-size scaling relations in the neighborhood of the stationary critical point λ_C:

$$m = L^{-\beta/\nu} m_0(L^{1/\nu}\epsilon) \; ; \; \mathcal{X} = L^{\gamma/\nu} \mathcal{X}_0(L^{1/\nu}\epsilon) \, ,$$

$$U = U_0(L^{1/\nu}\epsilon),$$

where $\epsilon = (\lambda - \lambda_C)/\lambda_C$, λ can be T or q, m_0, $\mathcal{X}_0$ and U_0 are scaling functions, and β, γ and ν are the respective critical exponents.

Results

- Increasing the additive probability p, we add new shortcuts to our square lattice, and when $p = 1$ all the sites have coordination number $z = 5$, at this point we have the best results obtained, and some of the thermodynamic quantities representing the AF to paramagnetic P phase, and transition from F to P phase can be shown.

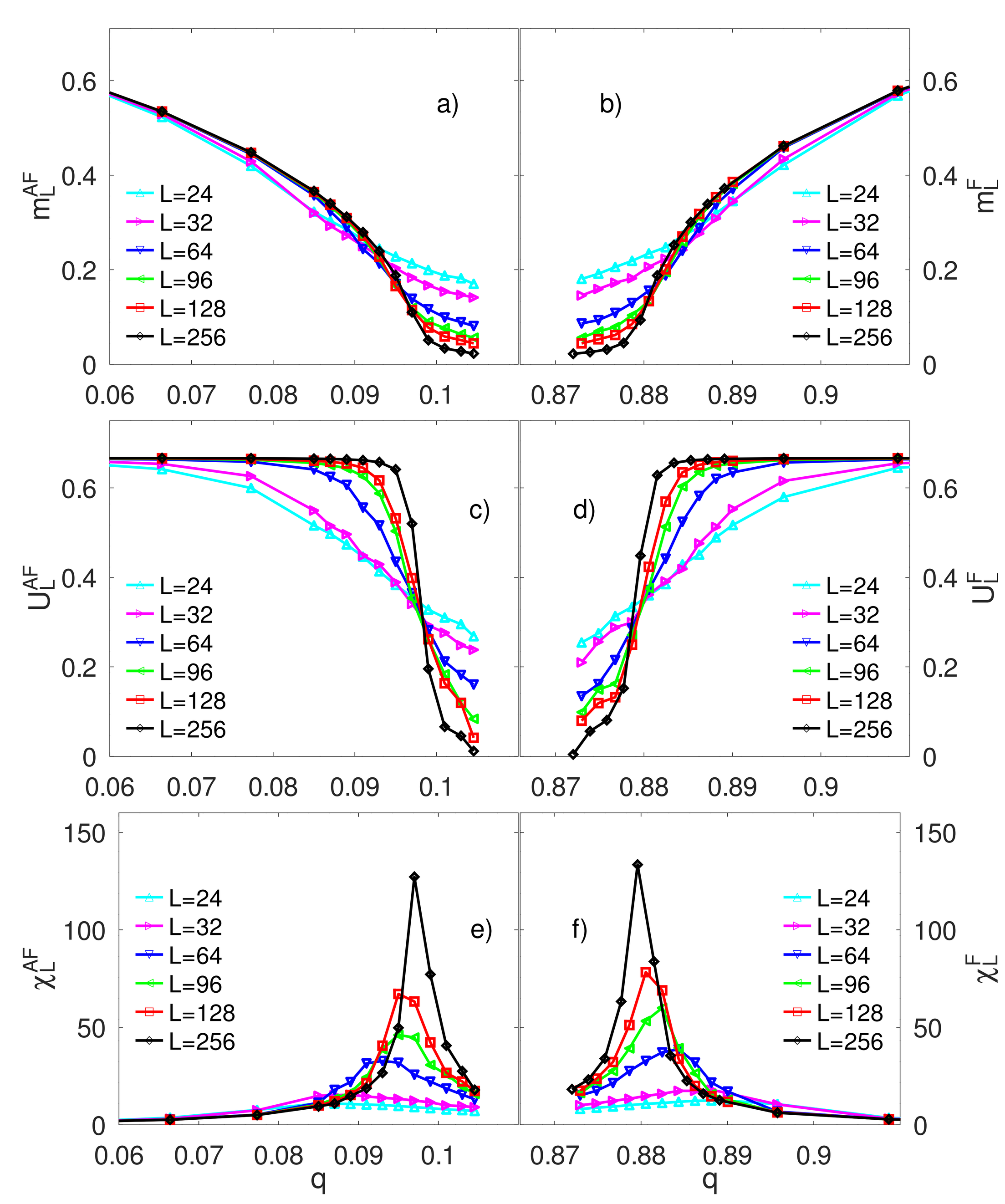

Fig. 1 - With some lattice sizes L we have in a) magnetization m_L^{AF} versus competition probability q, and b) $m_L^F \times q$. c) reduced fourth-order Binder cumulant $U_L^{AF} \times q$, where the crossing point of the curves gives us the critical point $q_C^{AF} = 0.0982 \pm 0.0002$, and d) $U_L^F \times q$, with $q_C^F = 0.8791 \pm 0.0003$. e) magnetic susceptibility $\chi_L^{AF} \times q$, were taking a fit of their peaks as a function of the inverse of L, we also have estimates for the critical point, $q_C^{AF}(L \to \infty) = 0.09774 \pm 0.0016$, and e) $\chi_L^{AF} \times q$ with $q_C^F(L \to \infty) = 0.8796 \pm 0.0013$. Here we have fixed $T = 1$ and $p = 1$.

- Behavior of thermodynamic quantities as a function of temperature T.

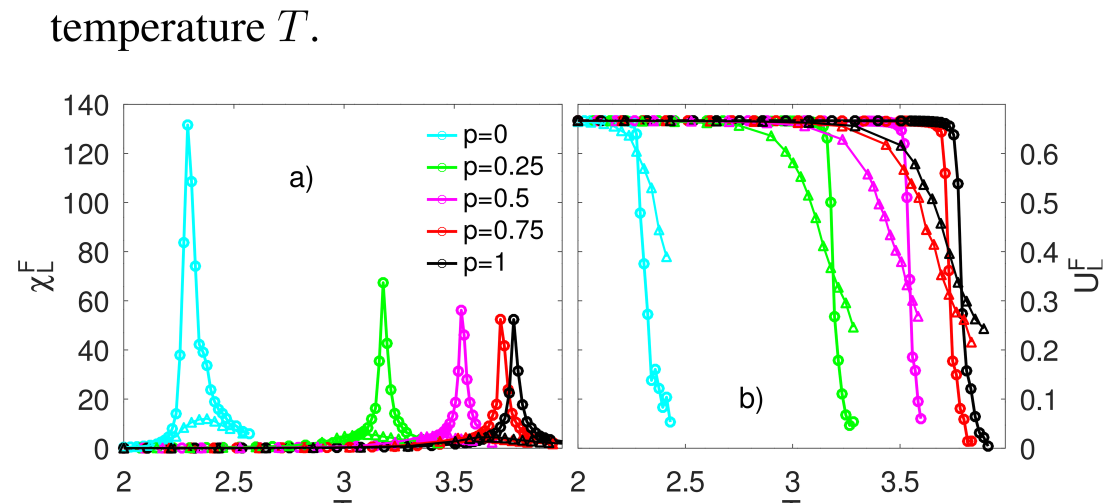

Fig. 2 - For some values of p, $L = 256$ (circles), $L = 24$ (triangles) and $q = 1$, in a) we have magnetic susceptibility $\chi_L^F \times T$ and b) shows the reduced fourth-order Binder cumulant $U_L^F \times T$.

- We have built the phase diagram based on the several measurements of critical points in the possible phase transitions.

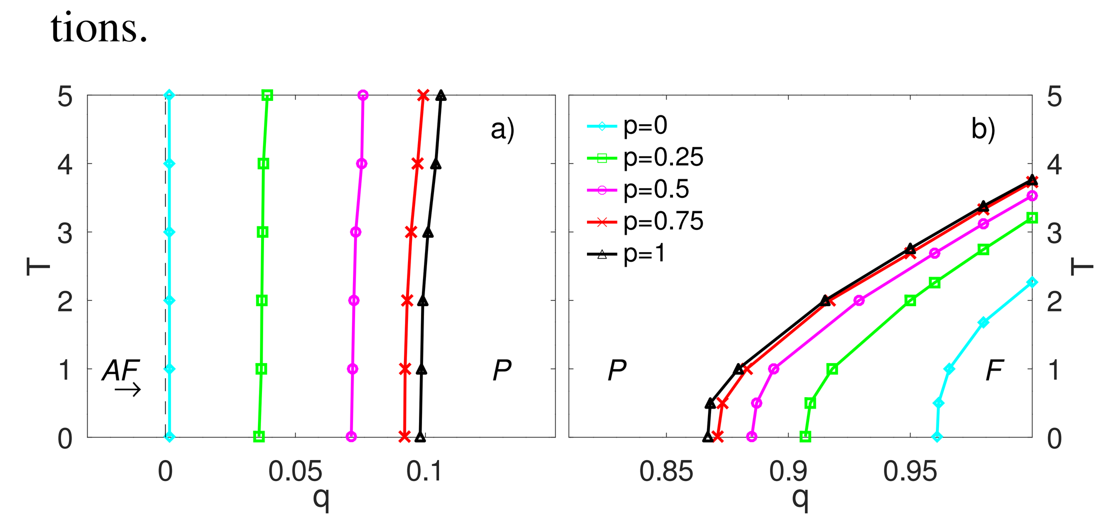

Fig. 3 - a) shows the region of q where we have found the transition from AF to P phase for some values of p, while in b) we have the region of the diagram where we have observed a phase transition between P and F phases, for the same values of p.

- Critical exponents β, γ and ν by collapsing the magnetization and magnetic susceptibility curves data.

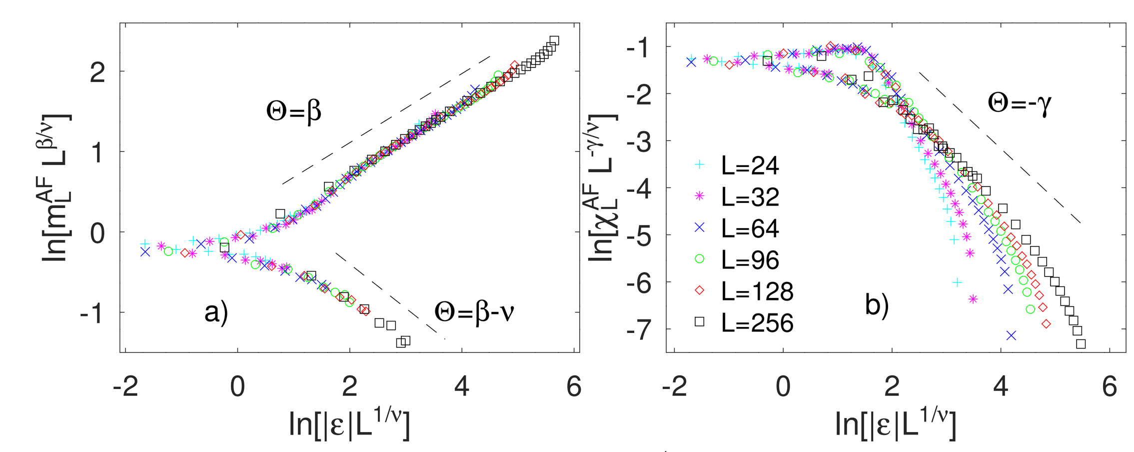

Fig. 4 - With fixed $p = 1$, $T = 1$ and $q_C^{AF} = 0.09774$, in a) we have the natural logarithm of the magnetization scaling function for some lattice sizes L, where by using $\beta = 0.42 \pm 0.02$ and $\nu = 0.98 \pm 0.04$, we obtained the best collapse of the data. b) in the same conditions with the magnetic susceptibility scaling function, the best collapse occurs in $\gamma = 1.08 \pm 0.06$ and $\nu = 0.99 \pm 0.06$. Where Θ is the slope of the collapsed data.

- Relationship between the critical exponents β, γ and ν, and the addition probability p.

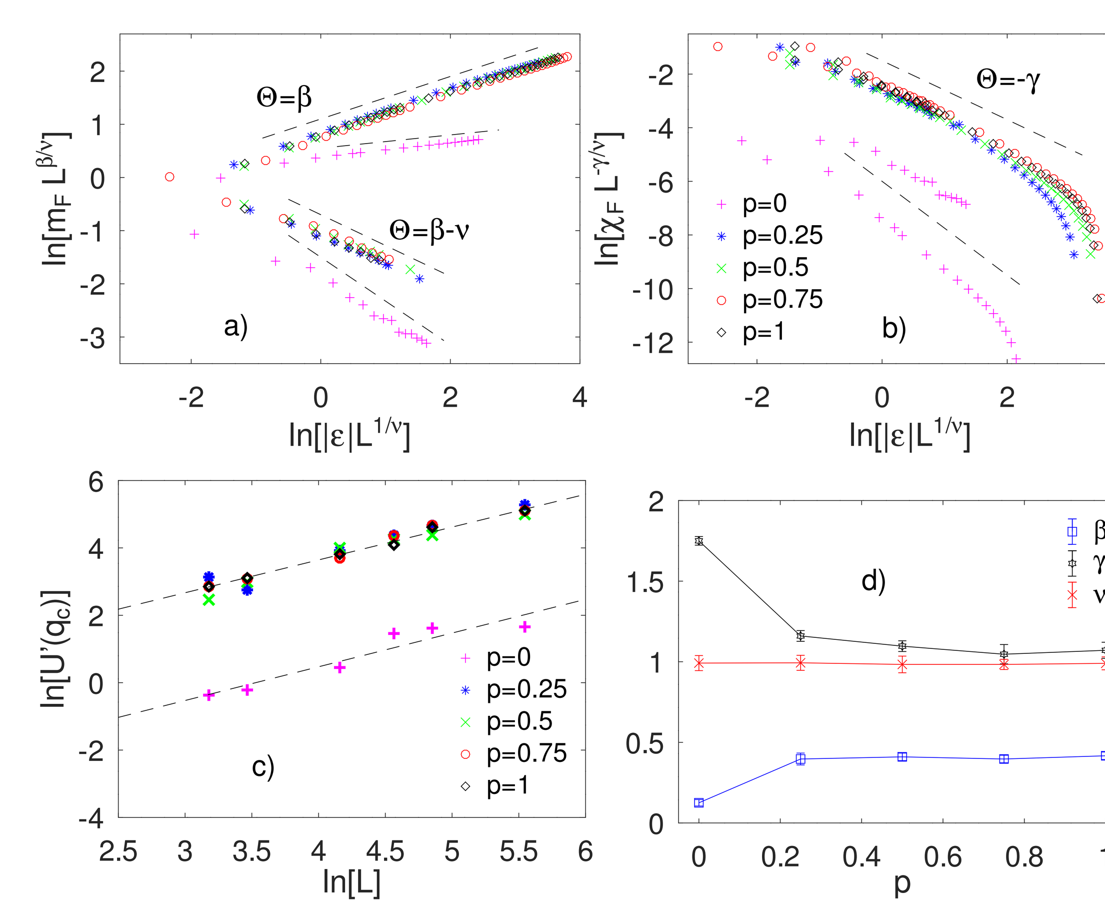

Fig. 5 - a) and b) contains collapsed curves of magnetization and magnetic susceptibility respectively, as a function of q in F phase, for some values of p and fixed $T = 1$, $L = 256$ and $q_C^F = 0.8796$. With these curves we have that for $p > 0$, $\beta = 0.4 \pm 0.05$, $\gamma = 1.1 \pm 0.09$ and $\nu = 1.0 \pm 0.05$ but for $p = 0$, $\beta = 0.125 \pm 0.02$, $\gamma = 1.75 \pm 0.03$ and $\nu = 1.0 \pm 0.04$. c) fit of the derivative reduced fourth-order Binder cumulant near the critical point $U'(q_C^F)$ for six lattice sizes L, where the inverse of the slope for these p values is $\nu = 1.0 \pm 0.08$. d) average over all critical exponents obtained, as a function of p.

Conclusions

- With competing dynamics we were able to find a spatial self-organized structure through the two types of phase transitions, from AF to P phase and from P to F phase, where both are seen by the magnetization of the system.
- Changing the coordination number by adding long-range interactions to network sites with probability p, changes the critical behavior of the system.
- Increasing p is also increased the region of the ordered AF and F phases, with the F phase being dependent on the temperature T and probability q, while AF phase depends only on the competition probability q.
- In SWN, the behavior of thermodynamic quantities near the critical point, characterized by the critical exponents, differs from the Ising model in regular square lattice, which are known exactly.

References

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Acknowledgements

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