Lab9

November 17, 2017

1 Lab de Circuitos - Preparatório 9

1.1 Rafael Rubim Cabral - 1511068

1.1.1 1)

Cálculo da função de transferência $H_Z(s)$:

$$Z_1(s) = \frac{1}{sC}//(Ls+Z) = \frac{1}{\dfrac{1}{Ls+Z}+sC} = \frac{Ls+Z}{1+LCs^2+ZCs}$$

$$V_2(s) = V(s)\frac{Z_1(s)}{R+Z_1(s)}$$

$$V_Z(s) = V_2(s)\frac{Z}{Ls+Z}$$

$$H_Z(s) = \frac{V_Z(s)}{V(s)} = \frac{Z_1(s)}{R+Z_1(s)}\frac{Z}{Ls+Z} = \frac{Z_1 Z}{RLs+ZR+Z_1Ls+Z_1Z} = \frac{Z}{\dfrac{RLs}{Z_1}+\dfrac{ZR}{Z_1}+Ls+Z}$$

$$H_Z(s) = \frac{Z}{\dfrac{RLs+RL^2Cs^3+ZRLCs^2}{Ls+Z}+\dfrac{ZR+ZRLCs^2+Z^2RCs}{Ls+Z}+Ls+Z}$$

$$H_Z(s) = \frac{ZLs+Z^2}{RLs+RL^2Cs^3+ZRLCs^2+ZR+ZRLCs^2+Z^2RCs+(Ls+Z)^2}$$

$$H_Z(s) = \frac{ZLs+Z^2}{RL^2Cs^3+ZRLCs^2+ZRLCs^2+L^2s^2+RLs+Z^2RCs+2ZLs+ZR+Z^2}$$

$$H_Z(s) = \frac{ZLs+Z^2}{RL^2Cs^3+(2ZRLC+L^2)s^2+(RL+Z^2RC+2ZL)s+(ZR+Z^2)}$$

Vamos considerar a fase da fonte $\phi = 0$. Logo temos o fasor $\dot{V} = Ae^{j.0}$. O fasor da saída $\dot{V}_Z$ será:

$$\dot{V}_Z = H_Z(j\omega_0)\dot{V}$$

$$H_Z(j\omega_0) = \frac{jZL\omega_0+Z^2}{-jRL^2C\omega_0^3-(2ZRLC+L^2)\omega_0^2+(RL+Z^2RC+2ZL)j\omega_0+(ZR+Z^2)}$$

$$H_Z(j\omega_0) = \frac{Z^2+jZL\omega_0}{[ZR+Z^2-(2ZRLC+L^2)\omega_0^2]+j[(RL+Z^2RC+2ZL)\omega_0-RL^2C\omega_0^3]}$$

1.1.2 a)

$Z = Z_0e^{j\phi}$ puramente resistivo: $Z = R+j.0 \rightarrow \phi = 0 \rightarrow Z = Z_0$

$$H_Z(j\omega_0) = \frac{Z_0^2 + jZ_0L\omega_0}{[Z_0R + Z_0^2 - (2RLZ_0C + L^2)\omega_0^2] + j[(RL + Z_0^2RC + 2LZ_0)\omega_0 - RL^2C\omega_0^3]}$$

Logo $\dot{V}_Z = H_Z(j\omega_0)\dot{V} = H_Z(j\omega_0)A$ e então $V_Z(t) = \mathbb{R}(H_Z(j\omega_0)Ae^{j\omega_0 t})$

$$V_Z(t) = \mathbb{R}\left(\frac{(Z_0^2 + jZ_0L\omega_0)Ae^{j\omega_0 t}}{[Z_0R + Z_0^2 - (2RLZ_0C + L^2)\omega_0^2] + j[(RL + Z_0^2RC + 2LZ_0)\omega_0 - RL^2C\omega_0^3]}\right)$$

1.1.3 b)

$Z = Z_0e^{j\phi}$ puramente capacitivo: $Z = 0 - j.X \rightarrow \phi = \dfrac{3\pi}{2} \rightarrow Z = -jZ_0$

$$H_Z(j\omega_0) = \frac{-Z_0^2 + Z_0L\omega_0}{[2LZ_0\omega_0 - Z_0^2 - L^2\omega_0^2] + j[(RL - Z_0^2RC)\omega_0 - Z_0R - RL^2C\omega_0^3 + 2RLZ_0C\omega_0^2]}$$

Logo $\dot{V}_Z = H_Z(j\omega_0)\dot{V} = H_Z(j\omega_0)A$ e então $V_Z(t) = H_Z(j\omega_0)Ae^{j\omega_0 t}$

$$V_Z(t) = \mathbb{R}\left(\frac{(-Z_0^2 + Z_0L\omega_0)Ae^{j\omega_0 t}}{[2LZ_0\omega_0 - Z_0^2 - L^2\omega_0^2] + j[(RL - Z_0^2RC)\omega_0 - Z_0R - RL^2C\omega_0^3 + 2RLZ_0C\omega_0^2]}\right)$$

1.1.4 c)

$Z = Z_0e^{j\phi}$ puramente indutivo: $Z = 0 + j.X \rightarrow \phi = \dfrac{\pi}{2} \rightarrow Z = jZ_0$

$$H_Z(j\omega_0) = \frac{-Z_0^2 - Z_0L\omega_0}{[-2LZ_0\omega_0 - Z_0^2 - L^2\omega_0^2] + j[(RL - Z_0^2RC)\omega_0 + Z_0R - RL^2C\omega_0^3 - 2RLZ_0C\omega_0^2]}$$

Logo $\dot{V}_Z = H_Z(j\omega_0)\dot{V} = H_Z(j\omega_0)A$ e então $V_Z(t) = H_Z(j\omega_0)Ae^{j\omega_0 t}$

$$V_Z(t) = \mathbb{R}\left(\frac{(-Z_0^2 - Z_0L\omega_0)Ae^{j\omega_0 t}}{[-2LZ_0\omega_0 - Z_0^2 - L^2\omega_0^2] + j[(RL - Z_0^2RC)\omega_0 + Z_0R - RL^2C\omega_0^3 - 2RLZ_0C\omega_0^2]}\right)$$

2) As simulações não puderam ser realizadas pois a licença da PUC do circuitlab não está funcionando e não encontrei outro simulador que gere gráficos com desenho de escala. Seguem as contas:

Adotando nos 3 casos:

$R = 100\Omega$

$L = 1\mu H$

$C = 1\mu F$

$\omega_0 = 100 rad/s$

$f = \dfrac{\omega_0}{2\pi} = 15,92Hz$

$T = \dfrac{1}{f} = 62,66ms$

$A = 10V$

1.1.5 a)

$Z_R = 100\Omega$

$Z = Z_0 = 100\Omega$

$$V_Z(t) = \mathbb{R}\left(\frac{(Z_0^2 + jZ_0L\omega_0)Ae^{j\omega_0 t}}{[Z_0R + Z_0^2 - (2RLZ_0C + L^2)\omega_0^2] + j[(RL + Z_0^2RC + 2LZ_0)\omega_0 - RL^2C\omega_0^3]}\right)$$

$$V_Z(t) = \mathbb{R}\left(\frac{(10^4 + j10^{-2})10e^{j10^2t}}{[10^4 + 10^4 - (2.10^{-8} + 10^{-12})10^4] + j[(10^{-4} + 1 + 2.10^{-4})10^2 - 10^{-16}10^6]}\right)$$
$$V_Z(t) = \mathbb{R}\left(\frac{10^4e^{j.0}10e^{j10^2t}}{[-10^{-8}] + j[(10^2 + 3.10^{-2}) - 10^{-10}]}\right)$$
$$V_Z(t) = \mathbb{R}\left(\frac{10^5e^{j10^2t}}{[-10^{-8}] + j[10^2]}\right)$$
$$V_Z(t) = \mathbb{R}\left(\frac{10^5e^{j10^2t}}{10^2e^{j\frac{\pi}{2}}}\right)$$
$$V_Z(t) = 10^3\sin\left(10^2t - \frac{\pi}{2}\right)$$

1.1.6 b)

$Z_C = 1\mu F$

$$Z = -jZ_0 = \frac{-j}{\omega_0 Z_C} = \frac{-j}{10^2 10^{-6}}$$
$$Z_0 = 10^4$$
$$V_Z(t) = \mathbb{R}\left(\frac{(-Z_0^2 + Z_0L\omega_0)Ae^{j\omega_0t}}{[2LZ_0\omega_0 - Z_0^2 - L^2\omega_0^2] + j[(RL - Z_0^2RC)\omega_0 - Z_0R - RL^2C\omega_0^3 + 2RLZ_0C\omega_0^2]}\right)$$
$$V_Z(t) = \mathbb{R}\left(\frac{(-10^8 + 1)10e^{j10^2t}}{[2 - 10^8 - 10^{-8}] + j[(10^{-4} - 10^4)10^2 - 10^6 - 10^{-16}10^6 + 2.10^{-2}]}\right)$$
$$V_Z(t) = \mathbb{R}\left(\frac{10e^{j10^2t}}{[-10^8] + j[-2.10^6]}\right)$$
$$V_Z(t) = \mathbb{R}\left(\frac{10e^{j10^2t}}{10^8e^{j.0}}\right)$$
$$V_Z(t) = 10^{-7}\sin\left(10^2t\right)$$

1.1.7 c)

$Z_L = 1\mu H$

$$Z = jZ_0 = j\omega_0 Z_L = j10^2 10^{-6}$$
$$Z_0 = 10^{-4}$$
$$V_Z(t) = \mathbb{R}\left(\frac{(-Z_0^2 - Z_0L\omega_0)Ae^{j\omega_0t}}{[-2LZ_0\omega_0 - Z_0^2 - L^2\omega_0^2] + j[(RL - Z_0^2RC)\omega_0 + Z_0R - RL^2C\omega_0^3 - 2RLZ_0C\omega_0^2]}\right)$$
$$V_Z(t) = \mathbb{R}\left(\frac{(-10^{-8} - 10^{-8})10e^{j10^2t}}{[-2.10^{-8} - 10^{-8} - 10^{-8}] + j[(10^{-4} - 10^{-12})10^2 + 10^{-2} - 10^{-16}10^6 - 2.10^{-14}10^4]}\right)$$
$$V_Z(t) = \mathbb{R}\left(\frac{-2.10^{-7}e^{j10^2t}}{[-4.10^{-8}] + j[2.10^{-2} - 4.10^{-10}]}\right)$$
$$V_Z(t) = \mathbb{R}\left(\frac{-10^{-7}e^{j10^2t}}{[-2.10^{-8}] + j[10^{-2} - 210^{-10}]}\right)$$
$$V_Z(t) = \mathbb{R}\left(\frac{-10^{-7}e^{j10^2t}}{10^{-2}e^{j\frac{\pi}{2}}}\right)$$
$$V_Z(t) = -10^{-5}\sin\left(10^2t - \frac{\pi}{2}\right)$$

1.1.8 3)

1.1.9 a)

A frequência de ressonância é igual a $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{3.10^4 10^{-6} 10^{-9}}} = \dfrac{1}{\sqrt{30}.10^{-6}} = \dfrac{\sqrt{30}.10^6}{30}$ rad/s

1.1.10 b)

A frequência de ressonância é a frequência para a qual a soma das impedâncias do capacitor e do indutor é igual a 0. Como a impedância equivalente do circuito RLC é $Z_R + Z_L + Z_C$, então a impedância equivalente do circuito valerá $Z_R = R$. Isso significa que no regime senoidal permanente a corrente estará em fase com a fonte e o circuito pode ser enxergado como um circuito puramente resistivo.

```
In [ ]:
```