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Optimal offering strategies in a pay-as-bid energy balancing market with ahead-of-time offers

by

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Abstract

In recent years, energy markets have seen an increasing participation of renewable generation that has brought electricity prices down and has contributed with cleaner and more sustainable energy sources. However, the integration of renewable comes with several technical and economical challenges for the system to tackle. One of the main challenges for the system is to compensate for the stochastic nature of renewables (i.e. wind and solar), where forecast errors are frequent and occur in very short time scales. The last leads the market to require energy regulation constantly to match supply and demand, resulting in highly volatile energy prices in short-term markets.

In this context, participation in short-term markets is not trivial and requires for agents to develop strategies in order to profit accordingly from it, and provide the flexibility required. The present thesis focuses in developing optimal offering strategies for an energy producer that participates in an electricity balancing market with a pay-as-bid pricing scheme. The producer is represented by a cluster of controllable units (i.e. thermal and storage) and has to submit optimal up and down regulation offering curves in order to maximize the expected profit, in a context of price uncertainty where a price forecast is available. In addition, offering curves must be submitted for periods ahead of time, before knowing the result of recent offerings made to the market.

A stochastic three-stage optimization model is proposed for the agent to decide for optimal offerings, and schedule the cluster accordingly to be able to deliver in the real-time the committed energy. Additionally, a dynamic programming model is implemented to explore future price scenarios and compute a value function for the producer's flexibility (i.e. capacity to modify production in response to variability) on a given hour. The output of the DP model serves as a "look-ahead" feature to the stochastic model, with the objective of modifying the offering curves in such way that the expected profit made throughout a set of sequential hours is improved. Finally, a tuned model is proposed which improves profits when compared to a model which completely ignores future prices.

Acknowledgments

I want to express my gratitude to my main supervisor Professor Ken McKinnon, who dedicated his time and interest to find the best answers to the questions this project posed.

The starting point of this thesis is based on my co-supervisor's work, Nicolo Mazzi. I would like to thank him for introducing me to these topics and giving out very close and dedicated supervision.

My heartfelt thanks to Amalia, Antonia and Colomba.

Own Work Declaration

I declare that this thesis was composed by myself and that the work contained therein is my own, except where explicitly stated otherwise in the text.

(Rafael Sacaan A.)

Contents

List of Tables

List of Figures

1 Introduction

1.1 Energy markets

Electricity, in economic terms, may be considered as an atypical commodity: it is not easily traceable nor storable. These properties pose several economic and physical challenges to regulation entities and participants in power systems, in order to organize the production and delivery of electrical energy to end consumers. In past years, a single electric utility was responsible for coordinating the generation of electric power, transmitting the energy to substations and, from there, distributing it to final users. In many cases, this vertically integrated structure led to price discrepancies among end customers and usually to a conservative stand towards system planning and energy supply. As a consequence, this approach resulted in a reliable but costly and inefficient operation [1] that motivated a massive process of deregulation in power systems starting in the 1980s and 1990s. According to [5], deregulation in electricity markets has been globally the largest when compared to deregulation processes carried out in other industries.

The process of deregulation in power systems consists in unbundling the vertical structure described before in three independent activities (i.e. generation, transmission and distribution) giving place to a new framework. According to [13], these sectors can be described as follows. In the generation sector, producers compete to provide electrical energy to medium/large consumers. On the basis that there are no identifiable economies of scale in the business nor in technology, natural conditions are given for full competition. On the other hand, the transmission sector is recognized to behave as a natural monopoly where economies of scale can take place, thus, requiring for governmental regulation. The transmission system is then organized by a non-profitable entity and acts as a platform where generation and demand needs to be matched in real-time according to the commercial agreements made by the participants. The transmission system is then open to all participants through the payment of fees for its usage, and it is responsible for providing transparent and non-discriminatory price signals. Finally, the distribution sector is in charge of buying energy from large producers and delivering it to end customers, and develops as a geographical monopoly. Beside distribution companies, there exists other type of customers which participate in the energy market for supply (e.g. industrial consumers).

As mentioned before, electrical energy is not easily stored in large quantities (and it is also yet not economical with current technology), hence, it must be generated exactly at the moment when it is demanded. This fact poses important physical challenges for the system to have a safe, continuous and stable operation. For this same reason and due to physical constraints of conventional generators, it is important for these units to schedule ahead of time their production in order to aim for technical feasibility and cost-efficient results. However, in the scope of a power system as a whole, demand forecast errors and system contingencies (e.g. failure in generation unit, congestion or failure in transmission lines) may occur and require deviations in committed generators or may even require new generators to be brought up to service. These adjustments are settled throughout time in sequential windows where the length of these get shorter as the time of delivering the energy gets closer. Therefore, marketplaces ahead of time are built on top of the previous months, days and hours of energy delivery, where bids and offers are transacted in order to ensure that demand is satisfied reliably and sufficiently. There are two important agents in this context: an Independent System Operator (ISO), which is in charge of ensuring the optimal technical operation of the power system, and a Market Operator (MO) which collects bids and offers from participants and clears quantities and prices in the market according to specified procedures.

In the context of an unbundled power system framework, a set of sequential energy marketplaces take place which can usually be classified into two main categories: futures markets and pools [2]. Futures markets are described as auction markets where participants agree to buy and sell physical energy or financial products (i.e. derivatives) on a specified future date, which spans between one week to years before the product delivery. Different to futures markets, markets closer to the date of delivery are characterized mainly by presenting high price volatility. Therefore, futures markets provide an instance for participants to hedge against price uncertainty until the time of delivery. Beside futures markets, bilateral agreements among sellers and purchasers are allowed, where a producer can sell products to consumers or retailers outside futures markets. On the other hand, auction places oftenly called "pools" are platforms where energy is exchanged on a short-term basis. A pool may be constructed typically as follows: i) a day-ahead market, ii) multiple shorter-term markets or adjustment markets, and iii) a balancing market which ensures to match demand and generation in real-time. Producers and customers/retailers interact in these markets by submitting offers/bids for energy while the Market Operator (MO) is in charge of clearing prices and traded quantities through market-clearing procedures that satisfy an optimal techno-economical operation of the system. As mentioned in [2], in most electricity markets pool prices present the following characteristics: non-stationary behavior (i.e. in their observed mean and variance), high volatility and high presence of outliers. These properties set hard conditions to properly generate accurate price forecasts.

1.2 Uncertainty in balancing markets

There are several sources of uncertainty in balancing markets that drive energy prices to fluctuate constantly and abruptly. In the first place, generation and transmission components of power systems are sensitive to unexpected physical failures which can make them unavailable on short notice. These events may require unscheduled generation units to deliver unsupplied energy quickly (e.g. gas-fired or diesel turbines) which come with a higher cost compared to units that are physically constrained to do so (e.g. coal-fired turbines). On the other hand, even though consumer's demand follow a relatively stationary pattern over different time horizons (i.e. multi-seasonal behavior), it is hard to predict exactly for real-time operation. Demand forecasts present deviations which can be typically driven by weather conditions and calendar events among others. However, the main cause of uncertainty now is due to the fact that more renewable generation participates in the system.

In recent years, with the increasing integration of renewable energy, changes in the economical and technical behavior of power systems have taken place. Considering the fact that energy produced from renewable generators has marginal costs equal to zero, regions with high penetration of renewables can be characterized by periods of low prices. Prices can even become negative under special circumstances. For example, for a conventional generator (i.e. physically constrained) it may be more economical to pay for a load to accept their energy than rather to deal with shut-down periods during its operation. Renewable generation (i.e. wind and solar) is categorized as non-dispatchable generation: production depends directly on the behavior of the natural resource and, therefore, the power production is partially controllable. For this reason and because of the impossibility to successfully forecast renewable generation, non-dispatchable producers need to participate in balancing markets an communicate in real-time the deviations from their committed production in previous markets.

Deviations declared by non-dispatchable producers in balancing markets entail the MO and ISO to cover the surplus or shortages of energy throughout the different "pools". For this reason, dispatchable (i.e. controllable) generators are specially relevant to the system because they are able to provide bulk and reliable energy. Furthermore, these units can provide the flexibility needed to cover the levels of intermittency present in the system. As long as a conventional producers can detect price signals in the balancing market, their participation will enable and incentive renewables generation. Consequently, optimal offering and operational strategies must be developed to enable dispatchable generators to profit and enhance not only their participation, but the overall efficiency in the balancing markets.

1.3 Optimal offering strategies in balancing markets

In literature, different work has been done to find operational and economical strategies that support the design of optimal offerings in short-termed energy markets. In [6], a framework is proposed to conjunctly schedule the operation of a combined heat and power (CHP) plant and a wind farm in the day-ahead and balancing market, with the objective of maximizing profits in both markets. Similarly, in [3] a wind farm is complemented by energy storage systems (ESS) to perform arbitrage in balancing markets and correct deviations form the day-ahead market produced by stochastic generation. In [11], a decision making tool is proposed for a storage system which participates in the day-ahead an balancing markets as a price-making agent, and additionally in the reserves market. In this context, the optimal operation of the storage is described when the objective is profit maximization, and also how interactions between the different markets changes operational decisions. In [7], a model is proposed to find strategical offerings for a price-taker agent that participates in the day-ahead market organized as a power exchange. The approach is to hedge risk against infeasible operational scenarios that may occur as a consequence of the acceptance of offers for certain price scenarios. Offering curves are built based on confidence intervals rather than on exact price forecasts. In [12] a stochastic model is provided for optimal bidding of a thermal generator that participates in sequential short-termed markets: day-ahead, the automatic generation control (AGC) and balancing market. The producer behaves as a price-taker in the first two markets, and as a price-maker in the balancing market.

However, in [9] the authors describe a model in which a price-taker producer maximizes profit through the design of optimal offering curves under a pay-as-bid pricing scheme in a balancing market, providing a linear formulation. This work is used as the starting point for the present work.

1.4 Thesis contribution

The contribution of this paper is to propose a three-stage stochastic model that builds up optimal up and down regulation offering curves for a price-taker producer in the balancing market. The balancing market modelled in this paper is similar to the UK's energy spot market, where offerings are made sequentially ahead of time. Specifically, offers have to be submitted for the time block that corresponds to three hours away from the real-time. This property poses several challenges: the producer has to submit offering curves before knowing if the curves submitted in the past, which correspond to the next two upcoming hours, are going to be accepted by the market. In addition, the producer must link an operational schedule through sequential hours in order to fulfill the committed energy in the past hours.

Furthermore, a dynamic programming (DP) model is proposed as a "look-ahead" feature of the stochastic model. The DP model explores price scenarios from a forecast available to the producer, and evaluates a value function which prices the amount of storage and the operating level of a ramp-constrained thermal unit for a given hour in a time horizon. This feature informs the stochastic model in such way that the available energy capacity of the cluster is increased when prices are high in order to improve blind-making offers (i.e. no future price exploration).

2 Modelling offering strategies in the UK electricity balancing market.

Deregulation in electricity markets has brought up several challenges to market and system operators, as well as for market participants. By removing a centralized organization, the disaggregation of power systems (i.e. generation, transmission and distribution) has opened space to competition and for investors to take part in the different markets. As mentioned in Section 1.1, the generation sector presents economic characteristics that can provide a scenario of full competition. In this sense, producers and consumers/retailers must interact continuously in a marketplace that is technically operated by an Independent System Operator (in Europe) and a Market Operator that ensures a transparent and cost-efficient economic exercise.

Close to the time of electricity delivery, several markets with different time horizons take place. The purpose of each of these markets is to provide a marketplace for producers and customers/retailers to adjust their offers/bids through time as different uncertainties affect their planned schedules or forecasts. Uncertainty in this context can come in many different forms: failure in transmission components or generation units, transmission congestion or demand forecast. In addition, nowadays the participation of renewable generation offers clean and inexpensive energy, but its intermittent nature (i.e. wind and solar) imposes a large uncertainty factor in the system's operation.

Usually, short-term settlement markets occur sequentially in the following way: i) the day-ahead market, ii) intra-day market and iii) balancing market. In Europe, generally all of these take the form of a power exchange, which is an open trade market for participants to offer/bid quantities of energy freely to the market trading platform [8]. Then, the Market Operator is responsible for clearing the market: matching the aggregate demand curve with the aggregate supply curve and determining the price and quantity (i.e. cleared for that predefined time interval).

In this context, participation in short-term markets is not trivial and requires agents to develop strategies in order to profit from it. The main problem of a producer is to submit offering curves (i.e. multiple energy price-quantity pairs) for future time periods that will maximize the expected profit of the energy sold, given that prices in the future are uncertain. In addition, the offers must follow an operational schedule that ensures that the producer can deliver the committed amount of energy when the market requires it. In addition, if the producer controls a storage unit, then he can benefit from market arbitrage: charging the storage unit when prices are low to sell energy in a future period when prices are high.

The present thesis focuses on developing optimal offering strategies for an energy producer that participates in the UK electricity balancing market. Some assumptions and constraints are made in order to simplify the development of the problem. Firstly, the producer is characterized by having a risk-neutral position towards his participation in the market, and is also treated as a price-taker agent. Secondly, a power exchange structure is considered to represent the balancing market, as it is the structure found in the UK day-ahead and balancing markets, and which also usually corresponds to most European markets. Finally, the producer agent is only allowed to operate controllable generating units (i.e. dispatchable generation), not those that have any endogenous uncertainty such as wind or solar generation. Consequently, the producer has to submit offer curves to the balancing market to maximize his profit, which means he has to solve a scheduling and planning problem for the generating units owned. The generation cluster can be composed by a single storage unit, a unique thermal unit, or the combination of both.

In Section 2.1 short-term markets are described, where uncertainty is reflected in the balancing market prices. Then, Section 2.2 describes the structure of the balancing market price forecast available to the producer in the form of a scenario tree. Section 2.3 describes the pay-as-bid pricing scheme mechanism. In Section 2.4 the cluster of energy units controlled by the producer are described. Finally, Section 2.5 describes the stochastic offering model of the producer.

2.1 Electricity energy short-term markets.

In this thesis, short-term trading is modelled by two sequential and separate marketplaces in time: first, the day-ahead market settlement and then, the balancing market. The day-ahead market is taken into account given that a producer may have contracted energy in this market in order to participate later on in the balancing market. When the balancing market takes place, adjustments can be offered over the contracted energy in the day-ahead according to the balancing market requirements. Also, it is important to mention that the day-ahead market provides a transparent and reliable reference price for electricity due to its high liquidity. Other intra-day markets which may also interact with the above mentioned markets are neglected for simplicity. In Sections 2.1.1 and 2.1.2 both markets are further explained along with the modelling considerations made.

2.1.1 The day-ahead market.

The day-ahead market is an instance for buyers and sellers to plan their operations by trading energy the day before its actual delivery. In this auction, a single offer or bid corresponds to a single quantity q of energy priced at a certain level p for a particular time period in the day-ahead. A single bid from buyer b corresponds to the pair (p_b^{bid},q_b^{bid}), where p_b^{bid} is the highest price buyer b is willing to pay for quantity q_b^{bid}. Similarly, producer g can submit offer (p_g^{off},q_g^{off}) where p_g^{off} is the minimum price he is willing to accept to sell the offered quantity q_g^{off}. Participants may submit multiple offers/bids for any of the trading blocks in the next day before a specified time called gate closure. Afterwards, the MO is in charge of collecting and aggregating every energy bid for each trading block, and accept the necessary amount of offers to match this demand. In this way, the MO clears the market and returns the total quantity traded in each block and its associated price.

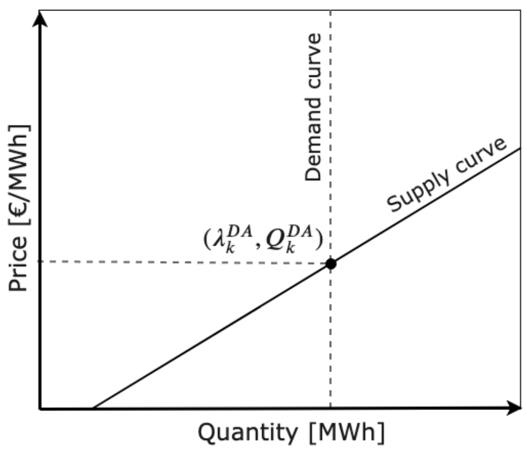


Figure 1: Day-ahead market: Demand and supply curves clearing for time block k.

In this thesis, the day-ahead market is considered to be structured by 24 sequential time blocks, each corresponding to one hour. In the case of the UK, the hourly day-ahead market is called the 'EPEX SPOT UK Power Auction'. Also, there is a shorter termed day-ahead market that is called the 'UK Half Hour Day-Ahead 15:30 Auction'. As its name suggests, this market accounts for half-hour intervals in the day-ahead with a gate closure occurring at 15.30.

Let us define K as the set of trading blocks available in the day-ahead market, and k the index of each of these blocks, where $k = 1, 2, 3, ..., 24$. Therefore, each hour k represents an exact hour of the day. The sequence of actions in the day-ahead market is the following: participants submit anonymously their offers/bids for any of the k blocks of the day-ahead before gate closure and are not allowed to see the offers/bids of the rest of the participants. Also, producers/consumers can submit multiple offers/bids for a given block, therefore, having the possibility to build up an individual supply/demand curve for each of these. Immediately after gate closure occurs, the MO collects every offer/bid for each of the blocks in K and builds up an aggregated demand and supply curve (figure 1). Next, the MO clears a quantity and price for each of the k day-ahead trading blocks, where λ_k^{DA} represents the day-ahead clearing price for block k and Q_k^{DA} the total quantity of energy traded during block k, as shown in figure 2. For simplicity, it is assumed that every accepted offer in the day-ahead is priced at the corresponding clearing market price (i.e. uniform pricing scheme). Finally, the MO clears every trading block in the day-ahead market independently and simultaneously.

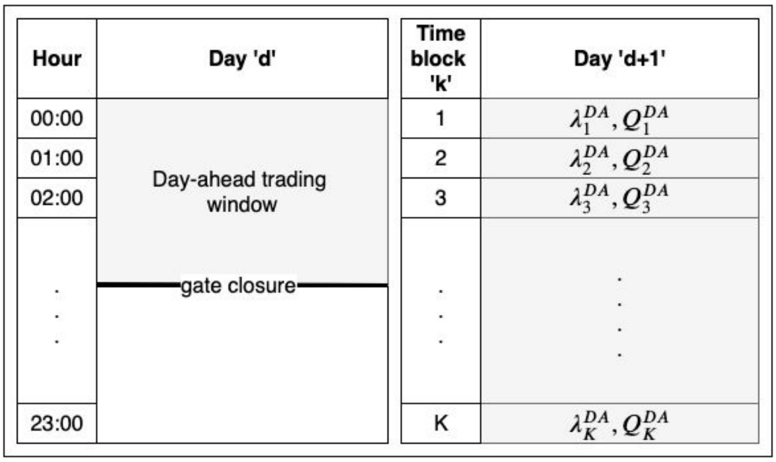


Figure 2: Day-ahead market: the MO collects offers and bids for each block k in the day-ahead before gate closure and clears energy prices λ_k^{DA} for each hour of the next day.

In general, we can suppose that a producer that has actively participated in the day-ahead market will have submitted multiple offers for each block k. If as a result of market clearance an offer is accepted, the producer will be committed to deliver the quantity offered in real-time during block k in the next day. Let q_k^{DA} represent the contracted quantity by a single producer in the day-ahead market for every trading block $k \in K$.

In this thesis, the contracted quantities q_k^{DA} will be considered as fixed. This means that the producer has already submitted his offers in the day-ahead market and it has been revealed already which of these were accepted by the MO. Beyond this point, no further uncertainty is attached to them. These assumptions are considered to simplify the offering problem and to maintain the focus of the present work in the interaction of the producer with the balancing market.

2.1.2 The balancing market.

The balancing market, as mentioned before, corresponds to a power exchange marketplace that has the same structure as the day-ahead market. It is the last trading floor that allows the system to match demand and supply. However, differently from the day-ahead market, the balancing market is cleared by the System Operator (SO) as it is very close to real-time and the system's constraints become more relevant. Also differently, it operates under a pay-as-bid pricing scheme.

In the UK, several electricity balancing markets or spot markets exist, called "EPEX SPOT" markets. These are categorized according to their time resolution: half-hour, one hour, two hours or four hours. In this thesis, an hourly balancing market composed by 24 blocks is considered.

Trading windows in the balancing market correspond to the same set K of hourly blocks found in the day-ahead market. When in time k, the System Operator knows from the day-ahead market that an aggregated quantity of generation has been committed to exactly match the demand predicted at the day-ahead gate closure. Hours before real-time delivery, different sources of uncertainty (e.g. demand forecast) will force further adjustments. In this context, let D_k^{DA} denote the aggregated demand quantity cleared during the day-ahead market for hour k, and D_k^{RT} the aggregated demand quantity seen close to real-time by the SO for hour k. Then, the demand deviation seen by the SO for hour k will be:

$$\delta_k^{demand} = D_k^{RT} - D_k^{DA}, \quad \forall k \in K \tag{2.1}$$

In the same way, uncertainty can affect generation that has already been committed (e.g. forecast errors in stochastic generation). Let N_k^G be the set of every single producer that has committed energy during the day-ahead for block k, where each producer in the set is indexed by g. Also, let O_{gk}^{RT} be the real-time generation of producer g during block k and O_{gk}^{DA} be the contracted energy in the day-ahead market of producer g for hour k. Then, the total deviation in production seen by the SO for hour k will be:

$$\delta_k^{gen} = \sum_{g=1}^{N_k^G} (O_{gk}^{RT} - O_{gk}^{DA}), \quad \forall k \in K \tag{2.2}$$

Quantities δ_k^{demand} and δ_k^{gen} can either take positive or negative values. Finally, the total deviation that the SO will foresee just before block k occurs will be:

$$\delta_k^{total} = \delta_k^{demand} - \delta_k^{gen}, \quad \forall k \in K \tag{2.3}$$

The resulting deviation δ_k^{total} (eq. 2.3) can take positive or negative values. The main objective of the balancing market is to adjust the deficit or surplus of energy supply close to real-time through a market mechanism. The process in which the SO adjusts supply and demand in the balancing market is usually called regulation. In the case when δ_k^{total} is equal to zero, demand and supply are matched together and no regulation is needed. If δ_k^{total} is positive, δ_k^{total} represents the unfulfilled demand during time k, and therefore, more production will be required in addition to the already committed energy during the day-ahead. This process is called up-regulation. The final case occurs when δ_k^{total} is negative, which means that there is a surplus of energy (i.e. energy committed during the day-ahead) and producers will be required to lower their production in order to match demand. This process is called down-regulation.

The balancing market is a platform where given a committed production schedule for every producer whose offers where accepted in the day-ahead for every block in K, it allows producers to submit offers for up and down regulation so that they can adjust close to real-time imbalances required by the SO. From the producer's perspective, each single producer is allowed to submit multiple offers into a "pool" of offers that correspond to each hour in K. The first action in the balancing market is the submission of offers by producers for a given hour k, before the balancing market gate closure for hour k. Submissions are performed blindly, this is, a producer cannot see other producer's offers. In this way, each single producer is able to build up their own supply curve in the marketplace for each of the hours. From the SO's point of view, a "pool" of anonymous up and down regulation offers will be submitted when at gate closure for each hour in K. Let I_k denote the collection of up regulation offers received by the SO for hour k, and J_k the collection of down regulation offers received by the SO for hour k in the balancing market. The *i-th* up regulation offer is defined by the pair $(\pi_{ik}^{up}, q_{ik}^{up})$, where π_{ik}^{up} represents the price of the offer and q_{ik}^{up} the quantity of the offer. In a similar manner, the *j-th* down regulation offer for hour k is denoted by the pair $(\pi_{jk}^{dw}, q_{jk}^{dw})$.

If the total deviation δ_k^{total} foreseen by the SO for a given hour k is greater than zero, the market will require an up regulation adjustment to match the unfulfilled quantity d_k^{total}. Then, the SO builds one aggregate up-regulation offer curve from the set of I_k up regulation offers and will neglect down regulation offers from set J_k. Up-regulation offers are then ordered increasingly according to their price (i.e. merit order) as shown in figure 3. Once the offer curve is built, a new price λ_k^{BM} is cleared, which corresponds to the price realization for hour k in the balancing market. Accordingly, the quantity Q_k^{BM} is cleared which is equal to $Q_k^{DA} + \delta_k^{tot}$.

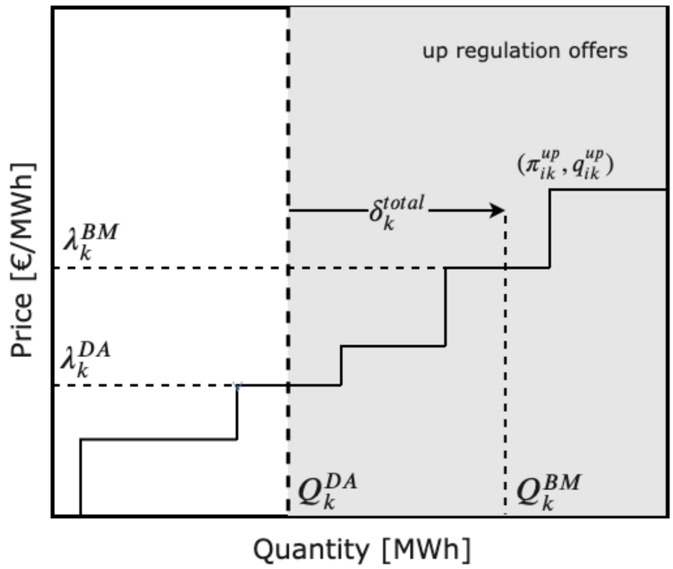


Figure 3: Balancing market: up regulation supply curve aggregated by the SO.

Similarly, down regulation is required when q_k^{tot} takes a negative value and generation has to be decreased to fulfill the surplus of energy contracted in the day-ahead market. In this case, an aggregate curve of down regulation offers is constructed by the SO over every down regulation offer in J_k according to an inverse merit order. For a down regulation offer $(\pi_{jk}^{dw}, q_{jk}^{dw})$, the price π_{jk}^{dw} is the amount a producer is willing to accept to decrease its production in q_{jk}^{dw} units in relation to the energy contracted in the day-ahead. For example, if a producer has contracted to produce $q_k^{DA} = 1$ for a price of $\lambda_k^{DA} = 100$ in the day-ahead market, the producer will obtain a profit of 100 from the day-ahead market. Now, if for that same hour k in the balancing market the producer gets accepted a down regulation offer of $q_{jk}^{dw} = 1$ for a price of $\pi_{jk}^{dw} = 80$, the producer "buys back" from the SO the single unit of energy, resulting in a profit of -80 in the balancing market. The producer will not incur in operational costs now that he has withdrawn his offer, and the total operation will result in a profit of 20 (i.e. $\lambda_k^{DA} - \pi_{jk}^{dw}$). In this sense, as the price of the down regulation offer decreases, more profit is to be obtained by the producer if the offer is accepted by the market, explaining the fact of an inverse merit order curve for the aggregate down regulation offers.

In this thesis, an assumption is made for prices in the balancing market with respect to the price cleared during the day-ahead market for a given hour. It is assumed that if the market requires up regulation for hour k, the prices considered in the set of up regulation offers I_k will need to be greater or equal than the cleared price in the day-ahead market, λ_k^{DA}. This means that producers will submit up regulation offers with prices higher than the one cleared for the day-ahead market for the same hour. The underlying assumption is that additional energy for the system implies incurring larger marginal costs (i.e. turning on generating units that have higher marginal costs). Inversely, when down regulation is required, prices in J_k will need to be smaller or equal than λ_k^{DA}. In [10] it is stated that situations in which these conditions do not hold are infrequent.

A different but important aspect of the balancing market is timing: different milestones are set sequentially for actions to take place. The present work attempts to model the UK balancing market. There are 24 hourly trading blocks that span a complete day: time block number 1 starts at hour 00:00 and time block number 24 ends by 23:59 of the same calendar day. Each market participant

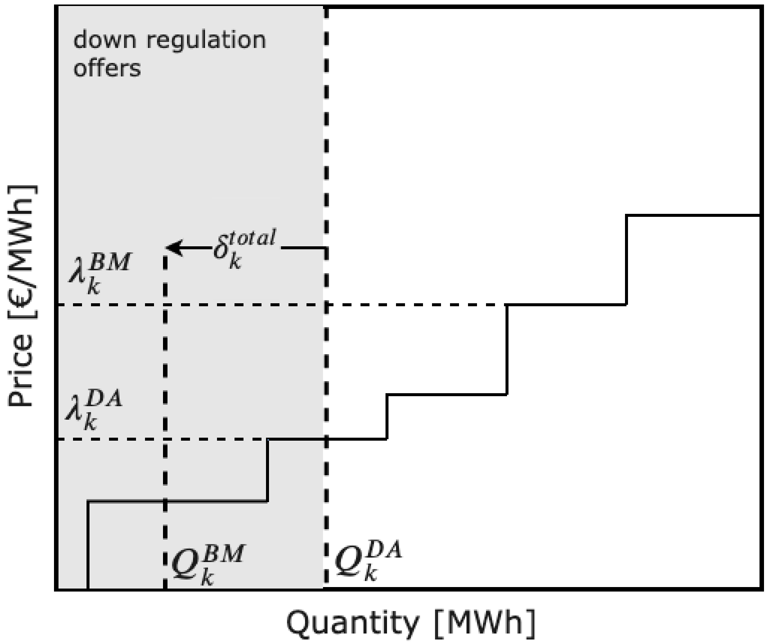


Figure 4: Balancing market: down regulation supply curve aggregated by the SO.

is allowed to submit their offer/bids anytime up to two hours (i.e. two time blocks) before the hour in which delivery starts. As shown in figure 5, a producer that is situated in time k will see a gate closure by the end of period k. This gate closure corresponds to offers that can be submitted for time block $k+3$. Afterwards, when in $k+1$, the producer will see a gate closure for offers corresponding to time block $k+4$, and so on. This means that producers will submit up and down regulation offer curves starting at least three blocks ahead of time. The delay described in the offering process presents several challenges to the producer. Firstly, the producer must submit offers for block $k+3$ without having knowledge on the outcome of the curves submitted for blocks $k+1$ and $k+2$. Essentially, the producer does not know which market prices will be revealed during blocks $k+1$ and $k+2$, hence, does not know in advance which of the already submitted offers will be accepted during these blocks. Secondly, the producer must build up offers for interval $k+3$ not knowing what will be the operation during intervals $k+1$ and $k+2$. Additionally, what operations are feasible in one period is dependant on the operation of the immediate past period, and so on sequentially through time.

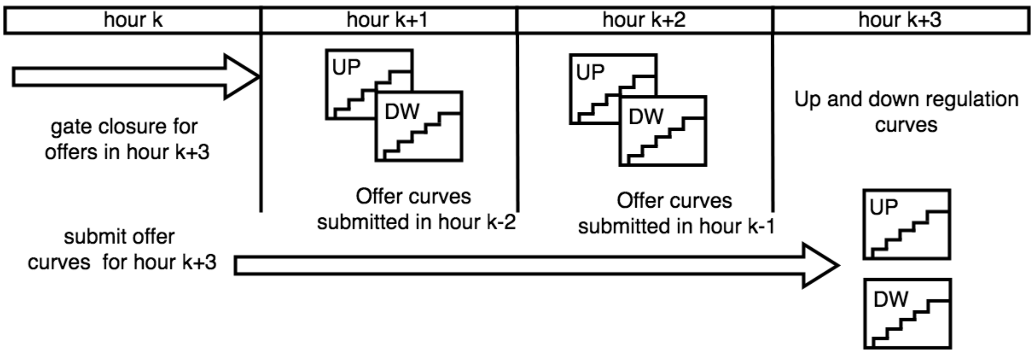


Figure 5: Balancing market: offer submissions and gate closure.

2.2 Scenario tree: balancing market prices

A fundamental aspect for a participant in the balancing market in order to build optimal offering curves are prices. In this case, balancing market prices λ_k^{BM} for each hour $k \in K$ are exogenous to the producer and are uncertain. Day-ahead market prices λ_k^{DA} for $k \in K$ are assumed to be known by the producer and fixed, meaning that there is no uncertainty attached to them. Hence, the producer needs to select as an input a set of sequential balancing market price realizations, each with an associated probability of occurrence. This set will represent the possible "states of the world" and will heavily influence the design of the regulation curves offered in the market, and coherently, the profit obtained.

In this thesis, balancing market price scenarios are artificially generated and taken as an input to the producer's offering model. The focus of this work is to find a way to evaluate different offering strategies based on the profit they generate rather than emulating exact profits from a real balancing market. The methodology for generating price scenarios is beyond the scope of this thesis. A fundamental model to generate sequences of day-ahead and balancing market prices is taken from [9]. In Section II of this paper, an explanation is given on which are the main mechanisms to generate prices and how they interact with each other. It is important to note that the only underlying source of price uncertainty taken into account is wind stochasticity, which is modelled over a real data set of wind power forecasts.

The scenario tree considered in this work is constructed as follows. Each balancing market price λ_k^{BM} is a random variable that follows $f^{\lambda} : \mathbb{R} \to \mathbb{R}^+$. In order to obtain a tractable model, the distribution of each random variable λ_k^{BM} is discretized for each $k \in K$. The discretization process is done in such way that for each hour k there are $\{\lambda_{k,n}^{BM} : n \in N_k\}$ price scenarios, where N_k denotes the set of balancing market price scenarios for hour k. Each hour k contains the same fixed number of price scenarios.

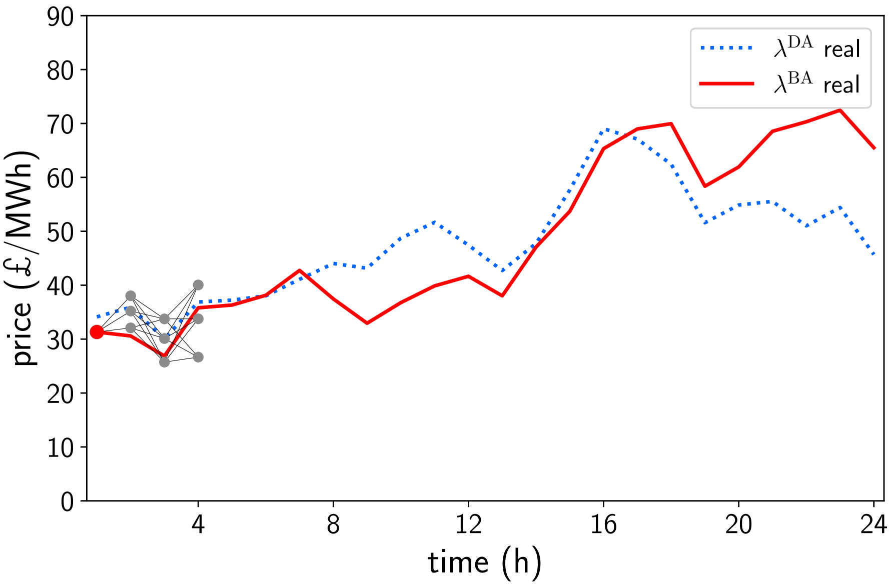


Figure 6: Scenario generation: A day-ahead market price curve and a balancing market price curve is generated with hourly resolution for 24 hours (i.e. one calendar day). For each hour k, a balancing market price forecast is available for the upcoming 3 hours (dots in the figure). The gray curves illustrate the range of prices the producer sees when standing in hour $k = 1$. In this case, three price scenarios are generated for each hour.

The scenario generator constructs data with an hourly resolution for 24 sequential hours, that is, it simulates price realizations for a complete day. A curve with the realization of day-ahead market prices is generated, and another curve with the balancing market price realizations is also generated (see figure 6). Day-ahead market prices are always available to the producer, as they have been cleared the day before simultaneously. At the start of hour k, price λ_k^{BM} for hour k is revealed to the producer. Then, a fully connected graph is built with the price forecast for hours $k + 1$, $k + 2$ and $k + 3$. Let us recall that in Section 2.1.2 it is mentioned that offer curves are submitted with a three hour delay, which explains the three hour forecast. This forecast is conditional to the fact that the producer is standing at the start of hour k in the real-time. In figure 6 we can see an example forecast when the producer is standing in hour $k = 1$ and three price scenarios are generated for each future hour within the forecast horizon. Once hour k has been revealed, the producer will move onto hour $k+1$, where a set of scenarios will be available for the next 3 upcoming hours, repeating this process by stepping from one hour to the next. Each new set of scenarios generated does not need to be consistent with the previous set of scenarios. Additionally, a matrix is generated which contains the probabilities of transitioning from one node in hour k to any other node in hour $k+1$, for $k \in K$.

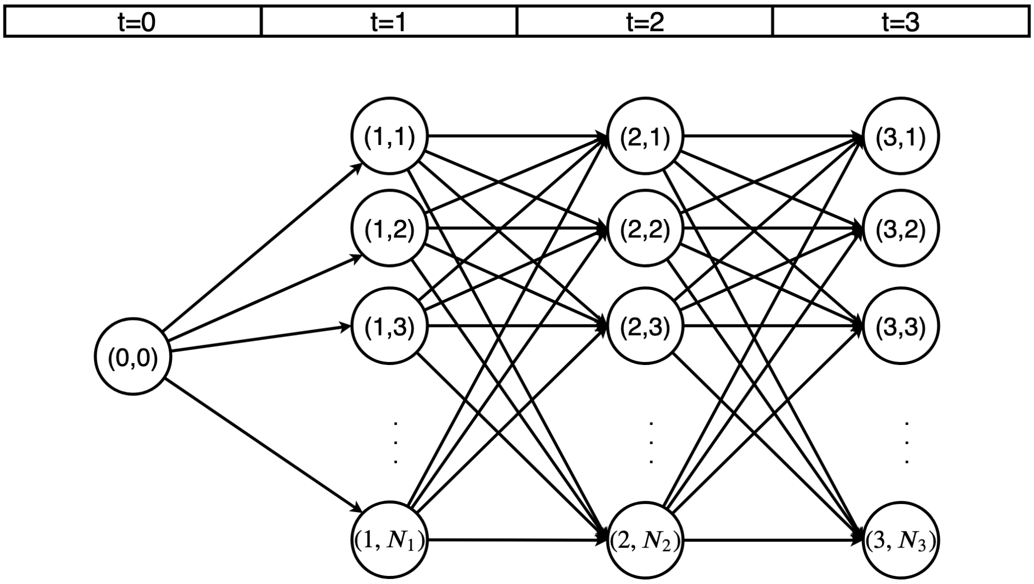


Figure 7: Scenario tree for price realization in the balancing market through time periods.

Let us consider hourly blocks $k \in K$ as an upper level index, where each k corresponds to a given hour of the day. Then, let us denote by $t \in T$ a lower level index, where the set of hours t unfold when the producer is standing in hour k of the day, and correspond to the hours of the forecast. Hence, once the producer is in hour k of the day, a forecast will be unfolded starting in hour $t = 0$ (i.e. the real-time) until the last hour of the forecast. In other words, when the producer is "inside" the forecast, hours are indexed by t.

Once the forecast is revealed to the producer, the complete set of nodes in the tree can be denoted by $\{(t, n) : t \in T, n \in N_t\}$ where N_t denotes the set of price scenarios (i.e. nodes) in hour t. The set $T = \{0, 1, 2, 3\}$ denotes the hours seen by the producer when looking at the forecast (i.e. up to three hours ahead of time). In figure 7 an example of a scenario tree is shown, where for each hour a total number of N_t scenarios is provided, except for hour 0 (i.e. current hour). Then, each node (t, n) is associated to a single balancing market price scenario $\lambda_{t,n}^{BM}$ which has a probability of occurrence equal to $\pi_{t,n}$, where $\sum_{n=1}^{N_t} \pi_{t,n} = 1$, $\forall t \in T$.

2.3 Offering curves under pay-as-bid payment scheme

In Section 2.1.2 the balancing market mechanism was described in a simplified manner from the perspective of the SO. In this section, the offering mechanism will be described from a single producer's point of view. The main goal of a producer in the balancing market is to maximize profit by selling energy in each of the independent and sequential trading intervals. The trading mechanism for producers to sell energy is by submitting multiple offers (i.e. price-quantity pairs) in each trading interval k, each of which can be accepted or rejected by the market. These offers can be divided into two: up regulation and down regulation offers. Therefore, the producer can submit one up regulation supply curve by submitting multiple up regulation offers for a given hour, and can proceed in the same way in order to submit a down regulation supply curve.

Taking into account the uncertainty in future balancing market prices, the producer intuitively will consider offering as much energy as possible in hours where forecasted prices are high in order to maximize profit. If the producer additionally controls a storage unit, he will support his strategy by buying energy to fill the storage when price levels are low. To fulfill the offered energy afterwards in the real-time, the producer must cope with the technical constraints that the generating units present (e.g. ramping constraints, charge/discharge rate of storage). In this paper, a thermal and a storage unit is considered. These pose a maximum bound on the quantity that can be generated and delivered in each interval, specially considering that the amount of energy stored may change across the hours. The above also implies that the operation of units must remain linked sequentially through time. For example, a thermal unit with a small ramping rate (i.e. "slow") will not be flexible enough to respond to large demand variations through sequential periods. Also, when a storage unit is taken into account, a charging schedule must be planned so that energy can be charged or discharged when balancing market prices are high.

A single offering curve consists in a non-decreasing collection of offers. Let us consider that for a given hour t, the set of possible price scenarios is $N_t = 1, 2, .., N$. Then, the price realization for that hour, λ_t^{BM}, can take the value of any price from the set $\lambda_{t,1}, \lambda_{t,2}, ..., \lambda_{t,N}$. For each of the price scenarios in N_t, a quantity $q_{t,n}$ can be allocated, in order to assign an offer to each of the prices. Then, a set of offers (i.e. an offering curve) is constructed: $\{(\lambda_{t,1}, q_{t,1}), (\lambda_{t,2}, q_{t,2}), ..., (\lambda_{t,N}, q_{t,N})\}$. The main idea for the producer is to fix the price of each of his offers for period t to every price scenario $\lambda_{t,n}^{BM}$ forecasted for that time period. Consequently, the producer will have to face the problem of finding the optimal quantities $q_{t,n}$ associated to each of the prices in $\lambda_{t,n}$ in order to maximize the expected profit.

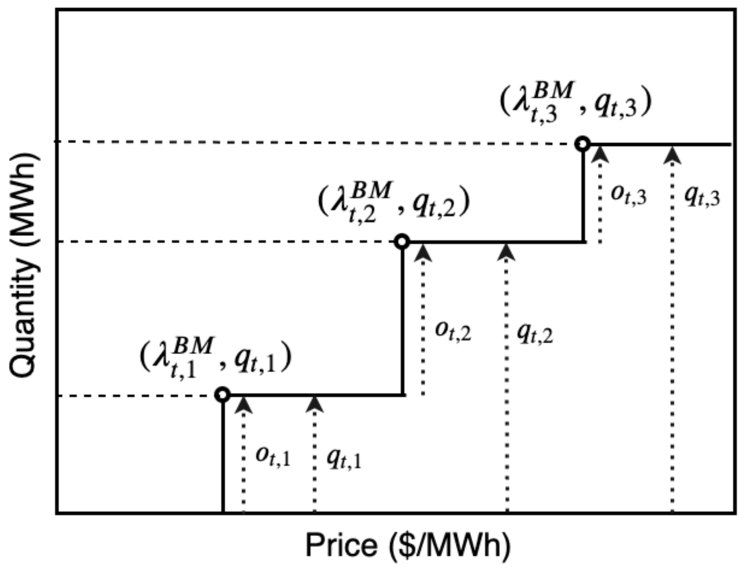


Figure 8: Offering curve example where offer prices are fixed to price scenarios found in that period to build each offer block.

As an example, let us consider the offering curve shown in figure 8. In this case, for a certain hour t three price scenarios are considered: $\lambda_{t,1}$, $\lambda_{t,2}$ and $\lambda_{t,3}$. As a result, the producer can submit an offering curve that consists in the following pairs: $(\lambda_{t,1}, q_{t,1})$, $(\lambda_{t,2}, q_{t,2})$ and $(\lambda_{t,3}, q_{t,3})$. Once the offers have been submitted, the SO may accept or reject each of these according to the price cleared in hour t, λ_t^{BM}.

Once the offers have been submitted for a given hour and a balancing market price has been revealed for that hour (i.e. SO has cleared the market), a profit is obtained by the producer for his offerings. In this thesis, the mechanism in which each producer is remunerated follows a pay-as-bid payment scheme. This scheme is found, for example, in the UK balancing market. In a pay-as-bid scheme, each accepted offer is remunerated at the same price indicated in the corresponding offer. An alternative and commonly found payment scheme is uniform pricing. Here, every accepted offer is priced at the balancing market price realization. The following mathematical model of a pay-as-bid scheme is based on Mazzi's proposal described in [9], where a linear formulation is introduced. For its implementation, additional variables, $o_{t,n}$, must be included. These represent the incremental quantity between two adjacent offers (i.e. $q_{t,n} - q_{t,n-1} = o_{t,n}$) and are always non-negative due to the non-decreasing nature of the offers as prices increase. Therefore, the total quantity $q_{t,n}$ offered for price scenario $\lambda_{t,n}$ in period t can be computed as:

$$q_{t,n} = \sum_{n'} M_{tnn'} o_{t,n'}, \quad \forall n \in N_t \tag{2.4}$$

where $M_{tnn'}$ is defined as the acceptance matrix for period t.

$$M_{tnn'} = \begin{cases} 1 & \text{if } \lambda_{t,n} \geq \lambda_{t.n'} \\ 0 & \text{otherwise.} \end{cases} \tag{2.5}$$

The acceptance matrix indicates which offers $(\lambda_{t,n'}, q_{t,n'})$ are accepted if $\lambda_{t,n}$ corresponds to the balancing market price realization. Accordingly, the corresponding profit $\rho_{t,n}$ for scenario n in hour t is:

$$\rho_{t,n} = \sum_{n'} M_{tnn'} o_{t,n'} \lambda_{t,n'} \quad , \forall n \in N_t \tag{2.6}$$

The profit under scenario n computed in (eq.2.6) calculates the summation over the profit of each independent offer block accepted, priced at its corresponding level. Following the example illustrated in figure 9, for a given hour t where three price scenarios are considered, a different profit will correspond to each node according to the acceptance of each of the independent offer blocks.

Finally, the expected profit for hour t can be calculated as follows:

$$\mathbb{E}[\rho_t] = \sum_{n} \pi_{t,n} \rho_{t,n} \tag{2.7}$$

where $\pi_{t,n}$ is the probability of scenario n occurring in hour t.

2.4 The producer: a cluster of energy units.

In this work we consider the case where the cluster consists of two units: one thermal generator (e.g. diesel, gas-fired turbine) and one storage unit. The cluster of units interacts as a whole with the system, meaning that either the system will see the cluster as a single unit. The cluster has two states of operation: it can be injecting energy to the system or either absorbing energy from the system. When the cluster is in state of injection, either the thermal unit is generating energy or the storage unit is discharging energy to the system, or the combination of both. Therefore, the cluster will be selling energy to the market. When the cluster is absorbing energy from the system, the storage unit will be charging itself directly from the system (i.e. buying energy from the market). In both cases, the energy flow between the cluster and the system corresponds to the quantity contracted in the day-ahead market plus the regulation offers that are accepted in the market, for a given hour.

The cluster's operation is constrained by the physical restrictions that each of the units impose. Let us recall that T is the set of hours $t = 0, 1, 2, 3$. Then, for each node $\{(t, n) : t \in T, n \in N_t\}$ in the scenario tree, a set of decision variables will describe the behavior of the thermal and storage units.

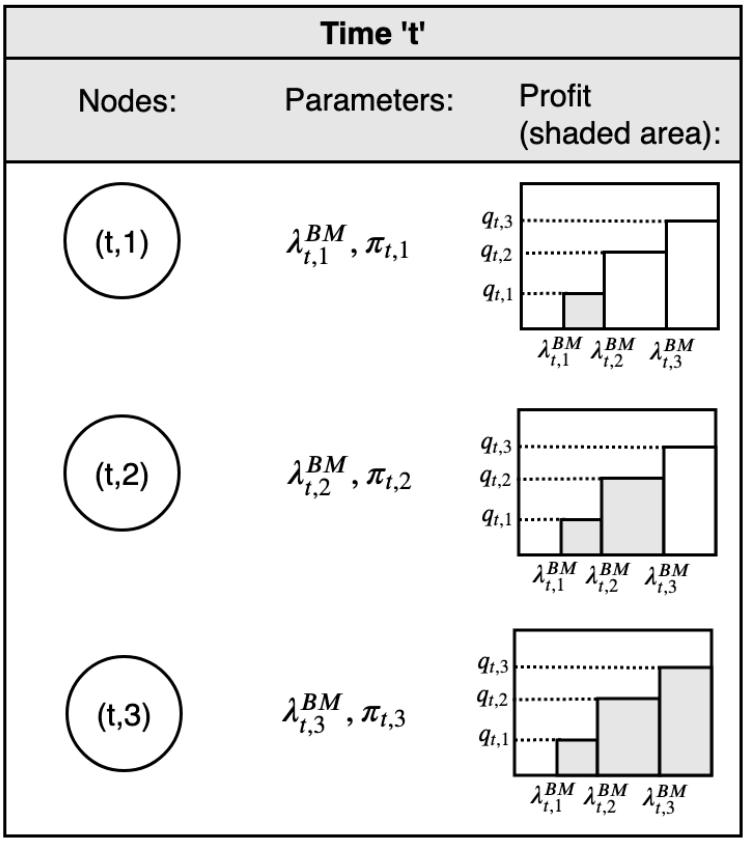


Figure 9: Offering curve profits: three price scenarios (with its corresponding probabilities, $\pi_{t,n}$) are considered in hour t, where each has an associated offer $q_{t,n}$. As a result, a profit is calculated for each price scenario.

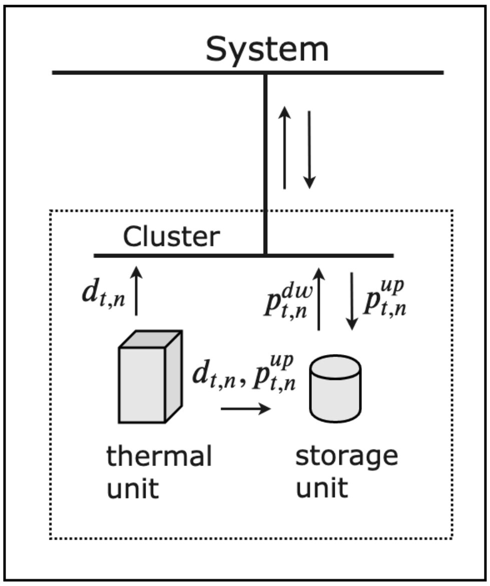


Figure 10: Interaction between the cluster and the system are illustrated, as well as intra-cluster interactions.

Let us define the following set of variables that describe each unit's operation:

$l_{t,n}$ = level of the storage unit [MWh] at the beginning of hour t in node n.
$p^{up}_{t,n}$ = total charging quantity of the storage unit [MWh] during hour t in node n.
$p^{dw}_{t,n}$ = total discharging quantity of the storage unit [MWh] during hour t in node n.
$d_{t,n}$ = total energy production of the thermal unit [MWh] during hour t in node n.
$c_{t,n}$ = production cost of the thermal unit [£/MWh] during hour t in node n.

Additionally, a set of binary variables are implemented to model the decision of turning on/off the thermal unit across every time interval. These are:

$$\begin{aligned}
u_{t,n} &= \text{commitment status of the thermal unit in hour t in node n (binary).}\\
y_{t,n} &= \text{start-up status of the thermal unit in hour t in node n (binary).}\\
z_{t,n} &= \text{shut-down status of the thermal unit in hour t in node n (binary).}
\end{aligned}$$

The parameters that describe the operation of the thermal unit are as follows:

$$\begin{aligned}
\overline{D}^{max} &= \text{maximum capacity of the thermal unit [MW]}\\
\overline{D}^{min} &= \text{minimum level of the thermal unit [MW]}\\
\overline{R}^{up} &= \text{ramp up limit of the thermal unit [MW/h]}\\
\overline{R}^{dw} &= \text{ramp down limit of the thermal unit [MW/h]}\\
C &= \text{marginal cost of the thermal unit [£/MWh].}\\
C_0 &= \text{fixed cost of the thermal unit [£].}\\
C_{st} &= \text{fixed start-up cost of the thermal unit [£].}\\
C_{sh} &= \text{fixed shut-down cost of the thermal unit [£].}
\end{aligned}$$

The parameters that describe the operation of the storage unit are as follows:

$$\begin{aligned}
\overline{L}^{max} &= \text{maximum level of the storage [MW]}\\
\overline{L}^{min} &= \text{minimum level of the storage [MW]}\\
\overline{P}^{up} &= \text{maximum charging rate of the storage [MWh]}\\
\overline{P}^{dw} &= \text{minimum charging rate of the storage [MWh]}\\
\eta &= \text{round-trip efficiency of the storage unit.}
\end{aligned}$$

To describe the following constraints, a notation needs to be introduced. Let us define $a(t,n)$ as the ancestor or parent node of (t,n). Then, the ancestor node of node (t,n) will be a node $n \in N_{t-1}$ which precedes or gives birth to node (t,n). Each node has a unique ancestor node, with the exception of node $(0,0)$ which has none.

The set of constraints for the thermal unit is described as follows:

$$d_{t,n} \geq u_{t,n}\overline{D}^{min}, \quad \forall t \in T, \forall n \in N_t \tag{2.8}$$
$$d_{t,n} \leq u_{t,n}\overline{D}^{max}, \quad \forall t \in T, \forall n \in N_t \tag{2.9}$$
$$d_{t,n} - d_{a(t,n)} \leq \overline{R}^{up}, \quad \forall t \in T, \forall n \in N_t \tag{2.10}$$
$$d_{a(t,n)} - d_{t,n} \leq \overline{R}^{dw}, \quad \forall t \in T, \forall n \in N_t \tag{2.11}$$
$$u_{t,n} - u_{a(t,n)} \leq y_{t,n}, \quad \forall t \in T, \forall n \in N_t \tag{2.12}$$
$$u_{a(t,n)} - u_{t,n} \leq z_{t,n}, \quad \forall t \in T, \forall n \in N_t \tag{2.13}$$
$$d_{t,n} \geq 0, \quad \forall t \in T, \forall n \in N_t \tag{2.14}$$
$$u_{t,n}, y_{t,n}, z_{t,n} \in \{0,1\}, \quad \forall t \in T, \forall n \in N_t \tag{2.15}$$

Constraints (2.8) and (2.9) define the operation range of the thermal unit. Constraints (2.10) and (2.11) describe the maximum/minimum ramping limits of the thermal unit. Constraints (2.12) and (2.13) describe the start-up/shut-down decisions. Finally, constraint (2.14) define the non-negativity condition for the generating quantity of the thermal unit, and (2.15) describe the binary nature of the commitment and start-up/shut-down variables. Additionally, values for the initial conditions are required to be defined. These are:

$$d_0 \;=\; \text{initial operating state of the thermal unit [MW]}$$
$$u_0 \;=\; \text{initial commitment state of the thermal unit (binary)}$$

Similarly, the set of constraints for the storage unit is described as follows:

$$l_{t,n} = l_{a(t,n)} + \eta\, p^{up}_{t,n} - p^{dw}_{t,n}, \quad \forall t \in T, \forall n \in N_t \tag{2.16}$$
$$l_{t,n} \leq \overline{L}^{max}, \quad \forall t \in T, \forall n \in N_t \tag{2.17}$$
$$l_{t,n} \geq \overline{L}^{min}, \quad \forall t \in T, \forall n \in N_t \tag{2.18}$$
$$p^{up}_{t,n} \leq \overline{P}^{up}, \quad \forall t \in T, \forall n \in N_t \tag{2.19}$$
$$p^{dw}_{t,n} \leq \overline{P}^{dw}, \quad \forall t \in T, \forall n \in N_t \tag{2.20}$$
$$l_{t,n},\, p^{up}_{t,n},\, p^{dw}_{t,n} \geq 0, \quad \forall t \in T, \forall n \in N_t \tag{2.21}$$

Constraint (2.16) describes the resulting level of the storage in each node after deciding either charging or discharging the storage, which depends also on $l_{a(t,n)}$ (i.e. level of the storage by the end of hour $t-1$ in node $a(n)$). Constraints (2.17) and (2.18) describe the maximum/minimum permitted level of the storage, and constraints (2.19) and (2.20) the maximum/minimum discharge rate of the storage. Finally, constraint (2.21) described the non-negative nature of the decision variables of the storage unit.

The production cost associated to the cluster is explained solely by the thermal unit as follows:

$$c_{t,n} = C_0\, u_{t,n} + C\, d_{t,n} + C_{st}\, y_{t,n} + C_{sh}\, z_{t,n}, \quad \forall t \in T, \forall n \in N_t \tag{2.22}$$

To sum up, the set of operational constraints of the cluster are described in (2.8) - (2.21), which is denoted by Ω^{op}.

2.5 The producer's stochastic offering model.

2.5.1 Model description

In this Section, a stochastic optimization model is proposed for the producer to construct optimal offering curves (i.e. up and down regulation curves) in the balancing market. Here, the producer is modelled as a single agent who participates in the balancing market with the objective of maximizing his profit through the submission of offering curves, and the resulting acceptance/rejection of the offers by the SO. It is assumed that this producer has already participated in the day-ahead market, hence, has already a defined production schedule for the upcoming 24 hours. As a result, the producer will have to deliver on each hour the quantity contracted in the day-ahead market plus the regulation offer accepted by the balancing market.

Accordingly, this section will describe the problem that the producer confronts while standing at a given hour k of the day, where a forecast is unfolded. The hours in the forecast are t-indexed and correspond to hours $t = 0, 1, 2, 3$, where $t = 0$ corresponds to the real-time (i.e. hour k of the day). The problem consists in solving one optimization model that outputs two optimal offering curves (i.e. up and down regulation) for hour $t = 3$. These curves must be submitted during hour $t = 0$ before gate closure and the producer must simultaneously decide the units' operation in periods $t = 0, 1, 2, 3$.

The sequence of actions that the agent must take are described as follows. Firstly, when the producer is standing in hour $t = 0$, a forecast which contains balancing market price scenarios for the next upcoming three hours (i.e. $t = 1$, $t = 2$ and $t = 3$) is available. These forecast corresponds to the scenario tree described in Section 2.2 and shown in figure 7. For clarity, the scenario graph can also be represented as a scenario tree, as shown in figure 11, where a set of two price scenarios is available per hour. It is important to recall that prices in the figure follow that: $\lambda^{BM}_{2,1} = \lambda^{BM}_{2,3}$ and $\lambda^{BM}_{2,2} = \lambda^{BM}_{2,4}$, $\lambda^{BM}_{3,1} = \lambda^{BM}_{3,3} = \lambda^{BM}_{3,5} = \lambda^{BM}_{3,7}$ and $\lambda^{BM}_{3,2} = \lambda^{BM}_{3,4} = \lambda^{BM}_{3,6} = \lambda^{BM}_{3,8}$.

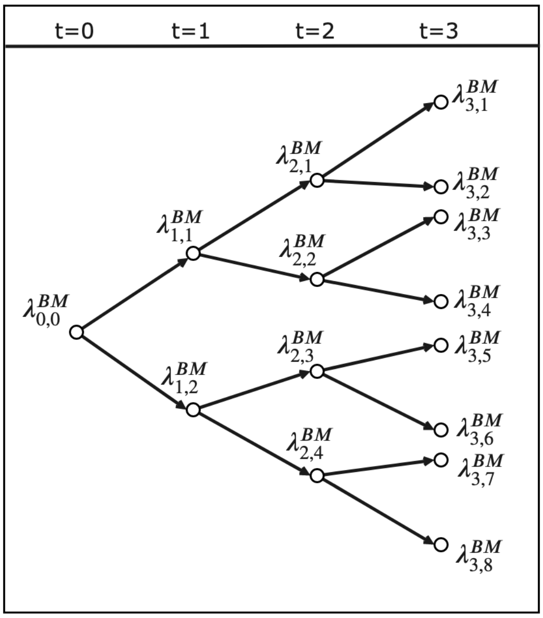


Figure 11: Scenario tree when two price forecasts are available per hour.

When in node $(0,0)$, a balancing market price realization, $\lambda^{BM}_{0,0}$, is revealed to the producer. Also, it is assumed that two offering curves have already been submitted in the past for hour $t = 0$. Then, price $\lambda^{BM}_{0,0}$ is compared to the day-ahead market price cleared for that same hour (λ^{DA}_{0}) to check if the market requires regulation. Three cases may arise:

Case 1: $\lambda^{BM}_{0,0} \geq \lambda^{DA}_{0}$, up regulation is required.
Case 2: $\lambda^{BM}_{0,0} = \lambda^{DA}_{0}$, no regulation is required.
Case 3: $\lambda^{BM}_{0,0} \leq \lambda^{DA}_{0}$, down regulation is required.

According to the market's requirement for regulation, a quantity is going to be cleared for the producer to deliver in the real-time (i.e. $t = 0$). If no regulation is required, then the producer must deliver the quantity contracted in the day ahead, q^{DA}_0. If up regulation is required, then price $\lambda^{BM}_{0,0}$ is intersected with the corresponding up regulation offer curve made for $t = 0$, and a quantity $q^{up}_{0,0}$ is cleared. Then, the producer is committed to deliver a total quantity of $q^{DA}_0 + q^{up}_{0,0}$ in hour $t = 0$. Similarly, when down regulation is required, the producer is committed to deliver a total quantity of $q^{DA}_0 - q^{dw}_{0,0}$ in hour $t = 0$. In other words, according to the price realization in the balancing market the producer will know which of his offers were accepted by the market.

After the producer has identified the total quantity of energy he is contracted to deliver, he must decide how the cluster will deliver the energy. This means choosing values for the following decision variables: $d_{0,0}$, $p^{up}_{0,0}$ and $p^{dw}_{0,0}$. Hence, in node $(0,0)$ the following balance constraint must be met:

$$q^{DA}_0 + q^{up}_{0,0} - q^{dw}_{0,0} = d_{0,0} + p^{dw}_{0,0} - p^{up}_{0,0} \tag{2.23}$$

Consequently, the producer will need to schedule a tentative plan for the units' decision variables for the hours $t = 1,2,3$. This plan will be tentative because it is dependant on the complete set of price forecasts revealed when in hour k. For hour $t = 1$, the following set of price scenarios can be seen: $\{N(1,n_1) : \forall n_1 \in N_1\}$. For any node $N(1,n_1)$, a price scenario λ^{BM}_{1,n_1} is associated. Similarly as before, it is assumed that offer curves for $t = 1$ have been submitted in the past. Again, the producer will identify a quantity to deliver in $t = 1$ through the same procedure as explained in the case of

$t = 0$, this time using the price λ_1^{DA}, and must choose to decide how to deliver the energy.

For hour $t = 2$, the producer will see nodes $\{N(2, n_2) : \forall n_2 \in N_2\}$. The producer will repeat the procedure implemented during $t = 1$.

Finally, the producer must submit his offers for $t = 3$. As described in Section 2.3, the producer must allocate quantities q_{3,n_3} to every price scenario forecast for $n_3 \in N_3$. The set of offers can be described by $\{(\lambda_{3,n_3}^{BM}, q_{3,n_3}) : \forall n_3 \in N_3\}$, which build up an offer curve. More precisely, the producer is required to submit the following:

$$\text{Up regulation curve: } \{(\lambda_{3,n_3}^{BM}, q_{3,n_3}^{up}) : \forall n_3 \in N_3\} \tag{2.24}$$

$$\text{Down regulation curve: } \{(\lambda_{3,n_3}^{BM}, q_{3,n_3}^{dw}) : \forall n_3 \in N_3\} \tag{2.25}$$

It is important to notice that a plan for all units' generation is made simultaneously at time $t = 0$ for periods $t = 0, 1, 2, 3$, along with the construction of the offer curves for hour $t = 3$. The producer is only committed by the offer curves submitted (i.e. inputs) for hours $t = 0, 1, 2$. The unit's generation at $t = 1, 2, 3$ are just tentative decisions which can be revised at later steps in the rolling horizon.

2.5.2 Three-stage stochastic offering model

Summing up, the three-stage stochastic optimization model for the producer can be described as follows:

$$\max_{\Theta} \sum_{t\in T} \sum_{n\in N_t} \pi_{t,n} \left(\lambda_t^{DA} q_t^{DA} + \rho_{t,n}^{up} - \rho_{t,n}^{dw} - c_{t,n} \right) \tag{2.26}$$

subject to:

$$q_t^{DA} + q_{t,n}^{up} - q_{t,n}^{dw} = d_{t,n} - p_{t,n}^{up} + p_{t,n}^{dw} + v_{t,n}^{+} - v_{t,n}^{-}, \qquad \forall t \in T, \forall n \in N_t \tag{2.27}$$

$$q_{t,n}^{up} = \sum_{n'\in N_t} M_{tnn'}^{up}\, o_{t,n'}^{up}, \qquad \forall t \in T, \forall n \in N_t \tag{2.28}$$

$$q_{t,n}^{dw} = \sum_{n'\in N_t} M_{tnn'}^{dw}\, o_{t,n'}^{dw}, \qquad \forall t \in T, \forall n \in N_t \tag{2.29}$$

$$\rho_{t,n}^{up} = \sum_{n'\in N_t} M_{tnn'}^{up}\, \lambda_{t,n}^{BM}\, o_{t,n'}^{up}, \qquad \forall t \in T, \forall n \in N_t \tag{2.30}$$

$$\rho_{t,n}^{dw} = \sum_{n'\in N_t} M_{tnn'}^{dw}\, \lambda_{t,n}^{BM}\, o_{t,n'}^{dw}, \qquad \forall t \in T, \forall n \in N_t \tag{2.31}$$

$$o_{t,n}^{up} = 0 \quad \text{if} \quad \lambda_{t,n}^{BM} \leq \lambda_t^{DA}, \qquad \forall t \in T, \forall n \in N_t \tag{2.32}$$

$$o_{t,n}^{dw} = 0 \quad \text{if} \quad \lambda_{t,n}^{BM} \geq \lambda_t^{DA}, \qquad \forall t \in T, \forall n \in N_t \tag{2.33}$$

$$\begin{aligned} c_{t,n} = C_0\, u_{t,n} + C\, d_{t,n} + C_{st}\, y_{t,n} + C_{sh}\, z_{t,n} \\ + M_p(v_{t,n}^{+} + v_{t,n}^{-}), \qquad \forall t \in T, \forall n \in N_t \end{aligned} \tag{2.34}$$

$$d_{t,n},\, p_{t,n}^{up},\, p_{t,n}^{dw},\, l_{t,n},\, u_{t,n},\, y_{t,n},\, z_{t,n} \in \Omega^{op}, \qquad \forall t \in T, \forall n \in N_t \tag{2.35}$$

$$o_{t,n}^{up},\, o_{t,n}^{dw}, v_{t,n}^{+}, v_{t,n}^{-} \geq 0, \qquad \forall t \in T, \forall n \in N_t \tag{2.36}$$

where:

$$\Theta = \{q_{t,n}^{up},\, q_{t,n}^{dw},\, o_{t,n}^{up},\, o_{t,n}^{dw},\, d_{t,n},\, p_{t,n}^{up},\, p_{t,n}^{dw},\, l_{t,n},\, c_{t,n},\, u_{t,n},\, y_{t,n},\, z_{t,n}, v_{t,n}^{+}, v_{t,n}^{-} : \forall t \in T, \forall n \in N_t\} \tag{2.37}$$

$$M_{tnn'}^{up} = \begin{cases} 1 & \text{if } \lambda_{t,n}^{BM} \geq \lambda_{t.n'}^{BM} \text{ and } \lambda_{t,n}^{BM} > \lambda_t^{DA} \\ 0 & \text{otherwise.} \end{cases} \tag{2.38}$$

$$M_{tnn'}^{dw} = \begin{cases} 1 & \text{if } \lambda_{t,n}^{BM} \leq \lambda_{t.n'}^{BM} \text{ and } \lambda_{t,n}^{BM} < \lambda_t^{DA} \\ 0 & \text{otherwise.} \end{cases} \tag{2.39}$$

In the above model, the objective is described by eq. (2.26). The objective consists in maximizing the expected profit in $t = 3$ through the submission of offering curves, and the corresponding operation according to market clearance for nodes in stages $t = 0, 1, 2$. For hours $t = 0, 1, 2$, quantities $q_{t,n}^{up/dw}$ will be cleared out from offering curves submitted in the past for each corresponding hour. Therefore, the stochastic model requires an input of six offering curves: one pair of up/down regulation curves for each $t = 0, 1, 2$. Then for $t = 3$, the producer must decide for every $q_{t,n}^{up/dw}$ in order to submit optimal up/down offering curves. For each node, the profit earned during the day-ahead market is included.

Constraint (2.27) describes the equilibrium between the energy produced and the energy offered in the market, as described in Section 2.5.1. Note that variables $v_{t,n}^{+}$ and $v_{t,n}^{-}$ have been included. These represent a "virtual generator" which helps to achieve model feasibility in exceptional cases.

Constraints (2.28) and (2.29) compute the up/down regulation offers submitted for every price scenario (t, n) when $t = 0, 1, 2$. Consequently, the set $\{o_{t,n}^{up}, o_{t,n}^{up} : t = 0, 1, 2, \forall n \in N_t\}$ corresponds to input values. When $t = 3$, the set $\{o_{3,n}^{up}, o_{3,n}^{up} : \forall n \in N_3\}$ holds the offers to be submitted in hour $t = 3$. In consequence, constraints (2.30) and (2.31) compute the expected revenue of selling energy in the balancing market under a pay-as-bid scheme.

Constraints (2.32) and (2.33) ensure that up/down regulation offers are considered only if required by the system. Constraint (2.34) computes the cost of generation, including the cost of the "virtual generator". Constraint (2.35) describes the operational constraints of the cluster and constraint (2.36) the non-negative property of offering variables and the virtual generator's production.

Finally, constraint (2.37) describe the complete set of variables of the model and constraints (2.38) and (2.39) the acceptance matrices for each price scenario (t, n) in the balancing market.

3 Gaining knowledge from future price scenarios in the balancing market

In Section 2.5, a stochastic optimization model to generate and schedule optimal offering curves in the balancing market has been proposed. To run the model, the producer will need to have participated and contracted energy in the day-ahead market for the current hour and each of the three upcoming hours, represented by the quantities $\{q_t^{DA} : t = 0, 1, 2, 3\}$. Additionally, the producer must have submitted offering curves for hours $t = 0, 1, 2$, which correspond to inputs in the stochastic model. Most importantly, when the producer is standing in hour k of the day, a balancing market price realization for the current hour (i.e. $t = 0$) and price scenarios for the the three upcoming hours (i.e. $t = 1, 2, 3$) are going to represent the states of the world for which the model will base the decision making process.

Until this point, the stochastic model is blind to the future: it does not know in which hours $k \in K$ prices are going to be high or low. Considering that the producer controls a storage unit, it is not trivial for the producer to know when the storage needs to be charged and discharged optimally with the objective of maximizing the expected profit. Intuitively, the producer will want to charge the storage using the thermal unit when prices are low, as the thermal unit has a fixed and a marginal cost of producing. In the other hand, the producer will want to have a maximum capacity of energy available when the prices are high (i.e. storage full) to offer the largest amount of energy possible and make profit out of it. Besides the storage level, ramping constraints in the thermal unit can also be considered. Similarly as with the storage, if the status of the thermal unit remains high it will be able to offer its maximum capacity when required. If prices are high and the status of the thermal unit is in a low point (i.e. generating close to its minimum capacity), the producer will not be able to take full advantage of the capacity of the thermal unit.

When the stochastic model does not have any knowledge of future balancing market prices, it will always decide to make offers using all of its available capacity in $t = 3$, if up regulation is required by the market. This means, by the end of period $t = 3$ the storage unit will be left empty, and the thermal unit will be left in a high operational point. Accordingly, if the offers in $t = 3$ get accepted by the market, the producer will face $t = 4$ with a low storage level. Now, if a high price occurs in $t = 4$ and the market accepts an up regulation offer, the producer may not have enough capacity to fulfill this offer, given that the offer was designed considering an amount of available storage, for example.

For this reason, a "look-ahead" feature is proposed as an input to the stochastic model. The feature consists in a value function (VF) that corresponds to $t = 4$, where a value of flexibility is assigned to the decision variables with respect to the future. In other words, the value function will quantify which is the value of having a certain level of storage and a certain operating level of the thermal unit at the start of hour $t = 4$. In order to build this value function, an exploration must be performed over future price scenarios beyond $t = 4$. This exploration must evaluate the expected profit of making offers in each price scenario for a given time horizon. For example, if for a given future time horizon many down regulation cases (i.e. low prices) are seen during the exploration, the value function in $t = 4$ may present low values for high levels of the storage and thermal units, as there is no necessity of having extra capacity available because little or no profit is to be made in the future by offering extra quantities of energy. In the other hand, if high prices are foreseen during the exploration, the value function will output high values for high levels of storage and the thermal unit. This means that the producer will schedule its operation and make his offerings considering to leave the highest amount possible of storage and level of the thermal unit, as this extra capacity will be highly priced in future hours.

3.1 The flexibility value function

The value function is constructed using dynamic programming (DP). First, a forecast for the upcoming twelve hours is required to effectuate the exploration over future prices. As prices in the balancing market are highly volatile, it is not trivial to select a correct amount of future hours in the forecast. The choice of a twelve-hour forecast is an arbitrary decision in this work, and analyzing the sensibility of the model with respect to the time span of the forecast is out of the scope of the present thesis. Then, the price scenarios that the producer will have available when in hour k are the same as explained in Section 2.2, but extended to twelve hours. This new scenario tree is shown in figure 12.

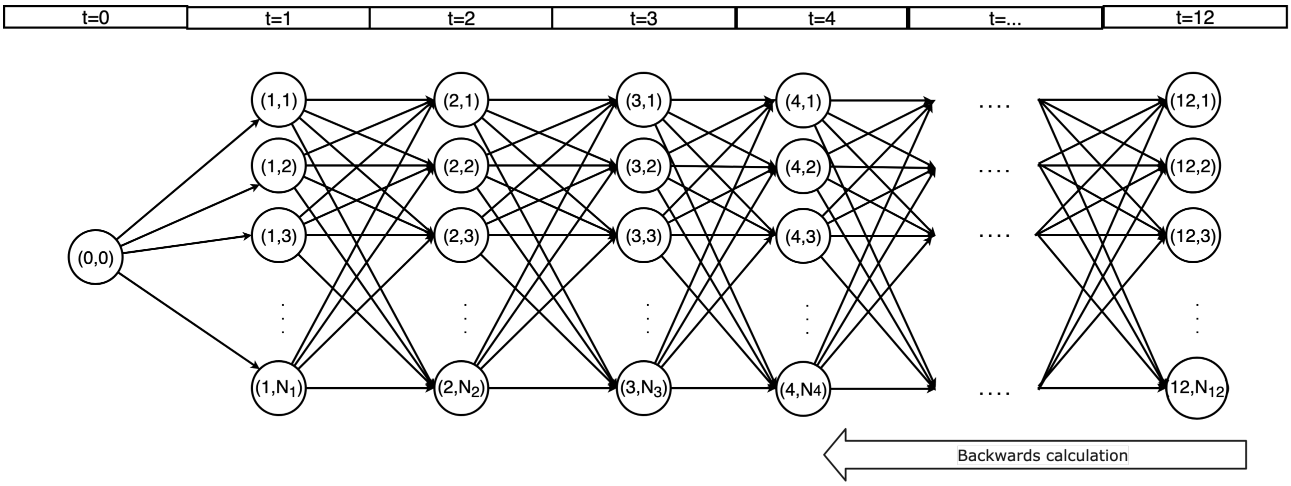


Figure 12: Set of scenario forecasts for a 12 hour time horizon.

According to figure 12, a backwards pass from $t = 12$ until $t = 4$ is performed. During the sweep through price scenarios, optimal offerings are made in each node and the value function is iteratively evaluated. In order to perform the evaluation, a few key considerations are taken into account. First, variables are discretized in order to follow DP criteria. Secondly, no delay is considered for the offerings during the DP evaluation. This means, for each node in a given hour, an offer is made and immediately sold to the market. The last assumption is made for the sake of simplicity. Thirdly, a uniform pricing scheme is considered for the DP process. In DP, each state is dependant uniquely on the last state it came from, and decisions are taken according to this assumption. This means, when in a current state (i.e. price scenario), no information is available about other price scenarios for the same hour. Hence, it is not possible to implement a pay-as-bid pricing scheme, as we cannot solve a problem that involves multiple nodes at once. Finally, the resulting value function will represent an approximation of the value of having a level of storage and of operation of the thermal unit, and in no way represents an exact measure of the value of flexibility.

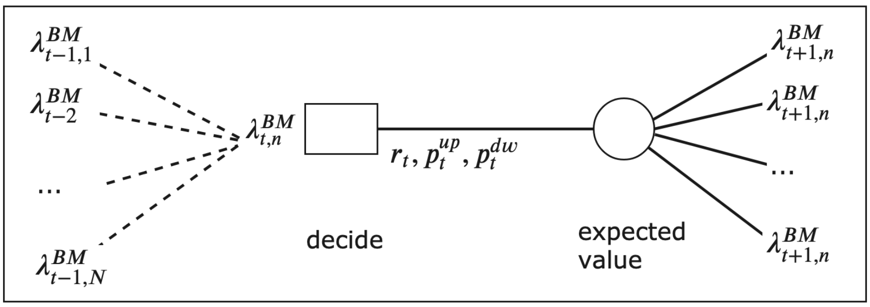


Figure 13: Sequence of actions in the DP model.

The DP model implemented to evaluate the value function is described as follows. The model will go through each price scenario for each hour in the forecast horizon. The set of visited nodes is described by $\{\lambda^{BM}_{t,n} : t = 4, 5, 6, ..., 12, n \in N_t\}$. When standing in a given node, the sequence of actions is shown in figure 13. First, a transition is made from any node $\lambda^{BM}_{t-1,n}$ to node $\lambda^{BM}_{t,n}$. Once the price for time t is revealed, the producer has to make a decision. The main decision variables are:

$$r_{t,n} \quad : \quad \text{ramping quantity in time } t \text{ when in node } n \tag{3.1}$$

$$p^{up}_{t,n} \quad : \quad \text{quantity charged to the storage in time } t \text{ when in node } n \tag{3.2}$$

$$p^{dw}_{t,n} \quad : \quad \text{quantity discharged from the storage in time } t \text{ when in node } n \tag{3.3}$$

These last set of decision variables determine the values for the intrinsic state variables, which are:

$$d_{t,n} \quad : \quad \text{operational level of the thermal unit at the start of time } t \text{ when in node } n \tag{3.4}$$

$$l_{t,n} \quad : \quad \text{storage level at the start of time } t \text{ when in node } n \tag{3.5}$$

Then, for a given price in t and each possible intrinsic state $\{(d_t, l_t, \lambda^{BM}_{t,n}) : d_t \in \mathbb{D}, l_t \in \mathbb{L}, n \in N_t\}$, where $\mathbb{D}$ and $\mathbb{L}$ are sets with the possible values for each corresponding variable, an optimal decision is found. The optimal decision for each intrinsic state maximizes the profit of an objective function. The decision made implies that the value of the intrinsic states in the next hour will be d_{t+1} and l_{t+1}. Therefore, the objective function also accounts for the future expected value of being in state $(d_{t+1}, l_{t+1}, \lambda^{BM}_{t+1})$. In this way, for each hour t, a three-dimensional discrete function is evaluated (i.e. $VF(d_t, l_t, \lambda^{BM}_t)$), as shown in figure 14. Proceeding to evaluate this function backwards in time, value function for hour $t = 4$ will serve as the "look-ahead" feature for the stochastic model.

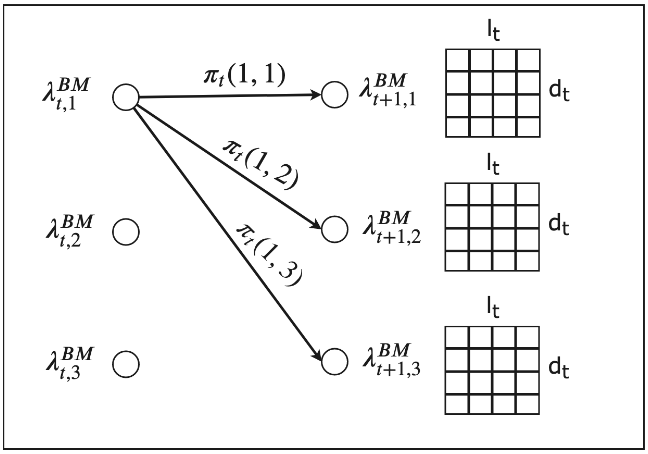


Figure 14: A three dimensional function is evaluated in the DP model in each hour in the horizon.

The DP model is described as follows. First we can define the value function as:

$VF_t(d_t, l_t, \lambda^{BM}_t)$: the future expected value of having a storage level of l_t and a thermal operational point of d_t at the beginning of hour t, when in price scenario λ^{BM}_t.

Then, for a given hour in the forecast, $VF_t(d_t, l_t, \lambda_t^{BM})$ will be computed for every $d_t \in \mathbb{D}$, $l_t \in \mathbb{L}$, $n \in N_t$ according to the following model:

$$\max_{\Gamma} \lambda_t^{DA} q_t^{DA} + \lambda_t^{BM} (q_t^{BM} - q_t^{DA}) - C\, d_{t+1} + \sum_{n}^{N_{t+1}} \Pi_t(\lambda_t^{BM}, \lambda_{t+1}^{BM})\, VF_{t+1}(d_{t+1}, l_{t+1}, \lambda_{t+1}^{BM}) \quad (3.6)$$

subject to:

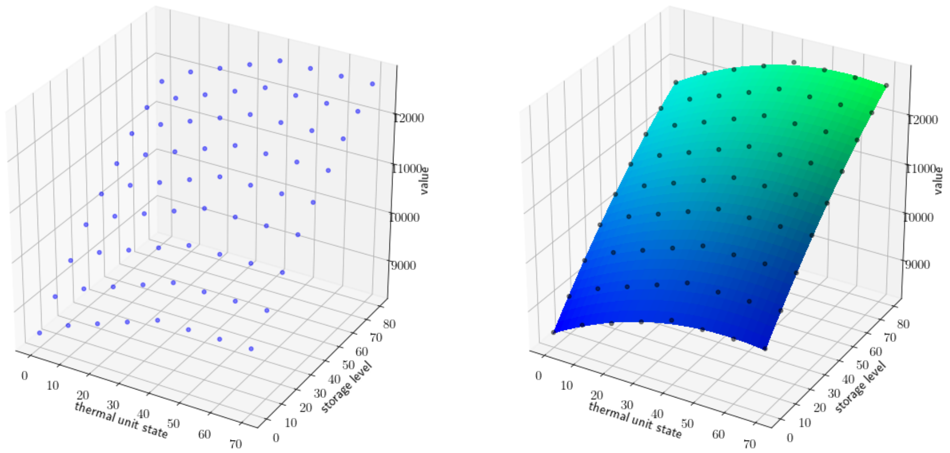
$$\begin{aligned}
q_t &= d_t + r_t + p_t^{up} - p_t^{dw} && (3.7)\\
d_{t+1} &= d_t + r_t && (3.8)\\
l_{t+1} &= l_t + p_t^{up} - p_t^{dw} && (3.9)\\
d_t + r_t &\leq \overline{D}^{max} && (3.10)\\
d_t + r_t &\geq \overline{D}^{min} && (3.11)\\
l_t - p_t^{dw} + p_t^{up} &\leq \overline{L}^{max} && (3.12)\\
l_t - p_t^{dw} + p_t^{up} &\geq \overline{L}^{min} && (3.13)\\
r_t &\leq \overline{R}^{up} && (3.14)\\
r_t &\geq -\overline{R}^{dw} && (3.15)\\
p_t^{up} &\leq \overline{P}^{up} && (3.16)\\
p_t^{dw} &\leq \overline{P}^{dw} && (3.17)\\
q_t \geq q_t^{DA} &\quad \text{if} \quad \lambda_t^{BM} \geq \lambda_t^{DA} && (3.18)\\
q_t \leq q_t^{DA} &\quad \text{if} \quad \lambda_t^{BM} \leq \lambda_t^{DA} && (3.19)\\
l_t, d_t, p_t^{up}, p_t^{dw} &\geq 0 && (3.20)\\
r_t &\quad \text{free} && (3.21)\\
l_t, d_t, r_t, p_t^{up}, p_t^{dw} &\in \mathbb{Z} && (3.22)\\
\Gamma &= \{q_t, l_t, d_t, r_t, p_t^{up}, p_t^{dw}\} && (3.23)
\end{aligned}$$

Equation (3.6) describes the objective function, where the function $\Pi_t(\lambda_t^{BM}, \lambda_{t+1}^{BM})$ is the probability of transitioning from price scenario λ_t^{BM} to λ_{t+1}^{BM}, when in hour t. Constraint (3.7) describes the total quantity sold in the market. Constraint (3.8) and (3.9) compute the intrinsic state's values for the next hour. Constraints (3.10)-(3.13) describe the operating limits of the thermal and the storage units, respectively. Constraints (3.14) and (3.15) describe the ramping constraints of the thermal unit, and constraints (3.16) and (3.17) describe the maximum charging/discharging rate of the storage unit. Constraints (3.18) and (3.19) describe the up/down regulation requirement of the market, thus, restricting appropriately the corresponding quantity of energy sold to the market. Constraint (3.20) restrict every variable to be non-negative, except the ramping rate, where in (3.21) it is allowed to take negative values as well. Finally, in (3.22) every variable is restricted to be an integer number.

3.2 Value function approximation by least squares

Once the DP model has obtained a value function $VF_4(d_t, l_t, \lambda_t^{BM})$ for every node $n \in N_4$, the latter can be used as an input to the stochastic model. However, the value function is a discrete function, given that the DP problem is constrained to work over an integer domain. One last step is needed in order to approximate the value function in such a way that the stochastic model can see a continuous function.

A least squares algorithm is implemented in order to approximate the collection of discrete values computed for $VF_4(d_t, l_t, \lambda_t^{BM})$ to a quadratic surface. This algorithm is widely known and its mathematical description is out of the scope of this work. A quadratic surface has been selected to



(a) Discrete value function

(b) Quadratic surface approximation.

Figure 15: Quadratic surface fitted by least squares to value function, for a given price scenario λ_t^{BM}.

approximate the value function for two reasons. Firstly, the function is convex and its shape is heavily influenced by the mix of scenarios and operational parameters of the units. Secondly, solvers can easily solve quadratic optimization problems, hence, the resulting function can be treated as an input in an optimization model (i.e. the stochastic model).

The equation that describes the quadratic surface to fit is the following:

$$f(x, y) = \alpha_1 x^2 + \alpha_2 y^2 + \alpha_3 xy + \alpha_4 x + \alpha_5 y + \alpha_6 \tag{3.24}$$

The least squares algorithm finds the values for the set of parameters $A = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$ in such way to minimize the mean squared error between every point in the value function and the quadratic surface. In this way, for a given price scenario, the discrete function can be fitted to a continuous surface as shown in figure 15.

3.3 Value function as look-ahead feature in offering strategy

As a result of the DP "look-ahead" in price scenarios, a continuous quadratic function in the form of $VF_4(d_t, l_t, \lambda_t^{BM})$ is available to the producer. In the stochastic model, variables $l_{3,n}$ and $d_{3,n}$ represent the level of storage an operational point in the thermal unit, respectively, by the end of hour $t = 3$. This means, they are obtained as a result of making decisions in $t = 3$. Hence, variables $l_{3,n}$ and $d_{3,n}$ will represent the level of the units in the beginning of $t = 4$. Therefore, the "look-ahead" feature can be plugged into the stochastic model by adding the following constraints:

$$l_{3,n} = VF_4(d_t, l_t, \lambda_t^{BM}), \qquad \forall n \in N_3 \tag{3.25}$$

$$d_{3,n} = VF_4(d_t, l_t, \lambda_t^{BM}), \qquad \forall n \in N_3 \tag{3.26}$$

Additionally, the value function $VF_4(d_t, l_t, \lambda_t^{BM})$ is added to the objective function. With constraints (3.25) and (3.26), the objective function must find a balance between the profit that can be made in $t = 3$ by emptying the level of the storage and offering a larger quantity to the market, and the expected future profit of saving that amount of storage for $t = 4$.

4 Offering strategy over a rolling horizon

A three-stage stochastic model has been proposed for a producer to submit offering curves for a period three hours ahead of time. When the producer is in hour k of the day, a forecast is revealed for the three next upcoming hours. This forecast is t-indexed and contains the balancing market price realization for $t = 0$ (i.e. hour k of the day) and price scenarios for each of the three upcoming hours $t = 1, 2, 3$. Then, the producer must simultaneously plan a tentative schedule for the hours $t = 0, 1, 2$ and submit an offer curve for hour $t = 3$.

Once the producer has submitted offering curves when standing in hour k of the day, he moves on to hour $k + 1$. Here, again a balancing market price realization is revealed for hour $t = 0$ (i.e. hour $k + 1$ of the day) and a new updated forecast will be revealed for the three next upcoming hours, $t = 1, 2, 3$ (i.e. hours $k + 2$, $k + 3$, $k + 4$ of the day). Given that the current forecast has different price scenarios than the forecast available when in hour k, a new tentative schedule will be planned for the units' operation for hours $t = 0, 1, 2, 3$ simultaneously, along with the construction of the corresponding offering curves. The three-hour schedule planned during hour $k + 1$ may differ form the one planned when in hour k.

It is important to notice that for each single hour problem (i.e. the three-stage stochastic model), the producer is committed to the offering curves submitted for hours $t = 0, 1, 2$ and must construct a new offering curve for $t = 3$. Also, the model requires initial conditions. These correspond to the level of the storage (i.e. $L0$) and the thermal units' operational point (i.e. $D0$) at the start of hour $t = 0$.

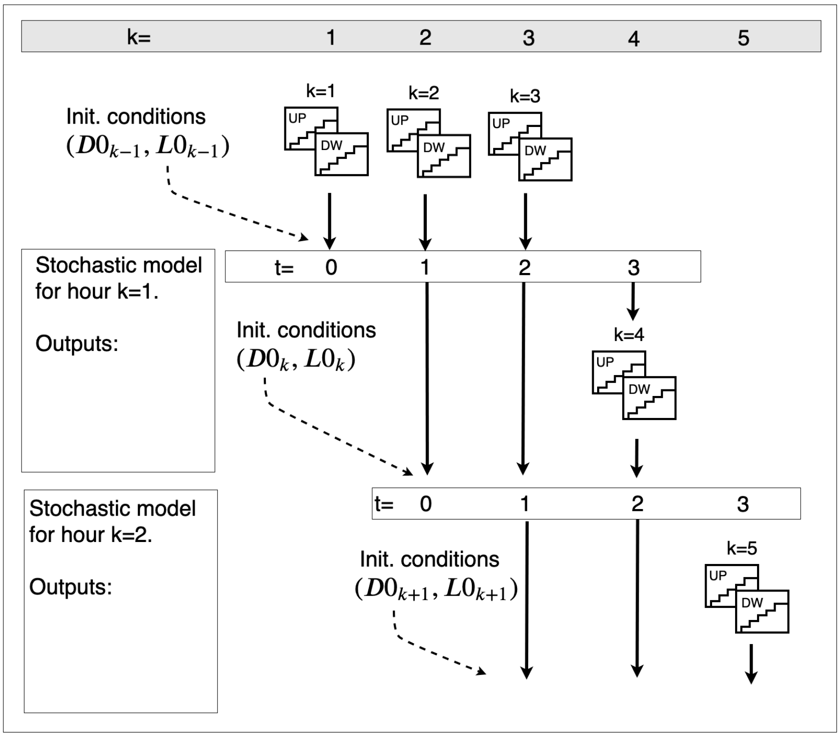


Figure 16: Rolling horizon setup where sequential stochastic models are chained together.

In this way, a rolling horizon across hours is simulated. The inputs of the stochastic model when in hour k will be offering curves (i.e. up and down regulation, already constructed in the past) for hours k, $k+1$ and $k+2$, and initial conditions $L0_{k-1}$ and $D0_{k-1}$. The output of the stochastic model will be offering curves for hour $k+3$ of the day, plus the resulting units' operation in hour k (i.e. $L0_k$ and $D0_k$). Then, when standing in hour $k + 1$, the model requires initial conditions $L0_k$ and $D0_k$ and offering curves for hours $k + 1$, $k + 2$ and $k + 3$. The output will result in offering curves for hour $k + 4$ and the resulting units' operation $L0_{k+1}$ and $D0_{k+1}$. Consequently, in this same fashion the stochastic model for hour k is linked with the stochastic model for hour $k+1$, as shown in figure 16.

In each stochastic model, the producer builds a tentative plan for hours $t = 0, 1, 2, 3$. The decision taken for the first hour, $t = 0$, will correspond to a real-time decision. This means, when a balancing market price is revealed to the producer during this hour, he must decide how to produce the quantity he has contracted with the market (i.e. day-ahead commitment and accepted regulation offers for hour $t = 0$). This decision reports a profit for hour $t = 0$, which corresponds to hour k of the day. Also, this decision leaves the storage and thermal units under a new operational state. Hence, the resulting state of the units will correspond to the initial values of the next rolling horizon (i.e. $k + 1$).

5 Case study: results and discussion

In this Section a small test case is proposed in order to run the stochastic model with and without the "look-ahead" feature over a series of sequential hours. In this context, we will describe which is the effect of the "look-ahead" feature on the offering curves' shape, and which is the impact in the profit made by the producer in each hour.

In our test case, the producer will see a balancing market price forecast containing four price scenarios per hour. The forecast spans for the next twelve upcoming hours. Also, it is assumed that the producer has contracted energy in the day-ahead market, hence, has to deliver a certain amount of energy in each hour. This quantity is fixed and the same for every hour, where $q_k^{DA} = 30$ [MWh], $\forall k \in K$. The technical characteristics of the thermal and storage units are described in table 1 and table 2.

$\overline{D}^{max}$	$\overline{D}^{min}$	$\overline{R}^{up/dw}$	C	C_0	C_{st}	C_{sd}
MW	MW	MW/h	£	£/Mwh	£	£
70	0	30	0	25	200	200

Table 1: Characteristics of the thermal unit.

$\overline{L}^{max}$	$\overline{L}^{min}$	$\overline{P}^{up/dw}$	η
MW	MW	MW/h	
80	0	30	1

Table 2: Characteristics of the storage unit.

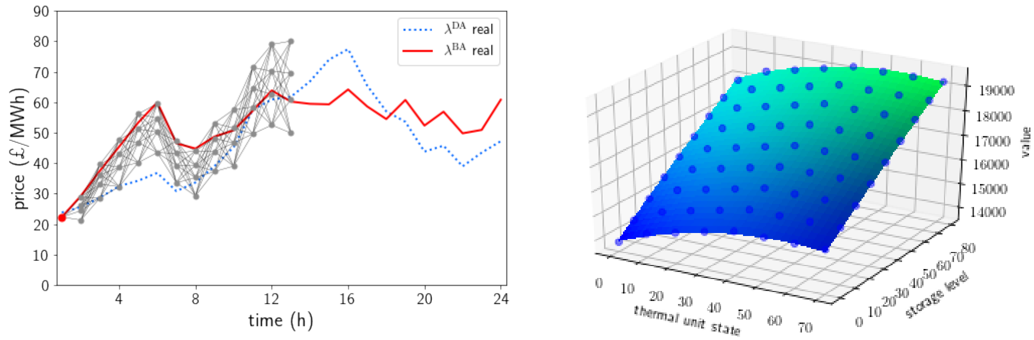
The penalization factor for the "virtual" unit is 10^5 £/MWh. In the DP model, variables are discretized with a resolution of 10 MW.

5.1 Offering for a future hour.

In this Section, the stochastic model is tested for any single hour of a day and we test how the look-ahead feature changes the shape of the curves. A scenario generator is available as described in Section 2.2, which can simulate day-ahead and balancing market price scenarios, as well as balancing market price forecasts when in any given hour. Also, the scenario generator outputs multiple simulated days (i.e. 24 hour time spans) based on real data, each of which contains balancing market forecasts and day-ahead market price realizations.

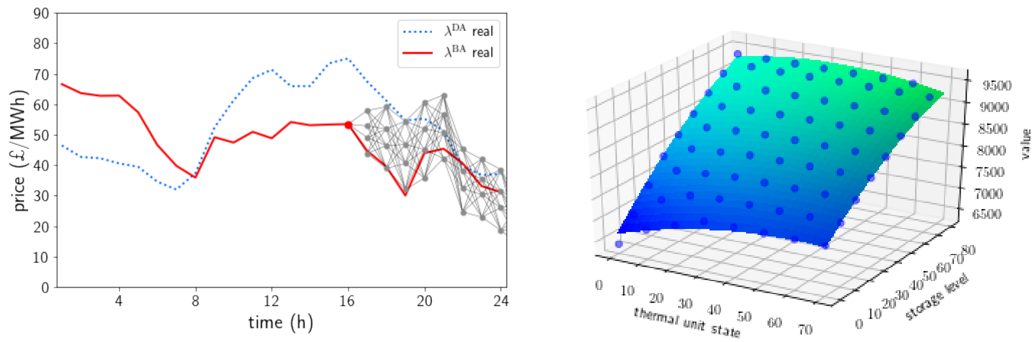
Firstly, let us explore the look-ahead feature. The latter mainly consists in the value function that is returned from the DP model, which explores price scenarios that range between three and twelve hours ahead of time. In general terms, two extreme cases can be identified. The first case corresponds when the price forecast predicts that in the majority of the future price scenarios, up regulation is required. Figure 17 illustrates a value function when the last case occurs. In the other hand, the opposite can happen. Figure 18 describes a case when the majority of future scenarios in the forecast will require down regulation.

It can be observed that when the forecast sees a large percentage of up regulation cases, the value function can double the value function that corresponds to the forecast of down regulation cases. This means that having a high storage level and a high operational point of the thermal unit is more appreciated when the market in the future is expected to require up regulation more frequently rather than down regulation. This event may lead the stochastic offering model to pullback from offering all its available capacity when constructing the curves, and saving that amount for the next hour k in the



(a) Forecast for hour 1 in day 2. (b) Value function when in hour 1 in day 2.

Figure 17: Value function when the forecast for a given hour predicts up regulation cases in the majority of the future scenarios.



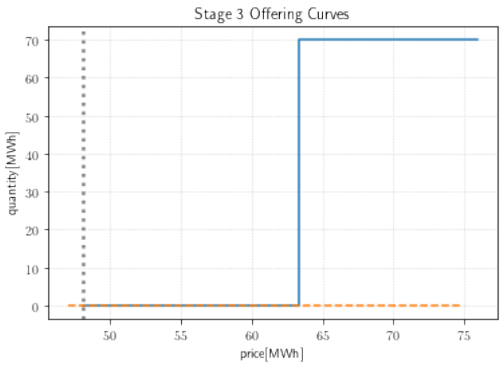
(a) Forecast for hour 16 in day 1. (b) Value function when in hour 16 in day 1.

Figure 18: Value function when the forecast for a given hour predicts down regulation cases in the majority of the future scenarios.

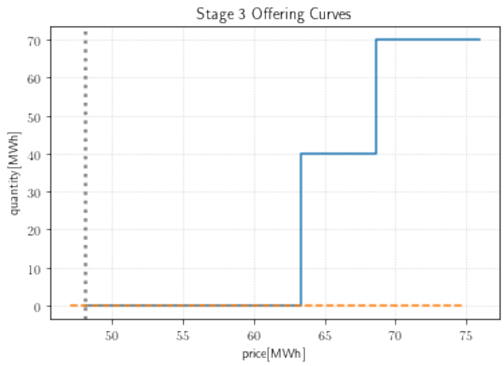
rolling horizon. In the case when down regulation is expected, saving capacity for the future is less appreciated, hence, the producer may find more profitable to offer a large part or all of its capacity in the current planning schedule. However, in the last context few up regulation cases still exist which may drive the quadratic surface to price flexibility in the future. It is important to recall that the profit reflected in the value function corresponds to the generation of the quantity contracted in the day-ahead market plus the generation that corresponds to regulation offers. For this reason, the value function never reaches zero or negative value.

The effect that the look-ahead feature can have offer the construction of the offering curves is illustrated in figure 19. In this example, the stochastic model is in hour $k = 1$ of the day and sees a forecast which consists mainly in price scenarios that require up regulation. Figure 19a shows the resulting offering curve when the stochastic model does not consider the look-ahead feature, which consists in a single step of 70 MW when the balancing market price is equal or higher than £63.2. When the look-ahead feature is considered, the offering curve shown in figure 19b is submitted. This piece-wise function is described also in eq. 5.1. We can observe the expected behavior as explained in the last paragraph.

$$q_4^{up}(\text{look-ahead}) = \begin{cases} 70 & \text{if } \lambda_4^{BM} \geq 74.92 \\ 70 & \text{if } 72.01 \leq \lambda_4^{BM} < 74.92 \\ 70 & \text{if } 68.61 \leq \lambda_4^{BM} < 72.01 \\ 40 & \text{if } 63.2 \leq \lambda_4^{BM} < 68.61 \\ 0 & \text{if } \lambda_4^{BM} < 63.2 \end{cases} \tag{5.1}$$



(a) Offering curve for hour 4 without look-ahead feature.



(b) Offering curve for hour 4 with look-ahead feature.

Figure 19: Offering curves constructed by the stochastic model for a single hour: blind model vs. model with look-ahead feature .

In the last example, the offer curves illustrated correspond to hour $k = 4$ of a day. In both cases, offer curves for $k = 1, 2, 3$, which are inputs to the stochastic model, are null (i.e. no regulation offers) and initial conditions are $D0_0 = 30$ MW and $L0_0 = 30$ MW. Then, the stochastic model simultaneously decides how to deliver energy for the current hour $k = 1$ and builds a tentative schedule for the next three hours $k = 2, 3, 4$, including the submission of the offer curves for hour $k = 4$. Regarding the real time decision (i.e. $k = 1$), both models identify that the quantity to be delivered corresponds to the quantity contracted in the day-ahead, as no regulation is offered for hour $k = 1$. The "look-ahead" model decides to produce 35 MWh with the thermal unit, where 30 MWh satisfy the day-ahead commitment and 5 MWh are injected to the storage unit. Therefore, the units' state for the next rolling horizon will be $D0_1 = 35$ MW and $L0_1 = 35$ MW. Differently, the "blind" model decides to satisfy the day-ahead commitment only by discharging the storage unit. As a result, the units' state for the next rolling horizon will be $D0_1 = 0$ MW and $L0_1 = 0$ MW. Therefore, the real-time decision in the "look-ahead" model is more expensive than the "blind" model, which in this example is zero.

The effect of the look-ahead feature has two facets. In one hand, the model may not offer its full capacity when constructing the offer curves, hence, can have a spare amount of capacity available constantly across the hours in the rolling horizon. This will allow the producer to submit higher offers when high prices realize from the market, improving his profit. In the other hand, the operation required to sustain this behavior is more expensive than exhausting the storage unit and maintaining the thermal unit at a low operational point. However, this is not always the case, as when the value function informs the model that low prices are seen in the future the units' state is maintained at lower levels which results in cheaper operational costs. Recalling that the value function represents an estimate of the future value of the units' operational state, most importantly because the pay-as-bid scheme and the delays in offering is not captured, the signal it provides to the stochastic model may be erratic.

5.2 Sources of infeasibility

In some cases, the stochastic model will not be able to schedule successfully a tentative operation and construct offer curves, leading to infeasible results. Two sources of infeasibility are identified which affect different areas of the scenario tree. These are explained in the following paragraphs.

5.2.1 Forecast mismatch case.

While the stochastic model has to construct offer curves three hours ahead of time, it also has to simultaneously plan a tentative schedule in order to achieve operational feasibility. This tentative schedule for hours $t = 0, 1, 2$, previous to the construction of the offer curves, is constrained by the offer curves submitted in the past for hours $t = 0, 1, 2$. Let us recall that when the producer is in hour k of the day, a forecast is revealed. Then, if the producer moves onto hour $k+1$, a new updated forecast is available, which does not necessarily match the previous one. This means that the design of the offer curves submitted in the past are completely dependant on the price scenarios seen at that moment, and may not be feasible to achieve when the tentative schedule is revised in the future. As a consequence, offering curves submitted in a given hour of the day may impose large constraints to the upcoming plannings (i.e. next rolling horizons) in the context of a new and unknown price forecast set.

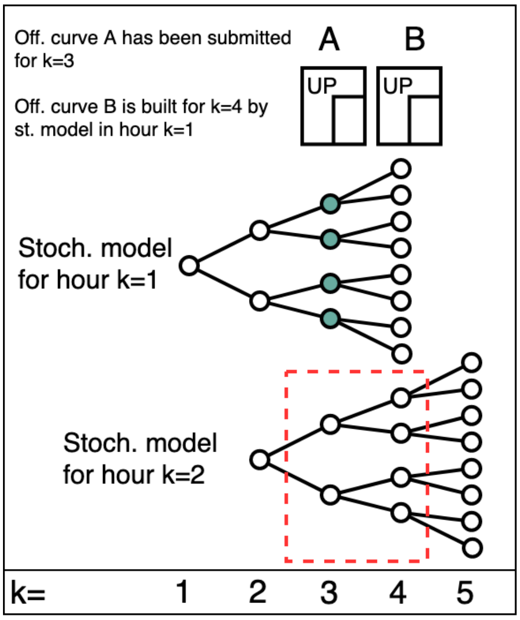


Figure 20: Infeasibility is generated in a model which is constrained by offer curves and a real-time decision made in the past with a price forecast set that mismatches the current forecast set. In hour $k = 1$, the model constructed offer curve B being blind to the acceptance of offer A in the market, as every price scenario was below the price of offer A. Also, a real-time decision was made in the same context. The model in hour $k = 2$ then is passed a set of initial conditions and the offer curves submitted in the past, and the price forecast set foresees that both offers A and B are going to be accepted in certain cases. As a result, the model will not be able to commit to the contracted energy required by the market.

Figure 20 helps illustrate the following example. Let us consider the stochastic model for hour $k = 1$ of a day, where no regulation offers have been submitted for $k = 1, 2$ and one up regulation offer, A, has been submitted for hour $k + 3$. Let us assume that for every node in $k = 3$ of this tree, each of the prices is lower than the price in offer A. This means that the tentative schedule and the design of the offer curves for $k = 4$ will never see that up regulation offer A is accepted by the market in any of the nodes in $k = 3$. Hence, up regulation offer B is generated considering that more capacity is available rather than in the case if offer A got accepted. When in the next rolling horizon, $k = 2$, the updated forecast may present scenarios in $k = 3$ that accept offer A and in the same path that accept offer B in $k = 4$. This situation presents large constraints for the scheduling of the units as the initial available capacity in $k = 2$ may not be enough to cover two sequential periods ($k = 3, 4$) of high demand, given that this circumstances were not foreseen by the previous scheduling.

This case of infeasibility can affect nodes of the scenario tree that stand in hours $t = 0, 1, 2$. If an infeasiblity occurs in $t = 0$, the real-time operation of the producer is affected, meaning that he must deviate from the commitments contracted with the market. Hence, a penalty has to be paid directly for every unit that the "virtual" generator injects or absorbs, affecting largely the profits in hour $t = 0$ or k of the day. Otherwise, if an infeasibility occurs in $t = 1, 2,$, the producer does not incur in a real penalty. However, this event can force the producer to deviate from the contracted energy required by the market when he has to make a real-time decision (i.e. decisions for $t = 0$ in the stochastic model).

5.2.2 Fully connected graph

The stochastic model can get infeasible in the last stage, $t = 3$, which corresponds to the construction of the offer curves. As the price forecast set corresponds to a scenario tree which is a fully connected graph, every node in a given hour may lead to any other node in the next time stage. As a consequence, the stochastic model is heavily constrained as it has to decide for quantities for each price scenario identified in $t = 3$ for every predecessor node located in time $t = 2$. Additionally, for each price scenario (i.e. node of the tree) in $t = 3$ a unique quantity must be allocated. For this reason the "virtual" generator turns on to help balance the nodes which cannot satisfy the energy balance constraint. Furthermore, let us recall that both, the thermal and storage units, have ramping constraints. This situation grows as the number of prices scenarios increases for each time stage.

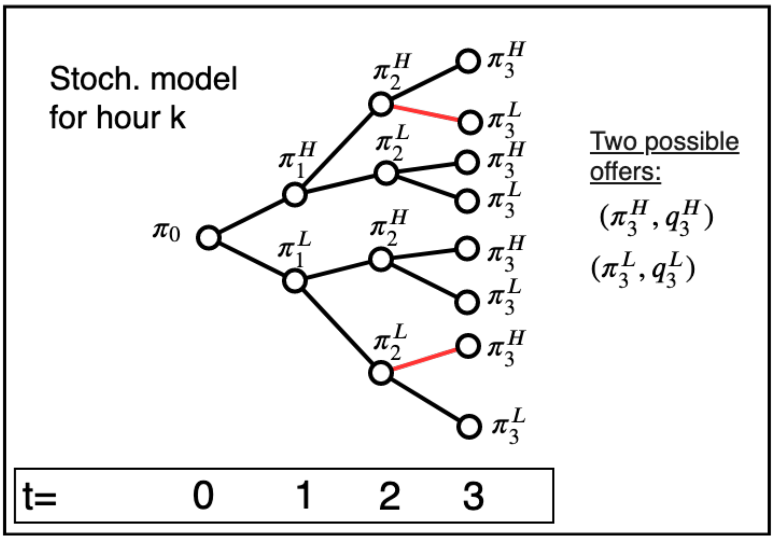


Figure 21: Infeasibilities may arise in the construction of the offering curves when combining operational constraints (i.e. ramping) with the fact that a single quantity has to be allocated to each price scenario, considering that price scenarios are repeated across the end of the branches in the tree.

Figure 21 supports the following example. Let us suppose two price scenarios are forecasted for each future hour, a high price λ_t^H and a low price λ_t^L. Also, two quantities are needed to be offered, q_3^H and q_3^L. Let us consider the scenario where there is no storage capacity available, and q_2^H is more than two thermal unit's ramp apart to q_2^L. In this case, the feasible production range offered from for λ_2^H does not intersect with the feasible production range when in λ_2^H, leading to an infeasible problem.

5.3 Rolling horizon.

Considering the test case described in Section 5, the stochastic model is ran sequentially over the hours in a rolling horizon fashion. For this, the scenario generator has simulated and created 60 sequential days. Each day contains hourly day-ahead market prices and balancing market price realizations. When in any given hour of a day, a price scenario forecast is revealed for the upcoming 12 hours, as well as the day-ahead market prices. For example, when standing in hour 24 of the day, a forecast and day-ahead market price realizations corresponding to the next day are provided to the producer. However, each day is treated independently, this means, the rolling horizon does not ran across days, instead, a rolling horizon is performed over the hours of each day. In order to implement this setup, border conditions are settled for the first and last hours of the day. For the first hours (i.e. $k = 1, 2, 3$), no regulation is offered by the producer. For the last hours of the day ($k = 22, 23, 24$), the producer does not submit offer curves and instead the only objective is to deliver the contracted energy in a cost-efficient way.

The model was ran on a machine with a 2.5GHz Intel Core i5 processor and 8 GB RAM. The running time statistics for the test case (4 price scenarios per hour) is shown in table 3. The look-ahead model consists, firstly, in running the DP exploration feature and subsequently the least squares approximation for the value function. Then, the stochastic model is ran. This process is sequentially repeated over 24 hours. Differently, the blind models only consists in the stochastic model, as the look-ahead feature is not used. Running times for the look-ahead feature and the value function approximation by least squares are computed as a whole and also shown in table 3.

	Look-ahead model secs.	Blind model secs	DP and LST.SQRS secs
Max. time	98.18	30.12	3.03
Avg. time	90.35	32.54	3.25
Min. time	86.01	35.66	4.23

Table 3: Running time statistics for a rolling horizon over 24 sequential hours for the look-ahead model, the blind model and the DP model plus the least squares computation.

Two cases are tested: first, the look-ahead model (i.e. the DP model and the stochastic model) and the blind model (i.e. stochastic model, no look-ahead feature). When both models are run for every generated day, 17 out of the 60 days present at least one hour where the producer is not able to deliver the contracted energy with the market during the real-time. This results in large monetary penalties which distort the analysis of the rolling horizon for the complete day, and is the reason why these days are set apart. For each of these days, a fixed set of hours conduce both models equally to an inability to stay feasible in the real-time. These infeasibilities correspond, as explained in Section 5.2, to forecast mismatches across sequential hours. This is reinforced by the fact that 4 price scenarios are considered in each hour, not having enough resolution of the complete range of prices that may occur.

	Look-ahead model £	Blind model £
Total profit in 43 days	2,155,237.26	2,123,703.16

Table 4: Accumulated profit of the producer made in 43 days when participating in the balancing market, considering the earnings of the day-ahead market.

Therefore, the results of the remaining 43 generated days are compared. These days are free of real-time infeasibility cases. Firstly, the total profit per day for each model is computed. This value corresponds to the profit made in each hour of the day in the day-ahead market plus the profit made in the balancing market, minus the cost of the operation of the cluster. Then, the total profit of the day is computed by summing the profit made in each of the 24 hours of the day. The results are illustrated in figure 22. The “alpha” tag in the legend of the figure refers to the weighting factor of the value function in the objective of the stochastic model. When $alpha = 1.0$, the look-ahead feature is considered in its entirety, and when $alpha = 0.0$, the look-ahead feature is taken to zero, hence, not considered at all bu the stochastic model.

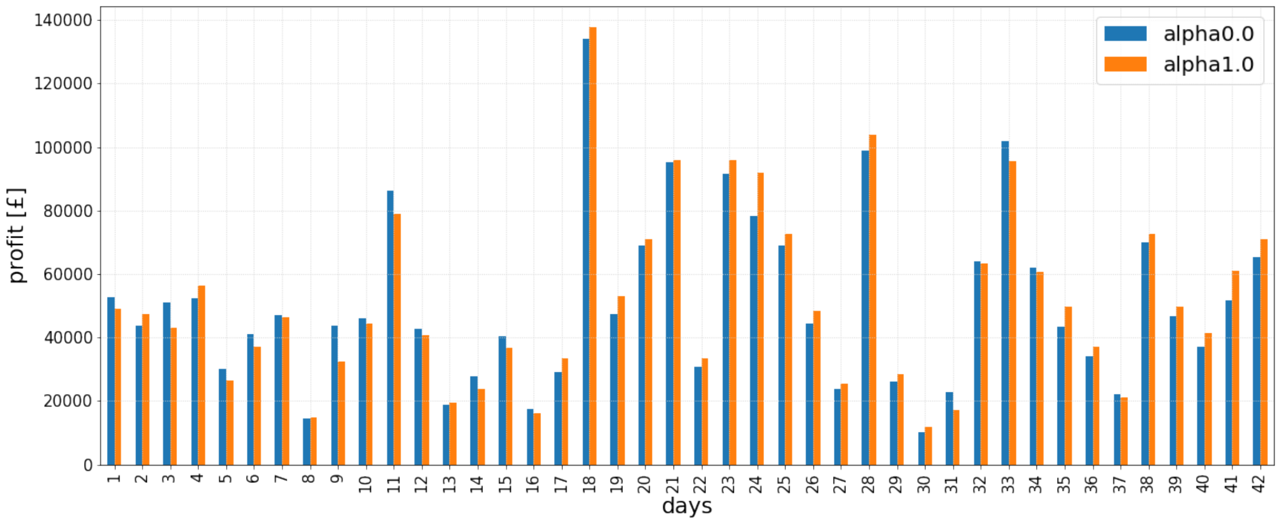


Figure 22: Daily total profit reported by the look-ahead and the blind models over the set of 43 days.

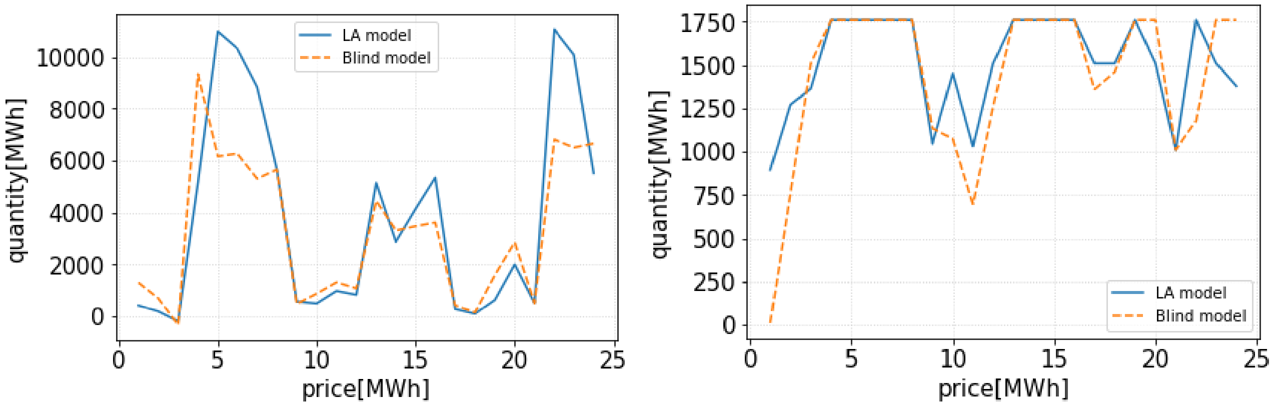
The accumulated total profits made in the 43 days is shown in table 4 for both models. We can observe that the look-ahead model reports £31 534.10 extra profits for the look-ahead model. In percentages, this amount represents an extra 1.484% profit.

In figure 22 we can observe that a large difference in profit is detected in day 24. In figure 23 the hourly profits and operational costs in day 24, when using both models, is illustrated. We can identify a peak offerings in hours 5 and 22 of the day. In general, we can see from figure 23b that the look-ahead model operates at higher costs, as it is always generating spare energy which is stored in the battery. This is clearly noted in hour 10, where a peak in the costs in the look-ahead model tells that storage is being charged. In figure 23a we can see that hour 10 corresponds to low offerings and profits, hence, of low prices. It is in this instance when the problem performs arbitrage: charges the storage when prices are low, to offer larger amounts in the upcoming hours (hour 16, in this case). This situation is repeated in the range of hours 5-8 and 22-24, where the look-ahead model outperforms the blind model. Even though both models identify peaking prices, we can see that the peaks in the look-ahead model are higher consistently during periods of high prices.

The opposite is observed in day 11, where the blind model generates more profit than the look-ahead model, as illustrated in figure 24. Here, we observe that the look-ahead model holds back from making an up regulation offer in hour 8, for the sake of offering over a peak price which occurs in hour 10. In this range of hours the blind model is consistently giving up regulation to the market, making in total more profit than the look-ahead model. In this case, the look-ahead model favored offering at high prices at the cost of offering low for the previous hours, which in result was a worse decision than making a steady profit over the range of hours. Even though the look-ahead was able to identify high prices and submit high offer curves, overall the performance is still worse than the blind model.

5.4 Tuning the α parameter.

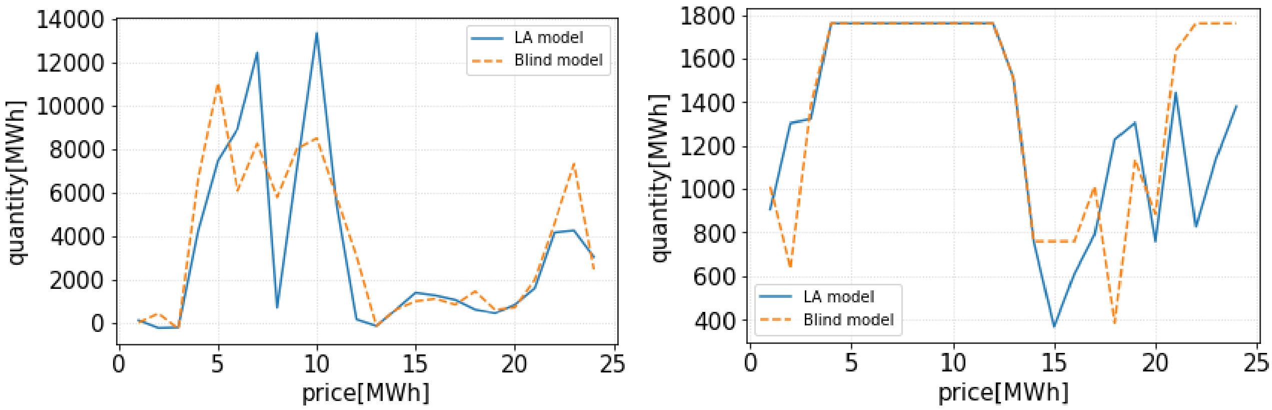
In the last section we observed that the look-ahead model may behave some times in a conservative fashion: when having a positive price signal from the look-ahead feature, it generates spare energy at a high operational cost and which can lead to low offerings in the present period, but betting that a large offer at a high price in the upcoming hours may result in a better cost-efficient decision. In the



(a) Hourly profits in day 24.

(b) Hourly operational costs in day 24.

Figure 23: Day 24 comparison of the look-ahead model and the blind model.



(a) Hourly profits in day 11.

(b) Hourly operational costs in day 11.

Figure 24: Day 11 comparison of the look-ahead model and the blind model.

other hand, the blind model is steadily offering through time, but misses the chances of making higher profits by identifying periods of high prices and making higher offers with the help of spare capacity for those hours.

Therefore, a new model is tested. In this model, the value function is weighted by a factor α in the objective function of the stochastic model. Given that it is not possible to calculate an optimal value for the parameter α, arbitrarily a value of 0.5 is assigned to it in order to construct the new model. Let us use the blind model as a benchmark, which resulted in worse profits over the set of days when compared to the look-ahead model.

	Blind model £	Look-ahead model £	Alpha 0.5 model £
Total profit in 43 days	2,123,703.16	2,155,237.26	2,224,838.56

Table 5: Accumulated profits of the producer made over a set of 43 days when participating in the balancing market, considering the earnings of the day-ahead market.

In table 5, we can observe the profits made with the tuned model (i.e. when the value function in the stochastic model is weighted by a factor of 0.5) over the set of 43 days. We can see that it reports $\pounds 101,135.40$ more profit than the benchmark (i.e. blind model). In percentage, it represents a 4.76%

extra profit when compared to the benchmark.

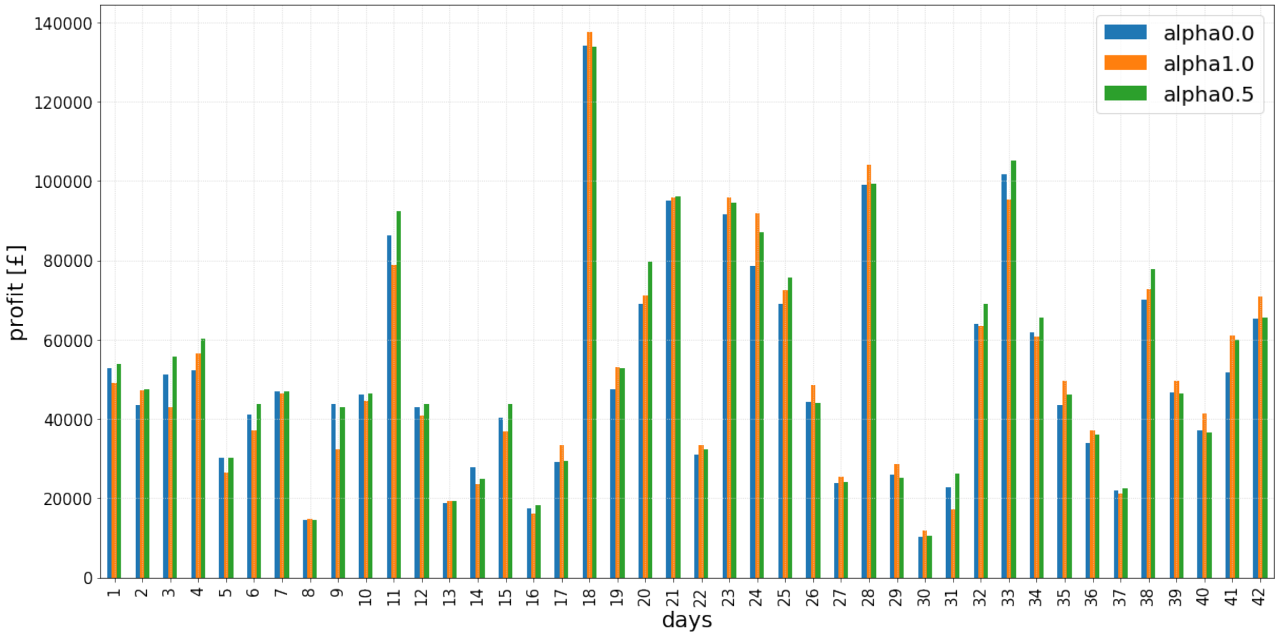


Figure 25: Daily total profits reported by the look-ahead model, the blind model and the tuned model over the set of 43 days.

We can observe that, in general, the tuned model does not perform worse, over a whole day, than the other two models. Let us explore again day 11 (figure 27), where the tuned model outperforms the other two. For day 11, the tuned model follows closely the blind model's shape, which is better than the look-ahead model. It not only follows its shape, but improves the performance in hours 6, 12, 21 and 22.

In day 20 (figure 28) we can see the tuned model offering at higher levels than the other two models, clearly without falling into more expensive operational costs. Let us observe the prices in day 20 in figure 26. Given that the price forecasts are very close together, the three models are able to clearly identify the peak prices of the day (i.e. hours 6, 10 and 21).

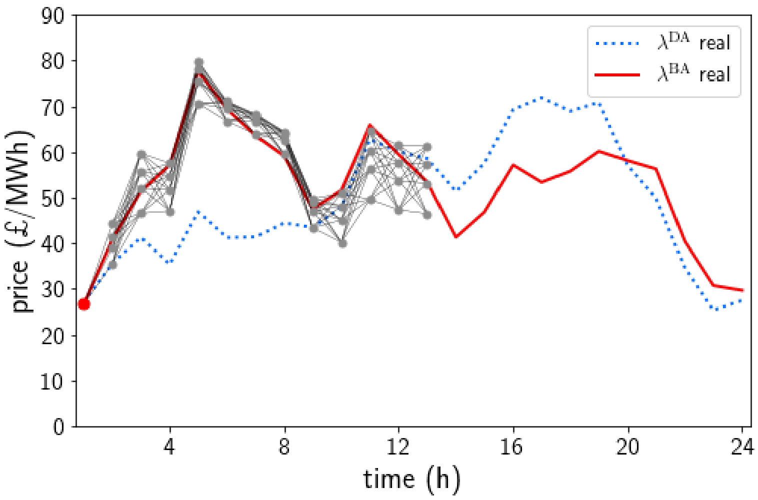
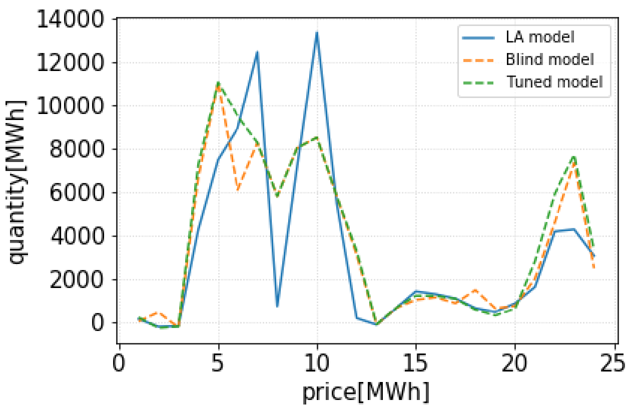
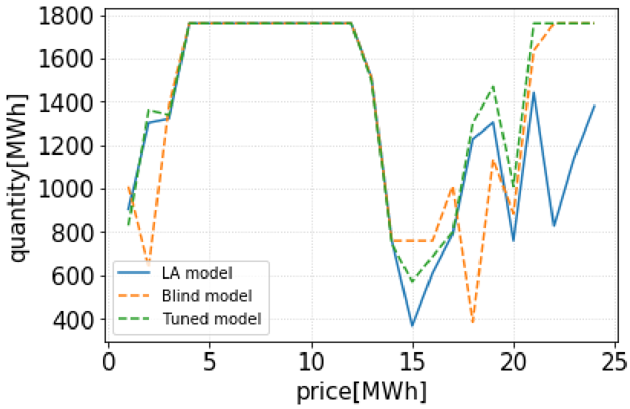


Figure 26: Day-ahead and balancing market prices of day 19.

Finally, we can conclude that the tuning parameter is relevant to adjust the price signal that the look-ahead feature is feeding to the stochastic model. When tuned to a value of 0.5, the model is less conservative than the look-ahead model, but still is conservative enough to save flexibility for the times of the day when high prices are expected.

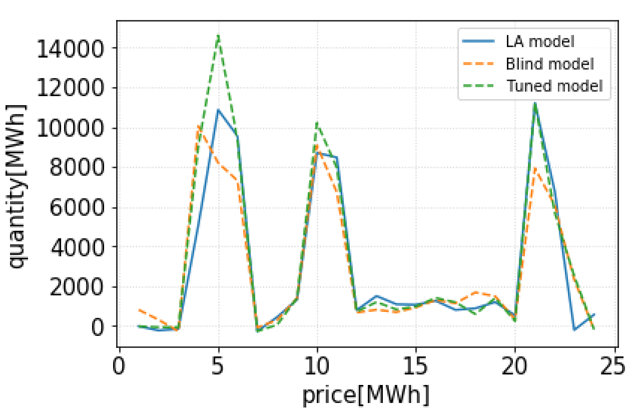


(a) Hourly profits in day 11.

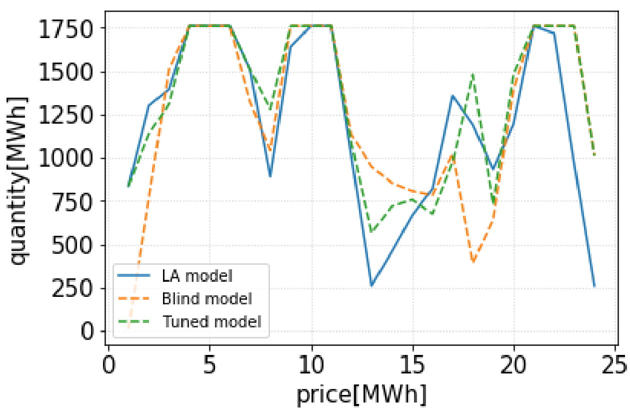


(b) Hourly operational costs in day 11.

Figure 27: Day 11 comparison of the look-ahead model and the blind model.



(a) Hourly profits in day 20.



(b) Hourly operational costs in day 20.

Figure 28: Day 20 comparison of the look-ahead model and the blind model.

6 Conclusions and further works

This work presents a stochastic optimization model where a producer needs to optimally offer up and down regulation to an electricity balancing market, where offers have to be submitted three hours ahead of the time of delivery. The objective of the producer is to maximize his expected profit under the uncertainty of balancing market prices. The producer is a price-taker agent that may have participated before in the day-ahead market, having contracted an amount of energy to be produced in future hours. For a given hour of the day the market reveals a balancing market price realization, and besides, a forecast for the next three upcoming hours is also available to the producer. This forecast consists in a set of price scenarios for each of the next three hours, represented through a fully connected graph (i.e. any price scenario is connected to every possible price scenario in the next hour). A cluster that consists in a thermal and a storage unit is controlled by the producer, where he can take advantage of the flexibility of the storage unit by performing arbitrage in the market. In this context, the stochastic model plans a tentative schedule for the units' operation for the next three hours, where simultaneously offer curves are constructed for the third hour ahead in time. After the curves are submitted to the market, the producer moves on to the next hour of the day and solves again a new stochastic model, with new price forecasts for the future. In this fashion, the producer moves sequentially through the hours, where every time revises the operation schedule of the units for the three upcoming hours and submits two new offer curves (i.e. up and down regulation).

A "look-ahead" feature is proposed as an input to the stochastic model. The stochastic model by default does not have any information on future prices, hence, schedules a plan that supports offering all of the available capacity through the offer curves. The "look-ahead" feature consists in a dynamic programming (DP) model which explores an extended forecast set of prices (twelve hours ahead of the real-time) and builds a value function. This value function represents the expected future profit that can be made in the balancing market for every combination of the operating states of the units (i.e. storage level and operational point of the thermal unit). In other words, it tells how valuable it is in the future to stay flexible in the current time (i.e. holding a positive amount of available capacity after every operational decision).

A tuned stochastic model is proposed, which makes use of the look-ahead feature by weighting it by an α factor in the objective of the stochastic model. The tuned model reports an improvement of 4.7% when compared with the stochastic model that submits offers ignoring future prices, over a set of 43 days where day-ahead and balancing market prices are generated.

However, further improvements to the look-ahead function may be implemented in order to generate more precise signals to the stochastic model. In the first place, the pay-as-bid pricing scheme could be represented in the value function. This is not possible through dynamic programming, but could be done using approximate dynamic programming (ADP). Through this approach, a continuous approximation of the value function may be directly obtained. Furthermore, a stochastic dual dynamic programming (SDDP) may be used to model the value function, as it has been applied to several hydro-management scheduling problems. Secondly, the "look-ahead" feature may be extended to include the delays in the sequential offering process. An approximate approach has been proposed in the present work in order to tackle the above mentioned problems by weighting the value function in the objective of the stochastic model by a factor ranging between zero and one.

Other issues arise with respect to the stochastic model. Infeasibilities may occur as a consequence of the mismatch of the price forecasts when in different hours of the day. To tackle this problem, price scenarios could be treated inside certain confidence intervals, rather than as fixed numbers. In this way, price forecasts may be treated probabilistically and large differences between different price forecasts may become less relevant. Yet another source of infeasibility is due to the fact that a set of price forecasts is represented as a fully connected graph. This property enables to use dynamic programming in the scenario tree and build the "look-ahead" feature. However, it comes at the cost of constraining the construction of offering curves, as a unique quantity has to be allocated to each price scenario identified. Because the graph is fully connected, price scenarios are repeated across the end of the branches of the tree.

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Appendices

A Code.

The code that backups the present work can be found in the link: https://github.com/rafasacaan/msc-offering-strategies/