MD. Rafat Ahmed
Reg. NO: 2018831028

Machine learning Assignment - 1 :

Q.1 : Here, $f(z) = \log_e(1+z)$

Where, $z = x^T x$, $x \in R^d$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} \qquad x^T = \begin{bmatrix} x_1, & x_2 & \cdots & x_d \end{bmatrix}$$

$$\therefore \quad x^T x = \left[x_1^2 + x_2^2 + \cdots \cdots x_d^2 \right]$$

Apply chain Rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dx}$$

$$= \frac{d}{dz}\left(\log_e(1+z)\right) \cdot \frac{d}{dx}(x^T x)$$

$$= \frac{d}{dz} \log_e(1+z) \cdot \frac{d}{dx}\left(x_1^2 + x_2^2 + \cdots x_d^2\right)$$

$$= \frac{1}{1+z} \cdot \frac{d}{dz}(z) \cdot \left(2x_1 + 2x_2 + \cdots 2x_d\right)$$

$$= \frac{1}{1+z} \cdot 2\left(x_1 + x_2 + \cdots + x_d\right)$$

$$= \frac{2}{\quad} \sum^{d} x_i$$

Question; 2

$$f(z) = e^{-z/2}$$

where; $z = g(y)$

$$g(y) = y^T s^{-1} y$$

$$y = h(x) = \cancel{\quad} x - u$$

using chain rule

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$\frac{df}{dz} = \frac{d}{dz}\left(e^{-z/2}\right) = -\frac{e^{-z/2}}{2}$$

$$\frac{dz}{dy} = \frac{d}{dy}\left(y^T s^{-1} y\right)$$

$$= \lim_{h \to 0} \frac{g(y+h) - g(y)}{h}$$

$$= \lim_{h \to 0} \frac{(y^T + h) s^{-1} (y+h) - y^T s^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{(y^T s^{-1} + h s^{-1})(y+h) - y^T s^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{y^T s^{-1} y} + y^T s^{-1} h + h s^{-1} y + h^2 s^{-1} - \cancel{y^T s^{-1} y}}{}$$

$$= \lim_{h\to 0} \frac{y^T s^{-1} h + h s^{-1} y + h^2 s^{-1}}{h}$$

$$= \lim_{h\to 0} \frac{h\left(y^T s^{-1} + s^{-1} y + h s^{-1}\right)}{h}$$

$$= \lim_{h\to 0} \; y^T s^{-1} + s^{-1} y + h s^{-1}$$

$$= y^T s^{-1} + s^{-1} y + \lim_{h\to 0} (s^{-1} h)$$

$$= y^T s^{-1} + s^{-1} y$$

$$\therefore \frac{dy}{dx} = (x - \mu) = 1$$

$$\therefore \frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= \frac{e^{-z/2}}{2} \left(y^T s^{-1} + s^{-1} y\right) \cdot 1$$

$$= \frac{e^{-z/2}}{2} \cdot \frac{1}{s} \left(y^T + y\right)$$

(Ans)