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Chapter 4.

Probability and Statistics

UNIT 4.

Statistical Hypothesis Testing

Samsung Innovation Campus

Unit 4. Statistical Hypothesis Testing

What this unit is about:

- You will learn about the principles of hypothesis testing.
- You will learn about the t-test for one sample.
- You will learn about the t-test for independent samples and paired samples.
- You will learn about the different kinds of Chi-squared test.
- You will learn about the F-test.

Expected outcome:

- Ability to apply the different kinds of hypothesis testing methods.

How to check your progress:

- Coding Exercises.
- Quiz.

Chapter 4.

Probability and Statistics

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Hypothesis Testing (1/8)

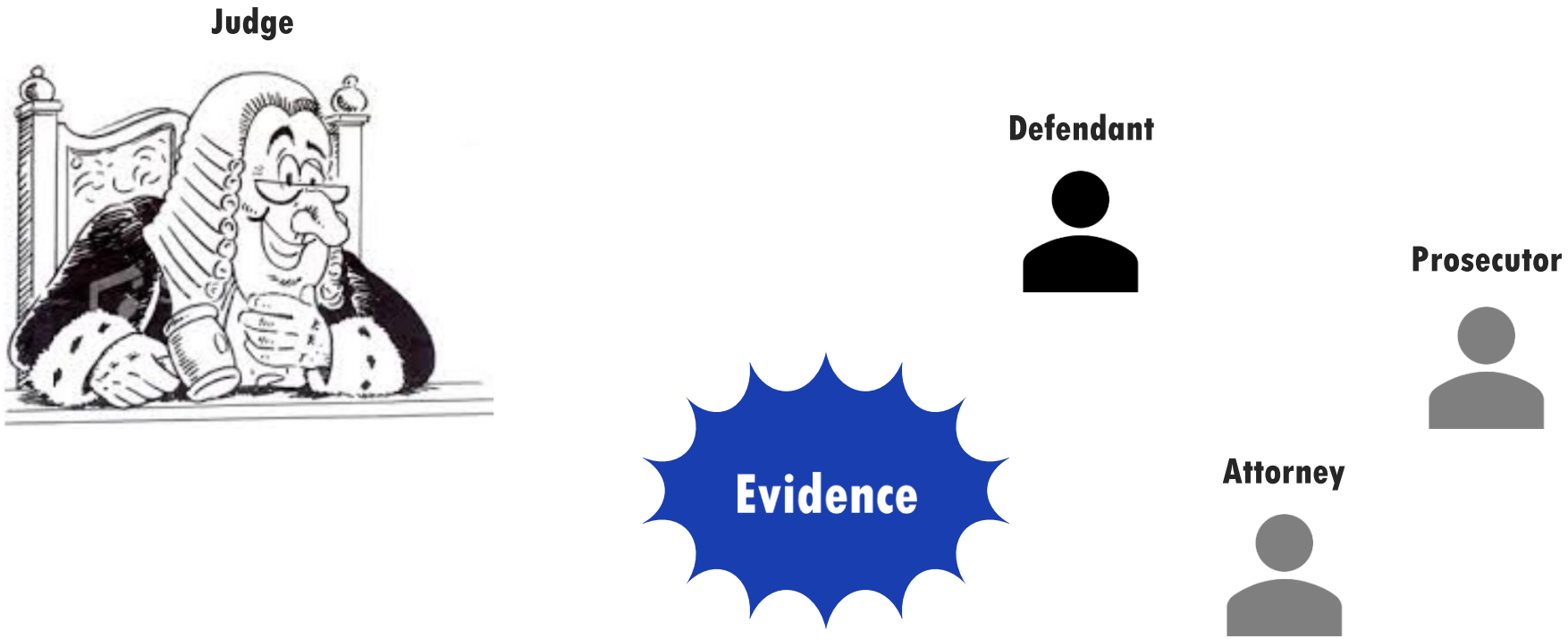
- Principles of the hypothesis testing in statistics:
  - Tests a hypothesis about the population parameters using the sample statistics.

Ex) "The average sleep time of adults in USA is 7 hours." ⇐ True or False?

Ex) "The average annual wage in Islamabad is Rs. 85,000." ⇐ True or False?

Hypothesis Testing (2/8)

Principles of the hypothesis testing in statistics:



Hypothesis Testing (3/8)

- Principles of the hypothesis testing in statistics:
  - The defendant is assumed innocent until proven otherwise.
  - The defendant doesn't need to prove his/her innocence.
  - The main objective of the trial is to prove the guiltiness of the defendant.
  - The judge delivers the verdict based on the presented evidence.

Hypothesis Testing (4/8)

Principles of the hypothesis testing in statistics:

- Null hypothesis $H0$: what is assumed until proven otherwise.
- Alternative hypothesis $H1$: what needs to be proven.
- Test statistic: a quantity used as the evidence; calculated from the sample.
- p-value: probability of observing the current test statistic or more extreme one assuming that the null hypothesis is true.

  a) If the p-value is small: weakens the assumption of the null hypothesis.

  b) If the p-value is large: strengthens the assumption of the null hypothesis.

Hypothesis Testing (5/8)

- Principles of the hypothesis testing in statistics:
  - Significance level α:

    a) It's the reference probability used when deciding whether the p-value is small enough or not.

    b) If p-value $\geq \alpha$, the H_0 is maintained.

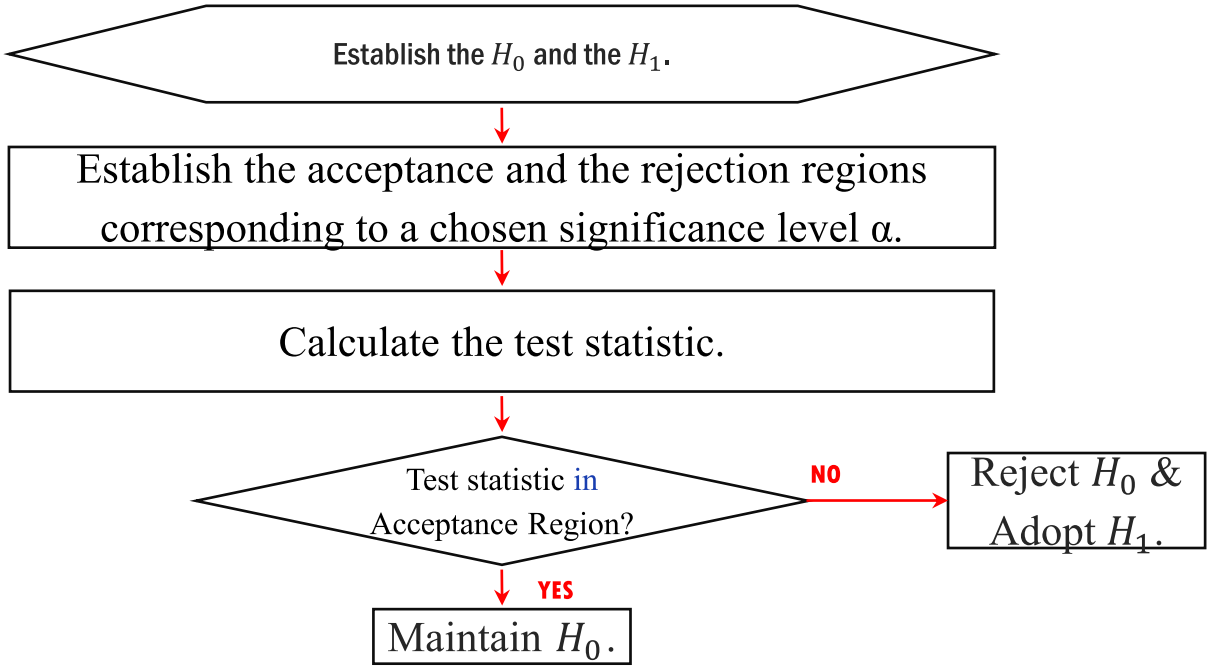
    If p-value $< \alpha$, the H_0 is rejected in favor of the H_1.

    c) This can also be interpreted as the maximum probability of rejecting the H_0 even when it's true.

    d) $1-\alpha$ is the probability that the true H_0 is retained.
  - Statistical power: probability of rejecting the false H_0 in favor of H_1.

Hypothesis Testing (6/8)

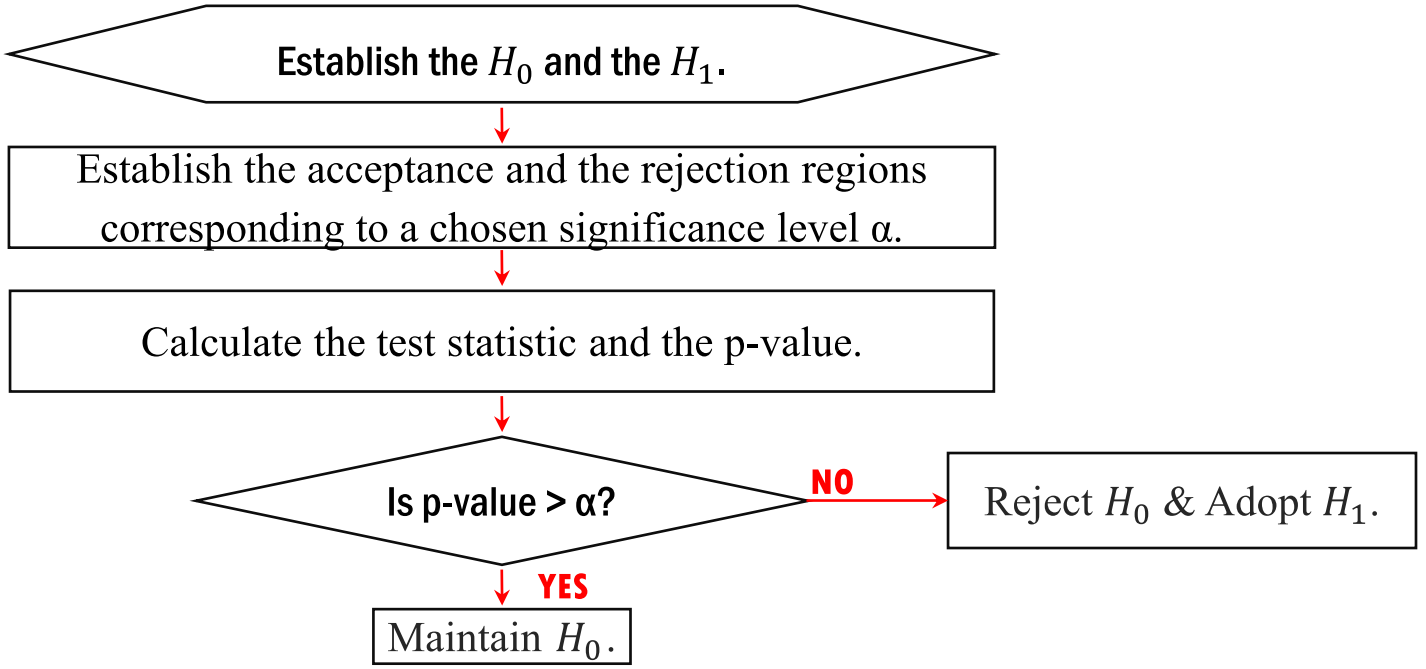
- Principles of the hypothesis testing in statistics:
  - Procedure using the test statistic:



Hypothesis Testing (7/8)

Principles of the hypothesis testing in statistics:

- Procedure using the p-value:



Hypothesis Testing (8/8)

Principles of the hypothesis testing in statistics:

TEST RESULT \ ACTUALLY	H_0 IS TRUE	H_0 IS FALSE
H_0 RETAINED	**Correct Decision.** Probability $= 1 - \alpha$	**Type 2 error.** Probability$= \beta$
H_0 REJECTED & H_1 ADOPTED	**Type 1 error.** Probability $= \alpha$	**Correct Decision.** Probability $= 1 - \beta$

Probability and Statistics

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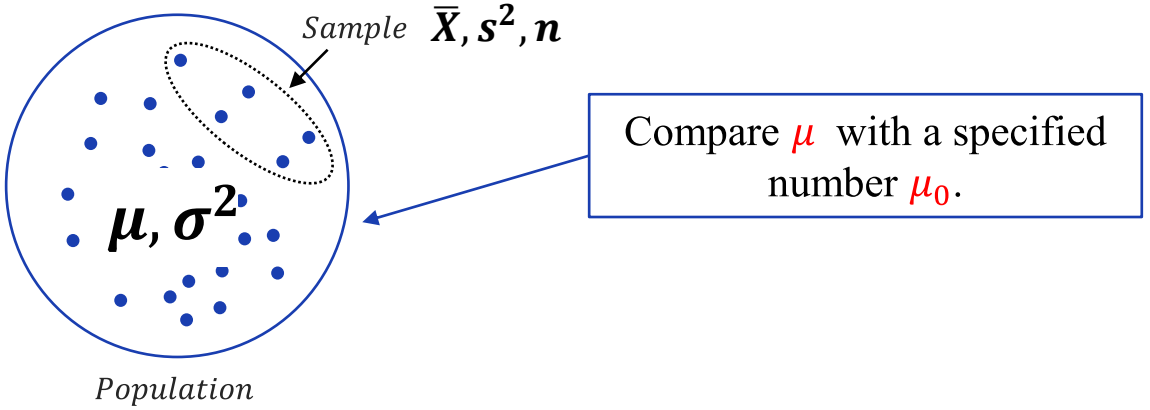
Hypothesis Test of the Means (1/8)

- One sample t-test:
  - There is one population and one sample.



  - Student-t distribution is used to interpret the test statistic calculated as following:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

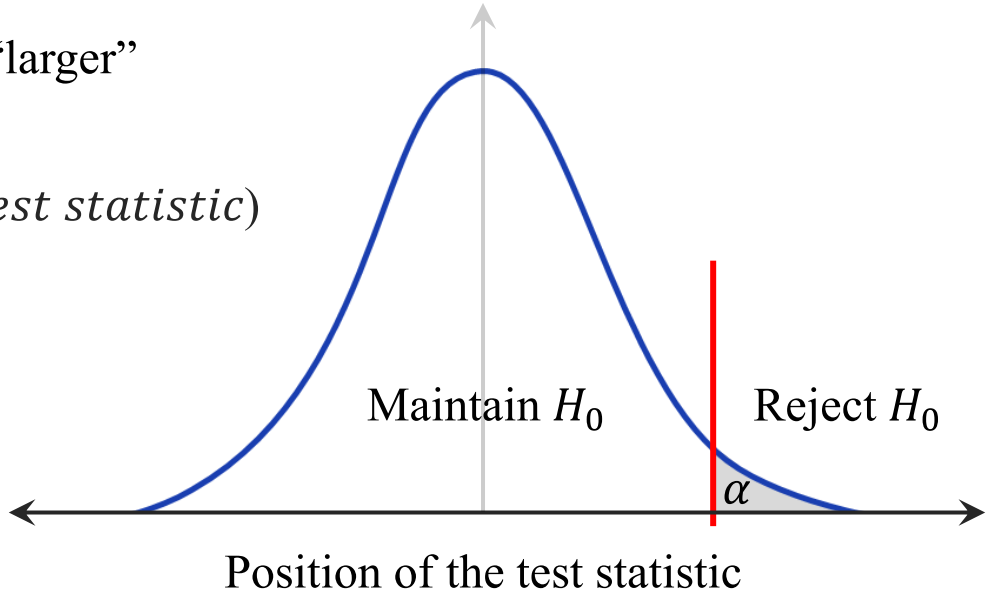
Hypothesis Test of the Means (2/8)

- One sample t-test:
  - Right tail test:

$H_0 : \mu \leq \mu_0$

$H_1 : \mu > \mu_0$ "larger"

  - p-value = $P(X > test\ statistic)$



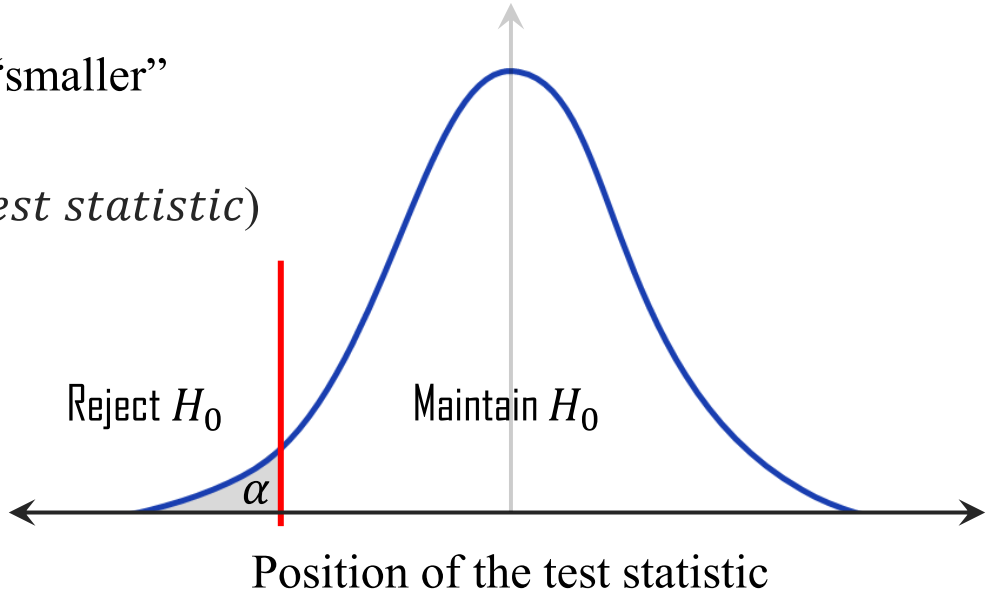
Hypothesis Test of the Means (3/8)

- One sample t-test:
  - Left tail test:

$H_0 \ : \ \mu \geq \mu_0$

$H_1 \ : \ \mu < \mu_0$ "smaller"

  - p-value = $P(X < test\ statistic)$



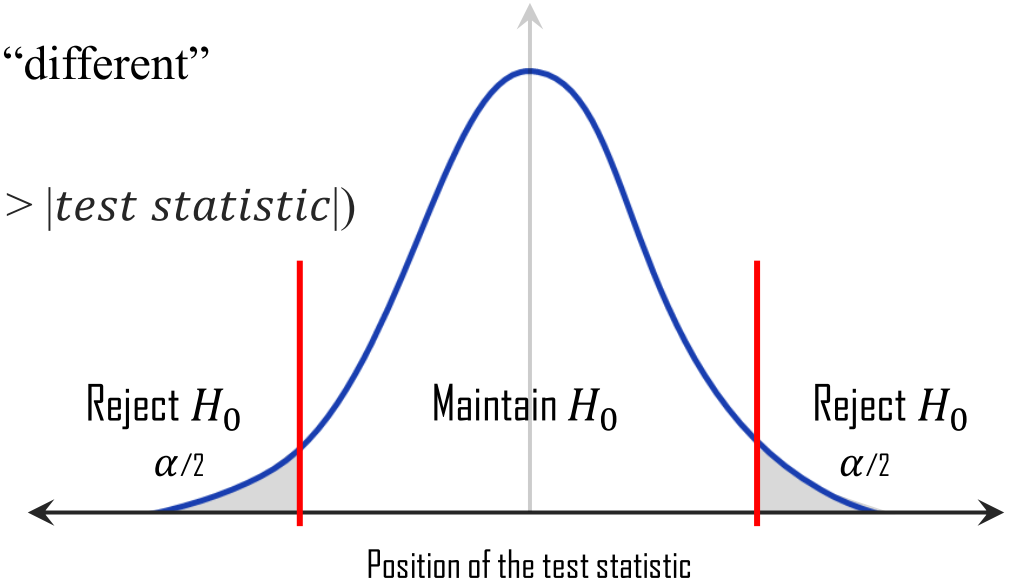
Hypothesis Test of the Means (4/8)

- One sample t-test:
  - Two tail test:

$H_0 : \mu = \mu_0$

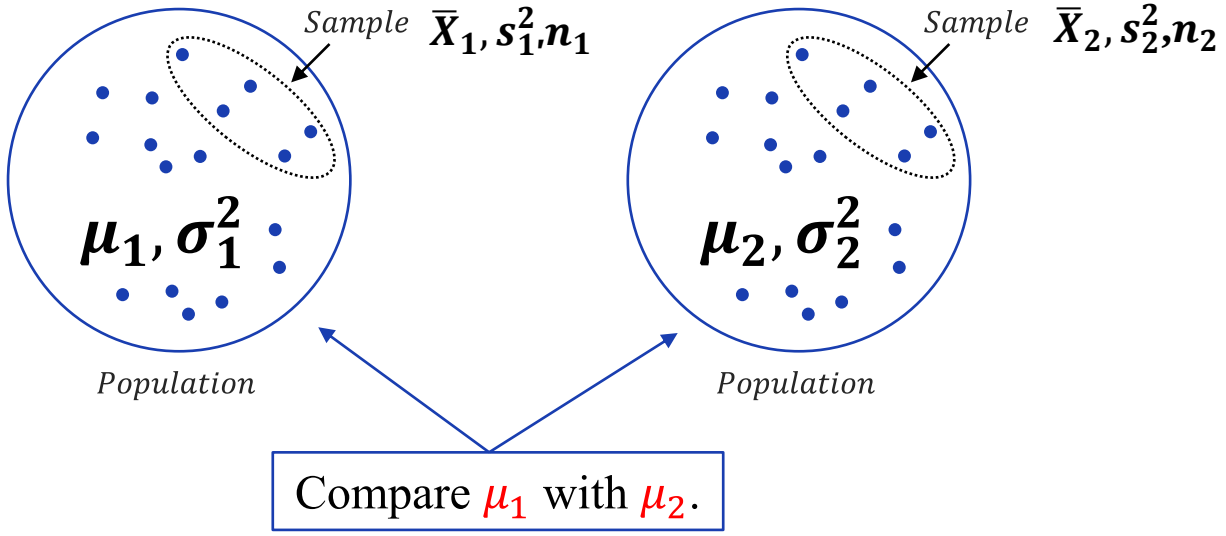
$H_1 : \mu \neq \mu_0$ "different"

  - p-value $=2 \times P(X > |test\ statistic|)$



Hypothesis Test of the Means (5/8)

- Independent two sample t-test:
  - There are two populations and two samples.



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Hypothesis Test of the Means (6/8)

Independent two sample t-test:

- Right tail test:

$$H_0 : \mu_1 - \mu_2 \leq 0 \quad \Leftrightarrow \quad \mu_1 \leq \mu_2$$
$$H_1 : \mu_1 - \mu_2 > 0 \quad \Leftrightarrow \quad \mu_1 > \mu_2$$

- Left tail test:

$$H_0 : \mu_1 - \mu_2 \geq 0 \quad \Leftrightarrow \quad \mu_1 \geq \mu_2$$
$$H_1 : \mu_1 - \mu_2 < 0 \quad \Leftrightarrow \quad \mu_1 < \mu_2$$

- Two tail test:

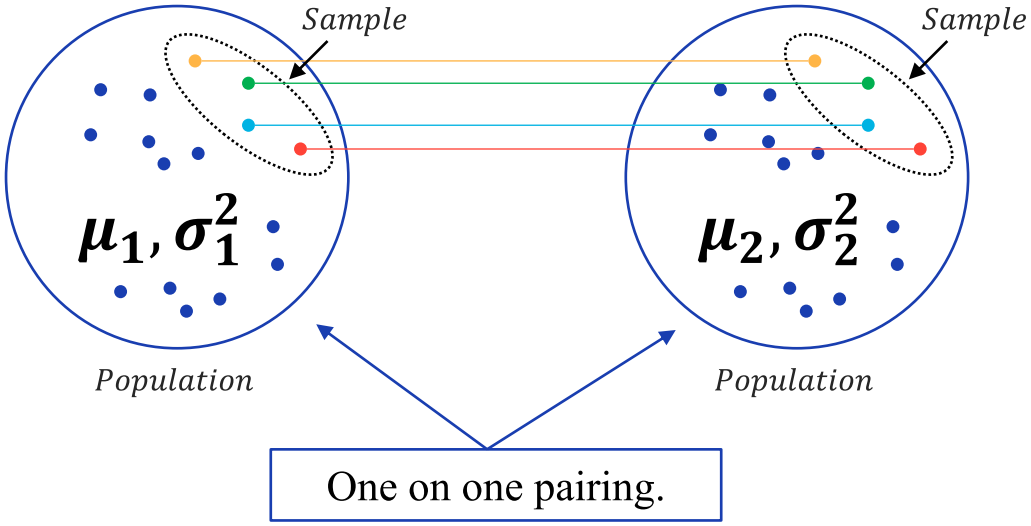
$$H_0 : \mu_1 - \mu_2 = 0 \quad \Leftrightarrow \quad \mu_1 = \mu_2$$
$$H_1 : \mu_1 - \mu_2 \neq 0 \quad \Leftrightarrow \quad \mu_1 \neq \mu_2$$

You should consider two cases:
1) Equal variances.
2) Unequal variances.

Hypothesis Test of the Means (7/8)

Paired two sample t-test:

- There are two populations and two samples. There is "one on one" pairing.



Ex) Change in the blood pressure of the same test subjects before and after taking a new drug.

Hypothesis Test of the Means (8/8)

- Analysis of Variance (ANOVA):
  - So far with the t-test, we had one or two groups (samples).
  - ANOVA can detect differences in the means of two or more groups.

    Null hypothesis $\boldsymbol{H_0}$: $\mu 1 = \mu 2 = \mu 3 = \ldots$

    Alternative hypothesis $\boldsymbol{H_1}$: There is at least one case where $\mu i \neq \mu j$
  - Assumptions: 1) The distribution of the data is Normal.

    2) The group variances are the same.

    3) The groups are independent from each other.
  - F distribution is used to calculate the p-value.

Coding Exercise #0307

Follow practice steps on 'ex_0307.ipynb' file.

Hypothesis Test of the Frequencies (1/4)

- Chi-squared test for one way table:
  - One way table or "frequency table" summarizes a categorical variable.
  - Compares the observed frequencies with a given model (expected frequencies).

    Null hypothesis H_0 : The observed frequency table and the expected model agree.

    Alternative hypothesis H_1 : The observed frequency table and the expected model are different.
  - Also called the "Goodness of fit test".

Hypothesis Test of the Frequencies (2/4)

- Chi-squared test for one way table:
  - The test statistic is:

$$test\ statistic = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

  - E_i are the expected frequencies and O_i are the observed frequencies.
  - k is the number of categories or types.
  - The test statistic follows the Chi-square distribution of degree of freedom = $k - 1$.

Hypothesis Test of the Frequencies (3/4)

- Chi-squared test for two way table:
  - A contingency table summarizes two categorical variables.

    Ex) Confusion matrix (machine learning).
  - Uses the frequencies to test the existence of relationship between two categorical variables.

    Null hypothesis H_0 : The categorical variables are independent.

    Alternative hypothesis H_1 : The categorical variables are not independent.
  - Also called “Independence test”.

Hypothesis Test of the Frequencies (4/4)

Chi-squared test for two way table:

- The test statistic is:

$$test\ statistic = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

- E_i are the expected frequencies and O_i are the observed frequencies.
- r is the number of rows and c is the number of columns in the two way table.
- The test statistic follows the Chi-square distribution of degree of freedom = $(r-1) \times (c-1)$.

Coding Exercise #0308

Follow practice steps on 'ex_0308.ipynb' file.

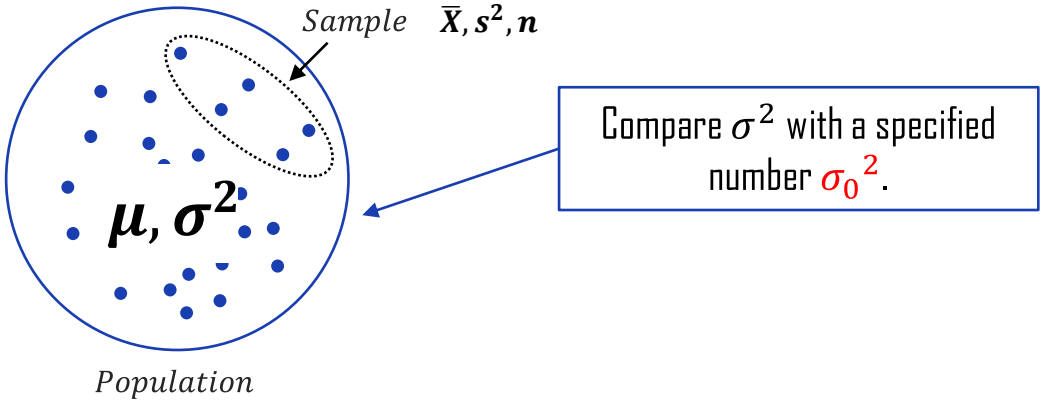
Coding Exercise #0309

Follow practice steps on 'ex_0309.ipynb' file.

Hypothesis Test of the Variances (1/4)

One sample t-test:

- There is one population and one sample.



Hypothesis Test of the Variances (2/4)

Chi-squared test of variance:

- There are left tail test, right tail test and two tail test. ⇐ Just like in t-test.
- The test statistic is calculated as: (n= sample size)

$$test\ statistic = \frac{(n-1)S^2}{\sigma_0^2}$$

- The test statistic follows the Chi-square distribution of degree of freedom = n – 1.

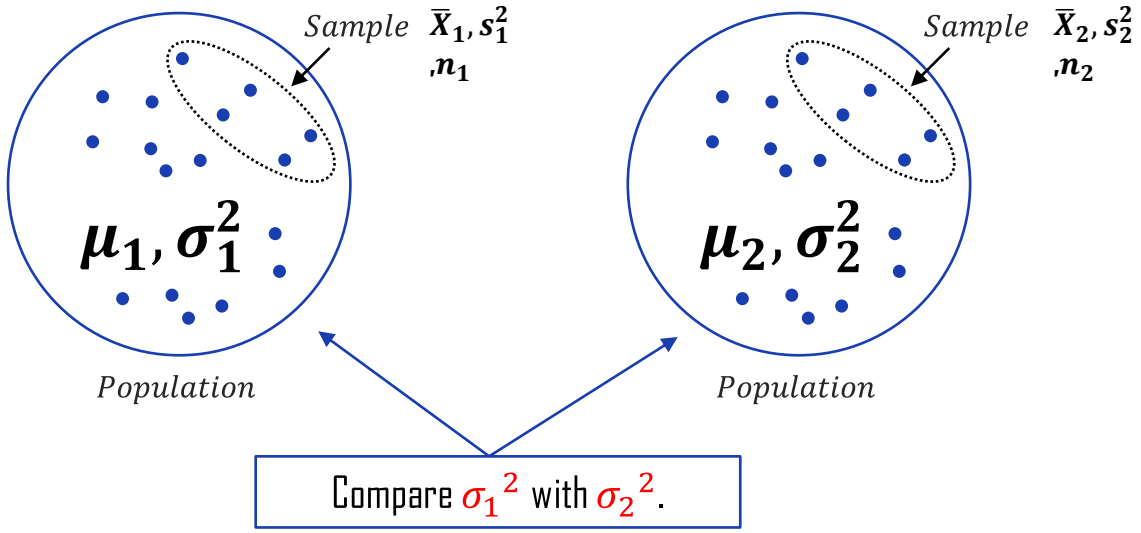
Coding Exercise #0310

Follow practice steps on 'ex_0310.ipynb' file.

Hypothesis Test of the Variances (3/4)

F-test of variance ratio:

- There are two populations and two samples.



Hypothesis Test of the Variances (4/4)

F-test of variance ratio:

- There are left tail test, right tail test and two tail test.
- The test statistic is calculated as a ratio of the sample variances:

$$test\ statistic = \frac{S_1^2}{S_2^2}$$

- The test statistic follows the F distribution $F(n_1 - 1, n_2 - 1)$. Here n_1 and n_2 are the sample sizes.

Hypothesis Test Summary

Here, this summarizes the hypothesis tests we have covered so far:

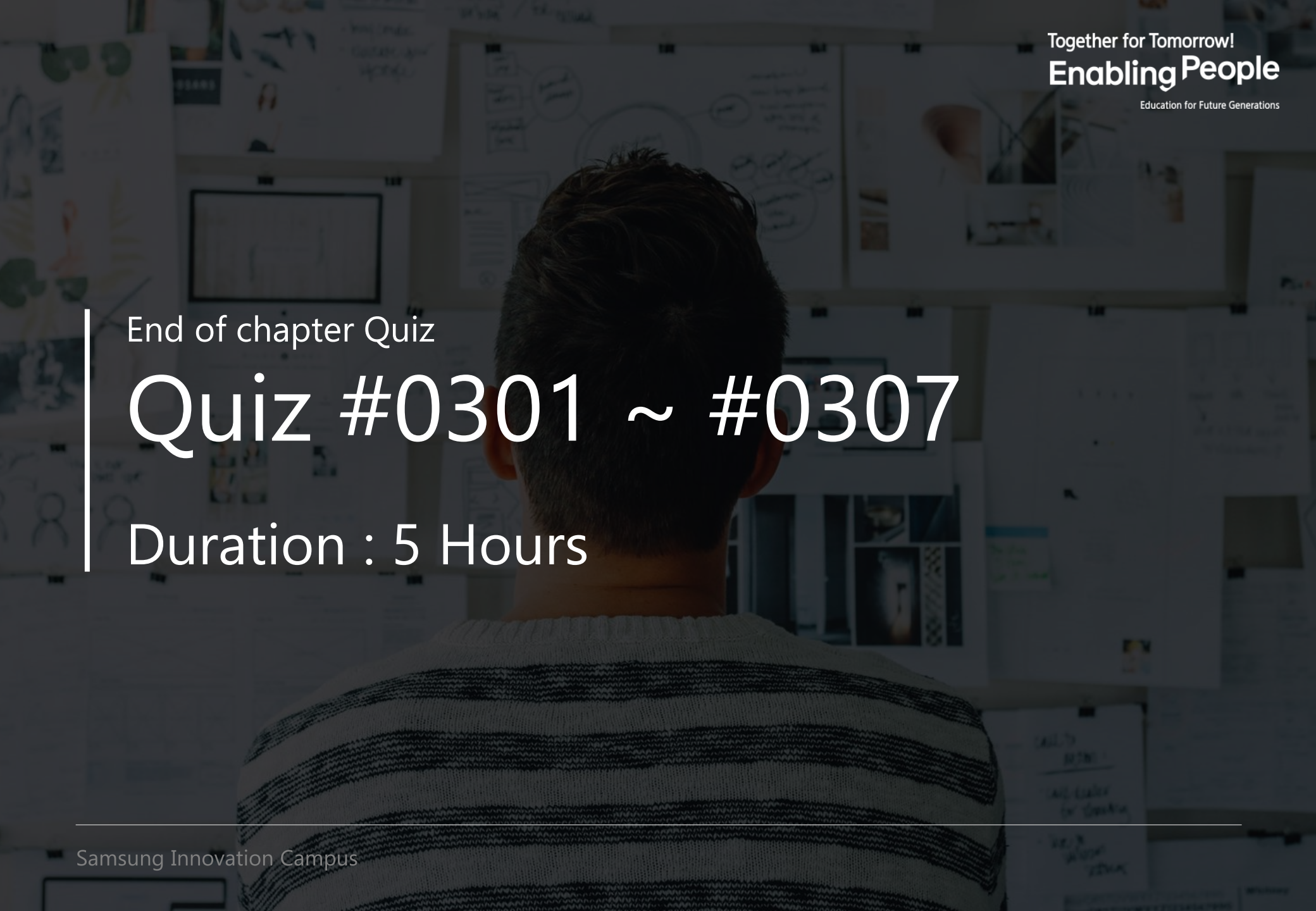
HYPOTHESIS TEST	PROBABILITY DENSITY DISTRIBUTION
One sample t-test, Independent two sample t-test, Paired sample t-test.	Student-t
ANOVA	F
Chi-squared test of one way table, Chi-squared test of two way table.	Chi-square
Chi-squared test of variance.	Chi-square
F-test of variance ratio.	F

Coding Exercise #0311

Follow practice steps on 'ex_0311.ipynb' file.

Coding Exercise #0312

Follow practice steps on 'ex_0312.ipynb' file.



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End of chapter Quiz

Quiz #0301 ~ #0307

Duration : 5 Hours

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