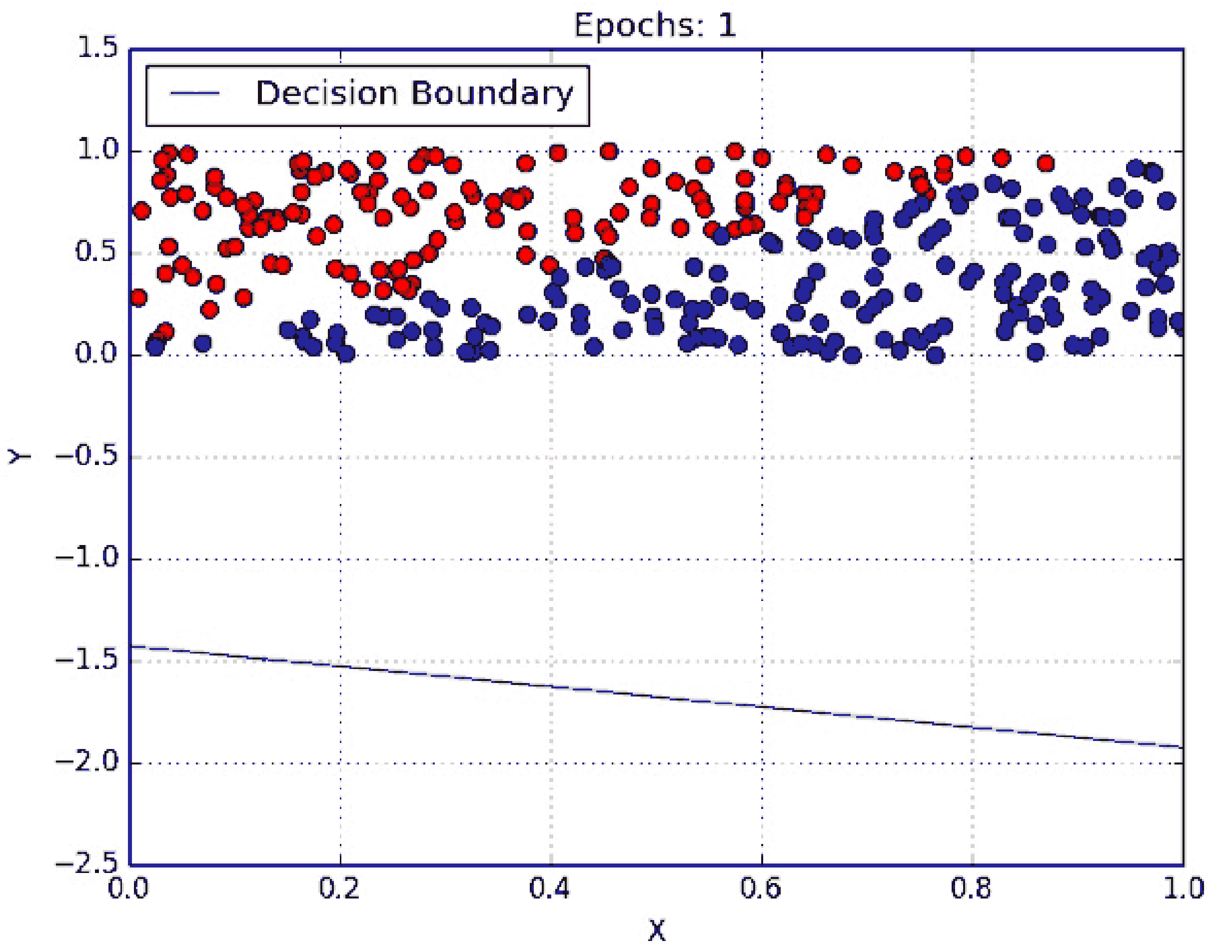
Introduction to Logistic Regression

Ayush Pant Jan 22, 2019 · 5 min read

Introduction

In this blog, we will discuss the basic concepts of Logistic Regression and what kind of problems can it help us to solve.



GIF: University of Toronto

spam, Online transactions Fraud or not Fraud, Tumor Malignant or Benign. Logistic regression transforms its output using the logistic sigmoid function to return a probability value.

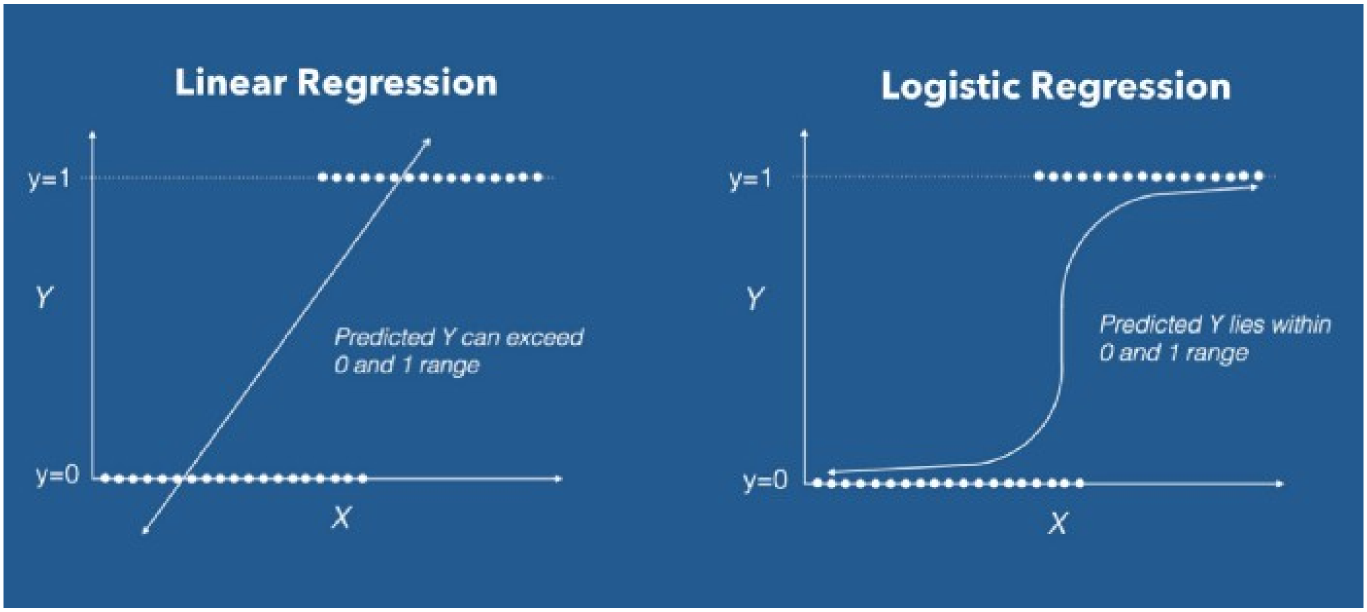
What are the types of logistic regression

1. Binary (eg. Tumor Malignant or Benign)

2. Multi-linear functions failsClass (eg. Cats, dogs or Sheep's)

Logistic Regression

Logistic Regression is a Machine Learning algorithm which is used for the classification problems, it is a predictive analysis algorithm and based on the concept of probability.



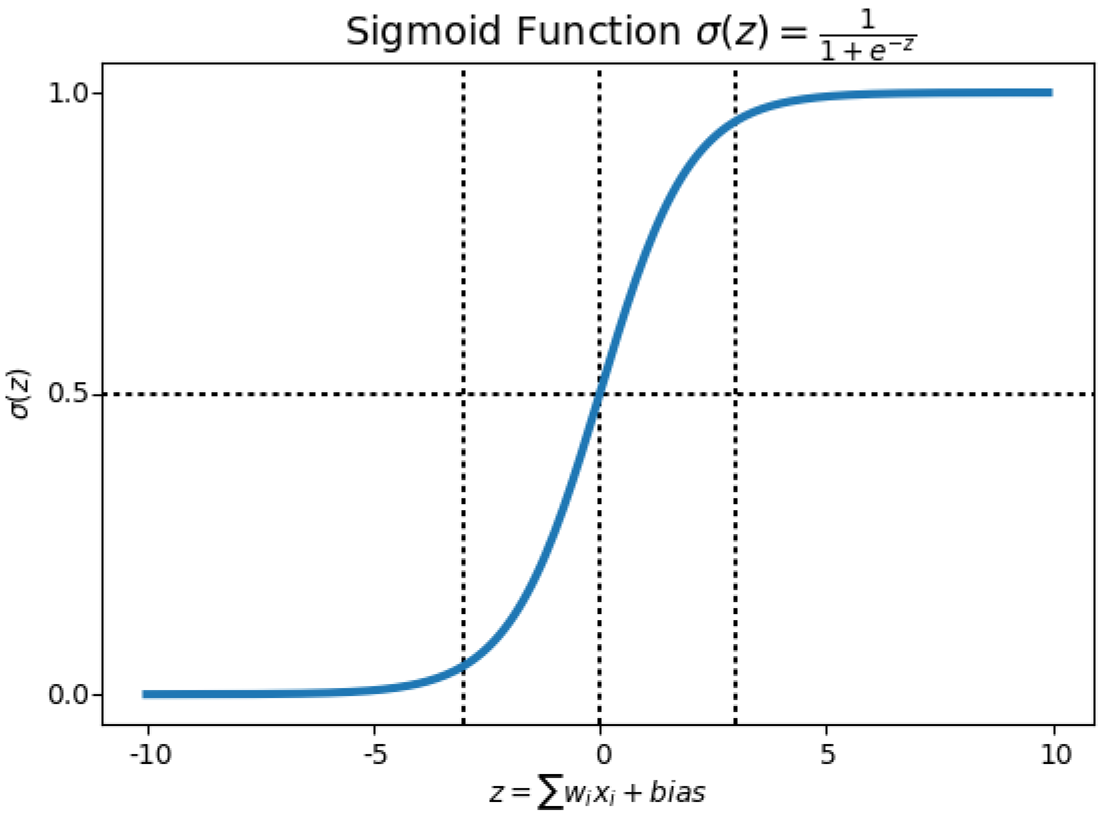
Linear Regression VS Logistic Regression Graph| Image: Data Camp

We can call a Logistic Regression a Linear Regression model but the Logistic Regression uses a more complex cost function, this cost function can be defined as the '**Sigmoid function**' or also known as the 'logistic function' instead of a linear function.

The hypothesis of logistic regression tends it to limit the cost function between 0 and 1. Therefore linear functions fail to represent it as it can have a value greater than 1 or less than 0 which is not possible as per the hypothesis of logistic regression.

$$0 \leq h_\theta(x) \leq 1$$

Logistic regression hypothesis expectation function maps any real value into another value between 0 and 1. In machine learning, we use sigmoid to map predictions to probabilities.



Sigmoid Function Graph

$$f(x) = \frac{1}{1+e^{-(x)}}$$

Formula of a sigmoid function | Image: Analytics India Magazine

Hypothesis Representation

When using *linear regression* we used a formula of the hypothesis i.e.

> $h\Theta(x) = \beta_0 + \beta_1 X$

For logistic regression we are going to modify it a little bit i.e.

We have expected that our hypothesis will give values between 0 and 1.

$Z = \beta_0 + \beta_1 X$

$h\Theta(x) = \text{sigmoid}(Z)$

i.e. $h\Theta(x) = 1/(1 + e\text{^}-(\beta_0 + \beta_1 X)$

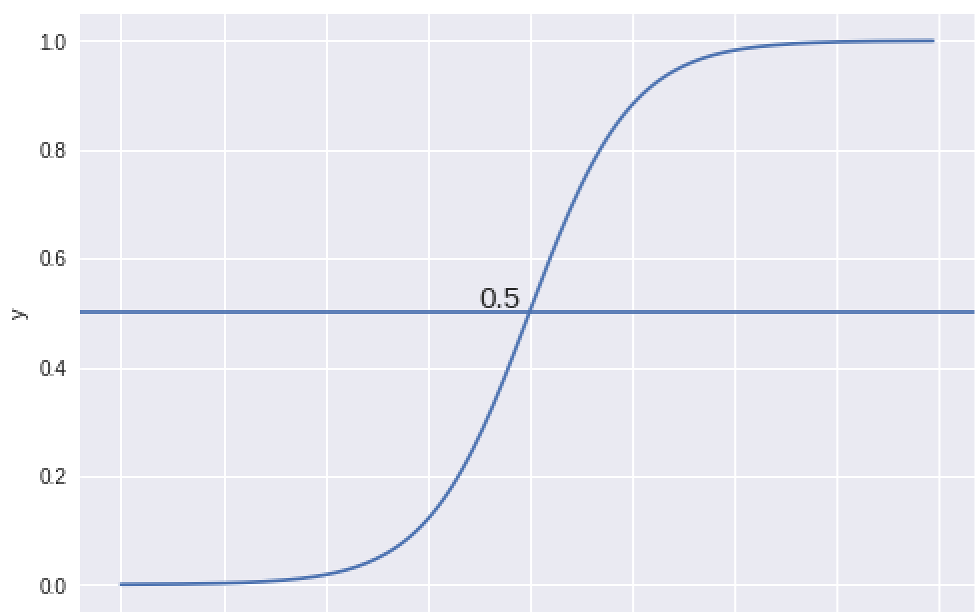
$$h\theta(X) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 X)}}$$

The Hypothesis of logistic regression

Decision Boundary

We expect our classifier to give us a set of outputs or classes based on probability when we pass the inputs through a prediction function and returns a probability score between 0 and 1.

For Example, We have 2 classes, let’s take them like cats and dogs(1 — dog , 0 — cats). We basically decide with a threshold value above which we classify values into Class 1 and of the value goes below the threshold then we classify it in Class 2.



Example

As shown in the above graph we have chosen the threshold as 0.5, if the prediction function returned a value of 0.7 then we would classify this observation as Class 1(DOG). If our prediction returned a value of 0.2 then we would classify the observation as Class 2(CAT).

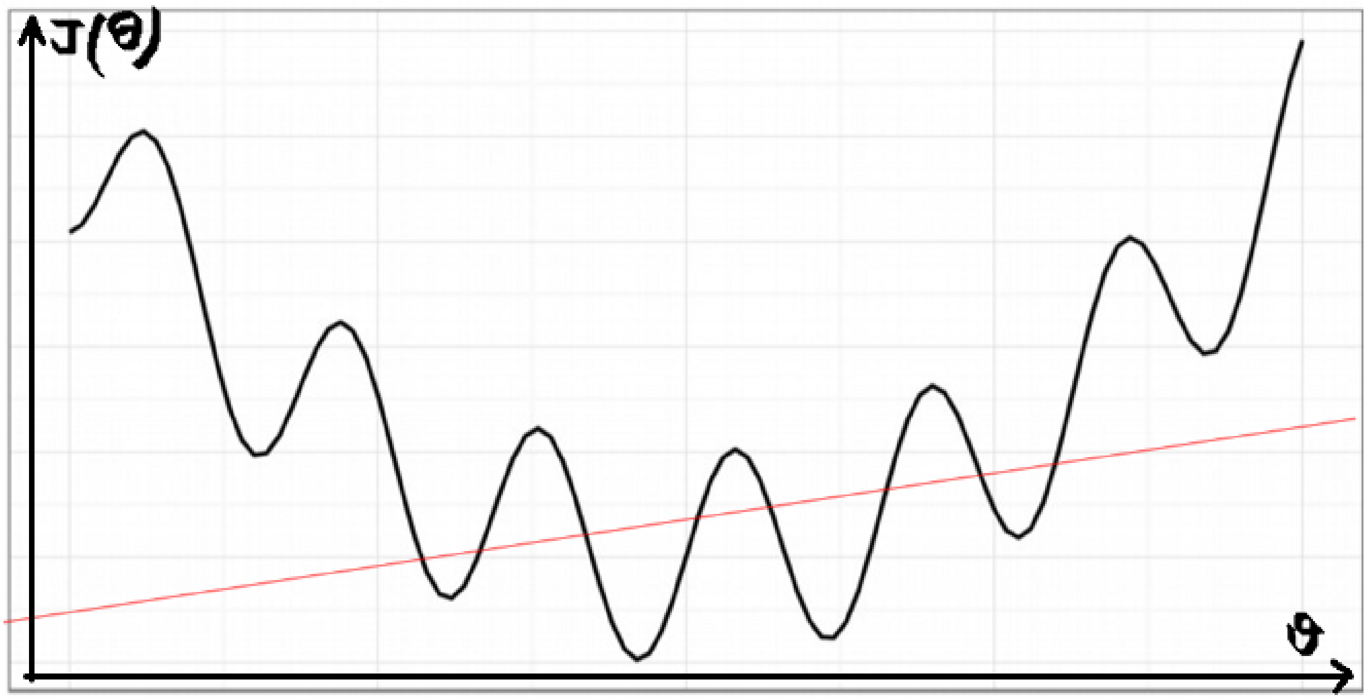
Cost Function

We learnt about the cost function $J(\theta)$ in the *Linear regression*, the cost function represents optimization objective i.e. we create a cost function and minimize it so that we can develop an accurate model with minimum error.

$$J(\theta) = \frac{1}{2}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2.$$

The Cost function of Linear regression

If we try to use the cost function of the linear regression in 'Logistic Regression' then it would be of no use as it would end up being a **non-convex** function with many local minimums, in which it would be very **difficult** to **minimize the cost value** and find the global minimum.

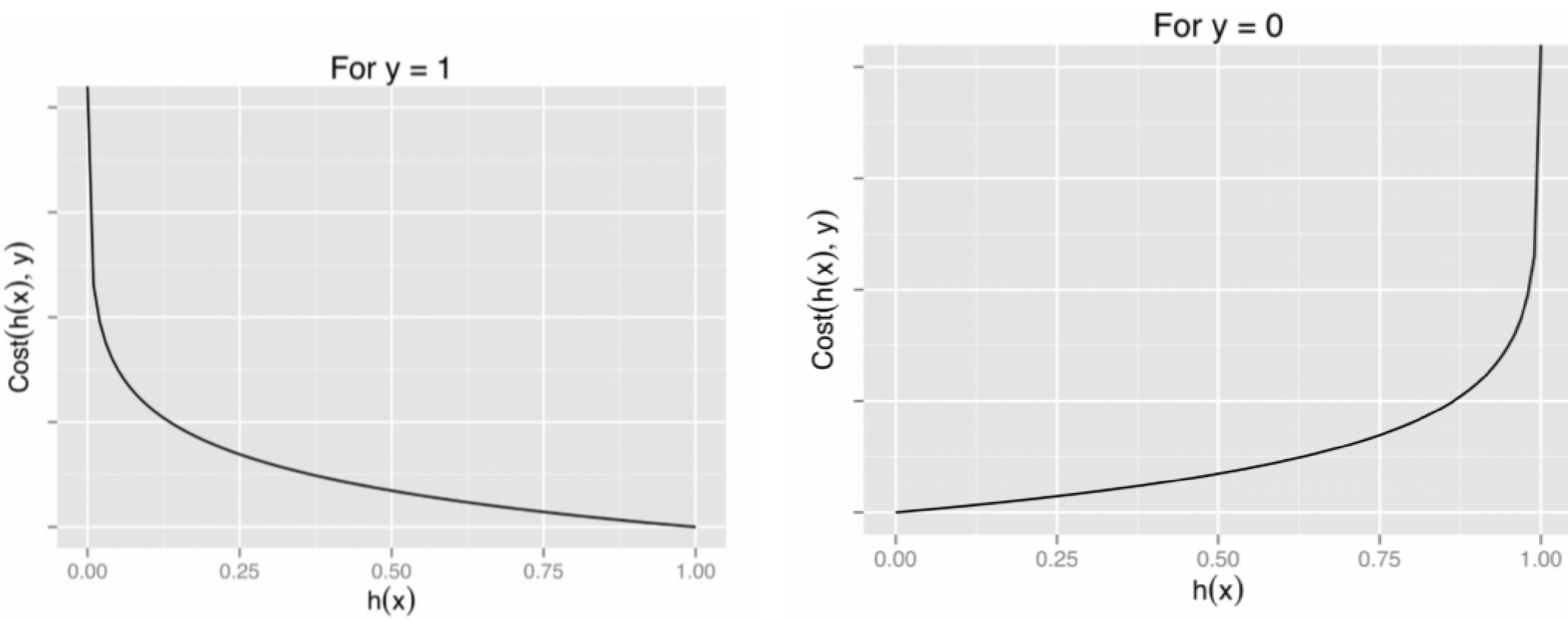


Non-convex function

> −log(hθ(x)) if y = 1
>
> −log(1−hθ(x)) if y = 0

$$Cost(h_\theta(x), y) = \begin{cases} -log(h_\theta(x)) & \text{if } y = 1 \\ -log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

Cost function of Logistic Regression



Graph of logistic regression

The above two functions can be compressed into a single function i.e.

$$J(\theta) = -\frac{1}{m}\sum\left[y^{(i)}\log(h\theta(x(i))) + \left(1 - y^{(i)}\right)\log(1 - h\theta(x(i)))\right]$$

Above functions compressed into one cost function

Gradient Descent

Now the question arises, how do we reduce the cost value. Well, this can be done by using **Gradient Descent.** The main goal of Gradient descent is to **minimize the cost value.** i.e. min J(***θ***).

$$\theta j := \theta j - \alpha \frac{\partial}{\partial \theta j} J(\theta)$$

Objective: To minimize the cost function we have to run the gradient descent function on each parameter

Want $\min_\theta J(\theta)$:

Repeat {

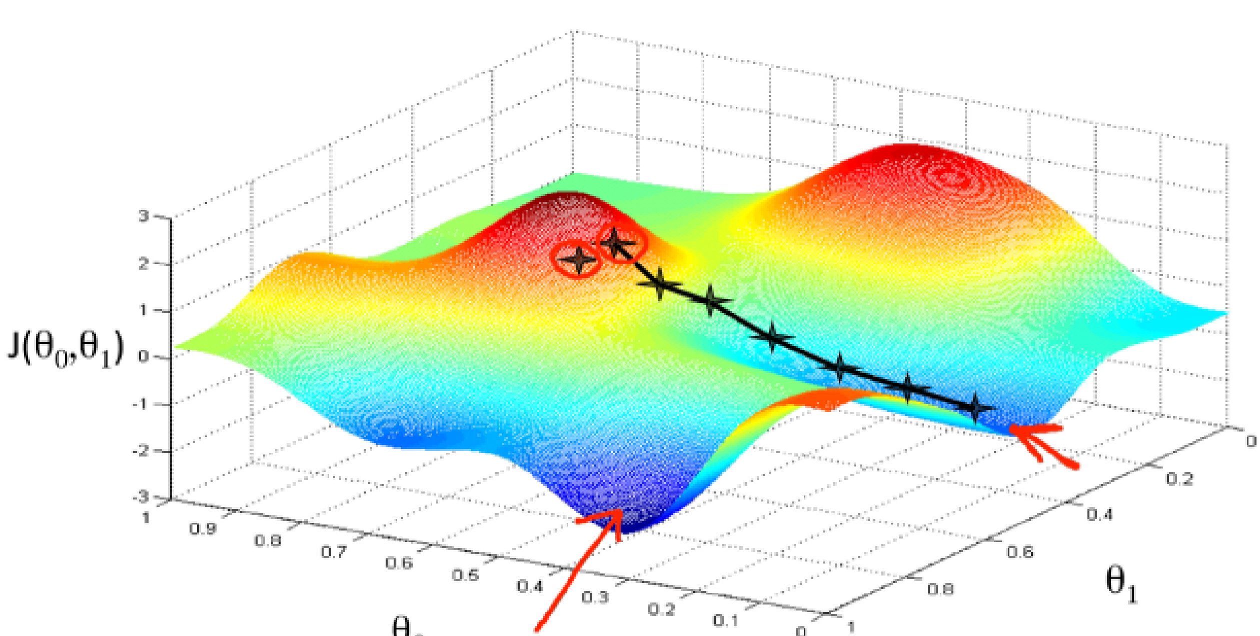
$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(simultaneously update all θ_j)

}

Gradient Descent Simplified | Image: Andrew Ng Course

Gradient descent has an analogy in which we have to imagine ourselves at the top of a mountain valley and left stranded and blindfolded, our objective is to reach the bottom of the hill. Feeling the slope of the terrain around you is what everyone would do. Well, this action is analogous to calculating the gradient descent, and taking a step is analogous to one iteration of the update to the parameters.



Gradient Descent analogy

Conclusion

In this blog, I have presented you with the basic concept of Logistic Regression. I hope this blog was helpful and would have motivated you enough to get interested in the topic.

Machine Learning Logistic Regression Supervised Learning Unsupervised Learning Cost Function