

# Analysing the effect of Pollution Pricing Act on Carbon Emissions in Canada

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## Abstract

The paper employs a DSGE framework to assess the effectiveness of a carbon tax policy on Global Greenhouse Gas emissions. Using a four sector model including households, an intermediary energy sector, government and a final goods sector, we solve for the equilibrium conditions of the economy. Augmenting this framework with an emission generating function allows us to model the carbon emissions generated by the final goods sector. We attempt to calibrate this model to the Canadian economy, and analyse impulse responses from exogenous tax and price shocks on GHG emissions. We find qualitative evidence in favor of a tax shock policy over a price shock policy. Tax shocks only affect the carbon emissions level unlike price shock policies that can have a depressing effect on consumption of the household.

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## 1. Introduction

The growing concern around climate change and its implications for a sustainable future has been central to policy decisions in leading economies. There is a growing consensus to treat this as a Global crisis that demands a Global response. Pacts such as the Kyoto Protocol and the Paris Climate Agreement are testament to the global nature of the problem. The reduction of GHG emissions is an undeniable responsibility of all nations. However, it is possible to argue that added responsibility falls on developed nations like Canada, the United States and China who have contributed mammoth shares to the global carbon footprint. Canada's policies to reduce GHG emissions have not been succesful in the recent past. It failed to achieve the targets set by the Kyoto Protocol and under the Conservative Prime

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Minister, Stephen Harper decided to withdraw from the agreement in 2011. The target that they failed to achieve was a 6% reduction in GHG within a ten year window. Currently, with the Liberal Party in power Canada faces the possibility of failing to meet the Paris Climate Agreement as well. In light of this, the government has adopted a dynamic carbon tax plan in 2018 that is expected to increase by \$10 annually starting from \$20 per tonne of GHG emission. Formally, this policy is referred to as the Greenhouse Gas Pollution Pricing Act and was put into effect in 2018. There is a trade-off to such policies on economic growth. Recent academic literature on growth has adopted numerous models to simulate the effects of such policy on growth. This paper adopts a DSGE framework to simulate the transition path of GHG concentration in the atmosphere under this policy.

Our objective is to simulate the effect of such a policy on GHG concentration and the effectiveness of the policy in light of the objectives set by the Paris Climate Agreement. Following Greiner et al. (2014) we adopt a ‘damaged welfare function’ that models increase in GHG concentrations within the atmosphere as a disutility. The paper aims to calibrate the model to the Canadian economy, and undertake a numerical analysis of the steady state GHG levels given a stochastic shock. Unlike Greiner et al. (2014), we use Bellman equations to solve the optimization problem. We then use the dynare package in Matlab for a numerical approximation of our model. We also present some areas of possible improvement in modelling from recent literature. Numerous models have been adopted to simulate and analyse the emissions of GHG into the atmosphere. The paper presents a brief analytical survey of such models. It also presents an analytical framework specific to the Canadian Economy. This will be useful for analysing future policy measures pertaining to the reduction of GHG emissions.

In the following sections the paper will highlight some of the relevant literature on this issue. The paper will then specify the model adapted from Greiner et al. (2014) in Section 3. Section 4 will discuss the calibration methodology, Section 5 will look at the results of the simulation and finally Section 6 will provide some concluding recommendations.

## **2. Relevant Literature**

While Nordhaus (1974) frames the climate change problem as a environmental resource management problem, the problem is essentially an intertem-

poral decision problem. This idea is best described by Nordhaus (1993) as the management of our common geophysical and biological resources. What sets it apart from a typical intertemporal decision problem is the length of time between the choice and the trade-off from the choice. The impact of consuming our natural resources today is not going to be felt in our lifetime. So maybe an intergenerational decision problem is a more appropriate terminology. The point is that there is some merit to thinking of this problem in this perspective.

The classical models of economic growth are flexible enough to include these natural resources as an input into a production function. A standard Cobb-Douglas is sufficient to represent a two factor production process. Maurer and Semmler (2015) use two alternative types of energy (polluting and non-polluting) as the only factors into the production process. This is somewhat intuitive as energy is a homogenous good which makes the two alternatives perfect substitutes. It can be argued that such a model is somewhat restrictive as it ignores labor choices. Chan (2019) shows that incorporating labor as a choice within the welfare function leads to results different from those predicted in previous literature. In particular they find that the dampening effect of an environmental tax shock is inherent in the set up of a closed economy model. Papers like Annicchiarico and Di Dio (2015) on the other hand, have adopted the New Keynesian approach to modelling the effect of environmental policies. Adopting the New Keynesian framework allows us to analyse the effects of price stickiness on environmental policies.

Other researchers have taken a multi-sectoral approach to modelling the impact of GHG concentration on economic growth within a DSGE framework. Niu et al. (2018) for instance, uses energy as an intermediate sector for the final goods sector. Augmenting a carbon emission function to the set-up, allows the model to capture the dynamics of GHG concentration caused by production in the final goods sector. It is possible to be critical of certain aspects of their set up. Specifically, the manner in which it models the carbon emission function represents an inherently inverse relationship between government taxes and share of fossil fuel based energy use. It would be more interesting to include an alternative source of energy to capture the trade-off between the two sources. Allowing for appropriate choice variables can then help determine the dynamic paths undertaken by the two alternative forms of energy.

Annicchiarico and Diluiso (2019) also uses a similar set-up with energy as an intermediate sector. Their approach is different from Niu et al. (2018) in that they use a damage function on the final goods sector to model the harmful effects of GHG concentration in the atmosphere. Greiner et al. (2014), Maurer and Semmler (2015) would argue that a more representative modelling of GHG concentration would be as a damage to the welfare function. This seems like a more intuitive approach to modelling the impact of GHG concentration in the atmosphere. The most immediate impact is likely to be reduced productivity due to poorer environmental conditions. Air, water and soil are all natural resources that contribute to our health and productivity. A depletion of these natural resources from increased GHG concentration is likely to have both direct and indirect impacts on labor productivity. Poorer air quality for instance, can have a direct impact on the respiratory functions of a worker<sup>1</sup>. Other indirect impacts of reduced quality of natural resources can be tracked through lower crop yields; a consequence of poorer soil and water quality. Vasilev (2018) also adopts a similar approach where instead of modelling carbon emission as a disutility to the welfare function they represent the ‘lack’ of pollution as a source of utility in the welfare function. This is also done within a DSGE framework.

The manner in which we model the impact of GHG concentration on the households intertemporal decision problem is particularly interesting and can have significant implications on the dynamics of the model. We have provided some intuition behind why there might be an effect on the household’s decision problem due to increased GHG concentration<sup>2</sup>. In fact, Greiner et al. (2014) shows that the steady state levels of GHG concentration from central planner’s solution is different from a laissez-faire solution. They go on to show that this difference stems from the difference in the dis-utility parameter from increased pollution between the representative household and the state. Essentially, this is recapturing the fact that increased GHG concentration can have negative externalities which are not accounted for in the disutility parameter of the representative household. If both the state and the representative household had the same disutility parameter for higher carbon emissions the steady state solutions would be equivalent. This notion is in

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<sup>1</sup>Fakir et al. (2018) finds considerable empirical evidence that respiratory health is a significant factor in determining productivity among rural farmers in Bangladesh.

<sup>2</sup>See Maurer and Semmler (2015) and Greiner et al. (2014)

congruence with economic theory.

However, there are significant difficulties to including such a disutility parameter into the welfare function<sup>3</sup>. Despite the intuitive appeal of modelling the effect of GHG concentration in this manner, there are empirical obstacles which make such models infeasible for empirical analyses. How do we go about calibrating the  $\eta$  parameter for numerical analysis? Greiner et al. (2014) does not provide any insight into how they calibrate this parameter. Given that this is a crucial parameter in our analysis of the dynamics of GHG concentration, this is a significant problem in this model, especially for empirical analysis.

A possible strategy to tackling this problem can be found through using computational strategies such as Genetic Algorithms<sup>4</sup>. Recent literature on macroeconomic models have started to loosen the representative agent assumption to allow for heterogeneity in the household decision making problem using Genetic Algorithm. It is useful to think of the algorithm as a learning by doing process where the agents have no prior information about the economy. Instead they learn by making choices in each period and observing the payoff of their choices from their welfare function. While this allows us to loosen the perfect foresight assumption, it also allows us to model heterogeneity in the household decision making problem. This computational approach to optimizing the welfare function can be useful in tackling the aforementioned problem. We can for instance assign a randomly generated vector of  $\eta$  into our initial population of agents and allow the agents to decide their own choice of  $\eta$  along with other choice variables. This will allow us to observe the dynamic of  $\eta$ , or the evolution of household preferences for greater increase in GHG concentration. Since the Genetic Algorithm allows for open form solutions to the optimization problem we need not worry about additional equations to solve for  $\eta$ . A larger string length should allow for making an additional choice in the optimization problem. Any other approach to solving this problem has to be rooted in Microeconomic theory as they pertain to preferences ordering.

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<sup>3</sup>Specifically we are referring to a damaged welfare function here of the CES form where the factor  $G^{-\eta}$  describes the disutility from increase in GHG concentrations with  $\eta$  being the disutility parameter.

<sup>4</sup>See Holland (1975).

### 3. Theoretical Model

Following the best practices in pervious literature we adopt the DSGE framework with three sectors, the government, households, an energy sector and the final goods sector. Usually, solving the social planner's problem would be appropriate. If there are no externalities in the model this will be equivalent to solving the laissez-faire problem. However, the existence of pollution within the model means that a government is necessary to correct the externality. Following the specifications in Niu et al. (2018), we incorporate a government into the framework.

#### 3.1. The Household

Households choose between consumption,  $c_t$  and labor,  $l_t$  in each period to solve their intra-temporal problem. They also choose  $k_{t+1}$  for the next period. There are no bond markets in this model. The household therefore, chooses  $\{c_t, k_{t+1}, l_t\}$  to maximize the following welfare function:

$$U_0 = E_0 \sum_{t=0}^{\infty} B^t [\ln(c_t) + \eta \ln(1 - l_t)] \quad (1)$$

$B^t$  refers to the discount factor and  $E_0$  is a mathematical expectation of future variables.  $\eta$  captures the utility from leisure (or the disutility from labor). The household maximizes  $U_0$  with respect to the following budget constraints:

$$c_t + x_t \leq w_t + r_t k_t \quad (2)$$

$$k_{t+1} = x_t + (1 - \delta_t) k_t \quad (3)$$

Here  $w_t$  is the income earned from supplying labor and  $r_t$  is the rent earned on capital. The left hand-side capture the expenditures in the form of consumption and investment ( $x_t$ ). The second constraint describes the capital accumulation process for the household.

The first order conditions of this problem yields the following conditions:

$$w_t(1 - l_t) = \eta c_t \quad (4)$$

$$c_{t+1} = B^t c_t (1 + r_t - \delta) \quad (5)$$

### 3.2. Energy sector

The energy sector produces  $E_t$  or energy output which is a function of  $K_t^e$  (the investment in capital in this sector). The production function is given as follows:

$$E_t = (K_t^e)^\gamma \quad (6)$$

$\gamma$  represents the elasticity of energy output with regard to capital investment in this sector. The firms in this sector have to maximize the following profit function:

$$\pi_t^e = p_t E_t - r_t K_t^e \quad (7)$$

The relevant first order conditions are:

$$r_t = \gamma \left( \frac{p_t E_t}{K_t^e} \right) \quad (8)$$

Here  $p_t$  refers to the price of energy and  $r_t$  is the return on capital investment. We let  $p_t$  be an exogenous AR(1) process in our model.

### 3.3. Government

The government tries to correct the externality problem caused by carbon emissions in the market. The government wants to impose a tax,  $\tau_t$  to internalize the negative externality generated by increased GHG concentration in the economy. Specifically, the government faces the following constraint:

$$G = \tau_t G H G_t \quad (9)$$

The rate of taxation,  $\tau_t$  is an exogenous variable that undergoes an AR(1) process given by:

$$\tau_t = \mu \tau_{t-1} + \epsilon_t \quad (10)$$

where  $\epsilon \sim N(0, \sigma_\tau^2)$

### 3.4. Final goods sector

Finally, we specify the final goods sector. This sector represents firms who use factor inputs from the intermediary energy sector, the labor market and the capital market. Essentially the firms choose  $\{l_t, k_t, E_t\}$  to maximize their profit function. The following production function is adopted from Niu et al. (2018):

$$Y = A_t k_t^\alpha l_t^\omega E_t^{1-\alpha-\omega} \quad (11)$$

Note that the production function is concave in energy inputs. The technology parameter  $A_t$  follows an AR(1) process as follows:

$$A_t = dA_{t-1} + \mu_t \quad (12)$$

The profit function of the final goods sector is given by:

$$\pi_t = A_t k_t^\alpha l_t^\omega E_t^{1-\alpha-\omega} - w_t l_t - r_t k_t - p_t E_t - \tau GHG_t \quad (13)$$

The  $\tau$  refers to tax paid by the firm for carbon emissions. This tax is collected by the government sector who then redistributes it in a lump-sum fashion to the household. The first order conditions yield the following equilibrium conditions for the final goods sector:

$$r_t = \alpha \left( \frac{Y}{k_t} \right) \quad (14)$$

$$w_t = \omega \left( \frac{Y}{l_t} \right) \quad (15)$$

$$p_t = (1 - \alpha - \omega) \left( \frac{Y}{E_t} \right) - \tau GHG_t \quad (16)$$

The final goods sector is also responsible for the emission of GHG into the atmosphere. Following Fan et al. (2016), we adopt the following function to track the dynamics of carbon emission from this sector:

$$GHG_t = F_t E_t \quad (17)$$

Where the term,  $F_t$  refers to the share of fossil fuel based energy from the total share of energy used within the economy. We want this term to be inversely related to the tax rate,  $\tau_t$  set by the government. So a higher tax rate should lead to a lower share of fossil fuel based energy in the total energy profile of the economy. The evolution of this term is given by the following expression:

$$F_t = \tau_t^\psi \quad (18)$$

Where  $\psi < 0$  to capture the inverse relationship between a carbon tax and the use of fossil fuel based energy.

We linearize the model around the equilibrium conditions. Using a combination of econometric estimations and literature review we pick parameter values that closely represent the Canadian data. We then analyze a tax shock and a pricing shock policy to analyze their effect on GHG emissions in the economy. The following section elaborates on the calibration process.



#### 4. Calibration of the Model

Table 1: Calibrating Parameters

Parameter	Value	Parameter Description	Source
$\beta$	0.999	Household discount factor.	World Bank Data
$\eta$	0.31	Share of leisure in utility function.	Dib (2003)
$\delta$	0.22	Depreciation rate of capital.	Statistics Canada
$\alpha$	0.493	Capital's share of output.	Mendoza (1991) <sup>5</sup>
$\omega$	0.349	Labor's share of output.	Dib (2003)
$\gamma$	0.349	Capital's share of energy output.	Statistics Canada
$\psi$	-1	Elasticity of tax shock on energy.	Li et al. (2014)
$\mu$	0.5	Sustainability coefficient of tax shock.	-
$\nu$	0.5	Sustainability coefficient of price shock.	-
d	0.5	Sustainability coefficient of productivity shock.	-

The household's discount factor is based off an annual interest rate of 4.8% in Canada as given by World Bank Data. The parameter  $\gamma$  has been calculated based on the share of capital investment on machinery and equipment in the energy sector over its total GDP contribution. Data on the following items were collected from the Statistics Canada web page. All parameters are computed with 2018 as the base year. Since we are estimating the effect of a tax shock policy beginning from 2018, we have chosen this as our base year. The parameter  $\psi$  is of great interest since it determines the responsiveness of non-renewable energy consumption from the tax shock. Due to a time constraint we refrain from econometric estimation of this parameter and refer to the literature instead. This is true for other parameters as well. One of the major limitations of this simulation is the inability to parameterize the shock sustainability parameters. We pick the value of 0.5 to model the nature of periodical increments in carbon tax proposed under the Pollution Pricing Act, 2018. We pick the same value for a price shock to maintain comparability of the two policies. A more empirical approach to calibrating the parameters might yield more robust inferences. However, as we mention there was a severe time constraint.

In the next section, we analyse the results of a tax policy shock and a price shock through the economy. We compare these two policies and look at GHG emission trends projected by the Canadian Government. We look at

the GHG emission reductions expected under the Paris Climate Agreement as the reference case.

## 5. Numerical Analysis & Results

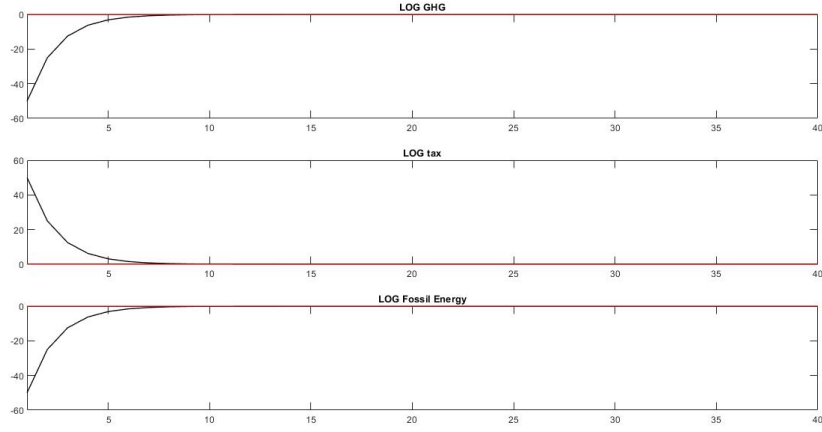


Figure 1: Impulse Responses of a tax shock

All variables are in the deviations from steady state form. The Impulse Response Functions show that a 50 basis point tax shock leads to a 50-basis point reduction in GHG emissions. This is not surprising since the share of non-renewable energy resources depends inversely on the tax shock. A tax shock thus leads to a lower  $F_t$  which implies a lower  $GHG_t$ . We see a 1-to-1 relationship between the two variables because of the parameter  $\psi = 1$  initialisation. Li et al. (2014) finds empirical evidence in support of this parameter value.

After the initial shock, the share of fossil-fuel based energy begins to return to the steady state. The shock only lasts upto 7 periods before dissipating. As the simulations show, the incidence of a carbon-tax policy falls entirely on the consumer of Energy, the final goods sector. This would imply a production shock as well since Energy is an input in the production function of the final goods sector. However, it is interesting to note that the shock does not have an affect on the output of the final goods sector. One reasonable explanation is that as fossil fuel based energy consumption

falls, firms substitute it with renewable energy sources. It is useful in this context to remember that  $E_t(k_t^e)$ . Energy output is independent of  $F_t$  (the share of fossil-fuel based energy).

The Canadian government projects a 2030 target of 673 megatonne of  $CO_2$  equivalent. In 2018, Canada's GHG emission in absolute level terms was 729 Mt  $CO_2$  eq. with a projection of 716 Mt  $CO_2$  eq for 2019. A persistent tax shock of this nature should reduce the GHG emission in absolute level terms significantly. However, the mechanism in which tax affects the GHG emission levels is suspect. It basically operates indirectly through  $F^6$ . This is where the calibration of the model parameters play an important role. Despite evidence in the literature it seems unlikely that  $\psi = 1$  is the correct parameterization. Given more time it would be interesting to attempt an econometric estimation of this parameter. Another limitation of this model lies in the way GHG is defined. A more representative modelling of the emission generating function could present more informative dynamics.

Due to the unreliability of the parameter it would be spurious to make any quantitative statements about the tax shock policy tool. It is clear however, that the tax shock only operates through one channel in this model. The tax shock does not have effects on the final goods sector.

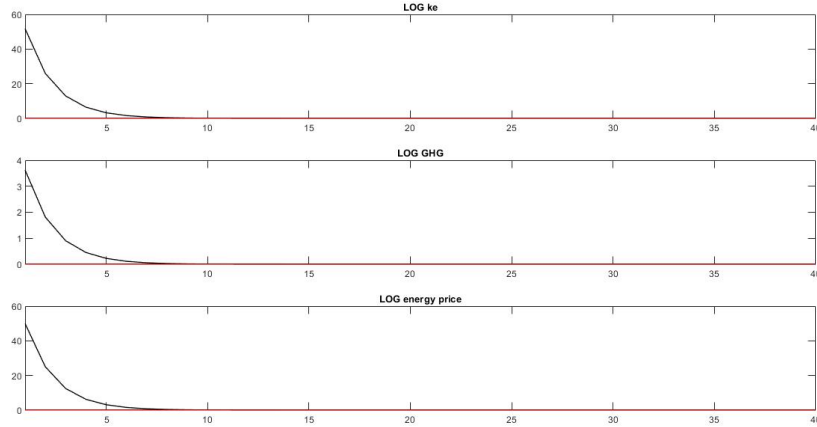


Figure 2: Impulse Responses of a price shock

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<sup>6</sup>Remember that  $F_t = \frac{1}{\tau_t^\psi}$

We next analyse the effect of a positive shock on energy prices. Energy prices in our model were an exogenous shock following an AR(1) process. We set the same value (0.5) for the sustainability parameter of both shocks for comparability. Again, it is important to point out the empirical limitations of the paper. The use of a better calibration approach could have allowed for empirical inferences. The dynamics generated by the price shock in the economy are still interesting. As figure.2 shows, we see a positive covariance between a price shock and GHG emissions. This is quite intuitive. A positive shock on energy prices would increase profits,  $\pi_t^e$ , for firms in the energy sector. This would lead to higher investment in capital,  $k_t^e$ , in the energy sector. This is supported by the impulse responses. A higher energy output in the economy would also lead to higher GHG emission levels. It is quite clear in this case that a price shock is not the appropriate measure to emission abatement.

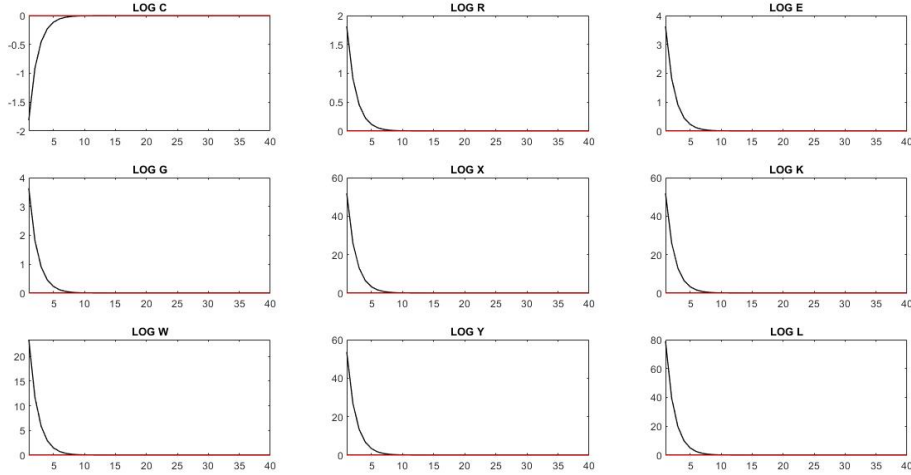


Figure 3: Impulse Responses of a price shock

Unlike the tax shock, the price shock has policy implications for other variables in the economy. The price shock policy tool surprisingly leads to an increase in output. One would expect an increase in energy prices to depress output. However, this is not the case in the impulse responses. It is not clear why that is. One possible explanation could be that output elasticity of energy is not very high. Capital is also pushed above the steady

state level because of the price shock. Similarly, labor supply goes above the steady state levels as well. This makes some sense. Since the cost of energy is exogenously raised, the other factor inputs (labor and capital) take up a higher share of the total output. This would consequently mean that price of these factors would go up as well. This pattern is consistent with the impulse responses. We see a one period positive deviation in both wage and interest rates. Interest rates are driven upwards by two different types of capital here  $(k_t^e, k_t)$ .

Consumption on the other hand falls initially and gradually returns to the steady state as the shock subsides. This is somewhat analogous to the effect of a oil price shock on consumption. Increasing energy prices can lead to uncertainty about the future<sup>7</sup>. Consequently, households will be incentivized to cut back on current consumption.

## 6. Conclusion

The paper concludes with qualitative remarks about the tax policy shock. It is apparent that a tax shock is the appropriate policy tool for the emission abatement objectives of the government. In this regard a carbon tax policy is the appropriate policy measure. However, the magnitude of the reduction in emission levels is not clear from the analysis of the impulse responses. This is mainly because the best approaches were not adopted in the calibration of the model. Despite the spuriousness in the calibration of the model, the qualitative results should not be too different. A tax shock would lead to lower fossil fuel based energy consumption. Furthermore, the effects of the tax shock are limited to the emission levels and energy consumption only. It does not impact consumption unlike a price shock policy.

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<sup>7</sup>Changes in oil price can cause added noise in the exchange rate leading to greater uncertainty among households

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