COD':

Calculated oxygen demand, normally is equal to the *chemical oxygen demand (COD).*The ratio of oxygen required for full oxidation of the cellular carbon per unit weight of cells.

True yield (Y): page 149

The fraction $f_s°$ can be converted into mass units, such as g cell produced/g COD' consumed. When expressed in mass units, it is termed the true yield and given the symbol Y. The conversion from fs° to Y is

Y = $f_s°$ *(Mc* g cells/mol cells)/ [(nee- eq/mol cells) (8 g COD/e- eq donor)]

Mc =the empirical formula weight of cells

ne= the number of electron equivalents in an empirical mole of cells

COD = the donor mass is expressed as COD.

X_a= active organism

b = the decay rate

Y_n= net yield

The net yield is less than *Y,* because some of the electrons originally present in the substrate must be consumed for energy of maintenance. When considering net yield, the portion of electrons used for synthesis is *fs* rather than *fs°,* and the portion for energy generation is *fe* rather than fe^0 . Still, the sum of *fs* and *fe* equals 1, and $fs < fs^0$, while $fe > fe^0$.

Electron donor = the "food" substrate for organisms.

1) Organic matter.
2) reduced inorganic compounds, such as ammonia and sulfide

Electron acceptor

1) under aerobic conditions = molecular oxygen (02)
2) Under anaerobic conditions = nitrate, sulfate, and carbon dioxide.

*** In some cases, organic matter is used as the electron acceptor, as well as the electron donor, and the reaction is then termed fermentation***

Order of preference for electron acceptors =

Oxygen>nitrate> sulfate>carbon dioxide (methanogenesis)>fermentation

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Bacterial growth → Energy production ➔ electron donor
Bacterial growth → Cellular synthesis ➔

- R_c = half-reaction for synthesis (Table 2-4)
- acceptor half-reactions
- R_a= acceptor half-reactions (Table 2-4) for the five most common electron acceptors: 0_2, $N0_3$, Fe_3^+, SO_4^{3-}, and CO2
- R_e = The energy reaction,
- R_d= donor half-reaction
- R_s = synthesis reaction

$$R_e = R_a - R_d$$
$$R_s = R_c - R_d$$
$$R = f_e R_a + f_s R_c - R_d$$

** The equation represents the net consumption of reactants and production of products when the microorganisms consume one electron equivalent of electron donor.

***Ammonium is the preferred nitrogen source

One equivalent of oxygen is 8 g of 0_2, one equivalent of any electron donor is equivalent to an OD of 8 g as O_2 (8 g OD/e^- eq)

Microbial yield from substrate utilization

1- energy reaction ➔ creates high-energy carriers (ATP)
2- energy carriers are "spent" to drive cell synthesis or cell maintenance

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1- energy costs of cell synthesis and
2- the energy lost in transfers

ΔG_s= the energy required to synthesize one equivalent of cells from a given carbon source when the nitrogen source is ammonium.
ΔG_p= The energy required to convert the carbon source to pyruvate
ΔG^0_c= free energy of the carbon source as the electron donor (table 2.3)
ΔG_{pc}= The energy required for Pyruvate carbon to converted to cellular carbon = 3.33 kJ per gram cells * electron equivalent of cell (PG173)

$$\Delta G_p = 35.09 - \Delta G_c^{0'}$$

$$\Delta G_s = \frac{\Delta G_p}{\varepsilon^n} + \frac{\Delta G_{pc}}{\varepsilon}$$

***$\Delta G_p \leq 0$ ➔ n= -1 & $\Delta G_p > 0$ ➔ n= +1

Heterotrophic bacteria= carbon source is the electron donor
Autotrophic reactions = carbon source is inorganic carbon (more energy)

ΔG_r= the free energy released per equivalent of donor oxidized for energy generation.
A =the equivalents of donor used for energy production per equivalent of cells formed

$$A = -\frac{\frac{\Delta G_p}{\varepsilon^n} + \frac{\Delta G_{pc}}{\varepsilon}}{\varepsilon \Delta G_r}$$

$$f_s^0 = \frac{1}{1+A} \quad \text{and} \quad f_e^0 = 1 - f_s^0 = \frac{A}{1+A}$$

Microbial Kinetic

- Rate-limiting **substrate** is the **electron donor**

Bacterial growth kinetic (*Monad equation*):

$$\mu_{\text{syn}} = \left(\frac{1}{X_a}\frac{dX_a}{dt}\right)_{\text{syn}} = \hat{\mu}\frac{S}{K+S}$$

in which

μ_{syn} = specific growth rate due to synthesis (T^{-1})
X_a = concentration of active biomass ($M_x L^{-3}$)
t = time (T)
S = concentration of the rate-limiting substrate ($M_s L^{-3}$)
$\hat{\mu}$ = maximum specific growth rate (T^{-1})
K = concentration giving one-half the maximum rate ($M_s L^{-3}$)

$$\mu = \hat{\mu}/2 \text{ when } K = S.$$

b = endogenous-decay coefficient (T^{-1})
μ_{dec} = specific growth rate due to decay (T^{-1}).
f_d = fraction of the active biomass that is biodegradable

net specific growth rate of active biomass (μ)

$$\mu = \frac{1}{X_a}\frac{dX_a}{dt} = \mu_{\text{syn}} + \mu_{\text{dec}} = \hat{\mu}\frac{S}{K+S} - b$$

r_{ut} = rate of substrate utilization ($M_s L^{-3} T^{-1}$)
$\hat{q}$ = maximum specific rate of substrate utilization ($M_s M_x^{-1} T^{-1}$)

$$r_{\text{ut}} = -\frac{\hat{q}S}{K+S}X_a$$

$$\hat{\mu} = \hat{q}Y$$

r_{net} = the net rate of active-biomass growth ($M_x L^{-3} T^{-1}$).

$$\mu = r_{\text{net}}/X_a = Y\frac{\hat{q}S}{K+S} - b$$

m = maintenance-utilization rate of substrate ($M_s M_x^{-1} T^{-1}$).

$$\mu = Y\left(\frac{\hat{q}S}{K+S} - m\right)$$

$$\left[\theta_x^{\min}\right]_{\lim} = \frac{1}{Y\hat{q} - b}$$

$$S_{\min} = K\frac{b}{Y\hat{q} - b}$$

$$Y_n = Y\frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$$

$$f_s = f_s^0\frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$$

Some biological processes receive significant inputs of biomass active in degradation of the substrate

θ_x is now defined with the denominator being the gross active biomass output rate (QX_a) minus the input rate (QX_a^0). Thus, the denominator remains the net production rate of new active biomass.(PG204)

$$S = K\frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)}$$

$$X_a = \frac{\theta_x}{\theta}\left[Y(S^0 - S)\frac{1}{1 + b\theta_x}\right] \qquad \theta_x = \mu^{-1} = \frac{X_a V}{QX_a - QX_a^0}$$

When θ_x is: (PG 190)

$$\theta_x = \frac{\text{active biomass in the system}}{\text{production rate of active biomass}} = \mu^{-1} \qquad \theta_x = \frac{VX_a}{QX_a} = \theta$$

$$S = K\frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)}$$

$$X_a = Y\left(\frac{S^0 - S}{1 + b\theta_x}\right)$$

$$\text{UAP} = -\frac{(\hat{q}_{\text{UAP}}X_a\theta + K_{\text{UAP}} + k_1 r_{\text{ut}}\theta)}{2} + \frac{\sqrt{(\hat{q}_{\text{UAP}}X_a\theta + K_{\text{UAP}} + k_1 r_{\text{ut}}\theta)^2 - 4K_{\text{UAP}}k_1 r_{\text{ut}}\theta}}{2}$$

$$\text{BAP} = \frac{-(K_{\text{BAP}} + (\hat{q}_{\text{BAP}} - k_2)X_a\theta)}{2} + \frac{\sqrt{(K_{\text{BAP}} + (\hat{q}_{\text{BAP}} - k_2)X_a\theta)^2 + 4K_{\text{BAP}}k_2 X_a\theta}}{2}$$

Mass balance on CSTR with sludge recycle: (PG301)

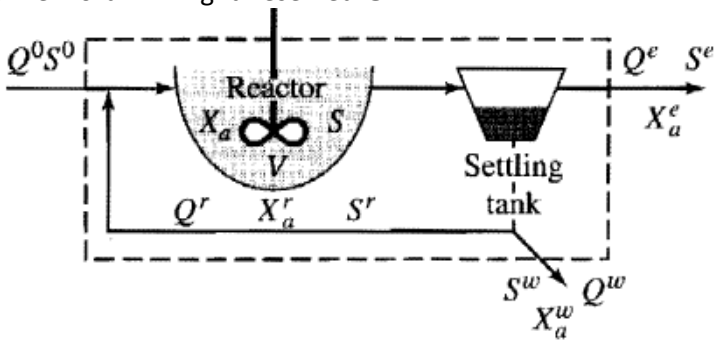

Figure 5.8 CSTR with sludge recycle

X^r_a = The concentration of microorganisms in this recycle line
X^e_a = The concentration of microorganisms in the effluent
X^w_a = waste sludge to be removed

$$\theta_x = \frac{\text{active biomass in the system}}{\text{production rate of active biomass}}$$

$$\theta_x = \frac{X_a V}{Q^e X_a^e + Q^w X_a^w} \qquad \frac{1}{\theta_x} = Y\frac{\hat{q}S}{K+S} - b$$

$$X_a = \frac{\theta_x}{\theta}\frac{Y(S^0 - S)}{1 + b\theta_x} \qquad S = K\frac{1 + b\theta_x}{\theta_x(Y\hat{q} - b) - 1}$$

Table 5.2 Summary of applicable equations for a CSTR with settling and recycle of microorganisms (operating at steady state, treating a soluble substrate, and with no input of active biomass)

Hydraulic Detention Time (θ):

$$\theta = \frac{V}{Q^0} \qquad \textbf{[3.20]}$$

Solids Retention Time, SRT (θ_x):

$$\theta_x = \frac{X_a V}{X_a^e Q^e + X_a^w Q^w} \qquad \textbf{[5.35]}$$

SRT at which microorganism washout results ($\theta_x^{\min}$), and the limit thereto:

$$\theta_x^{\min} = \frac{K + S^0}{S^0(Y\hat{q} - b) - Kb} \qquad S \to S^0 \qquad \textbf{[3.26]}$$

$$\left[\theta_x^{\min}\right]_{\lim} = \frac{1}{Y\hat{q} - b} \qquad S \to \infty \qquad \textbf{[3.27]}$$

Reactor or Effluent Substrate Concentration ($S = S^e$):

$$S = K\frac{1 + b\theta_x}{\theta_x(Y\hat{q} - b) - 1} \qquad \textbf{[5.39]}$$

Reactor Minimum Substrate Concentration ($S_{\min}$):

$$S_{\min} = K\frac{b}{Y\hat{q} - b} \qquad \theta_x \to \infty \qquad \textbf{[3.28]}$$

Reactor Active Microorganism Concentration (X_a):

$$X_a = \theta_x\frac{X(-r_{ut})}{1 + b\theta_x} \qquad \textbf{[5.40]}$$

$$X_a = \frac{\theta_x}{\theta}\frac{Y(S^0 - S)}{1 + b\theta_x} \qquad \textbf{[5.43]}$$

Reactor Inert Microorganism Concentration (X_i):

$$X_i = \frac{\theta_x}{\theta}\left[X_i^0 + X_a(1 - f_d)b\theta\right] \qquad \textbf{[5.46]}$$

Reactor volatile suspended solids concentration (X_v):

$$X_v = X_i + X_a$$

$$X_v = \frac{\theta_x}{\theta}\left[X_i^0 + \frac{Y(S^0 - S)(1 + (1 - f_d)b\theta_x)}{1 + b\theta_x}\right] \qquad \textbf{[5.47]}$$

Active Biological Sludge Production Rate (r_{abp}):

$$r_{abp} = \frac{X_a V}{\theta_x} \qquad \textbf{[5.45]}$$

Total Biological Solids Production Rate (r_{tbp}):

$$r_{tbp} = \frac{X_v V}{\theta_x} \qquad \textbf{[5.48]}$$

θ_x^d = Design θ_x

$$\theta_x^d = \mathrm{SF}\left[\theta_x^{\min}\right]_{\lim}$$

Table 5.3 Implied safety factors for typical biological treatment design loadings

Loading	Implied SF
Conventional	10–80
High Rate	3–10
Low Rate	>80

Activated sludge:

The food-to-microorganism ratio **(F/M)** is:

$$\text{F/M} = \frac{Q^0 S^0}{VX} \qquad \textbf{[6.1]}$$

in which

F/M = food-to-microorganism ratio, kg BOD or COD applied per day per kg of total suspended solids in the aeration tank
Q^0 = influent wastewater stream flow rate (m^3/d)
S^0 = influent wastewater concentration (BOD or COD in mg/l)
V = aeration-tank volume (m^3)
X = total suspended solids concentration in aeration tank (mg/l)

If volatile suspended solids, rather than total suspended solids are used, then Equation 6.1 is slightly modified to

$$\text{F/M}_v = \frac{Q^0 S^0}{VX_v} \qquad \textbf{[6.2]}$$

in which

F/M_v = food-to-microorganism ratio on volatile solids basis, kg BOD or COD per day per kg of volatile suspended solids in aeration tank
X_v = volatile suspended solids concentration in aeration tank (mg/l)

Table 6.2 Typical process loading factors and θ_x^d values for various activated sludge process modifications

	Normal Ranges for Various Factors					
Process Modification	**Volumetric kg BOD_5/m^3-d**	**MLSS mg/l**	**F/M_v kg BOD_5/ kg X_v-d**	**Typical BOD_5 Removal Efficiency**	**Typical θ_x^d d**	**Safety Factor***
Extended Aeration	0.3	3,000–5,000	0.05–0.2	85–95[B]	>14	>70
Conventional						
Conventional	0.6	1,000–3,000	0.2–0.5	95	4–14	20–70
Tapered Aeration	0.6	1,000–3,000	0.2–0.5	95	4–14	20–70
Step Aeration	0.8	1,000–3,000	0.2–0.5	95	4–14	20–70
Contact Stabilization	1.0	A	0.2–0.5	90	4–15	20–75
Modified Aeration	1.5–6	300–600	0.5–3.5	60–85[B]	0.8–4	4–20
High-Rate Aeration	1.5–3	5,000–8,000	0.2–0.5	95	4–14	20–70

*Assumed value of growth coefficients: Y = 0.65 g cells/g BOD_5, b = 0.15 d^{-1}.
A: Contact tank typically has 1,000–3,000 mg/l; stabilization tank typically has 5,000–10,000 mg/l.
B: Higher efficiency is based upon soluble effluent BOD_5.
SOURCE: Lawrence and McCarty (1970).

X= Mixed-liquor suspended solids concentration, or *X,* to select for the aeration tank
SVI = is defined as the volume in milliliters occupied by 1 g of the suspended solids after settling.

Nitrification

Nitrification conditions:

1- considered impossible for low-water temperatures(if t low => θ high Pg. 252)
2- highly efficient, as long as the SRT is maintained well above θ^{min}_{x}
3- Sufficient dissolved oxygen is present.

Other important features of nitrification.

1- It creates a major oxygen demand.
2- It produces almost two strong-acid equivalents per mole of NH4 removed.
3- Slow growth rate

*** The two disadvantages are overcome by ensuring that the nitrifiers have a long SRT, typically greater than 15 d, although larger values may be needed in the presence of toxic materials, a low D.O. concentration, or low temperature.

Anammox Process (Anaerobic Ammonium Oxidation)

- to anaerobically oxidize NH_4^+-N to N_2, not to $N0_2^-$

Anammox Process Conditions:

1- exceptional biomass retention (to give a very long SRT)
2- stable operation
3- presence of nitrite
4- lack of oxygen
5- lack of donors that could cause the reduction of nitrite via denitrification

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Denitrification

- Denitrification is the dissimilatory reduction of $N0_3^-$ or $N0_2^-$ to (mainly) N_2 gas

$NO_3^- + 2\,e^- + 2\,H^+ = NO_2^- + H_2O$	Nitrate Reductase
$NO_2^- + e^- + 2\,H^+ = NO + H_2O$	Nitrite Reductase
$2\,NO + 2\,e^- + 2\,H^+ = N_2O + H_2O$	Nitric Oxide Reductase
$N_2O + 2\,e^- + 2\,H^+ = N_2(g) + H_2O$	Nitrous Oxide Reductase

Accumulation of intermediates:

1- Very low concentrations of the electron donor
2- too high concentrations of D.O. concentration
3- pH values outside the optimal range of7 to 8 can lead to accumulation of intermediates

** High NO_3^--N levels occurred in waters that had little or no BOD, Thus, research addressed exogenous electron donors and carbon sources. Simple compounds that can be purchased in bulk quantity were evaluated: methanol, acetate, glucose, ethanol, and a few others.
Because methanol (CH30H) was relatively inexpensive, it gained widespread use, and a very large database on methanol has been developed. Being a one-carbon compound (Pg. 551)

** The heterotrophic denitrifiers have kinetic characteristics similar to aerobic heterotrophs.

Heterotrophic Denitrifiers VS Autotrophic Nitrifiers:

1- nitrifiers much slower growers, and require substantially longer solids retention times
2- maximum nitrification rates require a high D.O. concentration, while high D.O. concentration slows or stops denitrification

Denitrification by activated sludge:

1- The reactor is design to minimize aeration.
2- supplementation with electron donor is required

** Rule of thumb is 4 g *BODL/g* NO_3^--N removed through denitrification. Extra electron donor must be supplied if 0_2 enters the system.

Phosphorous removal

Enhanced biological phosphorus removal:

1- *Anaerobic bioreactor:* Electron acceptors-particularly 02 and N03-must be excluded to the maximum degree possible so that BOD oxidation is insignificant in this reactor. (bacteria are able to take up simple organic molecules and sequester PHB, hydrolysis of poly P, release of phosphorous)
2- Ample electron acceptors are available through aeration, which directly supplies O2 and al1ows generation of NO3-, if nitrification occurs.(ATP is generated)