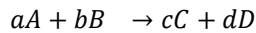


$$\text{Steady States} \Rightarrow \frac{dC}{dt} = 0$$



$$\Delta C_A = C_A - C_{A0} = \text{final} - \text{initial}$$

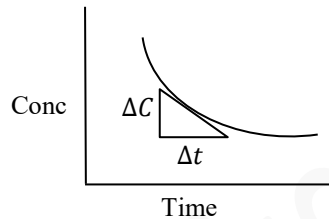
$$\frac{\Delta C_A}{a} = \frac{\Delta C_B}{b} = -\frac{\Delta C_C}{c} = \frac{\Delta C_D}{d}$$

$$\Rightarrow C_B = C_{B0} + (C_A - C_{A0}) \frac{b}{a}$$

$$\text{reaction rate}_A = k \cdot [A]^a \cdot [B]^b \leftarrow \text{if elementary}$$

$$\text{rate} = \frac{\Delta C}{\Delta t} = \frac{dC}{dt}$$

$$\frac{r_A}{a} = \frac{r_B}{b} = \frac{-r_C}{c} = \frac{-r_D}{d}$$



$$\text{one body reaction: } A \rightarrow C \Rightarrow A = A_0 e^{-kt}$$

$$\text{binary reaction (2body): } A + B \rightarrow C \Rightarrow r_A = -kC_A C_B$$

$$\text{Ternary reaction (3body): } 2A + B \rightarrow C \Rightarrow r_A = -kC_A^2 \cdot C_B$$

Mass Balance:

Rate of j in- Rate of j out + Net Generation of j = Rate of accumulation of j

$$QC_{j0} - QC_j + r_j^* V = \frac{dc_j^*}{dt}$$

$$\text{a) Batch: homogenous} \rightarrow r_j = r_j^* \text{ and } C_j = C_j^* \text{ and } Q = 0$$

$$QC_{j0} - QC_j + r_j^* V = \frac{dc_j^*}{dt} \Rightarrow r_j = \frac{dc_j^*}{dt} (mb + rating)$$

$$\begin{array}{l} \downarrow \\ \text{No mixing} \Rightarrow Q=0 \end{array} \quad \begin{array}{l} \downarrow \\ r_j = -kC_j (\text{from chem}) \end{array}$$

Where r_j^* is the volume average rate of formation of some species j. And C_j^* is the average concentration of j.

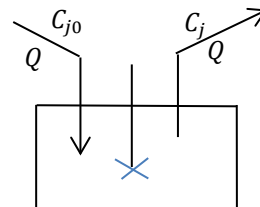
b) **Plug flow:** (=Moving batch reactor)

$$\text{SS Condition} \Rightarrow \frac{dC}{dt} = 0 \quad r_j = \frac{dC}{dt} \quad \tau = \frac{V}{Q} = \frac{x}{u}$$

c) **CSTR**

Well mixed

$$\text{SS} = \frac{dC}{dt} = 0 \Rightarrow C_{j0} - C_j = -r\tau \quad \tau = \frac{V}{Q}$$



5) Reaction kinetics

Batch

$$\text{a) } A \rightarrow P \Rightarrow C_A = C_{A0} \times e^{-kt}$$

$$\text{b) } A + A \rightarrow P \Rightarrow \frac{1}{C_A} - \frac{1}{C_{A0}} = k(t - t_0)$$

$$c) A + B \rightarrow P \Rightarrow C_{A0} \neq C_{B0} \Rightarrow \frac{1}{C_{B0} - C_{A0}} \ln \frac{C_A C_{B0}}{C_A (C_{B0} - C_{A0} + C_A)} = k(t - t_0)$$

$$C_{A0} = C_{B0} \Rightarrow \frac{1}{C_A} - \frac{1}{C_{A0}} = k(t - t_0)$$

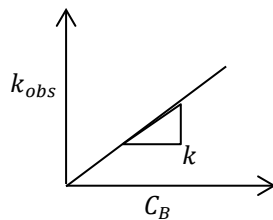
- ✓ Lakes are CSTR
- ✓ Rivers are Plug flow

Batch

1) method of excess

if $C_B \gg C_A \Rightarrow -r_A = k C_A C_B = k_{obs} C_A \quad A + B \rightarrow P \quad \text{Focus on A}$

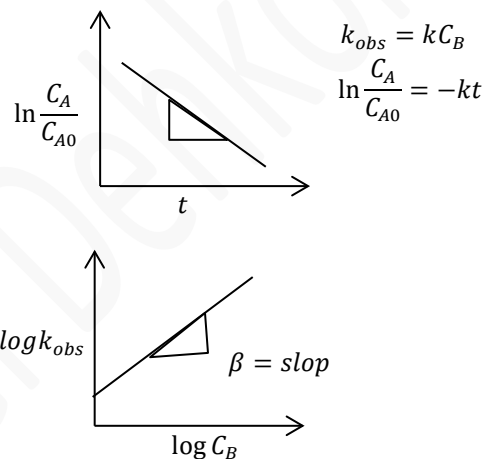
→ if first order: $k_{obs} = k C_B$



→ if not first order: $k_{obs} = k C_B^\beta$

$$\log k_{obs} = \log k + \beta \log C_B$$

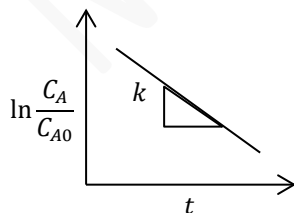
$y = \text{int} + \text{slope}$



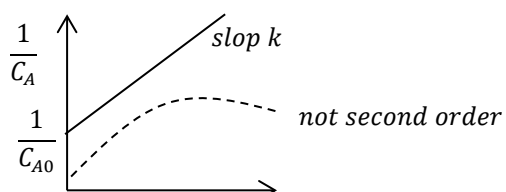
2) integral method

- ✓ guess → plot C vs t
- ✓ coupled to method of excess

– guess: first order: plot $\ln \frac{C_A}{C_{A0}}$ Vs t : line, slope $k \quad \ln \frac{C_A}{C_{A0}} = -kt$

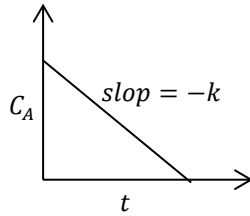


– guess: second order: $\frac{1}{C_A} - \frac{1}{C_{A0}} = kt$



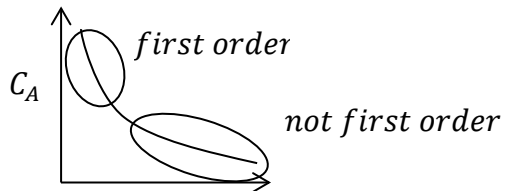
t

– guess: Zero order $C_A - C_0 = -kt$

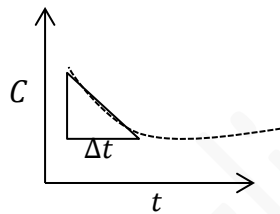


3) method of initial rate:

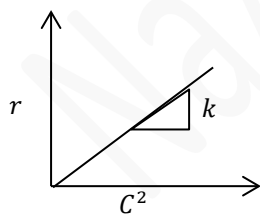
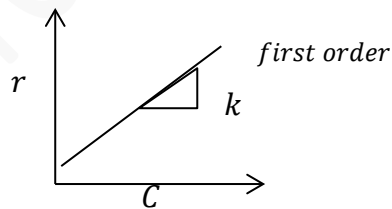
- ✓ for initial data
- ✓ true for surface reactions



4) Approximate rate as $\frac{\Delta C}{\Delta t}$



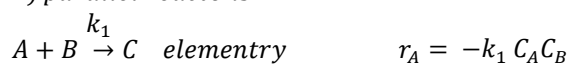
$r = kC$ line?



second order
 $r = kC^2$ line?

Reaction schemes:

1) parallel reactions:



$$r_{net} = -k_1 C_A C_B + -k_1 C_A C_D$$

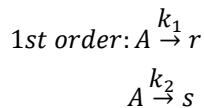
* $k_1 \gg k_2 \Rightarrow$ second reaction is negligible \Rightarrow rate = **first**

overall: $2A + B + D \rightarrow C + E$ $\left\{ \begin{array}{l} - \text{help with stoich? No} \\ \text{rate of reac A is degrading with 2 dif} \\ - \text{Implies wrong mechanisms} \\ \text{wrong rate expression} \end{array} \right.$

$$\Delta C_{A1} = \Delta C_B \quad \& \quad \Delta C_{A2} = \Delta C_D$$

$$\Delta C_A = \Delta C_{A1} + \Delta C_{A2} = \Delta C_B + \Delta C_D = -\Delta C_C - \Delta C_E$$

Basic parallel reaction:



$$\frac{dC_A}{dt} = -r_{A,net} = -k_1 C_A - k_2 C_A = -(k_1 + k_2) C_A \quad \text{first order}$$

$$\ln \frac{C_A}{C_{A0}} = (k_1 + k_2)t \Rightarrow C_A = C_{A0} \cdot e^{-(k_1 + k_2)t}$$

a) Batch

$$C_R - C_{R0} = \frac{k_1 C_{A0}}{k_1 + k_2} (1 - e^{-(k_1 + k_2)t}) \quad \text{Same for S} \quad C_S - C_{S0} = \frac{k_2 C_{A0}}{k_1 + k_2} (1 - e^{-(k_1 + k_2)t})$$

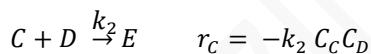
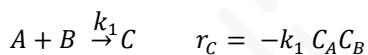
same for s.

b) CSTR

$$\tau = \frac{C_{A0} - C_A}{-r_A} = \frac{C_{A0} - C_A}{(k_1 + k_2) C_A}$$

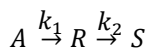
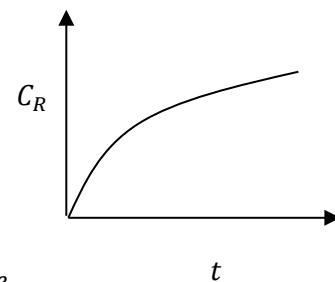
$$R = \frac{C_{R0} - C_R}{-k_1 C_A} \quad \text{Same for S with K2} \quad S = \frac{C_{S0} - C_S}{-k_1 C_A}$$

2) consecutive reactons: (focus on c)



$$r_{net} = -k_1 C_A C_B + -k_2 C_C C_D$$

$A + B + D \rightarrow E$ $\left\{ \begin{array}{l} - \text{Yes Etoichiometry "E" has to form this way} \\ - \text{No (for kinetics) because: C accumulate over time} \end{array} \right.$

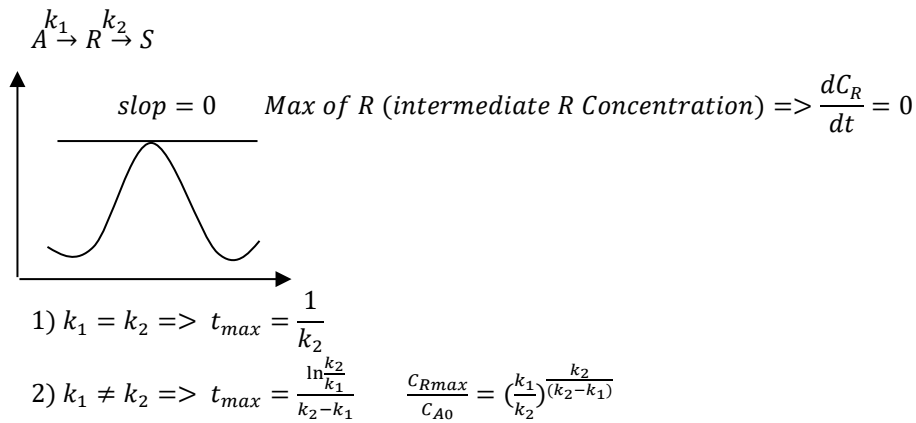


$$k_1 \neq k_2 \rightarrow \frac{C_R}{C_{A0}} = \left(\frac{k_1}{k_2 - k_1} \right) (e^{-k_1 t} - e^{-k_2 t})$$

$$k_1 \neq k_2 \rightarrow \frac{C_S}{C_{A0}} = 1 + \frac{k_1}{k_1 - k_2} e^{-k_2 t} - \frac{k_2}{k_2 - k_1} e^{-k_1 t}$$

$$k_1 = k_2 \rightarrow \frac{C_R}{C_{A0}} = k_1 e^{-k_2 t} t$$

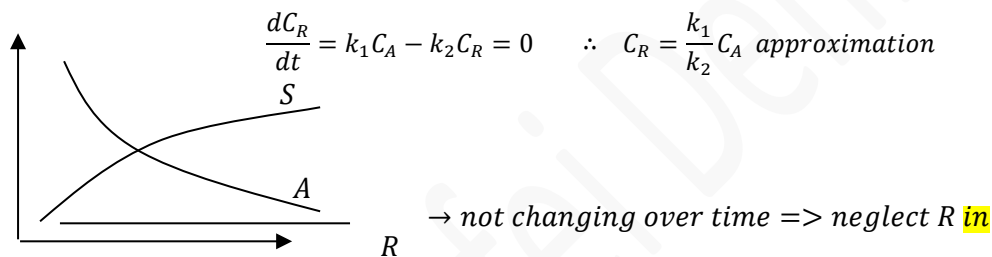
$$k_1 = k_2 \rightarrow \frac{C_S}{C_{A0}} = 1 - e^{-k.t} - k_1 e^{-k_2 t} t$$



if $k_2 \gg k_1$ $A \xrightarrow{\text{slow } k_1} R \xrightarrow{\text{faster } k_2} S \Rightarrow \text{look like } A \xrightarrow{\text{slowest } k_1} R$

always: $C_A = C_{A0}e^{-k_1t}$

no R accumulation $\Rightarrow \Delta C_A = \Delta C_S \Rightarrow C_S = C_{A0}(e^{-k_1t} - 1)$



–adjusting k to temperature:

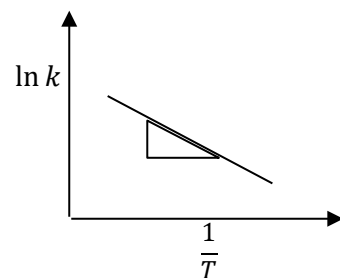
$$R = 8.314 \frac{J}{\text{mole} \cdot K}$$

$$KAe^{-\frac{E}{RT}} \rightarrow k \text{ vs } t \rightarrow \ln K = \ln A + \ln e^{-\frac{E}{RT}}$$

$$\frac{k_{T1}}{k_{T0}} = e^{\left(\frac{E}{R}\right)\left[\frac{1}{T_2} - \frac{1}{T_1}\right]}$$

E= Activation energy

A= Frequency Factor



–single PF vs. single CSTR:

PF have greater removal for same flow and same conversion(removal)

$$\tau_{PF} < \tau_{CSTR} \quad \therefore V_{PF} < V_{CSTR}$$

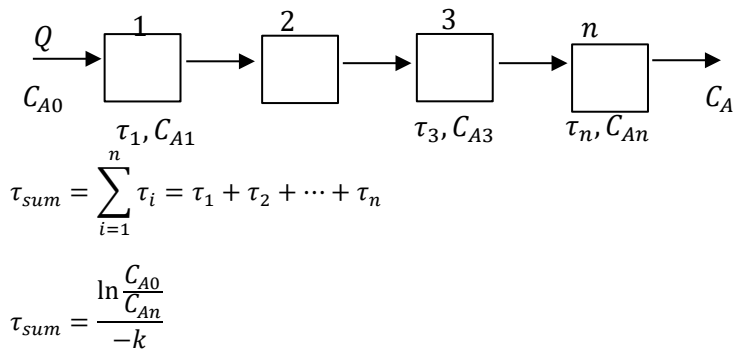
Rivers = PF

lakes = CSTR

PF in series

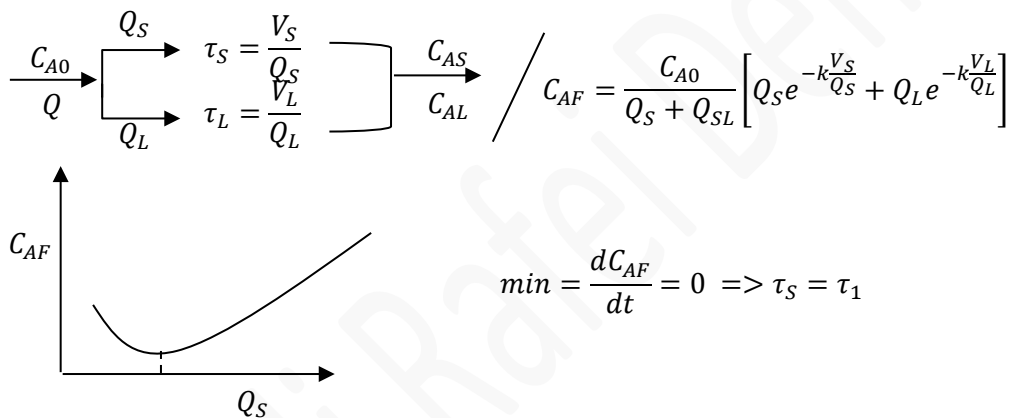
–order of reactors does not matter for dif size PF, dife V, diff τ

–order doesn't depend on the reaction order



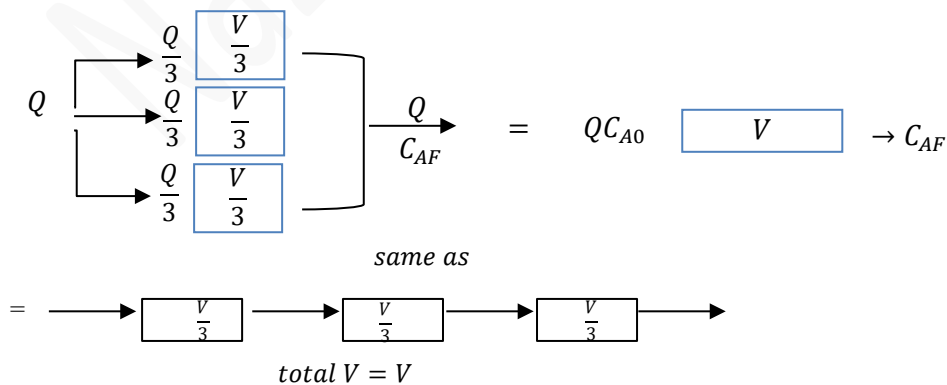
PF in parallel

$$C_{AS} = C_{A0} e^{-k\tau_S}$$

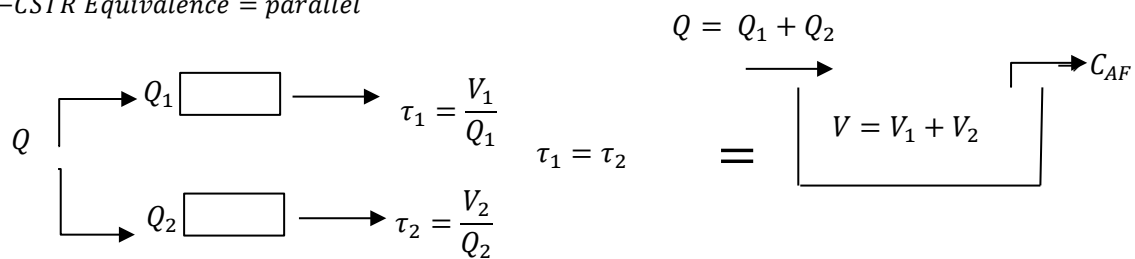


–series PF = parallel PF If they have same volume also same as one PF with volume V_S

if $\tau_1 = \tau_2 = \dots = \tau_{tPF}$



–CSTR Equivalence = parallel



–CSTR series

Laminar Flow: In fluid dynamics, laminar flow is characterized by fluid particles following smooth paths in layers, with each layer moving smoothly past the adjacent layers with little or no mixing. At low velocities, the fluid tends to flow without lateral mixing, and adjacent layers slide past one another like playing card

Terminal velocity is the maximum velocity attainable by an object as it falls through a fluid

Settling velocity = terminal velocity

Where,

d_p =particle diameter

v_s =settling velocity

P_l =density of liquid

P_p = density of particle

μ = viscosity

C_D = drag coeff,

Ideal mixing

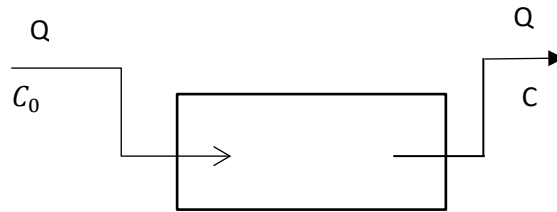
CSTR without reaction

$$C_0 = \frac{M}{V}$$

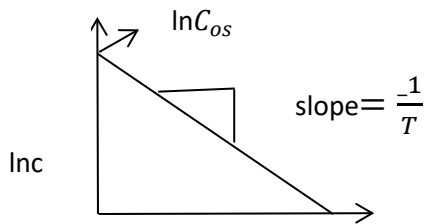
C_{os} = concentration of dye in tank at $t=0^+$

M_T = mass of dye added.

C_0 = dye concentration in influent



Mass balance on tank at $t=0^+$

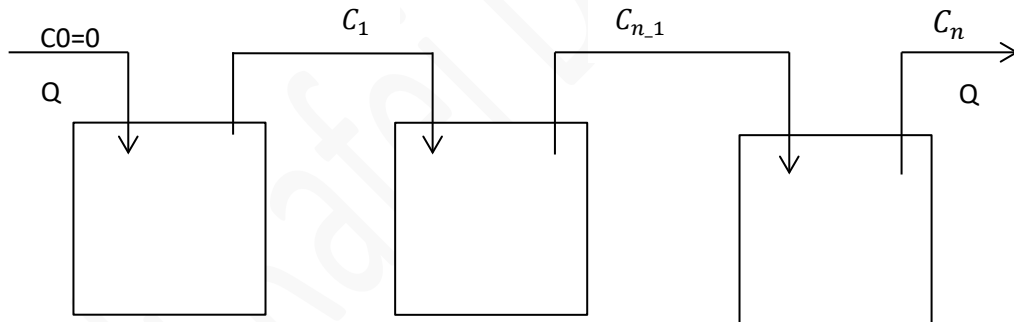


$$C = C_{os} e^{-\frac{t}{\tau}} \Rightarrow \ln C = \ln C_{os} - \frac{t}{\tau}$$

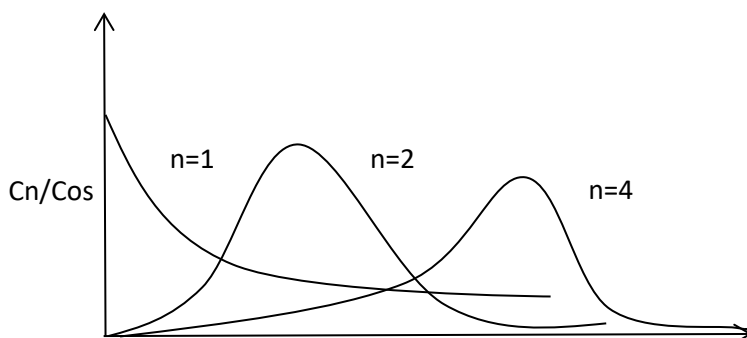
Rule; 3 residence time flushing to have 95%

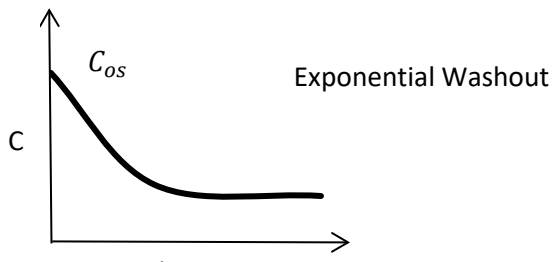
$$C = (1 - 0.95)C_{os} \quad t = 3T$$

Multiple well mixed tank with equal T



$$C_n = C_{os} \frac{t^{n-1}}{\tau^{n-1}} \left[\frac{1}{(n-1)!} \right] e^{-\frac{t}{\tau}}$$

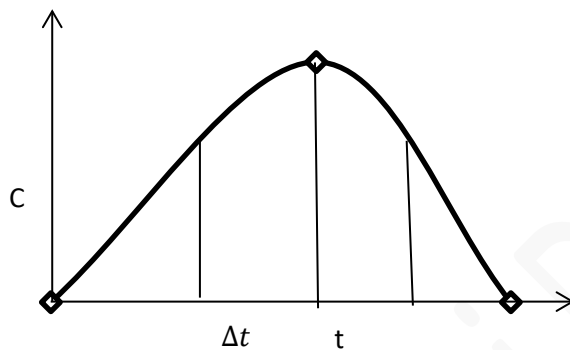




Ideal mixing plus kinetics

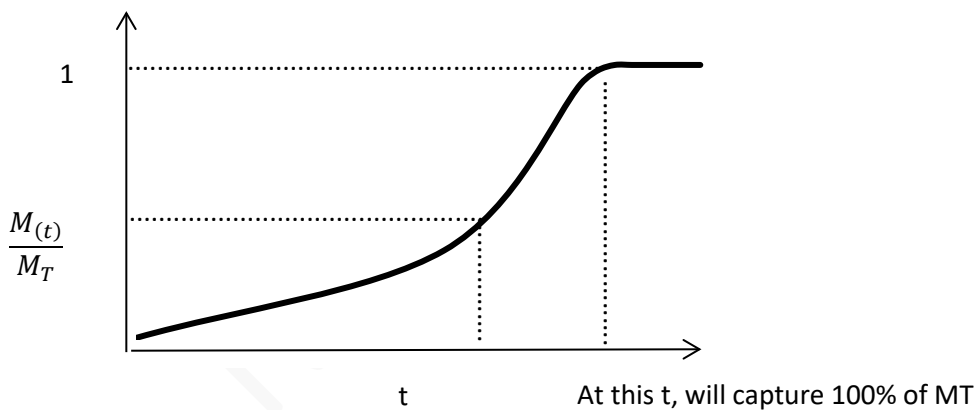
$$\ln \frac{C}{C_0} = -\left(\frac{1}{T} + k\right)t$$

$$T = \frac{V}{Q} \quad \text{or} \quad T = \frac{\int_0^\infty t c dt}{\int_0^\infty c dt} = \frac{\sum t_i c_i \Delta t_i}{\sum c_i \Delta t_i}$$



Mass of tracer recovered

$$M_t = \int_0^\infty c dt = Q \sum C_i \Delta t_i$$



Fraction of tracer recovered by $t_i = M_{(ti)} = \int_0^{t_i} c dt = Q \sum_0^{t_i} C_i \Delta t_i$

*We want 100% removal

Probability density:

$$P(x) = \frac{dp}{dx}, \quad dp \rightarrow \text{probability}, \quad p(x) \rightarrow \text{probability density}$$

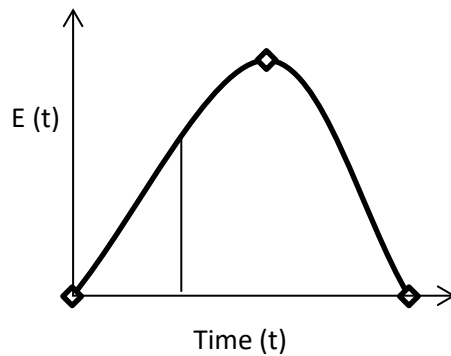
$$P(x) = \frac{dp}{dx} \Rightarrow \int_{x_1}^{x_2} p(x) dx, \quad \int_{-\infty}^{\infty} p(x) dx = 1 = 100\%$$

E=exit age of fluid packets

Fraction of fluid removed up to time t = Fraction or percent (or probability!) of fluid with exit age $t < t_x$ (or 0 to t)

$$\int_0^t E(t)dt = \frac{M(t)}{M_T} = \frac{\text{Mass tracer removed by } t}{\text{total mass tracer}} = \frac{\int_0^t c dt}{\int_0^\infty c dt}$$

$$\int_0^\infty E(t)dt = 1$$



$$\bar{T} = \frac{\sum t_i c_i \Delta t_i}{\sum c_i \Delta t_i}$$

$$\text{Exit age of a fluid packet at time } t = E(t_i) = \frac{c_i}{\sum c_i \Delta t_i}$$

$$\text{Fraction of fluid with exit age between } t_1 \text{ and } t_2 = \int_{t_1}^{t_2} E(t)dt = E(t)\Delta t$$

$$**\bar{T} = \text{average hydraulic resident time and it is when } p=0.5 = \sum_{t=0}^{\bar{T}} E(t_i)\Delta t_i$$

QS: what is T_{10} ? This is the youngest age of 10% of fluid

Answer: Area under $E(t)$ vs t diagram till 10% or in cumulative $E(t)$ vs t . point of 10% in probability axes and then you can find t on the other Axis.

$$C_A = C_{A1} E(t_1)\Delta t_1 + \dots = \sum_{n=1}^{\infty} C_{An} E(t_n)\Delta t_n$$

-collision rate

$$\text{Collision rate} = N_{ij} = \frac{\# \text{collision}}{\text{time} \cdot \text{volume}} = K_{ij} C_i C_j = B_{ij} C_i C_j$$

C_i = concentration of particle i

$K_{ij} = \beta_{ij} = f$ (particle size, density, fluid density, fluid mixing)

α = Collision efficiency \Rightarrow (usually we assume ~ 1)

$N_{ij} = \alpha N_{ij}$ N_{ij} = rate of collision

-what coagulants do, is make $\alpha = 1$. All collisions result in sticking.

$$\text{-net rate of k-type particles} = N_{kt} = N_{ij} = \alpha K_{ij} A_i A_j \frac{\#}{\text{time}}$$

-Fick's law:

Amount (moles, volume) of particles moving = V

Flux = A = Net rate of particles moving throw an area

$$\frac{V}{A} = \frac{(p_1 - p_2)}{T} \times D \rightarrow \text{Diffiusion constant}$$

$G = \text{Gradient}$ = Change in pressure (particles in over a distance a volume)

$$\text{Flux} = \frac{\# \text{or mass}}{\text{area} \cdot \text{time}}$$

$$\text{Flux} = J_{i \rightarrow j} = D_i \frac{dni}{dr}$$

r = radius of particles

D_i = diffusiry coefficient

$$D_j = \frac{KT}{6\pi\mu r_i}$$

K = Bozaman's constant = $1.38 \times 10^{-16} \frac{\text{dyne-cm}}{\text{k}}$

T = kelvin

$$\mu = \text{Viscosity of water} = 0.01005 \frac{\text{dyne-s}}{\text{cm}} @20$$

① Aggregate rate due to brown

$$N_{ij} = \alpha \left[\frac{2}{3} \cdot \frac{KT(d_i + d_j)}{3\mu d_i d_j} \right] n_i n_j = \alpha K_{ij} n_i n_j \rightarrow K_{ij} \text{ fluid shear}$$

n_i = number of i concentration motion

d = diameter of the particle

② Aggregate due to fluid shear

$$N_{ij} = \left[\frac{1}{6} (d_i + d_j)^3 G_l \right] n_i n_j \rightarrow K_{ij} \text{ fluid shear}$$

③ Aggregate due to differential sedimentation

$$N_{ij} = \underbrace{\left[\frac{4g(p_p - p_l)}{18\mu} |r_i^2 - r_j^2| \right]}_{|v_i - v_j|} \underbrace{\left[\pi (r_i + r_j)^2 \right]}_{\text{Cross section collision area}} n_i n_j \rightarrow K^{\text{sed}} = K_{ij} \text{ fluid shear due to differential sedimentation}$$

-units

$$n = \text{number concentrations} = \frac{\#}{\text{vol}} \text{ cm}^{-3} \text{ or } \text{L}^{-1}$$

$$K_{ij} = \frac{\text{cm}}{\text{s}}$$

$$N_{ij} = K_{ij} n_i n_j = \text{s}^{-1} \text{ cm}^{-3}$$

✓ Which mechanism is important?

Depends on G value (mixing) & size of primary particles.

a)

→ for uniform particles: $d_i = d_j$

$$\frac{N}{N} = \frac{2kT}{MG} \frac{1}{d_i^3}$$

→ Brownian motion would important one smaller particles

→ Fluid shear - good for large area.

b)

→ in a flocculator →

$$\frac{N_{ij}^{\text{brown}}}{N_{ij}^{\text{fluid shear}}} = \frac{4KT}{M} \times \frac{1}{G} \times \frac{1}{d_i} \times \frac{1}{d_j} \times \frac{1}{d_i + d_j} < 1 \text{ always}$$

=> Fluid shear is the only important.

-overall rate expressions.

$$N_{KT} = N_{KT}^{brown} + N_{KT}^{fluid} + N_{KT}^{sed}$$

- if we use radius instead of diameter we will have:

$$N_{ij}^{fluid\ sher} = \frac{1}{6} (d_i + d_j)^3 G n_i n_j = \frac{4}{3} n_i n_j G (r_i + r_j)^3$$

$$\Phi_j = n_j \left(\frac{4}{3} \pi r_j^3 \right) = \frac{\text{vol of all } j}{\text{total suspension vol}} = \text{volume fraction concentration of particles.}$$

$$\Rightarrow N_{ij} = \frac{\alpha G}{\pi} Q_j n_i = K_{nk} = \text{rate} = \text{first order reaction.}$$

- for constant floc volume \Rightarrow

$$N_{IT} = K_A G n_1 \quad \text{and} \quad K_A = \frac{\alpha}{\pi} \Phi_j$$

$$\text{Where } K_A = \text{flocculation constant} = \frac{\alpha}{\pi} \sum_2^c Q_K$$

$$\text{First order: } N_{IT} = K_A G n_i$$

G = mixing speed

Q_j = Amount or dose of flocs

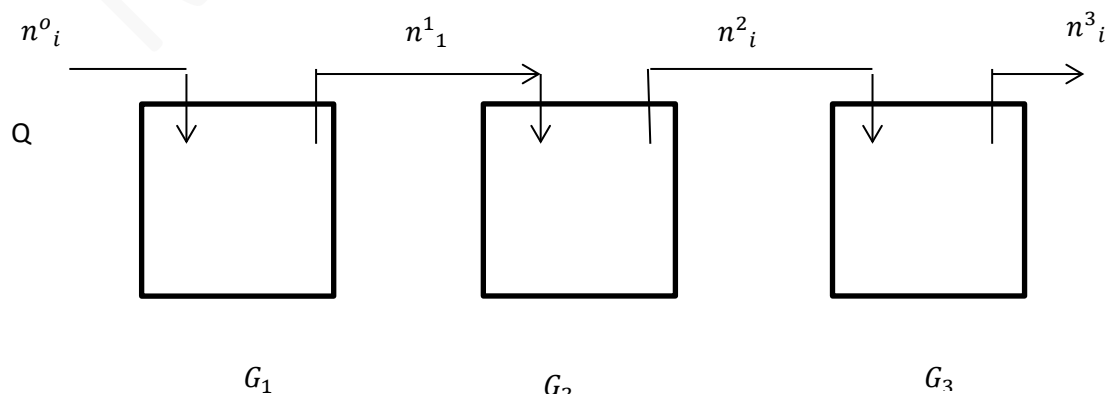
Rate $\propto (G, \neq \text{conc}_i, Q_j)$

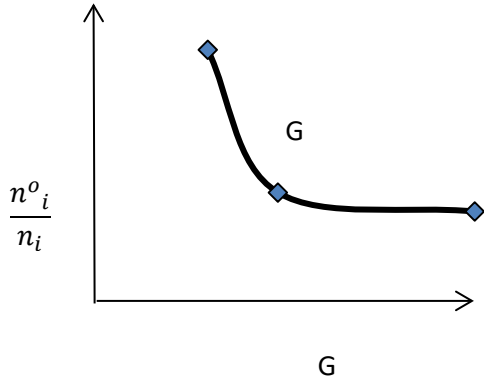
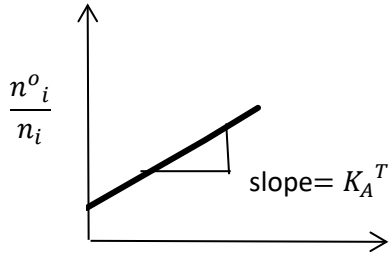
$$\text{-density } p_{fbc} = \frac{\text{mass of floc}}{\text{vol floc}} = \frac{m_i}{v_i}$$

-steady states means over time nothing would change.

$$-T = \frac{n_i^0 - n_i}{K_A G n_i} \Rightarrow \text{for CSTR just like } T = \frac{C_A^0 - C_A}{-r_{exit}}$$

-series CSTRs





$$\frac{n_i^o}{n_i} = 1 + K_A G_1 T$$

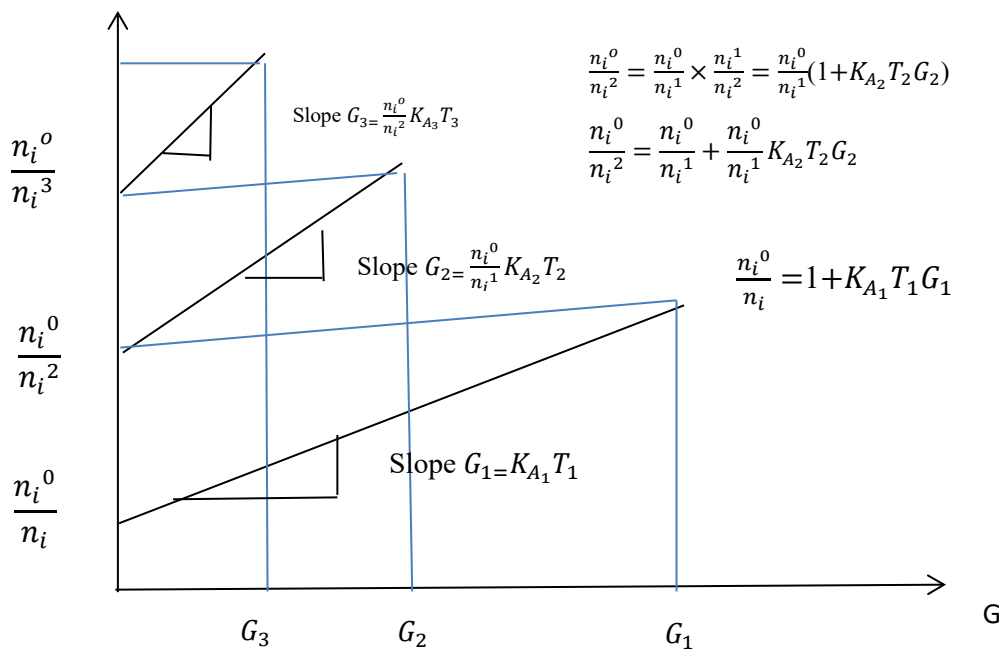
$$\frac{n_i^o}{n_i^1} = 1 + K_A G_1 T_1$$

$$\frac{n_i^1}{n_i^2} = 1 + K_A G_2 T_2$$

$$\frac{n_i^2}{n_i^3} = 1 + K_A G_3 T_3$$

$$\frac{n_i^o}{n_i^3} = \frac{n_i^o}{n_i^1} \times \frac{n_i^1}{n_i^2} \times \frac{n_i^2}{n_i^3} = \frac{n_i^o}{n_i^2} (1 + K_{A_3} T_3 G_3)$$

$$\frac{n_i^o}{n_i^3} = \frac{n_i^o}{n_i^2} + \frac{n_i^o}{n_i^2} K_{A_3} T_3 G_3$$



Strategy

1. Floc shear curve known
2. Calculate K_A, T independantly
3. Calculate slope.
4. Draw line until it interrupts
5. Find G value find intercept
6. Repeat for other CSTRs

Setting velocity:

$$V_s = \sqrt{\frac{\frac{4}{3} g (p_p - p_l) d_p}{C_o p_e}} \quad (1)$$

-Reynolds number= $R_e = \frac{d_p V_s p_l}{\mu}$

- 1) Laminer region = smoth $R_e < 1$ $C_o = \frac{24}{R_e}$
- 2) Transition zone: $1 < R_e < 1000$ $C_o = \frac{24}{R_e} + \frac{3}{\sqrt{R_e}} + 0.34$
- 3) Turbulent: $R_e > 1000$ $C_o = 0.44$

-Terminal velocity if laminar flow ($R_e < 1$)

- a) $V_s = \frac{g(p_p - p_e) d^2}{18\mu}$
- b) calc R_e now
- c) calc C_o new
- d) calc v_s with $V_s(1)$

e) is v_s new = $v_{s_{frist}}$ $\begin{matrix} ? \xrightarrow{yes} \text{done} \\ ? \xrightarrow{No} \text{go back to b} \end{matrix}$

1) Design base for a single design particle

$$V_c = \frac{Q}{A_s} \quad \text{Dssign eq}$$

↘ Top surface area

1-Q given

2-calc A_s

3-choose $h \approx 2-3$ m

4-calc $T = \frac{h}{v_c}$

5- $T = \frac{V}{Q}$

6-V, $h \Rightarrow$ calc W, L

-removal is independent of depth.

$$\text{-\%removal} = \sum_{i=1}^{\text{all particles } v > v_c} x_i + \sum_{i=1}^{\text{all particles } v < v_c} \frac{v_i}{v_c} x_i$$

- Settling velocity distributions (continuous)

- Batch settling test
- Measure particle concentration (or mass or turbidity) at certain depth z over time.
- Calculate velocity = $\frac{Q}{t}$ and Fraction of particles. “F(v)” at each time point

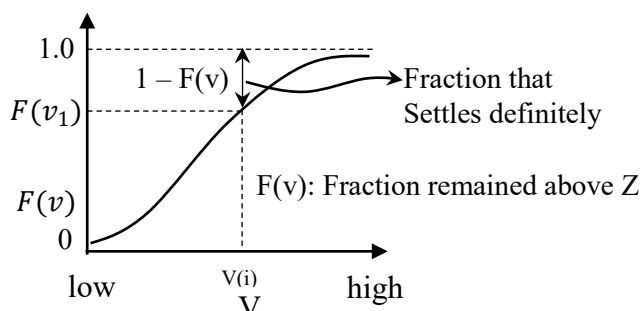
$$F(v)_{particle} = \frac{n(t)}{n_0} = \text{Fraction of remaining.}$$

- Plot F(v) vs V on a graph.

* we can say at $t = x$, F(v) percent of particles above Z have $V_{settle} < \text{velocity at that time.}$

- Fraction remaining at this velocity = F(v_i)

- Fraction settled at this velocity = 1- F(v₁)



$$\% \text{ removal} = 1 - F(v) + \frac{1}{v_c} \int_0^{v_c} v d F(v) \quad (\text{Continuous Case})$$

$$v_c = \frac{Q}{A} \Rightarrow F(v_c) \quad \text{and} \quad \text{interpolate}$$

Sand filter

- Evaluation of collector *efficiency* = $\eta_c = \frac{\text{rate of Capture}}{\text{rate of approach}}$

$$\text{rate of approach} = q_{flow} \times [\text{particle}] = \frac{\text{vol}}{\text{time}} \times \frac{\#}{\text{vol}} = \frac{\#}{\text{time}}$$

Porosity of Sand = 0.3 – 0.4

$$\text{Rate of approach} = C U_{\infty} \frac{\pi}{4} D_c^2$$

$$\checkmark \text{ Diffusion} = \eta_D = 0.9 \left[\frac{KT}{\mu dp D_c U_{\infty}} \right]^{2/3}$$

$$K = \text{boltzman constant} = 1.38 \times 10^{-23} \frac{m^2 kg}{S^2 K}$$

$\mu = \text{viscosity} =$

$$\checkmark \text{ Interception} = \eta_k = \frac{3}{2} \left(\frac{dp}{D_c} \right)^2$$

$$\checkmark \text{ Sedimentation} = \eta_s = \frac{(P_p - P_l) g dp^2}{18 \mu U_{\infty}}$$

$P_p = \text{density of particle}$

$f_i = \text{water density} = 1 \text{ g/cm}^3$

$\mu = \text{viscosity} = 1.002 \times 10^{-3} \text{ kg/ms}$

$g = 9.81 \text{ m/s}^2$

→ Approach velocity

Total collector efficiency

$$\eta_c = \eta_{ic} + \eta_D + \eta_s$$

* We have a dead zone in design. (Poor performance)

If you want to improve performance:

want η_D higher \uparrow : $D_c \downarrow$ smaller, U_{∞} Smaller

want η_{sed} higher : U_{∞} Smaller

$-r_c =$ Rate of particle removal from water per unit bed volume

$r_c =$ rate of particle generation in water per unit bed volume.

unit: $\frac{\#}{\text{time} \cdot \text{bed volume}}$

$$\text{Flux of particles} = U_{\infty} C = \frac{L}{T} \frac{\#}{L^3} = \frac{\#}{TL^2}$$

$$-r_c = \left[\alpha \times \eta_c \frac{3}{2} \times \frac{1 - \epsilon^{-\text{epsilon}}}{D_c} \right] U_{\infty} C = \lambda U_{\infty} C$$

$constant = \lambda = \text{filter coefficient}$

- Fraction of particles striking a collector which stick = $\alpha = 0.9-1$ % that sticks

- Rate at which particles strike a single collector (rate of capture) = $\eta_c U_\infty C \frac{\pi}{4} D_c^2$

$\sum \text{Mechanisms}$ \swarrow $\begin{matrix} \checkmark \text{ How the filter is designed} \\ \checkmark \text{ Water quality} \end{matrix}$

- Number of collectors per unit bed volume = $\frac{\# \text{ cdl}}{\text{bed vol}}$

$$= \frac{\text{total collector volume}(Vs)}{\underbrace{\text{single collector volume}}_{\text{Sphere } \frac{\pi}{6} D_c^3}} \times \frac{1}{\text{bed volume}} = \frac{1 - \epsilon}{V_{tot} \frac{\pi}{6} D_c^3}$$

$\frac{v_s}{V_{tot}} = 1 - \epsilon \rightarrow \text{porosity}$

- Mass balance on particles in water inside the filter.

$$\left. \begin{aligned} U_\infty \frac{dc}{dx} - r_c &= \frac{dc}{dt} t \\ \frac{dc}{dx} &= -\lambda c \quad \text{first order} \\ C &= C_0 e^{-\lambda x} \end{aligned} \right\} \text{Filter equation}$$

✓ For a non-spherical collector

$$\lambda_{non-sphere} = \frac{s}{4} \alpha \eta_c \frac{1 - \epsilon}{D_{ce}}$$

$$\lambda_{sphere} = \alpha \eta_c \frac{3}{2} \frac{1 - \epsilon}{D_c}$$

$D_{ce} =$

$S = \text{shape factor} = \frac{\sigma}{\Psi}$

$\Psi = \text{sphericity} = 3 - \sigma$

$\Psi_{sphere} = 1$

✓ Decrease porosity due to particle build up which changes the filter coefficient.

$$\epsilon_{new} = \epsilon_0 - \sigma$$

$\sigma = \text{rate of particles build up in the filter}$

✓ Particle accumulation stops when:

Rate of shearing = rate of attachment minimum porosity ~ 0.25 – 0.3

✓ Particle accumulation rate:

$\sigma = \text{Volume of particles added per unit bed volume}$

$$\sigma = \frac{\text{Volume of particles}}{\text{bed volume}_{\text{total}} = \text{Volume} = V_{\text{collectors}} + V_{\text{pores}}}$$

$r_\sigma = \text{rate of particles volume added per unit bed volume} = r_\sigma$

Porosity as a function of deposit: $\varepsilon = \varepsilon_0 - \sigma$

- Mass balance

$$\frac{d\sigma}{dt} = r_\sigma$$

Rate of volume particle that is lost from water = - r_c

$r_\sigma = \text{rate of particle accumulation on media} - r_c$

$$\frac{\text{mass part}}{\text{vol time}} = \frac{\text{vol part}}{\text{vol time}} \Rightarrow \frac{-r_c}{P_p} = r_\sigma$$

$$\begin{aligned} r_c &= -U_\infty \lambda c \\ c &= c_0 e^{-\lambda x} \end{aligned} \Rightarrow \frac{d\Delta}{dt} = r_\sigma = \frac{-r_c}{P_p} = U_\infty \frac{\lambda c_0 e^{-\lambda x}}{P_p}$$

$$\frac{c_0}{P_p} = \frac{\text{vol particles}}{\text{vol water}} = \phi_0 = \text{volume fraction concentration}$$

$$\frac{d\sigma}{dt} = U_\infty \lambda \phi_0 e^{-\lambda x}$$

Integrate rate for any X , consider 1 X at a time

$t_0 = 0 \quad \sigma_0 = 0$ no particles on media @ $t_0 = 0$

$$\sigma - \sigma_0 = \lambda U_\infty \phi_0 e^{-\lambda x} (t - t_0)$$

✓ Hydraulics of flow – throw porous media:

- Head loss due to friction.

Fair – hatch:

$$h_f = \frac{k\nu}{g} \left(\frac{S}{D_c}\right)^2 L U_\infty \frac{(1 - \varepsilon)^2}{\sigma^3}$$

$k = 5 - 6 \Rightarrow \text{depends on media (given)}$

$$\nu = \frac{\mu}{\rho_e} \longrightarrow \text{Density of liquid}$$

$$R = \frac{D_0 \varepsilon}{\frac{G}{\Psi} (1 - \varepsilon)}$$

Equation to work with

- 1) Particle removal rate from water

$$c = c_0 e^{-\lambda x}$$

- 2) Particle accumulation rate on media

$$\sigma = \lambda U_{\infty} \phi_0 e^{-x\lambda} t \frac{cm^3 \text{ part } v}{cm^3 \text{ bed } v}$$

- 3) Porosity changes

$$\epsilon = \epsilon_0 - \sigma$$

- 4) λ will change and it's not cte over t and X.

$$\lambda = f(\sigma) \Rightarrow \lambda = f(x, t) \Rightarrow \lambda = \alpha \eta_c \frac{3}{2} \frac{1 - \epsilon}{D_c}$$

- 5) Fair – hatch equation

$$ht$$

$$\frac{dm}{dt} = +DA \frac{dc}{dx}$$

D = diffusion coefficient (table)

$$\frac{\text{mass}}{\text{time} \times \text{area}} = \text{Flux} = N$$

- Kinetics = very fast \Rightarrow reach equilibrium in seconds (no need to model)
- Slow kinetics – time to reach equilibrium. (reality, model)

Gas transfer from gas phase to water phase:

$$K_L = \frac{D_L}{Y_L}$$

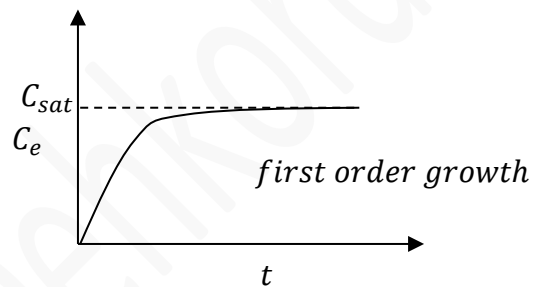
\swarrow Hypothetical liquid film thickness

$$C_i = C_{sat} = 9 \text{ mg/e @ 25}$$

$$\frac{dC_L}{dt} = k_L^a (C_{sat} - C_L) \Rightarrow \text{first order.}$$

Predict dissolve gas behavior

$$C_L = C_{sat} - (C_{sat} - C_{L_0})e^{-k_L^a t}$$



*You have to measure K_{La} for every type of compound and different waters different condition.
 \rightarrow dif compound

$K_L^a \rightarrow$ Dif condition

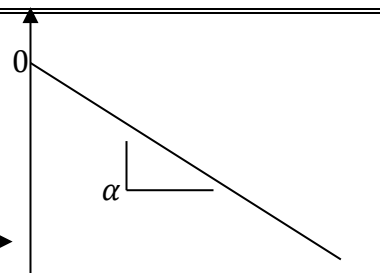
$$K_L^a|_{T_2} = K_L^a|_{T_1} \theta^{(T_2 - T_1)} \quad \theta = 1.024 \text{ usually}$$

$T \uparrow \Rightarrow K_L^a \uparrow \Rightarrow$ Faster rate of diffusion

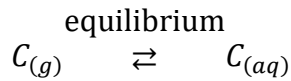
$$C_{L0} = DO_0$$

$$\text{diagram} = \ln \left(\frac{C_{sat} - C_e}{C_{sat} - C_{e_0}} \right) \text{ vs } t$$

$$\text{slope} = -K_L^a = \alpha$$



Henry's law:



$$K_H = \frac{C_{(aq)} = \frac{mg}{e}}{C_{(g)} = atm} = \frac{M}{atm}$$

We should look at table and units

$$\text{Dimensionless henry's constant} = H = \frac{C_g}{C_e} \text{ or } = \frac{C_{aq}}{C_g}$$

$$\text{If: } K_H = \frac{mole}{L.atm} \text{ and } H = \frac{gas}{Liquid} : K_H = \frac{1}{HRT}$$

$$R = \text{gas constant} = 0.082 \frac{L.atm}{mol K}$$

1- Aeration in a CSTR:

**Most basins are CSTR.

All SCTR are steady states \Rightarrow accumulation = 0

$$N_A = K_L(C_e^{sat} - C_e)$$

$$C_L = \frac{C_{L0} + \tau K_L^a C_L^{sat}}{1 + \tau K_L^a} \quad \tau = \frac{-C_L}{C_L K_L^a - K_L^a C_L^{sat}}$$

$$\tau = \frac{V_L}{Q_L}$$

2- Stripping in a CSTR:

Steady state $\Rightarrow acc = 0$

$$C_L = \frac{C_{L0}}{1 + \tau K_L^a}$$

Solubility of atmospheric oxygen in water at 25°C

$$C_L^{sat} = K_H p = 8.5 \frac{mg}{L} O_2$$

$p = \text{partial pressure of atmosphere} = 0.21 atm$

$$H = 32.4$$

1- Aeration in a batch reactor:

$in - out + flux XA = accumulation$

$$\frac{m}{t} - \frac{m}{t} + \frac{m}{tA} \times A = \frac{m}{t}$$

$$N_A A = \frac{dC_e}{dt} V_e$$

Design equation=

$$\ln \left[\frac{C_L^{sat} - C_L}{C_L^{sat} - C_{L0}} \right] = -k_{Lat}$$

2- Stripping in a batch reactor:

$$\text{Unsteady states} = \frac{dC}{dt} \text{ داریم}$$

$$C_e = C_{e0} e^{-k_e at}$$

$$\ln \frac{C_e}{C_{e0}} = -k_e at \Rightarrow \text{first order}$$

Stripping in first order decay.

Steady states in batch

$$\overset{0}{in} + \overset{0}{out} \pm rxn = \overset{0}{acc}$$

$$aeration + decay = 0$$

$$K_L(C_L^{sat} - C_L)A - K_{decay}C_LV_L = 0$$

$$K_{H_{Ozone}} = 90.9 \frac{atm}{mol/L} = \frac{P_{O_3}}{C_L^{sat}}$$

$$1 \text{ mol } O_3 = 48,000 \text{ mg } O_3$$

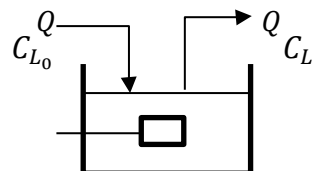
$$C_L = \frac{K_e^a C_L^{sat}}{K_e^a + K_{decay}}$$

CSTR with aeration and decay (Ozonation)

reaction rate

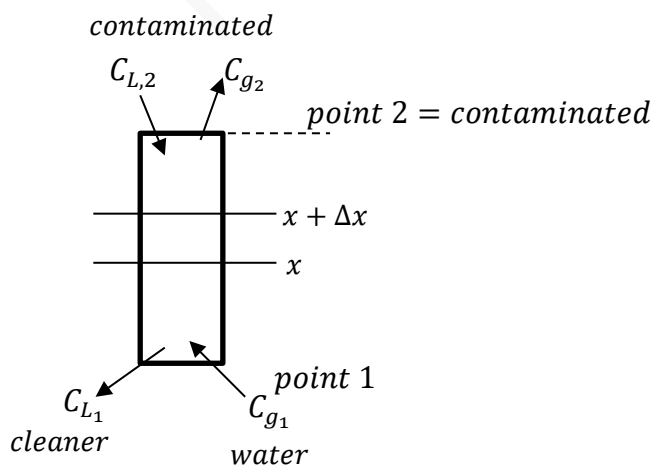
$$C_{L0} = 0$$

$$C_L = \frac{K_L s^a C_L^{sat}}{\frac{1}{\tau} + K_L^a + K_{decay}}$$



$$K_L^a = \text{gas transfer rate constant}$$

Stripping tower design:



$H \times C_{L2} = \text{Max possible value of } C_{g2} \text{ (best performance)}$

$$H = \frac{C_g}{C_L} (\text{dimensionless})$$

Contaminations are leaving water phase and enter the gas phase.

$$F_R = \text{Fraction removed from water} = \frac{C_{L2} - C_{L1}}{C_{L2}}$$

$$H \frac{Q_g}{Q_L} = F_R = R = \text{Stripping factor}$$

$$R = \text{stripping factor} = \frac{H Q_g}{Q_L}$$

Air Stripping:

Model 1: equilibrium control (fastest kinetics, maximum possible removal)

Model 2: mass transfer limitation (not at equilibrium)

Slow transfer kinetics limit us

$$a = \frac{\text{Area all bubbles} = A}{\Delta \text{volume bed} = \Delta x A_{xs}} \Rightarrow A = a \Delta x A_{xs}$$

Steady states \Rightarrow accumulation = 0

$$L = \text{Loading rate} = \frac{Q_L}{A_{xs}}$$

$$\underbrace{\int_{C_{L1}}^{C_{L2}} \frac{dC_L}{C_L - C_L^{sat}}}_{NTU} = \underbrace{\frac{K_e a}{L}}_{\frac{1}{HTU}} \underbrace{\int_{x=Z_1 \text{ point1}}^{x=Z_2 \text{ point2}} dx}_{\Delta Z}$$

NTU = #of transfer units

HTU = height of transfer unit

ΔZ = Height of tower

$$NTU \times HTU = \Delta Z = Z$$

$$Z = \left[\frac{Q_L/A_{xs}}{K_e^a} \right] \left[\frac{R}{R-1} \ln \left(\frac{\frac{C_{L2}(R-1)+1}{C_{L1}}}{R} \right) \right]$$

Rules:

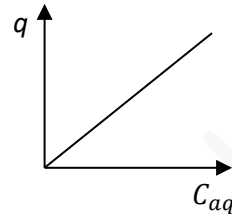
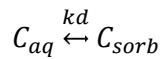
- ✓ $\frac{Q_g}{Q_L} = 25 - 75, \text{choose } 25$
 - ✓ $\frac{Q_C}{A_{xs}} - 20 \quad gal/min/ftz \quad L = \frac{Q_L}{A_{xs}}$
 - ✓ $\frac{Z}{dia} = 5 - 20$
 - ✓ $Z = 15 - 20m \quad \checkmark$
-
-

How to do lab data and find k

$$C_{\text{Sorbed}} = q = \frac{mg \text{ Sorbate}}{mg \text{ Sorbent}}, \frac{\mu \text{ mol}_c \text{ Sorbate}}{m^2 \text{ Sorbant}}$$

$$C_{aq} = C_{eq} = C_{\text{equilibrium}} = \frac{mg}{e}, MM$$

linear model:



$$k_d = \text{linear partitioning coefficient} = \frac{C_{sorb}}{C_{\text{equilibrium}}} = \frac{q}{C_{aq}}$$

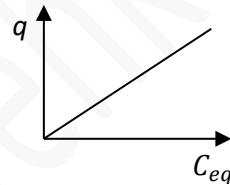
Freudlich Model: → non-linear behavior

assure: dif types of sites

$$q = k_F C_{eq}^n$$

$$n = 1: \text{linear}$$

$$n < 1: \text{curvilinear (mostly)}$$

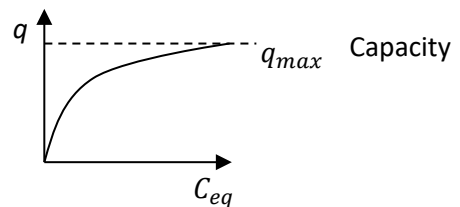


lungmuir Model:

assure 1 type of site which has a maximum sorption

$$q = q_{\max} \frac{k_L C_{eq}}{1 + k_L C_{eq}}$$

$$k_L = \text{partitioning coefficient}$$



sorption isotherms:

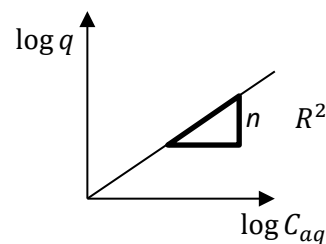
$C_o = \text{pollutant initial concentration (Sanc)}$

$$C_{eq} < C_o \Rightarrow q = \frac{C_o - C_{eq}}{\frac{\text{mass Ac}}{mg}} \times V \rightarrow$$

$$C_o - C_{eq} = \text{concentration that is sorbed} = \frac{mg}{e}$$

for k ⇒ linearize freundlich:

$$\log q = \log k_F + n \log C_{eq}$$



linearize lung muir:

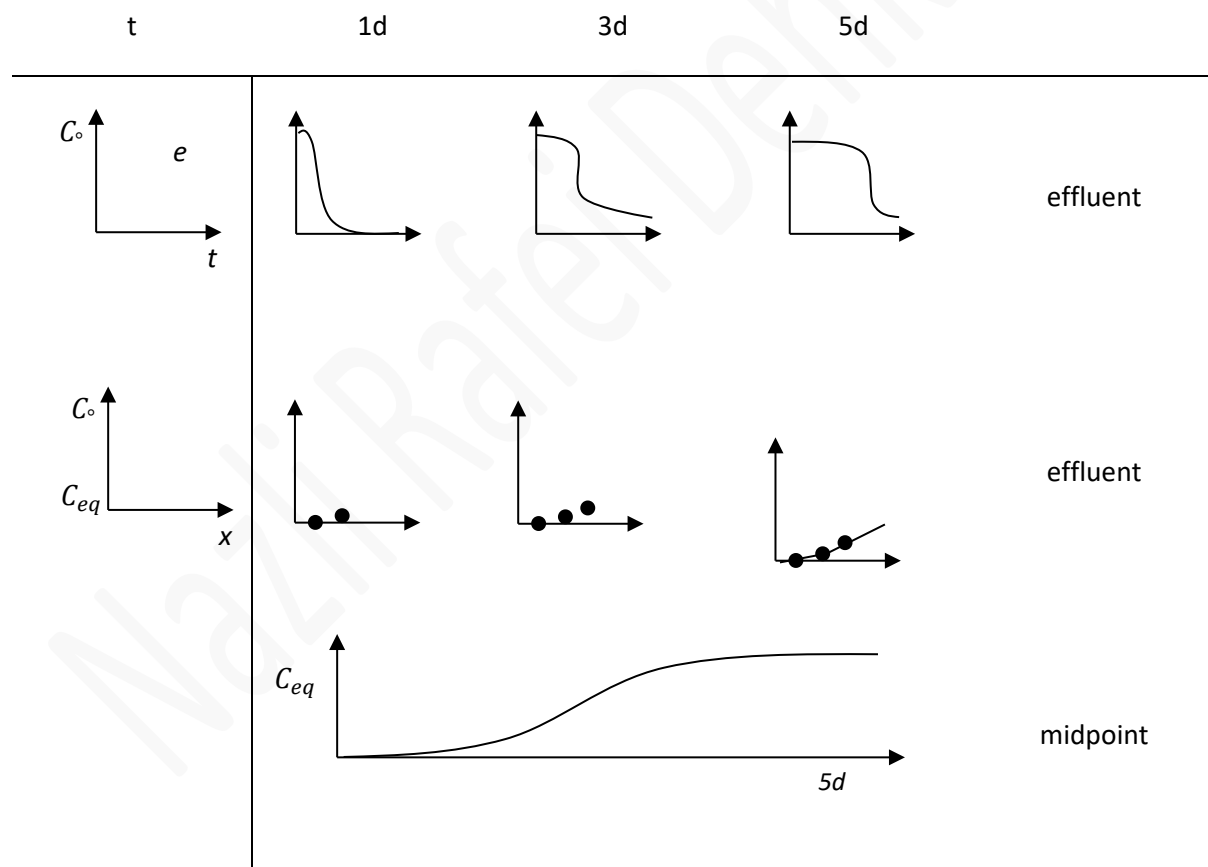
$$\frac{1}{q} = \frac{1}{q_{max}k_L} \times \frac{1}{C_{eq}} + \frac{1}{q_{max}}$$

$$porosity \ \epsilon, \eta, e = \frac{V_{voids}}{V_{totabed}} = \frac{V_{water}}{V_{totabed}}$$

$$bulk \ density \ P = \frac{kg \ solid}{L \ total \ Volume}$$

break through = no more clean bed

$$C = 10\%C_0$$



1) Accumulation terms: advection + dispersion – Sorption – react

$$V \frac{dC}{dt}$$

$$\text{velocity} = u_x = \frac{\Delta x}{\Delta t} = \frac{x}{t}$$

- *advections* = due to fluid movement. = $\frac{-V}{t} \Delta C = -V u_x \frac{dC}{dx}$
- *dispersion* = random molecular motion through different path
- *diffusion* = gradient of concentration

$$\text{Dispersion} = \underbrace{D_x}_{\text{Specific for contaminant}} \frac{d^2 C}{dx^2} V$$

$$D_x = \underbrace{\propto_x}_{0.01-1.0} u_x \Rightarrow \text{at faster velocity we can have more dispersion}$$

$$\text{sorption} = -V \frac{dC_{\text{sorb}}}{dt} \frac{\rho_b}{n}$$

$$n: \text{porosity} = \frac{L_{\text{water}}}{L_{\text{total}}}$$

$$\rho_b = \text{bulk density} = \frac{\text{kg}}{L_{\text{total}}}$$

$$\text{material density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{bulk density} = \frac{\text{mass}}{\text{bed volume (volume of cylinder)}}$$

$$C_{\text{sorb}} = k_d C \rightarrow \text{linear isotherm}$$

$$\text{Retardation coefficient (factor)} = R = \left(1 + \frac{k_d \rho_b}{n}\right) = \frac{\text{velocity of water}}{\text{velocity of contaminant}}$$

packed bed Reactor:

$$HRL = \frac{Q}{A_{xs}}$$

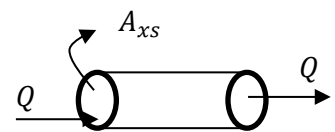
$$\text{pore velocity} = V_{\text{por}} = \frac{HLR}{\underbrace{E}_{\text{porosity}}}$$

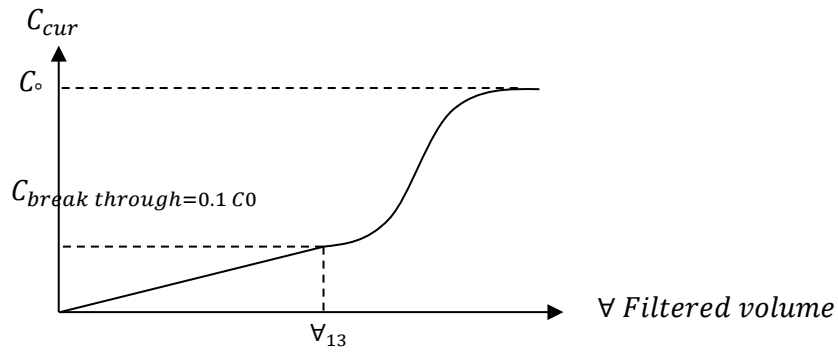
$$q_A = \text{solid phase conc} = \frac{\text{mas of absorbate}}{\text{mas of absorbent}}$$

$$C_A = \text{liquid phase conc of absorbate}$$

$$q_A = k_d C_A$$

$$k_d = \text{partition coeff} \frac{\text{mg/kg}}{\text{mg/L}} = \frac{L}{\text{kg}}$$





$$\text{carbon usage rate} = \frac{M_{GAC}}{V_B} = CUR$$

V_B = break through volume for certain mass of carbon

CUR = how much carbon is needed to treat certain volume of water.

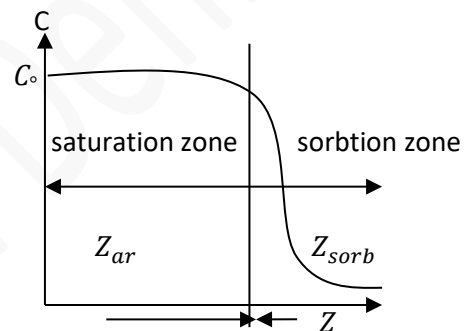
$$V_{sorb\ zone} = \frac{Z_{sat_1}}{t_1} = \frac{Z_{sat_2}}{t_2} = \dots$$

$$V_{sorb\ zone} = \frac{Q C_0}{\rho A_{xs} (1 - \eta) q_{sat}}$$

Length of colum = Z

$$Z = Z_{sat} + Z_{sorb}$$

$$Z_{sat} = V_{sorb\ zone} t_{treatment}$$



Design:

- 1) assume $Z_{sorb} \text{ (start)} = 1m$
- 2) calculate $Z = V_{sorb} \times t_{treatment} + Z_{sorb}$
- 3) pilot test: $\frac{t_{total} - t_{sat}}{t_{total}} = \frac{Z_{sorb}}{Z} = X\%$
- 4) calculate "real" $Z_{sorb} = X\% \times Z$ (Z comes from step 2)
- 5) if $Z_{sorb} < 1m$

- 1) Do the test until time to $C_{eff} = C_0$

2) *then we call this time total $t \Rightarrow t = t_{total}$*

3) *time to get 10% C_0 is going to be t_{sat}*

4) *when $C_{eq} = C_0 \Rightarrow \text{max. condition} \Rightarrow \text{max sorbed conce} = q_{sat}$*

5) $\rho_{GAC} \approx 1.2 - 1.4 \frac{g}{cm^3}$

6) $A_{xs} = \frac{\pi}{4} d^2$