

# A Numerical Revolution: Mathematical innovation and the growth of pre-modern European economies

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02/19/2025

## Abstract

The accumulation of knowledge and its application to human needs are discontinuous processes leading to economic growth through innovation and change. In this paper, we focus on the diffusion of mathematical innovation in pre-modern Europe (13th-16th century). Using an original dataset of over 1000 manuals of practical arithmetic, we produce historiographical and empirical evidence of the economic importance of the switch from Roman to Hindu-Arabic numerals (0 to 9) among European economic agents. We show that, by laying the foundations for commercial innovations observed from the 13th century onwards, this numerical revolution had a positive and significant effect on the growth of European cities. The economic importance of practical arithmetic applied to commerce and mechanical arts is confirmed by quasi-experimental exercises and placebo tests considering mathematical knowledge diffused via other channels.

**JEL codes:** O3; O4; N13; N3

**Keywords:** Human capital; knowledge diffusion; learning; economic growth

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# 1 Introduction

Until the late middle ages, people in Europe relied on tallies, finger reckoning and Roman numerals to represent numbers, and on reckoning boards to carry out calculations (Ifrah 2000). While these systems were effective for calculating addition and subtraction with natural numbers, they were cumbersome for multiplications and divisions, and for handling large and rational numbers. By overcoming the limitations of traditional European systems, the adoption of Hindu-Arabic numerals in European commercial practices, which began in Southern Europe in the 13th century, enabled new applications of mathematics to practical economic activities.

Thanks to the principle of place value and to the symbol for zero, Hindu-Arabic numerals (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) provided a system that was at the same time a reckoning tool and a notation technique to record numbers. The principle of place value, in fact, made it possible to use a limited set of symbols (i.e. just ten figures) to both represent any number, and to make algorithmic calculations with them (Ifrah 2000, 679).

Moreover, as Hindu-Arabic numerals were introduced into Europe together with the notation for fractions, they made it possible to represent and to calculate with virtually any rational number. It is indeed significant that the first European economic agents to adopt Hindu-Arabic numerals were also the inventors of fundamental business innovations based on calculations with rational numbers. These innovations included the bill of exchange, modern insurance contracts, and the management of international businesses based on the distribution of profits and losses according to shares – all characteristic features of the ‘commercial revolution’ of the 13th century, and fundamental stepping stones in the development of modern market institutions (De Roover 1953). Highlighting this coincidence, Max Weber argued that, while the positional numeral system had been invented in ancient India, it was put at the service – “in den Dienst” – of the development of “capitalism” once it reached Europe.<sup>1</sup>

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1. Weber noted this in the *Vorbemerkung* ('prefatory remark') to his *Die protestantische Ethik und der Geist des Kapitalismus*. The German text reads as follows: “Gerechnet, mit Stellenzahlen gerechnet, Algebra getrieben haben auch die Inder, die Erfinder des Positionszahlensystems, welches erst in den *Dienst* des sich entwickelnden Kapitalismus im Abendland trat, in Indien aber keine moderne Kalkulation und Bilanzierung

However, the importance of this relevant mathematical innovation is seldom discussed in detail in the economic history literature.

In this paper, we study the relationship between the diffusion of Hindu-Arabic numerals among economic agents and the growth of pre-modern European economies. The paper expands upon the broader theme concerning the role of knowledge in the process of economic growth (Aghion and Howitt 1992; Romer 1990; Grossman and Helpman 1993; Jones 1995; Weitzman 1998). The kind of new knowledge we are dealing with entails a codified component – it is contained and preserved through time in manuscript and printed texts – as well as a tacit component (Polanyi 1966; Dasgupta and David 1994) that resided in and was developed by the people who adapted and transmitted this knowledge as a foundation for advanced commercial skills. As recently stressed by Berkes and Nencka (2024), increasing access to technical knowledge can play a key role in the process of economic growth.

A stream of studies has documented the fundamental role played by advanced human capital in the early modern period, exploring both the contribution of practically-minded intellectuals and that of socially humbler figures, such as artisans, workers, and other practitioners – what Mokyr has described as the emergence of ‘useful knowledge’ (Berg 1994; Mokyr 2002; Berg 2007; Hilaire-Pérez 2007; Allen 2009; Squicciarini and Voigtländer 2015; Mokyr 2017). Kelly, Mokyr, and Ó Gráda have stressed the crucial complementarity between the applications of scientific knowledge to practical problems and the dexterity of the labour force in the run up to the British industrial revolution (Kelly et al. 2014, 2023). Kelly and Ó Gráda have recently argued that early modern ‘mathematical practitioners’ were key actors in these phenomena, showing that practical mathematics is a crucial field to document the relationship between the accumulation of knowledge and economic growth (Kelly and Ó Gráda 2022).

Building on previous work, we are going to extend Mokyr’s framework to the pre-industrial

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schuf” (Weber 2016, 115). English versions translate the term ‘Stellenzahlen’ as ‘decimals’, making Weber’s explicit reference to the positional numeral system more difficult to spot: “Calculation, even with decimals, existed also in the algebra of India, where the decimal system was discovered. Yet in India it never led to modern calculation and accounting methods; this mode of calculation was first placed into *operation* only in the West’s developing capitalism” (Weber 2012, 158–59).

era, showing that also the late medieval and early modern period was characterised by a process of growth through the accumulation of knowledge and its economic applications, contributing to the literature on the determinants of European long-run growth. The nature of the relevant knowledge involved in this process was new mathematical and practical knowledge applied to commerce. We focus on the diffusion of practical arithmetic among European practitioners over the period 1300-1600 and we explore its importance over the long run, documenting the deep historical roots of a body of knowledge that played a crucial role in the industrial revolution (Kelly and Ó Gráda 2022). In documenting the economic relevance of this knowledge, we illustrate the characteristics of the texts that allow us to reconstruct the spread practical arithmetic. Thanks to the tradition of practical arithmetic manuals, Hindu-Arabic numerals spread among European practitioners, enabling the application of new mathematical knowledge to commerce and the mechanical arts, and the development of advanced economic practices.

The paper combines historiographical evidence with econometric analyses of the relationship between the diffusion of Hindu-Arabic numerals and city-level growth. The study is based on a new and original dataset of over a thousand manuals of practical arithmetic. The dataset was constructed from existing catalogues as well as from extensive archival research. Contrary to previous studies, this dataset includes both manuscript and printed sources, making it possible to extend the period of analysis before the introduction of the printing press.

After presenting the historiographical framework, firstly, we provide systematic descriptive results by means of pooled and panel OLS regressions with as good a set of controls as can be applied in this empirical context. Secondly, we employ quasi-experimental methodologies, such as instrumental variables and staggered difference-in-difference analyses, to explore the possible causal and long-term effect of this ‘numerical revolution’ on the growth of European cities. To show that the phenomena we observe were linked specifically to the application of mathematics to commerce, we also provide a placebo test on the effects of the diffusion of the same mathematics in a different context, using astronomical texts that circulated, rather

than among economic actors, in European universities and monasteries.

## 2 Literature review and historical framework

### 2.1 Hindu-Arabic numerals as mathematical innovation

There is a growing interest in investigating the role played by ‘useful knowledge’ and skills in the pre-industrial period (Mokyr 2017; Mokyr et al. 2022). Van Zanden and others started to explore this theme from a variety of perspectives, suggesting that ‘human capital’ played a relevant role in European growth since the late middle ages (Baten and Van Zanden 2008; Van Zanden 2009). Buringh and Van Zanden (2009) estimated that Europe developed a consistently growing production of texts – both manuscript and printed – since the middle ages, raising the possibility that Europe developed a ‘knowledge economy’ starting from that period. Epstein has stressed the importance of technical change and the transmission of technical knowledge in premodern Europe (Epstein 2013). More recently, it has been argued that ‘human capital formation’ was ‘the driver of growth’ in the pre-industrial period, and that diverging economic outcomes of different European areas (the so-called ‘little divergence’) can be accounted for on the basis of the accumulation of ‘human capital’ (De Pleijt and Van Zanden 2016).

Focussing on the relationship between institutional and economic change, De la Croix et al. (2018) analysed the role of European institutions that secured the transmission of knowledge outside closed kinship systems, and Cantoni and Yuchtman (2014) argued that it is possible to find a causal relationship between the presence of a university and the establishment of new markets in 14th-century Germany. Boerner et al. (2021) explored the role of mechanical clocks in late medieval Europe, and Dittmar (2011) investigated the role of the printing press in the process of European growth, arguing that the introduction of this new general-purpose (information) technology had a causal effect on the growth of European cities.

These studies, however, have not focused on a significant phenomenon of European late-medieval economic history, namely the spread of advanced commercial skills since the onset

of the so-called ‘commercial revolution’ of the late middle ages. As it marked the origins of European economic expansion after the collapse of the western Roman Empire, this revolution was a turning point in European economic history (Lopez 1976). De Roover was the first to focus on the wave of financial and organisational innovations that were introduced during that period. He argued that their development marked a “complete or drastic change” in European commercial practices, which he compared in scope to the industrial revolution (De Roover 1953).

During this period, the first international commercial-banking companies appeared, based on a new kind of partnership contract – the *compagnia* (Goldthwaite 2009; Tognetti 2015; Padgett and McLean 2006). In order to manage these complex organisations, international merchant-bankers invented the foundational principle of modern accounting, i.e. double-entry bookkeeping (De Roover 1956; Melis 1972; Goldthwaite 2015). As they relied heavily on long-distance trade, these merchants also introduced the first insurance contracts (Ceccarelli 2020). Thanks to an accounting system integrated across different cities, these merchants were able to make international transfers by means of interlocking accounting operations registered in the ledgers of two branches. This is the origin of the bill of exchange, a fundamental financial innovation that made it possible to both move money internationally without moving specie, and to extend short-term credit under the guise of a double operation of exchange (De Roover 1953; Lane and Mueller 1985; Bell et al. 2017). While the amount of precious metal shipped across the main European trading centres did not significantly change, the bill of exchange made it possible to drastically expand the monetary supply (Spufford 1988, 254–55; Lopez 1976, 72; Bolton and Guidi-Bruscoli 2021).

These developments characterised European business practices over the long run.<sup>2</sup> These commercial practices required little capital investment apart from the training necessary to handle them, and studying the mechanisms that made them possible is a highly appropriate

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2. The bill of exchange remained the most important means for international exchange well into the 19th century; insurance contracts evolved until reaching today’s modern form; and double-entry bookkeeping is still today practiced following the principles developed in the 13th and 14th centuries.

angle to address the broader question of the role of knowledge as an engine of growth in pre-industrial Europe.

The adoption of Hindu-Arabic numerals was a key enabling technology for the development of these commercial innovations. The positional numeral system reached Europe relatively late, i.e. in the late middle ages (Katz et al. 2016; Folkerts and Kunitzsch 1997; Folkerts 2001). Together with developing international merchant-banking, double-entry bookkeeping, insurance contracts, and the bill of exchange, the merchants of the commercial revolution were also the first European economic agents to adopt the positional numeral system (Van Egmond 1976; Heeffer 2011; Ambrosetti 2008; Danna 2021). This was a major advancement over Roman numerals. Roman ‘fractions’ were based on the duodecimal subdivision of Roman units and only enabled the representation of a limited set of rational numbers: those based on the factors of twelve (Yeldham 1927; Maher and Makowski 2001). To address the problem of calculating multiplication and division, ‘reckoners’ usually relied on counter abacuses and multiplication tables. Conversely, thanks to the concept of place value and to the symbol for zero, Hindu-Arabic numerals provide a numeral system that is at the same time a reckoning tool and a notation to record numbers. The principle of place value makes it possible to use an efficient set of symbols (i.e. only ten figures) to both represent any number and to make algorithmic calculations with them (Ifrah 2000, 679). Moreover, the contemporaneous adoption from the Arabic world of the notation for fractions increased the potential for new uses of the positional numeral system, as it made it possible to represent and make calculations with virtually any rational number.

Given these characteristics, it is easy to see why merchant-bankers of the commercial revolution adopted Hindu-Arabic numerals for their calculations. Since using bills of exchange implies dealing with the slightest differences between exchange rates, one can wonder how (and to what extent) it would have been possible to develop such a financial instrument in the absence of a numerical notation that allowed to handle any rational number. A similar argument can be made about the calculations related to insurance contracts and to the

establishment of commercial *compagnie* (consider the distribution of profits and losses according to shares).<sup>3</sup> In other words, it would have been particularly cumbersome to handle these calculations and record these numbers in the absence of Hindu-Arabic numerals and fractions (Ifrah 2000).

## 2.2 Fibonacci's *Liber abaci*, practical arithmetic manuals and the 'abacus' schools

Leonardo Pisano's (better known as Fibonacci) *Liber abaci* provides early evidence of the knowledge exchanges between Italian maritime republics and the western-Arabic world that led 13th-century Italian merchants to become the first European economic agents to adopt Hindu-Arabic numerals. First completed in 1202, the *Liber abaci* is an advanced mathematical text that provides a synthesis of the Arabic and the European mathematical traditions, and cannot be considered as a text exclusively addressed to merchants (Giusti and D'Alessandro 2020). However, together with advanced mathematics, the *Liber abaci* also deals with the fundamentals of arithmetic, as it introduces the positional numeral system and the algorithms necessary to calculate with the ten figures. Moreover, it explicitly applies these mathematical tools to the solution of practical and commercial problems. As such, the *Liber abaci* documents the transfer of mathematical knowledge between mercantile communities that occurred across the shores of the Mediterranean in the first half of the 13th century (Giusti and Petti 2002).

While there is no direct evidence of the first use of Hindu-Arabic numerals for economic purposes, they surely had been adopted by the end of the 13th century. During this period, Italian city states developed a system of vocational schools. Among these schools, there were the so-called 'abacus schools', which secured the intergenerational transmission of practical

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3. Primary sources provide ample evidence that these merchants actually performed similar calculations. For example, exchange rates in bills of exchange and account books were recorded in terms of a certain amount and *fraction* thereof of currency x for currency y (Bolton and Guidi-Bruscoli 2021). In commercial companies, partners usually received shares of profits according to quotas specified as *fractions*, or as a certain number of *soldi* and *denari* (and also *fractions* thereof) of a *lira* (the standard monetary unit of medieval Europe, equivalent to 20 *soldi*, with a *soldo* equivalent to 12 *denari*) (Goldthwaite 2015, 634).

arithmetic skills (Black 2007; Grendler 1989). These were lay and vernacular schools where pupils between the age of ten and thirteen were trained to use Hindu-Arabic numerals, learned how to make calculations with them, and how to apply these mathematical skills to solve practical problems. Although there were some regional differences (especially between Tuscan and Venetian cities), the curricula of these schools did not significantly diverge in terms of mathematical subjects (Grendler 1989; Goldthwaite 1972; Ulivi 2008).

These schools were founded in most central-northern Italian cities, and there is evidence of their presence also in some centres in the south. Fees to attend these schools were low enough to make it possible also for children of small shopkeepers to afford a training in abacus mathematics (Van Egmond 1977; Goldthwaite 1972). Abacus masters were mathematical practitioners who belonged to a sort of urban middle class and were also employed as expert reckoners, for example in purveying and in auditing accounts for both the city state and for private citizens (Ulivi 2002; Van Egmond 1977; Goldthwaite 1972). Giovanni di Bicci de' Medici, the founder of the Medici bank, consulted abacus masters to learn about new algebraic methods for unsolved forms of quadratic and cubic equations (Ulivi 2015), which were crucial to calculate, for example, compound interest rates.

The most important documentation we have from these schools are practical arithmetic manuals. These texts are mostly written in vernacular, i.e. the working language of merchants and practitioners. As the vast majority of the known authors of these manuals were abacus masters, it is reasonable to think that these texts were mainly used within abacus schools. The structure of these texts is also coherent with this hypothesis, as abacus manuals mainly consist of long lists of worked examples, organised into sections according to their domain of application (e.g. problems of conversion, of exchange, of division of profits and losses, etc.) (Bocchi 2017, 10–65; Franci 2015).

These texts had a wide social circulation. Estimates on the average enrolment of pupils in abacus schools range from 25 to 50 students (Grendler 1989, 72; Van Egmond 1976, 105–13).<sup>4</sup>

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4. While Grendler estimated an average enrolment of 25 to 40 pupils per abacus school, Van Egmond argued that this number ranged between 40 and 50 students per school.

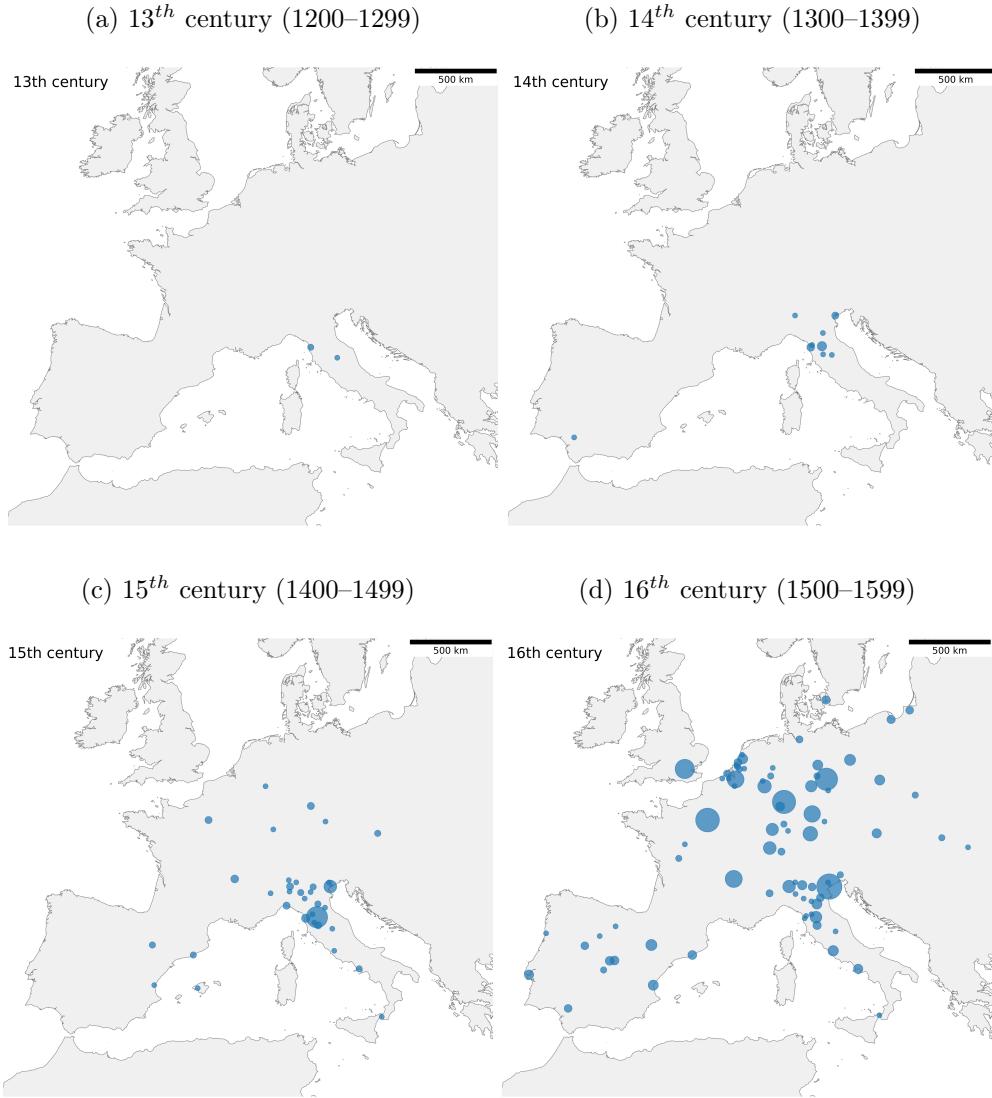
Even by using the lower estimate, it is easy to see that in the span of a decade each abacus manual used in a school reached over a hundred pupils. As a consequence, abacus manuals used within schools often had a readership of a few generations of abacus masters, and an audience of several hundreds of students. This makes abacus manuals distinctively public sources and, therefore, a particularly suitable source of evidence to study the diffusion of the mathematical skills they contain. Moreover, as abacus schools preceded the apprenticeship stage, they provided a foundational mathematical training to a wide range of would-be merchants and practitioners.

### 2.3 The diffusion of practical arithmetic in Europe

Practical arithmetic manuals were mostly used within schools, and their publication in a new city tended to be associated with the foundation of practical arithmetic schools. This is true not only for the Italian tradition of abacus mathematics, but for the entire European tradition (which we will call, more generally, the European tradition of practical arithmetic). The patterns of diffusion in Europe of practical arithmetic manuals – described in intertemporal maps in figure 1 – have economic relevance in their own right, but are also important for our research strategy, because we will use them in the construction of the instrumental variable applied in Section 4.2.1.

As is discussed in detail in Section A.2, practical arithmetic manuals spread incrementally from the Mediterranean to northern Europe between the 14th and the 16th centuries. The chronology of their diffusion shows that data on practical arithmetic manuals does not provide a measure for economic activity in general, but rather a proxy for the spread of specific commercial skills. If this was not the case, we would observe practical arithmetic manuals with Hindu-Arabic numerals appear simultaneously in the main European trading and industrial centres of the 13th and 14th centuries — i.e. not only in Italy and southern Europe, but also in the Low Countries, Hansa cities, and England. Practical arithmetic manuals, instead, appeared in these areas only in the 16th century. As the exchange of ideas and the movement of people were the key channels through which this practical knowledge spread,

Figure 1: Diffusion of Hindu-Arabic manuals across European cities.



*Notes:* The circle size is proportional to the number of manuals published in the city in that century.

the European diffusion of practical arithmetic did not follow the maritime routes of international trade, but rather inland channels based on proximity, as can be observed in figure 1. The choice to invest in practical arithmetic skills in cities of new adoption was often the result of the exposure of a local mercantile community to advanced commercial practices used abroad. In some cases, the authors of the first manuals were themselves the very first adopters of advanced commercial techniques. After learning these techniques abroad, these

early adopters wrote texts to foster their dissemination in their area of origin. Fibonacci's preface to his *Liber abaci* can be read as a deliberate choice to stimulate the introduction of Arabic arithmetic in the western-Latin world (Giusti and D'Alessandro 2020, 4).<sup>5</sup> Jan Ympyn Christoffels, who wrote the first treatise on bookkeeping published in northern Europe, had spent a decade in Venice, where he had been sent by his merchant father to learn business practices and accounting. Ympyn's work, published in 1543 in Antwerp in both Netherlandish and French, was quickly translated into English and printed in London in 1547 (Yamey and Bywater 1982).

In other cases, it can be documented that these developments were sparked when a generation of local merchants moved to an area of previous adoption not to trade, but to get an education in advanced commercial techniques. These early adopters then backed the foundation of practical arithmetic schools in their original cities. The first *Rechenschulen* were founded in Upper Germany after the first generation of German international merchant-bankers had moved to Venice and other Italian cities to gain a cutting-edge business education (Braunstein 2016, 342–44; Weissen 2000). Jacob Fugger, Lucas Rem and Matthäus Schwarz are a few examples of a number of German merchants who moved to Italy in their teen-ages to master advanced financial and accounting practices in the early 16th century (Braunstein 1992; Häberlein 2012). The first *Rechenschulen* served as the institutional arrangement to secure the diffusion across Upper German mercantile communities of the skills first acquired abroad by these early adopters.

As a consequence, it is common to find that the first practical arithmetic masters active in a city moved (or were called) to that city from an area of previous adoption with the aim of spreading mathematical knowledge.<sup>6</sup> The appearance of a practical arithmetic manual,

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5. “Summam huius libri quam intelligibilius potui in quindecim capitulis distinctam componere laboravi, fere omnia que inserui certa probatione ostendens, ut ex tam perfecto pre ceteris modo hanc scientiam appetentes instruantur, et gens latina de cetero, sicut hactenus absque illa minime inveniatur”.

6. The first master documented in the north of Italy – Maestro Lotto, who was appointed as public abacus master in Verona in 1284 – moved to that area from Tuscany, as he was originally from Florence (Black 2007, 227). German *Rechenmeister* played a key role in the diffusion of the teaching of practical arithmetic in the Low Countries. For example, lists of practical arithmetic masters in 16th-century Antwerp report German names in around 50% of cases (Meskens 2013, 91–94). The diffusion of practical arithmetic

therefore, occurred when merchants decided to invest in local practical knowledge, in order to secure the diffusion of useful knowledge, and the inter-generational transmission of advanced mathematical skills. The outcome of this investment was the appointment of practical arithmetic masters, who often came from areas of previous adoption, and the foundation of the first local practical arithmetic schools. Once a school was established, a market for practical arithmetic texts opened up, which led to the publication of the first practical arithmetic manuals in that city. This, in turn, accelerated the social (local) diffusion of knowledge, as the first manual was used to train a new generation of merchants and practitioners who would apply advanced mathematical techniques when they became active on the market. Working in a way comparable to a general-purpose technology, Hindu-Arabic numerals made possible the diffusion of advanced commercial techniques and enabled further experimentations in the world of the mechanical arts, fostering entrepreneurial activity. This is the framework in which we explain the publication of practical arithmetic manuals in a new city and its lagged influence on urban growth.

### 3 Data and variables

#### 3.1 Practical arithmetic manuals

We rely on a new dataset of over 1290 practical arithmetic manuals which was compiled relying both on previous literature and on extensive archival research. The dataset gathers over 350 manuscripts and over 940 printed manuals written from the late 13th century to 1600. These manuals were written by more than 340 authors, who were active in over 120 cities of western Europe scattered between Austria, Denmark, England, France, Germany, Hungary, Italy, Low Countries, Poland, Portugal, Spain, and Switzerland. The database records texts written in the following languages: Castilian, Catalan, Danish, English, French, German, Greek, Hebrew, Hungarian, Italian, Latin, Netherlandish, Polish, Portuguese, and

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to the northern Low Countries, and especially to Amsterdam, was triggered by the migration of Antwerp's Protestant citizens – that included a number of *rekenmeesters* – in the aftermath of the Dutch revolt (52–55). Humphrey Baker's *The welspring of sciences* (1574) attests that, in the third quarter of the 16th century, practical arithmetic skills were still mainly taught by masters coming from the continent (Otis 2017).

Provençal.

This database provides the first reconstruction of the European tradition of practical arithmetic and shows that the diffusion of these texts was a continuous and piecemeal process. The evidence provided in this database contributes to existing literature in two ways. First, as it covers a period of four centuries, it provides a long-run analysis which is based not on estimates of book production, but on clearly identifiable primary sources (Baten and Van Zanden 2008). Second, as it is based on both manuscript and printed sources, this database makes it possible to identify a continuous transmission of mathematical knowledge from the commercial revolution of the 13th century to the onset of the European ‘little divergence’. Moreover, for the reasons discussed above, this database covers a particularly significant subset of total European book production, providing a good proxy to study the spread of commercially-oriented knowledge. This tradition of vernacular practical arithmetic, in fact, documents the long-run roots of the mathematical knowledge that Kelly and Ó Gráda (2022) have recently discussed as a crucial factor in the industrial revolution.

As we cover the period from the late 13th century to 1600, the dataset includes both manuscript and printed sources. While manuscript sources are considered individually (with each manuscript being considered as an independent document), printed manuals have been recorded at the book-edition level. This choice is due to three independent reasons: (1) the different characteristics of manuscript and printed sources, (2) the varying social circulation of European practical arithmetic manuals, both manuscript and printed, and (3) the nature of the evidence extant, as discussed in appendix A.1.

Books offer a reliable source of evidence to reconstruct the diffusion of knowledge. The cost of shipping printed books was often sufficiently high to make it more convenient to reprint a text in a new city, rather than to ship large numbers of volumes across cities. In other words, printed books spread through new editions rather than through inter-city trade (Dittmar 2011, 1140–41). These considerations also apply to manuscript texts, as the high flexibility of the manuscript medium made these sources less standardised than printed

documents. Manuscript texts, in fact, tend to reflect the interests and features of the local context in which they were written, making them less fit for inter-city trade than printed books. This is essential to corroborate the reliability of our measure of the spread of practical knowledge.

### 3.2 Variables and samples

We exploit this novel dataset to investigate the importance of the diffusion of practical mathematics for the growth of European cities. Due to the lack of per capita income data for historic cities, we follow extant literature (Acemoglu et al. 2005; Bairoch et al. 1988; Dittmar 2011) and use population data – which can be considered a good indicator of cities’ well-being and technological advancement – as a proxy for economic growth. The population of European cities in pre-modern Europe is, indeed, well documented at the century-level. To perform our analysis, we combine the Buringh (2021) dataset on the population of European cities from the 8th to the 21th century<sup>7</sup> with more detailed population estimates for Italian cities by Malanima (1998) and Malanima (2011).<sup>8</sup> Our results are, however, robust to the use of different data samples, as we discuss in section 5.

Since we collect practical arithmetic manuals written in the 13th–16th centuries and we are interested in studying their effects on urban growth, we focus on population data in the 14th–17th centuries. Moreover, we consider only the twelve countries (following the 20th-century classification) that are present in the manuals’ dataset: Austria, Denmark, England, France, Germany, Hungary, Italy, Low Countries, Poland, Portugal, Spain, and Switzerland. The resulting dataset provides information about population growth in 827 European cities, with 1797 city-century total observations. We select the manuals published in these 827 cities from the entire database of practical arithmetic manuals, and we obtain 1037 texts written in 93 cities. Table 1 provides more details about the distribution of European cities in our sample across countries and centuries. The number of observed cities grows considerably

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7. These data contain a large share (almost 50%) of observations obtained by simple linear imputation. Our choice was to retain only the more reliable data, and hence we removed imputed observations.

8. Dataset available at: [Italian urban population 1300–1861](#), Paolo Malanima, 2005.

during the centuries: while we have information on population growth for only 282 cities in the 14th century, the same data are available for 731 cities in the 17th century. To better

Table 1: Historic cities and historic cities with practical arithmetic manuals across centuries and 20th-century countries.

Century Country	Total historical cities				Cities with manuals			
	14th	15th	16th	17th	13th	14th	15th	16th
Austria	1	1	8	10	0	0	1	1
Denmark	1	1	2	7	0	0	0	1
England	15	19	34	54	0	0	0	1
France	32	30	45	61	0	0	2	4
Germany	36	46	83	101	0	0	4	21
Hungary	1	4	4	4	0	0	0	2
Italy	141	126	159	246	2	8	23	22
Low Countries	25	36	49	53	0	0	0	12
Poland	3	4	18	21	0	0	0	3
Portugal	2	2	7	6	0	0	0	2
Spain	20	23	71	161	0	1	4	10
Switzerland	5	6	6	7	0	0	0	2
Total	282	298	486	731	2	9	34	81

*Notes:* The left-side table reports the number of historic cities in which we observe urban growth data across centuries and 20th-century countries. The bottom line summarizes the total number of cities per century. The right-side table reports the size of the subset of cities with at least one practical arithmetic manual across centuries and 20th-century countries.

estimate the effect of the diffusion of practical mathematics on population growth, we also consider in the analysis a number of aspects that could affect urban economies in pre-modern Europe. First of all, we control for the adoption of printing technology in the city. The first European movable-type printing press was introduced by Johannes Gutenberg in Mainz around 1450. In the following years, printing technology spread across the main European cities and substantially contributed to the decrease in book prices and to the dissemination of merchants' manuals and intellectual works.<sup>9</sup> In our analysis, we also consider the presence of printing in European cities during the 16th century, so as to cover our entire period of interest. To create an indicator of the presence of at least one printing press in a city, we

9. Dittmar (2011) finds a positive impact of the adoption of printing in 1450–1500 on city growth during the 16th century.

rely on the *Incunabula Short Title Catalogue*, for what concerns the books published up to 1500, and on the *Universal Short Title Catalogue*, for data after 1500.<sup>10</sup>

In addition to controlling for the presence of printing technology, we follow previous literature and include in the analysis a series of dummy variables that capture cities' cultural, political, and geographical status. Firstly, we include a variable indicating the presence of a university in the city, as a proxy for its intellectual activities and cultural development. This variable may vary across centuries. We extract this information from De Ridder-Symoens and Rüegg (1992), and obtain observations on 105 different universities in our sample of European cities during the 14th–17th centuries. To capture the political status of cities, we define a dummy variable indicating whether the city was a state capital. We integrate the data on state capitals in pre-modern Europe by Bosker et al. (2013) with historical sources to check whether cities were state capitals throughout the century and not only at its beginning. As expected, this variable changes over time, and we count 54 capitals in our period of observation. Concerning geographical location, we control for features – such as the presence of roads and ports – that may favour trade with other cities. We obtain information about the presence of Roman roads in the proximity of each city by integrating data by Bosker et al. (2013) with the geolocalisation of Roman roads.<sup>11</sup> We also define a variable accounting for the presence of a navigable river close to the city, once again relying on the information collected in Bosker et al. (2013). This data has been compared with the information about European cities in Buringh (2021) and checked manually for the remaining cities. Finally, we identify cities with seaports by distinguishing between Mediterranean, Atlantic, Baltic, and North Sea ports. We use data by Acemoglu et al. (2005) and Buringh (2021) as main sources of information on these city characteristics. Table A1 in appendix A summarises descriptive statistics concerning dependent, independent, and control variables in our panel

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10. The *Incunabula Short Title Catalogue* is available at: [ISCT](#), maintained by the British Library (2016). In February 2022, the database collected 26297 printed books published before 1500 in 283 European cities. The *Universal Short Title Catalogue*, instead, is available at: [USTC](#), hosted by the University of St Andrews (2022). In February 2022, we identified 311665 printed books published between 1500 and 1600 in 448 cities.

11. We measure the distance between cities and Roman roads and consider cities no more than 2.5 km away from those roads. Geolocalised data are from [Harvard Center for Geographic Analysis - Roman World](#).

dataset. Each observation refers to city-century pairs.

## 4 Econometric analysis and results

### 4.1 Descriptive evidence

To estimate the importance of the diffusion of practical mathematics in pre-modern European economies, we focus on the analysis of population growth, as a proxy for economic growth, with respect to the diffusion of practical arithmetic manuals across European cities during the previous century (13th – 16th centuries). We, therefore, estimate the following model by an ordinary least square (OLS) regression, pooling together all centuries of interest:

$$\log(\text{End-of-century pop}_{c;t}) = \beta_0 + \beta_1 \text{Manuals}_{c;t-1} + \gamma \mathbf{X}_{c;t} + \delta_t + \phi_a + \epsilon_{c;t}, \quad (1)$$

where  $c$  is a European city,  $t$  is the century (from the 14th to the 17th century),  $a$  is the country the city belongs to,  $\mathbf{X}_{c;t}$  are a set of controls at the city level,  $\delta_t$  and  $\phi_a$  capture, respectively, century and country (following the 20th-century classification) fixed effects, and  $\epsilon_{c;t}$  is the error term. In all estimates, we report clustered standard error by country. We lag the variable *Manuals* because some time had to pass since the first appearance of manuals for the new knowledge to diffuse in the commercial community and produce tangible changes. To better understand the economic importance of the diffusion of practical arithmetic manuals in a given city, we provide a set of alternative variables to measure the presence of manuals in that city: the *number of manuals* in the previous century, the *number of manuals in the second half* of the previous century, and, finally, a dummy variable that captures *the presence of at least one manual* in the previous century. Due to the skewed nature of distributions of the number of manuals, these variables are log-transformed.

Following previous studies (Acemoglu et al. 2005; Dittmar 2011), controls at the city level include the presence, during century  $t$ , of a university or state capital, as well as access to navigable rivers or seaports. They also include a dummy variable that signals the presence of the technology of printing during the century in which we observe practical arithmetic manuals ( $t - 1$ ). Finally, we control for the logarithm of population at the beginning of the

century to account for the initial level of economic development.

Table 2 reports the result in specifications (1)–(3). For all the indicators capturing practical arithmetic manuals, we observe a positive and significant association between the diffusion of these texts and the city population at the end of the subsequent century. More in detail, a 10% increase in the number of manuals in the century  $t - 1$  leads to a 1.24% increment in city population during century  $t$ . This association is greater if we consider texts from the second half of the century: a 10% increase in the number of manuals in the second half of the century is associated with a 1.71% increase in city population. The presence of at least one manual in the city during the previous century increases the city population by 12.47%.<sup>12</sup> To address possible sources of endogeneity, such as the omitted variable bias, and because poolability tests for fixed effects suggest the presence of city effects, we exploit the panel nature of our database, and we control for unobserved urban characteristics by replicating the previous analysis with city and century fixed effects:

$$\log(\text{End-of-century pop}_{c;t}) = \beta_0 + \beta_1 \text{Manuals}_{c;t-1} + \gamma \mathbf{X}_{c;t} + \delta_t + \phi_c + \epsilon_{c;t}, \quad (2)$$

where  $c$  is a city,  $t$  is the century,  $\mathbf{X}_{c;t}$  are a set of controls at the city-century level,  $\delta_t$  and  $\phi_c$  capture, respectively, time and city fixed effect, and  $\epsilon_{c;t}$  is the error term. All estimations report clustered standard errors at the city level. Overall, panel estimates confirm previous results. Even when we control for unobserved urban characteristics, we find a positive and significant association between practical arithmetic manuals and city growth, as reported in table 2 in specifications (4)–(6). Indeed, an increase of 10% in the number of texts in circulation in a given century is associated with an increment of 0.83% in city population in the following century. As before, the manuals published in the second half of the century are particularly relevant: an increment of 10% in the number of manuals in circulation in the second part of the previous century leads to a 1.12% increase in urban population. However, in this case, the association estimated through the dummy variable is positive but

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12. Following Kennedy (1981), we compute the percentage impact of the dummy variable as  $g^* = 100 \cdot [\exp(\hat{c} - \frac{1}{2}V(\hat{c})) - 1]$ , where  $\hat{c}$  is the estimated coefficient and  $V(\hat{c})$  is its variance.

Table 2: Practical arithmetic manuals and city population at the end of the following century.

	log(End-of-century population <sub>t</sub> )					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Nb. of manuals <sub>t-1+1</sub> )	0.124*** (0.029)			0.083** (0.035)		
log(Nb. of manuals 2 <sup>nd</sup> half-century <sub>t-1+1</sub> )		0.171*** (0.031)			0.112*** (0.041)	
Manuals dummy <sub>t-1</sub>			0.144*** (0.053)			0.086 (0.055)
Printing press <sub>t-1</sub>	0.126*** (0.033)	0.124*** (0.033)	0.129*** (0.034)	0.036 (0.042)	0.031 (0.042)	0.042 (0.043)
University	0.083*** (0.040)	0.086*** (0.040)	0.086** (0.041)	0.004 (0.064)	0.008 (0.064)	0.004 (0.064)
Capital	0.407*** (0.068)	0.403*** (0.067)	0.418*** (0.070)	0.277*** (0.099)	0.274*** (0.098)	0.275*** (0.100)
log(Beginning-of-century population <sub>t</sub> )	0.695*** (0.019)	0.693*** (0.019)	0.703*** (0.019)	0.293*** (0.038)	0.291*** (0.037)	0.299*** (0.038)
Geographical controls						
Country FE	Yes	Yes	Yes	No	No	No
City FE	Yes	Yes	Yes	No	No	No
Century FE	No	No	No	Yes	Yes	Yes
Observations	1,797	1,797	1,797	1,797	1,797	1,797
R <sup>2</sup>	0.735	0.736	0.733	0.895	0.895	0.895

*Notes:* This table presents pooled OLS estimates of equation 1 (specifications 1–3) and panel fixed-effect OLS estimates of equation 2 (specifications 4–6). The dependent variable is the logarithm of the population level at the end of century  $t$ , with  $t$  ranging between the 14<sup>th</sup> and 17<sup>th</sup> centuries. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in the century  $t - 1$ . Control variables include the presence of printing presses in the century  $t - 1$ , university, state capital and geographical variables (Roman roads, navigable rivers, and seaports). We also control for the logarithm of the city population at the beginning of the century  $t$ , and we add country (in pooled OLS), city (in panel fixed-effect OLS) and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the country level and reported in parentheses. Significance level: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

not significant.

## 4.2 Quasi experiments

### 4.2.1 Instrumental variable analysis

We propose a quasi-experimental design based on the introduction of an instrumental variable (IV) that mirror the diffusion of practical arithmetic manuals across Europe and across centuries. As discussed in previous sections, we observe an incremental diffusion of practical arithmetic manuals from the south to the north of Europe. While central Italy was the centre of diffusion for these manuals in the 13th century, new centres of diffusion emerged over the centuries, corresponding to the first translations of practical arithmetic manuals in regional vernaculars. As the translation of this mathematical knowledge into a new vernacular language boosted its circulation in the new linguistic area, centres of new diffusion tended to coincide with the first appearance of a manual in a new vernacular language. For example, the anonymous *Die maniere om te leeren cyffren*, printed in Bruxelles in 1508, was both the first practical arithmetic manual published in the Low Countries and the first text of this tradition to be written in Netherlandish. In a similar way, the English tradition of practical arithmetic started off with the first translations of French and Netherlandish texts into English (Williams 2012).

To track the diffusion of practical arithmetic manuals, we construct historical linguistic maps by associating languages to historic states available from the Euratlus project<sup>13</sup> and identify the first practical arithmetic manual in each linguistic area. Table 3 reports the cities and centuries of the first occurrence of texts written in local vernacular in each linguistic area.<sup>14</sup> As practical arithmetic manuals tend to spread by proximity rather than other channels (including maritime trade routes), cities close to diffusion centres are more likely to adopt them. Building on these premises, we define our instrumental variable as the distance between the city under investigation and the city of the same linguistic area where a practical arithmetic

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13. [euratlus.com](http://euratlus.com).

14. Table 3 reports cities for which population data are available.

Table 3: Centres of diffusion of practical arithmetic manuals across linguistic areas.

City	Linguistic Area	Century
Perugia	Italy	1200
Sevilla	Spain	1300
Bamberg	Germany	1400
Lyon	France	1400
Valencia	Catalonia	1400
Bruxelles	Low Countries	1500
Debrecen	Hungary	1500
Kobenhavn	Denmark	1500
Krakow	Poland	1500
Lisboa	Portugal	1500
London	England	1500

*Notes:* The table reports the city and linguistic area of the first occurrence of practical arithmetic manuals written in local vernacular in each linguistic area.

manual in the local vernacular appeared first. If no manual had yet appeared in a linguistic area in the century under investigation, the instrumental variable is equal to the minimum distance from the already existing centres of diffusion. As we are in a panel setting and practical arithmetic manuals have spread across linguistic areas along different centuries, this instrument varies over time, following the diffusion of practical arithmetic manuals from the south to the north of Europe. Specifically, in the 13th century, the instrument is equal to the distance between the focal city and Perugia (i.e., the centre of diffusion for manuals in Italian). With the emergence of a new centre of diffusion in the 14th century (i.e., Seville), the instrument returns the minimum value between the distance of the focal city from Perugia and the distance of the focal city from Seville. Considering the emergence of new centres of diffusion, an analogous algorithm is applied for the following centuries. By employing this instrument in panel fixed-effect two-stage least square (2SLS) estimations, we exploit the variation (reduction) of the distance from practical arithmetic manuals in the local language to explain the probability of observing a manual in the city under investigation.

This instrument has the desirable properties of mirroring the diffusion of practical arithmetic manuals and of being likely correlated to the number of those manuals in different cities and

centuries. At the same time, the distance from the first centre of diffusion in each linguistic area and its variations over centuries are exogenous from the determinants of urban growth. Indeed, the first appearance of manuals was not directly linked to economic trends in general but to the appearance of the specific business practices of the commercial revolution. Late medieval Europe had at least two areas characterized by considerable economic dynamism: the city-states of central-northern Italy and the cities of the Hanseatic League. Initially, practical arithmetic manuals only circulated in Italian city-states, where the commercial revolution originated. Hansa cities, on the contrary, were late adopters of this mathematics. This corresponds with their late adoption of the financial and accounting techniques of the commercial revolution, confirming the reading of practical arithmetic manuals as a good proxy for the toolkit of the commercial revolution. Moreover, the cities in which practical arithmetic manuals first appeared were not necessarily the main economic centres of their regions. The Italian first city of diffusion – Perugia – was a more peripheral town than Pisa, which in turn was by far not the biggest city in its region in the 13th century (Florence, Naples, Venice and Milan were at least twice as big as Pisa). Bamberg – the first city for the German language – had a declining population between 1300 and 1500, and Debrecen – the first city for the Hungarian language – was smaller than Pest and Buda in the 16th century. In several cases, also including Seville, cities did not display superior growth performance before the appearance of manuals, but as manuals translated into local vernacular appeared in these cities, they became epicentres of the regional diffusion of this practical mathematics. As the dynamics of diffusion of practical arithmetic manuals is independent of the dynamics of economic growth in Europe, the distance from the first centre of diffusion in each linguistic area is a good candidate to instrument the number of practical arithmetic manuals in our exercise.<sup>15</sup>

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15. Let us consider, for instance, German cities. In the 13th century, they were relatively close to one of the main economic centres of that time, i.e. Hansa cities, but quite far from Perugia (the only diffusion centre of manuals in that century). While the closest main economic centre (Hansa cities) remains the same over centuries, the value of the instrument for German cities diminishes over centuries following the spread of practical arithmetic manuals from the south to the north of Europe and the appearance of the first German practical arithmetic manual in Bamberg.

Table 4: Instrumental variable estimates. First stage.

	log(Nb. of manuals <sub>t-1+1</sub> ) (1)	log(Nb. of manuals 2 <sup>nd</sup> half-cent <sub>t-1+1</sub> ) (2)	Manuals dummy <sub>t-1</sub> (3)
log(Distance from 1 <sup>st</sup> man <sub>t-1+1</sub> )	-0.125*** (0.038)	-0.106*** (0.039)	-0.067*** (0.012)
Printing press <sub>t-1</sub>	0.250*** (0.045)	0.219*** (0.040)	0.175*** (0.026)
University	-0.045 (0.099)	-0.075 (0.078)	-0.034 (0.047)
Capital	-0.100 (0.103)	-0.042 (0.101)	-0.084 (0.087)
log(Beginning-of-century pop <sub>t</sub> )	0.094*** (0.031)	0.089*** (0.028)	0.036** (0.016)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,797	1,797	1,797
R <sup>2</sup>	0.563	0.547	0.570

*Notes:* This table presents panel fixed-effect OLS estimates on European cities in the 14<sup>th</sup>–17<sup>th</sup> centuries. Dependent variables are the logarithm of the number of practical arithmetic manuals in the city in century  $t - 1$ , the logarithm of the number of manuals in the second half of the century, and a dummy variable which signals the presence of manuals in century  $t - 1$ . *Distance from first manual* refers to the distance of the city from the location of the first occurrence of practical arithmetic manuals in the local vernacular. Control variables include the presence in the city of printing presses in century  $t - 1$ , university, and state capital. We also control for the logarithm of the city population at the beginning of the century  $t$ , and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Therefore, we estimate a panel fixed-effect 2SLS regressions for the three different indicators of the presence of manuals. The first-stage results (table 4) show that the distance from the first manual in the area is significantly and negatively correlated to the presence and the number of manuals in the city, confirming the importance of regional centres for the diffusion of these texts across Europe. As shown in section 5, we do not observe the same patterns when we consider – instead of the distance from cities in which practical arithmetic manuals appeared first – the distance from a random selection of the linguistic area’s main economic centres. This exercise provides additional evidence on the independence of the diffusion of practical arithmetic manuals from economic determinants, supporting the validity of our

instrument. Moreover, the high values of F tests for the IV signal that this distance is a strong instrument. The occurrence of practical arithmetic manuals is also associated with the presence of printing press since several manuals, especially later ones, were printed texts.

Table 5: Instrumental variable estimates. Practical arithmetic manuals and city population at the end of the following century.

Dependent Variable:	log(End-of-century population <sub>t</sub> )		
	(1)	(2)	(3)
log(Nb. of manuals <sub>t-1</sub> +1)	0.466*** (0.159)		
log(Nb. of manuals 2 <sup>nd</sup> half-century <sub>t-1</sub> +1)		0.553*** (0.198)	
Manuals dummy <sub>t-1</sub>			0.868*** (0.323)
Printing pres <sub>t-1</sub>	-0.073 (0.061)	-0.077 (0.065)	-0.108 (0.073)
University	0.016 (0.080)	0.036 (0.078)	0.024 (0.076)
Capital	0.324*** (0.107)	0.301*** (0.102)	0.350*** (0.129)
log(Beginning-of-century population <sub>t</sub> )	0.248*** (0.038)	0.242*** (0.039)	0.261*** (0.038)
<i>Fixed-effects</i>			
City	Yes	Yes	Yes
Century	Yes	Yes	Yes
Observations	1,797	1,797	1,797
R <sup>2</sup>	0.879	0.879	0.874
F-test (1 <sup>st</sup> stage)	77.2	73.3	74.6

*Notes:* This table presents panel fixed-effect 2SLS estimates of the following model:  $\log(\text{End-of-century pop}_{c,t}) = \beta_0 + \beta_1 \text{Manuals}_{c,t-1} + \gamma \mathbf{X}_{c,t} + \delta_t + \phi_a + \epsilon_{c,t}$ . The dependent variable is the logarithm of the population level at the end of century  $t$ . The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in the century  $t - 1$ . We instrument these variables with the city's distance from the location of the first occurrence of practical arithmetic manuals in the local vernacular. Control variables include the presence of printing presses in the century  $t - 1$ , university, and state capital. We also control for the logarithm of city population at the beginning of the century  $t$  and add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Table 5 reports the second-stage results. Overall, these estimations support a plausibly

causal interpretation of the positive and significant effects of the spread of practical arithmetic on the population of European cities. More specifically, a 10% increase in the number of manuals leads to a 4.66% increase in city population at the end of the following century. The effect is stronger for manuals published in the second half of the century: the same increment in the number of manuals corresponds to a 5.53% increase in city population. Overall, cities with at least one manual increased their urban population by 74.56% in the subsequent century. Compared with the panel fixed-effect OLS estimates, the results obtained by introducing the instrumental variable show a stronger effect of practical arithmetic on growth. An explanation of this difference lies in the ability of this instrument to detect fine-grained differences in the adoption and spread of Hindu-Arabic numerals in commercial practices. First, the distance from the first centre of adoption reduces the attenuation bias resulting from the possible presence of missing practical arithmetic manuals in nearby cities. Second, the number of practical arithmetic manuals published in the city is arguably an underestimation of the actual degree of integration of Hindu-Arabic numerals into local commercial practice. By capturing, instead, patterns of diffusion of practical arithmetic across European cities on the basis of distance, the instrument allows us to distinguish between cities with the same number of manuals, but with differences in local knowledge assimilation, thus generating more accurate estimates of the economic effect of this numerical revolution.

#### 4.2.2 Event study

It can be documented that arithmetic manuals appeared as a result of deliberate urban policies to invest in local practical knowledge and that these investments were designed to promote growth. Several Italian city-states chose to devote part of their budget to fund abacus schools. This possibly dates back to the very origin of the tradition of abacus mathematics, as in 1241 the republic of Pisa provided Fibonacci with an annual salary in compensation for his knowledge, identifying him as a “magister” (Ulivi 2011, 256–57). In the 14th century, the city of Pistoia justified the foundation of its abacus school stating that without practical arithmetic its merchants and artisans could not have run their businesses in appropriate

ways (“Sine scientia abaci mercatores et artifices utiliter et bene se exercere non possunt”) (Grendler 1989, 22). Similar developments can be observed in other European cities, such as Lyon.<sup>16</sup>

To corroborate our findings and quantify the effect of the adoption of practical arithmetic manuals on city growth in the long run, we design an additional quasi-experiment by adopting a difference-in-difference (DiD) approach. In our case, treatment, i.e. instances of the first appearance of a practical arithmetic manual in a city, are distributed over four centuries (13th–16th). As there are more than two time periods, we face a setting with staggered treatment rollout. Recent advancements in the DiD literature highlight several issues associated with a simple generalization of the DiD methodology in case of staggered treatment timing and suggest new methodologies to obtain consistent estimates (see, among others, Baker et al. 2022 and Roth et al. 2023).

To correct for the bias induced by the staggered nature of practical arithmetic manual adoption that would arise employing a standard DiD, we use the approach developed by Callaway and Sant’Anna (2021) that generalises the parallel trends assumption to multi-period settings. Callaway and Sant’Anna (CS) is a highly flexible approach that ensures the use of clean controls (i.e., cities that did not receive the treatment before the considered point in time) also in the case of staggered adoption, and allows treatment effect heterogeneity by computing individual cohort-time-specific treatment effects and then aggregating them to produce overall effect estimates. Differently from other staggered DiD estimators, CS can be applied to unbalanced panel data and is robust to parallel-trend assumptions that hold conditionally on covariates, such as the city population at the beginning of the century. Since we are dealing with historical data at the city level and population data time series might contain a few missing values – especially for early centuries – resulting in an unbalanced panel, this estimator is particularly suited for our exercise.

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16. In the second half of the 15th century, Lyon founded a number of practical arithmetic schools in order to boost the skills of its merchants vis-a-vis foreign merchant-bankers. The Lyonnaise merchant François Garin, for example, showed the awareness of this gap in competencies among the local mercantile community in the 1460s, when he urged his son to master the new mathematical techniques (Dubuis 1978, 94–95).

In this exercise, we extend the time series of population data to cover, when possible, the period from the 11th to the 19th century. We consider as treated only cities that adopted practical arithmetic manuals in the 14th-16th centuries due to the lack of information on urban population in the period before treatment for cities in which manuals appeared in the 13th century (Pisa and Perugia).<sup>17</sup> We then apply CS in an event-study framework and estimate the following coefficients for each period  $e$  ( $-3 \leq e \leq 3$ ) by comparing treated and never-treated cities:<sup>18</sup>

$$\theta_{es}(e) = \sum_{g \in \mathcal{G}} \mathbb{1}\{g + e \leq \mathcal{T}\} P(G = g | g + e \leq \mathcal{T}) ATT(g, g + e), \quad (3)$$

where  $\mathcal{G}$  is the set of possible treatment periods,  $G$  is the time period when a unit becomes treated,  $\mathcal{T}$  is the number of periods ( $t \in \{1, \dots, \mathcal{T}\}$ ),  $P$  is the probability of being treated in a given period,  $ATT(g, t)$  is the group-time average treatment effect. It is defined as:

$$ATT(g, t) = \mathbb{E}[Y_t(g) - Y_t(0) | G_g = 1], \quad (4)$$

where  $G_g$  is a binary variable equal to one if a unit is first treated in period  $g$  and  $Y_t$  is the outcome at period  $t$ . We also control for the logarithm of the population at the beginning of the century to ensure a valid comparison between treated and control cities.

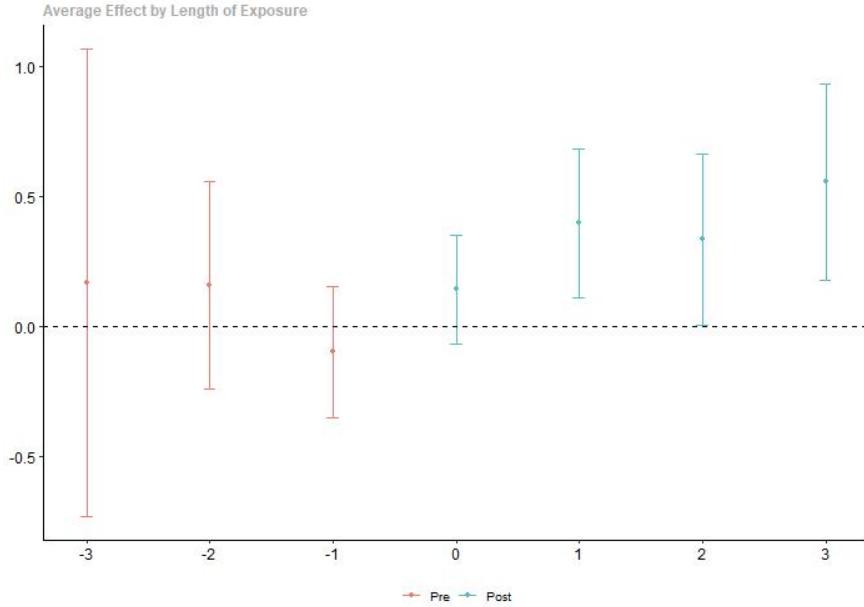
The results, reported in figure 2, show that there is no significant effect, as expected, during the century of adoption of the manuals, while the effect is positive and significant (confidence intervals are 95%) for the following three centuries. Specifically, the introduction of practical arithmetic manuals leads to a 39.8% increase in the city population in the century following the first manual adoption. This effect is persistent in the following centuries and equal to 33.6% and 55.8% after two and three centuries, respectively. The overall average treatment effect on the population of treated cities (i.e., the weighted average of ATT) in the three centuries after the first introduction of a manual is positive, significant and equal to 0.359 (std. error: 0.109).

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17. To ensure clean controls in the analysis, we remove Pisa and Perugia from our sample. However, results are unchanged if we include those cities as treated in the 13th century.

18. Analogous results are obtained when we compare treated and not-yet-treated cities.

Figure 2: Event study



*Notes:* The figure shows DiD coefficients estimated following the equation 3 for European cities. Treatment is defined as the first appearance of a practical arithmetic manual in the city. Control cities are those never treated in the period of observation. The dependent variable is the logarithm of city population at the end of the century and we control for the logarithm of city population at the beginning of the century. Treatment occurs at time 0. Confidence intervals are 95%. Standard errors are clustered at the city level.

### 4.3 Placebo test: Ptolemaic astronomy manuscripts

It can be argued that the effects attributed to practical arithmetic manuals are part of a more general effect of mathematical knowledge that could have diffused via alternative channels. To show that this is not the case, and therefore rule out this alternative explanation, we design a placebo test in which we analyse the influence on urban growth of arithmetical knowledge diffused in universities and monasteries. The rationale of this test is to explore the importance for economic performance not just of the diffusion of knowledge in general, but also, and more specifically, of its social circulation and its application to economic problems. To do so, we focus on the circulation of Hindu-Arabic numerals in theoretical – as opposed to practical – sources. While the main evidence of this paper is provided by data on the diffusion of vernacular manuals that circulated among practitioners, in this section we focus on mathematical texts that circulated among scholars.

In fact, the vernacular tradition of practical arithmetic was not the only tradition spreading

Hindu-Arabic numerals in Europe. The initial circulation of the positional numeral system in Europe started in 12th-century *al-Andalus* with the first Latin translations of Arabic mathematical works (Folkerts and Kunitzsch 1997; Folkerts 2003). These translations were carried out by European scholars proficient in both Arabic and Latin, and their subsequent copies constitute the tradition of the so-called Latin *algorismi*. These texts are brief mathematical primers which were used to show how to use Hindu-Arabic numerals to people who could access a Latin education. While they include fundamentally the same mathematical knowledge as vernacular manuals, Latin *algorismi* are theoretical texts, and generally do not deal with the practical applications of mathematics. As such, they were quickly adopted in the main centres of European learning – i.e., monasteries and universities – as they were relevant for scholars interested in advanced calculations. These were needed primarily to solve problems arising in astronomy, such as accurate calendrical calculations. As a consequence, Latin *algorismi* are often found in manuscripts which also include texts on astronomy (Allard 1991; Nothaft 2014). As they were primarily circulating in monasteries and universities, the geographical spread of Latin *algorismi* was different from that of practical arithmetic texts. While these texts include the same mathematical theory as practical arithmetic manuals – the same ‘technology’ – they circulated in different social and institutional settings, and their mathematical knowledge was applied to solve different problems. As observed in the incipit of a Florentine practical arithmetic manual written around 1457, the learned mathematics spread by Latin *algorismi* was of little use to economic agents active on the market.<sup>19</sup> We, therefore, use evidence on the diffusion of this Latin tradition to investigate the impact of the same factor as that of practical arithmetic (i.e. Hindu-Arabic numerals) applied in different social and institutional settings and to the solution of different problems. Specifically, we

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19. This is Francesco di Carlo de Macigni's *Libretto alla praticha della merchantantia*, preserved in Florence, Biblioteca Mediceo-Laurenziana, ms. Ash. 352. In the incipit of the text, Francesco remarked: “uno che voglia essere marchatante non ha bisogno d'inparare a misurare le stelle e pianeti e 'l chorso de' pianeti e movimenti de' cieli, ma solo ha bisogno d'intendere la merchantantia e lle ragioni ch'elli adoperano in essa merchantantia”, i.e. “One who wants to become a merchant does not need to learn how to measure stars and planets and their course or that of the skies, but only needs to know well the trade of merchants and the problems which are needed in their business”.

replicate the previous analyses by considering the diffusion in European cities in the 13th–16th centuries of Latin astronomical texts that use advanced arithmetic, relying on data on the Latin tradition of Ptolemaic texts.<sup>20</sup>

Table 6: Ptolemy’s manuscripts and city population at the end of the following century.

	log(End-of-century population <sub>t</sub> )		
	(1)	(2)	(3)
log(Nb. of Ptolemy’s man <sub>t-1</sub> +1)	-0.082 (0.069)		
log(Nb. of Ptolemy’s man 2 <sup>nd</sup> half-century <sub>t-1</sub> +1)		-0.089 (0.070)	
Ptolemy’s manuscripts dummy <sub>t-1</sub>			-0.085 (0.066)
Printing pres <sub>t-1</sub>	0.065 (0.043)	0.063 (0.043)	0.064 (0.043)
University	-0.005 (0.063)	-0.001 (0.063)	-0.005 (0.063)
Capital	0.260** (0.102)	0.269*** (0.101)	0.258** (0.102)
log(Beginning-of-century population <sub>t</sub> )	0.303*** (0.039)	0.303*** (0.039)	0.304*** (0.039)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,797	1,797	1,797
R <sup>2</sup>	0.895	0.895	0.895

*Notes:* This table presents panel fixed-effect OLS estimates on the placebo dataset. The dependent variable is the logarithm of the population level at the end of the century  $t$ , with  $t$  ranging between the 14<sup>th</sup> and 17<sup>th</sup> centuries. The *Ptolemy’s manuscripts* variables are indicators of the presence of Ptolemy’s manuscripts in the city in century  $t - 1$ . Control variables include the presence of printing presses in century  $t - 1$ , university, and state capital. We also control for the logarithm of the city population at the beginning of century  $t$ , and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: \* $p < 0.1$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$

As in the case of Latin *algorismi*, Europe came in contact with Ptolemy’s works thanks to the first translations of his texts from Arabic into Latin. For example, one of Ptolemy’s main works – the *Almagest* – was rediscovered by Western Europe thanks to the translations from Arabic to Latin carried out by Gerard of Cremona in late 12th-century *al-Andalus*

20. There is no complete survey of all surviving Latin *algorismi*, but evidence of the Latin Ptolemaic tradition is a reliable proxy of the circulation of Arabic arithmetic in the centres of learning in Europe.

(Kunitzsch 1990). Ptolemy’s works were among the most advanced astronomical texts of the time and required advanced mathematical skills to be understood.

We retrieve information on Ptolemy’s manuscripts written in Latin from the data collected by the *Ptolemaeus Arabus et Latinus* (PAL) project, which provides a reconstruction of the entire Latin *corpus Ptolemaicus*, i.e. of all Latin manuscripts which include works by or attributed to Ptolemy (Folkerts and Lorch 2000; Kunitzsch 2004).<sup>21</sup> The database provides information about the estimated date, origin, and provenance of 681 manuscripts belonging to the Ptolemaic tradition. By exploiting the information on the origin of manuscripts provided by the PAL project, we can locate Ptolemaic manuscripts in 57 cities (10 countries) of our database. We then replicate the analysis of the previous sections by replacing practical arithmetic manuals with Ptolemaic manuscripts. The results are reported in table 6 and show no significant effects of the presence of Ptolemaic manuscripts on urban growth. This rules out the academic channel as a possible alternative mechanism of growth.<sup>22</sup>

## 5 Additional robustness tests

We report a series of additional tests to further stress the robustness of our results and provide additional details on the phenomenon we are analysing.

**Fine-grained economic data.** We use more fine-grained data to explore the relationship between practical arithmetic and economic growth. Specifically, we leverage data on real wages from Serafinelli and Tabellini (2022), based on evidence gathered by Allen (2001). This information is more specific to economic development than urban population, is available for a long period for a sample of 26 cities in several European countries, and is collected per

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21. Data are available here: [PAL: Latin Manuscripts](#).

22. We cannot replicate the instrumental variable exercise in this setting since the diffusion of astronomical texts was not conditioned by the availability of manuals of practical arithmetic translated into the local vernacular. Moreover, the diffusion of astronomical manuscripts relied much less on proximity since the network of European centres of learning was significantly more integrated than that of vocational schools, and Latinate scholars were much more mobile than vernacular teachers. The spread of astronomical manuscripts was not multicentric but, in the beginning, linear from West to East, it did not require any translation, and was limited to universities and monasteries with natural science scholars. This diffusion process conflicts with our instrumental variable, designed to mirror the multicentric diffusion of practical arithmetic manuals in different languages across space and time.

decade rather than per century. We replicate the staggered DiD exercise using real wages of skilled workers as our outcome variable and using decades (rather than centuries) as our time periods. Data on skilled wages make it possible to investigate the relationship between practical arithmetic manuals and a more direct measure of commercial development because practitioners trained in practical arithmetic certainly feature among skilled workers, and were active in numerically-intensive sectors such as commerce and banking. Although the number of cities is limited compared to population data, we estimate a 15.9% increase in the real wages of skilled workers during the three decades following the introduction of practical arithmetic manuals in cities adopting these texts (see figure B1).<sup>23</sup>

**Controlling for city longitude and latitude.** In pooled OLS estimates, it is possible to further control for city location and spatial dependency by replacing country fixed effects with the *longitude* of the city, its *latitude*, and *their interaction*. Results, reported in table B1, are qualitatively unchanged.

**Controlling for spatial correlation.** While it is not possible to control for longitude and latitude in panel fixed-effect estimates, we can account for potential dependency based on spatial proximity by introducing Conley standard errors (Conley 1999, 2008) in our analysis. Given the limited state capacity of late medieval cities and the small average size of their territorial sovereignty, we consider the potential spatial effect in a radius of 50 km around the city. Results, reported in table B2, are comparable to those obtained in our main estimates (table 2), in which we use standard errors clustered at the city level. In both cases, the number of practical arithmetic manuals – in the entire century or in the second half of the century – is positively (coefficients are unchanged) and significantly associated with the city growth of the following century.

**Heterogeneous effects by city size.** We test whether the results are driven by city size. Table B3 shows that small cities, i.e. cities with up to 3,000 inhabitants at the beginning

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23. It is worth mentioning that the average treatment effect on real wages of unskilled workers resulting from a staggered DiD exercise is not significant.

of the century, benefit the most from the presence of practical arithmetic manuals.<sup>24</sup> This table presents panel fixed-effect estimates in which we interact the *Manuals* variables with a dummy variable indicating small cities. The interaction term is positive and significant for all possible indicators of the presence of manuals, while the coefficients associated with our main dependent variables remain qualitatively unchanged.

**Matched sample estimations.** To compare entities with similar characteristics, we match cities with and without practical arithmetic manuals in the period of observation, based on the presence of university and state capital, country, list of centuries in which we have information on city population, average population in the observed period, and geographical features (Roman roads, seaports, navigable rivers). The resulting sample includes 93 cities with at least one practical arithmetic manual in 13<sup>th</sup>-16<sup>th</sup> centuries and 93 cities without these texts but with similar characteristics. We use several matching strategies – i.e., nearest neighbour matching without replacement based on the propensity score difference, nearest neighbour matching based on Mahalanobis distance, and optimal pair matching – and all results are very similar to the ones reported in table 2.

**Instrumental variable falsification test.** We test the validity of our IV by replacing it with a placebo. The placebo variable definition mimics that of our main IV, as reported in section 4.2.1, but considers the distance from the most important economic centres rather than the centres of the first diffusion of the manuals. Specifically, we consider the distance of a city from large centres in the same linguistic area in the previous century. We define large centres as cities in the top 5% of the population distribution of the century. We then randomly select a large centre for every linguistic area in each century and use the distance from that centre as the IV for the cities in the linguistic area. If there are no large centres in the linguistic area, we compute the distance from the closest large centre of other linguistic areas. Table B7 reports the first stage of this placebo test on the IV and shows that the distance from the large cities of the linguistic area is not significantly correlated with the

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24. 11% of city-century observation has 3,000 inhabitants or fewer. Results are robust for different thresholds on the population distribution.

number of practical arithmetic manuals in the city. Contrary to our main IV, the distance from large cities is not a valid instrument and does not capture the patterns of diffusion of practical arithmetic manuals. Results are robust for different definitions of large cities.

**Alternative sample selection.** We replicate the previous analyses by considering only the Buringh (2021) dataset on European cities' population, without complementing it with data on Italian cities by Malanima. The sample includes information on 773 cities in twelve countries and 1536 city-century observations. Among those cities, 90 have at least one practical arithmetic manual in the observed period. While this dataset contains fewer cities and less precise information on small Italian cities, all previous results are confirmed since they remain qualitatively unchanged, and we report the most relevant ones in the appendix.

## 6 Conclusions

This paper explores the idea that the opportunities for growth depend on the socially distributed applications of knowledge to solving emergent problems and devising new tools and techniques, which in turn open new avenues for economic development and generate economic value. We have focused on the diffusion of a major mathematical innovation in commercial practices, and its implications for knowledge production and development. We have provided both qualitative evidence and quantitative analyses of the influence of the diffusion of Hindu-Arabic numerals on city-level growth: the historiographical methods shed light on the underlying economic mechanism, and the econometric analyses provide robust empirical evidence.

With respect to other forms of science and technology, mathematics provides less immediate illustrations of discontinuities associated, for example, with industrial revolutions. Yet, mathematical tools share some of the characteristics identified in general-purpose technologies (Bresnahan and Trajtenberg 1995; Jovanovic and Rousseau 2005; Bresnahan 2010) and can indeed produce paradigmatic change (Dosi 1988) in the structure and evolution of economic systems. The statistical and data-science foundations of a variety of advanced digital technologies may come to mind. There are of course other examples. Borjas and Doran

(2012), for instance, have studied the migration of Soviet mathematicians to North America, and the impact of their distinctive mathematical knowledge. In his path-breaking paper on the economics of scientific research, Nelson (1959) spoke eloquently about the importance of Maxwell’s equation for technical progress. The particular type of innovation we have studied in this paper – the introduction of Hindu-Arabic numerals – was an instrument that provided a range of very practical uses for the institutions of modern markets, and can indeed be understood as the diffusion of a general-purpose technology. The same innovation circulating in a narrower subset of the European population and applied to the solution of more theoretical problems – in our analysis exemplified by manuscript texts of Ptolemaic astronomy circulating among European scholars – generated no visible productivity gains. We argue that the socially distributed diffusion in Europe of this kind of mathematics and of the practical knowledge that was associated with it was indeed a numerical revolution, without which the prospects of economic growth would have been different.

The adoption of this particular body of knowledge was organized in society through purposeful learning activities in institutions, the practical arithmetic schools, which formed a vernacular curriculum parallel to that of the universities, and provided a key channel for the diffusion of mathematical innovations. It is important to stress that, as we have shown in sections 2 and 3, practical arithmetic texts reached, at different times for different places, a wide social circulation through a series of bottom-up decisions that did not follow academic hierarchies. While the tradition of the Latin *algorismi* was dominated by the names of a few authors,<sup>25</sup> the European tradition of practical arithmetic comprises hundreds of ‘minor’ authors who were mathematical practitioners active in the world of the mechanical arts. These actors, who are the ancestors of the mathematical practitioners studied by Kelly and Ó Gráda (2022), incrementally contributed to the development of a mathematical/commercial culture which eventually brought about measurable economic effects.

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25. Such as Alexander of Villedieu and Johannes de Sacrobosco.

societies unleashed the economic potential of the new techniques of calculation. These became a sort of ‘social technology’ (Nelson and Sampat 2001) for devising new ways to manage trade, make commercial decisions, design instruments, find routes at sea, improve precision, calculate proportions and exchanges, predict, survey, etc. One is left to wonder what type of market economy would have developed without this particular application of human intellect. In the end, this combination of mathematics, commercial practices, and their social circulation proved to be a crucial engine of epistemic, technical, and social changes which may help us reflect on the complex dynamics of ‘modern’ economic development.

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# A Data appendix

*For Online Publication*

## A.1 Database of practical arithmetic manuals: description and sources

Running from the late 13th century to 1600, the database of practical arithmetic manuals includes both manuscript and printed sources. Each manuscript source is considered in the dataset as an independent document, while printed manuals are recorded at the book-edition level. This choice is due to three independent reasons: (1) the different characteristics of manuscript and printed sources, (2) the varying social circulation of European practical arithmetic manuals, both manuscript and printed, and (3) the nature of the evidence extant. With regards to reason (1), while there is a degree of variation between copies of the same edition of a printed text (and this is particularly true for early incunabula), these differences are not comparable to the diversity presented by manuscript copies of the same text. This is due to the malleability of the manuscript medium in comparison with the standardisation determined by movable-type printing. This is particularly true for practical sources such as practical arithmetic manuals, because manuscript practical texts were often not copied with the aim of reproducing the original text, but, rather, with the aim of meeting their users' individual needs (Bocchi 2017; Long 2009).

As it was common for manuscript manuals to be handed down across generations of users, manuscript manuals often show sections added by different hands at different moments in time. Just to mention a few examples, the earliest Venetian manual presents these characteristics, as it is a notebook written by several hands starting from the 1340s which covers arithmetical operations with fractions, a variety of commercial problems, a section on practical geometry, and one on commercial customs, weights, and measures.<sup>26</sup> With its 18 surviving manuscript copies, Maestro Benedetto's *Trattato d'abaco* (c. 1465) was probably the most influential manuscript abacus manual of the 15th century. These copies show a high degree of variation, as copyists selected the parts which were most interesting for them,

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26. Florence, Biblioteca Riccardiana, ms. Ricc. 2161.

and arranged the sections according to their needs (Franci 2003; Ulivi 2002; Van Egmond 1980). A late example of the highly distinctive characteristics of each manuscript manual is the so-called memorandum-book of Richard Hill, a grocer from London who in the first half of the 16th century compiled a manuscript which mixes Latin, French, and English, gathering transcriptions of popular carols, chronicles, tables of exchange for wool cloth across the Channel, personal notes, and a text on practical arithmetic.<sup>27</sup> This high specificity of each manuscript manual justifies the choice to consider copies of the same text as independent items in the database.

With regards to the social circulation of these texts – i.e. reason (2) – the ‘social reach’ of each practical arithmetic manual tended to decrease with time. As we have seen, early manuscript manuals were probably texts which circulated in the hands of practical arithmetic masters. This makes these texts distinctively public sources, which were transmitted across generations of masters, and whose mathematical contents ‘reached’ hundreds of students. This is not limited to the Italian tradition of abacus mathematics. One of the earliest documents attesting a French-vernacular tradition of practical arithmetic is a manuscript written by two early practical arithmetic masters active in the south of France in the 1470s, Barthélemy de Romans and Mathieu Préhoude. The latter was possibly one of the first practical arithmetic masters in Lyon, and the former’s pupil (Spiesser 2000; Spiesser 2003; Hay 1988).<sup>28</sup> One of the first texts to attest the knowledge of algebra in Germany is an anonymous manuscript written in 1481 which was used by several generations of German *Rechenmeister* (Vogel 1981; Folkerts 2002).<sup>29</sup>

With time, there is an increasing number of manuscript manuals written or owned by scribes who were not masters. For example, we have evidence of 15th-century manuals written by a rag trader (1464)<sup>30</sup> and by a painter (c. 1480),<sup>31</sup> as well as of manuscripts owned by a

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27. Oxford, Balliol College, ms. 354.

28. Cesena, Biblioteca Malatestiana, ms S-XXVI-6.

29. Dresden, Sächsische Landesbibliothek, Codex Dresdensis C 80.

30. Bologna, Biblioteca Universitaria, ms. 1612.

31. Florence, Biblioteca Mediceo-Laurenziana, ms Ash 359 (291).

carpenter (1479),<sup>32</sup> and by a medical practitioner (c. 1460).<sup>33</sup> This is again not limited to the Italian tradition, as there is evidence that manuscript manuals were circulating among Catalan merchants from the late 15th century (Rey 2004).<sup>34</sup> As the first printed practical arithmetic manual was published in 1478,<sup>35</sup> and as the production of printed practical arithmetic manuals did not take off until the last decade of the 15th century, this increasing social circulation of practical arithmetic texts predated the effects on this tradition of the introduction of printing, which, as a consequence, acted as a rapid accelerator of a pre-existing trend. Thanks to much lower prices, printed manuals of practical arithmetic became affordable to a wider public, and therefore increasingly circulated in the hands of practitioners and students. While the printing press dramatically expanded the number of copies in circulation, the ‘social reach’ of practical arithmetic manuals decreased, as they moved from being public texts (in the case of early manuscript manuals used to lecture in front of generations of students) to increasingly private documents (e.g. printed manuals used by a single individual).

The third reason to record printed manuals at the book-edition level relates to the evidence extant. While we have good evidence regarding the editions of early printed European texts, with over 27,000 editions known before 1501, we have details about their print runs in only 160 cases. Moreover, these are often editions with special characteristics, making this evidence exposed to selection bias (Nuovo 2013, 104). As a consequence, the widespread scholarly practice is to set the focus of analysis to the book-edition level, and to consider the number of reprints as a proxy for the success of an early modern printed text. For example, scholarly catalogues, both printed and digital, typically include information at the book-edition level (Smith 1908; Smith 1939; Van Egmond 1980; Hoock 1991; Navarro Brotons 2000). For all these reasons concerning the different characteristics of manuscript and printed texts, the social circulation of practical arithmetic texts, and the available evidence on printed editions, considering manuscript documents at the copy-level and printed docu-

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32. Florence, Biblioteca Marucelliana, ms. A. c. s. 47.

33. Florence, Biblioteca Mediceo-Laurenziana, ms. Ash 1128.

34. Arxiu Històric de Mallorca, ms Diversos 37B/2. Antigua signatura: C108.

35. This is the anonymous *Aritmetica di Treviso*.

ments at the edition-level seemed the preferable choice.

Both secondary and primary sources were used in the construction of the database of practical arithmetic manuals. Catalogues are the most important secondary source. With respect to the Italian abacus tradition, the database includes all the evidence from the catalogue edited by Van Egmond (1980), which remains the most comprehensive source of its kind documenting this tradition. Even though this catalogue is relatively more thorough on manuscript sources than printed books (it does not provide summaries of contents for the latter), whenever possible, these sources were consulted directly, either through direct inspection or through digitised copies, and their content was manually recorded. Moreover, several findings of successive research in the field were integrated in the evidence provided by Van Egmond (Bocchi 2017; Franci 2015; Franci 2003; Ulivi 2011; Ulivi 2002; Long 2009). With respect to the European tradition, the database includes a variety of sources. Their foundation is the study by Smith (1908), a monumental work based on extensive archival research efforts. This source needed to be treated with care, and required the definition of precise criteria since it also includes texts that are arguably no part of a European tradition of practical arithmetic. Firstly, our database excludes all texts that do not present Hindu-Arabic numerals. Secondly, among the texts that do use Hindu-Arabic numerals, we retained only those that are part of the practical arithmetic tradition. This implies the exclusion of the early modern reprints of classical, as well as early medieval, sources (e.g. all the reprints of Boethius' *Arithmetica* and all the early modern texts in this tradition, jointly with works related to numerology and the computi for the calculation of the calendar).<sup>36</sup> Thirdly, the database includes only texts that were relevant for practice in that they 1) contained practical applications of mathematics; 2) were explicitly addressed to practitioners, or 3) could have been used in activities of commercial training. The implication is that purely theoretical works developed in universities (e.g. most of the algorismi tradition) did not enter

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36. The computi were texts that taught how to calculate the calendar and were widely used, particularly in ecclesiastical contexts, for the calculation of important dates, such as Easter. They constituted a relevant channel of diffusion for Hindu-Arabic numerals and in a number of cases they preceded by a considerable margin the spread of practical arithmetic Nothaft 2014.

the database. Theoretically-oriented algebraic texts were only included if they contained at least some practical applications.<sup>37</sup>

Relevant online repositories, depending on the kind of text under investigation, were used to check Smith's records. These included: the *Incunabula Short Title Catalogue* (ISTC), the *Universal Short Title Catalogue* (USTC), the *English Short Title Catalogue* (ESTC), the online catalogue of the Bibliothèque Nationale de France and their digitalisation project (Gallica), the Münchener DigitalisierungsZentrum of the Bayerische Staatsbibliothek, and the Biblioteca Virtual Miguel de Cervantes (Cervantes Virtual). These research tools, which were unavailable to both Smith and Van Egmond, allowed for a significant expansion of the information provided on every text, including identification of new texts by the same author, identification of new authors, and discovery of 1) editions not included in older catalogues, 2) holding institutions and 3) classmarks of original copies. Moreover, it was possible to identify and report every digital reproduction. When referring to these research tools, the risk of circularity was avoided by including their additional evidence only if it was based on information richer or independent from the sources that were already being consulted.

The research protocol we have just described (i.e. the criteria for inclusion in the database and the use of online repositories) was applied to the consultation of all catalogues. The texts cited in Smith (1939) were included only if it is clear that they had been consulted by Smith himself or if the USTC provided expanded the information on them, therefore corroborating Smith's information. Navarro Brotons (2000) contained valuable information on Iberian sources, whereas Hoock (1991) helped to consolidate the evidence available for central and northern Europe. The information provided by these sources was integrated by means of an extensive list of specialistic papers and local studies.

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37. Among the relevant examples, the *De arte supputandi libri quattuor* by Cuthbert Tonstall (London, 1522) is included in the database despite its predominant theoretical focus because it is the first text published in England that was entirely dedicated to arithmetic. Similarly, Petrus Ramus' *Arithmeticae libri tres* (first ed. Paris, 1555) is included because given the importance attributed by the author to applied mathematics Angelini 2008. Among the texts that are excluded even though they were of demonstrable practical relevance are the manuals concerning finger reckoning. Some of the late manuals on finger reckoning present Hindu-Arabic numerals, but belong to a different tradition than that of practical arithmetic.

Moreover, one of the authors of this paper personally visited archives in Florence and in Bologna, as well as archives in the British Library in London, the Cambridge University Library in Cambridge (UK) and the Bibliothèque nationale de France in Paris. The visits generated first-hand evidence and new documentation on early primary sources – of great interest – of Italian and European practical arithmetic traditions. Whenever possible, during the visits, the historian inspected the content of the texts. Through a combination of direct inspection of original or digitised copies, and through secondary studies, this work has generated data on the contents of 1051 texts (82% of all recorded texts).

Figure A1 provides an example of an unpublished 14th-century manual of practical arithmetic from Florence. The folios represent, respectively, a) an introductory page providing conversion tables between monetary units and a table of contents; b) multiplication tables; c) a problem asking to calculate the duration of the same maritime voyage using different types of sails. These examples are taken from one of the most important abacus manuals written in 14th-century Florence: Paolo dell’Abaco’s *Trattato di tutta l’arte dell’abaco*, written around 1339.

Figure A2 gives an example of a printed manual of practical arithmetic, provided by Adam Ries’ *Rechenung auff der linien und federn* (1527). With over one hundred reprints, this was the most successful practical arithmetic manual of 16th-century Germany. The image reproduces, respectively, a) the title page of the manual; b) the page introducing Hindu-Arabic numerals; c) the section dedicated to problems of exchange.

How representative is this sample of texts? We cannot provide a definitive answer to this question because the size of the population of reference is unknown. It is extremely difficult to estimate the sample’s representativeness considering that we do not know how many manuals of practical arithmetic were written in Europe between the 13th and the end of the 16th century. We do know, however, that the Italian abacus tradition is reasonably well documented during the first part of the observation period (14th and early 15th centuries). The same cannot be said for other parts of Europe, e.g. Spain and the south of France,

Figure A1: Example of a 14th-century Florentine manual of practical arithmetic: Paolo dell'Abaco's *Trattato di tutta l'arte dell'abaco* (c. 1339). Source: Florence, Biblioteca Nazionale Centrale, ms. Fondo Nazionale, II, IX, 57, fols. 19v, 25v, 121v.

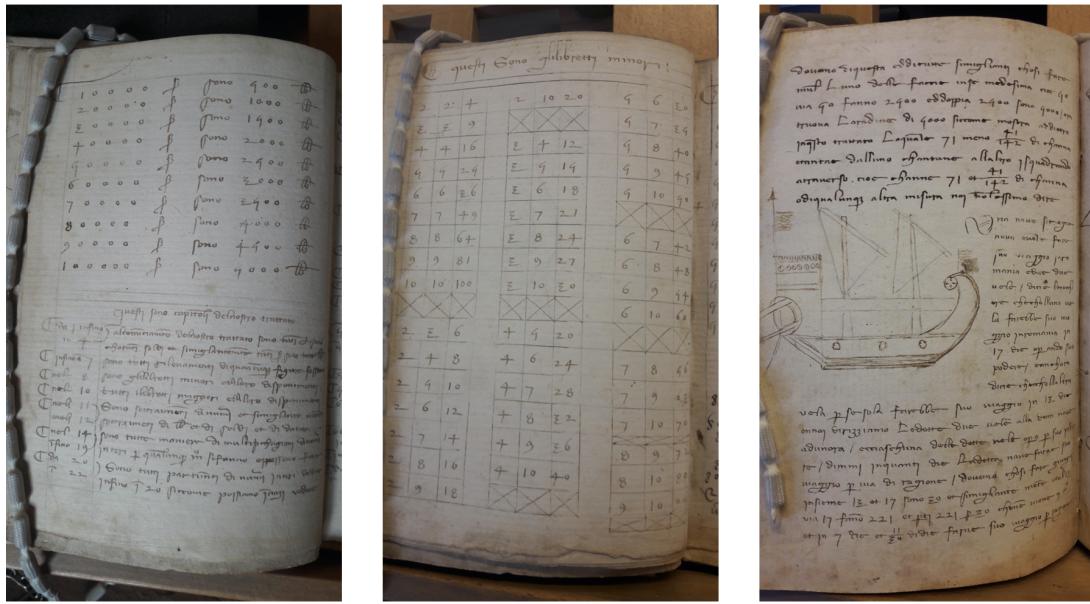
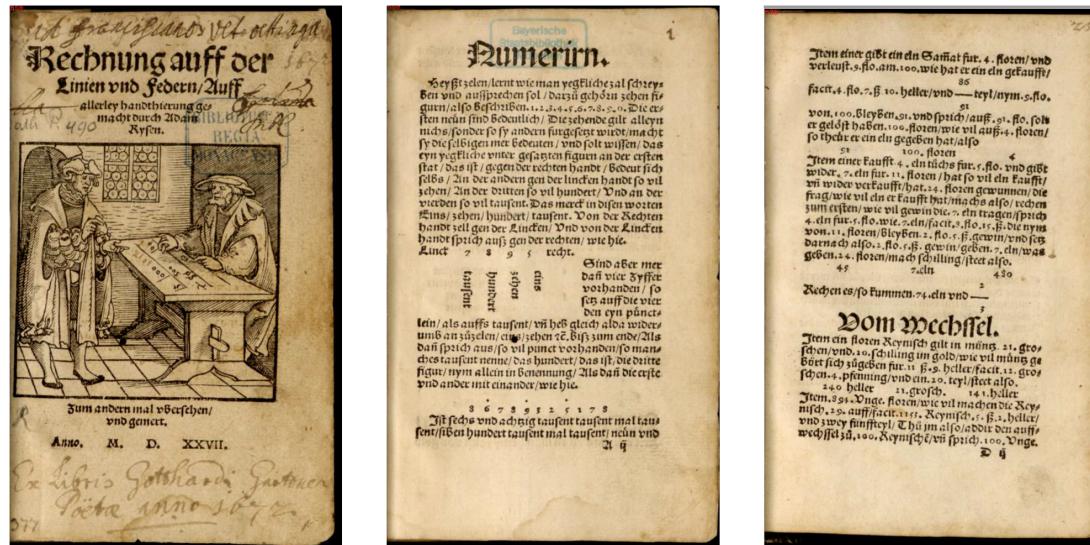


Figure A2: Example of a printed practical arithmetic manual: Adam Ries (1527) *Rechenung auff der linien und federn*, fols. ir, 1r, 25r. Image: Bayerische Staatsbibliothek, M unchener DigitalisierungsZentrum



where there is evidence of far fewer manuscripts than the case of Italy. As we have already mentioned, we do not have a good picture of the mathematical exchanges that may have

taken place in the early phase in the Mediterranean region because several sources may not have survived to the present day. It is possible that the manuscript material of the very early stages of the diffusion of European practical mathematics may be lost, except for relevant Italian sources. Of the known surviving manuscripts from the pre-printing age, our database offers very good coverage.

The quantity and quality of available sources increases considerably as we enter the age of the printing press. An overwhelming amount of the texts was produced in this period. Although it is conceivable that some materials are missing, the most reliable repositories currently available were used to compile detailed information on these stages of diffusion, and whenever possible each and every record has been manually double-checked.<sup>38</sup> The sources concerning the printing-press-era manuscripts from outside of Italy have not been studied as comprehensively as would be desirable.

Nevertheless, it is important to emphasise that 1) known sources of European printed texts can probably be considered as a representative sample of their respective traditions,<sup>39</sup> and 2) the database provides a highly satisfactory coverage of them. Crucially, the quality of the data is even better for Italy, for which the data can be considered a reliable sample of both the manuscript and printed traditions.

To summarise, in the case of Italy, both the manuscript and printed traditions have been studied and are relatively well preserved. The database offers good coverage of both. As far as the broader European region is concerned, few early manuscripts have been preserved, but the known sources have been studied in detail and recorded in the database. Conversely, manuscript material from the printing-press-era has not been comprehensively studied and, as a consequence, is probably not recorded in full. This, however, does not represent a concern, as evidence on the circulation of practical arithmetic at the city-level in this period

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38. The database relies on sources that cover most western-European areas. The data collected is arguably stronger for Italian, German, English printed texts, given the amount of studies that have addressed the areas (especially Italy) and the quality of the work that has been done on cataloguing and digitising primary sources (especially for Germany and England).

39. Information on editions of early European printed texts is abundant, as we know of over 27,000 editions of printed texts between 1450 and 1500 (Nuovo 2013, 104).

is provided by printed sources. Printed European material is instead abundant: it can be considered qualitatively and quantitatively representative of its tradition, and has been thoroughly recorded in the database. Given the type of historical sources, the database contains as good a sample as possible of the known sources, and one that is arguably representative for the purposes of our study.

## A.2 Historical evidence on the diffusion of practical arithmetic manuals in Europe

Practical arithmetic manuals and practical arithmetic schools appeared in Italian city states between the late 13th and the 14th centuries, as is discussed in 2.2. In other European areas, these manuals emerged in later periods. In France, an early vernacular manual (1470s) was written by one of the first practical arithmetic masters active in Lyon, Barthélemy de Romans (Spiesser 2000). De Romans was soon followed by Nicholas Chuquet, the author of the important *Triparty en la science des nombres* (1484), who consolidated the tradition of Lyonnaisse practical arithmetic schools (Benoit 1988; Flegg et al. 1985). These first manuscript texts were followed by a series of printed manuals, which grew steadily as Lyon became a key hub of European finance and trade. In Germany, the first manuscripts of practical arithmetic were written in the 1450s in Regensburg, under the influence of Venetian sources (Vogel 1954). These were followed by the first printed vernacular manuals (the *Rechenbücher*), published in Bamberg by Ulrich Wagner in 1482-1483 (Vogel 1980; Schröder 1996).<sup>40</sup> Following these first texts, the tradition of German *Rechenbücher* grew exponentially in subsequent decades together with the spread of the *Rechenschulen*, which provided the foundational training for generations of German merchants (Gebhardt and Albrecht 1996; Denzel 2002). Following the publication of *Die maniere om te leeren cyffren* in Brussels (1508) and in Antwerp (1510), the manuscript by the *rekenmeester* Christianus van Varenbraken (c. 1532) documents that also in the Low Countries the publication of these manuals was coupled with

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40. Wagner was a practical arithmetic master (*Rechenmeister*) who ran a school in Nuremberg in the second half of the 15th century.

the foundation of practical arithmetic schools (Kool 1988; Gärtner 2000, 244–47). These schools flourished first in Antwerp and, subsequently, in Amsterdam (Meskens 2013). After the first manuals printed in the 1520s and 1530s under the influence of French and Dutch texts, the English tradition of practical arithmetic manuals took off in 1543 with the publication of Robert Recorde's *The Ground of Artes* (Williams 2012). The popularity of these texts grew together with the foundation of practical arithmetic schools in London by the first generation of English masters (Woodbridge 2003, 3–5).

This chronology shows that data on practical arithmetic manuals does not provide a measure for economic activity in general, but rather a proxy for the spread of specific commercial skills. If this was not the case, we would observe practical arithmetic manuals with Hindu-Arabic numerals appear simultaneously in the main European trading and industrial centres of the 13th and 14th centuries – i.e. not only in Italy and southern Europe, but also in the Low Countries, Hansa cities, and England.

For example, both Florence and Bruges were key economic centres in 14th-century Europe, as both cities had a flourishing wool and cloth industry and relied on an extensive trade network (Goldthwaite 2009; Murray 2009). However, while the 14th century was the heyday of Florentine abacus mathematics – as marked by a booming production of manuscript manuals – in Bruges the first practical arithmetic manuals with Hindu-Arabic numerals were published only in the second half of the 16th century.<sup>41</sup> Significantly, this late appearance of vernacular practical arithmetic manuals corresponds with the fact that international finance in 14th-century Bruges was dominated by Florentines, and with the late adoption among Bruges merchants of the business practices of the commercial revolution (De Roover 1948).<sup>42</sup>

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41. The first practical arithmetic manual published in Bruges was probably Adriaen van der Gucht's *Cijfer bouck, inhoudende vele nieuwe, fraye, ende gherievighe practijcken van arithmetica*, published in 1569.

42. Similar considerations can be made about other important trade centres of northern Europe. Lübeck, for example, despite being among the leading Hanseatic cities in the 14th century – with its port providing a hub for maritime trade across the North Sea – had its first practical arithmetic manuals published only in the second half of the 16th century. In other prominent Hanseatic commercial cities, such as Bremen and Lüneburg, no practical arithmetic manuals with Hindu-Arabic numerals were published before 1600. England was also a late adopter of Hindu-Arabic numerals in commercial practices, as the first manuals of practical arithmetic in England were published in London in the 1530s (Williams 2012). It may not be by chance that, until the late 15th century, calculations of exchange proved ‘of the greatest difficulty’ for

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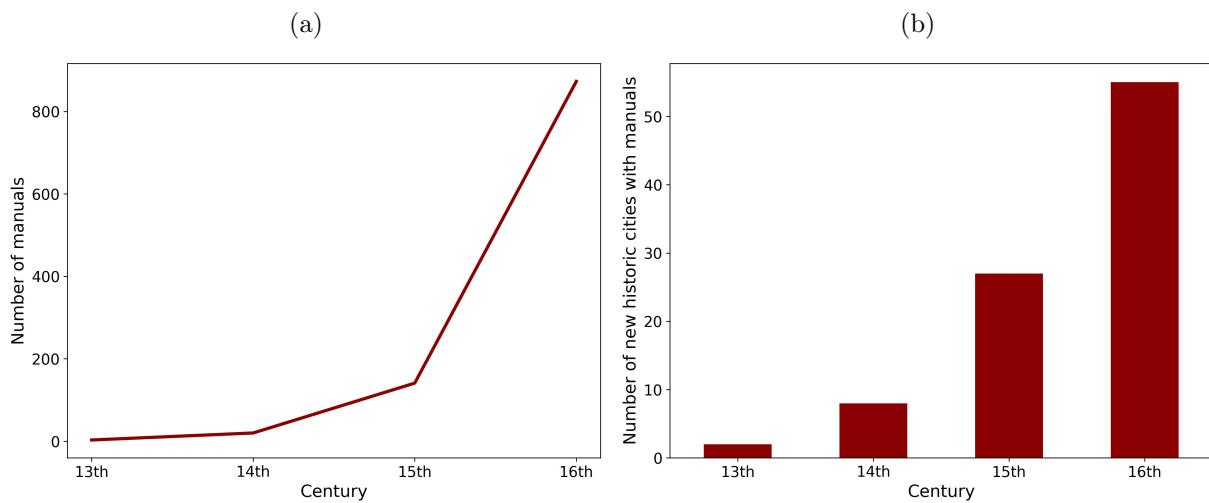
English merchants active in international trade (Hanham 1985, 165–66).

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### A.3 Additional descriptive statistics

Figure A3 reports the diffusion, in terms of the number of manuals and number of new cities that adopt those manuals, of practical mathematics during the period of interest. Combined with the information provided in table 1, this figure shows that for the period under investigation the spread of practical arithmetic manuals in Europe started in the 13th century with 3 manuals in 2 Italian cities and ended in the 16th century with 873 manuals in 81 cities across twelve European countries.

Figure A3: Number of practical arithmetic manuals and number of new cities with manuals in centuries.



Tables A1 and A2 report the descriptive statistics and correlation matrix of variables used in the regression analysis.

Table A1: Descriptive statistics

(a) Continuous variables

Variable	Mean	Std	Min	Max	Count
End-of-century population (in thousands)	13.30	27.19	1	575	1797
Beginning-of-century population (in thousands)	11.34	17.58	1	300	1797
Nb. of manuals	0.58	5.04	0.00	101	1797
Nb. of manuals 2 <sup>nd</sup> half century	0.38	3.49	0.00	67	1797

(b) Binary variables

Variable	N	%	Count
Manuals dummy	126	7.01	1797
Printing pres	448	24.93	1797
University	225	12.52	1797
Capital	135	7.51	1797
Roman road	979	54.48	1797
Navigable river	905	50.36	1797
Mediterranean port	181	10.07	1797
Atlantic port	27	1.50	1797
Baltic port	27	1.50	1797
North Sea port	112	6.23	1797

Table A2: Correlation matrix

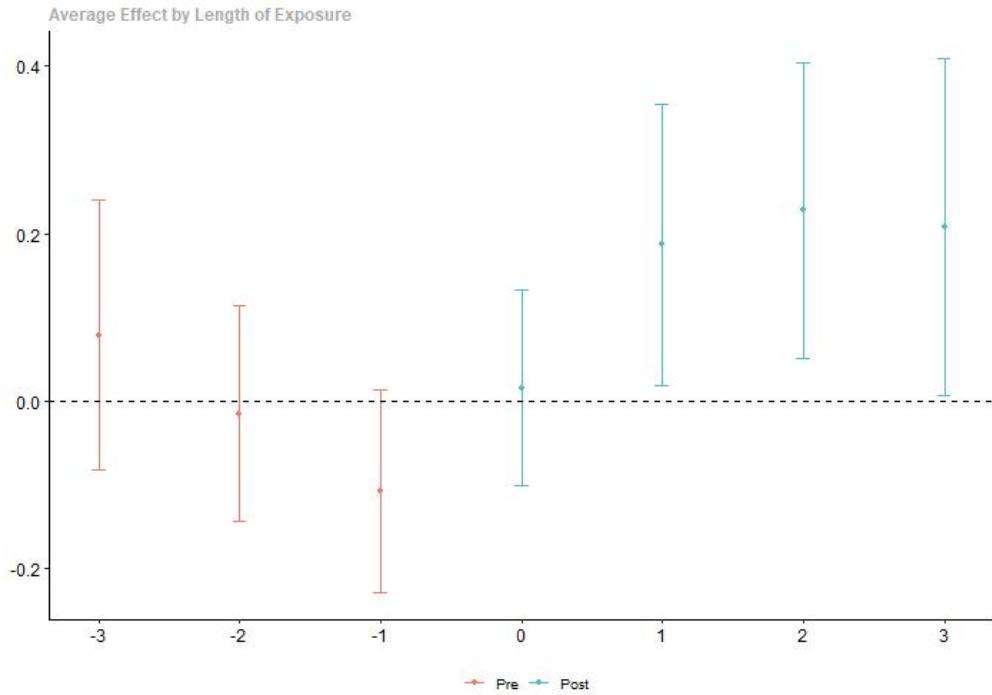
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1: End-of-century population	1.00	0.86	0.47	0.49	0.39	0.27	0.24	0.45	0.14	0.11	0.07	0.10	0.03	0.07
2: Beginning-of-century population	0.86	1.00	0.46	0.44	0.44	0.29	0.30	0.46	0.18	0.11	0.10	0.11	0.02	0.01
3: Nb. of manuals	0.47	0.46	1.00	0.97	0.42	0.19	0.10	0.16	0.02	0.07	0.02	-0.00	-0.00	0.02
4: Nb. of manuals 2 <sup>nd</sup> half	0.49	0.44	0.97	1.00	0.40	0.18	0.09	0.15	0.02	0.07	0.01	-0.01	0.00	0.03
5: Manuals dummy	0.39	0.44	0.42	0.40	1.00	0.41	0.26	0.23	0.09	0.13	0.00	0.04	0.02	0.00
6: Printing pres	0.27	0.29	0.19	0.18	0.41	1.00	0.32	0.14	0.13	0.22	-0.04	0.03	0.02	-0.00
7: University	0.24	0.30	0.10	0.09	0.26	0.32	1.00	0.23	0.18	0.15	0.04	-0.02	0.05	-0.07
8: Capital	0.45	0.46	0.16	0.15	0.23	0.14	0.23	1.00	0.10	0.12	0.09	0.03	0.02	-0.03
9: Roman road	0.14	0.18	0.02	0.02	0.09	0.13	0.18	0.10	1.00	0.11	0.19	0.06	-0.14	-0.12
10: Navigable river	0.11	0.11	0.07	0.07	0.13	0.22	0.15	0.12	0.11	1.00	-0.29	-0.05	-0.01	-0.01
11: Mediterranean port	0.07	0.10	0.02	0.01	0.00	-0.04	0.04	0.09	0.19	-0.29	1.00	-0.04	-0.04	-0.09
12: Atlantic port	0.10	0.11	-0.00	-0.01	0.04	0.03	-0.02	0.03	0.06	-0.05	-0.04	1.00	-0.02	-0.03
13: Baltic port	0.03	0.02	-0.00	0.00	0.02	0.02	0.05	0.02	-0.14	-0.01	-0.04	-0.02	1.00	-0.03
14: North Sea port	0.07	0.01	0.02	0.03	0.00	-0.00	-0.07	-0.03	-0.12	-0.01	-0.09	-0.03	-0.03	1.00

## B Robustness checks

*For Online Publication*

**Fine-grained economic data.** We study the effect of the introduction of practical arithmetic manuals by considering more fine-grained economic data than city-level population. We use evidence on cities' average real wage of skilled workers at the end of the decade following the appearance of a practical arithmetic manual. Data on average real wages are retrieved from Serafinelli and Tabellini (2022), based on evidence gathered by Allen (2001), and are available for 26 cities in 8 European countries. As observations are at the decade level but the publication of practical arithmetic manuals in a city is usually less frequent (the same manuals were used for multiple decades), we only focus on the staggered DiD exercise, which relies on the first occurrence of practical arithmetic manuals in a city. Figure B1 shows the event study performed on this sample. As almost all cities on which we have data on real wages of skilled workers eventually received the treatment, we consider not-yet-treated cities as a control group. The average treatment effect on the average real wages of skilled workers in treated cities is positive, statistically significant and equal to 0.159 (Std. Error: 0.054).

Figure B1: Event study for average real wage of skilled workers.



*Notes:* The figure shows DiD coefficients estimated following the equation 3 for European cities (time units are decades in this case). Treatment is defined as the first appearance of a practical arithmetic manual in the city. Control cities are those never treated in the period of observation. The dependent variable is the logarithm of the average real wage of skilled workers at the end of the decade. Treatment occurs at time 0. Confidence intervals are 95%. Standard errors are clustered at the city level.

**Controlling for city longitude and latitude.** Table B1 reports the results of pooled OLS estimates with the city's longitude, its latitude and their interaction as control variables.

Table B1: Practical arithmetic manuals and city population at the end of the following century - Pooled OLS estimates with controls for city's longitude and latitude and time fixed effects.

	log(End-of-century population <sub>t</sub> )		
	(1)	(2)	(3)
log(Nb. of manuals <sub>t-1+1</sub> )	0.110*** (0.029)		
log(Nb. of manuals 2 <sup>nd</sup> half-century <sub>t-1+1</sub> )		0.158*** (0.032)	
Manuals dummy <sub>t-1</sub>			0.124** (0.053)
Printing pres <sub>t-1</sub>	0.163*** (0.033)	0.159*** (0.034)	0.166*** (0.034)
University	0.061 (0.042)	0.063 (0.042)	0.064 (0.042)
Capital	0.374*** (0.066)	0.369*** (0.065)	0.384*** (0.067)
Longitude	0.041 (0.030)	0.041 (0.030)	0.042 (0.030)
Latitude	0.005 (0.005)	0.005 (0.005)	0.006 (0.005)
Longitude × Latitude	-0.0008 (0.0007)	-0.0008 (0.0006)	-0.0008 (0.0006)
log(Beginning-of-century population <sub>t</sub> )	0.712*** (0.018)	0.709*** (0.018)	0.718*** (0.018)
Geographical controls	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,797	1,797	1,797
R <sup>2</sup>	0.722	0.723	0.720

*Notes:* This table presents pooled OLS estimates of equation 1. The dependent variable is the logarithm of the population level at the end of century  $t$ . The *Manuals* variables are indicators of the presence of practical arithmetic manuals in century  $t - 1$ . Control variables include the presence of printing presses in century  $t - 1$ , university, state capital and geographical controls (Roman roads, navigable rivers, and seaport). We control for the log city population, longitude, and latitude. We add century fixed effects. Heteroskedasticity-robust standard errors are clustered at the country level and reported in parentheses. Significance level: \* $p < 0.1$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$

**Controlling for spatial correlation.** Table B2 shows the panel fixed-effect estimates with Conley standard errors, which control for potential spatial correlation in the data.

Table B2: Practical arithmetic manuals and city population at the end of the following century - Panel fixed-effect estimates with Conley standard errors.

	log(End-of-century population <sub>t</sub> )		
	(1)	(2)	(3)
log(Nb. of manuals <sub>t-1</sub> +1)	0.083** (0.035)		
log(Nb. of manuals 2 <sup>nd</sup> half-century <sub>t-1</sub> +1)		0.112*** (0.042)	
Manuals dummy <sub>t-1</sub>			0.086 (0.057)
Printing pres <sub>t-1</sub>	0.036 (0.045)	0.031 (0.045)	0.042 (0.046)
University	0.004 (0.062)	0.008 (0.062)	0.004 (0.061)
Capital	0.277*** (0.106)	0.274*** (0.105)	0.275** (0.107)
log(Beginning-of-century population <sub>t</sub> )	0.293*** (0.042)	0.291*** (0.042)	0.299*** (0.043)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,797	1,797	1,797
R <sup>2</sup>	0.895	0.895	0.895

*Notes:* This table presents panel fixed-effect OLS estimates of equation 2 accounting for spatial correlation in the data. The dependent variable is the logarithm of the population level at the end of century  $t$ , with  $t$  ranging between the 14<sup>th</sup> and 17<sup>th</sup> centuries. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in century  $t - 1$ . Control variables include the presence of printing presses in century  $t - 1$ , university, and state capital. We also control for the logarithm of the city population at the beginning of century  $t$ , and we add city and century fixed effects. Conley standard errors (50 km) are reported in parentheses. Significance level:  
\* $p < 0.1$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$

**Heterogeneous effects across cities.** Table B3 reports panel fixed-effects estimates in which we study the interaction between the *Manuals* variables and a dummy variable that indicates cities with up to 3,000 inhabitants.

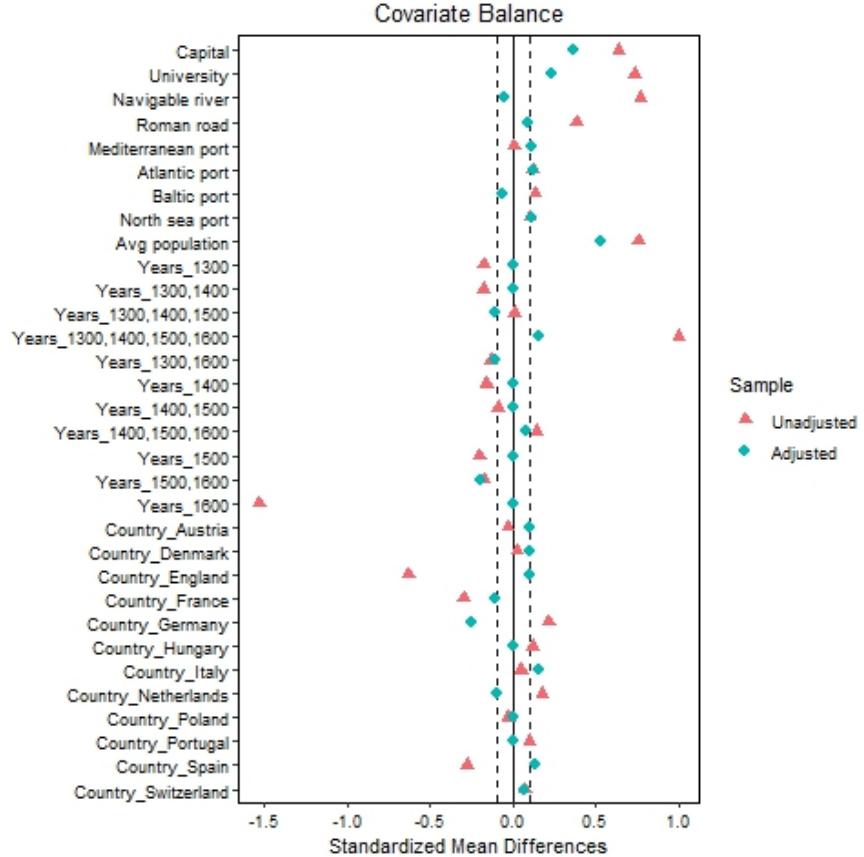
Table B3: Practical arithmetic manuals and city population at the end of the following century in small cities (up to 3,000 inhabitants).

	log(End-of-century population <sub>t</sub> )		
	(1)	(2)	(3)
log(Nb. of man <sub>t-1</sub> +1)	0.080** (0.035)		
log(Nb. of man <sub>t-1</sub> +1) × Small city	0.618*** (0.105)		
log(Nb. of man 2 <sup>nd</sup> half-century <sub>t-1</sub> +1)		0.110*** (0.041)	
log(Nb. of man 2 <sup>nd</sup> half-century <sub>t-1</sub> +1) × Small city		0.589*** (0.107)	
Manuals dummy <sub>t-1</sub>			0.077 (0.055)
Manuals dummy <sub>t-1</sub> × Small city			0.403*** (0.091)
Printing pres <sub>t-1</sub>	0.036 (0.042)	0.032 (0.042)	0.043 (0.043)
University	0.003 (0.064)	0.007 (0.064)	0.002 (0.064)
Capital	0.277*** (0.099)	0.274*** (0.098)	0.275*** (0.100)
Small city	0.010 (0.064)	0.008 (0.064)	0.023 (0.064)
log(Beginning-of-century population <sub>t</sub> )	0.297*** (0.042)	0.294*** (0.041)	0.307*** (0.042)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,797	1,797	1,797
R <sup>2</sup>	0.895	0.896	0.895

*Notes:* This table presents panel fixed-effect OLS estimates of Equation 2 in which we include an interaction term between the variable *Manuals* and a dummy variable that captures the small size of the city (cities up to 3,000 inhabitants). The dependent variable is the logarithm of the population level at the end of the century *t*, with *t* ranging between the 14<sup>th</sup> and 17<sup>th</sup> centuries. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in century *t* – 1. Control variables include the presence of printing presses in century *t* – 1, university, and state capital. We also control for the logarithm of the city population at the beginning of century *t*, and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

**Matching.** To identify pairs of cities with similar features, we compute a propensity score, estimated with logistic regression, in terms of the presence of university and state capital, country, list of centuries in which we have information on city population, average population in the observed period, and geographical features (Roman roads, seaports, navigable rivers). The resulting sample includes 93 cities with at least one practical arithmetic manual in 13<sup>th</sup>-16<sup>th</sup> centuries and 93 cities without this kind of text in the observed period. Cities are matched through a nearest neighbour matching without replacement based on the propensity score difference. Figure B2 shows the difference between the characteristics of cities with and without manuals before and after the nearest neighbour matching based on propensity score difference.

Figure B2: Distance between cities with and without practical arithmetic manuals before and after matching.



Results are reported in table B4 and, once again, they confirm the importance of practi-

cal arithmetic manuals for city growth. The estimated association between the presence of practical arithmetic manuals and urban growth in the following century is similar to the one obtained in table 2. However, by reducing the sample of cities and balancing places with and without texts, in this set of estimates, the coefficient associated with the dummy variable is positive and significant. This is a useful complement to the results obtained with the number of manuals in the interpretation of the economic importance of practical mathematics. Results are confirmed when considering alternative matching strategies such as nearest neighbour matching based on Mahalanobis distance and optimal pair matching. These additional results are presented in tables B5 and B6.<sup>43</sup>

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43. It is interesting to notice that, in these empirical designs, there is no effect associated with the presence of the printing press.

Table B4: Matching estimates. Practical arithmetic manuals and city population at the end of the following century.

	log(End-of-century population <sub>t</sub> )		
	(1)	(2)	(3)
log(Nb. of manuals <sub>t-1</sub> +1)	0.087** (0.035)		
log(Nb. of manuals 2 <sup>nd</sup> half-century <sub>t-1</sub> +1)		0.116*** (0.040)	
Manuals dummy <sub>t-1</sub>			0.099* (0.054)
Printing pres <sub>t-1</sub>	0.024 (0.060)	0.016 (0.061)	0.025 (0.062)
University	0.020 (0.062)	0.026 (0.062)	0.016 (0.062)
Capital	0.332*** (0.089)	0.329*** (0.088)	0.331*** (0.089)
log(Beginning-of-century population <sub>t</sub> )	0.384*** (0.059)	0.378*** (0.058)	0.396*** (0.060)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	617	617	617
R <sup>2</sup>	0.891	0.892	0.890

*Notes:* This table presents panel fixed-effect OLS estimates of Equation 2 on matched cities. Matching is performed with the nearest neighbour matching based on the propensity score difference among cities. The dependent variable is the logarithm of the population level at the end of century  $t$ . The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in the century  $t - 1$ . Control variables include the presence of printing presses in the city in century  $t - 1$ , university, and state capital. We also control for the logarithm of the city population at the beginning of the century  $t$ , and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Table B5: Matching based on Mahalanobis distance. Practical arithmetic manuals and city population at the end of the following century.

	log(End-of-century population <sub>t</sub> )		
	(1)	(2)	(3)
log(Nb. of manuals <sub>t-1+1</sub> )	0.093*** (0.035)		
log(Nb. of manuals 2 <sup>nd</sup> half-century <sub>t-1+1</sub> )		0.122*** (0.040)	
Manuals dummy <sub>t-1</sub>			0.105* (0.054)
Printing pres <sub>t-1</sub>	0.020 (0.058)	0.013 (0.058)	0.022 (0.059)
University	0.036 (0.065)	0.042 (0.065)	0.032 (0.065)
Capital	0.348*** (0.089)	0.344*** (0.088)	0.346*** (0.090)
log(Beginning-of-century population <sub>t</sub> )	0.391*** (0.059)	0.386*** (0.057)	0.404*** (0.059)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	602	602	602
R <sup>2</sup>	0.897	0.898	0.896

*Notes:* This table presents panel fixed-effect OLS estimates of Equation 2 on matched cities. Matching is performed with the nearest neighbour matching based on Mahalanobis distance among cities. The dependent variable is the logarithm of the population level at the end of century  $t$ . The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in century  $t - 1$ . Control variables include the presence of printing presses in century  $t - 1$ , university, and state capital. We also control for the logarithm of the city population at the beginning of century  $t$ , and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: \* $p < 0.1$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$

Table B6: Optimal pair matching. Practical arithmetic manuals and city population at the end of the following century.

	log(End-of-century population <sub>t</sub> )		
	(1)	(2)	(3)
log(Nb. of manuals <sub>t-1</sub> +1)	0.093*** (0.035)		
log(Nb. of manuals 2 <sup>nd</sup> half-century <sub>t-1</sub> +1)		0.122*** (0.041)	
Manuals dummy <sub>t-1</sub>			0.107** (0.054)
Printing pres <sub>t-1</sub>	0.047 (0.061)	0.040 (0.061)	0.048 (0.062)
University	0.028 (0.063)	0.034 (0.063)	0.023 (0.062)
Capital	0.335*** (0.089)	0.332*** (0.088)	0.334*** (0.090)
log(Beginning-of-century population <sub>t</sub> )	0.369*** (0.061)	0.363*** (0.060)	0.382*** (0.062)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	617	617	617
R <sup>2</sup>	0.890	0.891	0.889

*Notes:* This table presents panel fixed-effect OLS estimates of Equation 2 on matched cities. Matching is performed with an optimal pair matching based on the propensity score difference among cities. The dependent variable is the logarithm of the population level at the end of century  $t$ . The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in century  $t - 1$ . Control variables include the presence of printing presses in century  $t - 1$ , university, and state capital. We also control for the logarithm of the city population at the beginning of century  $t$ , and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: \* $p < 0.1$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$

**Instrumental variable falsification test.** Table B7 shows the first stage of 2SLS regression, in which we test a placebo for the instrumental variable: we replace the distance from the first city with practical arithmetic manuals written in local vernacular in a linguistic area during the previous century with the distance from large cities of the linguistic area (if any) – i.e. cities in the top 5% of the population distribution – in the previous century.

Table B7: Falsification test. First stage.

	log(Nb. of man <sub>t-1+1</sub> ) (1)	log(Nb. of man 2 <sup>nd</sup> half-century <sub>t-1+1</sub> ) (2)	Manuals dummy <sub>t-1</sub> (3)
log(Distance large city <sub>t-1+1</sub> )	0.015 (0.043)	0.011 (0.030)	0.026 (0.017)
Printing pres <sub>t-1</sub>	0.282*** (0.044)	0.246*** (0.039)	0.191*** (0.025)
University	-0.030 (0.097)	-0.062 (0.076)	-0.025 (0.047)
Capital	-0.121 (0.107)	-0.060 (0.098)	-0.094 (0.089)
log(Beginning-of-century pop <sub>t</sub> )	0.119*** (0.036)	0.110*** (0.034)	0.049*** (0.018)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,797	1,797	1,797
R <sup>2</sup>	0.545	0.529	0.554

*Notes:* This table presents the first stage of panel fixed-effect 2SLS estimates, in which we use the distance from a large city of the linguistic area, i.e. a city in the top 5% of the population distribution, as an instrumental variable. The dependent variables are indicators of the presence of practical arithmetic manuals in the city in century  $t - 1$ , with  $t$  ranging between the 14<sup>th</sup> and 17<sup>th</sup> centuries. Control variables include the presence of printing presses in century  $t - 1$ , university, and state capital. We also control for the logarithm of the city population at the beginning of century  $t$ , and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: \* $p < 0.1$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$

**Different sample selection.** Tables B8, B9, and B10 report the main results derived by a replication of the previous analyses on a sub-sample of data, i.e. considering only observations in the Buringh (2021)'s dataset of European cities' population. Figure B3 shows the event study performed on this sample. The overall ATT is positive, statistically significant and equal to 0.461 (Std. Error: 0.125), corresponding to a 46.1% increase in the city population during the three centuries following the first adoption of practical arithmetic manuals.

Table B8: Sub-sample. Practical arithmetic manuals and city population at the end of the following century – Panel fixed-effect estimates.

	log(End-of-century population <sub>t</sub> )		
	(1)	(2)	(3)
log(Nb. of manuals <sub>t-1+1</sub> )	0.071* (0.038)		
log(Nb. of manuals 2 <sup>nd</sup> half-century <sub>t-1+1</sub> )		0.101** (0.043)	
Manuals dummy <sub>t-1</sub>			0.067 (0.063)
Printing pres <sub>t-1</sub>	0.014 (0.048)	0.009 (0.049)	0.021 (0.050)
University	-0.019 (0.073)	-0.016 (0.074)	-0.019 (0.072)
Capital	0.392*** (0.110)	0.388*** (0.108)	0.391*** (0.111)
log(Beginning-of-century population <sub>t</sub> )	0.309*** (0.042)	0.306*** (0.041)	0.314*** (0.042)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,536	1,536	1,536
R <sup>2</sup>	0.905	0.905	0.905

*Notes:* This table presents panel fixed-effect OLS estimates of equation 2. The dependent variable is the logarithm of the population level at the end of the century  $t$ , with  $t$  ranging between the 14<sup>th</sup> and 17<sup>th</sup> centuries. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in century  $t - 1$ . Control variables include the presence of printing presses in century  $t - 1$ , university, and state capital. We also control for the logarithm of the city population at the beginning of century  $t$ , and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Table B9: Sub-sample. Practical arithmetic manuals and city population at the end of the following century - Instrumental variable estimates.

	log(End-of-century population <sub>t</sub> )		
	(1)	(2)	(3)
log(Nb. of manuals <sub>t-1</sub> +1)	0.396** (0.157)		
log(Nb. of manuals 2 <sup>nd</sup> half-century <sub>t-1</sub> +1)		0.427** (0.168)	
Manuals dummy <sub>t-1</sub>			0.788** (0.316)
Printing pres <sub>t-1</sub>	-0.073 (0.064)	-0.068 (0.063)	-0.107 (0.072)
University	-0.015 (0.090)	0.002 (0.086)	-0.015 (0.083)
Capital	0.421*** (0.117)	0.396*** (0.110)	0.451*** (0.146)
log(Beginning-of-century population <sub>t</sub> )	0.268*** (0.043)	0.268*** (0.043)	0.281*** (0.042)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,536	1,536	1,536
R <sup>2</sup>	0.894	0.897	0.889
F-test (1 <sup>st</sup> stage)	88.4***	81.4***	92.1***

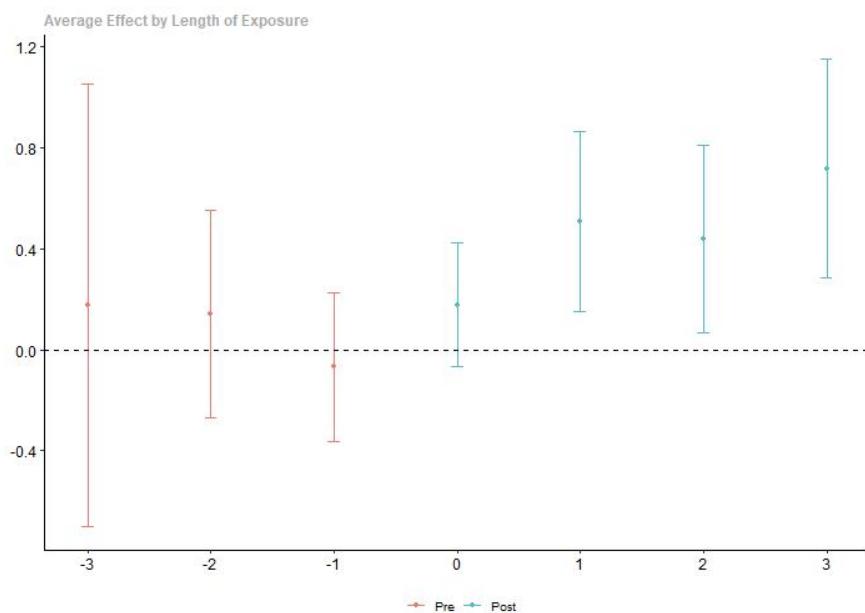
*Notes:* This table presents panel fixed-effect 2SLS estimates of Equation 2. The dependent variable is the logarithm of the population level at the end of century  $t$ . The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in the century  $t - 1$ . We instrument these variables with the distance of the city from the location of the first occurrence of practical arithmetic manuals in the local vernacular. Control variables include the presence of printing presses in the century  $t - 1$ , university, and state capital. We also control for the logarithm of the city population at the beginning of the century  $t$ , and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Table B10: Sub-sample. Ptolemy's manuscripts and city population at the end of the following century.

	log(End-of-century population <sub>t</sub> )		
	(1)	(2)	(3)
log(Nb. of Ptolemy's man <sub>t-1</sub> +1)	-0.100 (0.070)		
log(Nb. of Ptolemy's man 2 <sup>nd</sup> half-century <sub>t-1</sub> +1)		-0.091 (0.072)	
Ptolemy's manuscripts dummy <sub>t-1</sub>			-0.127* (0.071)
Printing pres <sub>t-1</sub>	0.042 (0.050)	0.038 (0.050)	0.043 (0.049)
University	-0.030 (0.072)	-0.025 (0.072)	-0.031 (0.072)
Capital	0.374*** (0.113)	0.387*** (0.111)	0.372*** (0.113)
log(Beginning-of-century population <sub>t</sub> )	0.318*** (0.043)	0.317*** (0.043)	0.318*** (0.043)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,536	1,536	1,536
R <sup>2</sup>	0.905	0.905	0.905

*Notes:* This table presents panel fixed-effect OLS estimates on the placebo dataset. The dependent variable is the logarithm of the population level at the end of the century  $t$ , with  $t$  ranging between the 14<sup>th</sup> and 17<sup>th</sup> centuries. The *Ptolemy's manuscripts* variables are indicators of the presence of Ptolemy's manuscripts in the city in century  $t - 1$ . Control variables include the presence of printing presses in century  $t - 1$ , university, and state capital. We also control for the logarithm of the city population at the beginning of century  $t$ , and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Figure B3: Sub-sample. Event study.



*Notes:* The figure shows DiD coefficients estimated following the equation 3 for European cities (sample from Buringh 2021). Treatment is defined as the first appearance of a practical arithmetic manual in the city. Control cities are those never treated in the period of observation. The dependent variable is the logarithm of city population at the end of the century and we control for the logarithm of city population at the beginning of the century. Treatment occurs at time 0. Confidence intervals are 95%. Standard errors are clustered at the city level.