$Logical\ Equivalence\ Laws:$

When you are asked to use logical equivalence laws to show something, your answer must have the following form. Each step applies only a single law. Each step is justified - using the table on the right of slide 10 and the table on slide 11 (Fig. 7.11) of week6.part1.logicII.pdf. Example:

$$\begin{split} (A \wedge B) &\to (C \vee \neg D) \\ &\equiv \neg (A \wedge B) \vee (C \vee \neg D) \\ &\equiv (\neg A \vee \neg B) \vee C \vee \neg D \end{split}$$
 (imp. elim.)
$$(De Morgans)$$

Resolution:

When running resolution, list all the sentences in the KB (numbered), then proceed with resolution steps, listing which sentences take part.

Example:

1	P	
2	Q	
3	$\neg P \lor R$	
4	$\neg Q \lor \neg R \lor S$	
5	$\neg S$	
6	R	(1,3)
7	$\neg R \lor S$	(2,4)
	:	

1. (6 points) True or False. State the correctness of each statement. You may want to use the definitions of entailment on p. 214 and 222 of the book.

 $\alpha \models \beta$ iff in every model is which α is true, β is also true.

For any sentences α and β , $\alpha \models \beta$ iff $(\alpha \rightarrow \beta)$ is valid.

(a) (1 point) False \models True

True

(b) (1 point) True \models False

False

(c) (2 points) $p \leftrightarrow q \models p \lor q$

False

PETQ = PVQ

= (P > 9) M(9 > P) |= PV2

=(7PV9)1(7qVP) =PV9

When Pand 9 are all False, it means Trave = False, which is False

(d) (2 points) $(p \lor q) \land \neg (p \to \neg q)$ is satisfiable.

True

(PV2) 17(P->79)

= (Pva) 1 7(7pv7g)

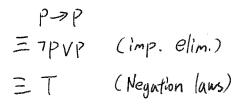
=(Pvq) 1(P1q)

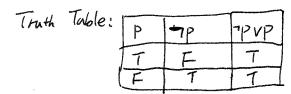
when Pistrue and Dis True, it is satisfiable, so it's True

- 2. (5 points) Consider a world that has 4 propositions p, q, r, s. How many models are there for the following sentences?
 - (a) (1 point) $p \wedge r$ 4. We keep P = r = True, and no natter how r and s change, PAr is True. Therefore the total models should be 2×2 by r and s θ values.
 - (b) (2 points) $(p \wedge r) \vee (q \wedge \neg s)$ when $(P \wedge r) = True$, there are also 4 models when $(q \wedge \neg s) = True$, there are also 4 models there is same models in these 8 models, $p = r = q = \neg s = True$ Therefore, the total models are 4 + 4 - 1 = 7
 - There are two stamodels of q and s when $(q \leftrightarrow s)$ is false. There is only one model of $p \not = r$ whon $(p \rightarrow r)$ is false. There are two models of q, s, p, r for $p(q \leftrightarrow s) \lor (p^2 \rightarrow r)$ is false. There are two models of q, s, p, r for $p(q \leftrightarrow s) \lor (p^2 \rightarrow r)$ is false. There are $z^4 = |b|$ models for all the combinations of q, s, p, r. Therefore, there are |b-2| = |4| models for $(q \leftrightarrow s) \lor (p \rightarrow r)$ are r. The answer is r.

3. (8 points) Determine whether each of the following sentences is valid, unsatisfiable, or neither. Verify the determination using both equivalence rules (Method 1) and a truth table (Method 2).

(a) (2 points) $p \to p$





therefore, the determination is valid.

(b) (6 points)
$$(p \rightarrow q) \rightarrow (\neg s \rightarrow p)$$

Truth Table:

P	5	PVS
F	FT	TT
F	F	F
F	7	T

Therefore, the determination is not ralid and & unsatisfitable neither. The is satisfiable.

4. (5 points) Let a KB consist of the following sentences: Let a KB consist of the following sentences:

$$\begin{array}{ll} 1 & p \\ 2 & p \rightarrow q \\ 3 & (p \rightarrow q) \rightarrow (q \rightarrow r) \end{array}$$

Conver the knowledge base to CNF.

$$\begin{array}{l} (P : P) \\ (P : P) \\$$

=(P/17q)V(7qVr) (Re Morgan) =[P179)V79]Vr (associativity of V)

=[(PV7q) / (7qV7q)]VF (distributivity of V over 1)

三(PV7qvr) 1 (7qvr) (distributivity of V over 1)

3. PV79Vr

4. 79 Vr

5. (6 points) Resolution

Let a KB consist of the following sentences:

$$\begin{array}{l} 1 & \neg (P \wedge \neg Q) \vee \neg (\neg S \wedge \neg T) \\ 2 & \neg (T \vee Q) \end{array}$$

$$3 \quad U \to (\neg T \to (\neg S \land P))$$

Prove $\neg U$ using resolution refutation.

Below is the KB converted into CNF.

- (1) $\neg P \lor Q \lor S \lor T$
- $(2) \neg T$
- $(3) \neg Q$
- $(4) \neg U \lor T \lor \neg S$
- (5) $\neg U \lor T \lor P$

Show the resolution method; use one rule at a time, justifying each step.

6. (20 points) Resolution

A famous logic problem using the following propositional symbols to construct a KB:

Y "unicorn is mythical"

I "unicorn is immortal"

M "unicorn is mammal"

H "unicorn is horned"

G "unicorn is magical"

- (a) (4 points) Translate English sentences to logic. Think of the paragraph as the following sentences:
 - (i) If the unicorn is mythical, then it is immortal.
 - (ii) If the unicorn is not mythical, then it is a mortal mammal.
 - (iii) If the unicorn is either immortal or a mammal, then it is horned.
 - (iv) If the unicorn is horned, then it is magical.

(b) (4 points) Convert (a) to KB in CNF.

(c) (6 points) Can you prove the unicorn is magical? Answer the question by adding the negation of the query to the KB and run resolution.

To prove unicorn is magical, we can show KB 1 unicorn is not managical is unsultsfill show KB 17G is unsultstille

(d) (6 points) Can you prove the unicorn is horned? Answer the question by adding the negation of the query to the KB and run resolution.

To prove the aunicorn is horned, we can show KB 1714 is unsatisfiable

TY VI	
YVM	2
I VH	3
MVH	4
THVG	5
714	6, (assumption)
JI /	7, (7, 6)
m	3, (4, 6)
17	9,(1,7)
7 /	10, (2,8)
1}	\$1, (9, (0)

There is a contradiction. Therefore, the I-I is True, unicorn is horned.