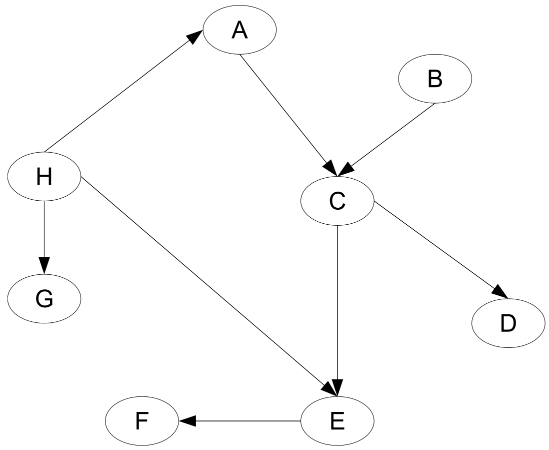
W 5

1. (6 points) Consider the following Bayesian network with variables A, B, C, D, E, F, G, H. Categorize the following as d-separated or d-connected with respect to the given BN.



(a) $A \perp\!\!\!\perp F \mid \{\ \}$ or $A \perp\!\!\!\perp F$

(b) $H \perp\!\!\!\perp D \mid A$

(c) $H \perp\!\!\!\perp D \mid \{A, E\}$

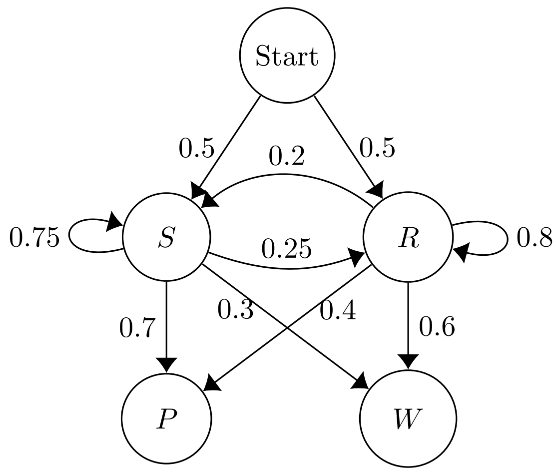
(d) $B \perp\!\!\!\perp G \mid \{\ \}$

(e) $B \perp\!\!\!\perp G \mid \{D\}$

(f) $A \perp\!\!\!\perp B \mid \{E, F\}$

2. (13 points) Hidden Markov Models

Consider the following simple model of the weather in November using an HMM. There is a unobserved variable, X_t, indicating that the weather today can be either snowing (S) or raining (R), and you observe, E_t, what the majority of people are wearing: parkas (P) or wellington boots (W). This is modeled in the following HMM:



(a) (3 points) Compute $P(X_1 = S, X_2 = R, X_3 = R, X_4 = S)$

(b) (4 points) Compute $P(X_1 = S \mid E_1 = P)$

(c) (6 points) Compute $P(E_1 = W, E_2 = W)$

3. (19 points) Hidden Markov Models II

You are modelling the state of your health $\{h$ - *healthy*, c - *coming down with something*, or u - *unwell* $\}$. Each day, you can observe whether you are sneezing s or not n to infer your state of health. Use the following information to model your health.

Initial Dist.

State	$P(X_1)$
h	0.9
c	0.1
u	0.0

Transition Model

X_t	X_{t+1}	$P(X_{t+1} \mid X_t)$
h	h	0.6
h	c	0.2
h	u	0.2
c	h	0.3
c	c	0.2
c	u	0.5
u	h	0.7
u	c	0.2
u	u	0.1

Emission Distribution

X_t	E_t	$P(E_t \mid X_t)$
h	n	0.8
c	n	0.3
u	n	0.5
h	s	0.2
c	s	0.7
u	s	0.5

(a) (2 points) What is $P(X_1 = h, X_2 = c, X_3 = u)$?

(b) (2 points) What is $P(X_1 = u, X_2 = c, X_3 = h)$?

(c) (4 points) On the first day, you did not sneeze, $E_1 = n$. What is the probability you are *coming down with something* given this observation?

(d) (8 points) You observe not sneezing on day 1 and sneezing on day 2. What's the probabilty of *coming down with something* given the sequence of observations?

(e) (3 points) You can confirm that you were healthy on days one and two. What is the probability that you will be unwell on day 3?

4. (12 points) Markov Decision Process

   A student selects fruit from a bowl of *Apples* - A and *Oranges* - O. At each state she has either an *Apple* or *Orange* in her hand. There is only one action, *Swap* - S, when she returns the fruit in her hand to the bowl, and selects a new piece of fruit, where $P(A) = 0.7$. The MDP has the following information:

   States: A - apple, O - orange
   Actions: S - swap
   Start state: A - apple
   there are no terminal states

   Let $R(A, S, O) = 3$ and $R(O, S, A) = 2$, with all other rewards = 0. Assume $\gamma = 0.5$

   (a) (12 points) Run value iteration for this MDP for three iterations and fill in the value estimates in the table. Show your work.

i	$V_i^*(A)$	$V_i^*(O)$
0	0	0
1		
2		
3		

(b) (3 points (bonus)) What are $V^*(A)$ and $V^*(O)$? *Hint: solve the Bellman equations*