W 4

1. Let A and B be two independent events with $P(A) = 0.4$ and $P(B) = 0.5$.

   (a) (2 points) What is $P(A \text{ and } B)$?

   $P(A \land B) = P(A) \times P(B) = 0.2$

   (b) (3 points) What is $P(\neg A \text{ or } \neg B)$?

   $P(\neg A \lor \neg B) = \neg P(A \land B) = 1 - P(A \land B) = 0.8$

2. Consider 4 bags of candy where the 1st bag has two Snickers, the 2nd bag has three Reese's peanut butter cups, the 3rd bag has two Snickers and three Reese's cups, the 4th bag has one Snickers and two Reese's.

   Assume a bag is selected randomly, then a piece of candy is selected randomly from the bag.

   Let B be the random variable corresponding to which bag is selected: $\{1, 2, 3, 4\}$. Let C be the random variable corresponding to the candy selected $\{Snickers, Reese\}$.

   (a) (1 point) What is the distribution of B? Present $P(B)$ as a table.

B	P(B)
1	0.25
2	0.25
3	0.25
4	0.25

   (b) (2 points) What is the conditional distribution $P(C|B)$? Present as a table.

B	C	P(C|B)
1	S	1
1	R	0
2	S	0
2	R	1
3	S	0.4
3	R	0.6
4	S	0.33
4	R	0.67

(c) (2 points) What is the joint probability of an outcome of the selecting the fourth bag and a Snickers bar? $P(B = 4, C = Snickers) =?$ *Hint: product rule*

$$P(B=4, C=Snickers) = P(C|B) \cdot P(B) = 0.33 \times 0.25 = 0.0825$$

(d) (3 points) What is the conditional probability that $P(B = 4|C = Snickers)$?

$$P(B=4|C=Snickers) = \frac{P(B,C)}{P(C)} = \frac{0.0825}{P(C)}$$

$$P(C=Snickers) = \sum_b P(C|B) \cdot P(B) = 0.25 \times 1 + 0.25 \times 0 + 0.25 \times 0.4 + 0.25 \times 0.33$$

$$= 0.25 \times (1 + 0 + 0.4 + 0.33) = 0.4325$$

$$P(B=4|C=Snickers) = \frac{0.0825}{0.4325} \approx 0.1907$$

3. Consider the following full joint distribution over 3 variables $\{A, B, C\}$

A	B	C	Prob.
a	b	c	0.108
a	b	$\neg c$	0.072
a	$\neg b$	c	0.012
a	$\neg b$	$\neg c$	0.008
$\neg a$	b	c	0.016
$\neg a$	b	$\neg c$	0.144
$\neg a$	$\neg b$	c	0.064
$\neg a$	$\neg b$	$\neg c$	0.576

(a) (1 point) $P(c)$.

$P(c) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2$

(b) (3 points) $\mathbf{P}(A)$.
Note, the bold-face "P" and capital "A" indicates that the question is asking for the probility distribution, not just a single value. For a binary variable like "A", this is often presented in the following form: $\langle$value for a, value for $\neg a\rangle$.

$P(A=a) = 0.108 + 0.072 + 0.012 + 0.008 = 0.2$

$P(A=\neg a) = 0.016 + 0.144 + 0.064 + 0.576 = 0.8$

(c) (3 points) $\mathbf{P}(C \mid a)$

$P(C|a) = \dfrac{P(C,a)}{P(a)}$

$P(C=c|a) = \dfrac{P(c,a)}{P(a)} = \dfrac{0.108+0.012}{0.2} = 0.6$

$P(C=\neg c|a) = \dfrac{P(\neg c,a)}{P(a)} = \dfrac{0.072+0.008}{0.2} = 0.4$

(d) (4 points) $\mathbf{P}(A \mid b \vee c)$

$$P(A \mid b \vee c) = \frac{P(A, b \vee c)}{P(b \vee c)}$$

$$P(A=a \mid b \vee c) = \frac{P(a, b \vee c)}{P(b \vee c)} = \frac{0.108 + 0.072 + 0.012}{0.108 + 0.072 + 0.012 + 0.016 + 0.144 + 0.064} = \frac{0.192}{0.416} \approx 0.462$$

4. (5 points) Consider two tests A and B for a virus. Test A is 95% effective at recognizing when the virus is present, but it has a 10% false positive rate (indicating that the virus is present, when it is not). Test B is 90% effective at recognizing when the virus is present, but has a 5% false positive rate. The two tests use independent methods of identifying the virus. The virus is carried by 1% of all people. Say that a person is tested for the virus using only one of the tests, and that test comes back positive for carrying the virus. Which test returning positive is more indicative of someone really carrying the virus? Show why.

Let V - be the patient has the virus, A - be test A returns positive, B - be test B returns positive.

$P(A|V)=0.95 \quad P(B|V)=0.9 \quad P(V)=0.01 \quad P(\neg V)=0.99 \quad P(A|\neg V)=0.1 \quad P(B|\neg V)=0.05$

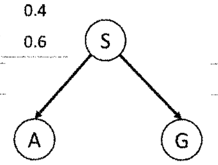
the accuracy is $P(V|A) = \frac{P(V,A)}{P(A)} = \frac{P(A|V)P(V)}{P(A)} = \frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.1 \times 0.99} \approx \frac{0.0095}{0.1085} \approx 0.0876$

the accuracy of test B is $P(V|B) = \frac{P(V,B)}{P(B)} = \frac{P(B|V) \cdot P(V)}{P(B)} = \frac{0.9 \times 0.01}{0.9 \times 0.01 + 0.05 \times 0.99} \approx 0.1538$

As a result, Test B is more indicative

5. Ana notices that people who drive SUVs (S) use a large amount of gas (G) and are in more accidents (A) than the national average. Ana generated the following Bayesian network:

S	P(S)
t	0.4
f	0.6



A	S	P(A| S)
t	*t*	0.75
f	*t*	0.25
t	*f*	0.2
f	*f*	0.8

G	S	P(G| S)
t	*t*	0.8
f	*t*	0.2
t	*f*	0.3
f	*f*	0.7

(a) (3 points) Compute $P(a, \neg s, g)$ using chain rule. Note, $P(a, \neg s, g)$ is a short-hand expression for $P(A = t, S = f, G = t)$.

$$P(a, \neg s, g) = 0.6 \times 0.2 \times 0.3 = 0.036$$

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(b) (6 points) Compute $P(a)$ using inference by enumeration.

$$\cancel{P(a) = \sum_S \sum_G P(a, S, G)}$$

$$\cancel{P(a) = \sum_S \sum_G P(a,S,G) = P(a,s,g) + P(a,s,\neg g) + P(a,\neg s, g) + P(a, \neg s, \neg g)}$$

$$\cancel{= 0.24 + 0.06 + 0.036 + 0.084}$$

$$\cancel{= 0.42}$$

$$P(a) = \cancel{P}\sum_S P(a|S) \cdot P(S) = P(a|\neg s) \cdot P(\neg s) + P(a|s) \cdot P(s)$$

$$= 0.2 \times 0.6 + 0.75 \times 0.4$$

$$= 0.42$$

(c) (6 points) Using conditional independence, compute $P(\neg g, a|s)$ and $P(\neg g, a|\neg s)$. Then, use Bayes Rule to compute $P(s|\neg g, a)$.

G and A are independent when given S. therefore, $P(\neg g, a|s) = P(\neg g|s) \times P(a|s)$
$= 0.2 \times 0.75 = 0.15$

$P(\neg g, a|\neg s) = P(\neg g|\neg s) \times P(a|\neg s) = 0.7 \times 0.2 = 0.14$

$$P(s|\neg g, a) = \frac{P(s, \neg g, a)}{P(\neg g, a)} = \cancel{\frac{P(\neg g, a|s)\cdot P(s)}{P(s)\times P(\neg g, a|s) + P(\neg g, a|\neg s)\times P(\neg s)}} = \cancel{\frac{0.15 \times 0.4}{0.15 + 0.14}} \cancel{\approx 0.2069}$$

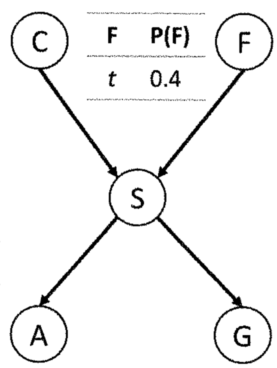
$$= \frac{P(\neg g, a|s) \times P(s)}{P(\neg g, a|s) \times P(s) + P(\neg g, a|\neg s) \times P(\neg s)} = \frac{0.15 \times 0.4}{0.15 \times 0.4 + 0.14 \times 0.6} \approx 0.4167$$

(d) (2 points) Additionally, Ana notes that two people drive SUVs, people from California (C) and people with large families (F). The following Bayesian network is formed including the new information.

C	P(C)
t	0.3

F	P(F)
t	0.4

C	F	P(S|C, F)
t	*t*	0.80
f	*t*	0.60
t	*f*	0.5
f	*f*	0.25



S	P(A| S)
t	0.75
f	0.2

S	P(G| S)
t	0.8
f	0.3

Use chain rule to compute $P(\neg g, a, s, c, \neg f)$.

$P(\neg g, a, s, c, \neg f) = P(\neg f) \times P(c) \times P(s|c, \neg f) \times P(a|s) \times P(\neg g|s)$
$= 0.6 \times 0.3 \times 0.5 \times 0.75 \times 0.2$
$= 0.0135$

(e) (4 points) Answer whether the following independences are True/False for the Bayesian network above.

i. $C \perp\!\!\!\perp G$ False

ii. $F \perp\!\!\!\perp A \mid S$ True

iii. $C \perp\!\!\!\perp F$ True

iv. $C \perp\!\!\!\perp F \mid A$ False