'Iolani Science Olympiad Pre-Nationals Practice

Division C

OPTICS TORTURE, ROUND 2

Name ______________________________

Honolulu
2025

1 The Big Bad Multiple Choice Questions

1. Which statement best explains why sub-wavelength pit depths on a CD ($\approx 1/4\lambda$) yield high-contrast readback signals?

a) They maximize specular reflection from lands.

b) They force a π-phase shift between pit and land reflections, creating destructive interference at the detector.

c) They reduce the spot size below the diffraction limit.

d) They enlarge the numerical aperture without changing lens diameter.

2. If a focus servo is tuned for maximum rejection of high-frequency spindle wobble, which failure mode is most likely to appear?

a) Excessive laser-diode relative intensity noise (RIN).

b) Crosstalk between focus and tracking servos.

c) Increased sensitivity to low-frequency disc tilt, causing burst uncorrectable errors.

d) Reduced tracking-error signal amplitude at the outer radius.

3. Residual birefringence in the polycarbonate substrate primarily affects readback quality by:

a) Changing pit depth through photo-elastic expansion.

b) Creating depolarization that increases shot noise.

c) Rotating the polarization state so differential pit/land reflectivity drifts, shifting the optimal slice level.

d) Narrowing the laser linewidth via optical feedback.

4. Why doesn't simply increasing the space between layers indefinitely solve optical cross-talk?

a) Servo bandwidth limits dominate first.

b) Larger spacing increases spherical aberration and degrades the focal spot on the target layer.

c) Fresnel reflections drop to zero, limiting signal.

d) Laser coherence length shortens with distance inside the disc

5. In a combo CD/DVD pickup, chromatic focal shift is principally mitigated by:

a) Servo firmware that changes focus-loop gain between 780 nm and 650 nm.

b) Switching objective lenses on a rotating turret.

c) Designing a single aspheric objective whose spherical aberration at 780 nm cancels chromatic defocus at 650 nm.

d) Increasing the pit depth of CD media during manufacture.

6. The Abbe diffraction-limited lateral resolution d of a transmitted-light microscope objective is best expressed as

a) $d = \frac{\lambda}{NA}$

b) $d = \frac{1.22\lambda}{NA}$

c) $d = \frac{\lambda}{4NA}$

d) $d = \frac{\lambda}{2NA}$

7. An oil-immersion objective has numerical aperture $NA = 1.40$. If the immersion oil has $n = 1.52$, the objective's half acceptance angle θ (to the optical axis) is closest to

a) 43°

b) 58°

c) 67°

d) 74°

8. Which pair of matched optical elements gives the phase-contrast microscope its characteristic image contrast?

a) Dark-field stop ordinary condenser lens

b) Annular ring stop & phase-retarding plate

c) Polarizer & analyzer crossed at 90°

d) Nomarski prism & compensator plate

9. In a confocal laser-scanning microscope, the pinhole is placed in a plane conjugate to the specimen to

a) correct chromatic aberration

b) eliminate spherical aberration

c) reject out-of-focus fluorescence

d) increase the numerical aperture

10. A polarizing microscope used for mineralogy reveals birefringence mainly because the analyzer is

a) parallel to the polarizer

b) set at Brewster's angle

c) crossed 90° to the polarizer

d) rotated continuously during imaging

11. A myopic eye has its far point at $R = -0.40$ m. Neglecting vertex distance, the spectacle lens power that will allow relaxed distant vision is

a) +2.5 D

b) -2.5 D

c) +0.40 D

d) -0.40 D

12. Astigmatism is corrected with a cylindrical lens whose axis is prescribed as 180°. This means the lens

a) has power along the 180° meridian only

b) has zero power along the 180° meridian

c) has equal power in all meridians

d) contains a prism oriented horizontally

13. A hyperopic patient requires a +4.00 D contact lens worn directly on the cornea. If instead a spectacle lens is fitted 12 mm in front of the eye, the approximate spectacle power needed is

a) +3.53 D

b) +4.00 D

c) +4.48 D

d) +5.00 D

14. Chromatic aberration of the human eye (longitudinal) causes blue focus to lie 1D in front of red focus. The principal physical origin is

a) dispersion of the cornea & lens

b) Stiles-Crawford effect

c) Purkinje shift

d) Pupil diffraction

15. The plate scale of a telescope is inversely proportional

a) focal length

b) mirror diameter

c) $f/\#$

d) aperture ratio

16. A 200 mm Newtonian reflector operating at f/5 suffers from coma. Converting it to a classical Cassegrain with f/15 primary–secondary combination primarily

a) reduces coma by increasing focal ratio

b) eliminates spherical aberration only

c) shortens physical tube length without optical change

d) improves MTF by removing central obstruction

17. An apochromatic refractor uses extra-low-dispersion glass and often a third element mainly to minimise

a) astigmatism & field curvature

b) secondary spectrum across visible wavelengths

c) oblique illumination vignetting

d) thermal expansion of the tube

18. Limiting visual magnitude m of a telescope under dark sky grows as $m \propto 2.5 \log_{10} D^2$. Doubling aperture therefore improves the limit by roughly

a) 0.75 mag

b) 1.50 mag

c) 2.50 mag

d) 4.00 mag

19. The Lagrange invariant Hu sets an upper bound on

a) field-of-view for a given entrance pupil

b) mirror surface accuracy

c) diffraction-limited spot size

d) secondary mirror obscuration fraction

20. A sextant reading shows an index error of +1.6'. To obtain the true altitude one must

a) add 1.6'

b) subtract 1.6'

c) double 1.6'

d) ignore, since it is negligible

21. In a marine sextant the two reflecting surfaces mean that the observed angle between objects equals

a) half the arc setting

b) the arc setting itself

c) twice the arc setting

d) arc + index correction

22. While measuring the altitude of the Sun, the sextant's "dip" correction compensates for

a) Earth's curvature relative to eye height

b) refraction in the sextant mirrors

c) sextant temperature expansion

d) observer's latitude change during sight

23. The f-number of a photographic lens is defined strictly as

a) aperture / focal length

b) focal length / aperture

c) numerical aperture / index

d) reciprocal of lens transmissivity

24. Rolling-shutter distortion in CMOS sensors arises because

a) photosites saturate non-linearly

b) each row is exposed sequentially over time

c) low-pass optical filter is misaligned

d) ADC quantization is temperature dependent

25. A Bayer-pattern sensor uses twice as many green pixels as red or blue primarily to

a) match human photopic luminance sensitivity

b) increase overall pixel fill-factor

c) simplify demosaicing math

d) reduce chromatic aberration

26. Depth-of-field (DoF) increases when

a) sensor size is larger at constant framing

b) f-number is decreased

c) circle of confusion criterion is loosened

d) subject distance is halved

27. Minimum deviation δ_{min} for a prism of apex angle A and refractive index n occurs when the internal ray is symmetric and is given by

a) $\delta_{min} = A(n-1)$

b) $\delta_{min} = 2\sin^{-1}(n \sin \frac{A}{2}) - A$

c) $\delta_{min} = 2\tan^{-1}(n)$

d) $\delta_{min} = A\sqrt{n^2 - 1}$

28. A direct-vision Amici prism produces zero net beam deviation at a design wavelength by combining glasses of

a) equal dispersion, equal n

b) high n, low dispersion and low n, high dispersion

c) identical Abbe numbers

d) zero thermo-optic coefficient

29. The critical angle for total internal reflection at a BK7–air interface ($n = 1.517$ at 589 nm) is closest to

a) 39.2°

b) 41.3°

c) 48.6°

d) 59.1°

30. A Porro-I binocular prism pair both inverts and reverts the image, yielding

a) erect, unreversed view

b) inverted, reversed view

c) erect but reversed view

d) inverted but unreversed view

31. Single-lens-reflex (SLR) cameras historically use a pentaprism rather than two mirrors because the pentaprism

a) adds zero optical path length

b) maintains image parity regardless of camera tilt

c) eliminates chromatic aberration from the viewfinder

d) provides variable magnification

32. In a Michelson interferometer, moving one mirror by $\lambda/4$ causes the output fringe pattern to shift by

a) 1/2 fringe

b) 1 fringe

c) 2 fringes

d) 4 fringes

33. A periscope using two parallel mirrors separated by distance d has its line-of-sight displaced by

a) $d \tan\theta$

b) $2d$

c) d

d) independent of d

34. Roof (Schmidt–Pechan) prisms in compact binoculars require phase-corrective coatings mainly to

a) boost reflectivity above 99%

b) compensate for interference between s- and p-polarized components

c) eliminate lateral chromatic aberration

d) increase eye relief

35. Kaleidoscopes create repeated mosaic patterns because three front-surface mirrors are set so that the sum of internal angles equals

a) 60°

b) 90°

c) 120°

d) 180°

36. A dielectric slab of index $n = 1.60$ and thickness d is placed in one arm of a Michelson interferometer operating at $\lambda_0 = 633\,\text{nm}$. As the slab is slowly withdrawn, 450 fringes cross the detector. What is d?

a) $89.0\,\mu\text{m}$

b) $178\,\mu\text{m}$

c) $356\,\mu\text{m}$

d) $712\,\mu\text{m}$

37. A linearly–polarized plane wave is incident at Brewster's angle on a loss-less dielectric. Which statement is true for the reflected beam?

a) It is purely s–polarized.

b) It is purely p–polarized.

c) It contains equal s and p components.

d) Its degree of polarization depends on wavelength.

38. The group-velocity dispersion parameter D of standard single-mode fiber at 1550 nm is about $17\,\text{ps}\,\text{nm}^{-1}\,\text{km}^{-1}$. For a 40 km link, a 2 nm spectral width pulse will broaden by roughly

a) 1.4 ps

b) 34 ps

c) 680 ps

d) 1.36 ns

39. A confocal Fabry–Pérot cavity of length L uses mirrors of radius $R = L$. Its free spectral range (FSR) relative to a planar-mirror cavity of the same length is

a) identical

b) half

c) double

d) $\sqrt{2}$ times larger

40. The resolving power $R = \lambda/\Delta\lambda$ of a grating used in Littrow is maximized primarily by

a) increasing groove density

b) blazing the grating at normal incidence

c) increasing illuminated length on the grating

d) operating at first diffraction order

41. In the Fraunhofer regime, the far-field pattern of a circular aperture is governed by the first zero of the Airy disk at angle $\theta \approx 1.22\,\lambda/D$. Doubling both wavelength and aperture diameter keeps θ

a) unchanged

b) doubled

c) halved

d) quadrupled

42. The minimum beam waist w_0 achievable by a Gaussian beam focused with an ideal lens is fundamentally limited by

a) diffraction only

b) lens material dispersion

c) spherical aberration

d) chromatic focal shift

43. A laser cavity supports three longitudinal modes separated by 250 MHz. The cavity length is approximately

a) 0.30 m

b) 0.60 m

c) 1.2 m

d) 3.0 m

44. Rayleigh scattering loss in silica fibers scales with wavelength as λ^{-4}. If loss is 0.20 dB/km at 1.3 μm, the expected loss at 0.65 μm is closest to

a) 0.8 dB/km

b) 3.2 dB/km

c) 12.8 dB/km

d) 51 dB/km

45. Which aberration remains even when both spherical aberration and coma are perfectly corrected in a rotationally symmetric lens?

a) Astigmatism

b) Chromatic axial

c) Field curvature

d) Distortion

46. In a Mach–Zehnder intensity modulator using $LiNbO_3$, half-wave voltage V_π is inversely proportional to

a) interaction length

b) refractive index

c) crystal thickness

d) optical wavelength squared

47. Dichroic beam splitters work by

a) metallic reflection

b) total internal reflection

c) multi-layer thin-film interference

d) birefringence

48. In optical pumping of a four-level laser, the lower laser level being well above the ground state primarily

a) increases threshold

b) decreases threshold

c) broadens linewidth

d) reduces quantum efficiency

49. The quarter-wave plate converts linearly polarized light to circular only when the incident polarization is

a) parallel to the fast axis

b) 45° to the fast axis

c) perpendicular to the slow axis

d) arbitrary

50. The Sellmeier equation expresses

a) reflection at interfaces

b) dispersion of refractive index

c) scattering cross-section

d) nonlinear coefficient

51. A corner-cube retro-reflector returns an incident ray

a) parallel but inverted

b) parallel and unreversed

c) antiparallel

d) at twice the incident angle

52. Fresnel number $N_F = a^2/(\lambda L)$ of an aperture of radius a observed at distance L determines

a) polarization ellipse

b) near- vs far-field diffraction regime

c) coherence length

d) Brewster angle

53. The critical angle in a step-index fiber equals $\theta_c = \sin^{-1}(n_2/n_1)$. Increasing core-cladding index difference Δ will

a) increase numerical aperture

b) decrease modal dispersion

c) decrease acceptance cone

d) leave NA unchanged

54. Optical time-domain reflectometry (OTDR) locates faults in fibers by measuring

a) transmitted power

b) back-scattered Rayleigh signal versus time

c) beat frequency between modes

d) chromatic dispersion

55. In a Kerr shutter, ultrafast gating is achieved through

a) two-photon absorption

b) photorefractive effect

c) intensity-dependent birefringence

d) electro-optic Pockels effect

56. For a thin lens with focal length f, the Newtonian form of the lens equation is $(x)(x') = f^2$, where x and x' are measured from

a) lens plane

b) focal planes

c) principal planes

d) nodal points

58. Coherence length L_c of a source with Lorentzian linewidth $\Delta\nu$ is

a) $c\Delta\nu$

b) $c/(\pi\Delta\nu)$

c) $c/(2\Delta\nu)$

d) $2c/\Delta\nu$

59. The Talbot distance for a grating of period p and wavelength λ is $z_T = p^2/\lambda$. Halving grating period yields a Talbot distance

a) unchanged

b) halved

c) quartered

d) doubled

60. Photonic crystal fibers confine light primarily via

a) total internal reflection in solid core

b) Bragg reflection from periodic cladding

c) metallic waveguide action

d) anti-resonant reflecting layer

61. Did you solve Cornell 2024? I didn't. The Jones matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ describes

a) a half-wave plate

b) a quarter-wave plate

c) a linear polarizer at 45°

d) a swap of x and y polarizations

62. In an achromatic doublet formed from crown and flint glasses, the key design condition is that the two elements

a) have equal focal lengths but opposite signs

b) satisfy $f_1 V_1 + f_2 V_2 = 0$ where V is Abbe number

c) are cemented with matching refractive indices

d) are both positive lenses of different curvatures

63. A plano-convex lens of focal length f is used at $2f$ object distance. The image is therefore

a) at $2f$ on the other side, inverted and same size

b) at $f/2$, erect and magnified 2x

c) at $2f$, erect and same size

d) at f, inverted and half size

64. The Fresnel reflection coefficient at normal incidence between glass ($n = 1.50$) and air is closest to

a) 2.0%

b) 4.0%

c) 8.0%

d) 16%

65. Huygens' principle explains straight-line light propagation by assuming each point on a wavefront produces

a) longitudinal waves only

b) secondary spherical wavelets

c) rays normal to surfaces

d) equal energy in all directions without interference

66. The numerical aperture (NA) of an objective immersed in water ($n = 1.33$) with half-angle $\theta = 60°$ is

a) 0.66

b) 1.15

c) 1.33

d) 2.66

67. For constructive interference in a soap film of thickness d (air on both sides), the condition is approximately

a) $2nd = m\lambda$ with no phase shift

b) $2nd = (m + \frac{1}{2})\lambda$ with one phase reversal

c) $nd = \lambda/4$

d) independent of n

68. In a diffraction grating the angular separation between successive maxima increases when

a) groove spacing increases

b) wavelength decreases

c) order number increases

d) illuminated width decreases

69. A 5 mW He–Ne laser beam is expanded from 1 mm to 5 mm diameter. Neglecting losses, beam intensity (W/m^2) becomes

a) 25x larger

b) 5x larger

c) unchanged

d) 25x smaller

70. Doubling the core radius of a step-index multimode fiber approximately multiplies the number of guided modes by

a) 2

b) 4

c) 8

d) 16

71. A pinhole camera with pinhole diameter too large suffers mainly from

a) diffraction blur

b) geometric blur

c) chromatic blur

d) astigmatism

72. A quarter-wave stack anti-reflection coating on glass works because the first reflected rays from the two layers

a) reinforce constructively

b) cancel destructively

c) convert to circular polarization

d) undergo total internal reflection

73. Laser speckle contrast is reduced by

a) spatially averaging many independent speckle patterns

b) narrowing the laser linewidth

c) increasing temporal coherence

d) focusing to diffraction limit

74. A dielectric prism used to deviate a beam introduces lateral dispersion because

a) n varies with wavelength

b) apex angle varies with temperature

c) mechanical stress rotates polarization

d) prism mass depends on wavelength

75. Circular dichroism measures differential absorption of

a) left- and right-hand circularly polarized light

b) s- and p-polarized light

c) different wavelengths

d) longitudinal modes

76. In a simple microscope, angular magnification increases when

a) focal length decreases

b) eye–lens distance increases

c) lens diameter decreases

d) object distance moves toward ∞

77. A CCD camera converts incident photons primarily into

a) phonons

b) electron–hole pairs

c) plasmons

d) excitons

78. The resolving power of a telescope limited by diffraction depends on

a) mirror diameter only

b) mirror diameter and wavelength

c) focal length only

d) magnification

79. Rayleigh criterion for resolution sets minimum angular separation at

a) $1.22\,\lambda/D$

b) $\lambda/2D$

c) $0.61\,\lambda/D$

d) $2.44\,\lambda/D$

80. A half-wave plate rotates plane polarization by an angle equal to

a) the angle between fast axis and input polarization

b) twice that angle

c) half that angle

d) always 90°

81. Fluorescence differs from phosphorescence mainly in

a) emission wavelength

b) required excitation source

c) excited-state lifetime

d) involving nuclear transitions

82. Coherent population inversion in a laser medium is achieved by

a) spontaneous emission

b) pumping more atoms to the upper than lower laser level

c) lowering cavity losses to zero

d) applying a quarter-wave plate

83. The Q-factor of an optical resonator is proportional to

a) stored energy divided by loss per cycle

b) input pump power

c) cavity volume only

d) cavity length squared

84. Optical isolators rely on the Faraday effect, which rotates polarization proportional to

a) magnetic field strength and medium length

b) electric field strength squared

c) temperature

d) incident intensity

85. In holography, the reference beam provides

a) phase conjugation of the object beam

b) a known phase to record interference fringes

c) temporal gating of exposure

d) amplitude modulation of the object

86. Diffraction efficiency of a volume Bragg grating increases when

a) index modulation depth increases

b) grating thickness decreases

c) incident angle deviates from Bragg angle

d) wavelength is far from design value

87. Optical coherence tomography (OCT) employs low-coherence interferometry to obtain

a) transverse resolution only

b) axial depth information

c) polarization contrast images

d) fluorescence lifetime maps

88. The Purcell effect describes modification of spontaneous emission rate by

a) magnetic field

b) surrounding optical cavity

c) temperature

d) pump polarization

89. An afocal Galilean telescope consists of

a) two positive lenses

b) positive objective and negative eyepiece

c) negative objective and positive eyepiece

d) two negative lenses

90. When linearly polarized light passes through a stress-birefringent plastic under crossed polarizers, coloured fringes appear due to

a) selective absorption

b) wavelength-dependent phase retardation

c) scattering

d) fluorescence

91. The Kerr effect produces an index change proportional to (this was on a tryout test!)

a) electric field strength squared

b) magnetic field strength

c) optical intensity inverse

d) temperature difference

92. Snell's law may be obtained from Fermat's principle by requiring

a) minimum optical path length

b) maximum mechanical work

c) constant phase velocity

d) equal angles of incidence and refraction

93. In white-light interferometry for surface metrology, height variation is measured from

a) fringe colour at each pixel

b) absolute fringe order

c) Doppler shift of scattered light

d) Brewster angle shift

94. A laser diode's threshold current decreases if

a) cavity losses are reduced

b) operating temperature rises

c) reflectivity of both facets is lowered

d) active region length is shortened dramatically

95. In a graded-index (parabolic) fiber with profile $n(r) = n_0\sqrt{1-(\alpha r)^2}$, all meridional rays—independent of launch angle—arrive at the same axial plane with identical group delay. This happens because

a) rays farther from the axis travel a longer geometric path but propagate through lower index, so time-of-flight errors cancel exactly.

b) the core–cladding interface forces every mode to equal phase velocity.

c) chromatic material dispersion negates the path-length difference.

d) modal coupling randomizes arrival times, averaging the delay.

96. The Goos–Hänchen lateral shift for an s-polarized He–Ne beam ($\lambda = 633$ nm) totally internally reflected from BK7 glass ($n = 1.517$) into air at $\theta = 45°$ is on the order of

a) 40 nm

b) 400 nm

c) 4 μm

d) 40 μm

97. For any passive, linear, lossless and non-magnetic optical device, Lorentz reciprocity requires that its forward– and reverse-transmission coefficients t_{ij} obey

a) $t_{ij} = t^*_{ij}$ (Hermitian)

b) $t_{ij} = t_{ji}$ (symmetric)

c) $|t_{ij}| = |t_{ji}|$ but phases unrelated

d) no specific relationship; reciprocity is broken without magneto-optics

98. The optical invariant (etendue) $G = A\Omega$ cannot decrease in any passive system. Which element — *by itself and idealized* — could in principle reduce the etendue of a beam?

a) A perfect thin lens at its focal planes

b) A homogeneous glass slab of index n

c) A tapered mirror-lined light pipe ("Winston cone")

d) None; Liouville's theorem forbids passive etendue reduction

99. Interfering a vortex (OAM) beam of topological charge ℓ with a tilted plane wave produces a fork dislocation in the interference fringes. The number of bright "tines" emerging from the fork equals

a) $|\ell|$

b) $2|\ell|$

c) $|\ell| + 1$

d) $2|\ell| + 1$

100. A dielectric shows no absorption ($k = 0$) below an electronic band-edge frequency ω_g. Causality via the Kramers–Kronig relations then demands that, for $\omega < \omega_g$, the refractive index $n(\omega)$

a) remains exactly unity (vacuum-like).

b) decreases monotonically as ω increases.

c) exhibits normal dispersion, rising as $\omega \to \omega_g^-$.

d) is indeterminate without measuring above ω_g.

2 Lasers, Masers, Tasers

2.1 The LASER

A continuous-wave neodymium-doped yttrium aluminum garnet (Nd:YAG) laser at $\lambda = 1064$ nm pumps a 10-mm-long lithium niobate ($LiNBO_3$) crystal for second-harmonic generation (SHG). The crystal is cut for type-1 phase matching at $T_0 = 25°C$ where its refractive indices satisfy $n(1064) = 2.223$ and $n(532) = 2.271$. Assume the effective nonlinear coefficient $d_{eff} = 4.0$ pm V^{-1} and the input fundamental power is $P_\omega = 1.0W$ in a $\mathrm{TEM}_0 0$ Gaussian beam of radius $w = 40\mu m$ (constant through the crsytal). Neglect depletion of the fundamental.

a) Name 2 uses of Nd:YAG lasers in ophthamology.

b) Show that perfect phase matching requires

$$\Delta k \equiv k_{2\omega} - 2k_\omega \Rightarrow n(532) = n(1064)$$

(c) At 25C the indices are equal within 1.5×10^{-3}. Compute the corresponding phase-mismatch Δk (m^{-1}).

(d) Define the coherence length $L_c = \pi/|\Delta k|$. Determine L_c and determine whether the 10-mm crystal is much longer than L_C.

(e) The SHG power for a plane-wave undepleted pump is

$$P_{2\omega} = \eta P_\omega^2, \quad \eta = \frac{2\omega^2 d_{eff}^2 L^2}{\epsilon_0 c^3 n_{2\omega} n_\omega^2 A} \mathrm{sinc}^2\left(\frac{\Delta k L}{2}\right)$$

with $A = \pi w^2$. Evaluate $P_{2\omega}$ for the 10-mm crystal at 25C.

(f) Lithium niobate's birefringence changes with temperature at roughly $dn/DT = 1.2 \times 10^{-5}\ \mathrm{K}^{-1}$ near 1064 nm. Estimate the temperature shift ΔT needed to bring Δk to zero. (Assume the same shift applies at 532 nm).

(g) Qualitatively sketch (no numbers needed) how $P_{2\omega}$ veries with temperature across several coherence-length fringes, and explain why operating exactly at the phase-matching temperature is far more critical for long crystals than for short ones.

2.2 The MASER

A molecular-beam ammonia (NH_3) maser operates on the inversion transition $|s\rangle \to |a\rangle$ at $\nu_0 = 23.87$ GHz. Inside the copper cavity (a right circular cylinder) the loaded quality factor is $Q_L = 5.0 \times 10^4$ and the mode volume is $V = 100$ cm^3 (I didn't even make you integrate to find it this time!) A state-selector hexapole injects only the upper-state molecules into the cavity. The beam parameters are as follows:

- Molecular flux (upper state): $F = 5.0 \times 10^{16} \mathrm{s}^{-1}$
- Beam velocity: $v = 1000$ m/s
- Beam cross-section in the cavity: $A = 1.0$ cm^2
- Stimulated-emission cross-section: $\sigma = 2.0 \times 10^{-15}$ m^2
- Cavity length along the beam: $L_b = 2.0$ cm

Assume (i) relaxation of the inversion is dominated by the molecules' transit time through the cavity, (ii) spontaneous emission is negligible, and (iii) every molecule remains inverted until it leaves the cavity.

(a) Calculate the angular frequency ω_0 and the photon energy $h\nu_0$.

(b) Find the photon decay time (energy lifetime) $\tau_c = \frac{Q_L}{\omega_0}$.

(c) Show that the small-signal power gain per cavity pass is $G = \sigma N_L$ where the column density $N_L = \frac{F\tau_t}{A}$ and $\tau_t = \frac{L_b}{v}$. Compute G.

(d) A one-pass fractional energy loss equals $1/Q_L$. Argue that the threshold condition is $G_{th} \approx 1/Q_L$. Determine the threshold flux and decide whether the maser oscillates with the given beam.

(e) Assuming the maser saturates until the stimulated-emission rate equals the pump of inverted molecules, estimate the steady-state output power.

2.3 The TASER

Raphael finally tries to talk to a woman for the first time. She immediately tases him. A simplified model of her TASER uses

- A boost-charged capacitor $C = 10\mu\text{F}$ charged to $V_0 = 300V$
- A fly-back step-up transformer with turns ratio $N_s : N_p = 30 : 1$
- An effective primary series resistance $R_p = 5.0\Omega$
- A pulse width $\tau_p = 100\mu\text{s}$ delivered every $T = 52$ ms
- A load (Raphael's body) that can be approximated by a fixed $R_{raph} = 1.2k\Omega$

Assume:

- the capacitor discharges only through $_p$ during the 100μs pulse
- transformer losses plus spark/arc losses reduce the energy that reaches Raphael to an overall efficiency $\eta = 0.30$
- once the pulse ends the capacitor is fully re-charged before the next pulse
- air breaks down at 3 kV $\cdot$ mm^{-1} at sea level.

(a) Compute the energy E_0 initially stored in the capacitor.

(b) Show that the capacitor voltage (and therefore primary current) obeys $V(t) = V_0 e^{-t/\tau}$ with time constant $\tau = R_p C$. Evaluate τ and the peak primary current I_{p0} at $t = 0$.

(c) For an ideal transformer the instantaneous secondary open-circuit voltage is $V_{s,oc}(t) = N_s V(t)$ and the current limit is $I_s(t) = I_p(t)/N_s$. At $t = 0$, find $V_{s,oc,0}$. Assuming Raphael's resistance limits the current, compute the delivered voltage $V_{raph0} = I_{s0} R_{raph}$ and the current I_{raph0}.

(d) Nearly all of E_0 is released during the pulse. The residual energy is $\exp(-2\tau_p/\tau) \approx 1.8\%$. Estimate the energy pulse that reaches raph, $E_{raph} = \eta E_0$, and the average output power $P_{avg} = E_{raph}/T$

(e) Raphael hasn't even gotten that close, but the frightened woman sets off the taser without touching Raphael. If the transformer were unloaded (no Raphael touching the taser), its peak secondary voltage would still be $V_{s,oc,0}$. Using the given breakdown field, estimate the maximum electrode gap across which a spark would jump at that voltage.

(f) International standards on ventricular-fibrillation risk treat exposures below $10J$ per pulse (for pulses $\leq$ 400 kHz) as non-fibrillating. Compare your E_{raph} to this limit and state the safety margin. Does Raph die?

3 A Slit Problem

A monochromatic plane-wave laser (wavelength λ=632 nm) illuminates an opaque mask in the $z = 0$ plane that contains two infinitesimally narrow slits separated by $d = 0.50$ mm along the x-axis (their centers are at $(\pm d/2, 0, 0)$).

Your event supervisor is a lazy bum who doesn't want to draw diagrams. Instead, he will provide information about every single region through which the light passes. This decomposition into regions instead of your typical intuitive illustration is something that your university professors will definitely do. Beyond the mask,

- Region 1 ($0 < z < t$): one of the slits is covered by a very thin glass wedge of refractive index $n = 1.50$. Its thickness increases linearly with y:

$$t(y) = t_0 + \alpha y, \quad t_0 = 1.0\mu\text{m}, \quad \alpha = 5.0 \times 10^{-3}$$

- Region 2 ($z \geq t$): free space again.

A flat observation screen is located a distance $L = 2.0$ m downstream, but the screen is tilted: rotated by a small angle of $\theta = 12^\circ$ about the y-axis, so its surface normal makes an angle θ with the +z axis.

Assume paraxial geometry whenever appropriate, neglect diffraction from slit width, and work entirely with path differences. Everywhere, keep the algebra exact until the final numerical substitutions.

(a) With $\theta = 0$ and $\alpha = 0$, derive the usual expression for the fringe spacing Δx_0 on a perpendicular screen at $z = L$.

(b) Now set $\alpha = 0$ (remove the wedge) but keep the screen tilt $\theta = 12°$.

(i) Show that the points of equal optical path difference now lie on straight lines in the (x, y) plane of the mask that satisfy

$$\Delta\ell(x, y) = \frac{d}{L}(x - y\tan\theta)\cos\theta$$

(ii) Hence find the effective fringe spacing Δx_θ measured along the mask's x-axis (i.e., projected back to the mask plane).

(iii) Evaluate Δx_θ numerically and compare to Δx_0. Explain qualitatively why the tilt stretches or compresses the pattern.

(c) Now consider the screen untilted ($\theta = 0$) but restore the glass wedge ($\alpha = 0$).

(i) Derive the additional phase shift $\varphi(y) = \frac{2\pi(n-1)}{\lambda}dy$

(ii) Show that the bright-fringe condition becomes

$$x = m\left(\frac{\lambda L}{d}\right) + \left[(n-1)\alpha\frac{L}{d}\right]y$$

so the fringes are oblique lines of slope $(n-1)\alpha\frac{L}{d}$ in the mask plane.

(iii) Compute this slope numerically (unitless) and state whether the fringes lean left or right as y increases.

(d) With both tilt ($\theta \neq 0$) present, the two skewing effects superpose.

(i) Write the total path difference $\Delta\ell_{tot}$ including both geometrical tilt and optical wedge. Show that the bright-fringe loci satisfy

$$x = m\left(\frac{\lambda L}{d}\right) + [k - \tan\theta\cos\theta]y$$

(ii) Choose α so the fringes run exactly parallel to the y-axis on the mask plane (i.e. the coefficient of y vanishes). Solve for the required α_{crit} in closed form.

(iii) Using the given $\theta = 12°$, compute α_{crit}. How does it compare with the actual $\alpha = 5.0 \times 10^{-3}$? State whether the present wedge over-cancels or under-cancels the tilt.

(e) Suppose you tune α exactly to α_{crit}. Describe what an observer scanning along y on the tilted screen would see. Do individual bright fringes stay fixed at y, or do they drift with z?

4 Bubbles!

A spherical soap bubble of radius $R = 5.0$ cm is suspended in still air. Its liquid film (refractive index $n = 1.33$) is thin compared with R and, because of gravity drainage, its thickness varies linearly with the vertical height h (measured upward from the lowest point on the bubble):

$$t(h) = t_0 - \gamma h, \quad t_0 = 650 \text{ nm}, \quad \gamma = 15 \text{ nm cm}^{-1}, \quad 0 \le h \le 2R$$

White daylight is incident from directly overhead. The observer is far enough away that every ray reaching their eye is effectively parallel to a viewing direction that makes an angle φ with the bubble's vertical axis ($\varphi = 0$ when the observer looks straight down at the top pole). From each point on the bubble the ray strikes at an incidence angle θ_i measured from the local surface normal. Neglect refraction inside the film, magnification effects, and multiple internal reflections. Take the speed of light in vacuum as c, but it will cancel in all results.

(a) Show, using the geometry of a sphere, that for a distant observer

$$\cos\theta_i = \frac{h}{R}\cos\varphi + \left(1 - \frac{h^2}{2R^2}\right)\sin\varphi$$

to first order in h/R (keep only terms $\le h^1$).

(b) Because the outer-surface reflection undergoes a π phase shift while the inner-surface reflection does not, the condition for constructive interference of the wavelength λ is

$$2nt(h)\cos\theta_i = (m + \frac{1}{2})\lambda, \quad m = 0, 1, 2, \cdots$$

Show that for normal viewing ($\varphi = 0$) this reduces to a simple linear relation between height h and fringe order m.

(c) For $\varphi = 0$ calculate the heights of the first bright red band $\lambda_R = 640$ nm) and the neighboring bright blue-green band ($\lambda_B = 500$ nm). Which one is higher on the bubble? (*Hint: keep m as small as possible*)

(d) Take $\varphi = 25°$. Determine the horizontal displacement x (measured along the observer's line of sight, projected back to the bubble surface) between the bright red bands seen at the bottom edge ($h = 0$) and at height $h = 1.0$ cm).

(e) Drainage makes the thickness gradient steeper at $\dot{\gamma} = +1.0$ nm cm^{-1}min^{-1} (γ increases linearly with time). With the observer fixed at $\varphi = 0$, find the vertical speed at which the bright red band in part (c) moves upward after 5 minutes.

5 The Final Problem.

A next-generation combo optical drive employs a **temperature-tunable external-cavity diode laser (ECDL)** to read both CD ($\lambda_1 = 780\,\text{nm}$) and DVD ($\lambda_2 = 650\,\text{nm}$) media without swapping optics. The ECDL's emission wavelength shifts at a rate of $\Delta\lambda/\Delta T = 0.10\,\text{nm}\,\text{K}^{-1}$. The temperature is controlled by a Peltier cooler with thermal dynamics modeled by

$$C_{\text{th}}\frac{dT}{dt} + G_{\text{th}}(T - T_{\text{amb}}) = P_{\text{el}}(t),$$

where C_{th} is the thermal capacitance, G_{th} the thermal conductance, and $P_{\text{el}}(t)$ the electrical heating power.

The collimated laser beam is focused by a single aspheric objective lens (focal length f, diameter D) of numerical aperture $\text{NA} = D/(2f)$. Due to chromatic dispersion, the focal plane shifts by

$$\Delta z_{\text{foc}} \approx M\Delta\lambda,$$

with a constant M determined by the lens material.

The focus-servo is implemented with a voice-coil actuator moving the objective lens along the z-axis. Its mechanical dynamics are

$$m\ddot{z} + c\dot{z} + kz = F_{\text{emf}}(t) + F_{\text{dist}}(t),$$

where the electromagnetic control force is $F_{\text{emf}} = K_p e(t) + K_d \dot{e}(t)$, and the disturbance is due to spindle wobble modeled as

$$z_{\text{wob}}(t) = A\sin(\omega_w t).$$

Finally, the readback signal suffers from timing jitter caused by residual focus error and the laser's intrinsic linewidth $\Delta\nu$. The bit clock is sinusoidal at frequency f_c, and timing jitter can be estimated from phase noise in the carrier.

(a) (i) Solve the thermal ODE for $T(t)$ when $P_{\mathrm{el}}(t)$ steps from 0 to P_0 at $t = 0$. Find the thermal time constant τ_{th} and the steady-state temperature rise ΔT.

(ii) To switch from λ_1 to λ_2, the laser must shift wavelength by $\Delta\lambda = \lambda_2 - \lambda_1$. Determine the required ΔT, and express P_0 in terms of G_{th} and the tuning coefficient.

(b) (i) Using the thin-lens formula and assuming first-order dispersion $dn/d\lambda$, derive an expression for M in $\Delta z_{\mathrm{foc}} = M\Delta\lambda$. (Hint: differentiate $1/f = (n-1)(1/R_1 - 1/R_2)$ with respect to λ)

(ii) The depth of focus is given by $\text{DoF} = \pm\lambda/(2\text{NA}^2)$. Determine the maximum NA such that $|\Delta z_{\text{foc}}| \leq \text{DoF}$ for switching between λ_1 and λ_2.

(c) (i) Write the closed-loop transfer function $H(s) = E(s)/Z_{\text{wob}}(s)$ in the Laplace domain, where $E(s)$ is the focus error and $Z_{\text{wob}}(s)$ is the wobble input.

(ii) Given a damping ratio $\zeta = 0.707$ and natural frequency $\omega_n = 2000\,\text{rad/s}$, choose appropriate values of K_p and K_d so that the characteristic equation

$$ms^2 + (c + K_d)s + (k + K_p) = 0$$

meets the desired specifications.

(iii) Compute $|H(i\omega_w)|$ for $\omega_w = 2\pi \times 100\,\mathrm{Hz}$, and determine the RMS focus error e_{rms} assuming $z_{\mathrm{wob}}(t)$ has amplitude A.

(d) (i) Assume that the RF carrier has instantaneous phase $\phi(t) = \phi_{\mathrm{focus}}(t) + \phi_{\mathrm{laser}}(t)$, where $\phi_{\mathrm{focus}}(t) = \alpha e(t)$ and $\phi_{\mathrm{laser}}(t)$ has PSD $S_{\phi,\mathrm{laser}}(f) = \Delta\nu/(\pi f^2)$. Write an expression for the total phase noise PSD.

(ii) Derive the RMS timing jitter:

$$\sigma_t \approx \frac{\sigma_\phi}{2\pi f_c},$$

where σ_ϕ^2 is the integrated phase noise from both sources. Express σ_t in terms of $e_{\mathrm{rms}}, \alpha, \Delta\nu$, and f_c.

(e) The read-out margin is defined as the ratio of signal-to-jitter spacing to the pit length L_p. Assuming a pit spacing corresponding to a bit period $T_b = 100$ ns and a nominal signal slice threshold that requires a maximum timing jitter of 5% of T_b, determine the maximum allowable combination of e_{rms} and $\Delta\nu$ to meet the margin, given your results from part (d).

ALMOST THERE... HERE'S SOME MORE SCRATCH PAPER FOR THAT LAST ONE.