

This is the answer key for the Kaiser Science Olympiad Invitational 2026 Division C Materials Science exam, written by Raphael Esquivel.

1. A sheet made of carbon atoms arranged in a honeycomb lattice is known as **graphene**. SWCNTs have the structure of a tube made by rolling up such a sheet. SWCNTs are an example of a **1 D** nanomaterial.
2. In the sheet, each carbon atom has **sp²** hybridization; three orbitals form σ bonds with neighboring carbon atoms in a planar hexagonal lattice, and one remaining p_z orbital (perpendicular to the plane) forms the π and π^* bonds responsible for **conduction**. The curvature introduced by rolling up the sheet alters orbital orientations. The p_z orbitals are no longer perfectly perpendicular to the surface, σ and π orbitals are no longer orthogonal, and there is a mixing between σ and π states. As a result, carbon atoms in carbon nanotubes (CNTs) exhibit partial **sp³** character, often described as $sp^2+\delta$ hybridization.
3. The bandgap is the energy difference between the valence band maximum and the conduction band minimum (or, alternatively, the minimum energy needed to excite an electron from a bound/valence state into a conducting state)
4. 0 eV
5. Circles the left band structure diagram
6. Fermi level (+1pt); represents electron chemical potential OR the thermodynamic work required to add one electron to a body (+1pt); at T=0K it is the boundary between occupied states (below) and unoccupied states (above) (+1pt)
7. In the context of band structures, Γ and Z represent high-**symmetry** points in the first **Brioullin** zone in reciprocal space. In the same way the **Bravais** lattice is divided up into Wigner-Seitz cells in real space, the reciprocal lattice is broken up into these zones.
8. Cutting lines. Award half credit for “allowed k-lines”, “quantized k-lines”, “quantization lines”

Plugging in and using the reciprocal relation, $\mathbf{K} \cdot \mathbf{C} = \frac{1}{3}(\mathbf{b}_1 - \mathbf{b}_2) \cdot (n\mathbf{a}_1 + m\mathbf{a}_2) = \frac{2\pi}{3}(n - m)$. The condition becomes $\frac{2\pi}{3}(n - m) = 2\pi\ell$ or $n - m = 3\ell$. Since this is true for $(n, m) = (3, 0)$, it follows that such SWCNTs are metallic.

9.
 - +3 points (full credit): shows work, obtains the condition n-m=3l and comes to conclusion that (n,m)=(3,0) is metallic
 - OR
 - +2 points: significant progress towards solution
 - OR
 - +1 point: attempts to take dot product between K and C
10. Conduction-band minimum and valence band-minimum occur at the same crystal momentum (+1pt). Electron-hole transitions can occur without a change in crystal momentum, so no phonon is required to conserve momentum (direct optical transitions without phonon assistance) (+1pt). Accept half credit for “an electron can directly emit a photon”
11. B
12. A
13. D

14. Photolithography (+1pt), top-down (+1pt, do not award if the answer is something other than photolithography)
15. A
16. A
17. A
18. C
19. C
20. A
21. D
22. A
23. B
24. B
25. B
26. D
27. C
28. C
29. C
30. B
31. A
32. B
33. The Wiedemann-Franz law says that $k/\sigma = LT$, so the electronic thermal conductivity k is on the order of 10^{-1} or 10^0 for each films (we can assume T is on the order of 10^3). Since the actual thermal conductivities are on the order of 10^3 , the chirality does not have a significant effect on thermal conductivity. MUST note that the electronic contribution to thermal conductivity predicted by Wiedemann-Franz is on the order of 0.1 or 1 to receive credit.
34. Photoluminescence is the emission of light from a material following optical excitation (+1pt). It occurs when an incident photon excites an electron from the valence band to the conduction band, creating an electron–hole pair. After excitation, the electron and hole may relax to the band edges and then recombine radiatively, emitting a photon (+1pt) with energy approximately equal to the bandgap energy (or, more precisely, the excitonic transition energy; accept either answer). (+1pt)
35. The two resonances correspond to excitation into a higher excitonic transition (+1pt) followed by relaxation and emission from the lowest bright exciton (+1pt).
36. Satellite spots are explained plausibly as bundles (or weakly coupled aggregates/clumps) of nanotubes of the same chirality, which redshift transitions slightly versus isolated tubes.
37. Longer diffusion length means excitons travel farther before recombining (+1pt). Longer diffusion length makes PL more sensitive to nanotube length because excitons are more likely to reach ends. (+1pt)
- 38.
- At short lengths, many excitons may reach the tube ends and are quenched nonradiatively; increasing length reduces the fraction of excitons lost to ends, so PL rises (+1pt).

- b. At long lengths and low power, end effects become negligible for excitons created near the center; PL approaches a plateau because adding more length doesn't significantly increase the fraction that recombines radiatively (+1pt).
 - c. At higher power, exciton-exciton annihilation shortens the effective lifetime, reducing diffusion length. With shorter diffusion length, ends matter less, so high PL can occur even for shorter L; hence the curve "turns on" earlier. (+1pt)
39. Linear increase means each absorbed photon creates one emitting exciton (+1pt). At high power, excitons interact and annihilate (+1pt). The physical process responsible for this change is exciton-exciton annihilation (+1pt).
40. 1 point per valid reasons (max 2 points):
- a. Defects localize excitons, preventing diffusion to quenching sites
 - b. Defect states lie below dark exciton states, making radiative decay more likely
 - c. Defects create new optically allowed transitions with higher PL efficiency
 - d. Localization enables single-emitter behavior at room temperature
41. Lists one good reason; exciton localization improves emission efficiency, stability at room temperature, reduced sensitivity to material size and disorder, redshifted emission, compatibility with nanophotonic structures, enabling single emitter behavior, etc
42. Aryl dopants convert local carbon bonding from sp^2 to sp^3 , breaking pi conjugation. This creates a localized potential well that traps excitons, preventing free diffusion. (In 1D systems, carriers cannot bypass defects, so a single localized site has a strong impact on exciton dynamics.)
43. Localized states lie energetically below delocalized excitons, causing red-shifted emission. Localization also reduces interaction with quenching sites, stabilizing emission at room temperature.
44. The periodic hole structure of the photonic crystal defines allowed cavity modes (+1pt). On-cavity emission is filtered and enhanced by cavity resonances, while off-cavity emission reflects the full ensemble of defect states (+1pt).
45. The cavity increases the local optical density of states at certain wavelengths, making radiative decay more probable. (+1pt) The emitter structure is unchanged; only the surrounding electromagnetic structure is modified. (+1pt)
46. Defect localization limits emission to discrete sites, while the cavity selectively enhances only those emitters that are spatially, spectrally, and polarization matched. This structural filtering suppresses contributions from other emitters.
47. Caffeine