

This is the answer key for the Kaiser Science Olympiad Invitational 2026 Division C Materials Science exam, written by Raphael Esquivel.

1. A sheet made of carbon atoms arranged in a honeycomb lattice is known as **graphene**. SWCNTs have the structure of a tube made by rolling up such a sheet. SWCNTs are an example of a **1** D nanomaterial.
2. In the sheet, each carbon atom has **sp²** hybridization; three orbitals form σ bonds with neighboring carbon atoms in a planar hexagonal lattice, and one remaining p_z orbital (perpendicular to the plane) forms the π and π^* bonds responsible for **conduction**. The curvature introduced by rolling up the sheet alters orbital orientations. The p_z orbitals are no longer perfectly perpendicular to the surface, σ and π orbitals are no longer orthogonal, and there is a mixing between σ and π states. As a result, carbon atoms in carbon nanotubes (CNTs) exhibit partial **sp³** character, often described as $sp^{2+\delta}$ hybridization.
3. The bandgap is the energy difference between the valence band maximum and the conduction band minimum (or, alternatively, the minimum energy needed to excite an electron from a bound/valence state into a conducting state)
4. 0 eV
5. Circles the left band structure diagram
6. Fermi level (+1pt); represents electron chemical potential OR the thermodynamic work required to add one electron to a body (+1pt); at $T=0K$ it is the boundary between occupied states (below) and unoccupied states (above) (+1pt)
7. In the context of band structures, Γ and Z represent high-**symmetry** points in the first **Brillouin** zone in reciprocal space. In the same way the **Bravais** lattice is divided up into Wigner-Seitz cells in real space, the reciprocal lattice is broken up into these zones.
8. Cutting lines. Award half credit for “allowed k-lines”, “quantized k-lines”, “quantization lines”

Plugging in and using the reciprocal relation, $\mathbf{K} \cdot \mathbf{C} = \frac{1}{3}(\mathbf{b}_1 - \mathbf{b}_2) \cdot (n\mathbf{a}_1 + m\mathbf{a}_2) = \frac{2\pi}{3}(n - m)$. The condition becomes $\frac{2\pi}{3}(n - m) = 2\pi\ell$ or $n - m = 3\ell$. Since this is true for $(n, m) = (3, 0)$, it follows that such SWCNTs are metallic.
9.

+3 points (full credit): shows work, obtains the condition $n-m=3l$ and comes to conclusion that $(n,m)=(3,0)$ is metallic

OR

+2 points: significant progress towards solution

OR

+1 point: attempts to take dot product between \mathbf{K} and \mathbf{C}
10. Conduction-band minimum and valence band-minimum occur at the same crystal momentum (+1pt). Electron-hole transitions can occur without a change in crystal momentum, so no phonon is required to conserve momentum (direct optical transitions without phonon assistance) (+1pt). Accept half credit for “an electron can directly emit a photon”
11. B
12. A
13. D

14. Photolithography (+1pt), top-down (+1pt, do not award if the answer is something other than photolithography)
15. A
16. A
17. A
18. C
19. C
20. A
21. D
22. A
23. B
24. B
25. B
26. D
27. C
28. C
29. C
30. B
31. A
32. B
33. The Wiedemann-Franz law says that $k/\sigma = LT$, so the electronic thermal conductivity k is on the order of 10^{-1} or 10^0 for each films (we can assume T is on the order of 10^3). Since the actual thermal conductivities are on the order of 10^3 , the chirality does not have a significant effect on thermal conductivity. MUST note that the electronic contribution to thermal conductivity predicted by Wiedemann-Franz is on the order of 0.1 or 1 to receive credit.
34. Photoluminescence is the emission of light from a material following optical excitation (+1pt). It occurs when an incident photon excites an electron from the valence band to the conduction band, creating an electron-hole pair. After excitation, the electron and hole may relax to the band edges and then recombine radiatively, emitting a photon (+1pt) with energy approximately equal to the bandgap energy (or, more precisely, the excitonic transition energy; accept either answer). (+1pt)
35. The two resonances correspond to excitation into a higher excitonic transition (+1pt) followed by relaxation and emission from the lowest bright exciton (+1pt).
36. Satellite spots are explained plausibly as bundles (or weakly coupled aggregates/clumps) of nanotubes of the same chirality, which redshift transitions slightly versus isolated tubes.
37. Longer diffusion length means excitons travel farther before recombining (+1pt). Longer diffusion length makes PL more sensitive to nanotube length because excitons are more likely to reach ends. (+1pt)
- 38.
- At short lengths, many excitons may reach the tube ends and are quenched nonradiatively; increasing length reduces the fraction of excitons lost to ends, so PL rises (+1pt).

- b. At long lengths and low power, end effects become negligible for excitons created near the center; PL approaches a plateau because adding more length doesn't significantly increase the fraction that recombines radiatively (+1pt).
 - c. At higher power, exciton-exciton annihilation shortens the effective lifetime, reducing diffusion length. With shorter diffusion length, ends matter less, so high PL can occur even for shorter L; hence the curve "turns on" earlier. (+1pt)
- 39. Linear increase means each absorbed photon creates one emitting exciton (+1pt). At high power, excitons interact and annihilate (+1pt). The physical process responsible for this change is exciton-exciton annihilation (+1pt).
- 40. 1 point per valid reasons (max 2 points):
 - a. Defects localize excitons, preventing diffusion to quenching sites
 - b. Defect states lie below dark exciton states, making radiative decay more likely
 - c. Defects create new optically allowed transitions with higher PL efficiency
 - d. Localization enables single-emitter behavior at room temperature
- 41. Lists one good reason; exciton localization improves emission efficiency, stability at room temperature, reduced sensitivity to material size and disorder, redshifted emission, compatibility with nanophotonic structures, enabling single emitter behavior, etc
- 42. Aryl dopants convert local carbon bonding from sp^2 to sp^3 , breaking pi conjugation. This creates a localized potential well that traps excitons, preventing free diffusion. (In 1D systems, carriers cannot bypass defects, so a single localized site has a strong impact on exciton dynamics.)
- 43. Localized states lie energetically below delocalized excitons, causing red-shifted emission. Localization also reduces interaction with quenching sites, stabilizing emission at room temperature.
- 44. The periodic hole structure of the photonic crystal defines allowed cavity modes (+1pt). On-cavity emission is filtered and enhanced by cavity resonances, while off-cavity emission reflects the full ensemble of defect states (+1pt).
- 45. The cavity increases the local optical density of states at certain wavelengths, making radiative decay more probable. (+1pt) The emitter structure is unchanged; only the surrounding electromagnetic structure is modified. (+1pt)
- 46. Defect localization limits emission to discrete sites, while the cavity selectively enhances only those emitters that are spatially, spectrally, and polarization matched. This structural filtering suppresses contributions from other emitters.
- 47. Capsaicin