Circuit Lab C

November 15-23, 2025

Instructions:

➔ You will have **50 minutes** to complete your exam. At the end of this time period, you must stop working. Partial credit will be awarded, so attempt as many questions as you can!

➔ **Reference Materials:** Please follow national Science Olympiad rules for reference materials. Do not consult the internet, artificial intelligence, or any outside sources.

➔ **Calculator:** You may use a Class III graphing calculator (see Science Olympiad rules for calculator classes)

School/Team Name: ____

Team Number: ____

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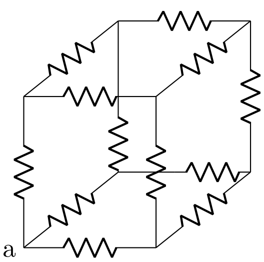
1 Multiple-Choice Questions (40 points)

1. (1 point) Michael Faraday's experiments in the 1830s revolutionized the understanding of electricity and magnetism. Historically, his most significant discovery during this period was:
   A. The mathematical equations unifying electricity and magnetism
   B. The induction of an electric current by a changing magnetic field
   C. The invention of the alternating current induction motor
   D. The measurement of electrostatic forces between charges
   E. The demonstration that chemical reactions could produce steady electric current

2. (3 points) (Tiebreaker #7) Consider a cube made of resistors, each identical to one another, having resistance R. What's the resistance between node a and node b below?



   A. R/2
   B. 7R/12
   C. 3R/4
   D. 5R/6
   E. R

3. (1 point) Which scientist is known for pioneering work in electrochemistry and inventing the first battery?
   A. Alessandro Volta
   B. Gustav Kirchhoff
   C. Charles Coulomb
   D. James Clerk Maxwell
   E. Nikola Tesla

4. (1 point) In the early 19th century, Andre-Marie Ampere's investigations were sparked by Orsted's discovery that a current-carrying wire deflects a compass needle. Historically, Ampere's main contribution was to:
   A. Demonstrate that electricity could be stored in chemical cells
   B. Establish quantitative laws relating forces between electric currents
   C. Invent alternating currents
   D. Build the first practical device for measuring electrical resistance
   E. Lay the foundations of electrodynamics by introducing the concept of the magnetic effect of current

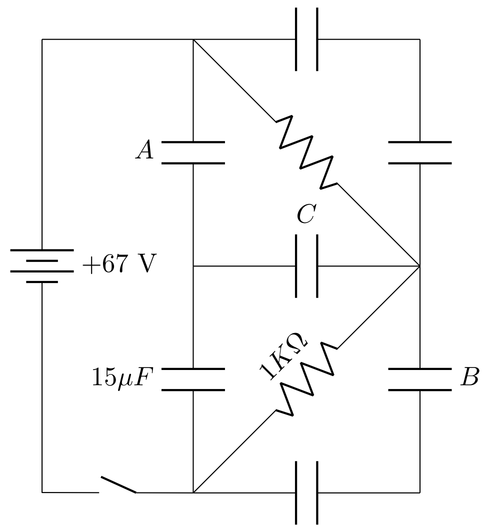
5. (1 point) A light-emitting diode (LED) produces light primarily due to:
   A. Thermal radiation from the heated junction
   B. The oscillation of free electrons in a conductor
   C. The resonance of photons within a crystal lattice
   D. Electrons recombining with holes across the p-n junction
   E. The breakdown of the depletion region under reverse bias

6. (1 point) Which of the following is a distinction of LEDs compared to conventional incandescent lamps?
   A. LEDs emit light in all directions equally
   B. LEDs require heating a filament to emit photons
   C. LEDs convert electrical energy into light with higher efficiency
   D. LEDs cannot operate at low voltages
   E. LEDs emit a continuous spectrum of light like sunlight

7. (1 point) The color of light emitted by an LED is determined by:
   A. The resistance of the p-n junction
   B. The applied forward voltage
   C. The temperature of the diode
   D. The thickness of the depletion region
   E. The bandgap energy of the semiconductor material

8. (2 points) What is the number of distinct resistances that can be produced from a circuit of 6 equal resistors using only series and parallel combinations?
   A. 22
   B. 32
   C. 53
   D. 64
   E. 128

The next 6 questions are about this circuit. Assume that all capacitors are of identical capacitance and all resistors are of identical resistance. The switch is initially open and all capacitors uncharged.



9. (1 point) What is the resistance across the circuit?
   - A. 0Ω
   - B. 67Ω
   - C. 500Ω
   - D. 1000Ω
   - E. 2000Ω

10. (1 point) The switch is closed. At steady state, what is the current through each resistor?
    - A. $0A$
    - B. $0.0335A$
    - C. $0.067A$
    - D. $35.5A$
    - E. The current is not identical in both resistors.

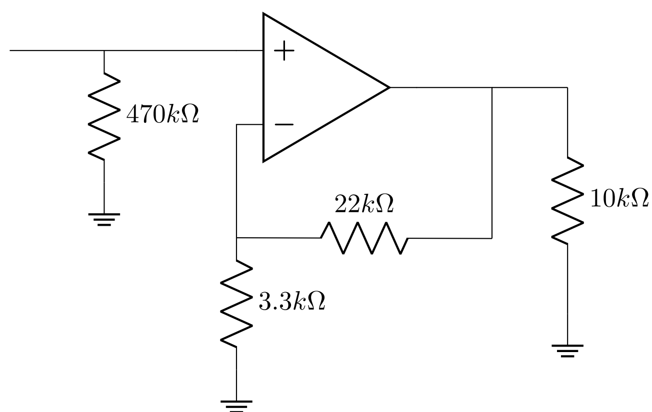
11. (1 point) At steady state, what is the current through the capacitor labeled A?
    - A. $0A$
    - B. $0.0335A$
    - C. $0.067A$
    - D. $15A$
    - E. $35.5A$

12. (1 point) At steady state, the charge on the capacitor labeled A is closest to:
    - A. $0C$
    - B. $2.5 \cdot 10^{-4}C$
    - C. $5.0 \cdot 10^{-4}C$
    - D. $1 \cdot 10^{-3}C$
    - E. $2.5 \cdot 10^{-3}C$

13. (1 point) At steady state, the voltage across the capacitor labeled B is closest to:
    A. $0V$
    B. $11.2V$
    C. $16.8V$
    D. $33.5V$
    E. $67V$

14. (2 points) At steady state, the charge on the capacitor labeled C is closest to:
    A. $0C$
    B. $2.5 \cdot 10^{-4}$ C
    C. $5.0 \cdot 10^{-4}$ C
    D. $1 \cdot 10^{-3}$ C
    E. $2.5 \cdot 10^{-3}$ C

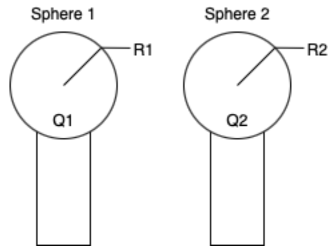
The next 2 questions are about the circuit below:



15. (2 points) What is the voltage gain of this circuit?
    A. 1.15
    B. -6.67
    C. 6.66
    D. 7.67
    E. 143

16. (2 points) What is the input impedance?
    A. 0
    B. $21k\Omega$
    C. $25.3k\Omega$
    D. $470k\Omega$
    E. ∞

2 conducting spheres of radius and charge of R_1 & Q_1 and R_2 & Q_2 respectively are shown in the diagram below. Use this information to answer the following questions.



17. (1 point) The spheres are brought next to each other and briefly make contact before being pulled away from each other. If the radii of the spheres are equal and the spheres have a charge of $+Q$ and $-2Q$ respectively, what are the final charges on spheres 1 and 2, respectively?
    A. $-Q$ and $+2Q$
    B. $-\frac{Q}{2}$ on each
    C. $-Q$ and 0
    D. 0 and $-Q$
    E. $+\frac{3Q}{2}$ and $-\frac{5Q}{2}$

18. (1 point) The spheres are brought next to each other and briefly make contact before being pulled away from each other. If $R_1 = 2R_2$ and the spheres have a charge of $+Q$ and $-2Q$ respectively, what are the final charges on spheres 1 and 2, respectively?
    A. $-\frac{Q}{3}$ and $-\frac{2Q}{3}$
    B. $-2Q$ and Q
    C. $-\frac{2Q}{3}$ and $-\frac{Q}{3}$
    D. 0 and $-Q$
    E. $+\frac{Q}{2}$ and $-\frac{3Q}{2}$

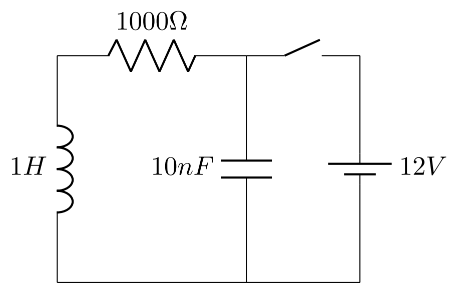
19. (1 point) Given that the spheres have a charge of $+Q$ and $-2Q$ respectively, what is $\frac{\sigma_1}{\sigma_2}$, the ratio of the spheres' surface charge density?
    A. $-\frac{R_1^2}{2R_2^2}$
    B. $-\frac{R_2}{2R_1}$
    C. $(\frac{R_1}{R_2})^2$
    D. $-\frac{R_1}{2R_1}$
    E. $-\frac{R_2^2}{2R_1^2}$

20. (1 point) The spheres are made to contact each other and a positively charged glass rod is brought near Sphere 1. If the spheres are identical and have initial charges of 0, when the rod is near Sphere 1 must have a ____________ charge and Sphere 2 must have a ____________ charge.
    A. negative, positive
    B. positive, negative
    C. neutral, neutral
    D. negative, negative
    E. neutral, positive

21. (1 point) The spheres are made to contact each other and a positively charged glass rod is brought near Sphere 1. Sphere 2 is then discharged to ground using a thin wire. The glass rod is then removed, and the spheres are separated. If the spheres are identical and have initial charges of 0, then Sphere 1 must have a ________ charge and Sphere 2 must have a ________ charge.

    A. negative, positive
    B. positive, negative
    C. neutral, neutral
    D. negative, negative
    E. neutral, positive

The following questions are about the circuit below. Assume the switch is initially open and all capacitors are initially uncharged.



22. (1 point) What is the steady-state current that passes through the capacitor?

    A. 12 mA
    B. 120 mA
    C. 12 A
    D. 0 A
    E. The circuit never achieves a steady state.

23. (1 point) What is the steady-state current that passes through the solenoid?

    A. 12 mA
    B. 120 mA
    C. 12 A
    D. 0 A
    E. The circuit never achieves a steady state.

24. (1 point) If the capacitor is a Barium Titanate ceramic capacitor, which of the following could be its dielectric coefficient?

    A. 0.5
    B. 6
    C. 38
    D. 7000
    E. 50000

25. (1 point) Using the dielectric coefficient from the last question what is the plate separation of the capacitor, given that plate area = $10mm^2$?

    A. 61.95 μm

    B. 25.33 μm

    C. 4 nm

    D. 5.63 μm

    E. 1.33 mm

26. (1 point) If the solenoid has 500000 turns per unit length, what is the magnetic field produced by it?

    A. 7.5×10^{-2} T

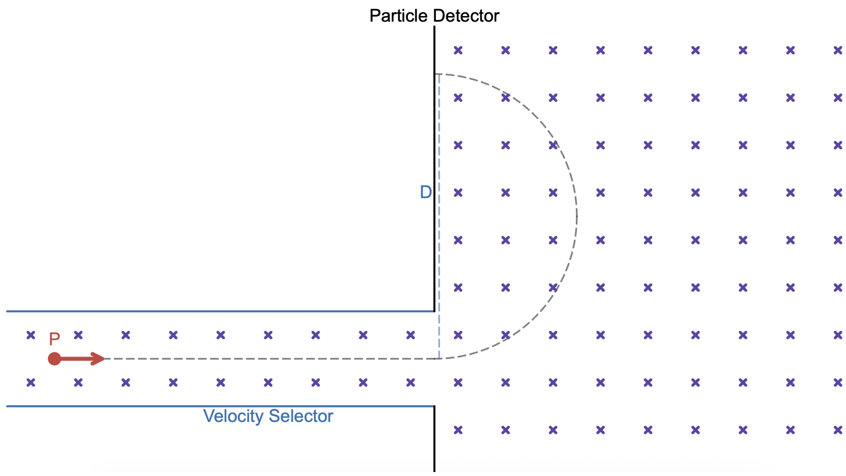
    B. 7.5×10^{-3} T

    C. 7.5×10^{-4} T

    D. 3.1×10^{-2} T

    E. 1.2×10^{-3} T

The magnetic field generated by the solenoid and the electric field generated by the capacitor are used in the velocity selector of a mass spectrometer, shown below.



27. (2 points) What is the velocity of the charged particles that exit the velocity selector?

    A. 1.25×10^6 m/s

    B. 8.85×10^9 m/s

    C. 1.59×10^4 m/s

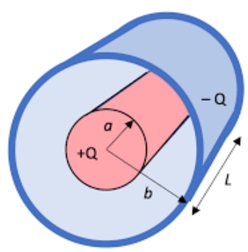
    D. 2.57×10^7 m/s

    E. 6.32×10^7 m/s

28. (2 points) A beam of singly charged ions ($q = +e$) exits the velocity selector undeflected at a speed of $v = 2.57 \times 10^7$ m/s. It then enters a uniform analyzer magnet of magnetic field $B_m = 1.00$ T and follows a circular path of radius $r = 1.00$ m to reach the detector. What is the mass of the ion? (Use $e = 1.602 \times 10^{-19}$ C.)
    - A. 6.24×10^{-24} g
    - B. 1.25×10^{-23} g
    - C. 6.24×10^{-27} g
    - D. 6.24×10^{-25} g
    - E. 3.12×10^{-27} g

29. (1 point) If the actual dielectric constant of the capacitor is twice what was originally assumed, what is the percent error of the mass you originally calculated?
    - A. 75%
    - B. 125%
    - C. 25%
    - D. 100%
    - E. 50%

For the next 3 questions, use the image below to help you answer questions about the electric field of the model shown at the various distances r from the center axis. (Hint: The electric field can be calculated using Gauss's Law, $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$ which, for electric fields normal to the surface, simplifies to $EA = \frac{q_{enc}}{\epsilon_0}$ where E is the electric field, A is the area, q_{enc} is the charge enclosed, and ϵ_0 is the universal constant.)



30. (1 point) Assuming the core is an insulator, what is the relationship between the magnitude of the E and r when $r < a$?
    - A. inverse-square
    - B. linear
    - C. 0
    - D. inverse
    - E. quadratic

31. (1 point) Calculate the electric field of the model when $a < r < b$. Express your answer in terms of Q, a, b, r, L, and universal constants.
    A. $\frac{Q}{4\pi\epsilon_0 r^2}$
    B. $\frac{Qr}{2\pi\epsilon_0 La^2}$
    C. 0
    D. $\frac{Q}{2\pi\epsilon_0 rL}$
    E. $\frac{Qr^2}{4\pi\epsilon_0 L}$

32. (1 point) Calculate the electric field of the model when $r > b$. Express your answer in terms of Q, a, b, r, L, and universal constants.
    A. $\frac{Q}{4\pi\epsilon_0 r^2}$
    B. $\frac{Qr}{2\pi\epsilon_0 La^2}$
    C. 0
    D. $\frac{Q}{2\pi\epsilon_0 rL}$
    E. $\frac{Qr^2}{4\pi\epsilon_0 L}$

2 Particle accelerator (15 points)

In 1932 Cockcroft and Walton, two young British physicists successfully operated the first high energy proton accelerator and succeeded in causing nuclear disintegration. Their experiment provided one of the earliest confirmations of the relativistic mass-energy relation. To achieve this, they created the Cockcroft-Walton (CW) generator, an electrical circuit which generates a high AC voltage from a low-voltage DC. They used this voltage multiplier cascade for most of their research, which in 1951 won them the Nobel Prize in Physics for the "Transmutation of atomic nuclei by artificially accelerated atomic particles."

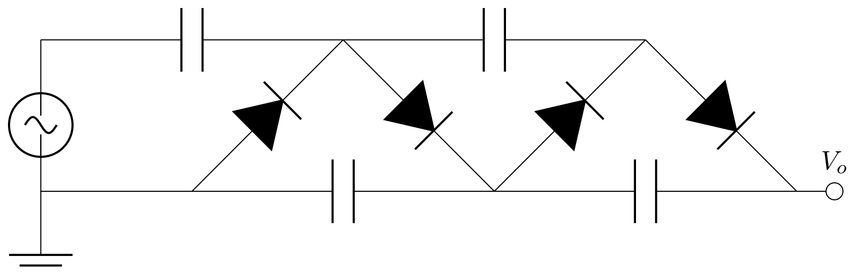


Figure 1: Two-stage Cockcroft-Walton multiplier

33. (1 point) (Tiebreaker #6) The CW generator is part of a larger class of electrical devices that converts AC to DC. What is the name of this class of electrical devices?

34. (2 points) Why are diodes necessary in the CW generator? What would happen to the charge distribution across the capacitors if the diodes were replaced with simple wires?

35. (2 points) Explain, using charge storage concepts, how capacitors in such a circuit can "add" voltages together. (Hint: Think about how a charged capacitor in series with a voltage source affects total potential difference.)

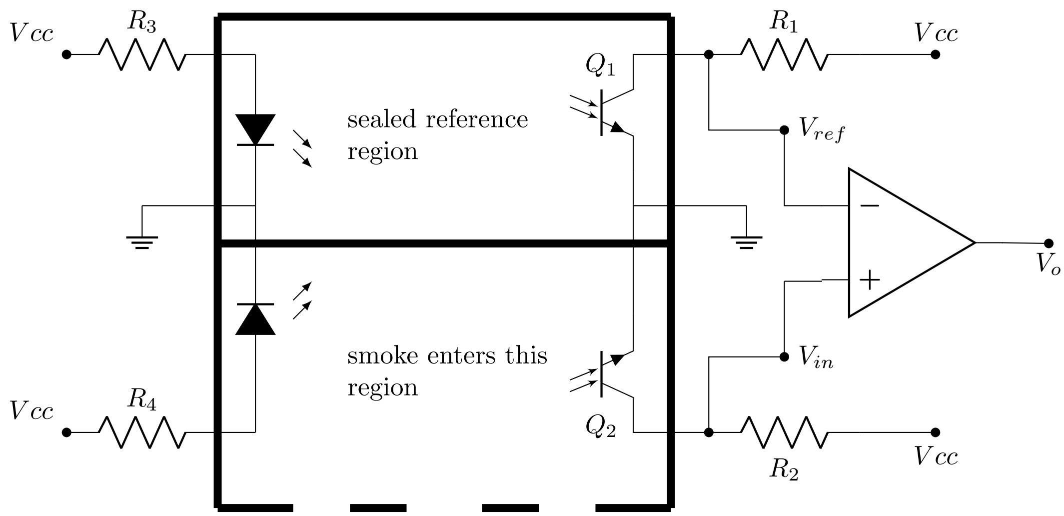
36. (2 points) (Tiebreaker #5) Using Kirchhoff's law, analyze two loops of a two-stage CW generator to show how voltages "stack up" during charging.

37. (4 points) (Tiebreaker #4) Assume all capacitors are initially uncharged, and the circuit in the figure is powered by an alternating voltage $\mathbf{V}_i$ such that $V_i = V_p \sin(\omega t + \pi)$. Derive the total output voltage under no-load conditions in terms of V_p.

38. (2 points) In practice, the output voltage of a CW generator decreases when a load (like a particle beam) is connected. Explain why this happens in terms of capacitor discharge and current flow.

39. (2 points) If a CW generator can multiply voltage far beyond the input source, why does this not violate energy conservation in the context of accelerating particle beams to high energies?

3 Smoke detector (13 points)



Above is a schematic for a simple smoke detector *chirps*. The upper (sealed) chamber provides a reference signal via phototransistor Q_1 and the lower (vented) chamber senses smoke via phototransistor Q_2. Each chamber contains its own LED that shines across the chamber; more smoke means less light at the phototransistor. Nodes V_{ref} and V_{in} are pulled up by R_1 and R_2 to Vcc and pulled down by the respective phototransistor currents to the ground. It uses an op-amp configured as a comparator, which asserts V_O high when $V_{in} > V_{ref}$.

Unless noted otherwise, take $Vcc = 5.0V$. Phototransistors have collector current $I_C = \beta I_{ph} + I_{leak}$ with $\beta = 100$, $I_{leak} = 50nA$ at $25°C$. In clean air at $25°C$ both chambers deliver photocurrent $I_{ph0} = 200\mu A$. In smoke, the sensing chamber's photocurrent scales to $I_{ph} = (1-\alpha)I_{ph0}$, where α is the fractional attenuation due to smoke. The sealed chamber stays at I_{ph0}. Ignore based currents and $V_{CE(sat)} unlesssaturationisexplicitlyinvoked.$

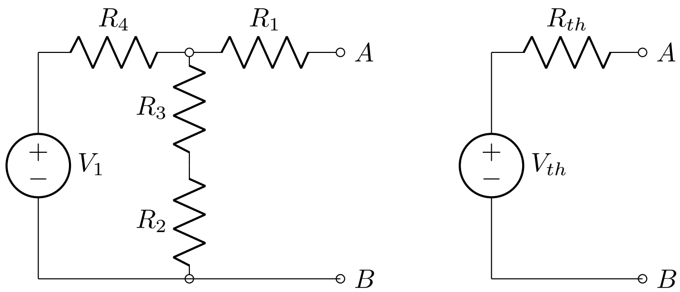
40. (3 points) (Tiebreaker #3) With clean air ($\alpha = 0$), choose equal pull-ups $R_1 = R_2 = R$ so that $V_{ref} = V_{in} = 2.0V$. Compute required R and verify the phototransistor operating region (are Q_1/Q_2 in saturation or active?).

41. (4 points) As it stands, our smoke detector trips for *any* impurity in air, which is not exactly a good thing. Add a positive-feedback resistor R_H from V_O (assume it swings 0-5V) to the non-inverting input (+). With R_2 unchanged, derive the two trip points $\alpha_{T,UP}$ (rising smoke) and $\alpha_{T,DN}$ (clearing smoke) in terms of R_2 and R_H *chirp*. Choose R_H so the width of the hysteresis $\alpha_{T,UP} - \alpha_{T,DN} = 0.10$ (i.e., attenuation band 10%) and report the numeric values.

42. (2 points) If the 9-V battery sags and the whole circuit runs from a reduced Vcc (the op-amp still swings $0 - Vcc$), what is the minimum Vcc that keeps both phototransistors in the forward-active region in clean air? Use resistor values from previous parts, or, if you were not able to obtain answers, feel free to leave your answer in terms of those quantities.

43. (4 points) You want to ignore very brief puffs of dust. Add a capacitor to create a first-order time constant of about $\tau \approx 20ms$ for the positive-feedback path so the comparator won't flip on sub-20ms events. If you place the capacitor across R_H (from V_o to V_{in}), what value C_f gives $\tau \approx 20ms$? If you instead place a capacitor from V_{in} to ground, what value C_g gives the same τ, and why is this a poorer choice? (if you were not able to obtain answers for R_H, R_2,etc, feel free to leave your answer in terms of those quantities)

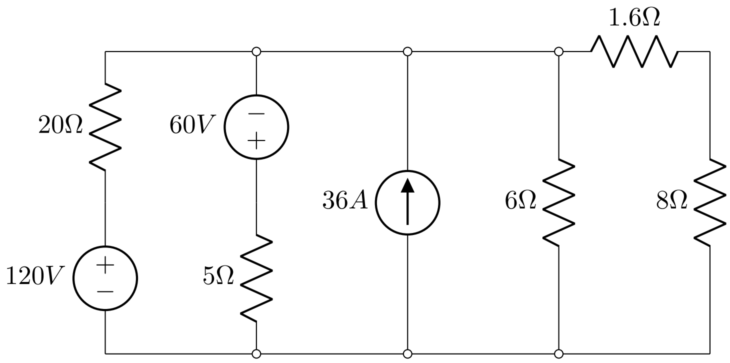
4 Circuit Analysis (12 points)



44. (4 points) Thevenin's theorem states that any linear network containing only voltage sources, current sources, and resistances can be replaced at terminals A-B by an equivalent combination of a voltage source V_{th} in a series connection with a resistance R_{th}. Given the circuit on the left, solve for the Thevenin-equivalent circuit, giving values V_{th} and R_{th} in terms of V_1, R_1, R_2, R_3, and R_4.

45. (3 points) Dual to Thevenin's theorem is Norton's theorem, which states that linear networks containing only voltage sources, current sources, and resistances can be replaced at terminals A-B by an equivalent combination of a current source I_{no} in parallel with a single resistor R_{no}. From the same circuit as the last question, solve for the Norton-equivalent circuit, giving values I_{no} and R_{no} in terms of V_1, R_1, R_2, R_3, and R_4.

46. (5 points) (Tiebreaker #2) Having these two theorems under your belt, compute the voltage across the 8Ω resistor for the circuit below, and note the direction of current.



5 Computers (20 points)

Silicon PN junctions form the basis of diodes and transistors. A transistor, made of two PN junctions, can act as a switch and is the building block of logic gates in computers. Logic gates are the fundamental units of digital circuits that process information using binary signals (0 = low voltage, 1 = high voltage). Transistors revolutionized the world of computers. First, we analyze a historically important computer that used relays.

47. (9 points) The Harvard Mark II (1947) implemented arithmetic and control with about 13,000 electromechanical relays.

A Mark II-class relay coil is modeled as an ideal series $R - L$ driven from a 48 V DC supply. Let $R = 200\Omega$, $L = 80$ mH. The armature motion is approximated as a fixed mechanical lag $t_m = 10$ ms after the coil current reaches the pickup threshold. Assume pickup occurs at $0.8I_\infty$ and dropout at $0.2I_\infty$, where $I_\infty = V/R$:

(a) (1 points) Derive the pickup time t_p and dropout time t_d referenced to the electrical step only, using $i(t) = I_\infty(1 - e^{-t/\tau})$ and $i(t) = I_\infty e^{-t/\tau}$ with $\tau = L/R$.

(b) (2 points) Include the mechanical lag and report the make and break delays.

(c) (1 point) If a combinational path traverses k relay stages in series, each requiring one make transition, give the worst-case propagation delay $T_p(k)$.

(d) (3 points) Starting from steady state at $t = 0^-$, the supply is opened at $t = 0$. Compare the electrical dropout time for these three cases placed across the coil:

(i) No clamp, but the driver limits the flyback to a constant $|v_{coil}| \approx 200V$ during decay (idealized)

(ii) Flyback diode with $V_F = 0.7V$ (assume ideal diode otherwise)

(iii) Zener clamp: diode + Zener such as $|v_{coil}| \approx 24.7V$ during decay

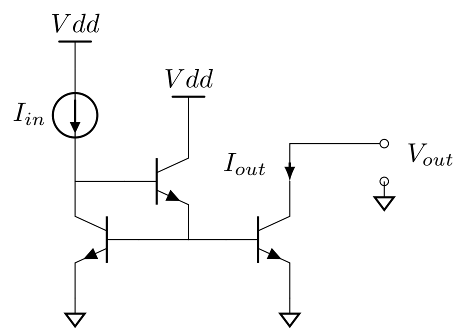
For each case, derive the governing equation and solve for t_d. Then report break delay.

(e) (7 points) (Tiebreaker #1) Consider a complementary metal-oxide semiconductor (CMOS) with $V = 1V$, $R_{on} = 1k\Omega$, $C_L = 40fF$; estimate (i) propagation delay and (ii) energy per output toggle for each, then decide if either can meet 1 MHz at $\leq$ 1 mW per gate and by how many orders of magnitude the better option wins. Use, for the relay, $t_p \approx \tau \ln 5 + t_m$, $E \approx \frac{1}{2}LI_p^2$ with $I_p = 0.8V/R$, and, for the CMOS, $t_p \approx 0.69R_{on}C_L$, $E \approx \frac{1}{2}C_LV^2$.

48. (2 points) Explain what happens at the atomic level in a silicon PN junction when it is forward biased versus reverse biased. Why does current flow more easily in one direction than the other?

49. (2 points) Consider an NPN transistor used as a switch. Describe the role of the base, collector, and emitter in controlling current flow, and explain why a small base current can control a much larger collector current.

50. (0 points) (Optional, not included in the Mason Invitational Satellite exam) A current mirror is an electronic circuit designed to copy an input current and provide that current to a load, often used to create a stable, constant current source or sink. By creating multiple copies of a stable reference current, current mirrors are essential in integrated circuits (ICs) for biasing amplifiers and are implemented using MOSFETs or bipolar junction transistors (BJTs). For the following BJT circuit mirror, express I_{out}/I_{in} as a fraction of expanded polynomials in current gain $\beta = \frac{I_C}{I_B}$, assuming all transistors are matched. Also, solve for the lower bound of V_{OUT} such that all transistors remain forward active in terms of base-emitter voltage V_{BE} and collector-emitter saturation voltage $V_{CE,sat}$. You cannot neglect base currents.



51. (0 points) (Optional, not included in the Mason Invitational Satellite exam) Draw a NAND logic gate using transistor logic.