Circuit Lab C

November 15-23, 2025

Instructions:

➔ You will have **50 minutes** to complete your exam. At the end of this time period, you must stop working. Partial credit will be awarded, so attempt as many questions as you can!

➔ **Reference Materials:** Please follow national Science Olympiad rules for reference materials. Do not consult the internet, artificial intelligence, or any outside sources.

➔ **Calculator:** You may use a Class III graphing calculator (see Science Olympiad rules for calculator classes)

School/Team Name: ____

Team Number: ___

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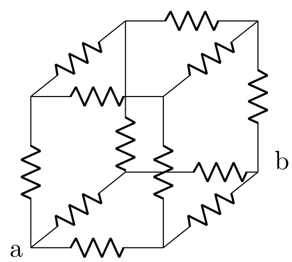
1 Multiple-Choice Questions (40 points)

1. (1 point) Michael Faraday's experiments in the 1830s revolutionized the understanding of electricity and magnetism. Historically, his most significant discovery during this period was:
   - A. The mathematical equations unifying electricity and magnetism
   - **B. The induction of an electric current by a changing magnetic field**
   - C. The invention of the alternating current induction motor
   - D. The measurement of electrostatic forces between charges
   - E. The demonstration that chemical reactions could produce steady electric current

2. (3 points) (Tiebreaker #7) Consider a cube made of resistors, each identical to one another, having resistance R. What's the resistance between node a and node b below?



   - A. R/2
   - B. 7R/12
   - **C. 3R/4**
   - D. 5R/6
   - E. R

3. (1 point) Which scientist is known for pioneering work in electrochemistry and inventing the first battery?
   - **A. Alessandro Volta**
   - B. Gustav Kirchhoff
   - C. Charles Coulomb
   - D. James Clerk Maxwell
   - E. Nikola Tesla

4. (1 point) In the early 19th century, Andre-Marie Ampere's investigations were sparked by Orsted's discovery that a current-carrying wire deflects a compass needle. Historically, Ampere's main contribution was to:
   - A. Demonstrate that electricity could be stored in chemical cells
   - **B. Establish quantitative laws relating forces between electric currents**
   - C. Invent alternating currents
   - D. Build the first practical device for measuring electrical resistance
   - E. Lay the foundations of electrodynamics by introducing the concept of the magnetic effect of current

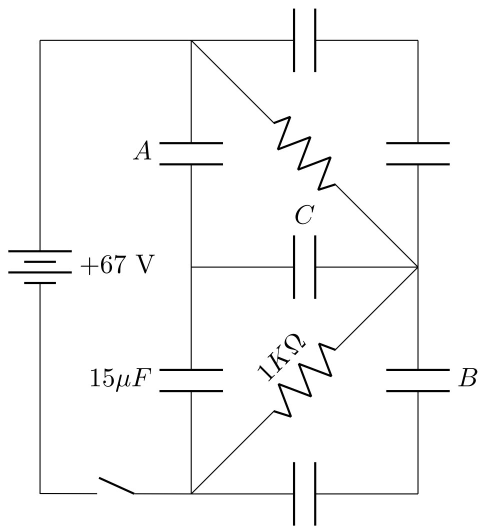
5. (1 point) A light-emitting diode (LED) produces light primarily due to:
   A. Thermal radiation from the heated junction
   B. The oscillation of free electrons in a conductor
   C. The resonance of photons within a crystal lattice
   D. Electrons recombining with holes across the p-n junction
   E. The breakdown of the depletion region under reverse bias

6. (1 point) Which of the following is a distinction of LEDs compared to conventional incandescent lamps?
   A. LEDs emit light in all directions equally
   B. LEDs require heating a filament to emit photons
   C. LEDs convert electrical energy into light with higher efficiency
   D. LEDs cannot operate at low voltages
   E. LEDs emit a continuous spectrum of light like sunlight

7. (1 point) The color of light emitted by an LED is determined by:
   A. The resistance of the p-n junction
   B. The applied forward voltage
   C. The temperature of the diode
   D. The thickness of the depletion region
   E. The bandgap energy of the semiconductor material

8. (2 points) What is the number of distinct resistances that can be produced from a circuit of 6 equal resistors using only series and parallel combinations?
   A. 22
   B. 32
   C. 53
   D. 64
   E. 128

The next 6 questions are about this circuit. Assume that all capacitors are of identical capacitance and all resistors are of identical resistance. The switch is initially open and all capacitors uncharged.



9. (1 point) What is the resistance across the circuit?
   A. 0Ω
   B. 67Ω
   C. 500Ω
   D. 1000Ω
   E. 2000Ω

10. (1 point) The switch is closed. At steady state, what is the current through each resistor?
    A. $0A$
    B. $0.0335A$
    C. $0.067A$
    D. $35.5A$
    E. The current is not identical in both resistors.

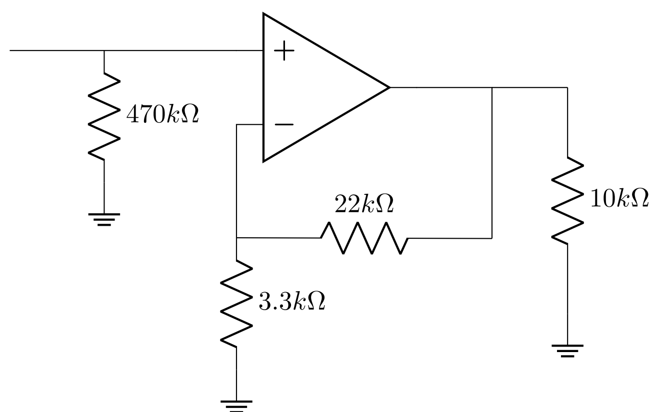
11. (1 point) At steady state, what is the current through the capacitor labeled A?
    A. $0A$
    B. $0.0335A$
    C. $0.067A$
    D. $15A$
    E. $35.5A$

12. (1 point) At steady state, the charge on the capacitor labeled A is closest to:
    A. $0C$
    B. $2.5 \cdot 10^{-4}C$
    C. $5.0 \cdot 10^{-4}C$
    D. $1 \cdot 10^{-3}C$
    E. $2.5 \cdot 10^{-3}C$

13. (1 point) At steady state, the voltage across the capacitor labeled B is closest to:
    A. $0V$
    B. $11.2V$
    C. $16.8V$
    D. $33.5V$
    E. $67V$

14. (2 points) At steady state, the charge on the capacitor labeled C is closest to:
    A. $0C$
    B. $2.5 \cdot 10^{-4}$ C
    C. $5.0 \cdot 10^{-4}$ C
    D. $1 \cdot 10^{-3}$ C
    E. $2.5 \cdot 10^{-3}$ C

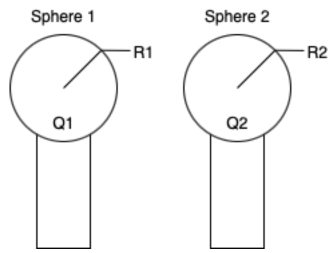
The next 2 questions are about the circuit below:



15. (2 points) What is the voltage gain of this circuit?
    A. 1.15
    B. -6.67
    C. 6.66
    D. 7.67
    E. 143

16. (2 points) What is the input impedance?
    A. 0
    B. $21k\Omega$
    C. $25.3k\Omega$
    D. $470k\Omega$
    E. ∞

2 conducting spheres of radius and charge of R_1 & Q_1 and R_2 & Q_2 respectively are shown in the diagram below. Use this information to answer the following questions.



17. (1 point) The spheres are brought next to each other and briefly make contact before being pulled away from each other. If the radii of the spheres are equal and the spheres have a charge of $+Q$ and $-2Q$ respectively, what are the final charges on spheres 1 and 2, respectively?
   A. $-Q$ and $+2Q$
   B. $-\frac{Q}{2}$ on each
   C. $-Q$ and 0
   D. 0 and $-Q$
   E. $+\frac{3Q}{2}$ and $-\frac{5Q}{2}$

18. (1 point) The spheres are brought next to each other and briefly make contact before being pulled away from each other. If $R_1 = 2R_2$ and the spheres have a charge of $+Q$ and $-2Q$ respectively, what are the final charges on spheres 1 and 2, respectively?
   A. $-\frac{Q}{3}$ and $-\frac{2Q}{3}$
   B. $-2Q$ and Q
   C. $-\frac{2Q}{3}$ and $-\frac{Q}{3}$
   D. 0 and $-Q$
   E. $+\frac{Q}{2}$ and $-\frac{3Q}{2}$

19. (1 point) Given that the spheres have a charge of $+Q$ and $-2Q$ respectively, what is $\frac{\sigma_1}{\sigma_2}$, the ratio of the spheres' surface charge density?
   A. $-\frac{R_1^2}{2R_2^2}$
   B. $-\frac{R_2}{2R_1}$
   C. $(\frac{R_1}{R_2})^2$
   D. $-\frac{R_1}{2R_1}$
   E. $-\frac{R_2^2}{2R_1^2}$

20. (1 point) The spheres are made to contact each other and a positively charged glass rod is brought near Sphere 1. If the spheres are identical and have initial charges of 0, when the rod is near Sphere 1 must have a __________ charge and Sphere 2 must have a __________ charge.
   A. negative, positive
   B. positive, negative
   C. neutral, neutral
   D. negative, negative
   E. neutral, positive

21. (1 point) The spheres are made to contact each other and a positively charged glass rod is brought near Sphere 1. Sphere 2 is then discharged to ground using a thin wire. The glass rod is then removed, and the spheres are separated. If the spheres are identical and have initial charges of 0, then Sphere 1 must have a __________ charge and Sphere 2 must have a __________ charge.

    A. negative, positive

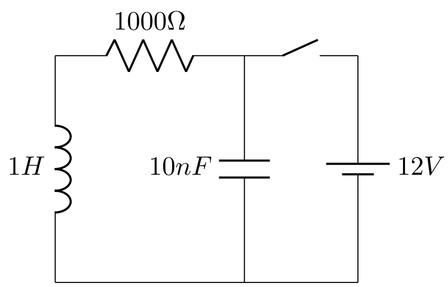
    B. positive, negative

    C. neutral, neutral

    D. negative, negative

    E. neutral, positive

The following questions are about the circuit below. Assume the switch is initially open and all capacitors are initially uncharged.



22. (1 point) What is the steady-state current that passes through the capacitor?

    A. 12 mA

    B. 120 mA

    C. 12 A

    D. 0 A

    E. The circuit never achieves a steady state.

23. (1 point) What is the steady-state current that passes through the solenoid?

    A. 12 mA

    B. 120 mA

    C. 12 A

    D. 0 A

    E. The circuit never achieves a steady state.

24. (1 point) If the capacitor is a Barium Titanate ceramic capacitor, which of the following could be its dielectric coefficient?

    A. 0.5

    B. 6

    C. 38

    D. 7000

    E. 50000

25. (1 point) Using the dielectric coefficient from the last question what is the plate separation of the capacitor, given that plate area = $10mm^2$?

    A. 61.95 μm

    B. 25.33 μm

    C. 4 nm

    D. 5.63 μm

    E. 1.33 mm

26. (1 point) If the solenoid has 500000 turns per unit length, what is the magnetic field produced by it?

    A. 7.5×10^{-2} T

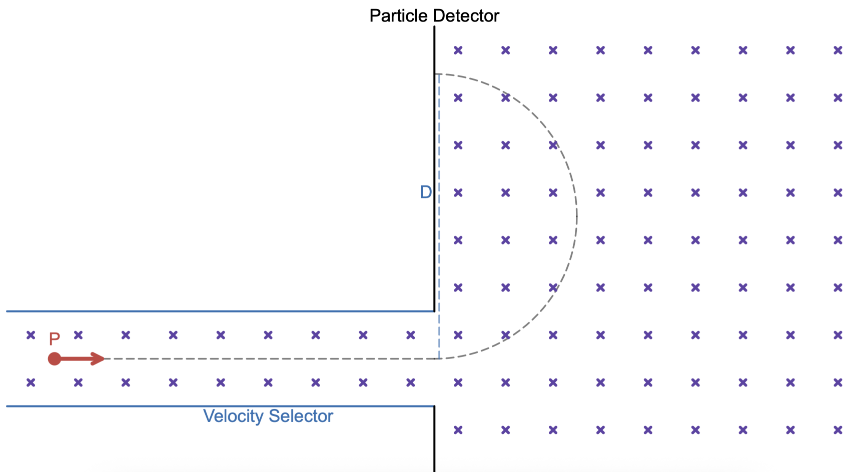
    B. 7.5×10^{-3} T

    C. 7.5×10^{-4} T

    D. 3.1×10^{-2} T

    E. 1.2×10^{-3} T

The magnetic field generated by the solenoid and the electric field generated by the capacitor are used in the velocity selector of a mass spectrometer, shown below.



27. (2 points) What is the velocity of the charged particles that exit the velocity selector?

    A. 1.25×10^6 m/s

    B. 8.85×10^9 m/s

    C. 1.59×10^4 m/s

    D. 2.57×10^7 m/s

    E. 6.32×10^7 m/s

28. (2 points) A beam of singly charged ions ($q = +e$) exits the velocity selector undeflected at a speed of $v = 2.57 \times 10^7$ m/s. It then enters a uniform analyzer magnet of magnetic field $B_m = 1.00$ T and follows a circular path of radius $r = 1.00$ m to reach the detector. What is the mass of the ion? (Use $e = 1.602 \times 10^{-19}$ C.)

    A. 6.24×10^{-24} g

    B. 1.25×10^{-23} g

    C. 6.24×10^{-27} g

    D. 6.24×10^{-25} g

    E. 3.12×10^{-27} g

29. (1 point) If the actual dielectric constant of the capacitor is twice what was originally assumed, what is the percent error of the mass you originally calculated?

    A. 75%

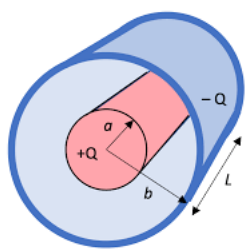
    B. 125%

    C. 25%

    D. 100%

    E. 50%

For the next 3 questions, use the image below to help you answer questions about the electric field of the model shown at the various distances r from the center axis. (Hint: The electric field can be calculated using Gauss's Law, $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$ which, for electric fields normal to the surface, simplifies to $EA = \frac{q_{enc}}{\epsilon_0}$ where E is the electric field, A is the area, q_{enc} is the charge enclosed, and ϵ_0 is the universal constant.)



30. (1 point) Assuming the core is an insulator, what is the relationship between the magnitude of the E and r when $r < a$?

    A. inverse-square

    B. linear

    C. 0

    D. inverse

    E. quadratic

31. (1 point) Calculate the electric field of the model when $a < r < b$. Express your answer in terms of Q, a, b, r, L, and universal constants.
   A. $\frac{Q}{4\pi\epsilon_0 r^2}$
   B. $\frac{Qr}{2\pi\epsilon_0 L a^2}$
   C. 0
   D. $\frac{Q}{2\pi\epsilon_0 r L}$
   E. $\frac{Qr^2}{4\pi\epsilon_0 L}$

32. (1 point) Calculate the electric field of the model when $r > b$. Express your answer in terms of Q, a, b, r, L, and universal constants.
   A. $\frac{Q}{4\pi\epsilon_0 r^2}$
   B. $\frac{Qr}{2\pi\epsilon_0 L a^2}$
   C. 0
   D. $\frac{Q}{2\pi\epsilon_0 r L}$
   E. $\frac{Qr^2}{4\pi\epsilon_0 L}$

2 Particle accelerator (15 points)

In 1932 Cockcroft and Walton, two young British physicists successfully operated the first high energy proton accelerator and succeeded in causing nuclear disintegration. Their experiment provided one of the earliest confirmations of the relativistic mass-energy relation. To achieve this, they created the Cockcroft-Walton (CW) generator, an electrical circuit which generates a high AC voltage from a low-voltage DC. They used this voltage multiplier cascade for most of their research, which in 1951 won them the Nobel Prize in Physics for the "Transmutation of atomic nuclei by artificially accelerated atomic particles."

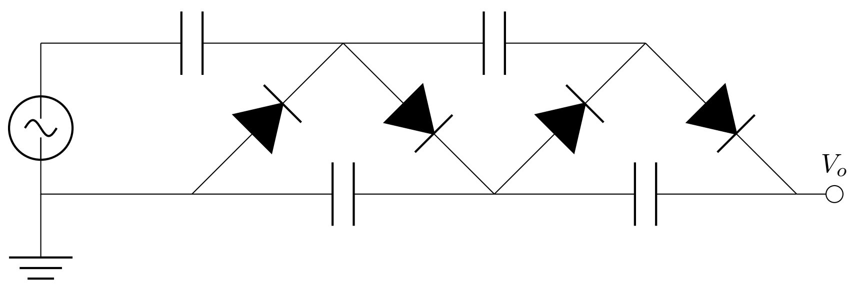


Figure 1: Two-stage Cockcroft-Walton multiplier

33. (1 point) (Tiebreaker #6) The CW generator is part of a larger class of electrical devices that converts AC to DC. What is the name of this class of electrical devices?

Solution: Rectifier

34. (2 points) Why are diodes necessary in the CW generator? What would happen to the charge distribution across the capacitors if the diodes were replaced with simple wires?

Solution: Diodes in a CW generator ensure unidirectional current flow so that each capacitor charges in sequence, building up higher voltages. If they were replaced with wires, the capacitors would discharge back and equalize, preventing any voltage multiplication.
+1 point: Mentions that diodes provide directionality / prevent discharge.
+1 point: Explains that without diodes, capacitors would equalize / voltage multiplication would not occur.

35. (2 points) Explain, using charge storage concepts, how capacitors in such a circuit can "add" voltages together. (Hint: Think about how a charged capacitor in series with a voltage source affects total potential difference.)

Solution: When a charged capacitor is placed in series with a voltage source, its stored charge produces a potential difference that adds to the source's voltage, so the total output is the sum of both. In a CW generator, multiple capacitors charged in sequence can therefore stack their voltages, effectively multiplying the input.
+1 pt: Mentions that a charged capacitor provides its own potential difference due to stored charge.
+1 pt: Explains that in series, this potential difference adds to the source (or other capacitors), creating a summed voltage.

36. (2 points) (Tiebreaker #5) Using Kirchhoff's law, analyze two loops of a two-stage CW generator to show how voltages "stack up" during charging.

Solution: During the positive peak, KVL around the loop (source → D1 → C1 → return) gives $V_s - V_{C1} \approx 0 \Rightarrow V_{C1} \approx V_{s,pk}$. On the opposite half cycle, KVL around (source → C1 (now a source) → D2 → C2 → return) gives $-V_s + V_{C1} - V_{C2} \approx 0 \Rightarrow V_{C2} \approx V_{C1} + V_{s,pk} \approx 2V_{s,pk}$ showing the voltages stack.
+1 pt: Writes correct KVL for first loop showing $V_{C1} \approx V_{s,pk}$
+1 pt: Writes correct second-loop KVL yielding $V_{C2} \approx V_{C1} + V_{s,pk} \approx 2V_{s,pk}$.

37. (4 points) (Tiebreaker #4) Assume all capacitors are initially uncharged, and the circuit in the figure is powered by an alternating voltage $\mathbf{V}_i$ such that $V_i = V_p \sin(\omega t + \pi)$. Derive the total output voltage under no-load conditions in terms of V_p.

Solution: Under no load, each diode conducts only on the half-cycle that forward-biases it, so every capacitor charges to (approximately) the input peak V_p. During subsequent peaks, previously charged capacitors appear in series with the source, so the charging path for the next capacitor sees a peak of V_p (source) + already-stacked peaks; this yields node voltages $0, 2V_p, 4V_p, \cdots$, and hence for an N-stage CW the output is $V_{out,NL} \approx 2NV_p$. In particular, for a two stage generator $V_{out,NL}$
+1 pt: States each capacitor charges to approximately V_p on its conducting half-cycle
+1 pt: Uses KVL/series stacking idea that the charge capacitor adds its potential to the source on the next charge path
+2 pts: Concludes $V_{out,NL} = 4V_p$

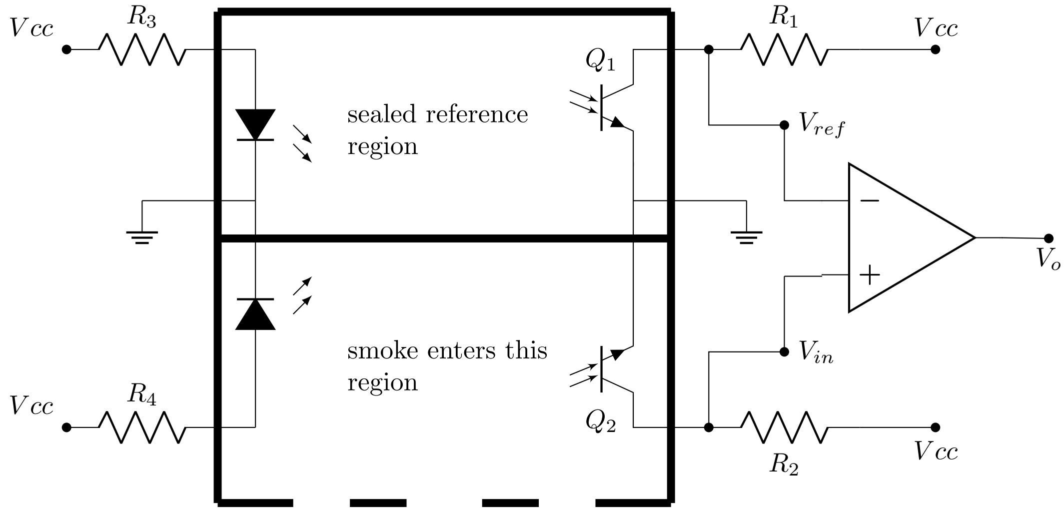
38. (2 points) In practice, the output voltage of a CW generator decreases when a load (like a particle beam) is connected. Explain why this happens in terms of capacitor discharge and current flow.

Solution: When a load is connected, current must be supplied from the capacitors during the parts of the AC cycle when diodes are reverse-biased. This causes the capacitors to partially discharge, so their voltages sag and the stacked output voltage falls below the ideal no-load value.
+1 pt: Mentions that the capacitors discharge into the load when current is drawn.
+1 pt: Explains that this discharge lowers the stored voltages, reducing the total stacked output.

39. (2 points) If a CW generator can multiply voltage far beyond the input source, why does this not violate energy conservation in the context of accelerating particle beams to high energies?

Solution: The CW generator increases voltage by stacking capacitor charges, but the total energy delivered to the beam comes from the AC source. As more charge (beam current) is drawn, the source must supply more energy, so energy is conserved.
+2pts: Any answer that the total energy delivered to the beam comes from the AC source, and that higher voltage $\not\Rightarrow$ higher energy.

3 Smoke detector (13 points)



Above is a schematic for a simple smoke detector *chirps*. The upper (sealed) chamber provides a reference signal via phototransistor Q_1 and the lower (vented) chamber senses smoke via phototransistor Q_2. Each chamber contains its own LED that shines across the chamber; more smoke means less light at the phototransistor. Nodes V_{ref} and V_{in} are pulled up by R_1 and R_2 to Vcc and pulled down by the respective phototransistor currents to the ground. It uses an op-amp configured as a comparator, which asserts V_O high when $V_{in} > V_{ref}$.

Unless noted otherwise, take $Vcc = 5.0V$. Phototransistors have collector current $I_C = \beta I_{ph} + I_{leak}$ with $\beta = 100$, $I_{leak} = 50nA$ at $25°C$. In clean air at $25°C$ both chambers deliver photocurrent $I_{ph0} = 200\mu A$. In smoke, the sensing chamber's photocurrent scales to $I_{ph} = (1-\alpha)I_{ph0}$, where α is the fractional attenuation due to smoke. The sealed chamber stays at I_{ph0}. Ignore based currents and $V_{CE(sat)} unless saturation is explicitly invoked.$

40. (3 points) (Tiebreaker #3) With clean air ($\alpha = 0$), choose equal pull-ups $R_1 = R_2 = R$ so that $V_{ref} = V_{in} = 2.0V$. Compute required R and verify the phototransistor operating region (are Q_1/Q_2 in saturation or active?).

Solution: Plugging values in the given formula gives collector current $I_{C0} \approx 20mA$. The node equation given by Kirchhoff's current law; the current going in is the current through the resistor R, and the current going out is the current taken by the collector (the current to the op-amp is negligible): $\frac{(Vcc-V)}{R} = I_C$, giving $R = 150\Omega$. If the device were saturated, $V_{CE} \approx V_{CE(sat)} \sim 0.2V$ giving current through R, $I_R = \frac{5-0.2}{150} \approx 32mA$ which exceeds the maximum active-mode current $\beta I_{ph} \approx 20mA$. Hence the transistor cannot supply that much; it settles where $I_R = I_C = 20mA$. Since V_{CE} is well abouve $V_{CE(sat)}$, both Q_1 and Q_2 operate in the forward-active region (not saturation)
+2pt: Gets correct R
+1pt: Notes both phototransistors are active
+1pt: Incorrect R, but used KCL at the appropriate node

41. (4 points) As it stands, our smoke detector trips for *any* impurity in air, which is not exactly a good thing. Add a positive-feedback resistor R_H from V_O (assume it swings 0-5V) to the non-inverting input (+). With R_2 unchanged, derive the two trip points $\alpha_{T,UP}$ (rising smoke) and $\alpha_{T,DN}$ (clearing smoke) in terms of R_2 and R_H *chirp*. Choose R_H so the width of the hysteresis $\alpha_{T,UP} - \alpha_{T,DN} = 0.10$ (i.e., attenuation band 10%) and report the numeric values.

Solution: At the + input (node V_{in}) the KCL at the instant of switching is

$$\frac{Vcc - V_T}{R_2} + \frac{V_O - V_T}{R_H} = \beta(1 - \alpha_T)I_{ph0} + I_{leak}$$

with $V_T = V_{ref} = 2V$. From the clean-air design (no feedback) $\beta I_{ph0} + I_{leak} = \frac{Vcc - V_T}{R_2}$. Eliminating βI_{ph0} gives $\alpha_T = -\frac{V_O - V_T}{R_H\left(\frac{Vcc - V_T}{R_2} - I_{leak}\right)}$; since $I_{leak} << \frac{Vcc - V_T}{R_2}$ this reduces to

$$\alpha_T \approx -\frac{R_2}{R_H}\frac{V_o - V_T}{Vcc - V_T}$$

so the two thresholds are

$$\alpha_{T,up} \approx \frac{R_2}{R_H}\frac{V_T - 0}{Vcc - V_T} = \frac{2}{3}\frac{R_2}{R_H}$$
$$\alpha_{T,dn} \approx \frac{R_2}{R_H}\frac{V_T - 5}{Vcc - V_T} = -\frac{R_2}{R_H}$$

with hysteresis width $\alpha_{T,up} - \alpha_{T,dn} \approx \frac{5}{3}\frac{R_2}{R_H}$. Setting up this width to be 0.10 yields $R_H = 2.5k\Omega$. The numeric trip points are thus $\alpha_{T,up} \approx 0.04$ and $\alpha_{T,dn} \approx -0.06$ (formal result; negative means it wont reset within $0 - 100\%$ attenuation).

An alternative way to obtain this result is by considering that the Vin node sees the Thevenin source

$$V_{th} = \frac{\frac{5}{R_2} + \frac{V_O}{R_H}}{\frac{1}{R_2} + \frac{1}{R_H}}, \quad R_{th} = R_2||R_H = \frac{R_2 R_H}{R_2 + R_H}$$

The phototransistor current (forward-active) is $I_C \approx \beta I_{ph} = 0.02(1-\alpha)$. The comparator trips when $V_{in} = V_{ref} = 2.0V$; $2.0 = V_{th} - I_C R_{th}$. We then have

$$\alpha_{T,up} = 1 - \frac{V_{th} - 2}{0.02R_{th}} = 1 - \frac{3R_H - 2R_2}{0.02R_2R_H}$$
$$\alpha_{T,dn} = 1 - \frac{5 - 2}{0.02R_{th}} = 1 - \frac{150(R_2 + R_H)}{R_2R_H}$$

giving hysteresis width $\alpha_{T,up} - \alpha_{T,dn} = \frac{250}{R_H}$, yielding the same numerical results as before.
+2pts: Correct trip points (or if the answer would have been correct if a correct answer had been obtained for the previous question)
+1.5pt: Correct R_H (or if the answer would have been correct as stated above
+0.5pt: Notes negative $\alpha_{T,dn}$ means the down-going threshold clips at $\alpha = 0$
+1pt: Incorrect trip points but attempts KCL at Vin or considers a Thevenin equivalent circuit

42. (2 points) If the 9-V battery sags and the whole circuit runs from a reduced Vcc (the op-amp still swings $0 - Vcc$), what is the minimum Vcc that keeps both phototransistors in the forward-active region in clean air? Use resistor values from previous parts, or, if you were not able to obtain answers, feel free to leave your answer in terms of those quantities.

Solution: In clean air each phototransistor draws 20mA. The input node is pulled up by R_2 (or R_1 for the reference) so $V_{in} = Vcc - I_C R_2 = Vcc - 3.0V$. To avoid saturation we need V_{CE} comfortably above the practical guard $\geq 0.2V$, therefore $Vcc_{min} \approx 3.0V + 0.2V = 3.2V$. If $Vcc < 3.2V$, the pull-up cannot sustain I_C without forcing the node toward the ground, pushing the phototransistors into saturation and degrading the comparator thresholds. +1pt: States that $V_{CE} > V_{CE(sat)}$ is needed and solves for $Vcc_{min} = 3.2V$ +1pt: Correctly writes $V_{in} = Vcc - I_C R_2$ (or equivalent for V_{ref}) with $I_C \approx 20mA$ +0.5pt: Incorrect answer but shows correct qualitative idea (voltage drop across resistor current) +0.5pt: Neglects saturation voltage but still solves for about $3V$ OR incorrect only because of incorrect resistor values from previous parts. NO CREDIT for no mention of active/saturation boundary or irrelevant condition.

43. (4 points) You want to ignore very brief puffs of dust. Add a capacitor to create a first-order time constant of about $\tau \approx 20ms$ for the positive-feedback path so the comparator won't flip on sub-20ms events. If you place the capacitor across R_H (from V_o to V_{in}), what value C_f gives $\tau \approx 20ms$? If you instead place a capacitor from V_{in} to ground, what value C_g gives the same τ, and why is this a poorer choice? (if you were not able to obtain answers for R_H, R_2,etc, feel free to leave your answer in terms of those quantities)

Solution: With C_f across R_H, the dominant RC forms in the feedback leg: $\tau \approx R_H C_f$. Thus $C_f \approx 8.0\mu F$. For the capacitor from input to ground, the node sees (to AC) two low-impedance paths to supply a rail: R_2 to Vcc and R_H to the op-amp output. The Thevenin resistance at V_{in} is roughly $R_{th} \approx R_2 || R_H \approx 141\Omega$. To get $\tau = R_{th} C_g \approx 20ms$, $C_g \approx 142\mu F$. This is a poorer choice because it is an order of magnitude larger, physically bulkier, and leakier. That large cap loads the sensor node, disturbing DC thresholds and phototransistor currents and injecting large transient currents when V_o toggles; it slows all changes at V_{in}, not just the positive feedback action, potentially degrading sensitivity and stability

+1pt: Correct C_f (or correctly wrote in terms of R_H or incorrect only due to incorrect R_H)

+1.5pts: Correct C_g (or correctly wrote in terms of R_H or incorrect only due to incorrect R_H)

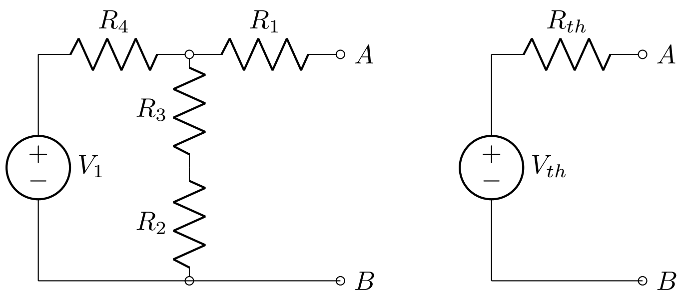
+0.5pt: Noted that the capacitor to ground is a poorer choice because it is bulkier or leakier

+0.5pt: Noted that the large cap loads the sensor node and disturbs DC thresholds

+0.5pt: Comments on negative effects of C_g on sensitivity and stability

+1pt: Incorrect C_g, but attempted using Thevenin resistance

4 Circuit Analysis (12 points)



44. (4 points) Thevenin's theorem states that any linear network containing only voltage sources, current sources, and resistances can be replaced at terminals A-B by an equivalent combination of a voltage source V_{th} in a series connection with a resistance R_{th}. Given the circuit on the left, solve for the Thevenin-equivalent circuit, giving values V_{th} and R_{th} in terms of V_1, R_1, R_2, R_3, and R_4.

Solution: Let $R_{23} = R_2 + R_3$, and label the junction where the three resistors meet node N. With no load, no current flows through R_1, so $V_A = V_N$. Node N is tied to V_1 through R_4 and to ground through R_{23}, so $V_{th} = V_A = \frac{V_1/R_4}{1/R_4+1/R_{23}} = V_1\frac{R_{23}}{R_{23}+R_4} = V_1\frac{R_2+R_3}{R_2+R_3+R_4}$. To obtain the resistance, zero the source V_1 (replace by a wire). Then from node N to B we have $R_4||R_{23}$. This is in series with R_1 between A and N, so $R_{th} = R_1 + (R_4||(R_2+R_3)) = R_1 + \frac{R_4(R_2+R_3)}{R_4+R_2+R_3}$.

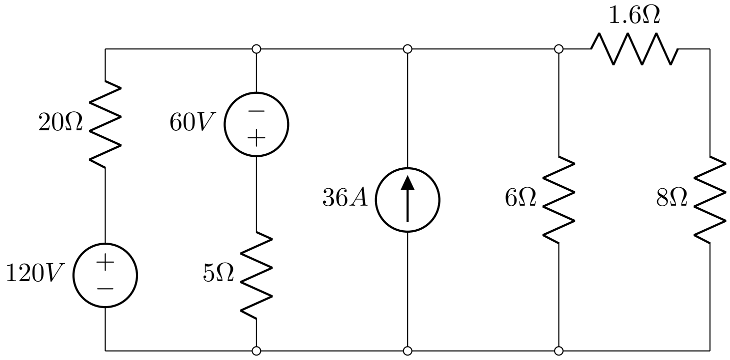
+2pts: Correct V_{th}
+2pts: Correct R_{th}
+1pt: Incorrect V_{th}, but made an honest attempt to use KVL

45. (3 points) Dual to Thevenin's theorem is Norton's theorem, which states that linear networks containing only voltage sources, current sources, and resistances can be replaced at terminals A-B by an equivalent combination of a current source I_{no} in parallel with a single resistor R_{no}. From the same circuit as the last question, solve for the Norton-equivalent circuit, giving values I_{no} and R_{no} in terms of V_1, R_1, R_2, R_3, and R_4.

Solution: Let $R_{23} = R_2+R_3$. The Norton resistance equals the Thevenin resistance $R_{no} = R_{th}$. The Norton current is the short-circuit current from A to B. With A shorted to B node N sees $R_1||R_{23}$ to ground so $I_{no} = I_{sc} = \frac{V_N}{R_1} = \frac{V_1(R_1||R_{23})}{R_1(R_4+R_1||R_{23})} = \frac{V_1(R_2+R_3)}{R_4(R_1+R_2+R_3)+R_1(R_2+R_3)}$. Equivalently, $I_{no} = V_{th}/R_{th}$.

+2pts: Correct I_{no}
+1pt: Correct R_{no} (or, if incorrect R_{no}, at least notes that $R_{th} = R_{no}$. No credit for obtaining the same wrong answer for resistance as the last question without explicitly stating $R_{th} = R_{no}$)
+1pt: Incorrect I_{no}, but used Ohm's law wiht V_{th} and R_{th}.

46. (5 points) (Tiebreaker #2) Having these two theorems under your belt, compute the voltage across the 8Ω resistor for the circuit below, and note the direction of current.



Solution: Combine the left network (all branches except the 1.6Ω-8Ω arm) into its Thevenin form seen at the top bus. For the open-circuit node voltage V_{th}, with node voltage V, $\frac{V-120}{20} + \frac{V+60}{5} + \frac{V}{6} - 36 = 0 \implies V = 72V$ so $V_{th} = 72V$. For the Thevenin resistance (sources killed: voltage sources become shorts, current sources become open) $R_{th} = (20||5||6) = 2.4\Omega$. Now this V_{th} in series with R_{th} feeds the series path $1.6\Omega + 8\Omega$. Total series resistance is $2.4 + 1.6 + 8 = 12\Omega$ so the current is $I = 72/12 = 6A$. The voltage across the 8Ω resistor is then $V_8 = I \cdot 8 = 48V$.

We could do this equivalently with Norton's theorem, with a little more work. We can replace everything to the left of the 1.6Ω resistor with its norton equivalent seen at the left side of that resistor which we label node A. The bottom rail is the ground. Short A to ground removing the 1.6Ω-8Ω. In the 20Ω+$120V$ branch, $I_1 = \frac{0-120}{20} = -6A$ (i.e. 6A upward). At the 60V+5Ω branch, $I_2 = \frac{0+60}{5} = 12A$ downward. At the 6Ω branch $I_3 = 0/6 = 0$. At the current source, $36A$ upward. KCL at node A (downward positive) gives $I_{short} = 36 - (I_1 + I_2 + I_3) = 30A$. For the norton resistance turn off the independent sources: short the two voltage sources, open the 36A source. Seen from node A we have $R_N = 20||5||6|| = 2.4\Omega$. Attaching the load $R_L = 1.6 + 8 = 9.6\Omega$, the node-$A$ voltage with a norton source is $V_A = I_N(R_N||R_L) = 57.6V$. The 1.6 and the 8 ohms are in series so the voltage across 8Ω is the divider share $V_A \cdot \frac{8}{1.6+8} = 48V$.

+4pts: Correct V_8
+1pt: Current flows downward
+1pt: Incorrect V_8 but honest attempt using KCL or KVL or Thevenin theorem or Norton theorem

5 Computers (20 points)

Silicon PN junctions form the basis of diodes and transistors. A transistor, made of two PN junctions, can act as a switch and is the building block of logic gates in computers. Logic gates are the fundamental units of digital circuits that process information using binary signals (0 = low voltage, 1 = high voltage). Transistors revolutionized the world of computers. First, we analyze a historically important computer that used relays.

47. (9 points) The Harvard Mark II (1947) implemented arithmetic and control with about 13,000 electromechanical relays.

A Mark II-class relay coil is modeled as an ideal series $R - L$ driven from a 48 V DC supply. Let $R = 200\Omega$, $L = 80$ mH. The armature motion is approximated as a fixed mechanical lag $t_m = 10$ ms after the coil current reaches the pickup threshold. Assume pickup occurs at $0.8I_\infty$ and dropout at $0.2I_\infty$, where $I_\infty = V/R$:

(a) (1 points) Derive the pickup time t_p and dropout time t_d referenced to the electrical step only, using $i(t) = I_\infty(1 - e^{-t/\tau})$ and $i(t) = I_\infty e^{-t/\tau}$ with $\tau = L/R$.

Solution: Pickup is given by $i(t_p) = 0.8I_\infty = I_\infty(1 - e^{-t_p/\tau})$. Simple algebra gives $t_p \approx 0.644ms$. Dropout is given by $0.2I_\infty = I_\infty e^{-t_d/\tau}$ so $t_d = 0.644ms$. No partial credit.

(b) (2 points) Include the mechanical lag and report the make and break delays.

Solution: Make delay (pickup+motion): $t_{make} = t_p + t_m \approx 10.644ms$, break delay (electrical dropout + motion) $t_{break} = t_d + t_m = 10.644ms$. +1pt for each correct answer, half credit if correct formula but incorrect due to part (a).

(c) (1 point) If a combinational path traverses k relay stages in series, each requiring one make transition, give the worst-case propagation delay $T_p(k)$.

Solution: $T_p(k) = k(t_p + t_m) = 10.644k$. **Half credit if incorrect due to part (a).**

(d) (3 points) Starting from steady state at $t = 0^-$, the supply is opened at $t = 0$. Compare the electrical dropout time for these three cases placed across the coil:

(i) No clamp, but the driver limits the flyback to a constant $|v_{coil}| \approx 200V$ during decay (idealized)

(ii) Flyback diode with $V_F = 0.7V$ (assume ideal diode otherwise)

(iii) Zener clamp: diode + Zener such as $|v_{coil}| \approx 24.7V$ during decay

For each case, derive the governing equation and solve for t_d. Then report break delay.

Solution: General KVL with a clamp imposing a reverse constant magnitude $V_C > 0$ **across the coil during decay gives** $L\frac{di}{dt} + Ri = -V_c \implies i(t) = \left(I_\infty + \frac{V_c}{R}\right)e^{-t/\tau} - \frac{V_c}{R}$ **(you can do RL circuits without calculus too). Set** $i(t_d) = 0.2I_\infty$ **to get** $t_d = \tau \ln\left(\frac{I_\infty + V_c/R}{0.2I_\infty + V_c/R}\right)$**. Now we apply to each clamp.**

(i) $V_c/R = 1.0A$ **so plug it all in and** $t_d = 0.0673ms$ **so break delay** $10.067ms$

(ii) $V_c = 0.7V \implies V_c/R = 0.0035A \implies t_d = 0.621ms \implies$ **break delay** 10.621**.**

(iii) $V_c = 24.7 \implies t_d = 0.300ms \implies$ **break delay** 10.300

+1pt: Sets up KVL correctly
+0.5pt: Attempts KVL incorrectly
+0.5pt: Gets the t_d **formula**
+0.5pt for each correct break delay (+1.5 max)

(e) (7 points) (Tiebreaker #1) Consider a complementary metal-oxide semiconductor (CMOS) with $V = 1V$, $R_{on} = 1k\Omega$, $C_L = 40fF$; estimate (i) propagation delay and (ii) energy per output toggle for each, then decide if either can meet 1 MHz at $\leq$ 1 mW per gate and by how many orders of magnitude the better option wins. Use, for the relay, $t_p \approx \tau \ln 5 + t_m$, $E \approx \frac{1}{2}LI_p^2$ with $I_p = 0.8V/R$, and, for the CMOS, $t_p \approx 0.69R_{on}C_L$, $E \approx \frac{1}{2}C_LV^2$.

Solution: For the relay, (i) $t_p \approx \tau \ln 5 + t_m \approx 10.644ms$, (ii) $I_p = 0.8V/R = 0.192A$, $E \approx \frac{1}{2}LI_p^2 = 1.475mJ$. At 1 MHz $t_p >> 1\mu s$ so cannot meet 1MHz (too slow). For power $P \approx Ef = 1.48 \times 10^3 W >> 1mW$ so far over. For the CMOS, (i) $t_p \approx 0.69R_{on}C_L \approx 27.6ps$, (ii) $E \approx \frac{1}{2}C_LV^2 = 20fJ$. At 1 MHz, speed meets easily the margin by 4.6 orders of magnitude; per-toggle energy budget at 1 MHz under 1 mW is 1 nJ; CMOS uses 20 fJ, i.e., 50,000× below budget (4.7 orders). So conclusively relay fails both speed and power by huge margins but CMOS wins. Against the relay specifically CMOS' energy per toggle is smaller by a factor of 10.9 orders and its delay is faster by 8.6 orders.

If you're wondering where the 0.69 numerical factor came in for the CMOS propagation delay, it's form the RC-time-constant. For a simple first order RC response, $V_{out}(t) = V_{DD}\left(1 - \exp\left(-\frac{t}{R_{on}C_L}\right)\right)$. Logic switching is considered complete when the output passes half the supply, $V_{out} = 0.5V_{DD}$. Solving for t gives $t = (R_{on}C_L)\ln 2 \approx 0.69R_{on}C_L$.

+1pt: Correct relay propagation delay
+1pt: Correct relay energy per output toggle
+2pts: Correct CMOS propagation delay
+1pt: Correct CMOS energy per output toggle
+2pt: Correct orders of magnitude
+1pt: Incorrect orders of magnitude but due to incorrect CMOS propagation delay

48. (2 points) Explain what happens at the atomic level in a silicon PN junction when it is forward biased versus reverse biased. Why does current flow more easily in one direction than the other?

Solution:

Forward bias: must mention depletion region decreases, carriers cross easily, or current flows freely. Reverse bias: must mention depletion region widens, barrier increases, and current is blocked except small leakage.

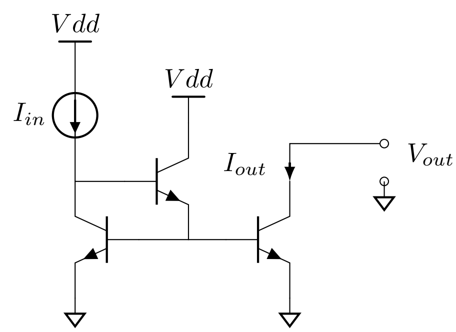
+1pt: Correct description of what happens under forward bias.
+1pt: Correct description of what happens under reverse bias.

49. (2 points) Consider an NPN transistor used as a switch. Describe the role of the base, collector, and emitter in controlling current flow, and explain why a small base current can control a much larger collector current.

Solution: Base controls current, emitter injects carriers, collector collects them. Carrier injection amplifies current and most electrons go from emitter to collector, not lost in the base.

+1pt: Correctly describes the roles of base, collector, and emitter.
+1pt: Explains why a small base current controls a larger collector current.

50. (0 points) (Optional, not included in the Mason Invitational Satellite exam) A current mirror is an electronic circuit designed to copy an input current and provide that current to a load, often used to create a stable, constant current source or sink. By creating multiple copies of a stable reference current, current mirrors are essential in integrated circuits (ICs) for biasing amplifiers and are implemented using MOSFETs or bipolar junction transistors (BJTs). For the following BJT circuit mirror, express I_{out}/I_{in} as a fraction of expanded polynomials in current gain $\beta = \frac{I_C}{I_B}$, assuming all transistors are matched. Also, solve for the lower bound of V_{OUT} such that all transistors remain forward active in terms of base-emitter voltage V_{BE} and collector-emitter saturation voltage $V_{CE,sat}$. You cannot neglect base currents.



51. (0 points) (Optional, not included in the Mason Invitational Satellite exam) Draw a NAND logic gate using transistor logic.