'Iolani Science Olympiad Invitational 2024

Division C

MATERIAL SCIENCE LAB KEY

Honolulu
2024

1 A rubber band

You have been given the following materials:

- Strand of thin rubber
- Strand of thick rubber
- Vernier caliper
- Spring scale
- Ruler

Using these materials, you will investigate some properties of rubber. You may use whichever strand you want for each part. **Show all calculations**.

2 Stress and strain

1. (3 points) Determine Poisson's ratio for the band.

Poisson's ratio is the ratio of lateral strain to longitudinal strain. 1 point for correct formula, 2 points for correct/plausible number.

2. (5 points) Draw a stress-strain curve for the rubber band, using at least 5 points. Use true stress.

True stress is the load force divided by the cross-sectional area at the time of measurement. Alternatively, one can find $\sigma_t = \sigma_{eng}(\varepsilon_{eng} + 1)$. 1 point for using stress on y-axis and strain on x-axis and labeling units for stress, 2 points for using true stress and not engineering stress, 2 points for using at least 5 data points.

3. (2 points) Draw a stress-strain curve using engineering stress.

Engineering stress is the load force divided by the initial cross-sectional area. 1 point for using stress on y-axis and strain on x-axis and labeling units for stress, 1 point for at least 5 data points for engineering stress.

4. (2 points) Find the yield strength using a strain offset of 0.002.

Yield strength can be found by drawing a line parallel to the linear part of the stress-strain curve and passing through the point $(0.002, 0)$. 2 points for calculating yield stress this way.

5. (3 points, tiebreaker) Calculate the shear modulus.

The elastic modulus E is the slope of the linear part of the stress-strain curve representing elastic deformation. Using the Poisson ratio ν from problem 1, the shear modulus G can be found using the equation

$$G = \frac{E}{2 + 2\nu}$$

1 point for calculation of elastic modulus, 2 points for calculation of shear modulus.

3 Hysteria!

6. (2 points) As load is applied, the stress-strain curve takes a certain path; however, during unloading, the stress-strain curve doesn't retrace its path. Identify and explain this phenomenon.

This is called hysteresis, and it's a result from friction happening inside the rubber as it is stretched, creating energy loss due to heat. Also accept "Mullins effect". 1 point for identifying, 1 point for explanation.

7. (3 points) Draw the unloading true stress-strain curve on your graph for problem 2.

3 points for doing it correctly.

8. (1 point) What does the area between the loading and unloading curves represent?

The area of the closed hysteresis loop is measures the amount of energy lost per cycle. 1 point for correct explanation.

4 A creepy problem

9. (5 points) Suppose a constant load of 10 N is suddenly applied to one of the strands. Using the Kelvin-Voigt model of viscoelastic materials, predict how long it would take for the strand's length to double as a function of the viscosity η. Show which strand was used.

The Kelvin-Voigt model represents a material as a purely viscous damper and a purely elastic spring connected in parallel. The effect of a constant stress σ_0 to the strain over time is governed by the following equation:

$$\varepsilon(t) = \frac{\sigma_0}{E}\left(1 - e^{-t/\tau_R}\right)$$

where t is time, E is elastic modulus, and the retardation time is given by $\tau_R = \frac{\eta}{E}$. Setting $\varepsilon(t) = 2$,

$$\begin{aligned} 2 &= \frac{10}{E}\left(1 - e^{-tE/\eta}\right) \\ 1 - \frac{E}{5} &= e^{-tE/\eta} \\ \ln\left(1 - \frac{E}{5}\right) &= -\frac{tE}{\eta} \\ t &= -\frac{\eta}{E}\ln\left(1 - \frac{E}{5}\right) = \frac{\eta}{E}\ln\left(\frac{5}{5-E}\right) \end{aligned}$$

2 points for using the creep equation, 3 points for the answer.

5 Chemistry problems from a physicist

For problems 10-12, assume that the lab is at 23° C.

10. (4 points) The Flory theory of rubber suggest rubber elasticity is governed by entropy changes. Assuming entropy changes linearly with stretch, find the change in entropy per unit change in length. Specify which strand was used.

For an ideal elastomer:

$$F = -k_B T \frac{\partial S}{\partial L}$$

$$\Rightarrow \frac{\partial S}{\partial L} = -\frac{F}{k_B T}$$

From linearity, $\partial S/\partial L$ is constant and represents the change in entropy per unit change in length. 1 point for the equation, 1 point for choosing a point within the elastic region, 2 points for the answer.

11. (5 points) Find the molecular mass between crosslinks using your data and previous answers.

The number density of network chains ν is related to the elastic modulus by

$$M_c = \frac{\rho RT}{G}$$

With R the ideal gas constant and ρ the polymer density. Due to Poisson's ratio being close to 0.5 for elastometric materials above their glass transition temperature, also accept the approximation

$$M_c = 3\frac{\rho RT}{E}$$

No partial credit.

12. (3 points) For the thick strand, what is the average force per polymer chain for a load force of 10 N?

The number density of network chains n is related to the molecular weight between crosslinks M_c by

$$n = \frac{\rho}{M_c} N_A$$

with ρ the density and N_A Avogadro's number. Therefore the average force per polymer chain is

$$F_{chain} = \frac{\sigma}{n}$$

No partial credit.