‘Iolani School
Physics Club

AN INTRODUCTION TO PARTICLE PHYSICS

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Summary

1 A PARTICLE ZOO

By convention sweet is sweet, bitter is bitter, hot is hot, cold is cold, color is color; but in truth there are only atoms and the void.

Democritus (c. 400 BC)

1.1 The Classical Period of Elementary Particle Physics (1897-1932)

Classical Model

We begin our story with the three best known subatomic particles: the **electron** (e^-), the **proton** (p^+), and the **neutron** (n). In 1914, Niels Bohr proposed a model of hydrogen in which an electron circled the proton, like a planet going around the sun, held in orbit. It was natural then to suppose that the nuclei of heavier atoms were composed of two or more protons bound together, supporting the same number of orbiting electrons. However, this was problematic, for the next heavier atom, helium, although carrying two electrons, weighs four times as much as hydrogen. In 1932, the electrically neutral neutron was discovered, explaining the extra weight of the helium atom.

Photon

In 1900, statistical mechanics, which had proved brilliantly successful when applied to most thermodynamical processes, yielded nonsensical results when applied to electromagnetic fields (the so-called ultraviolet catastrophe). In particular, Planck, in trying to explain the blackbody spectrum for the electromagnetic radiation emitted by a hot object, found that he could escape such results by assuming electromagnetic radiation is quantized, coming in little packages of energy

$$E = h\nu \tag{1.1}$$

where ν is the frequency of radiation and h is a constant which Planck eventually adjusted to fit the data.

In 1905, Einstein adapted Planck's idea and formula to explain the photoelectric effect: When electromagnetic radiation strikes a metal surface and electrons pop out. Einstein suggested that a light quantum hits an electron in a metal, giving up its energy; the excited electron then breaks through the material surface, losing in the process an energy w (the so-called work function of a material). The electron thus emerges with an energy

$$E \leq h\nu - w \tag{1.2}$$

This implies that electron energy is independent on the power of light and depends only on its wavelength; more power would mean more particles and more electrons popping out, but each individual electron corresponds to the energy of the incoming quanta. In 1923, A.H. Compton found that the light scattered from a particle at rest is shifted in wavelength according to

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta) \tag{1.3}$$

This is precisely the formula you get if you treat light as a particle of zero mass with energy given by Planck's equation and apply the laws of conservation of relativistic energy and momentum. We call this particle the **photon** γ. It was then concluded in quantum field theory that the electric field is quantized, and that we may picture the interaction as consisting of a stream of photons passing back and forth between the two charges, each charge continually emitting photons and continuously absorbing them. The same goes for any non contact force; where we classically interpret as action on a distance as mediated by a field, we now say that it is mediated by an exchange of particles (the quanta of the field).

1.2 The Middle Period (1930-1960)

Meson

But what holds the nucleus together? Positively charged protons should repel each other violently. The force, more powerful than the force of electrical repulsion, that binds the protons and neutrons together, is called the **strong force**. Although such a potent force is in nature, most everyday forces are electromagnetic or gravitational; strong force has a very short range. Gravitational and electromagnetic forces have infinite range, but the range of strong force is the size of the nucleus itself.[1]

In 1934, Yukawa assumed that the proton and neutron are attracted to one another by some sort of field. This field should be quantized. This quantum should then be analogous to the photon. Yukawa calculated its mass should be a sixth of the mass of a proton, or 300 times that of an electron. Yukawa's particle became known as the **meson** (middle-weight); the electron is called a **lepton** (light-weight) and the proton and neutrons are **baryons** (heavy-weight). The existence of these particles were confirmed by observation in cosmic rays in 1937.

However, there were discrepancies in that cosmic ray particles sometimes had the wrong lifetime and seemed significantly lighter than what Yukawa had predicted. In 1947, it was discovered that there were actually two middle-weight particles in cosmic rays: the π (or **pion**) and the μ (or **muon**).The true Yukawa meson is the π, which disintegrates long before reaching the ground. One of the decay products is the lighter and longer lived μ. In the search for Yukawa's meson, then, the μ had nothing to do with strong interactions; in fact, it behaves in every way like a heavier version of the electron, and properly belongs in the lepton family.

[1]This is known as Yukawa coupling, and the potential is given by $V(r) = -\frac{g}{4\pi}\frac{1}{r}e^{-\mu r}$ when mediated by a Yukawa particle of mass μ.

Antiparticle

For every kind of particle there must exist a corresponding ***antiparticle****, with the same mass but opposite electric charge*. The **positron**, discovered in 1931, is the anti-electron. The negatively charged antiproton was first observed experimentally in 1955, and the (neutral) antineutron was discovered the following year. Antiparticles are often denoted by an overbar (e.g. n for the neutron and $\bar{n}$ for antineutron), but they can sometimes also be denoted by the charge (e.g. e^- for the electron, e^+ for the positron). Some particles, such as photons, are their own antiparticle. Particle-antiparticle pairs can annihilate each other, producing photons. One notable rule in antiparticles is **crossing symmetry**. Suppose a reaction of the form

$$A + B \to C + D \tag{1.4}$$

is known to occur. Any of these particles can be crossed over to the other side of the equation, provided it is turned into its antiparticle, and the resulting interaction will also be allowed; for example,

$$A \to \bar{B} + C + D \tag{1.5}$$

would be allowed in our case. However, conservation of energy may veto a reaction that is otherwise permissible; if A weighs less than the sum of B, C, and D, then the decay described by Equation 1.5 cannot occur; similarly, if A and C are light whereas B and D are heavy, then $A + \bar{C} \to \bar{B} + D$ will not take place unless the initial kinetic energy exceeds a certain threshold value. That is to say, a crossed reaction is dynamically permissible, but it may or may not be kinematically allowed.

Neutrino

In beta decay, a radioactive nucleus A is transformed into a slightly lighter nucleus B, with the emission of an electron:

$$A \to B + e^- \tag{1.6}$$

Conservation of charge requires that B carry one more unit of positive charge than A. Thus the daughter nucleus B lies one position farther along on the period decay.

It is a characteristic of two-body decays of the form $A \to B + C$ that the outgoing energies are kinematically determined in the center of mass frame. By conservation of energy, if the parent nucleus A is at rest, so that B and e^- come out back-to-back with equal and opposite momenta, then conservation of energy dictates that the electron energy is

$$E = \left(\frac{m_A^2 - m_B^2 + m_e^2}{2m_A}\right) c^2 \tag{1.7}$$

Experimentally, however, emitted electrons may vary considerably in energy, and this equation determines only the maximum electron energy for a beta decay process. In the early 1930s, while Niels Bohr was ready to abandon the law of conservation of energy, Pauli proposed that another particle was emitted along with the electron, carrying off the missing energy. It had to be electrically neutral to conserve charge. In 1933, Fermi presented a theory of beta decay that incorporated Pauli's particle. From the fact that the observed electron energies range up to the value from Equation 1.7, it follows that the new particle must be extremely light. Fermi called it the neutrino. We now call it the **antineutrino**. In modern terminology, the fundamental beta decay process is

$$n \to p^+ + e^- + \bar{\nu} \tag{1.8}$$

Powell found similar irregularities in the decay of muons and pions, and concluded that Pauli's neutrino must also have gone undetected:

$$\pi \to \mu + \nu \tag{1.9}$$

Powell also found experimentally that the muon decay must be

$$\mu \to e + 2\nu \tag{1.10}$$

In the mid-1950, Cowan and Reines experimentally confirmed the existence of the neutrino. This was difficult, because neutrinos interact very weakly with matter; in fact, a neutrino with a moderate amount of energy could pierce through a thousand light years of lead.

In 1953, Konopinski and Mahmoud introduced a rule for determining which reactions would work by assigning a **lepton number** $L = +1$ to the electron, negative charge muon, and neutrino, and $L = -1$ to the positron, positive muon, and antineutrino. All other particles are given a lepton number of zero. They then proposed the **law of conservation of lepton number**–in any physical process, the sum of the lepton numbers before must equal the sum of the lepton numbers after. This is why we called the beta decay particle named by Fermi the antineutrino; the charged pion decays should really be written

$$\begin{aligned} \pi^- &\to \mu^- + \bar{\nu} \\ \pi^+ &\to \mu^+ + \nu \end{aligned} \tag{1.11}$$

and the muon decays are actually

$$\begin{aligned} \mu^- &\to e^- + \nu + \bar{\nu} \\ \mu^+ &\to e^+ + \nu + \bar{\nu} \end{aligned} \tag{1.12}$$

In the late 1950s, a new rule was introduced to explain why $\mu^- \to e^- + \gamma$ had never been observed, despite being consistent with the lepton rule. [2] Suppose there are two different kinds of neutrino; an **electron neutrino** ν_e and a **muon neutrino** ν_μ. We assign a **muon number** $L_\mu = +1$ to μ^- and ν_μ, and $L_\mu = -1$ to μ^+ and $\bar{\nu_\mu}$, and refine the conservation of lepton number to two separate laws–conservation of electron number and conservation of muon number–we can then account for all allowed and forbidden processes. Neutron decay becomes

$$n \to p^+ + e^- + \bar{\nu}_e \tag{1.13}$$

[2] As a general rule of thumb in physics, whatever is not expressly *forbidden* is *mandatory*.

and the pion and muon decays become

$$\begin{aligned}
\pi^- &\to \mu^- + \bar{\nu}_\mu \\
\pi^+ &\to \mu^+ + \nu_\mu \\
\mu^- &\to e^- + \bar{\nu}_e + \nu_\mu \\
\mu^+ &\to e^+ + \nu_e + \bar{\nu}_\mu
\end{aligned} \tag{1.14}$$

Strange Particle

In 1947, Rochester and Butler found that a cosmic ray striking a lead plate emitted a diverging π^+ and π^-. To explain this,, a new neutral particle with at least twice the mass of the pion was introduced; the K^0 (**kaon**).

$$K^0 \to \pi^+ + \pi^- \tag{1.15}$$

In 1949, Brown found a charged kaon:

$$K^+ \to \pi^+ + \pi^+ + \pi^- \tag{1.16}$$

In 1950, another neutral particle similar to the kaon was found by Anderson; this time, the products were a p^+ and a π^-. We call it the Λ;

$$\Lambda \to p^+ + \pi^- \tag{1.17}$$

The **lambda** belongs with the proton and the neutron in the baryon family.

Over the next few years, many more heavy baryons were discovered: the Σ, Ξ, Δ, and so on. This family of heavy baryons became collectively known as the **strange** particles. Pais found that they produced copiously (on the scale of 10^{-23}s) but decayed slowly (10^{-10}s); in modern language, he concluded that strange particle are produced by the strong force, but decay by the **weak force** (the one that accounts for beta decay and all other neutrino processes). This required that strange particles be produced in pairs (known as **associated production**). In 1953, Gell-Mann and Nishijima assigned to each particle a new property–**strangeness**–that is conserved in any strong interaction, but unlike others, is not conserved in a weak interaction. For example, if a pion and a proton collide, we might produce two strange particles

$$\begin{aligned}
\pi^- + p^+ &\to K^+ + \Sigma^- \\
&\to K^0 + \Sigma^0 \\
&\to K^0 + \Lambda
\end{aligned} \tag{1.18}$$

Here the Ks carry strangeness $S = +1$, and the Σs and Λs have $S = -1$, while ordinary particles have $S = 0$. However, we never produce just one strange particle; on the other hand, when these strange particles decay, strangeness is not conserved; for example,

$$\Lambda \to p^+ + \pi^- \tag{1.19}$$

is a weak process, and thus does not respect the conservation of strangeness.

1.3 The Periodic Table of Elementary Particles

The Eightfold Way

In 1961, Gell-Mann introduced a way to arrange the baryons and mesons according to their charge and strangeness. The eight lightest baryons fit into a hexagonal array, with two particles at the center.

Figure 1: Baryon Octet

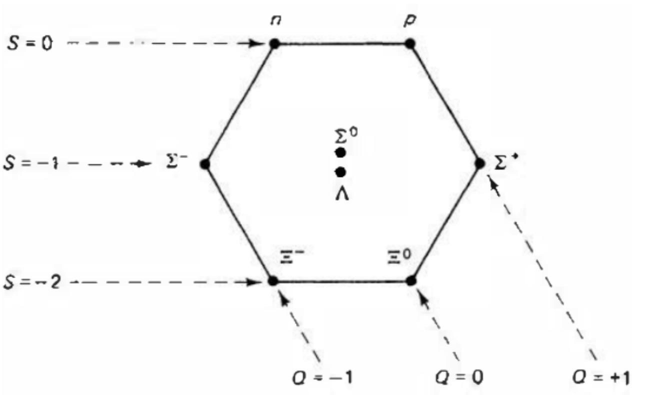
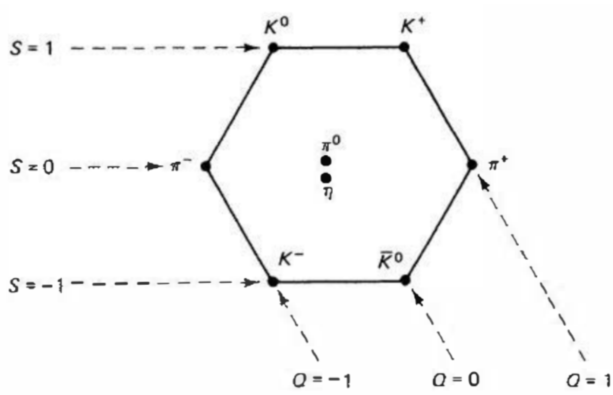


Figure 2: Meson Octet



Notice that the particles of like charge in Figure 1 lie along the downward-sloping diagonal lines; $Q = +1$ (in units of the proton charge) for the proton and the Σ^+, and so on. Horizontal lines associate particles of like strangeness. The eight lightest mesons fill a similar hexagonal pattern in Figure 2, forming the meson octet. Once again, diagonal lines determine charge and horizontal lines determine strangeness, but this time, the top line has $S = 1$, the middle line has $s = 0$, and the bottom line $S = -1$.

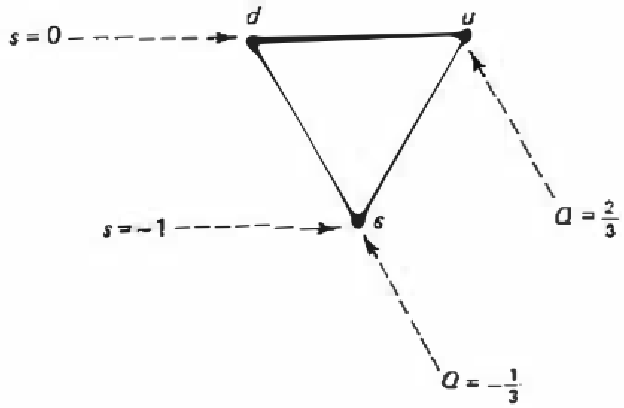
Mesons and baryons are collectively known as **hadrons**. Not every figure allowed by the Eightfold Way was a hexagon; over the next 10 years, every new hadron found a place in one of the Eightfold Way supermultiplets.

Quark

But why do hadrons fit into these bizarre patterns? In 1964, Gell-Mann and Zweig independently proposed that all hadrons are in fact composed of even more elementary constituents, which Gell-Mann called **quarks**. The quarks come in three types (or **flavors**) forming a triangular Eightfold Way pattern.

The u (up) quark carries a charge of $2/3$ and a strangeness of zero; the d (down) quark carries a charge of $-1/3$ and $S = 0$; the s (strange) quark carries a charge of $-1/3$ and $S = -1$. To each quark q, there corresponds an **antiquark** $\bar{q}$ with opposite charge and strangeness. There are two composition rules.

Figure 3: Triangular Eightfold Way pattern for quarks



1. Every baryon is composed of three quarks (and every antibaryon is composed of three antiquarks)

2. Every meson is composed of a quark and an antiquark

All we need to do is list the combinations of three quarks (or quark-antiquark pairs) and add up their charge and strangeness.

Table 1: The Baryon Decuplet

qqq	Q	S	Baryon
uuu	2	0	Δ^{++}
uud	1	0	Δ^{+}
udd	0	0	Δ^{0}
ddd	-1	0	Δ^{-}
uus	1	-1	Σ^{+}
uds	0	-1	Σ^{0}
dds	-1	-1	Σ^{-}
uss	0	-2	Ξ^{0}
dss	-1	-2	Ξ^{-}
sss	-1	-3	Ω^{-}

Table 2: The Meson Nonet

$q\bar{q}$	Q	S	Meson
$u\bar{u}$	0	0	π^{0}
$u\bar{d}$	1	0	π^{+}
$d\bar{u}$	-1	0	π^{-}
$d\bar{d}$	0	0	η
$u\bar{s}$	1	1	K^{+}
$d\bar{s}$	0	1	K^{0}
$s\bar{u}$	-1	-1	K^{-}
$s\bar{d}$	0	-1	$\bar{K}^{0}$
$s\bar{s}$	0	0	?

Of course, the same combination of quarks can go to make a number of different particles. The delta-plus and the proton are both composed of two u's and a d, for example. Just as the hydrogen atom has many energy levels, a given collection of quarks can bind together in many different ways; but whereas the various energy levels in the electron/proton system are relatively close together, so that we naturally think of them all as hydrogen, the energy spacings for different states of a bound quark system are very large, and we normally regard them as distinct particles.

However, no one has ever seen an individual quark. In the late 1960s and early 1970s, after numerous failed attempts at producing isolated quarks, the notion of quark confinement was introduced. Perhaps, quarks are absolutely confined within hadrons. This does not mean they are inaccessible to experimental study; with neutrino beams at CERN in the early 1970s, the results of 'deep inelastic scattering experiments', which were reminiscent of Rutherford's, showed that most incident particles pass through, whereas a small number bounce back. This means that charge of the proton is concentrated in small lumps; however, in the case of the proton, the evidence suggests three lumps.

To deal with the discrepancy between the quark model and the Pauli exclusion principle, which can be generalized to apply to all particles of half-integer spin, Greenberg in 1964 proposed that quarks not only come in three *flavors*, but in three **colors**: red, green, and blue. A red quark carries one unit of redness, zero of blueness, and zero greenness. A red antiquark caries minus one unit of redness and so on. Of course, this is just a handful of words that are used just to sort the types of quarks into discrete categories, and is not to be confused with the optical property of color; however, one convenient characterization of the particular quark combinations is that **all naturally occurring particles are colorless**, meaning that either the total amount of each color is zero, or all three colors are present in equal amounts (like when optical light beams of three primary colors combine to make white).

New flavor

In 1974, Ting observed the ψ (**psi**) meson. It was an electrically neutral, extremely heavy meson–more than three times the weight of a proton–and, remarkably, it lasted 10^{-20} seconds before disintegrating, which is astonishing in comparison to the typical lifetimes of hadron, which was in the order of 10^{-23} seconds. The true nature of this ψ meson was best explained in the months that followed by the quark model–that the ψ is a bound state of a new, fourth quark: the c (**charm**) and its antiquark. However, the ψ had no net charm; the search then began for a naked or bare charm, and many charmed hadrons were discovered in the following years.

In 1975, a new lepton was discovered–the **tau**. It has its own neutrino. 2 years later, a new heavy meson (the **upsilon**) was discovered and quickly recognized as the carrier of the fifth quark, b (for **beauty** or **bottom**): $\Upsilon = b\bar{b}$. Bare bottom baryons were found in the early 1980's, and the second in 2006, while bottom mesons were found in 1983.

Then, of course, a sixth quark came (t, for **truth**, or **top**). It was extraordinarily heavy, over 40 times the weight of a bottom quark; it was also too short-lived to form bound states–apparently there are no top baryons and mesons. The top quark's existence was not definitively established until 1995.

Intermediate Vector Boson

In 1933, Fermi treated the process of beta decay as a contact interaction, requiring no mediating particle; this was an excellent approximation at low energies due to the short range of the weak force, but this was bound to fail at high energies. There was a need for a supplement with a theory in which the interaction is mediated by the exchange of some particle, analogous to the photon; the mediator came to be known as the **intermediate vector boson**. However, its properties were unclear.

It was not until the emergence of the electroweak theory of Glashow, Weinberg, and Salam that a really firm prediction of the mass became possible. In this theory, there are three intermediate vector bosons; two of them charged (W^+, W^-) and one neutral (Z). They turned out to be nearly 100 times as heavy as a proton. In 1983, the discovery of the W was reported by Rubbia, and 5 months later he announced the discovery of the Z.

Gluon

Another phenomenon that needed explaining is why quarks are held together. The strong force is now said to be mediated by the **gluon** between quarks. Two quarks nearby each other exchange gluons in what's called a flux tube, connecting the two. Unlike electromagnetic force, the strength does not decrease as you move the gluons further apart; in fact, the more you stretch the flux tube, the more energy is held, until the flux tube effectively "breaks", by which point it will have enough energy to create new quarks on the broken ends. There are eight gluons; they carry color, and therefore, like quarks, should not exist as isolated particles. We can indirectly detect gluons only within hadrons or in colorless combinations with other gluons (glueballs).

Standard Model

In the current view, then, all matter is made out of leptons, quarks, and mediators. There are 6 leptons, classified by their charge Q, electron number L_e, muon number L_μ, and tau number L_τ.

They naturally fall into three generations, each new generation increasing in mass. Table 3 separates the first, second, and third generation, respectively, in descending order.

Table 3: Lepton Classification

Lepton	Q	L_e	L_μ	L_τ
e	-1	1	0	0
ν_e	0	1	0	0
μ	-1	0	1	0
ν_μ	0	0	1	0
τ	-1	0	0	1
ν_τ	0	0	0	1

There are also 6 antileptons, with all signs reversed; including these, there are really 12 leptons.

As for quarks, there are six flavors, classified by charge, strangeness (S), beauty (B), truth (T), upness (U), and downness D.

Again, all signs would be reserved on the table of antiquarks. Meanwhile, each quark and antiquark also comes in three colors, so there are 36 in total.

Finally, every interaction has its mediator; photon for the electromagnetic force, two Ws and a Z for the weak force, the graviton[3] for gravity, and the gluon for strong force. These all belong to the family of **bosons**[4]. The most recently discovered elementary subparticle is the **Higgs boson**

[3]Gravity is the weakest force in nature, so the existence of a graviton has yet to be confirmed

[4]Bosons and fermions depend on spin, which we may cover in a later lecture next year.

Table 4: Quark Classification

q	Q	D	U	S	C	B	T
d	$-1/3$	-1	0	0	0	0	0
u	$2/3$	0	1	0	0	0	0
s	$-1/3$	0	0	-1	0	0	0
c	$2/3$	0	0	0	1	0	0
b	$-1/3$	0	0	0	0	-1	0
t	$2/3$	0	0	0	0	0	1

(discovered in 2012), which essentially gives particles their mass and inertia. This is not to be confused with *weight*, which is a purely gravitational notion.

This brings us to a grand total of 61 elementary subparticles in the Standard Model.

2 PARTICLE DYNAMICS

2.1 Four Forces

There are four fundamental forces in nature: **strong, electromagnetic, weak, and gravitational**. They are listed in the following table in order of decreasing strength. Of course, the notion of strength

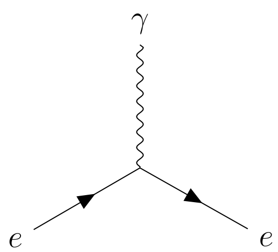
Table 5: Force Classification

Force	Strength	Theory	Mediator
Strong	10	Chromodynamics	Gluon
Electromagnetic	10^{-2}	Electrodynamics	Photon
Weak	10^{-13}	Flavordynamics	W and Z
Gravitational	10^{-42}	General Relativity	Graviton

of a force is an intrinsically ambiguous notion, as it depends on the nature of the source and how far away you are. This is just to show somewhat of a representation of the relative strengths.

2.2 Quantum Electrodynamics (QED)

Figure 4: Feynman Diagram of Electron-Photon Interaction



All electromagnetic phenomena are ultimately reducible to the elementary process in Figure 4 (a *Feynman diagram*).

In these figures, time usually flows *horizontally* from left to right, so the diagram reads: a charged particle, e, enters, emits (or absorbs) a photon γ, and exits. For the sake of the argument, we'll assume that the charged particle is an electron; it could as well be a quark or any lepton except a neutrino (neutrinos are neutral and do not experience any electromagnetic force).

To describe more complicated processes, we combine two or more replicas of this *primitive vertex*. You can snap them together, photon-to-photon or electron-to electron! In the latter case, you must preserve the direction of arrow. Figure 5 shows a process with a photon passing between two electrons,

Figure 5: Møller scattering

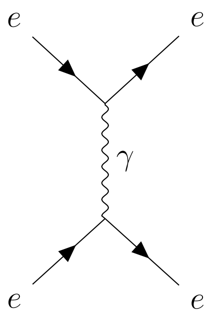


Figure 6: Bhabha scattering

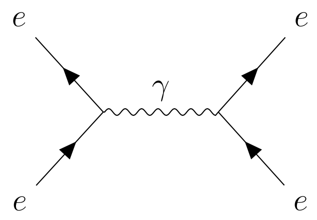
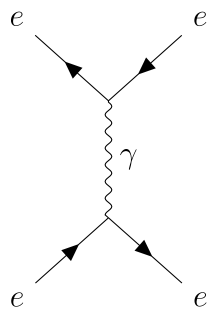


Figure 7: Also Bhabha scattering



which we recognize in classical theory as the Coulombic repulsion of like charges.

In Figure 6, a particle line running backward in time–an arrow pointing toward the left–is interpreted as the corresponding antiparticle going forward (the photon is its own antiparticle, hence why we don't need an arrow on the photon line). In this process an electron and a positron annihilate to form a photon, which in turn produces a new electron-positron pair.

Figure 7 also describes Bhabha scattering; both diagrams must be included in the analysis.

Notice that Bhabha and Møller scattering are related by crossing symmetry. *In terms of Feynman diagrams, crossing symmetry corresponds to twisting or rotating the figure*.

By allowing more vertices, the possibilities rapidly proliferate. The internal lines which begin and end within the diagram represent **virtual particles**. These represent the mechanism; virtual particles cannot be observed without entirely changing the process.

The Feynman rules for drawing Feynman diagrams enforce conservation of energy and momentum at each vertex, and hence for the diagram as a whole; the primitive QED vertex by itself does not represent a possible physical process. Within a larger diagram, however, these figures are perfectly acceptable, because, although energy and momentum must be conserved at each vertex, a virtual particle does not carry the same mass as the corresponding free particle.

We have been assuming that the charged particle in question is an electron, but it could as well be any charged particle.

In Figure 9, the $u/\bar{u}$ annihilated, producing two photons; one photon is kinematically forbidden, since we cannot construct it with Figure 4. From Table 4, this pair represents a π^0 meson; although this could be interpreted as the decay of the π^0, this is fundamentally a pair annihilation, in which the original pair happen to be bound together as a meson. This explains why π^0 has a lifetime 9 orders of

Figure 8: Various Feynman diagrams allowed by quantum electrodynamics using four vertices

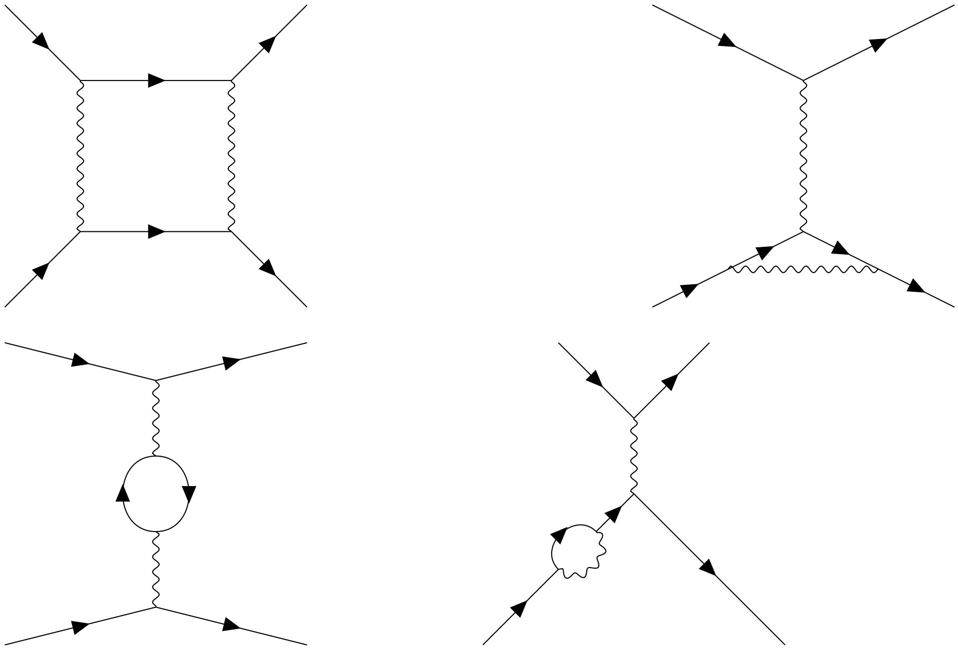
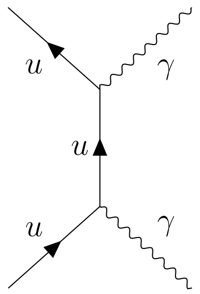


Figure 9: Annihilation of up quark-antiquark pair



magnitude smaller than its charged siblings, since it decays by an electromagnetic process, whereas the others have to await the weak interactions.

3 THE LAGRANGIAN IN QUANTUM FIELD THEORY

This section has been added to the lecture notes for fun. This will not be covered in the lecture. This is also very oversimplified for obvious reasons.

Recall from a previous meeting that the Lagrangian formulation of mechanics uses the principle of least action, where action is defined as[5]

$$S = \int_{t_1}^{t_2} dt L(\mathbf{q}, \mathbf{q}, t) \tag{3.1}$$

$\mathbf{q}$ represents the set of generalized coordinates required to define the configuration of the system:

[5] In this equation, the differential is written before the integrand. This is just notation used in physics; it is helpful when you're dealing with longer integrands or multiple integrals.

$\mathbf{q} : \mathbb{R} \to \mathbb{R}^N$ Mathematically, the principle is

$$\delta S = 0 \tag{3.2}$$

That is, the integral is minimized/stationary over the time interval. In mechanics we define the Lagrangian to be the difference between the kinetic energy and the potential energy.

In field theory there is an analogue. While Lagrangian mechanics analyzes the motion of discrete particles with finite degrees of freedom, field theory applies to continua and fields, which have infinite degrees of freedom. We replace the independent variable by an event in spacetime (x, y, z, t), or more generally a point s on a Riemann manifold.[6] The dependent variables are replaced by a value of that point in spacetime $\varphi(x, y, z, t)$; these are somewhat analogous to the generalized coordinates of mechanics, except the field comprises of a family dynamical variables $\varphi_x(t)$ defined at every point of space; that family is a function $\varphi(x, y, z, t)$ of all 4 spacetime coordinates. The Lagrangian L is the space integral[7]

$$L = \int d^3\mathbf{x}\mathcal{L}(\varphi, \nabla\varphi, \dot{\varphi}) \tag{3.3}$$

of the Lagrangian density $\mathcal{L}$. We then define the action as the four-dimensional spacetime integral of the Lagrangian density

$$S = \int dt \int d^3\mathbf{x}\mathcal{L}(\varphi, \nabla\varphi, \dot{\varphi}) \tag{3.4}$$

Similarly to mechanics, the equations of motion for a classical field come from the least action principle: among all the time evolutions of a field with given initial and final values, the evolution which obeys the field equation has the least action.

An example where we work with such a field is Newtonian gravity, wherein we obtain the Lagrangian density

$$\mathcal{L} = -\frac{1}{8\pi G}(\nabla\Phi^2 - \rho\Phi) \tag{3.5}$$

where Φ is the gravitational potential[8] and ρ is the mass density.

Anyways, the point I was leading to is that there is a Lagrangian fit for the standard model. Quantum field theory treats particles as excited states of their underlying quantum fields. These fields are the fermion fields, the electroweak boson fields, the gluon field, and the Higgs field, the particles constituting the latter should now be familiar. With the Lagrangian of all these, we can essentially describe everything we know about particle physics in one expression. While I won't go over the specifics of the math, you may marvel at its beauty in the appendix.

[6]Explaining Riemann manifolds is an extreme task that simply cannot be condensed into this simplified explanation of field-theory Lagrangians. Just take s to be a point in spacetime.

[7]We put an exponent on the d and we integrate over $\mathbf{x}$. This means that we integrate 3 times–once for every variable in $\mathbf{x} = (x, y, z)$ (so really, this is a triple integral).

[8]The gravitational potential is a field associating with each point in space the work per unit mass needed to move an object to that point from a fixed reference point; this is like electric potential, but using mass instead of charge.

4 APPENDIX

Figure 10: The Standard Model of Particle Physics in the Lagrangian Formalism.

$$
\begin{gathered}
\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4} g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \frac12 \partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac12 \partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \frac12 g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac12 g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \frac12 \partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac12 \partial_\mu \phi^0 \partial_\mu \phi^0 - \\
\beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac12 (H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \frac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M (H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-) - \\
\frac18 g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2) - \\
gMW^+_\mu W^-_\mu H - \frac12 g\frac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \\
\frac12 ig \left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\frac12 g \left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \frac12 g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\,(\frac{1}{c_w} Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+) - ig\frac{s_w^2}{c_w} MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - \\
W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w} Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \\
\frac14 g^2 W^+_\mu W^-_\mu \,(H^2 + (\phi^0)^2 + 2\phi^+\phi^-) - \frac18 g^2 \frac{1}{c_w^2} Z^0_\mu Z^0_\mu \,(H^2 + (\phi^0)^2 + 2(2s_w^2-1)^2\phi^+\phi^-) - \\
\frac12 g^2\frac{s_w^2}{c_w} Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \frac12 ig^2\frac{s_w^2}{c_w} Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac12 g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \frac12 ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1) Z^0_\mu A_\mu\phi^+\phi^- - \\
g^2 s_w^2 A_\mu A_\mu \phi^+\phi^- + \frac12 ig_s\, \lambda^a_{ij}(\bar q^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu - \bar e^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar\nu^\lambda(\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar u^\lambda_j(\gamma\partial + \\
m^\lambda_u)u^\lambda_j - \bar d^\lambda_j(\gamma\partial + m^\lambda_d) d^\lambda_j + igs_w A_\mu \left(-(\bar e^\lambda\gamma^\mu e^\lambda) + \frac23(\bar u^\lambda_j\gamma^\mu u^\lambda_j) - \frac13(\bar d^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\frac{ig}{4c_w} Z^0_\mu\{(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar e^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar d^\lambda_j\gamma^\mu(\frac43 s_w^2 - 1 - \gamma^5)d^\lambda_j) + \\
(\bar u^\lambda_j\gamma^\mu(1 - \frac83 s_w^2 + \gamma^5)u^\lambda_j)\} + \frac{ig}{2\sqrt2} W^+_\mu \left((\bar\nu^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa} e^\kappa) + (\bar u^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa} d^\kappa_j)\right) + \\
\frac{ig}{2\sqrt2} W^-_\mu \left((\bar e^\kappa U^{lep}{}^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar d^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\frac{ig}{2M\sqrt2}\phi^+ \left(-m^\kappa_e(\bar\nu^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar\nu^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\frac{ig}{2M\sqrt2}\phi^- \left(m^\lambda_e(\bar e^\lambda U^{lep}{}^\dagger_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar e^\lambda U^{lep}{}^\dagger_{\lambda\kappa}(1-\gamma^5)\nu^\kappa)\right) - \frac{g}{2}\frac{m^\lambda_\nu}{M} H(\bar\nu^\lambda\nu^\lambda) - \\
\frac g2 \frac{m^\lambda_e}{M} H(\bar e^\lambda e^\lambda) + \frac{ig}{2}\frac{m^\lambda_\nu}{M}\phi^0(\bar\nu^\lambda\gamma^5\nu^\lambda) - \frac{ig}{2}\frac{m^\lambda_e}{M}\phi^0(\bar e^\lambda\gamma^5 e^\lambda) - \frac14\, \bar\nu_\lambda\, M^R_{\lambda\kappa}\,(1-\gamma_5)\hat\nu_\kappa - \\
\frac14\, \bar\nu_\lambda\, M^R_{\lambda\kappa}\,(1-\gamma_5)\hat\nu_\kappa + \frac{ig}{2M\sqrt2}\phi^+ \left(-m^\kappa_d(\bar u^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar u^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\frac{ig}{2M\sqrt2}\phi^- \left(m^\lambda_d(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \frac g2\frac{m^\lambda_u}{M}H(\bar u^\lambda_j u^\lambda_j) - \\
\frac g2\frac{m^\lambda_d}{M}H(\bar d^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar u^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar d^\lambda_j\gamma^5 d^\lambda_j) + \bar G^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar G^a G^b g^c_\mu + \\
\bar X^+(\partial^2 - M^2)X^+ + \bar X^-(\partial^2 - M^2)X^- + \bar X^0(\partial^2 - \frac{M^2}{c_w^2})X^0 + \bar Y\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar X^0 X^- - \\
\partial_\mu\bar X^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar Y X^- - \partial_\mu\bar X^+ Y) + igc_w W^-_\mu(\partial_\mu\bar X^- X^0 - \\
\partial_\mu\bar X^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar X^- Y - \partial_\mu\bar Y X^+) + igc_w Z^0_\mu(\partial_\mu\bar X^+ X^+ - \\
\partial_\mu\bar X^- X^-) + igs_w A_\mu(\partial_\mu\bar X^+ X^+ - \\
\partial_\mu\bar X^- X^-) - \frac12 gM \left(\bar X^+ X^+ H + \bar X^- X^- H + \frac{1}{c_w^2}\bar X^0 X^0 H\right) + \frac{1-2c_w^2}{2c_w} igM \left(\bar X^+ X^0\phi^+ - \bar X^- X^0\phi^-\right) + \\
\frac{1}{2c_w} igM \left(\bar X^0 X^-\phi^+ - \bar X^0 X^+\phi^-\right) + igMs_w \left(\bar X^0 X^-\phi^+ - \bar X^0 X^+\phi^-\right) + \\
\frac12 igM \left(\bar X^+ X^+\phi^0 - \bar X^- X^-\phi^0\right) .
\end{gathered}
$$