Introduction to Further Topics in Physics

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1 Introduction

This lecture aims to address a wide variety of "knowledge gaps" that must be filled to maximize Physics Bowl performance. It will cover the following:

- Fluids
- Thermodynamics
- Waves and Sound
- Optics
- Modern Physics: Relativity and Radiation

Although this is a very wide spectrum of topics, Physics Bowl will only require a superficial understanding of these. A rigorous course in many of these topics would require mathematics beyond the high school level. We will thus address these topics with far less rigor than in previous lectures in the interest of brevity. Also, although much of geometrical optics requires no more than basic algebra, it is such a vast field that it is impossible to fit all of the content.

As requested, I have included some exercises that incorporate the topics of the lecture. They can be found at the end of each section. Special thanks to Emir Tataroğlu for helping write problems.

2 Fluids

Definition 2.1. A *fluid* is a liquid, gas, or other material that may continuously move and deform under an external force.

Definition 2.2. *Mass density* is the ratio of mass to volume.

$$\rho = \frac{m}{V} \tag{1}$$

Theorem 2.3. *(Archimedes' principle)* An object partially or fully submerged in a fluid experiences an upward *buoyant force* opposing its weight, given by

$$F = \rho g V \tag{2}$$

where V is the volume displaced.

Theorem 2.4. *(Bernoulli's principle)* An increase in the speed of a fluid occurs simultaneously with a decrease in static pressure or the fluid's potential energy.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 \tag{3}$$

Another statement of Bernoulli's principle is

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{ constant} \tag{4}$$

Definition 2.5. *Pressure* is the force applied perpendicular to the surface of an object per unit area over which that force is distributed.

$$P = \frac{F}{A} \tag{5}$$

Theorem 2.6. *(Stevin's law)* The pressure at a point h deep in a liquid is

$$p = \rho g h + p_0 \tag{6}$$

where p_0 is the *gauge pressure* (the ambient atmospheric pressure).

Exercise 2.7. *A 100 g cube with 5 cm long sides is placed in water ($\rho = 1g/cm^3$). What portion of the cube is submerged? Use $g = 10m/s^2$.*

Exercise 2.8. *Derive equation 6 from the equation 5.*

3 Thermodynamics

Theorem 3.1. *(Ideal gas law)* A container of volume V with n moles of gas of temperature T (measured in Kelvin) follows the equation

$$PV = nRT \tag{7}$$

where R is the *ideal gas constant.*

Theorem 3.2. *(First law of thermodynamics)* The internal energy change of a system (ΔU) can be defined in terms of the work W done on or by the system and the amount of heat Q supplied or withdrawn from the system in a thermodynamic process.

$$\Delta U = Q + W \tag{8}$$

In this convention, net energy transfers *to* the system are positive and net energy transfers *from* the system are negative.

Theorem 3.3. When a system expands in a *quasistatic process* (a thermodynamic process that happens slowly enough for the system to remain in internal physics thermodynamic equilibrium), the thermodynamic work done by the system on the surroundings is the product

$$W = P\Delta V \tag{9}$$

with P the pressure and ΔV the volume change.[1]

[1]More rigorously, $dW = PdV \Rightarrow dU = \delta Q + PdV$ where δQ is the inexact differential of an infinitesimal amount of heat supplied to the system from its surroundings.

Definition 3.4. The *specific heat* of a material is the heat required to raise the temperature of a unit mass of a given substance by one degree Kelvin/Celsius.

We get that the change in temperature of a sample ΔT when heat energy Q is added to it is

$$\Delta T = \frac{Q}{m \cdot s} \tag{10}$$

where s is the specific heat. The product $m \cdot s$ is the *heat capacity.* The energy required for a phase change is the *latent heat* of that phase change multiplied by mass:

$$Q = mL \tag{11}$$

Definition 3.5. For a given set of macroscopic variables, the *entropy* measures the degree to which the probability of the system is spread out over different possible microstates. In contrast to the macrostate, which characterizes plainly observable average quantities, a microstate specifies all molecular details about the system including the position and velocity of every molecule. The more such states are available to the system with appreciable probability, the greater the entropy. In statistical mechanics, entropy is a measure of the number of ways a system can be arranged, often taken to be a measure of "disorder" (the higher the entropy, the higher the disorder).

Theorem 3.6. *(Clausius theorem) For a thermodynamic system (e.g. heat engine or heat pump) exchanging heat with external thermal reservoirs and undergoing a thermodynamic cycle, the entropy*

$$\Delta S = \frac{Q}{T} \tag{12}$$

Exercise 3.7. *A special non-Hookean spherical balloon exerts a constant pressure on its contents such that they are held at a constant pressure P regardless of size. It has radius r at room temperature (293 K). It is brought into a fridge of 273 K. Assume the gas inside the balloon cools slowly. The gas' heat capacity is 5 $Jg^{-1}K^{-1}$.*

(i) What's the new radius of the balloon?

(ii) What's the change in internal energy of the gas?

Exercise 3.8. *A box contains n moles of a monoatomic ideal gas at volume $1m^3$, pressure P, and temperature T. The box magically expands to a volume of 10 m^3. What is the new pressure, P_f? Express P_f in terms of P.*

Exercise 3.9. *Assume the gas from exercise 3.8 is at a total mass M. A heat Q is now supplied to the gas in the container until it reaches a new temperature T_f. Assume the specific heat of the gas is 1 $Jg^{-1}K^{-1}$. Express T_f in terms of Q, T, M. What is the new pressure P_{f2} after the heat transfer? Express in terms of T, Q, M, n, R (not in terms of P).*

4 Waves and Sound

Definition 4.1. A *wave* is a propagating dynamic disturbance (change from equilibrium) of one or more quantities. Periodic waves oscillate repeatedly about an equilibrium (resting) value at some frequency. Frequency is defined as the reciprocal of the period of oscillation, and is measured in $Hz \equiv s^{-1}$.

Definition 4.2. The *wavelength* of a wave is related to the wave's speed and its frequency by

$$\lambda = \frac{v}{f} \tag{13}$$

Definition 4.3. *Transverse* waves cause the medium to move perpendicular to the direction of the wave. *Longitudinal waves* cause the medium to move parallel to the direction of the wave.

Definition 4.4. *Sound* is transmitted through gases and liquids as longitudinal waves. The speed of sound in air is roughly 343 m/s.

Definition 4.5. *The sound intensity level, usually measured in decibels (dB), is defined by*

$$L = 10 \log_{10} \left(\frac{I}{I_0} \right) \tag{14}$$

where I is the sound intensity and I_0 the reference sound density, which is 10^{-12} W/m^2 in air.

Theorem 4.6. *(Doppler effect)* The frequency of a sound wave is changed in relation to an observer who is moving relative to the source of the wave. For an observer at velocity v_o and a source of velocity v_s in a medium in which sound propagates at v has an observed frequency f_o related to the emitted frequency f_s by

$$f_o = f_s \left(\frac{v \pm v_o}{v \pm v_s} \right) \tag{15}$$

Use $+v_o$ for an observer moving toward the source, $-v_o$ for an observer moving away from the source, $+v_s$ for a source moving away from the observer, and $-v_s$ for a source moving towards the observer. One way you can think about this is the frequency gets higher when the source and observer move toward each other and lower when they move away from each other.

Exercise 4.7. *A fire truck's siren wails at* 100 *dB. You walk on the sidewalk at 1 m/s, while the truck moves in the direction opposite to you at 40 m/s. You have perfect pitch, determining that the siren makes an F# sound, which translates to 1498 Hz in physics terms.*

(i) What's the intensity of the siren?

(ii) What's the frequency of the siren?

(iii) Using your answer from part 2, what's the wavelength of the sound waves emitted by the sirens?

5 Optics

The photon is an elementary particle. It is a quantum of the electromagnetic field, including electromagnetic radiation such as light and radio waves, and the force carrier for the electromagnetic force. Photons are massless and always travel at the speed of light–about 3×10^8 m/s (often represented as c). They are best explained by quantum mechanics, and exhibit *wave-particle duality*, meaning their behavior features properties of both waves and particles.

As mentioned, light is electromagnetic radiation. Light is visible to humans when its wavelength is about 380-700 nanometers.[2] *Geometrical optics* represents the propagation of light in terms of rays that propagate in straight-line paths as they travel in homogeneous media. When a ray encounters an interface between two media, some of it will be *reflected*.

Theorem 5.1. *(Law of reflection)* The angle at which a ray is incident on a surface equals the angle at which it is reflected.

Whether the rays will all leave in the same direction (like a mirror), which is called specular reflection, or whether they get scattered in all directions, which is called diffuse reflection, matters on the nature of the material.

However, light is not always fully reflected. Sometimes, it permeates the interface. In a material, a photon travels at the speed of light until it hits an atom. The photon is then absorbed and re-emitted in the same direction, which takes a small amount of time. The more this happens, the slower the effective speed of propagation. can change its direction in a process called *refraction* to satisfy *Fermat's principle*, which states that the path taken by a ray between two given points is the path that can be traveled in the least time.

Definition 5.2. The *index of refraction* n of a material is defined as the factor by which the speed and wavelength of the radiation are reduced with respect to their vacuum values:

$$n = \frac{c}{v} \tag{16}$$

$$n = \frac{\lambda_0}{\lambda} \tag{17}$$

Notice that the frequency of the wave is not affected by the refractive index.

Theorem 5.3. *(Snell's law)* A ray travelling in a medium of index n_1 encountering an interface with another medium of index n_2 will change its course based on the following relation:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{18}$$

where θ_1 is the *angle of incidence*–the angle made between the incoming ray and the normal of the interface–and θ_2 is the *index of refraction*, which is the angle made between the transmitted ray and the normal of the interface.

A ray entering higher index bends toward normal, while a ray entering lower index bends away from normal.

Definition 5.4. If the incident index of refraction is higher than the other medium's, then there exists a *critical angle* of incidence at which the ray will be *totally internally reflected*, given by

$$\theta_c = \sin^{-1}\left(\frac{\theta_2}{\theta_1}\right) \tag{19}$$

For all $\theta_i > \theta_c$, the ray will be totally internally reflected.

[2]One nanometer is 10^{-9} m.

Object	Image			
Convex				
Location	Type	Location	Orientation	Relative Size
$\infty > s_o > 2f$	Real	$f < s_i < 2f$	Inverted	Minified
$s_o = 2f$	Real	$s_i = 2f$	Inverted	Same size
$f < s_o < 2f$	Real	$\infty > s_i > 2f$	Inverted	Magnified
$s_o = f$		$\pm\infty$		
$s_o < f$	Virtual	$\lvert s_i \rvert > s_o$	Erect	Magnified
Concave				
Location	Type	Location	Orientation	Relative Size
Anywhere	Virtual	$\lvert s_i \rvert < \lvert f \rvert$, $s_o > \lvert s_i \rvert$	Erect	Minified

Figure 1: The resulting images of lenses. Parallel rays are treated as objects or images at infinity.

One optical device that refracts light is the *lens*. It focuses or disperses a light beam.

Concave lenses are thinner in the middle than at the edges and have a shape that curves inward, resembling a cave. This design causes parallel light rays that pass through the lens to spread out or diverge. Because of this spreading effect, concave lenses are also known as diverging lenses.

Convex lenses, in contrast, are thicker in the middle than at the edges and curve outward. This shape allows the lens to bring parallel light rays together, or converge, after passing through the lens. Therefore, convex lenses are also referred to as converging lenses.

Definition 5.5. The *focal length* of a lens determines how strongly the system converges or diverges light and is given by

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \tag{20}$$

where s_o and s_i represent the object's distance from the lens and the distance of the formed image, respectively. A positive focal length indicates that a lens converges light. Initially parallel rays are brought together to a focus. A negative focal length indicates how far in front of the lens a point source must be located to form parallel rays.

Definition 5.6. The image of an object is defined as the collection of focus points of light rays coming from the object. A *real* image is the collection of focus points made by converging rays, while a *virtual* image is the collection of focus points made by extensions of diverging rays. In other words, a virtual image is found by tracing real rays that emerge from an optical device backward to perceived or apparent origins of ray divergences.[3]

[3] A good simulator to visualize this can be found at https://phydemo.app/ray-optics/simulator/

Definition 5.7. *Lateral magnification* is minus the ratio between an image's distance from the lens and the object's distance from the lens, or equivalently the image's height and the object's height.

$$M = -\frac{s_i}{s_o} = \frac{h_i}{h_o} \tag{21}$$

A negative magnification implies an inverted image.

A famous result from optics involve the single/double slit experiment, which deals with *diffraction*, an effect not accounted for in geometrical optics.

When light passes through a single narrow slit, it doesn't simply produce a straight line on the screen behind the slit. Instead, it spreads out and forms a pattern of light and dark bands known as a diffraction pattern. This effect occurs because the light waves emerging from the slit interfere with each other, a phenomenon that indicates light's wave-like nature.

When light passes through two closely spaced slits, it produces an interference pattern on the screen behind the slits. This pattern consists of a series of bright and dark bands. The bright bands occur where the light waves from the two slits arrive in phase and reinforce each other (constructive interference), while the dark bands occur where the waves are out of phase and cancel each other out (destructive interference). This experiment further supports the wave nature of light but also leads to questions about particle-like behavior when individual photons are observed going through the slits.

Theorem 5.8. The condition for constructive interference in the double slit experiment is as follows:

$$m\lambda = d \sin\theta \tag{22}$$

Here, m represents the order of the maximum–an integer representing the number of wavelengths by which the path length differs. λ is the wavelength of light, d is the distance between the two slits, and θ is the angle relative to the original direction of the light where constructive interference occurs.

This equation allows us to calculate the angles at which bright fringes (bands) will appear on the screen, demonstrating the relationship between the physical setup (slit separation and wavelength of light) and the observed interference pattern.

Theorem 5.9. As for single-slit experiments, we describe the diffraction pattern with the condition for the minima (dark fringes) with the following:

$$m\lambda = d \sin\theta \tag{23}$$

Here, m represents the order of the minimum, λ is the wavelength of light, d is the distance between the two slits, and θ is the angle relative to the original direction of the light at which minima occur.

The equation in theorem 5.8 is different from that in theorem 5.9 because it focuses on the condition for minima rather than maxima. In the context of a single slit, the minima are the result of destructive interference of light waves spreading out from different parts of the slit.

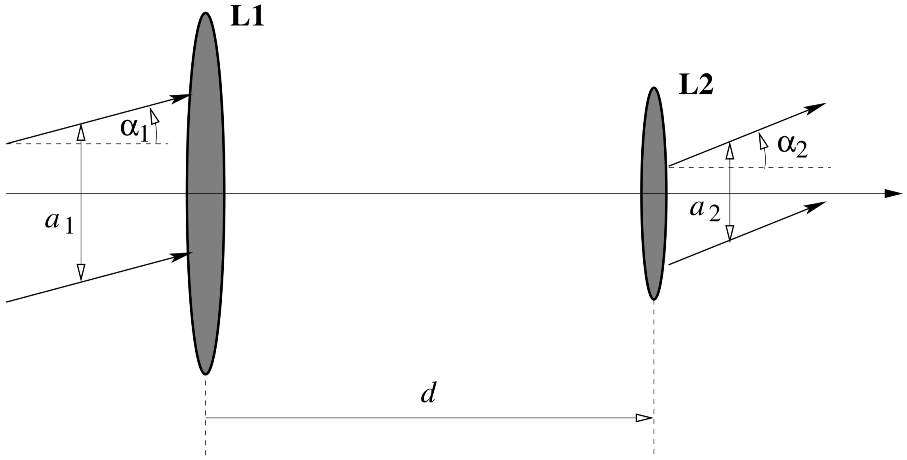

Figure 2: Setup for problem 5.11

Exercise 5.10. *In a Young's double slit experiment, narrow double slits* $0.2mm$ *apart diffract monochromatic light onto a screen placed* $1.5m$ *away. The distance between to fifth minima on either side of the zeroth order maximum is measured to be* $34.73mm$*. Determine the wavelength of the light.*

Exercise 5.11. *A parallel ray bundle with width* a_1 *and propagation angle* α_1 *is incident from the left on a two-lens system composed of two positive lenses* ***L1*** *(focal length* $f_1 > 0$*) and* ***L2*** *(focal length* $f_2 > 0$*) as shown in figure 2. Show that if the separation* d *between the two lenses equals the sum* $f_1 + f_2$ *of focal lengths, then a parallel ray bundle emerges from the system.*

6 Modern Physics: Radiation and Relativity

Albert Einstein came up with special relativity in 1905. Maxwell's equations from last meeting appeared to be incompatible with Newtonian mechanics. This observation led to the following two postulates:

- The laws of physics are invariant in all inertial frames of reference.
- The speed of light in vacuum is the same for all observers, regardless of the motion of the observer.

A consequence of this is that the behavior of objects observed to have a velocity that is a significant fraction of the speed of light differs slightly from what is observed in classical physics. Notably, bodies experience two main phenomena.

Definition 6.1. *The Lorentz factor* γ *is*

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{24}$$

with v *the relative velocity between the observer and the moving object.*

Theorem 6.2. *(Length contraction)* A moving object's length is shorter than its proper length, the length as measured in the object's own rest frame.

$$L = \frac{L_0}{\gamma} \tag{25}$$

Theorem 6.3. *(Time dilation)* A moving clock is observed to run slower than a clock in the observer's rest frame. The time elapsed on the moving clock $\Delta t'$ is related to the time on the rest clock Δt by

$$\Delta t' = \gamma \Delta t \tag{26}$$

Einstein also had many other famous discoveries that contributed to the field of *modern physics*.

Theorem 6.4. *(Mass–energy equivalence)* Energy and mass are related by the famous

$$E = \gamma m c^2 \tag{27}$$

Theorem 6.5. The relativistic momentum of an object is

$$p = \gamma m v \tag{28}$$

Corollary 6.6. *(Energy-momentum relation)* The total energy of a moving object is

$$E = \sqrt{p^2c^2 + m^2c^4} \tag{29}$$

Corollary 6.7. The relativistic kinetic energy of a moving object is

$$E_k = (\gamma - 1)mc^2 \tag{30}$$

Theorem 6.8. *(deBroglie relation)* The momentum of a photon is

$$p = \frac{h}{\lambda} \tag{31}$$

with h the Planck constant.

Corollary 6.9. *The energy of a photon is*

$$E = hf \tag{32}$$

Radioactive decay is the process by which an unstable atomic nucleus loses energy by radiation. A material containing unstable nuclei is considered radioactive. The three most common types of decay are alpha, beta, and gamma decay. Radioactive is a stochastic (i.e. random) process at the level of single atoms. According to quantum theory, it is impossible to predict when a particular atom will decay, regardless of how long the atom has existed. However, for a significant number of identical atoms, the overall decay rate can be expressed as a decay constant or as half-life. The decaying nucleus is called the parent radionuclide and the process produces at least one daughter nuclide. Except for gamma decay or internal conversion from a nuclear excited state, the decay is a nuclear transmutation resulting in a daughter containing a different number of protons or neutrons (or both). When the number of protons changes, an atom of a different chemical element is created.

Definition 6.10. *Alpha decay* involves an atomic nucleus that emits an *alpha particle*.The alpha particle is identical to the nucleus of a helium-4 atom, which consists of two protons and two neutrons.

Definition 6.11. *Beta decay* involves an atomic nucleus that emits a *beta particle*, which is a fast energetic electron or positron.

For example, the beta decay of a neutron transforms it into a proton by the emission of an electron accompanied by an antineutrino; or, conversely a proton is converted into a neutron by the emission of a positron with a neutrino in so-called positron emission. Neither the beta particle or its associated (anti)neutrino exist within the nucleus prior to beta decay, but are created in the decay process. By this process, unstable atoms obtain a more stable ratio of protons to neutrons. Beta decay is a consequence of weak force, which is characterized by relatively lengthy decay times. Nucleons are composed of up quarks and down quarks, and the weak force allows a quark to change its flavor by emission of a W boson leading to creation of an electron/antineutrino or positron/neutrino pair. For example, a neutron, composed of two down quarks and an up quark, decays to a proton composed of a down quark and two up quarks.

Definition 6.12. *Gamma rays* are produced during *gamma decay*, which normally occurs after the other forms of decay occur. When a radioactive nucleus emits an alpha or beta particle, the daughter nucleus that results is usually left in an excited state. It can then decay to a lower energy state by emitting a gamma ray photon. Gamma decay may also follow nuclear reactions such as neutron capture, nuclear fission, or nuclear fusion.

Exercise 6.13. *(Physics Bowl 2023 #50) An electron (e^-) is moving and has a kinetic energy of 1.00 MeV. It makes a head-on collision with a positron (e^+) initially at rest. In the collision the two particles annihilate each other and are replaced by two photons (γ) of equal energy, each traveling at angles θ to the electron's original direction of motion. The reaction is $e^- + e^+ \to 2\gamma$. Determine the angle of emission θ of each photon.*

Exercise 6.14. *Chad is born on the same day that his mom turns 40 years old. Chad stays on earth while his mom then goes on a trip far away at $v = 0.866c$ for 40 years in her own reference frame, and stops at a new planet extremely fast. Ignore the process of stopping. When the mother has stopped at said planet, what is the new age difference between them? Assume the earth is at rest.*

Exercise 6.15. *Chad has a meter stick of mass 10 kg. Frustrated that his mom left him, he instantaneously accelerates it to a velocity v. The meter stick moves at a constant velocity, and in doing so has an energy of $30c^2$ Joules in Bob's reference frame.*

(i) What is v in terms of c?

(ii) How long does Chad observe the meter stick to be?

(iii) What is the meter stick's energy as observed in its own frame?