Chapter 6

Scatterplots, Association, and Correlation

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Review: Comparing Variables

In previous chapters, we looked for relationships (associations) between variables by:

- Comparing categorical variables with contingency tables and stacked barplots
- Comparing numeric variables across groups with side-by-side boxplots
- Looked at how variables change over time with timeplots

In this chapter, we will:

- Look for relationships between two numeric variables

The Data

Recall the Motor Trend Cars data from previous chapters:

```
##                    mpg cyl disp  hp    wt  qsec vs     am
## Mazda RX4         21.0   6  160 110 2.620 16.46  V   auto
## Mazda RX4 Wag     21.0   6  160 110 2.875 17.02  V   auto
## Datsun 710        22.8   4  108  93 2.320 18.61  S   auto
## Hornet 4 Drive    21.4   6  258 110 3.215 19.44  S manual
## Hornet Sportabout 18.7   8  360 175 3.440 17.02  V manual
## Valiant           18.1   6  225 105 3.460 20.22  S manual
```

We might want to know:

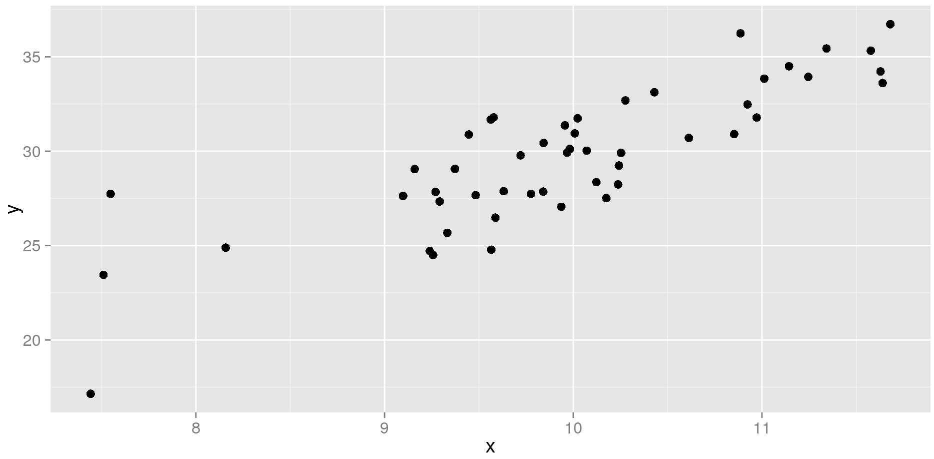
- Is there a relationship between engine displacement (size) and horsepower?
- Is weight related to fuel efficiency?

Overview

How to we find relationships between numeric (quantitative) variables?

- Visually: using **scatterplots**
- Numerically: using the **correlation coefficient**
- Usually, we do both
- In this course, we will only focus on **linear** relationships

Scatterplots



Scatterplots

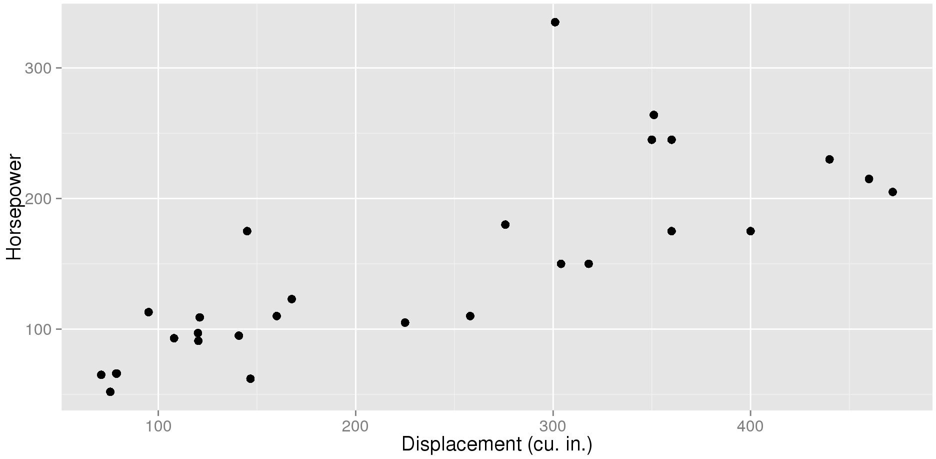
How to make scatterplots:

- Define one variable as the X variable, and one as Y
- Draw a point for each observation, using the values of the X and Y variables as coordinates
- Typically, the X variable is on the horizontal axis and the Y variable on the vertical axis

What we look for:

- Is there are trend or pattern?
- Are there any outliers or unusual points?

Horsepower vs. Displacement



Horsepower vs. Displacement

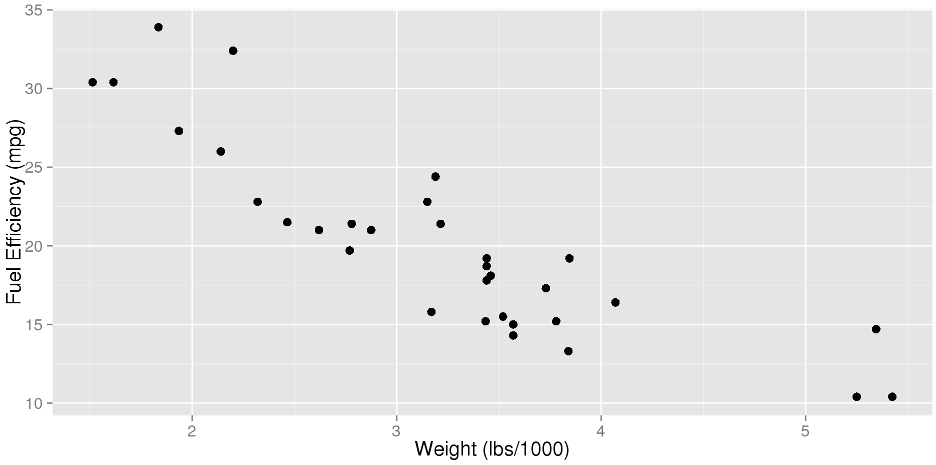
Is there a relationship?

- As engines get bigger, they tend to have more horsepower
- We call this a **positive** association

Are there any unusual points?

- There is a point well above the rest
- Notice that it's engine size is right in the middle (about 300 cu. in.), but its horsepower is larger than any other car

Weight vs. Fuel Efficiency



Weight vs. Fuel Efficiency

Is there a relationship?

- As cars get heavier, they tend to have lower fuel efficiency
- We call this a **negative** association

Are there any outliers?

- No points fall far away from the rest

Types of Relationships

There are many types of trends that can come up when we make scatterplots. In this class, we will focus on the most common:

- **Linear**: The trend can be described fairly well by a straight line
- **Non-linear**: Any other type of trend

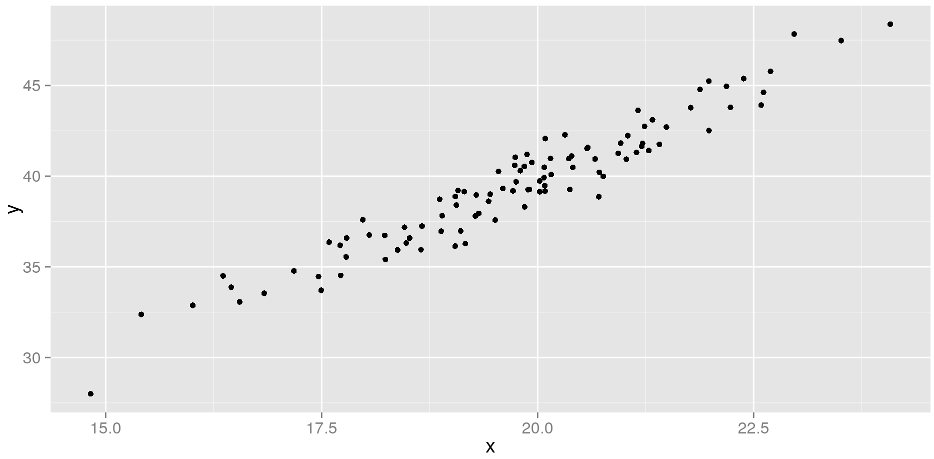
Directions of Relationships

- **Positive**: As one variable goes up, so does the other one
- **Negative**: As one variable goes up, the other goes down

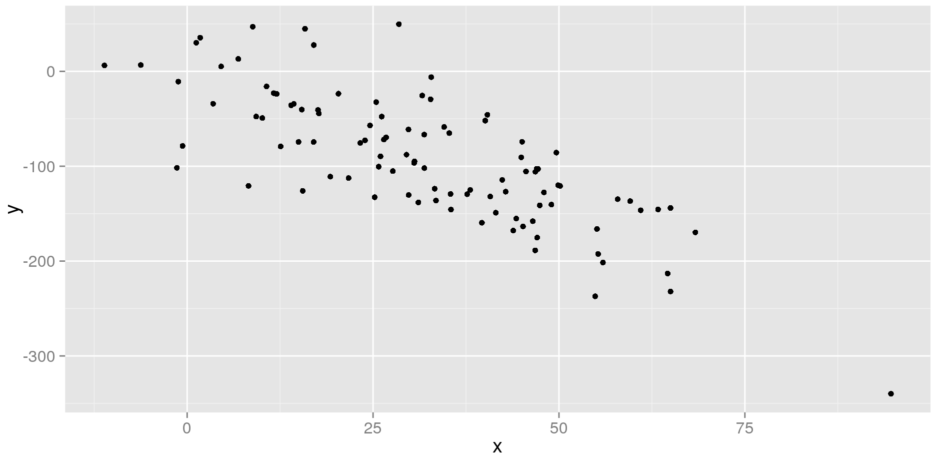
Why lines?

- In statistics, we often try to find the *simplest adequate method*. Lines are simple.

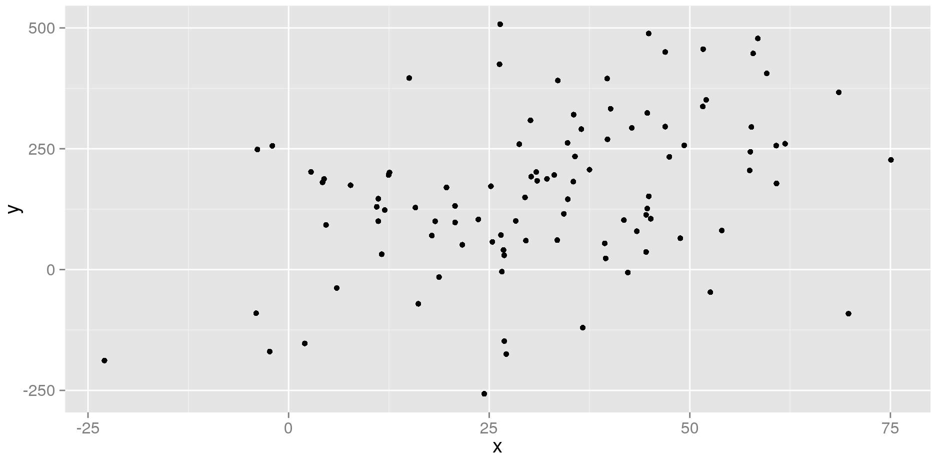
Strong Positive Linear Trend



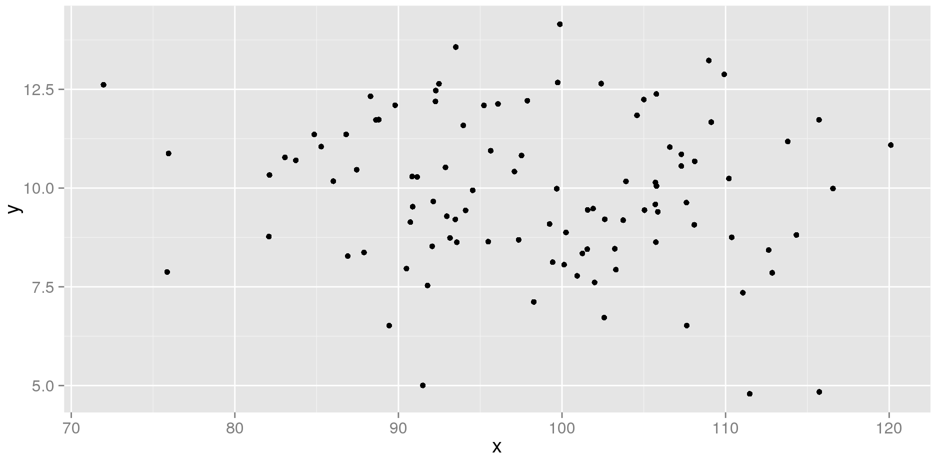
Moderate Negative Linear Trend



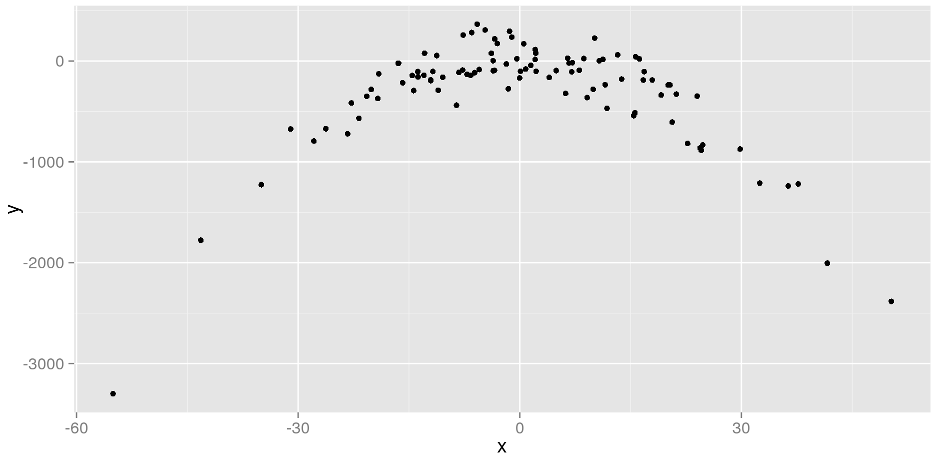
Weak Positive Linear Trend



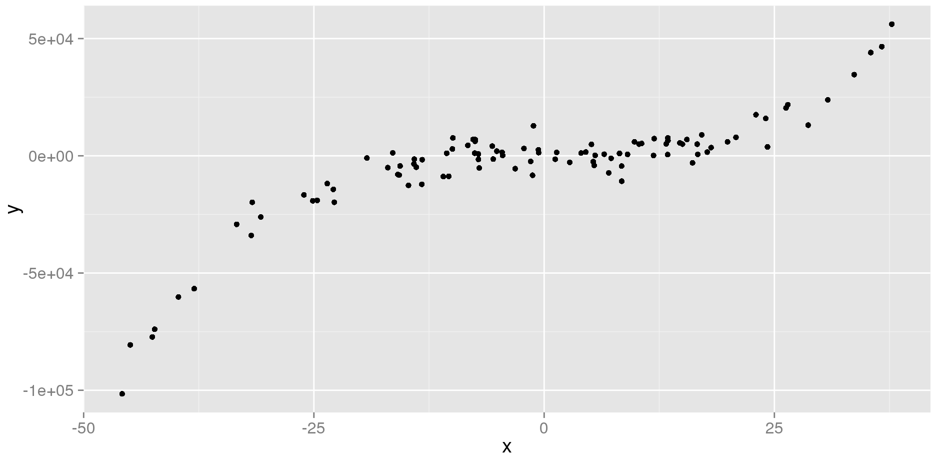
No Trend



Non-Linear Trend



Non-Linear Trend



Roles of Variables

How do we decide which is X and which is Y?

The X Variable is:

- The **explanatory** or **independent** variable.
- We want to know if changes in this variable *explains* changes in Y

The Y Variable is:

- The **response** or **dependent** variable
- We want to see if this variable *responds* when we change X

Which is which depends on what question we're asking.

Variable Role Examples

Horsepower vs. Engine Displacement

- It makes sense that giving a car a bigger engine gives it more power.
- We can't just give a car more horsepower, horsepower *responds* to changes we make to the car.
- Horsepower should be Y, and Engine Displacement should be X.

Fuel Efficiency vs. Weight

- When we make a car heavier, it should mean that it takes more fuel to move it.
- Fuel efficiency *responds* to changes in the properties of the car.
- Fuel Efficiency should be Y, and Weight should be X.

Measuring the Strength

How do we measure how strong the relationship is?

- We use the **correlation coefficient**
- $r = \frac{\sum z_y \times z_x}{n-1}$
- StatCrunch will find this for us

What is the correlation coefficient?

- r is the **strength of the linear relationship between two numeric variables**
- It tells us how well a straight line explains the relationship

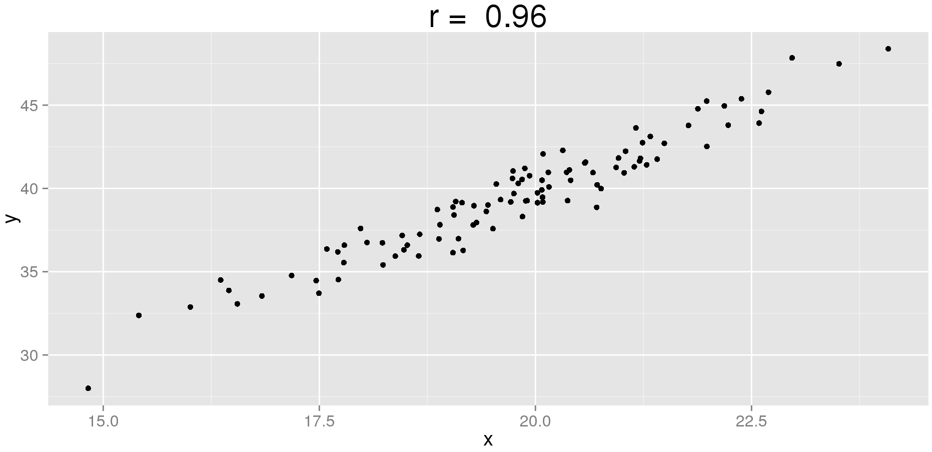
Interpreting Correlation

- $-1 \leq r \leq 1$
- The **value** of r tells us the strength
- The **sign** of r tells us the direction
- $r = 1$: the points make a **perfect** straight line with a **positive** slope
- $r = -1$: the points make a **perfect** straight line with a **negative** slope
- $r = 0$: there is no linear relationship at all

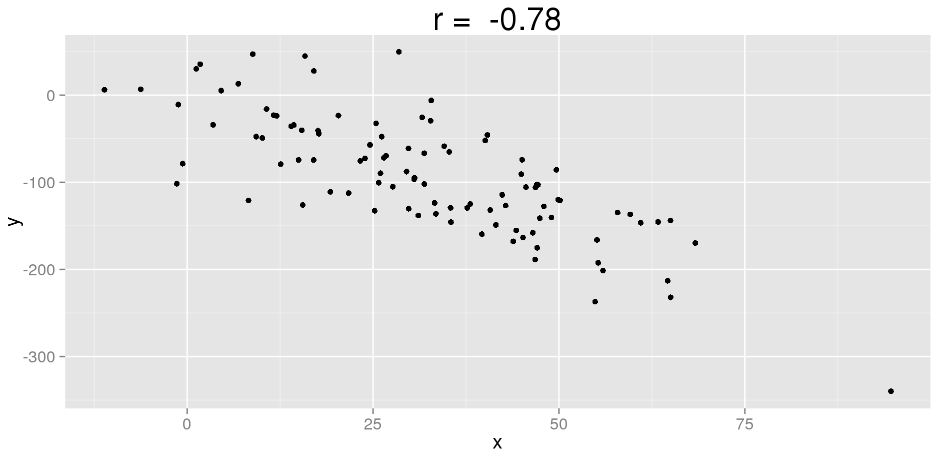
Notes:

- You can sometimes get high correlations even if the relationship isn't linear
- You should **always** see a scatterplot along with a correlation coefficient to know whether or not it's meaningful

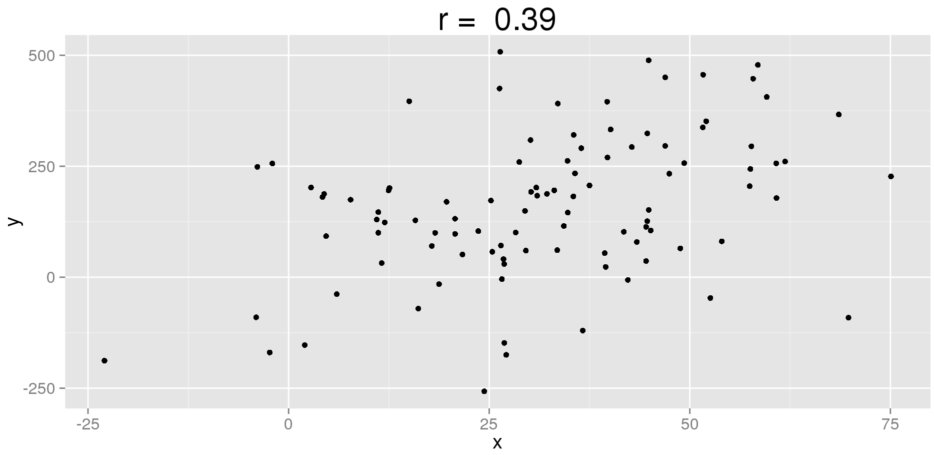
Strong Positive Linear Trend



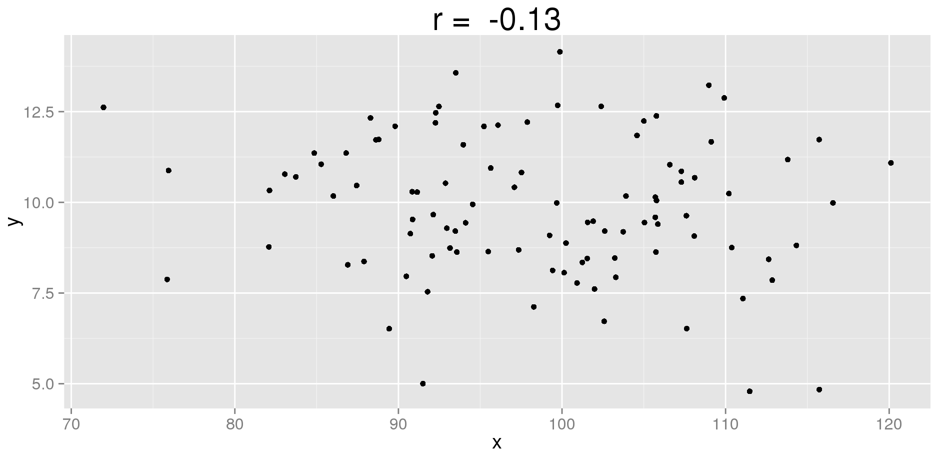
Moderate Negative Linear Trend



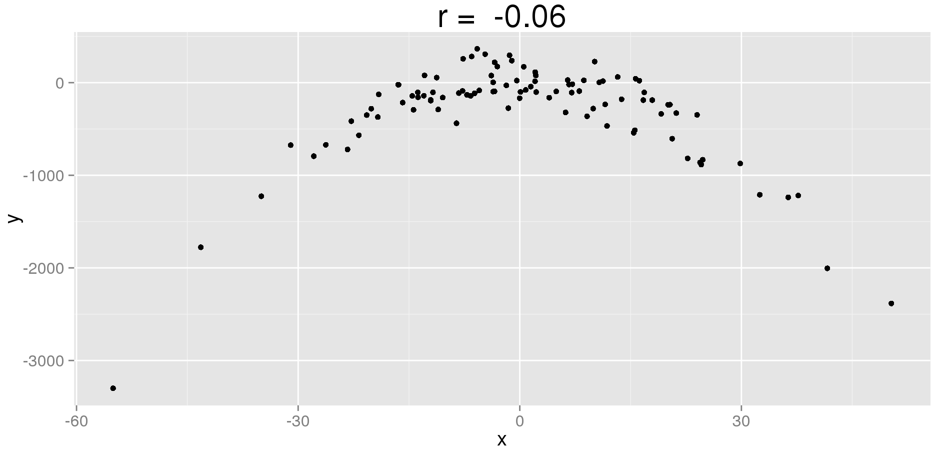
Weak Positive Linear Trend



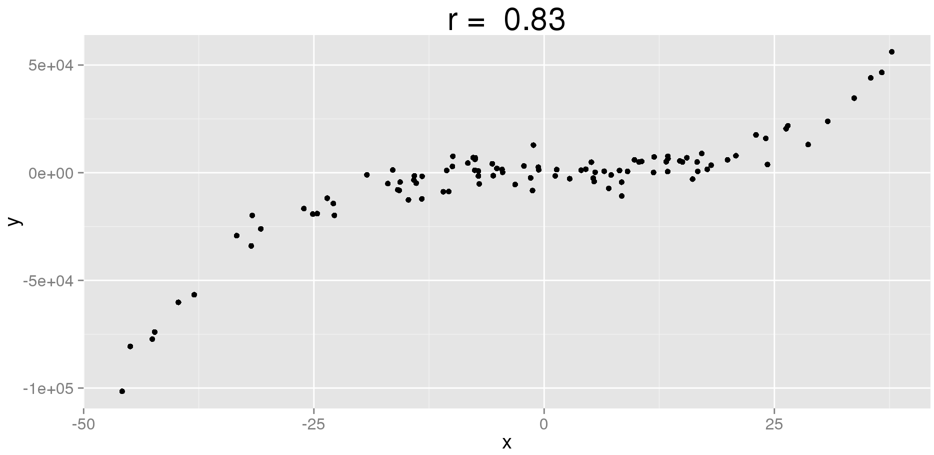
No Trend



Non-Linear Trend



Non-Linear Trend



Using the Correlation Coefficient

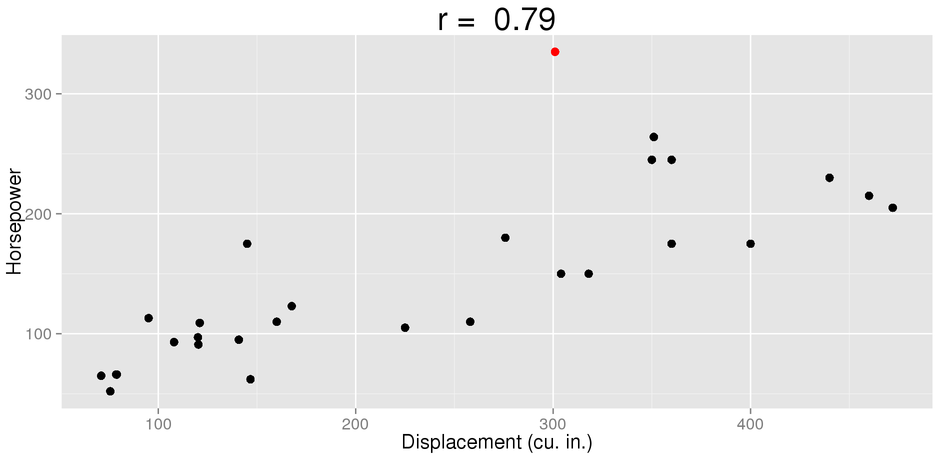
So how do we use r?

- First, make a scatterplot
- There needs to be a **linear** association, or r is meaningless
- Check the sign: is the relationship positive or negative?
- Check the value: how strong is the relationship?
- Are there outliers? The correlation is very sensitive to them.

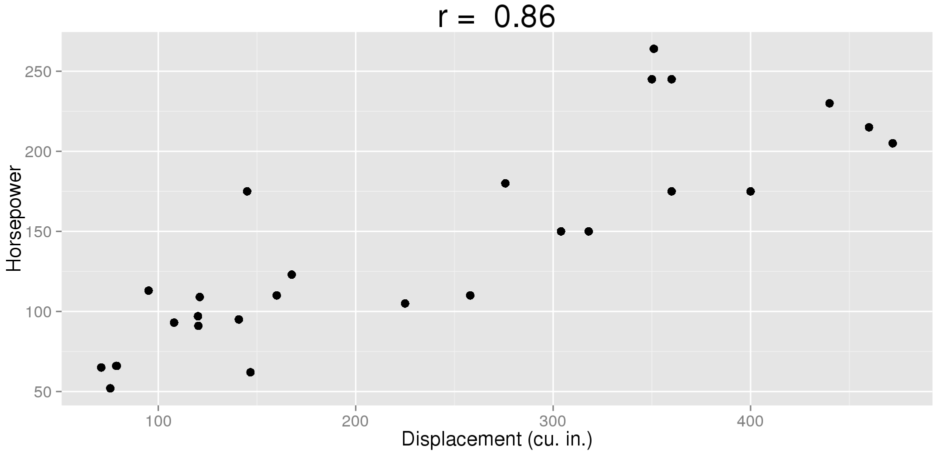
Note:

- We often use the terms **weak**, **moderate**, and **strong** to describe the relationship, but these are up to interpretation.

Horsepower vs. Engine Displacement



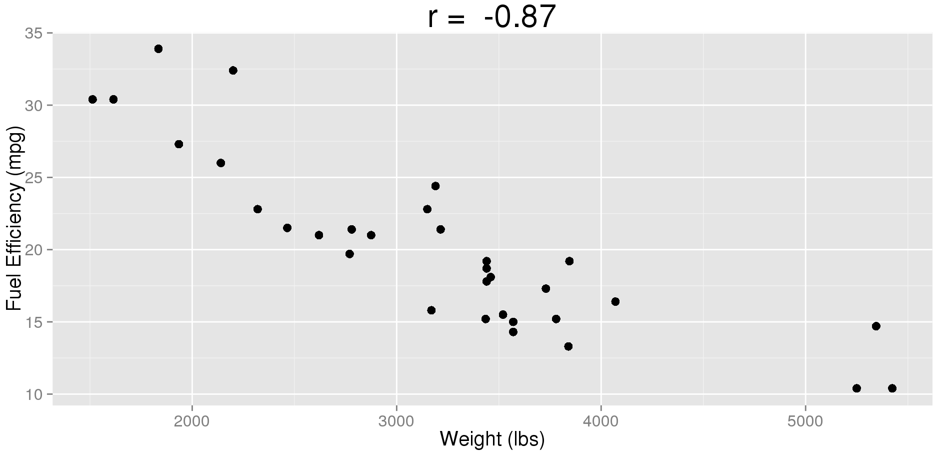
Horsepower vs. Engine Displacement



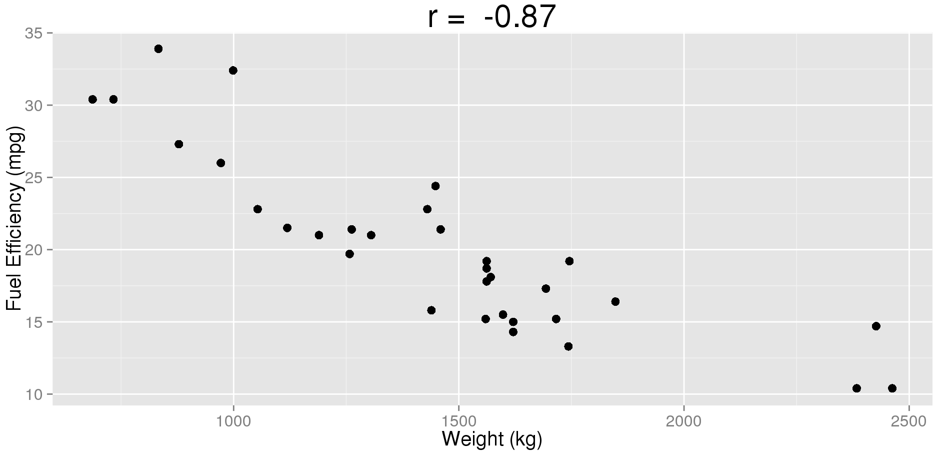
More Properties of Correlation

- r is **unitless**
- r is not affected by changes of center or scale
- If we change units, the correlation will not change (e.g., $lbs \rightarrow kg$)
- The correlation of X and Y is the same as the correlation between Z_x and Z_y (their z-scores)
- The correlation stays the same if we flip X and Y
- Correlation only applies to relationships between **numeric** variables. If there is an association involving categorical variables, it is **not** correlation.

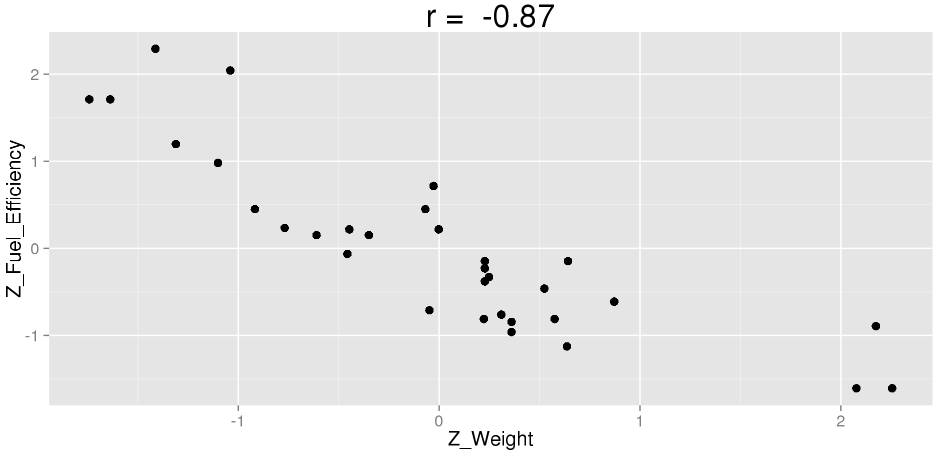
Weight (lbs) vs. Fuel Efficiency



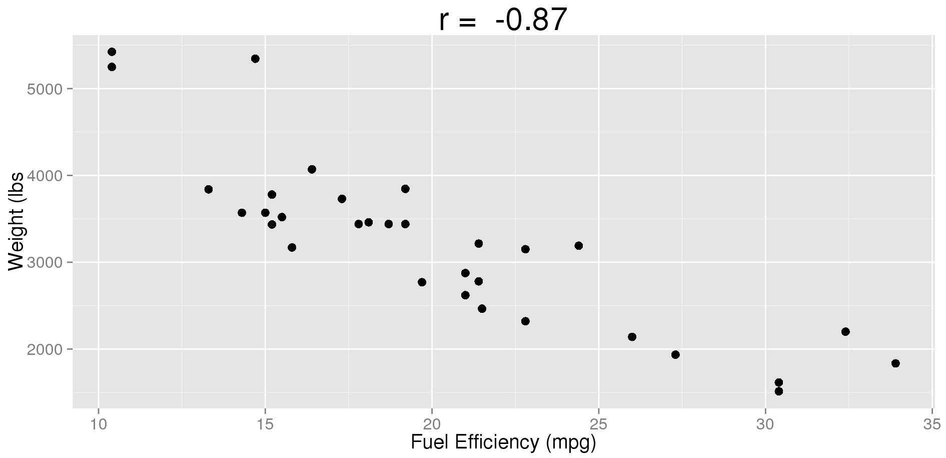
Weight (kg) vs. Fuel Efficiency



Weight vs. Fuel Efficiency (Z-Scores)



Fuel Efficiency vs. Weight



In StatCrunch

Scatterplots:

1. `Graph` $\rightarrow$ `Scatter Plot`
2. `X Column` $\rightarrow$ Select your explanatory (X) variable
3. `Y Column` $\rightarrow$ Selected your response (Y) variable
4. `Compute!`

Correlation:

1. `Stat` $\rightarrow$ `Summary Stats` $\rightarrow$ `Correlation`
2. `Select Column(s)` $\rightarrow$ Hold `Shift`/`Ctrl`/`Command` to select multiple variables (note: if you select more than two variables, it will find all pair-wise correlations)
3. `Compute!`

Correlation $\neq$ Causation

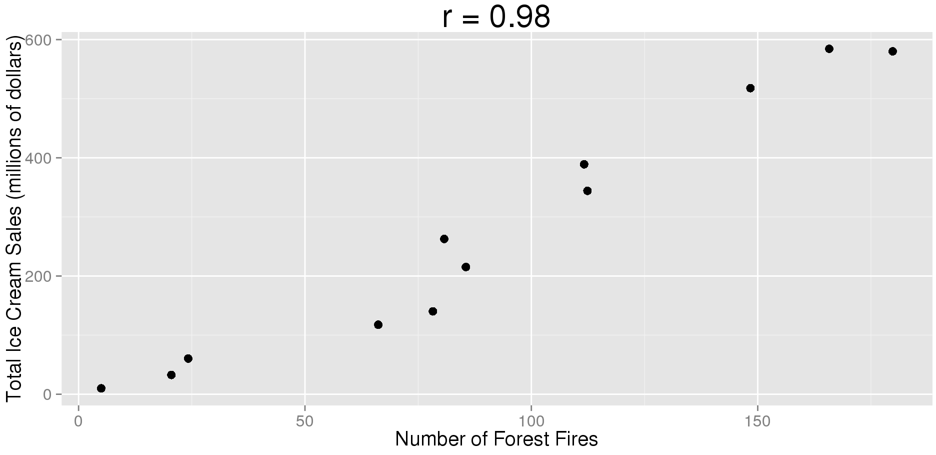
Must people are familiar with the phrase "correlation does not equal causation," but what does that really mean?

- Even if we find a correlation between two variables, it does not mean that one causes the other.
- This is especially common when two things both increase or decrease over time.
- Both may be caused by other, unknown variables.
- We call these unknown variables **lurking variables** or **confounding variables**.

For example:

- What if we looked at the correlation between national ice cream sales and the number of forest fires, recorded for each month of the year?

Ice Cream Sales and Forest Fires



Ice Cream Sales and Forest Fires

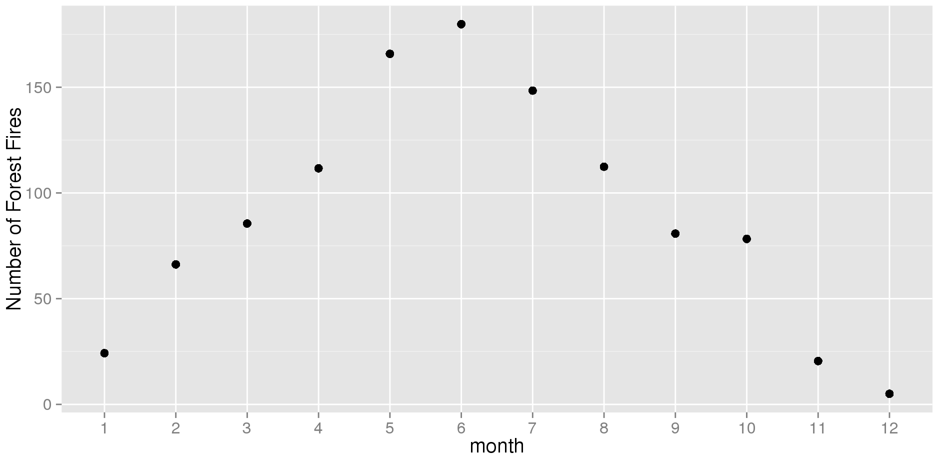
It certainly looks like there is a relationship.

- As the number of forest fires increase, the amount of ice cream being sold does as well
- If you open a pint of Ben & Jerries, does this light a patch of brush in California?
- The more likely explanation is the there is at least one lurking variable

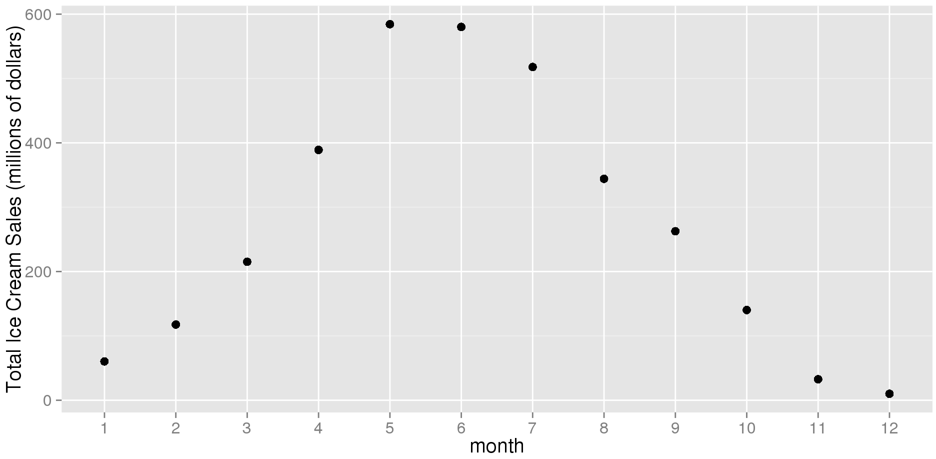
What could it be?

- Both could be related to the month in which the information was collected
- Additionally, certain months tend to be hotter and drier.
- Both of these conditions lead to people wanting ice cream and forest fires being easier to start

Forest Fires vs. Month



Ice Cream Sales vs. Month



Reporting Correlation

Employee Salaries and Productivity, $r = .8$:

- **Bad**: Raising salaries increases productivity.
- **Good**: Employees with higher salaries tend to be more productive.

Red Wine and Cholesterol, $r = -0.99$:

- **Bad**: This proves that drinking more red wine lowers cholesterol.
- **Good**: There is a strong negative association between red wine consumption and cholesterol level.

Parents' and Children's Education Levels (association, not correlation):

- **Bad**: A child that has two educated parents will graduate from college.
- **Good**: Children whose parents are educated are more likely to graduate from college

Summary

- We can use scatterplots to find relationships and outliers between two numeric variables
- The X variable is the **explanatory** variable
- The Y variable is the **response** variable
- A relationship between variables is called **association**
- We can measure the strength of a **linear relationship** between two **numeric variables** using **correlation**
- Correlation doesn't neccessarily imply causation, there may be lurking variables