From time series to complex networks: The visibility graph

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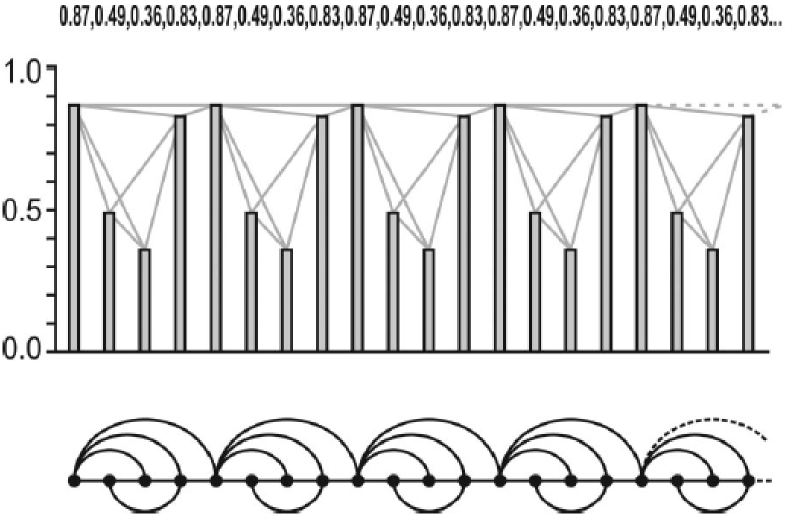
Visibility Graph



Visibility Graph

- Every node corresponds, in the same order, to series data.

- Two nodes are connected if visibility exists between the corresponding data, that is to say, if there is a straight line that connects the series data, provided that this “visibility line” does not intersect any intermediate data height.

Visibility Graph

Formally:
Two arbitrary data values (t_a, y_a) and (t_b, y_b) will have visibility, and consequently will become two connected nodes of the associated graph, if any other data (t_c, y_c) placed between them fulfills:

$$y_c < y_b + (y_a - y_b)\left(\frac{t_b - t_c}{t_b - t_a}\right) \tag{1}$$

The extracted graph is always:

- Connected;
- Undirected;
- Invariant.

Visibility Graph

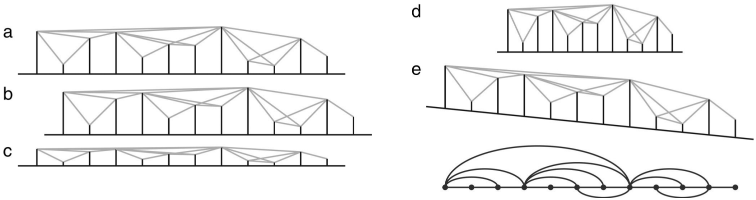


Figura 1: a) Original time series. b) Translation of the data. c) Vertical rescaling. d) Horizontal rescaling. e) Additional of a linear trend to the data. In all the cases the visibility graph remains invariant.

Visibility Graph

Questions:

1. Do the associated graph inherit some structure of the time series?
2. The process that generated the time series can be characterized by using graph theory?

Visibility Graph

1. Periodic series:
   - Associated graph is regular;
   - Discrete degree distribution;
   - Its regularity is conserved or inherited structurally in the graph.
2. Random series:
   - Associated graph is exponential random;
   - Exponential degree distribution.

Visibility Graph

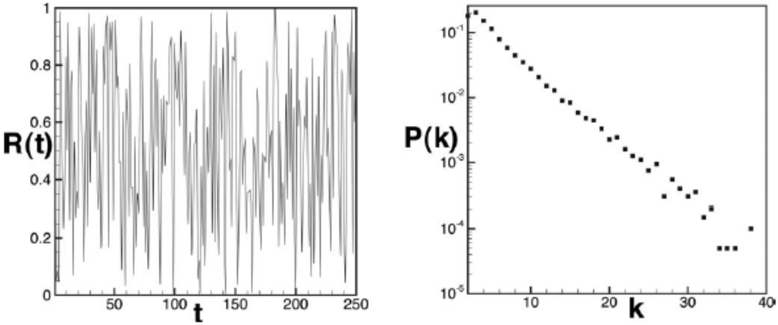


Figura 2: $R(t)$ is 10^6 values extracted from uniform distribution. (*Left*) First 250 values of $R(t)$. (*Right*) The beginning of the curve approaches the result of a Poisson process, the tail is clearly exponential. This behavior is due to data with large values (rare events), which are the hubs.

Visibility Graph

- Periodic series $\longrightarrow$ regular graphs;
- Random series $\longrightarrow$ Exponential random graphs;

Order and disorder structure in the time series seem to be inherited in the topology of the visibility graph.

Question:

- What kind of visibility graph is obtained from a fractal time series*?

*Objects which have a similar appearance when viewed at different scales.

Visibility Graph

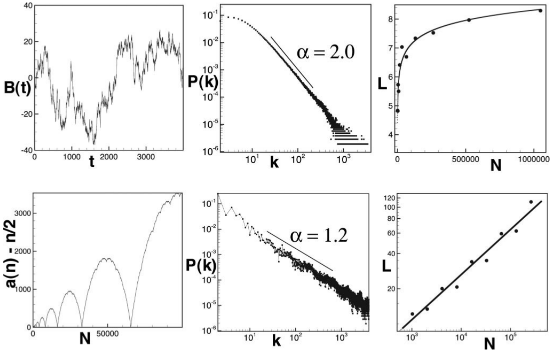


Figura 3: Fractal series. (*Upper*) Brownian motion series $B(t)$. This network shows small-world effect in addition to being scale-free com $L(N) = 1.21 + 0.51\log(N)$. (*Lower*) Conway series. This network is scale-invariant com $L(N) = 0.76N^{0.38}$. Both series have power laws degree distribution: $P(k) \sim k^{-\alpha}$.

Visibility Graph

	Graphs	**Degree Distribution**
Periodic	Regular	Discrete
Random	Random	Exponential
Fractal	Scale-free	Power Law

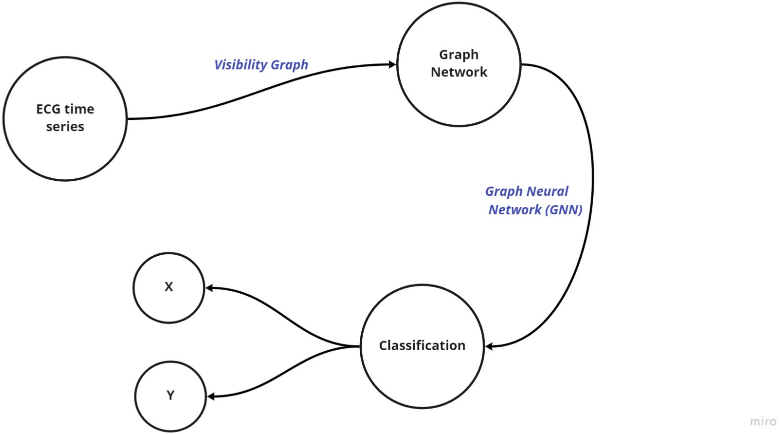
Visibility graph can actually distinguish different types of series.

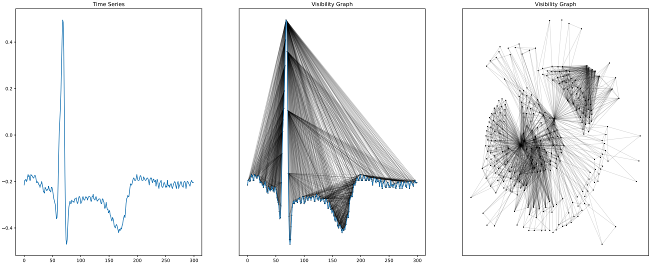
Visibility Graph

Remarks:

- Periodic series (T_1 and T_2) would have the same visibility graph, albeit being quantitatively different $\Rightarrow$ **Weighted Newtorks (slope of the visibility line)**;
- Undirected graphs $\Rightarrow$ **Directed graphs (**k_{in}, k_{out}**)**;
- Investigations in spatial location in chaotic dynamic systems, and human behavior time series;
- Scale-free evidencing small-world (Brownian) $\times$ Scale-free (Conway) - Fractal time series $\Rightarrow$ **Hub repulsion phenomenon**

My Project


ECG time series
Visibility Graph
Graph Network
Graph Neural Network (GNN)
Classification
X
Y
miro


Time Series
0.4
0.2
0.0
−0.2
−0.4
0
50
100
150
200
250
300
Visibility Graph
0
50
100
150
200
250
300
Visibility Graph

"That's all Folks!"