Wavelet Transform Based ECG Denoising Using Adaptive Thresholding

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ABSTRACT
Electrocardiogram (ECG) is a widely employed tool for the analysis of cardiac disorders and clean ECG is often desired for proper treatment of cardiac ailments. In the real scenario, ECG signals are usually corrupted with various noises during acquisition and transmission. As an important branch of wavelet transform, multiresolution has achieved good results in the noise reduction processing in many fields, such as ECG signal, voice signal, image signal and so on. However, multiresolution has strong dependence on the selection of wavelet threshold and wavelet function. In this paper, an adaptive wavelet threshold calculation and selection method is proposed. Based on the heuristic threshold optimization method, the adjustment factor of wavelet decomposition layer number and level influence is incorporated into the method. By dynamically adjusting the threshold calculation function for wavelet coefficients of each layer, more reasonable signal decomposition and noise reduction could be realized. The experimental results show that the proposed algorithm could achieve better performance in reducing the noise of ECG and could meet the needs of clinical application.

CCS Concepts
•Applied computing → Health informatics

Keywords
Wavelet transform; Multiresolution; Threshold Selection; ECG Denoise; S-median

ICBBS '18, June 23–25, 2018, Shenzhen, China
© 2018 Association for Computing Machinery.
ACM ISBN 978-1-4503-6409-6/18/06…$15.00
DOI: https://doi.org/10.1145/3239264.3239272

1. INTRODUCTION
The interpretation based on ECG signal can help doctors to make timely and accurate diagnosis of various cardiovascular disorders. It is important to reduce the non-repairable injury and mortality caused by the sudden cardiovascular disease. Whether it is artificial interpretation or intelligent interpretation based on various machine learning methods, the feature extraction and pattern classification of the waves of P, QRS and T included in the ECG signals should firstly reduce a large number of noise signals contained in the ECG signals, which affect the interpretation accuracy and effect of ECG signal collected from clinical data [1].

How to eliminate the noise interference in the ECG signal more accurately and effectively and improve the signal-to-noise ratio (SNR) of the clinical ECG signal has always been a popular research topic in the field. The commonly used methods for the noise reduction of ECG include bandpass filters [2], weighted mean filters [3], empirical mode decomposition [4], neural network [5], principal component analysis [6], independent component analysis [7], and adaptive dual threshold [8]. These methods have their own characteristics, advantages and limitations in the field of ECG signal filtering and noise reduction. For example, an adaptive filter and neural network based system requires additional reference signals and training stages, so it is not suitable for real-time applications; the statistical models of principal component analysis and independent component analysis are very sensitive to small changes in the signal which limit their long-term using; the dependence of empirical mode decomposition on inherent pattern functions is too strict and unsuitable for practical application and the intrinsic mode function often leads to a sharp decline in the performance of noise reduction.

As a branch of rapid development and application verification in applied mathematics, wavelet transform has been successfully applied to many fields and has achieved good results, such as seismic wave analysis, speech recognition, and image analysis. In 1988, Mallat, S. proposed the concept of multiresolution wavelet transform [9], which greatly promoted the research and application of wavelet. However, the multiresolution has a strong dependence on the selection of wavelet function, the setting of wavelet decomposition level and sensitivity of threshold selection. Especially for the threshold selection, it directly affects the results in the wavelet coefficients calculation process and the denoising effect of ECG signals. The relevant researchers have obtained some research results for the wavelet threshold selection method [8], [10], which could solve the application of different fields.

In this paper, an adaptive wavelet threshold calculation and selection method is proposed to solve the problem of ECG signal noise reduction by multiresolution wavelet transform. The method is based on the heuristic threshold optimization method to integrate the adjustment factor of wavelet decomposition layer number and level influence. By dynamically adjusting the threshold calculation function of wavelet coefficients for each decomposition layer, a more reasonable signal decomposition and noise reduction processing could be realized. Through the numerical experiment the proposed algorithm is tested for the effect of ECG signal noise reduction, and the value of the method in the application of ECG signal noise reduction is analyzed for clinical application.

2. PRINCIPLE OF WAVELET-BASED ECG SIGNALS DENOISING

Electrocardiogram (ECG) signals often contain various types of noise, which affect feature extraction and classification. Wavelet transform can represent the local information of ECG signal in time and frequency domain to adjust the low-frequency and high-frequency sub-band separately through time window and frequency window adjustment, so as to achieve the purpose of ECG signals denoising.

2.1 Noise in ECG Signals

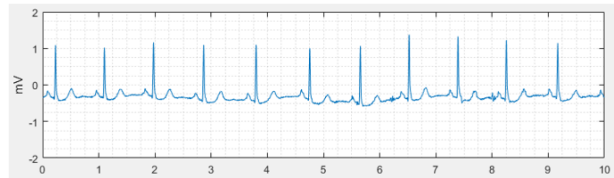
There are usually a large number of different types of noise interference in ECG collected by clinical practice, and these noises need to be removed before extracting and classifying ECG features. This section introduces several kinds of noise in ECG signal and analyzes its influence on the analysis of ECG signal.

(1) Baseline wander. In the process of ECG signal acquisition, the disturbance caused by the respiratory movement of the patient causes the baseline wander, which shows that the ECG signal presents an approximate sinusoidal waveform. The frequency of baseline wander of ECG signals is generally 0.05-2Hz, and the amplitude is about 15%. As shown in Figure 1 (b), the ECG signal with baseline wander is generally expressed like a relatively slow and approximate sinusoidal curve.

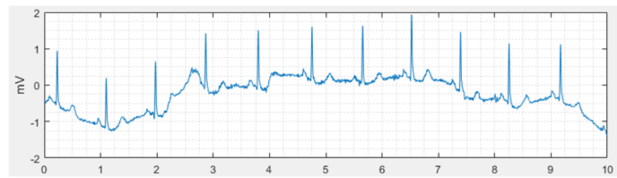
(2) Muscle artifact. The contraction of the muscles of the human body produces electromyographic signals, which will lead to irregular and rapid changes in the ECG signals. As shown in Figure 1 (c), a ECG signal containing muscle artifact is usually presented as a high frequency noise jamming signal with a lot of burrs.

(3) Electrode movement.

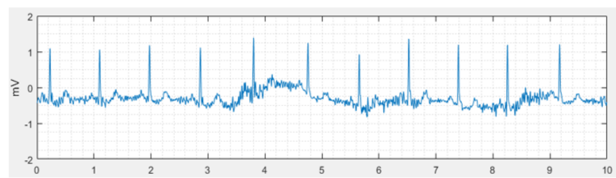
If the electrode attaching to the surface of the human body is moving or even falling off because of the movement of the human body, the waveform of the electrocardiogram signal will have a large irregular change. Even to the extent that the connection between the electrode and the human body is interrupted, the frequency interference of the 50Hz frequency would be clearly seen. Figure 1 (d) shows a ECG signal containing noise caused by electrode movement, and the change of the waveform is large and irregular.



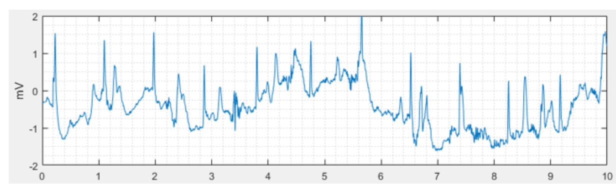
(a) Original ECG signal



(b) ECG signal added to baseline wander noise



(c) ECG signal added to muscle artifact noise



(d) ECG signal added to electrode movement noise

Figure 1. Different types of noise in ECG signals

Figure 1 shows a contrast diagram of ECG signals with different types of noise. The original ECG signals is the record of number 101 which obtained from MIT-BIH Arrhythmia Database [11]. The baseline wander (BW), muscle artifact (MA), and electrode movement (EM) noise are obtained from MIT-BIH Noise Stress Test Database and then are added to the original ECG signals.

2.2 Wavelet Transform

Wavelet transform is a series of scaling and shifting operations on the basis of wavelet function, which constitutes a standard orthogonal basis for $L^2(\mathbb{R})$. The formula of Continuous Wavelet Transform (CWT) is shown as formula (1):

$$\langle f(x),\psi_{a,\tau}\rangle = WT_f(a,\tau) = \frac{1}{\sqrt{a}}\int f(x)\psi^*\left(\frac{x-\tau}{a}\right)dx \qquad (1)$$

in which, $f(x)$ is the original signal function, $\psi(x)$ is wavelet function, $a,(a>0)$ represents scaling factor and τ represents shifting factor.

As CWT has strong correlation, so Discrete Wavelet Transform (DWT) is often used in practical application. Convert the factors of CWT as $a = a_0^m$ and $\tau = n\tau_0 a_0^m$ meanwhile set the values as $a_0 = 2$ and $\tau_0 = 1$, could obtain new wavelet function of DWT with better time-frequency localization as shown in formula (2):

$$\psi_{m,n}(x) = 2^{-m/2}\psi(2^{-m}x - n) \tag{2}$$

The scaling and shifting after the conversion constitutes an orthogonal base of the $L^2(\mathbb{R})$, and DWT uses a set of binary scale factors to extract the orthogonal basis of the signal from the wavelet function.

DWT decomposes the signal iteratively, and each iteration is decomposed from the middle value of input signal frequency. High pass filters (HPF) and low pass filters (LPF) are used for signal filtering respectively. As shown in Figure 2, it is a signal decomposition process of two layers of wavelet transform.

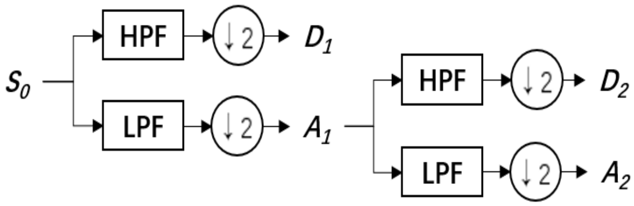


Figure 2. Signal decomposition process based on two layers wavelet transform

The input signal S_0 is filtered into high frequency sub-band signal D_1 through high pass filter (HPF) with down-sampling and low frequency sub-band signal A_1 through low pass filter (LPF) with down-sampling; and then A_1 is filtered into high frequency sub-band signal D_2 and low frequency sub-band signal A_2 through filters with down-sampling. The decompose process of input signal by wavelet transform could be presented as formula (3):

$$S_0 = D_1 + D_2 + A_2 \tag{3}$$

The characteristics of low frequency part of signal could be refined layer by layer through wavelet transform, which is referred as multiresolution wavelet transform, and could play a good effect on the noise reduction and feature extraction of ECG signals.

2.3 Threshold Selection

After multiresolution wavelet decomposition, wavelet coefficients can realize wavelet coefficient amplitude signal which is greater than the noise in a certain extent, and the above theoretical basis by setting the threshold will retain the effective signal subspace based on noise subspace removal, so as to achieve the purpose of reducing noise. The selection of wavelet threshold contains hard threshold and soft threshold method.

The hard threshold method preserves the signal subspace after the wavelet transform which is greater than the setting threshold value thr, and the wavelet coefficient of the noise subspace is set to zero which less than the threshold value. The formula for calculating the wavelet coefficient of hard threshold method is shown as formula (4):

$$\delta_{m,n}^{H}(x) = \begin{cases} 0 & |x| \le thr \\ x & |x| > thr \end{cases} \tag{4}$$

The soft threshold method sets the wavelet coefficient of the signal subspace greater than the set threshold as the result of step function Sgn of the difference between the coefficient and the threshold value, and the wavelet coefficient of the noise subspace is set to zero which less than the threshold value. The formula for calculating the wavelet coefficient of soft threshold method is shown as formula (5):

$$\delta_{m,n}^{S}(x) = \begin{cases} 0 & |x| \le thr \\ Sgn(x)(|x| - thr) & |x| > thr \end{cases} \tag{5}$$

The wavelet coefficients obtained by hard threshold method are not derivable and restricts its practical application in signal reconstruction. The wavelet coefficients obtained by soft threshold method are derivable, so it is widely applied in the field of signal denoising.

It can be seen that the selection of threshold plays a key role in the application of wavelet transform. Researchers have done a lot of works for the selection of threshold, for example, universal threshold method for signal denoising with a large number of Gauss white noise signals[12], heuristic threshold method based on threshold optimization[10], double threshold selection method[8] and so on. The above methods have their own advantages in image, voice and physiological signal denoising and expand the theory and application of wavelet transform.

3. WAVELET-BASED USING ADAPTIVE THRESHOLDING

Aiming at the application demand of remote ECG monitoring based on wireless body sensor network, this paper proposes an adaptive wavelet threshold selection method, which is used to solve the problem of noise reduction in clinical ECG signal containing a lot of real noise. First, the consideration of wavelet function for ECG noise reduction is introduced, and then the selection method of adaptive threshold and noise reduction process is presented.

3.1 ECG Oriented Wavelet Function Selection

The Daubechies (dbN) wavelet transform system is generated by selecting the square root of the minimum phase, and the best energy of the filter is concentrated near its supporting point with asymmetry. Although there are many scholars in the field of ECG signal denoising research choosing dbN wavelet as wavelet function, but it is limited by its symmetry and the effect of ECG signal denoising needs further improvement.

In order to improve the symmetry defect of dbN wavelet transform, the choice of square root needs to be optimized so as to get approximate linear phase and achieve greater symmetry. In the aspect of symmetry, one of the improved wavelet transforms is called "Symlet" wavelet, that is "approximately symmetric compactly supported biorthogonal wavelet", which achieves better spectrum information in signal and image processing. On the other hand, the wavelet function and scaling function forms of Symlet wavelet transform are closer to the waveform of ECG [10]. The wavelet function and scaling function forms of Sym4, Sym6 and Sym8 wavelet transform are shown in Figure 3.

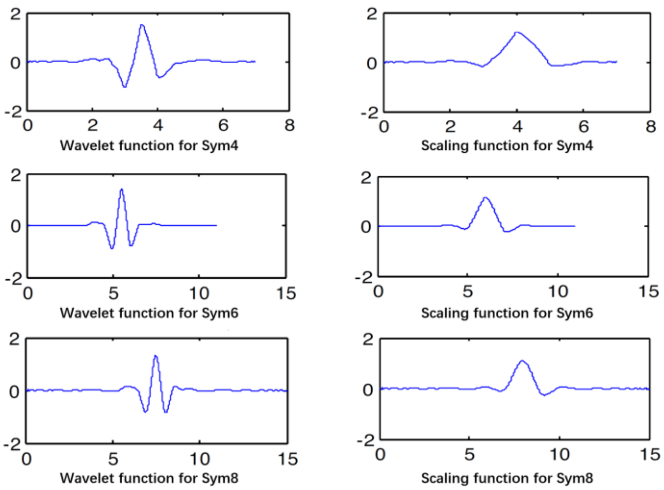


Figure 3. Wavelet function and scaling function forms of Sym4, Sym6 and Sym8

Based on the above considerations, this paper selects the Symlet wavelet transform to denoise the ECG signals.

3.2 Adaptive Threshold

The fixed threshold method often performs well on some signal test samples, while the performance of other test samples drops sharply. Heuristic threshold selection method based on optimization can adjust and select the threshold dynamically in the process of considering the noise characteristics and wavelet decomposition process[10]. In this paper, an adaptive threshold selection method is proposed based on the heuristic threshold method.

The signal denoising based on wavelet transform is divided into high and low frequency sub-bands according to the threshold in each layer of wavelet decomposition. If a globally consistent fixed threshold is adopted, it may lead to the removal of the same frequency band noise signal for different decomposition levels, resulting in the original signal deviating from the original shape. By introducing the adaptive function for threshold to dynamically adjust the threshold value, a more suitable threshold value for different frequency sub-bands could be obtained. A threshold selection method based on adaptive factor is shown as formula (6):

$$thr_{L,k} = \delta_k \sqrt{2\log(n)} \,/\, (S_{L,k} + b) \tag{6}$$

in which, $S_{L,k}$ is sub-band level parameter to reflect the threshold adjustment of different decomposition levels, $S_{L,k} = 2^{(L-k/L)}$, L is the total layer number of wavelet decomposition, k is the number of current decomposition layers; $\delta_k = median\,|x|/0.6745$, and b is the regulating factor.

This paper presents an adaptive threshold calculation method based on the adjustment factor of wavelet decomposition layer number and level influence to dynamically adjust the threshold value, as shown in formula (7):

$$thr_{L,k} = (k/L) \times \delta_k \sqrt{2\log(n)} \,/\, (S_{l,k} + b) \tag{7}$$

In the process of threshold selection, the threshold value is further adjusted by the adjustment factor $\theta = k/L$. With the increase of the number of decomposition layers, the threshold amplitude is gradually reduced and the resolution of the sub-band of the low frequency signal is improved. Because the effective parts of the ECG signal are mostly distributed in the stable low frequency part of the signal, this adaptive threshold selection method could deal with the wavelet decomposition of ECG signal more effectively and improve the noise reduction effect of ECG signal.

4. EXPERIMENT AND RESULTS

In order to test the effectiveness of the proposed method of ECG signal denoising based on adaptive wavelet threshold, this paper tests part of ECG signals in MIT-BIH Arrhythmia Database[11] and compares the performance of the Mean Square Error (*MSE*), the Normalized Mean Square Error (*NMSE*) and the Signal-to-Noise Ratio improvement (*SNRimp*). The experiments were implemented based on MATLAB 9.0.

With the variable $s^0(x)$ representing the collected ECG signal containing noise, $s^n(x)$ representing the recognized noise signal, and $s^d(x)$ representing denoise the ECG after noise reduction, we defined the performance evaluating indicator *MSE*, *NMSE* and *SNRimp*.

The formula for calculating Mean Square Error (*MSE*) is shown as (8):

$$MSE = \frac{1}{N}\sum_{x=0}^{N-1}(s^0(x) - s^d(x))^2 \tag{8}$$

The formula for calculating Normalized Mean Square Error (*NMSE*) is shown as (9):

$$NMSE = \frac{\sum_{x=0}^{N-1}(s^0(x) - s^d(x))^2}{\sum_{x=0}^{N-1}(s^0(x))^2} \tag{9}$$

The formula for calculating Signal-to-Noise Ratio improved (*SNRimp*) is shown as (10):

$$SNR_{imp} = 10\log\left(\frac{\sum_{x=0}^{N-1}(s^n(x) - s^0(x))^2}{\sum_{x=0}^{N-1}(s^d(x) - s^0(x))^2}\right) \tag{10}$$

4.1 Performance of Different Wavelet Functions

As mentioned above, this paper chooses Symlet wavelet as wavelet function to denoise ECG signals. In order to compare and analyze the effect of each wavelet in the Symlet wavelet system on the de-noising effect of ECG, a total of ten wavelet functions of Sym1-Sym10 were compared and analyzed, with the number of wavelet decomposition as four. The contrast analysis of different wavelet bases in the Symlet wavelet system is carried out to test SNR improvement of different types of noise, such as baseline wander (BW), muscle artifact (MA), and electrode movement (EM) noise in ECG signals. The performance of different wavelet functions is shown in Table 1.

Table 1. SNR_{imp} **using different Symlet wavelet functions**

Wavelet function	SNR_{imp}		
	BW added	**MA added**	**EM added**
Sym1	0.006	1.843	0.167
Sym2	0.013	1.924	**0.271**
Sym3	0.015	2.462	0.209
Sym4	**0.024**	**2.945**	0.231
Sym5	0.021	2.762	0.253
Sym6	0.016	2.229	0.257
Sym7	0.018	2.674	0.216
Sym8	0.013	2.343	0.167
Sym9	0.009	1.975	0.135
Sym10	0.005	2.068	0.114

From the performance of SNR improvement using Symlet wavelet functions as shown in Table 1, we could observe that different Symlet wavelet functions have different noise reduction effect. For instance, Syml4 has better performance on the test data with baseline wander noise and muscle artifact noise, while Sysm2 performs better on the test ECG signal with electrode movement, and maybe Sym4 could denoise the ECG signal containing noise which the change of the waveform larger and more irregular. Based on the experiment results for SNR improvement with different Symlet wavelet functions above, we choose Sym4 as our selected wavelet function for ECG signal noise reduction in this work.

4.2 Performance of Different Decomposition Levels

Another important factor for the structure design of DWT is the decomposition level. To compare the noise reduction effect for ECG signals with different number of decomposition levels, we decompose wavelet from 3 levels to 10 levels with Sym4 wavelet function to denoise the ECG signals with baseline wander, muscle artifact and electrode movement noise. The SNR improvement with different decomposition levels is shown as Table 2.

Table 2. SNR_{imp} **using different decomposition levels**

Decomposition level	SNR_{imp}		
	BW added	**MA added**	**EM added**
level 3	0.003	1.206	0.087
level 4	0.006	1.856	0.654
level 5	0.011	2.351	0.124
level 6	0.019	**3.126**	0.184
level 7	**0.023**	2.987	0.234
level 8	0.021	3.024	**0.264**
level 9	0.008	2.862	0.162
level 10	0.006	1.684	0.036

From the comparison of the influence of wavelet decomposition level on SNR improvement, it can be seen that the denoising result is better between level 6-8. If the decomposition level is too small or too many, the noise reduction effect is often not good. The analysis of the results can be considered as follows: if the number of wavelet decomposition layers is less, the signal to noise ratio is not much improved and the signal noise reduction effect is not obvious, and the enhancement effect of signal resolution may not be met; meanwhile, if the number of wavelet decomposition layers is more, the wavelet coefficients are often processed on the basis of soft threshold based on the wavelet coefficients and the information of the ECG signal is often lost, and the signal to noise ratio of the noise reduction is reduced. At the same time, the practical value of clinical application will be limited due to the increase of the calculation burden.

Based on the above experimental results and analysis, we choose Sym4 wavelet function and 6 decomposition levels for the process of ECG denoising.

4.3 Performance Using Different Threshold Selection Methods

In this section, we compare and analyze several commonly used wavelet threshold selection methods with the adaptive threshold selection method proposed in this paper. Three types of mixed noise of baseline wander (BW), muscle artifact (MA), and electrode movement (EM) are added to the MIT-BIH Arrhythmia Database record number 109. Different threshold selection methods were applied to denoise the signal and compared the Mean Square Error (*MSE*), the Normalized Mean Square Error (*NMSE*) and the Signal-to-Noise Ratio improved (*SNRimp*). The results of the experiment are shown in Table 3.

Table 3. Noise reduction effect using different threshold selection methods

Threshold selection	Denoising Performance		
	MSE	*NMSE*	SNR_{imp}
SURE[13]	0.004	0.073	0.005
universal[12]	0.026	0.196	0.153
minimax[14]	0.014	0.087	0.067
S-median[15]	**0.052**	0.286	0.197
proposed	0.049	**0.325**	**0.228**

From the experimental results of wavelet threshold selection with the proposed method, we can see that our method has better denoising performance in the Normalized Mean Square Error (*NMSE*) and the Signal-to-Noise Ratio improvement (*SNRimp*) and is slightly inferior to the S-median method in the aspect of Mean Square Error (*MSE*) of noise reduction. In general, the method based on optimal selection and adaptive threshold selection is generally better than the fixed threshold reduction method in denoising effect, but the adjustment of threshold selection calculation method may affect its performance on different test signals. Therefore, it is more critical to design a more suitable threshold calculation method for the design of ECG signals with different features.

5. CONCLUSIONS

The effect of ECG signal denoising plays an important role in the feature extraction and pattern classification of ECG signal, and it has a basic role in the application of ECG detection equipment especially for the wearable monitoring and application of telemedicine. In this paper, an adaptive wavelet threshold calculation and selection method is proposed for the denoising problem of ECG signal based on multiresolution wavelet function. The method is integrated with the adjustment factor of wavelet decomposition layer number and level influence on the basis of optimal threshold selection method. By dynamically adjusting the threshold value of wavelet coefficients for each layer, a more reasonable signal decomposition and noise reduction processing could be realized. The experimental results show that the proposed algorithm has a good performance in the noise reduction effect of ECG signals.

The signal analysis method based on wavelet decomposition has strong dependence on the selection of wavelet function, hierarchical level and threshold value, and the design of wavelet transform parameters directly affects the effect of signal noise reduction for the practical application scene. On the other hand, as a kind of individual difference, ECG signal has the characteristic of physiological signal data with sudden change at different time periods, and it is necessary to study the differences between individuals and different periods in the individual from the point of view of big data, and to explore the clinical assistant diagnosis method based on multiple physiological data in combination with multimodal data fusion technology.

6. ACKNOWLEDGMENTS

Our thanks to the support of National Import Research Priorities Program of China(2016YFB0801004) and Jiangsu Province Policy Guidance Program (Research Cooperation)-Prospective Joint Research Project (BY2016049-01).

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