Knowledge Representation

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Knowledge Representation

- **Knowledge base (KB):**

  - Knowledge that describe facts about the world in some formal (representational) language

  - Domain specific

- **Inference engine:**

  – A set of procedures that use the representational language to infer new facts from known ones or answer a variety of KB queries. Inferences typically require search.

  -Domain independent

Logics

A logic is a triple (L, S, R) where

- L, the logic's language, is a class of sentences described by a formal grammar
- S, the logic's semantics is a formal specification of how to assign meaning in the "real world" to the elements of L
- R, the logic's inference system, is a set of formal derivation rules over L
- There are several logics: propositional, first-order, higher-order, modal, temporal, intuitionistic, linear, equational, non-monotonic, fuzzy, . . .

We will concentrate on propositional logic and first-order logic

Knowledge Representation

- **Objective:** express the knowledge about the world in a computer-tractable form
- **Knowledge representation languages (KRLs) Key aspects:**

  - **Syntax:** describes how sentences in KRL are formed in the language

  - **Semantics:** describes the meaning of sentences, what is it the sentence refers to in the real world

  - **Computational aspect:** describes how sentences and objects in KRL are manipulated in concordance with semantic conventions Many KB systems rely on some variant of logic

Knowledge Representation

- Propositional Logic
- Predicate Logic
- Semantic Networks
- Frames
- Fuzzy Logic

Propositional Logic

- A formal method of reasoning, which represents knowledge, allowing automated inference and problem solving.
- Concepts are translated into symbolic representations which closely approximate the meaning. These symbolic structures can then be manipulated in programs to deduce various facts, to carry out a form of automated reasoning.
- Propositional logic is the simplest.
  - Symbols represent whole propositions (facts): P, Q, R, S, etc..
  - These are joined by logical connectives (and, or, implication) e.g., P   Q; Q   R
  - Given some statements in the logic we can deduce new facts (e.g., from above deduce R)

Propositional Logic: Examples

- My car is painted red.
- Snow is white.
- People live on the moon.
- **Logical connectives:**
- It is raining and the wind is blowing.
- I shall go there or ask kamal to visit him.
- If you study heard you will be successful.
- The sum of 20 and 30 is not 100.
- The car belongs to the VC is painted silver.

Propositional Logic

- Propositions are some elementary atomic sentences. Propositions may be either true or false. In propositional logic, a world is represented as knowledge using a list of facts.
- Formally propositional logic P:
- – Is defined by Syntax + interpretation + semantics of P
- **Syntax of PL :**
- symbol -> P | Q | R | S | ...
- atomic sentence -> TRUE | FALSE
- sentence -> atomic sentence | complex sentence
- complex sentences -> ~ sentence | (sentence ^ sentence) | (sentence v sentence) |(sentence → sentence) |
- (sentence ↔ sentence)
- Precedence relation operators: ~,^,v,→, ↔.

Syntax of Propositional Logic

- The syntax of propositional logic defines the allowable sentences for the knowledge representation. There are two types of Propositions:

1. **Atomic Propositions**
2. **Compound propositions**

- **Atomic Proposition:** Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.

- Example:

a) 2+2 is 4, it is an atomic proposition as it is a **true** fact.

b) "The Sun is cold" is also a proposition as it is a **false** fact.

- **Compound proposition:** Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives.

- Example:

a) "It is raining today, and street is wet."

b) "Ankit is a doctor, and his clinic is in Dhaka."

Propositional Logic: Semantics

- Propositions
  - Sentences and truth values
  - Propositional connectives and their truth tables
    - Negation: $\sim P$
    - Conjunction: $P \wedge Q$
    - Disjunction: $P \vee Q$ (inclusive or)
    - Implication: $P \rightarrow Q$
    - Equivalence: $P \leftrightarrow Q$
  - Other propositional connectives
    - $P \oplus Q$ (exclusive or), $P \downarrow Q$ (nor), $P \uparrow Q$ (nand),..

Semantics of Propositional Logic

Logical connectives are used to connect two simpler propositions or representing a sentence logically.

1. **Negation:** A sentence such as $\neg$ P is called negation of P. A literal can be either Positive literal or negative literal.

2. **Conjunction:** A sentence which has $\wedge$ connective such as, **P $\wedge$ Q** is called a conjunction.
   Example: Rohan is intelligent and hardworking. It can be written as,
   P= Rohan is intelligent,
   Q= Rohan is hardworking. → P $\wedge$ Q.

3. **Disjunction:** A sentence which has $\vee$ connective, such as **P $\vee$ Q**. is called disjunction, where P and Q are the propositions.
   Example: "Ritika is a doctor or Engineer",
   Here P= Ritika is Doctor. Q= Ritika is Doctor, so we can write it as **P $\vee$ Q**.

Semantics of Propositional Logic

4. Implication: A sentence such as $P \rightarrow Q$, is called an implication. Implications are also known as if-then rules. It can be represented as

If it is raining, then the street is wet.

Let P= It is raining, and Q= Street is wet, so it is represented as $P \rightarrow Q$

5. Biconditional: A sentence such as **$P \Leftrightarrow Q$ is a Biconditional sentence, example**

If I am breathing, then I am alive

P= I am breathing, Q= I am alive, it can be represented as $P \Leftrightarrow Q$.

Semantic Rules for Statements

Rule No.	True Statements	False Statements
1	T	F
2	~ f	~ t
3	t & t′	f & a
4	t or a	a & f
5	a or t	F or f′
6	a □ t	t □ f
7	f □ a	t □□ f
8	t □□ t′	f □□ t
9	f □□ f ′	

Properties of Statements

- **Satisfiable**: A statement is satisfiable if there is some interpretation for which it is true.
- **Contradiction**: A statement is said to be contradictory (unsatisfiable) if there is no interpretation for which it is true.
- **Valid**: A statement is valid if it is true for every interpretations. Valid statements are also called tautologies.
- **Equivalence**: Two sentences are equivalent if they have the same truth value under every interpretation.

Semantics & Interpretations

P	Q	$P \wedge Q$	$P \vee Q$	$\sim P$	$P \rightarrow Q$	$P \leftrightarrow Q$
f	f	f	f	t	t	t
f	t	f	t	t	t	f
t	f	f	t	f	f	f
t	t	t	t	f	t	t

Meaning of Statements

What would be the meaning of the following statement, if some interpretation imply true to *P*, false to *Q* and false to *R* ?:

$$((P \,\&\, \sim Q) \rightarrow R) \vee Q$$

Assignments:

1. Find the meaning of the statement:

$(\sim P \vee Q) \,\&\, R \rightarrow S \vee (\sim R \,\&\, Q)$

for each of the interpretations given below:

I_1: P is true, Q is true, R is false, S is true.

I_2: P is true, Q is false, R is true, S is true.

2. Determine whether each of the following sentence is
(a) satisfiable (b) contradictory, or (c) valid

S_1: $(P \,\&\, Q) \vee \sim (P \,\&\, Q)$

S_2: $(P \vee Q) \rightarrow (P \,\&\, Q)$

S_3: $(P \,\&\, Q) \rightarrow R \vee \sim Q$

S_4: $(P \vee Q) \,\&\, (P \vee \sim Q) \vee P$

S_5: $P \rightarrow Q \rightarrow \sim P$

S_6: $P \vee Q \,\&\, \sim P \vee \sim Q \,\&\, P$

Meaning of Statements 1

Statement: If the earth moves round the sun or the sun moves round the earth, then Copernicus might be a mathematician but wasn't an astronomer.

Let,

P = the earth moves round the sun = T

Q= the sun moves round the earth = F

R= Copernicus might be a mathematician = T

S= Copernicus was an astronomer = T

$(P \lor Q) \rightarrow (R \land {\sim}S)$

$= (T \lor F) \rightarrow (T \land {\sim}T)$

$= T \rightarrow F$

$= {\sim}T \lor F$

$= F$

So, the meaning of the statement is FALSE.

Meaning of Statements 2

Exercise:

Inspite of having French nationality, B. Russel was a critic of imperilism, then either he was not a bachelor or he was a universal lover.

Rules of Inference

- **Modus Ponens**

$$\frac{P \rightarrow Q,\ P}{\therefore Q} \qquad \{((P \rightarrow Q) \wedge P) \rightarrow Q\}$$

- **Modus Tollens**

$$\frac{P \rightarrow Q,\ \sim Q}{\therefore \sim P} \qquad \{((P \rightarrow Q) \wedge \sim Q) \rightarrow \sim P\}$$

- **Hypothetical Syllogism (H. S.)**

$$(P \rightarrow Q)\ \&\ (Q \rightarrow R)$$

$$\therefore P \rightarrow R$$

- **Disjunctive Syllogism (D. S.)**

$$\frac{(P \vee Q)\ \ \sim P}{\therefore Q}$$

Modus Ponens

- **Modus Ponens**

$$\frac{P \rightarrow Q,\ P}{\therefore Q} \qquad \{((P \rightarrow Q) \wedge P) \rightarrow Q\}$$

- **Example:**

 "If you have a current password, then you can log on to the network"
 "You have a current password“
Therefore:
"You can log on to the network"
This has the form:

$P \rightarrow Q$

P

$\therefore Q$

Modus Tollens

- **Modus Tollens**

$$\frac{P \rightarrow Q, \sim Q}{\therefore \sim P} \qquad \{((P \rightarrow Q) \wedge \sim Q) \rightarrow \sim P\}$$

- **Example:**

 You can't log into the network
 If you have a current password, then you can log into the network
Therefore
You don't have a current password.
This is an argument of the form:
$\neg Q$
$P \rightarrow Q$
$\therefore \neg$
P

Hypothetical Syllogism

- **Hypothetical Syllogism (H. S.)**

  $$\frac{(P \to Q)\ \&\ (Q \to R)}{\therefore P \to R}$$

$\{((P \to Q) \wedge (Q \to R)) \to (P \to R)\}$

- If I do not wake up, then I cannot go to work.
- If I cannot go to work, then I will not get paid.

Therefore,

If I do not wake up, then I will not get paid.

This is an argument of the form:

$P \to Q$

$Q \to R$

$\therefore$

$P \to R$

Disjunctive Syllogism

- **Disjunctive Syllogism (D. S.)**

$$\frac{(P \lor Q) \quad \sim P}{\therefore Q}$$

$$\{((P \lor Q) \wedge \sim P) \rightarrow Q\}$$

Example:

- The breach is a safety violation, or it is not subject to fines.
- The breach is not a safety violation.

Therefore,
it is not subject to fines.

This is an argument of the form:

$P \lor Q$

$\sim P$

$\therefore Q$

Biconditional Statement

- **Conditional Statement:**

  $P \rightarrow Q = \sim P \ \vee \ Q$

- **Biconditional Statement:**

  $P \leftrightarrow Q = (P \rightarrow Q) \ \& \ (Q \rightarrow P)$

- **Truth Table:**

P	Q	$P \rightarrow Q = \sim P \ \vee \ Q$	$Q \rightarrow P = \sim Q \ \vee \ P$	$P \leftrightarrow Q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Logical Equivalence

Equivalence	*Name*
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$p \vee (q \vee r) \equiv (p \vee q) \vee r$ $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws

Example 1

- **Prove that**

$$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q \equiv \neg(p \vee q).$$

We proceed as follows

$$\begin{aligned}
\neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{De Morgan's law} \\
&\equiv \neg p \wedge (\neg(\neg p) \vee \neg q) && \text{De Morgan's law} \\
&\equiv \neg p \wedge (p \vee \neg q) && \text{double negation law} \\
&\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) && \text{distributive law} \\
&\equiv F \vee (\neg p \wedge \neg q) && \text{since } \neg p \wedge p \equiv F \\
&\equiv (\neg p \wedge \neg q) \vee F && \text{commutative law} \\
&\equiv (\neg p \wedge \neg q) && \text{identity law} \\
&\equiv \neg(p \vee q) && \text{De Morgan's law.}
\end{aligned}$$

The above logical equivalence is proved.

Example 2

- You have given the following knowledge:

  If it is hot and humid, then it is raining

  If it is humid, then it is hot

  It is humid

Is it possible to infer “Is it raining”?

Solution:

- **Step 1: Defining Proposition:**

We can proceed as follows:

It is humid = P

It is raining=R

It is hot=H

- **Step 2: Defining Propositional Logic Expression**

Now we can represent the following knowledge as:

1. $(H \ \& \ P) \rightarrow R$
2. $P \rightarrow H$
3. P

Example 2 cont.…

- **Propositional Logic Expression**

1. $(H \& P) \rightarrow R$
2. $P \rightarrow H$
3. P

- **Reasoning using inference rule and Algebra:**

We can apply Modus Ponens in equation 2 and 3

$$\begin{array}{l} P \rightarrow H \\ \underline{P \qquad\qquad} \\ H \qquad\qquad \ldots\ldots\ldots(4) \end{array}$$

Now from equation 1 we can write,

$(H \& P) \rightarrow R = \sim (H \& P) \vee R$

$= (\sim H \vee \sim P) \vee R$ [De Morgan's Law]

$= (\sim T \vee \sim T) \vee R$ [using 4 and 3]

$= F \vee R$ [Idempotent]

$= R$ [using identity law]

= It is raining [Proved]

Exercise 1

- **You have given the following proposition**

– It is raining, it is snowing or it is dry.
– It is warm.
– It is not raining raining.
– It is not snowing.
– If the weather is nice, then it is good to walk.
– If the weather is dry and warm, the weather is nice.

Prove by resolution “It is good to walk.”

Drawbacks of PL

- Propositional logic isn't powerful enough as a general knowledge representation language.
- Impossible to make general statements. E.g., "all students sit exams" or "if any student sits an exam they either pass or fail".
- So we need predicate logic..

Thank you