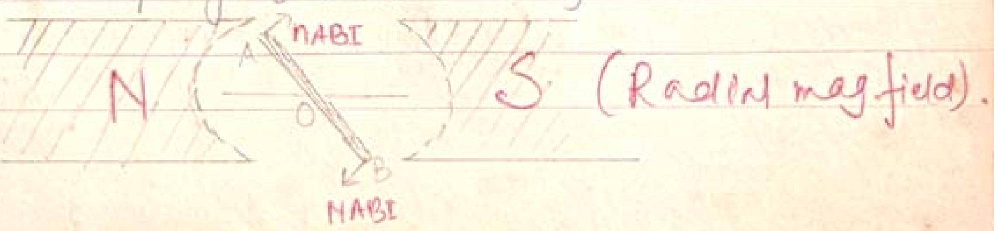
Ballistic Galvanometer (Suspended coil type).

Principle: A ballistic galvanometer is used for the measurement of charge. Here the charge to be measured must be passed through the coil, before the coil starts moving. This requirement can be fulfilled by choosing a suspension having a large time period of oscillation (T = 6 to 10sec). As the charge flows through the coil, it gives rise to a current due to which coil experiences a torque; which acts for a very very short interval. Hence the product of the torque and the time interval gives the impulse of the torque due to which the coil gets a jerk and starts rotating. When the initial K.E of the coil received from the impulse is completely used up in doing work, in rotating the coil against the restoring couple; the coil stops & due to the restoring couple it starts coming back to its original position. Thus the coil oscillates in the magnetic field. The deflection in the 1st throw (max^{m}) is noted from which charge can be calculated.

Construction: In the construction of ballistic galvanometer two requirements are to be fulfilled
(i) The time period of the suspension should be large between (6sec to 10sec) &
(ii) the damping should be very low.



A circular or rectangular coil of fine insulated copper or aluminium wire of about 10 to 15 turns is suspended from a torsion head T, by means of a suspension wire of quartz fibre so that torsional couple per unit twist 'C' is small. The M.I of the coil should be moderately large, so that time period of oscillation of the coil is between 6 to 10 sec. The coil is suspended in a radial field between the concave pole pieces of a strong magnet. As the coil rotates in the magnetic field an emf is induced across the coil which according to Lenz's law opposes the motion of the coil and this is known as electromagnetic damping. In order to minimize electromagnetic damping the coil should be wound on a frame of bamboo or wood. The whole suspension is enclosed in a metal case, provided with glass faces and the instrument is supported on levelling screws.

Theory: n = number of turns in the coil.
A = Area of the plane of the coil.
B = The induction of the magnetic field in which the coil is suspended.
Q = Total charge passed through the coil.

Let dQ be the charge flowing through the coil in time 'dt' measured at an instant of time t, $0 \leq t \leq$ where t_1 is the time taken by the entire charge to flow.

Let i = current flowing through the coil at that instant of time t.

$$i = \frac{dQ}{dt} \quad \text{or} \quad dQ = i\,dt \quad \text{— (1)}$$

The torque experienced by the coil at that instant of time 't' $= nABi$

The impulse of this torque in time dt (during which current i remains constant) $= nABi\,dt$
$= nAB\,dQ$

Total impulse of the torque $= \int_0^Q nAB\,dQ = nABQ$ — (2)

But we know that the impulse of the moment of force or the moment of the impulse is equal to the change in angular momentum.

I = M.I of the coil about the axis of suspension.
Let ω be the angular velocity with which the coil starts rotating, receiving the moment of the impulse

∴ Change in angular momentum $= I\omega - 0 = I\omega$. — (3)

Equating (2) and (3) :- $nABQ = I\omega$ — (4).

<u>To find ω:</u> Let ϕ be the deflection of the coil intermediate between 0 & θ, where θ = maximum deflection of the coil.

C = Torsional couple per unit twist of the suspension wire

∴ Restoring couple $= C\phi$

To rotate the coil further by an angle '$d\phi$' work done $dW = C\phi\, d\phi$

In order to rotate the coil by an angle θ, total work done $W = \int dW = \int_0^\theta C\phi\, d\phi = \frac{1}{2} C\theta^2$ — (5)

This work is done at the cost of K·E with which the coil started moving

i.e. $K \cdot E = \frac{1}{2} I\omega^2$ is converted into Strain P.E.

$$\frac{1}{2} I\omega^2 = \frac{1}{2} C\theta^2 \quad \text{or} \quad \omega = \sqrt{\frac{C}{I}}\,\theta \quad \text{— (6)}$$

putting (6) in (4) : $nABQ = I\sqrt{\frac{C}{I}} \cdot \theta$ — (7)

If T is the time period of oscillation of the coil

$$T = 2\pi\sqrt{\frac{I}{C}} \quad \therefore \sqrt{\frac{C}{I}} = \frac{2\pi}{T} \quad \text{— (8)}$$

putting (8) in (7) :-

$$Q = \frac{2\pi I}{T \cdot nAB} \cdot \theta$$

Eqn (7) $nABQ = \sqrt{IC} \cdot \theta$ — (7)

$$T = 2\pi\sqrt{\frac{I}{C}} \quad \text{or} \quad \sqrt{I} = \frac{T}{2\pi}\sqrt{C} \quad \text{— (8)}$$

putting eqn (8) in (7) :-

$$\boxed{Q = \frac{TC}{2\pi \cdot nAB} \cdot \theta}$$

putting $\frac{nAB}{C} = G$ = constant of the galvanometer

$$Q = \frac{T}{2\pi G} \cdot \theta \quad \text{— (9)}$$

Let θ_D be the steady deflection of the coil when a steady current i_d is passed through the coil.

Following the theory of dead-beat galvanometer:

$$nAB\, i_D = C\theta_D \quad \text{or} \quad \frac{C}{nAB} = \frac{i_D}{\theta_D} \quad \text{— (10)}$$

$$\therefore \boxed{Q = \frac{T}{2\pi} \cdot \frac{i_D\, \theta}{\theta_D}} \quad \text{— (11)}$$

Using eq^n (11) the charge can be calculated.

Effect of damping on Ballistic galvanometer:

(1). $I \cdot \frac{d^2\theta}{dt^2}$ = the retarding couple due to the moment of inertia I of the coil.

(2). $a\frac{d\theta}{dt}$ = the retarding couple due to damping which is assumed to be proportional to the angular velocity of the coil.

(3). $C\theta$ = the restoring couple due to the suspension fibre.

(4). $nABi = Gi$, the displacement couple due to current i

(5). Opposing e.m.f $e = -L\frac{di}{dt}$, where L is the inductance of the galvanometer and its circuit.

(6). The electromotive damping due to this emf $= nAB\frac{d\theta}{dt}$

$$= G \cdot \frac{d\theta}{dt}.$$

Hence

$$I\frac{d^2\theta}{dt^2} = -C\theta - a\frac{d\theta}{dt} - G\frac{d\theta}{dt}$$

For ballistic use, no steady current flows & hence $i=0$ & $L\frac{di}{dt}$ is neglected

$$\frac{d^2\theta}{dt^2} + \frac{(a+G)}{I}\frac{d\theta}{dt} + \frac{C}{I}\theta = 0 \quad \text{— (11)}$$

putting $\frac{a+G}{I} = 2k$; $\frac{C}{I} = \mu^2$ — (12)

putting eqⁿ (12) in (11) :-

$$\frac{d^2\theta}{dt^2} + 2k\frac{d\theta}{dt} + \mu^2\theta = 0 \quad \text{— (13)}$$

Eqⁿ (13) is second order differential eqⁿ and can be solved by 'D' operator method :

put $\frac{d}{dt} = D$; $\frac{d^2}{dt^2} = D^2$

$$\therefore [D^2(\theta) + 2k\,D(\theta) + \mu^2(\theta)] = 0$$

or $[D^2 + 2kD + \mu^2](\theta) = 0$ — (14)

The bracketed term is quadratic and hence has two roots. Let m_1 and m_2 be the roots of the equation

$$\therefore m_1 = -k + \sqrt{k^2-\mu^2} = -k + m$$

$$m_2 = -k - m \quad \text{Where } m = \sqrt{k^2-\mu^2}$$

$$m_1 + m_2 = -2k \; ; \; m_1 m_2 = \mu^2 \quad \text{— (15)}$$

putting eqⁿ (15) in (14) :-

$$\left[D^2 - (m_1+m_2) D + m_1 m_2 \right] (\theta) = 0$$

$$\left[D (D-m_1) - m_2 (D-m_1) \right] (\theta) = 0$$

$$(D-m_1)(D-m_2)(\theta) = 0$$

Either

$$(D-m_1)\,\theta = 0 \quad \text{or} \quad (D-m_2)\,\theta = 0$$

or $\frac{d\theta}{dt} = m_1\theta$ or $\frac{d\theta}{\theta} = m_1 dt$ Integrating both sides $\log\theta = m_1 t + \log A_1$ or $\theta = A_1 e^{m_1 t}$

Where A_1 is unknown constant of Integration. Similarly proceeding with the other solution $(D-m_2)(\theta) = 0$; $\theta = A_2 e^{m_2 t}$ where A_2 is another const. of Integⁿ

∴ General Solution of eqⁿ (13) :-

$$\theta = A_1 e^{m_1 t} + A_2 e^{m_2 t} \quad \text{—} (15)$$

Spl. Cases: (I): Let $K^2 > \mu^2$ i.e. $\left(\frac{a+G}{2I}\right)^2 > \frac{C}{I}$

$m = \sqrt{K^2-\mu^2} < K$ ∴ $m_1 = -K+m = (-)\text{ve}$.

$m_2 = -K-m = (-)\text{ve}$.

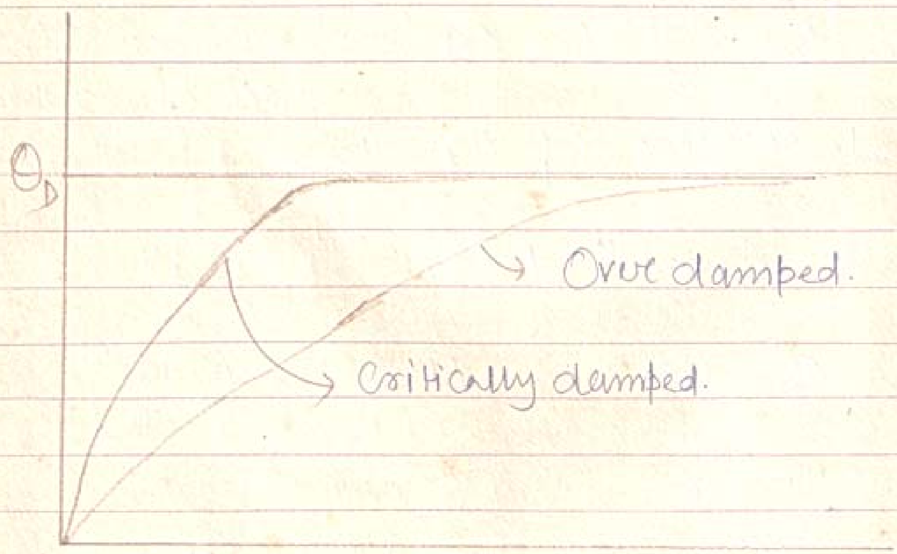
In equation (15) m_1 and m_2 both being (-)ve, θ decreases very rapidly in an exponential fashion and is said to be heavily damped. The motion is said to be non-oscillatory and the galvanometer is said to be dead beat.

Case II Let $k^2 = \mu^2 \left(\frac{a + m/R}{2I}\right)^2 = \frac{C}{I}$

$m = \sqrt{K^2 - \mu^2} = 0 \quad \therefore m_1 = -K = m_2.$

Eqn (15) becomes $\theta = e^{-Kt}(A_1 + A_2)$

or $\theta = Be^{-Kt}$ In this case the deflection steadily decreases to zero in an exponential fashion with lapse of time and the galvanometer is said to be critically damped. This means that the deflection of coil, goes to its final value without any swing or hesitation i.e it deflects to its final value in minm. time.



The resistance of the galvanometer circuit which satisfies the condition $K^2 = \mu^2$ for critical damping is known as critical resistance (m = coeff. of self induction).

$$\frac{\left(a + \frac{m}{R}\right)^2}{4I^2} = \frac{C}{I} \quad \text{or} \quad a + \frac{m}{R} = 2\sqrt{IC}$$

or $\frac{m}{R} = 2\sqrt{IC} - a$ or $R_c = \left[\frac{m}{2\sqrt{IC} - a}\right]$

Case III: Let $k^2 < \mu^2$ $\frac{(a + m/R)^2}{4I^2} < \frac{C}{I}$

$\therefore m = \sqrt{k^2 - \mu^2} = \sqrt{-1(\mu^2 - k^2)} = j\gamma$, $j = \sqrt{-1}$; $\gamma = \sqrt{\mu^2 - k^2}$ = real

Eq^n (15) becomes:

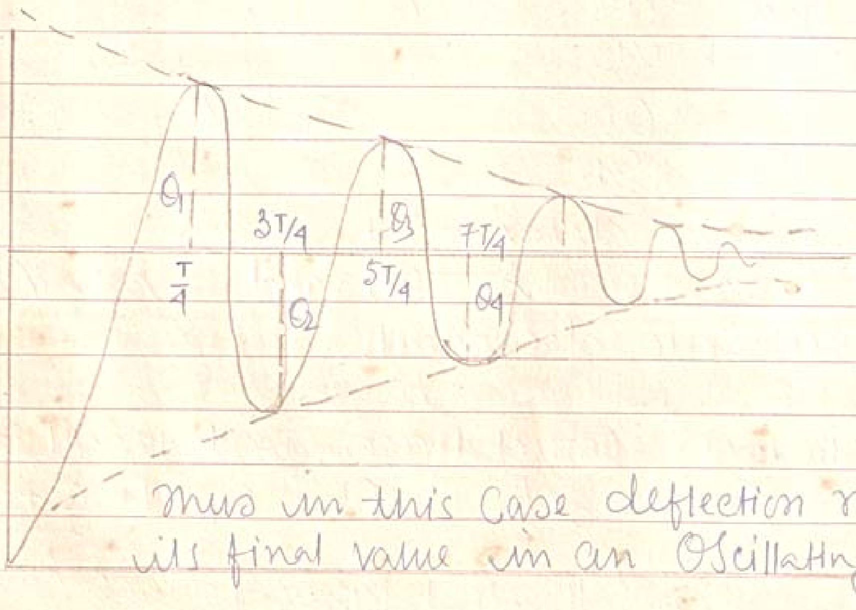
$$\theta = e^{-kt}\left[A_1 e^{mt} + A_2 e^{-mt}\right]$$

$$\text{or } \theta = e^{-kt}\left[A_1 e^{j\gamma t} + A_2 e^{-j\gamma t}\right]$$

$$\text{or } \theta = e^{-kt}\left[(A_1 + A_2)\cos\gamma t + j(A_1 - A_2)\sin\gamma t\right]$$

put $\left.\begin{aligned} A_1 + A_2 &= B\sin\delta \\ A_1 - A_2 &= B\cos\delta \end{aligned}\right\}$

$$\therefore \theta = Be^{-kt}\sin(\gamma t + \delta) \quad \text{—} (16).$$



Thus in this case deflection reaches to its final value in an oscillating fashion.

The amplitude of Oscillation Be^{-Rt} is not Constant but decreases with increase in time in an Oscillating fashion.

Thus for a galvanometer to be ballistic $R^2 < \mu^2$

i.e $\frac{(a+m/R)^2}{4I} < c$ — (17) | $9<10$, $4<5$

Equation (17) implies that (i) I should be large (ii) C should be small. (iii) R should be larger.

Let $\theta_1, \theta_2, \theta_3$ ------------ be the Successive amplitudes at instants $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$ respectively.

$\therefore \theta_1 = Be^{-R\frac{T}{4}}$; $\theta_2 = Be^{-R3T/4}$; $\theta_3 = Be^{-R \cdot 5T/4}$

$\therefore \frac{\theta_1}{\theta_2} = \frac{Be^{-RT/4}}{Be^{-R3T/4}} = e^{RT/2}$

$\frac{\theta_2}{\theta_3} = \frac{Be^{-R3T/4}}{Be^{-R5T/4}} = e^{RT/2}$

$\therefore \frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \text{----} = e^{RT/2} = \text{Const} = e^{\lambda/2} = d$ (say)

d is Known as decrement.

$\log_e d = \frac{\lambda}{2}$ $\therefore \lambda = 2\log_e d$; λ is Known as logarithmic decrement.

Let θ_c be the 1st correct throw; had there been no damping $\frac{\theta_c}{\theta_1} = e^{\lambda/2}$ or $\theta_c = \theta_1\left[1 + \frac{\lambda}{2} + \frac{\lambda^2}{4 \cdot 2!} + \cdots\right]$

λ being very small its higher powers can be neglected. $\theta_c = \theta_1[1 + \lambda/2]$ or $\theta_c =$