## Final Problems 4

Due Wednesday Nov 6.

1. For a string $x \in \{a,b\}^*$, let $p(x)$ be the shortest string $y$ such that $xy$ is a palindrome ($xy = \mathrm{rev}(xy)$). For example:

   - $p(ababbab) = baba$ since $ababbab \cdot baba$ is a palindrome but $ababbab \cdot y$ is not a palindrome for any $|y| \leq 3$.
   - $p(abbabba) = \epsilon$ since $abbabba$ is already a palindrome.
   - $p(aaaaab) = aaaaa$.

   Show that the following language is not a CFL:

   $$P = \{x\#y \mid x, y \in \{a,b\}^* \text{ and } y = p(x)\}$$

   Following the above examples, the following strings are in $P$:

   $$ababbab\#baba, \quad abbabba\#, \quad aaaaab\#aaaaa \in P$$

*Hint:* In the demon game, choose a string starting with $a^p b^p a^{p+1}\# \cdots$ Carefully consider all the cases for how the string may be chopped up.

2. Suppose you have a grammar in which all productions are of the following form:

   $$A \to aB \qquad \text{or} \qquad A \to \epsilon$$

   That is, the right-hand side of every production is either empty, or a single terminal followed by a single non-terminal. Show that a grammar of this form generates a **regular language**.

*Hint:* Construct an NFA to simulate this grammar. Try some examples to gain an intuition about what such a grammar does. Clearly describe the construction of the NFA, e.g.: "the states of the new NFA are ...; for each production of $this form in the grammar, I include an NFA transition $that ..."

3. Let bin be as before (e.g., bin(0101) = 5, bin(1111) = 15). Show that the following language is a CFL:

   $$B = \{x\#y \mid \mathrm{bin}(x) + 1 = \mathrm{bin}(\mathrm{rev}(y))\}$$

   Don't overlook the "rev" in there! For example, the following strings are in $B$:

   $$1001\#0101, \quad 1111\#00001, \quad 101010\#110101 \in B$$