

**Final Problems 3**

Due Friday Oct 20.

1. Show that the following languages are not regular:

- (a)  $\{xx \mid x \in \{a,b\}^*\}$
- (b)  $\{xy \in \{a,b\}^* \mid |x| = |y| \text{ and } \exists i : [i\text{th character of } x] = [i\text{th character of } y]\}$
- (c)  $\{a^n b^m \mid n < m \text{ and both } n \text{ and } m \text{ are multiples of } 4\}$
- (d)  $\{a^n b^m c^t \mid m < t \text{ or } m < n\}$

2. For any fixed integer  $k$  we can define the following regular language:

$$L_k = \{w \in \{a,b\}^* \mid \text{the } k\text{th-to-last character of } w \text{ is } b\}$$

For example,  $L_3 = (a+b)^* b (a+b)(a+b)$ . Recall that  $L_k$  can be described by an NFA with  $k+1$  states. Prove that any DFA for  $L_k$  must have at least  $2^k$  states.

*Hint:* use a pigeonhole principle argument similar to the first proof that  $\{a^n b^n \mid n \geq 0\}$  is non-regular, to show that any DFA with fewer than  $2^k$  states must make a mistake.