CS 321: Homework #0

This homework is for review / warmup only. Do these problems to make sure you are prepared for the course. You may optionally submit solutions, and the TAs & I will gladly provide feedback.

It's OK if you need to review some concepts to solve these problems (especially after summer break). However, it is a red flag if you find these problems impossible even after some light review.

1. Let $F(n)$ denote the nth Fibonacci number, defined as follows:

$$F(0) = 0; \qquad F(1) = 1; \qquad F(n) = F(n-1) + F(n-2), \text{ for } n \geq 2$$

Using this definition of the Fibonacci sequence, **give a formal, inductive proof** of the following identity:

$$\sum_{i=0}^{n} F(i) = F(n+2) - 1$$

2. Let $A = \{1, 2, 3\}$, $B = \{1, \{2, 3\}\}$, and $C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x \neq y\}$.

   (a) What is $A \times B$?

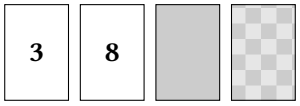
   (b) What is $\mathcal{P}(A) \setminus B$?

   (c) What is $\mathcal{P}(A) \setminus \mathcal{P}(B)$?

   (d) What is $(A \times A) \setminus C$?

   "$\times$" is the cartesian product operation, "$\mathcal{P}(X)$" is the powerset of a set X, "$\setminus$" is the set difference operation, and $\mathbb{Z}$ is the set of integers $\{\ldots, -1, 0, 1, \ldots\}$.

3. Four cards are on a table. Each card has a **number** (from 1 to 10) on one side and a **pattern** (checkerboard or solid) on the other side. The cards are currently in the following state:



   Someone has made a rule about cards:

   > *"If a card has an even number on one face, then it **must have** a solid pattern on the opposite face."*

   Which of the four cards (maybe none, maybe just one, maybe more than one) *must* be turned over to determine whether the rule is being followed?[1] **Why?**

4. Write an equivalent logical statement without using any negation operations ($\neg$):

$$\neg \forall x : \exists y : ([\neg f(x, y)] \wedge g(y))$$

   Hint: the final answer may involve an implication ($a \Rightarrow b$)

[1] It sounds like click-bait, but literally less than 10% of people get this question right (it's a famous psychology experiment about deductive reasoning). Subjects do much better (75% success) when you replace "card" with "person"; "even number" with "drinks beer"; "solid pattern" with "age over 21"; and ask them to verify whether the drinking age rule is being obeyed.