CS 321: Homework #4

Due: Monday Oct 23 at 9am, on Canvas

Homeworks should be **typed**. You can describe a DFA by giving its transition table (don't forget to indicate start state and accept states), or by drawing a state diagram. You can easily draw state diagrams using this web-based tool: http://madebyevan.com/fsm/.

> For reference, here is the *pumping lemma game* (for language A):
>
> 1. Adversary picks a number $p \geq 0$.
> 2. You pick a string $w \in A$, such that $|w| \geq p$.
> 3. Adversary breaks w into $w = xyz$, such that $|xy| \leq p$ and $y \neq \varepsilon$.
> 4. You pick a number $i \geq 0$. If $xy^i z \notin A$, then you win.
>
> If you can describe a strategy in which you always win, then A is not regular.

1. Show that the following languages are not regular. You can use the pumping lemma game, or you can use closure properties (or both).

   (a) $\{w \in \{a,b,c\}^* \mid \mathsf{num}(a,w) = \mathsf{num}(b,w) + \mathsf{num}(c,w)\}$
   In this problem $\mathsf{num}(a,w)$ means the number of a characters in the string w.

   (b) $\{a^n b^m c^k \mid n = m \text{ or } m = k\}$

   (c) $\{w \in \{a,b\}^* \mid \text{the length of } w \text{ is a square number}\}$
   This language contains all strings of length 1, 4, 9, 16, etc. *Hint:* after you pump, you'll want to show that the length of the resulting string is *not* a square. The best way to do this is to show that its length is *strictly* between consecutive squares n^2 and $(n+1)^2$ for some appropriate n.

   (d) $\{w \in \{a,b\}^* \mid w \neq \mathsf{rev}(w)\}$.
   In this problem $\mathsf{rev}(w)$ denotes the reverse of w (i.e., characters put in opposite order).