CS 321: Homework #3

Due: Monday Oct 16 at 9am, on Canvas

Homeworks should be **typed**. You can describe a DFA by giving its transition table (don't forget to indicate start state and accept states), or by drawing a state diagram. You can easily draw state diagrams using this web-based tool: http://madebyevan.com/fsm/.

1. Give regular expressions for the following languages:

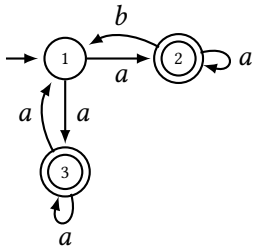
   (a) $\{w \in \{a,b\}^* \mid w$ contains substring ab an even number of times$\}$

   (b) $\{w \in \{a,b\}^* \mid w$ has an even number of a's **and** even number of b's$\}$
   Note: the a's and b's can come in any order, so strings like $abbbab$ should be accepted.

   To help the grader out (and to increase chance of partial credit), if you have a long regular expression, please identify small conceptual parts and explain what each part does.

2. Give an NFA for the set of strings matching $(0 + 1(01^*0)^*1)^*$

3. Give a regular expression equivalent to the following NFA:



4. Let $M = (Q, \Sigma, \delta, s, F)$ be a DFA.

   (a) Show that for any $q \in Q$ and $P \subseteq Q$, the following language is regular:

   $$\{w \in \Sigma^* \mid \delta^*(q,w) \in P\}$$

   Clearly describe a procedure to construct a DFA for this language, in terms of M.

   (b) If A is a language over alphabet Σ, define:

   $$\mathsf{undouble}(A) \stackrel{\text{def}}{=} \{w \in \Sigma^* \mid ww \in A\}.$$

   Show that if A is regular, then so is $\mathsf{undouble}(A)$.
   Example: if $A = \{\varepsilon, 0, 11, 0010, 0101\}$ then $\mathsf{undouble}(A) = \{\varepsilon, 1, 01\}$.
   Hint: First consider a simpler version where I fix the "middle" state q, so:

   $$\{w \in \Sigma^* \mid ww \in A \text{ and } \delta^*(s,w) = q\}$$

   In the "real" version of the problem, the "middle state" is not fixed.