

## CS 321: Homework #5

**Due: Monday Nov 6 at 9am, on Canvas**

Homeworks should be **typed**. You can describe a PDA by giving a transition table, or by drawing a state diagram. You can easily draw state diagrams using this web-based tool: <http://madebyevan.com/fsm/>.

1. Describe CFGs for the following languages:

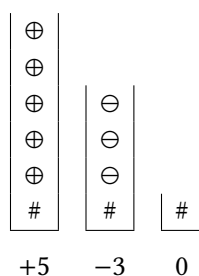
(a)  $\{a^n b^m c^k d^n \mid n \geq 0 \text{ and } m \neq k\}$

(b)  $\{a^n b^m \mid n \leq m \leq 2n\}$

2. Describe PDAs for the following languages:

(a)  $\{w \in \{a, b\}^* \mid 2 \cdot \#a(w) > 3 \cdot \#b(w)\}$ , where “ $\#a(w)$ ” denotes the number of  $a$ 's in  $w$ .

*Suggestion:* Use the PDA's stack to represent an integer counter, as follows:



Conceptually think of your PDA as an NFA with a counter. Keep track of  $2 \cdot \#a(w) - 3 \cdot \#b(w)$ . Carefully consider how to update the value in the counter (especially taking into account all cases of positive, zero, negative for the starting value of the counter, and the possibility of the counter's value changing signs).

(b) Describe a PDA for the following language:

$$\{xcy \mid x, y \in \{a, b\}^* \text{ and } \text{rev}(x) \text{ is a substring of } y\}$$

In other words, strings that contain a single  $c$  character, where the part of the string occurring before  $c$  appears reversed somewhere after the  $c$ .

*Example:* aaabb  $c$  abababbbaaabba is in the language.