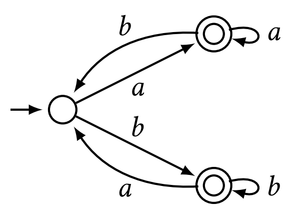
CS 321: Study guide

1. True or false? If true, justify. If false, give counterexample:
   (a) if A not regular then $\overline{A}$ is not regular either.
   (b) if A is regular and $A \cap B$ is not regular, then B is not regular
   (c) If A is not regular and B is not regular, then $A \cap B$ is not regular
   (d) If A is regular and $A \subseteq B$, then B is regular.
   (e) $a(aa)^* + (aa)^*$ and a^* are equivalent regular expressions

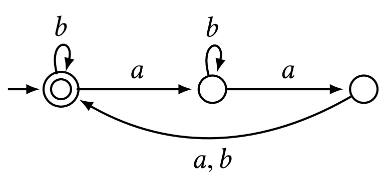
2. Show that the following are regular. This can involve describing an explicit DFA, NFA, or regular expression; it can also involve using known closure properties:
   (a) $\{w \in \{a, b\}^* \mid w$ contains abb as a substring but not $abba\}$
   (b) $\{w \in \{a, b\}^* \mid w$ contains at least 4 b's$\}$
   (c) $\{w \in \{a, b\}^* \mid w$ contains at most 4 b's$\}$
   (d) $\{w \in \{0, 1\}^* \mid w$ is a multiple of 5 in binary$\}$
   (e) $\{w \in \{0, 1\}^* \mid$ first two characters of w equal the last two characters of $w\}$.
   For example: $\underline{ab}bbb\underline{ab}$ is in the language, but $\underline{aa}aaa\underline{bb}$ is not.

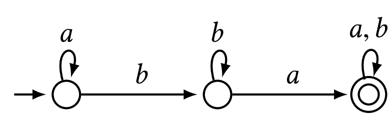
3. Regarding this NFA . . .



   (a) Convert it to an equivalent DFA
   (b) Convert it to an equivalent regular expression

4. Draw a DFA that accepts the intersection of these two languages:





5. Convert $(aa^*b + ba^*b)^*ba^*$ to an NFA

6. Show that the following languages are **not regular.** This can involve the pumping lemma and/or closure properties.

   The following box **will be included** on the exam. You don't have to memorize the pumping-lemma game:

For reference, here is the *pumping lemma game* (for language A):

1. Adversary picks a number $p \geq 0$.
2. You pick a string $w \in A$, such that $|w| \geq p$.
3. Adversary breaks w into $w = xyz$, such that $|xy| \leq p$ and $y \neq \varepsilon$.
4. You pick a number $i \geq 0$. If $xy^iz \notin A$, then you win.

If you can describe a strategy in which you always win, then A is not regular.

(a) $\{a^m b^n a^n b^m \mid m \geq 0 \text{ and } n \geq 0\}$

(b) $\{a^m b^n c^k \mid m < n < k\}$

(c) $\{a^m b^n c^k \mid m, n, k \text{ are all distinct}\}$

(d) $\{a^m b^n \mid m < n \text{ and both } n \text{ and } m \text{ are multiples of } 3\}$

(e) $\{w \in \{\texttt{[}, \texttt{]}\}^* \mid w \text{ is a string of balanced parentheses}\}$