CS 321: Homework #1

Due: Monday Oct 2 at 9am, on Canvas

Homeworks should be **typed**. You can describe a DFA by giving its transition table (don't forget to indicate start state and accept states), or by drawing a state diagram. You can easily draw state diagrams using this web-based tool: http://madebyevan.com/fsm/.

For problems 1-3, describe a DFA that accepts the given language. Describe in plain language **what is the purpose of each state.** It will probably help to give thoughtful names to the states.

1. $\{x \in \{a, b\}^* \mid$ last 5 characters of x are **not** $ababb\}$.

2. $\{x \in \{0, 1\}^* \mid \mathsf{bin}(x)$ is 1 more than a multiple of 5$\}$.

   $\mathsf{bin}(x)$ is the function from lecture, defined recursively as:

$$\begin{aligned} \mathsf{bin}(\varepsilon) &= 0 \\ \mathsf{bin}(wc) &= 2 \cdot \mathsf{bin}(w) + c, \qquad \text{for all } w \in \{0,1\}^* \text{ and } c \in \{0,1\} \end{aligned}$$

3. $\{x \in \{a, b\}^* \mid x$ contains at least 3 occurrences of the substring $aba\}$.

   Overlapping of substrings is allowed. For example, the string *abababa* should be accepted.

4. In lecture, we defined the extended transition function $\delta^* : Q \times \Sigma^* \to Q$ recursively in terms of δ, via:

$$\begin{aligned} \delta^*(q, \varepsilon) &= q \\ \delta^*(q, wc) &= \delta(\delta^*(q, w), c), \qquad \text{for all } w \in \Sigma^* \text{ and } c \in \Sigma \end{aligned}$$

   Using this definition, prove that:

$$\delta^*(q, xy) = \delta^*(\delta^*(q, x), y), \qquad \text{for all } x, y \in \Sigma^*$$

   In other words, the state that you get to by starting at q and reading xy, is the state that you get to by starting at q, reading x, then reading y.

   Hint: Use induction on the length of y. You can follow the examples for induction on strings from Erickson's notes section 1. However, in this case you should do induction by adding a character to the *end* of y (not beginning of y as in the Erickson examples). This makes things match the recursive definition δ^* better.