Final Problems 5

Due Monday Nov 20.

1. Show that if L is a CFL and R is a regular language, then $L \cap R$ is context-free.

Hint: Use a product construction on a PDA & DFA. Describe the construction abstractly and precisely: i.e., "given arbitrary PDA $M_1 = (Q_1, \ldots)$ and arbitrary DFA $M_2 = (Q_2, \ldots)$, construct a new PDA $M^* = (Q^*, \ldots)$ where $Q^* = \ldots$" You don't have to prove correctness of the new PDA.

2. A **counter automaton (CA)** is basically an NFA with an extra counter that holds an integer value (initially zero). Its transitions are of the form $p \xrightarrow{c,s,i} q$, where:

   - $c \in \Sigma \cup \{\epsilon\}$ is a character to be read from input (or ϵ),
   - $s \in \{<, >, =\}$ indicates a sign test (comparison against zero) on the current value of the counter,
   - $i \in \{-1, 0, +1\}$ is an increment to the counter.

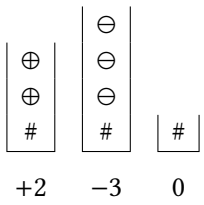
   So:

   - Transition $p \xrightarrow{a,>,+1} q$ means "if in state p, reading input character a, and counter is currently positive, move to state q and add 1 to the counter."
   - Transition $p \xrightarrow{\epsilon,=,0} q$ means "if in state p, and counter is currently zero, without consuming an input character, move to state q and don't change the counter."

   The CA can be nondeterministic, and it accepts if there is any valid sequence of transitions to reach an accept state after reading the entire input.

   Show that a CA accepts a context-free language.

Hint: Simulate a CA with a PDA. Use the stack to represent the counter, as follows:



   Carefully consider how to update the value in the counter, taking into account all possible cases. As before, make your construction abstract (general-purpose) and precise: e.g., "for every CA transition $p \xrightarrow{c,s,i} q$, create the following PDA transition(s): $\cdots$"

3. Show that $\{w \in \{a, b\}^* \mid 2 \cdot \#a(w) > 3 \cdot \#b(w)\}$ is context-free, where "$\#a(w)$" denotes the number of a's in w.

Hint: Construct a counter automaton!