CS 321: Homework #6

Due: Monday Nov 27 at 9am, on Canvas

Homeworks should be **typed**.

> For reference, here is the *CFL pumping lemma game* (for language A):
>
> 1. Adversary picks a number $p \geq 0$.
> 2. You pick a string $s \in A$, such that $|s| \geq p$.
> 3. Adversary breaks s into $s = uvwxy$, such that $|vwx| \leq p$ and $|vx| > 0$.
> 4. You pick a number $i \geq 0$. If $uv^i wx^i y \notin A$, then you win.
>
> If you can describe a strategy in which you always win, then A is not context-free.

1. Show that the following languages are **not** context-free. You can use pumping lemma or closure properties or a combination.

   (a) $\{xcy \mid x, y \in \{0,1\}^* \text{ and } \mathsf{bin}(x) + 1 = \mathsf{bin}(y)\}$
   Example: a string like $100111c101000$ would be in this language.

   (b) $\{a^n b^m c^k \mid k = mn\}$

2. Consider the following grammar:

$$S \to Sa \mid Sb \mid T$$
$$T \to aTa \mid bTb \mid cU$$
$$U \to aU \mid bU \mid \varepsilon$$

   (a) Convert this grammar to Chomksy normal form. Show your intermediate steps to increase chances of partial credit.

   (b) Design an equivalent PDA.

3. Show that $\{x \in \{a,b\}^* \mid x \neq \mathsf{rev}(x)\}$ is context-free. This is the set of all **non**-palindromes — the complement of the set of palindromes. **Explain** your solution (either CFG or PDA).