## First Equation

Original:

$$q = \frac{-1 + (1 + 4D_d[1 - Vu^{0.5} - (1 - V)u])^{0.5}}{2D_d} * q_{max}$$
$$\frac{dq}{du} = (q_{max}(1 + 4D_d(1 - Vu^{0.5} - (1 - V)u))^{-0.5}) * (-0.5Vu^{-0.5} + V - 1)$$

Utility:

q: 
$$((-1+(1+4*D*(1-V*u^0.5-(1-V)*u))^0.5)/(2*D))*qmax$$

$$diff(q,u): 1.0*qmax*(4*D*(-V*u^0.5 - u*(-V + 1) + 1) + 1)^0.5*(-0.5*V*u^0.5/u + V - 1)/(4*D*(-V*u^0.5 - u*(-V + 1) + 1) + 1)$$

Latex equivalent:

$$\frac{dq}{du} = 1.0 \frac{q_{max} \left(4D_d(-Vu^{0.5} - u(-V+1) + 1) + 1\right)^{0.5} \left(-0.5 \frac{Vu^{0.5}}{u} + V - 1\right)}{\left(4D_d(-Vu^{0.5} - u(-V+1) + 1) + 1\right)}$$

## Equivalent

## **Second Equation**

Original:

$$q = [1 - Vu^{0.5} - (1 - V)u] \cdot q_{max}$$
$$\frac{dq}{du} = q_{max} \cdot (-0.5Vu^{-0.5} + V - 1)$$

Utility:

$$q: (1-V*u^0.5-(1-V)*u)*qmax$$

$$diff(q,u): qmax*(-0.5*V*u^0.5/u + V - 1)$$

Latex equivalent:

$$\frac{dq}{du} = q_{max} \left( -0.5 \frac{Vu^{0.5}}{u} + V - 1 \right)$$

#### Equivalent

# Third Equation

Original:

$$q = q_{max} \cdot \frac{-1 + \left(1 + 4D_d \cdot \left[1 - 2vx\left(\frac{p_R}{p_x}\right)u^{0.5} - v(1 - x)(\frac{p_R}{p_x})^2u\right]\right)^{0.5}\right)}{2D_d}$$

$$\frac{dq}{du} = q_{max} \cdot \left(1 + 4D_d \cdot \left[1 - 2vx \frac{p_R}{p_x} u^{0.5} - v(1 - x) \left(\frac{p_R}{p_x}\right)^2 u\right]\right)^{-0.5} \cdot \left(-v \frac{p_R}{p_x} \left[x u^{-0.5} + \frac{p_R}{p_x} (1 - x)\right]\right)$$

Utility:

q: 
$$qmax*(-1+(1+4*D*(1-2*v*x*(pr/px)*u^0.5-v*(1-x)*(pr/px)^2*u))^0.5)/(2*D)$$

Latex Equivalent:

$$\frac{dq}{du} = 1.0 \frac{q_{max} \left(4D_d \left(-\frac{Vp_r^2 u(-x+1)}{p_x^2} - 2\frac{Vp_r u^{0.5} x}{p_x} + 1\right) + 1\right)^{0.5} \left(-\frac{Vp_r^2 u(-x+1)}{p_x^2} - 1.0\frac{Vp_r u^{0.5} x}{p_x u}\right)}{\left(4D_d \left(-\frac{Vp_r^2 u(-x+1)}{p_x^2} - 2\frac{Vp_r u^{0.5} x}{p_x} + 1\right) + 1\right)}$$

## **Equivalent**

## Fourth Equation

Original:

$$q = q_{max} \cdot \left[ 1 - 2vx \frac{p_R}{p_x} u^{0.5} - v(1-x) \left( \frac{p_R}{p_x} \right)^2 u \right]$$
$$\frac{dq}{du} = q_{max} \cdot \left( -v \frac{p_R}{p_x} \left[ xu^{-0.5} + \frac{p_R}{p_x} (1-x) \right] \right)$$

Utility:

q: 
$$qmax*(1-2*V*x*(pr/px)*u^0.5-V*(1-x)*(pr/px)^2*u)$$

$$diff(q,u): qmax*(-V*pr^2*(-x + 1)/px^2 - 1.0*V*pr*u^0.5*x/(px*u))$$

Latex equivalent:

$$\frac{dq}{du} = q_{max} \left( -\frac{Vp_r^2(-x+1)}{p_x^2} - 1.0 \frac{Vp_r u^{0.5} x}{p_x u} \right)$$

# Equivalent

All the original derivatives are equivalent to the ones differentiated by SymPy.