

$$\frac{dy}{dx} + P(t) \cdot y = Q(t)$$

$$t^3 y' + 3t^2 y = t^3 - t + 1$$

$$\Rightarrow y' + 3 \frac{1}{t} y = 1 - \frac{1}{t^2} + \frac{1}{t^3}$$

$$P(t) = \frac{3}{t}$$

$$Q(t) = 1 - \frac{1}{t^2} + \frac{1}{t^3}$$

$$\text{IF} = e^{\int \frac{3}{t} dt}$$

$$= e^{3 \ln t} = t^3$$

$$y \cdot t^3 = \int t^3 - t + 1 \, dt$$

$$\Rightarrow y \cdot t^3 = \frac{t^4}{4} - \frac{t^2}{2} + t + C$$

General Solution

$$y(1) = 1$$

$$1 \times 1 = \frac{1}{4} - \frac{1}{2} + 1 + C$$

$$\Rightarrow C = \frac{1}{4}$$

$$y t^3 = \frac{t^4}{4} - \frac{t^2}{2} + t + \frac{1}{4}$$

$$\Rightarrow y = \frac{t}{4} - \frac{1}{2t} + \frac{1}{t^2} + \frac{1}{4}$$

Particular Solution

