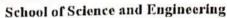
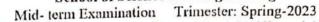
United International University





Course Title: Linear Algebra, Ordinary & Partial Differential Equations
/ Calculus and Linear Algebra

Course Code: Math 183/Math-2183 Marks: 30 Time: 1 Hour 45 Mins



(i)

The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. The curve passes through the point (3,5)

- (a) Find the equation of the curve.
- (b) Find the x coordinate of the stationary point.
- (c) State the values of x for which y is increasing.
- (ii) The total external surface area of a solid cylinder is 192π cm². The cylinder has a radius of r cm and a height of h cm.
 - (a) Express h in terms of r and hence show that the volume $V \text{ cm}^3$, of the cylinder is given by $V = \pi(96r r^3)$.
 - (b) Find the stationary value and determine whether it a maximum or a minimum.

 Q^2 [3+2+5=10]

Using chain rule find $\frac{\partial w}{\partial x}$, where

$$W = u^3 v + \sqrt{v}$$
, $u = \cos x + xy$ and $v = (x^2 + y)$

(ii), Given that $x^3 + 2xy - y^2 + 3x + 2y + 7 = 0$, find $\frac{dy}{dx}$.

The variables x and y are related by the function $f(x,y) = 3x^2 + xy - 9x + 2y^2 + 10y + 1$. Evaluate $f_{x'}$, f_{xx} , f_{xy} , $f_{y'}$ and f_{yy} and hance state the nature of the turning point.

Show that $y = e^{-2t}$ is the solution of the differential equation y'' - 4y = 0

(ij)

A liquid is heated so that its temperature is x (in degree centigrade) after t seconds. It is given that the rate of increase of x is proportional to (100 - x). The initial temperature of the liquid is 25° C.

- (a) Form a differential equation relating x, t and a constant of proportionality, k to model this information.
- (b) Solve the differential equation and obtain an expression for x in terms t and k.
- (c) After 180 seconds the temperature of the liquid is 85°C. find the value of k and hence find the temperature of the liquid after 200 seconds.

Solve the following differential equation.

$$t\frac{dx}{dt} + x = 3, \quad x(1) = 6$$