Mid - 231

1) a. $y' = 3\sqrt{x} - 3\frac{1}{\sqrt{x}}$

$= 3\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$

$y = \int 3\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$

$= 3 \cdot \left[\frac{2x^{\frac{3}{2}}}{3} - \frac{x^{-\frac{1}{2}+1}}{\frac{1}{2}}\right]$

$= 3 \cdot \left[\frac{2x^{3/2}}{3} - 2x^{1/2}\right]$

$= 2x^{3/2} - 6x^{1/2} + C$

$\Rightarrow 5 = 2 \times 3^{3/2} - 6 \times 3^{1/2} + C$

$\Rightarrow C = 5 - 2 \times 3^{3/2} + 6 \times 3^{1/2}$

$= 5$

$$\boxed{y = 2x^{3/2} - 6x^{1/2} + 5}$$

b) Stationary Point

$\hookrightarrow f'(x) = 0$

$y' = 3x^{1/2} - 3x^{-1/2}$

$0 = 3 \times \left(x^{1/2} - \frac{1}{x^{1/2}}\right)$

$\Rightarrow 0 = 3 \times \frac{x - 1}{x^{1/2}}$

$\Rightarrow 3(x-1) = 0$

$\therefore x = 1 \rightarrow$ stationary Point

(c)

$$y = 2x^{3/2} - 6x^{1/2} + 5$$

$$y' = 3x^{1/2} - 3x^{-1/2}$$

$$y' = 0 \Rightarrow x = 1$$

interval	sign of y'	Conclusion
$x < 1$	$(-)$	Decreasing in $(-\infty, +1]$
$x > 1$	$(+)$	Increasing. in $[1, \infty)$

11)

(a)

$$A = 2\pi r^2 + 2\pi r h$$

$$\Rightarrow 192\pi = 2\pi r(r+h)$$

$$\Rightarrow 2r(r+h) = 192$$

$$\Rightarrow r + h = \frac{96}{r}$$

$$\therefore h = \frac{96}{r} - r$$

$$V = \pi r^2 h$$

$$= \pi \times r^2 \times \left(\frac{96}{r} - r\right)$$

$$\boxed{V = \pi \times (96r - r^3)}$$

$$V = 96\pi r - \pi r^3$$

$$V' = 96\pi - 3r^2\pi$$

$$\Rightarrow 0 = 96\pi - 3r^2\pi$$

$$\Rightarrow 3r^2 = 96$$

$\therefore r = +4\sqrt{2} \rightarrow$ Stationary Val

$$V'' = 0 - 3\pi \times 2r$$
$$= -6\pi r$$
$r = 4\sqrt{2}$ $\quad V'' = -6\pi \times 4\sqrt{2}$
$$= (-)$$
$\downarrow$
maximum at $r = 4\sqrt{2}$

Q2

(i) $$\frac{\partial w}{\partial x} = \boxed{\frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial w}{\partial v} \cdot \frac{\partial v}{\partial x}}$$
$$= v \cdot 3u^2 \cdot (-\sin x + y)$$
$$+ \left(u^3 + \frac{1}{\cancel{2}\sqrt{v}}\right) \cdot (\cancel{2}x)$$
$$= -3u^2 v \sin x + 3u^2 vy$$
$$+ 2xu^3 + \frac{x}{\sqrt{v}}$$
$$= 3u^2 v (y - \sin x) + x\left(2u^3 + \frac{1}{\sqrt{v}}\right)$$

$$= 3u^2 v (y - \sin x) + x\left(2u^3 + \frac{1}{\sqrt{v}}\right)$$
$$= 3x(\cos x + xy)^2 v \cdot (y - \sin x)$$
$$+ 2x(\cos x + xy)^3$$
$$+ \frac{x}{\sqrt{x^2 + y}}$$

(ii) $$x^3 + 2xy - y^2 + 3x + 2y + 7 = 0$$
$$\Rightarrow 3x^2 + 2x\frac{dy}{dx} + 2y - 2y\frac{dy}{dx} + 3 + 2 \cdot \frac{dy}{dx} = 0$$
$$\Rightarrow \frac{dy}{dx}(2x - 2y + 2) + 3x^2 + 2y + 3 = 0$$
$$\Rightarrow \frac{dy}{dx} = -\frac{(3x^2 + 2y + 3)}{2x - 2y + 2}$$

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

$$= -\frac{3x^2+2y-0+3+0+0}{0+2x-2y+0+2}$$

$$= -\frac{3x^2+2y+3}{2x-2y+2}$$

(iii)

$f(x,y) = 3x^2 + xy - 9x + 2y^2 + 10y + 1$

$f_x = 6x + y - 9$

$f_{xx} = 6$

$f_{xy} = 1$

$f_y = 0 + x - 0 + 4y + 10$

$= x + 4y + 10$

$f_{yy} = 4$

3. (i) $y = e^{-2t}$ $\Rightarrow y' = -2e^{-2t}$

$\Rightarrow y'' = 4e^{-2t}$

$y'' - 4y = 0$

L.H.S $4e^{-2t} - 4e^{-2t}$

$= 0 =$ R.H.S

(ii)

(a) $\frac{dx}{dt} \propto (100-x)$

$\Rightarrow \frac{dx}{dt} = k(100-x)$ —— (i)

$t = 0\,s$ $x = 25°C$

$t = t\,s$ $x = x°C$

#Separable Equation $M(x,y)\,dx + N(x,y)\,dy = 0$

$$\downarrow$$

Soln: $\int A(x)\,dx + \int B(y)\,dy = 0$

b) $\frac{dx}{dt} = K(100-x)$

$\Rightarrow \int \frac{1}{(100-x)}\,dx = \int K\,dt$

$\Rightarrow \ln(100-x) = Kt + \ln c$

$\Rightarrow \ln(100-x) - \ln c = Kt$

$\Rightarrow \ln \frac{100-x}{c} = Kt$

$\Rightarrow \frac{100-x}{c} = e^{Kt}$

$\Rightarrow 100-x = ce^{Kt}$

$\Rightarrow x = 100 - ce^{Kt}$

(c) $t=0 \qquad x=25$

$t=180 \qquad x=85$

$25 = 100 - ce^{0}$

$\Rightarrow c = 100-25 = 75$

$85 = 100 - 75 \times e^{K\cdot 180}$

$\Rightarrow 75e^{K\cdot 180} = 15$

$\Rightarrow e^{180K} = \frac{1}{5}$

$\Rightarrow 180K = \ln\left(\frac{1}{5}\right)$

$\therefore K = \frac{\ln(\frac{1}{5})}{180} = -8.94 \times 10^{-3}$

$t = 200\,s$

$$x = 100 - ce^{kt}$$

$$= 100 - 75 \times e^{\frac{\ln(\frac{1}{5})}{180} \times 200}$$

$$= 87.4562$$

(iii)

$$t\frac{dx}{dt} + x = 3 \qquad \boxed{x(1) = 6}$$

$$\Rightarrow \frac{dx}{dt} = \frac{3-x}{t}$$

$$\Rightarrow -\int \frac{-1}{3-x}\,dx = \int \frac{1}{t}\,dt$$

$$\Rightarrow -\ln(3-x) = \ln t + \ln c$$

$$\Rightarrow \ln(3-x)^{-1} = \ln(ct)$$

$$\Rightarrow \frac{1}{3-x} = ct$$

$x = 6,\ t = 1$

$$\Rightarrow \frac{1}{3-6} = c$$

$$\Rightarrow c = -\frac{1}{3}$$

$$\Rightarrow \frac{1}{x-3} = \frac{1}{3}t$$

$$\Rightarrow 3 = t(x-3)$$

$$\Rightarrow xt - 3t = 3 \text{ (Ans)}$$

$$\Rightarrow \boxed{\begin{array}{l} x - 3 = 3 \\ \therefore x = 6 \end{array}}$$