Differential Equation ⟶ Solution Function(s) OR Functions

$\boxed{\text{order: The order of the highest Derivative.}}$

D.E Classification

- → Order
- → No of Unknown Functions.
- → Linearity

Linear	non-linear

$a(x)y' + b(x)y = c(x)$ Linear DE

→ The variable/ functions whose derivatives are present called $\boxed{\text{Dependent}}$

→ No multiplication of these dependent functions or its derivatives can be present in Linear DE

→ no transcendental Function of dependent variable.

(1) $t^2\left[\frac{d^2y}{dt^2}\right]^1 + t\cdot\left[\frac{dy}{dt}\right]^1 + 2y^1 = \sin t$

3 Check

$y \to$ Dependent.
$t \to$ Independent.

✓ → All y term and its derivative should have power (1)

✓ → No y and its derivative can be in multiplied form

✓ → No transcendental func. of y (logarithmic or exponential)

Linear

2. $(1+y^2)\frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = e^t$

$\Rightarrow \frac{d^2y}{dt^2} + y^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = e^t$

1 → First check Fail

↳ Non-Linear

$D \to y$

$I \to t$

3. $\frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + y = 1$

$D \to y$

$I \to t$

1 2 3

Linear Equation

Order = 4

4. $\frac{dy}{dt} + ty^2 = 0$

$D \to y$

$I \to t$

$\cancel{1}$ → First Check Out

Non Linear

order = 1

5. $\frac{d^2y}{dt^2} + \boxed{\sin(t+y)} = \sin t$

$D \to y$ $I \to t$

1 2 $\cancel{3}$

Non-Linear

order = 2

6. $\frac{d^3y}{dt^3} + t\frac{dy}{dt} + (\cos^2 t)\,y = t^3$

1 2 3 → Linear DE

order = 3

Verifying the given Sol^n

Mid Q

(14) $y' - 2ty = 1$

$y = e^{t^2} \cdot \int_0^t e^{-s^2} ds + e^{t^2}$

$= e^{t^2} (\int_0^t e^{-s^2} ds + 1)$

$y' = (I+1) e^{t^2} \cdot 2t$

$L.H.S = (I+1) 2t e^{t^2} - 2t \cdot e^{t^2} (I+1)$

$= (I+1) 2t e^{t^2} - (I+1) 2t e^{t^2}$

$= 0$

$y'' - 4y = 0$

$y = e^{-2t}$

$\Rightarrow y' = e^{-2t} \cdot (-2)$

$\Rightarrow y'' = -2 \cdot (-2) e^{-2t}$

$= 4e^{-2t}$

$4e^{-2t} - 4e^{-2t} = 0$ (showed)

Determining Value of r.

(16) $y'' - y = 0$

$\Rightarrow r^2 \cdot e^{rt} - e^{rt} = 0$

$\Rightarrow r^2 - 1 = 0$

$\Rightarrow r^2 = 1$

$\therefore r = \pm$

$y = e^{rt}$

$\Rightarrow y' = r e^{rt}$

$\Rightarrow y'' = r \cdot r \cdot e^{rt}$

$= r^r e^{rt}$

linear or non-linear $\boxed{\text{Partial}}$

(23) $u_{xxxx} + 2u_{xxyy} + u_{yyyy} = 0$

4th Order

Linear

(24) $u_t + \underline{u u_x} = 1 + u_{xx}$

↳ No Multi Allowed

Non linear 2nd order

Verify (Partial)

26.

$\alpha^r u_{xx} = u_t$

$u(x,t) = e^{-\alpha^r t} \sin x$

$u_x = e^{-\alpha^r t} \cos x$

$u_{xx} = -e^{-\alpha^r t} \sin x$

$u_t = \sin x \cdot (-\alpha^r) \cdot e^{-\alpha^r t}$

$L.H.S = -\alpha^r \times e^{-\alpha^r t} \sin x$

$R.H.S = -\alpha^r e^{-\alpha^r t} \sin x$

Solving First Order Differential Equation

Linear First Order

Form : $\frac{dy}{dt} + P(t)\,y = Q(t)$

Integrating Factor, $IF = e^{\int P(t)\,dt}$

$y \cdot IF = \int Q(t) \cdot IF \; dt + C$

2.1 (19) $t^3 \cdot y' + 4t^{3}y = e^{-t}$

$\Rightarrow y' + 4 \cdot \frac{1}{t} \cdot y = \frac{1}{e^{t} \cdot t^3}$

$y' + P(t) \cdot y = Q(t)$

$y \cdot IF = \int Q(t)\, IF \cdot dt$

$P(t) = \frac{4}{t}$

$Q(t) = e^{-t} t^{-3}$

$IF = e^{\int \frac{4}{t} dt} = e^{4 \cdot \ln t} = t^4$

$y \cdot t^4 = \int e^{-t} t^{-3} t^4 \; dt$

$= \int e^{-t} \cdot t\, dt$

$= t \cdot (-e^{-t}) - \int 1 \cdot (-e^{-t})\, dt$

$= -te^{-t} + \int e^{-t} dt$

$= -te^{-t} - e^{-t}$

$\Rightarrow y \cdot t^4 = -e^{-t}(1+t) + C$ ------(1)

$\hookrightarrow$ General Solution

$y(-1) = 0$

$t = -1$

$y = 0$

$0 = -e(1-1) + C$

$\therefore \boxed{C = 0}$

At (1)

$yt^4 + e^{-t}(1+t) = 0$

Separable Equation

$$M(x,y) \cdot dx + N(x,y) \cdot dy = 0$$

$$\Rightarrow M(x \cdot y) + N(x,y) \cdot \frac{dy}{dx} = 0$$

$$\downarrow \downarrow \downarrow$$

$$A(x)\, dx + B(y) \cdot dy = 0$$

Soln: $\int A(x)\, dx + \int B(y)\, dy = c$

(18) $y' = (e^{-x} - e^{x}) / (3+4y)$, $y(0) = 1$

$$\Rightarrow \int (3+4y)\, dy = \int (e^{-x} - e^{x})\, dx$$

$$\Rightarrow 3y + 2y^2 = -e^{-x} - e^{x} + c$$

$$\Rightarrow 2y^2 + 3y + e^{-x} + e^{x} = c$$

$y = 1 \quad x = 0$

$2 \times 1 + 3 + 1 + 1 = c$

$\therefore c = 7$

$$2y^2 + 3y + e^{-x} + e^{x} - 7 = 0$$

$$\Rightarrow y = \frac{-3 \pm \sqrt{9 - 8 \times (e^{-x} + e^{x} - 7)}}{2 \times 2}$$

$$= \frac{-3 \pm \sqrt{9 - 8e^{-x} - 8e^{x} + 56}}{4}$$

$$= -\frac{3}{4} \pm \frac{1}{4}\sqrt{65 - 8e^{-x} - 8e^{x}}$$

$$= -\frac{3}{4} \pm \frac{1}{4}\sqrt{9 - 8e^{-x} + 8e^{x}}$$