

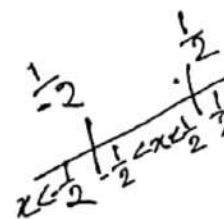
United International University
School of Science and Engineering
Mid-term Examination Trimester: Summer - 2022
Course Title: Calculus and Linear Algebra
Course Code: Math - 2183 Marks: 30 Time: 1 Hour 45 Minutes

Answer all the questions.

1. Consider the function $f(x) = 4x^3 - 3x + 5$, find

[10]

- (i) It's critical and inflection points.
- (ii) The x -intercepts and y -intercepts.
- (iii) The intervals on which $f(x)$ is increasing and decreasing.
- (iv) The intervals on which $f(x)$ is concave up and down.
- (v) Relative extrema of $f(x)$.
- (vi) Sketch the graph of $f(x)$.



2. (a) Consider, $w = 2p^3q^5 \cot 3r$, $p = \cos 2u$, $q = e^{u+v}$, $r = \ln(3uv)$.
Apply chain rule to find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$.

[4]

(b) Solve the following system of linear equation by Gaussian elimination method:

$$\begin{aligned} 5x - 2y + z &= 5 \\ -3x + y - 2z &= 4 \\ 2x + 3y + 3z &= -11 \end{aligned}$$

(c) Consider the function $u(x, y) = e^{2x} \sin 3y$. Prove that $u_{xy} - u_{yx} = 0$

[2]

3. (a) A rectangular field is to be bounded by a fence on three sides and by a straight stream on the fourth side. Find the dimensions of the field with maximum area that can be enclosed using 680 ft of fence. Also find the total cost if the garden is bounded on two sides by a fence costing 150 taka per ft and on the third side by a fence costing 200 taka per ft.

[5]

(b) Determine the dimensions of a rectangular box, open at the top, having a volume of 108 ft^3 , and requiring the least amount of material for its construction.

[5]