



United International University
School of Science and Engineering

Mid Term Examination Trimester: Spring-2019

Course Title: Linear Algebra, Ordinary & Partial Differential Equations

Course Code: Math 183 Marks: 30 Time: 105 min

There are 4 questions. Answer 1,4 and any one from the rest.

1. [CO1] a) Find the values of a for which the system below has unique, infinite or no solution. [3]

$$\begin{aligned}x + (a^2 - 1)y &= a \\ (a^2 - 4)y &= a - 1\end{aligned}$$

- b) Solve the system of homogenous linear equations. Show the result in vector form. Is there any non-trivial solution? [3]

$$\begin{aligned}-x_1 + 2x_2 - x_3 + x_4 &= 0 \\ 3x_1 - 2x_2 + x_3 - x_4 &= 0 \\ x_1 - 3x_2 + x_3 - 2x_4 &= 0\end{aligned}$$

- c) Solve the following system by row reduction method [4]

$$\begin{aligned}x + y + z &= 1 \\ -x + 2y - 2z &= -1 \\ x + y - 2z &= 2\end{aligned}$$

2. [CO2] a) Find the inverse of $A = \begin{bmatrix} 3 & -2 & 0 \\ 1 & 0 & 1 \\ -2 & 3 & 0 \end{bmatrix}$ by applying inversion algorithm (row reduction). [4]

- b) Find eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$. Also sketch the eigenspace in xy -coordinates. [5]

- c) Determine whether the matrix $A = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ has nontrivial fixed points. [1]

3. [CO2] a) Given $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -3 \\ 2 & 1 \\ -2 & 1 \end{bmatrix}$ [4]
- i) Show $(AB)^T = B^T A^T$ ii) Find A^{-3}

- b) Find $P(A)$ for $p(x) = x^2 - 5x + 5$ where $A = \begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$. [3]

- c) Find cofactor matrix of $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$. [3]

4. [CO3] a) Find the solution of the given differential equations [1]

i) $ty' + 2y = t^2 - t + 1, \quad y(1) = \frac{1}{2}$

ii) $y' = \frac{3x^2 - e^x}{2y - 5}, \quad y(0) = 1$

- b) Determine the values of r for which the differential equation $t^2 y'' - 6ty' + 10y = 0$ has solutions of the form t^r .