United International University
School of Science and Engineering
Mid Term Examination Trimester: Fall-2022
Course Title: Calculus and Linear Algebra
Course Code: Math 183/Math 2183 Marks: 30 Time: 105 min

Answer all the questions

1. For the function $f(x) = x^3 - 6x^2 + 2$ investigate, **[10]**

   (i) The $x-$intercept and $y-$intercept.
   (ii) Discuss the behavior at the infinity.
   (iii) The intervals on which $f(x)$ is increasing and decreasing.
   (iv) The intervals on which $f(x)$ is concave up and concave down.
   (v) The critical and inflection points.
   (vi) The relative maximum and minimum values of $f(x)$ by using the 1st and 2nd derivative test.

   Finally, Sketch the graph of $f(x)$ using the above-mentioned information.

2. (a) For the given function $f(x,y) = \cos(xy^2 - 4x)$, **[6]**
   (i) Find the slope of the surface $z = f(x,y)$in the x and y direction at the point $(2,1)$.
   (ii) Verify mixed second order partial derivatives are same or not?

   (b) Let $w = f(u)$ be a differentiable function and $u = \ln(xy) - x^2 - y^2$. **[2]**
   Then, find $\frac{\partial w}{\partial x} - \frac{\partial w}{\partial y}$.

   (c) Using appropriate chain rule find $\frac{\partial w}{\partial \theta}$, where $w = 4x^3 + 4y^3 + z^3$ with the assumptions $x = \rho \sin\theta \cos\emptyset$, $y = \rho \sin\theta \sin\emptyset$, and $z = \rho \cos\theta$. **[2]**

3. (a) Find the solution of the given differential equations: **[7]**
   (i) $t^3y' + 3t^2y = t^3 - t + 1, \quad y(1) = 1$
   (ii) $y' = \frac{y\,cosx}{1+2y^3}, \quad y(0) = 1$

   (b) Determine all the values of r for which the second-order differential equation $t^2y'' + 8ty' + 12y = 0$ has solutions of the form $t^r; t > 0$. **[3]**