EXCERCISE 13.3

Partial Derivatives

13.3: Definition-13.3.1,Examples: 1-5, 10-14 Exercise Set 13.3: 1-13, 25-52, 85-92, 95-104

1. Let $f(x, y) = 3x^3y^2$. Find

(a) $f_x(x, y)$ (b) $f_y(x, y)$ (c) $f_x(1, y)$
(d) $f_x(x, 1)$ (e) $f_y(1, y)$ (f) $f_y(x, 1)$
(g) $f_x(1, 2)$ (h) $f_y(1, 2)$.

2. Let $z = e^{2x} \sin y$. Find

(a) $\partial z/\partial x$ (b) $\partial z/\partial y$ (c) $\partial z/\partial x|_{(0,y)}$
(d) $\partial z/\partial x|_{(x,0)}$ (e) $\partial z/\partial y|_{(0,y)}$ (f) $\partial z/\partial y|_{(x,0)}$
(g) $\partial z/\partial x|_{(\ln 2,0)}$ (h) $\partial z/\partial y|_{(\ln 2,0)}$.

3–10 Evaluate the indicated partial derivatives. ■

3. $z = 9x^2y - 3x^5y$; $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

4. $f(x, y) = 10x^2y^4 - 6xy^2 + 10x^2$; $f_x(x, y), f_y(x, y)$

5. $z = (x^2 + 5x - 2y)^8$; $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

6. $f(x, y) = \dfrac{1}{xy^2 - x^2y}$; $f_x(x, y), f_y(x, y)$

7. $\dfrac{\partial}{\partial p}(e^{-7p/q}), \dfrac{\partial}{\partial q}(e^{-7p/q})$

8. $\dfrac{\partial}{\partial x}(xe^{\sqrt{15xy}}), \dfrac{\partial}{\partial y}(xe^{\sqrt{15xy}})$

9. $z = \sin(5x^3y + 7xy^2)$; $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

10. $f(x, y) = \cos(2xy^2 - 3x^2y^2)$; $f_x(x, y), f_y(x, y)$

11. Let $f(x, y) = \sqrt{3x + 2y}$.

(a) Find the slope of the surface $z = f(x, y)$ in the x-direction at the point $(4, 2)$.

(b) Find the slope of the surface $z = f(x, y)$ in the y-direction at the point $(4, 2)$.

12. Let $f(x, y) = xe^{-y} + 5y$.

(a) Find the slope of the surface $z = f(x, y)$ in the x-direction at the point $(3, 0)$.

(b) Find the slope of the surface $z = f(x, y)$ in the y-direction at the point $(3, 0)$.

13. Let $z = \sin(y^2 - 4x)$.

(a) Find the rate of change of z with respect to x at the point $(2, 1)$ with y held fixed.

(b) Find the rate of change of z with respect to y at the point $(2, 1)$ with x held fixed.

25–30 Find $\partial z/\partial x$ and $\partial z/\partial y$. ■

25. $z = 4e^{x^2y^3}$

26. $z = \cos(x^5y^4)$

27. $z = x^3 \ln(1 + xy^{-3/5})$

28. $z = e^{xy} \sin 4y^2$

29. $z = \dfrac{xy}{x^2 + y^2}$

30. $z = \dfrac{x^2y^3}{\sqrt{x + y}}$

31–36 Find $f_x(x, y)$ and $f_y(x, y)$. ■

31. $f(x, y) = \sqrt{3x^5y - 7x^3y}$

32. $f(x, y) = \dfrac{x + y}{x - y}$

33. $f(x, y) = y^{-3/2} \tan^{-1}(x/y)$

34. $f(x, y) = x^3e^{-y} + y^3 \sec \sqrt{x}$

35. $f(x, y) = (y^2 \tan x)^{-4/3}$

36. $f(x, y) = \cosh(\sqrt{x}) \sinh^2(xy^2)$

37–40 Evaluate the indicated partial derivatives. ■

37. $f(x, y) = 9 - x^2 - 7y^3$; $f_x(3, 1)$, $f_y(3, 1)$

38. $f(x, y) = x^2ye^{xy}$; $\partial f/\partial x(1, 1)$, $\partial f/\partial y(1, 1)$

39. $z = \sqrt{x^2 + 4y^2}$; $\partial z/\partial x(1, 2)$, $\partial z/\partial y(1, 2)$

40. $w = x^2 \cos xy$; $\partial w/\partial x\left(\frac{1}{2}, \pi\right)$, $\partial w/\partial y\left(\frac{1}{2}, \pi\right)$

41. Let $f(x, y, z) = x^2y^4z^3 + xy + z^2 + 1$. Find
(a) $f_x(x, y, z)$ (b) $f_y(x, y, z)$ (c) $f_z(x, y, z)$
(d) $f_x(1, y, z)$ (e) $f_y(1, 2, z)$ (f) $f_z(1, 2, 3)$.

42. Let $w = x^2y \cos z$. Find
(a) $\partial w/\partial x(x, y, z)$ (b) $\partial w/\partial y(x, y, z)$
(c) $\partial w/\partial z(x, y, z)$ (d) $\partial w/\partial x(2, y, z)$
(e) $\partial w/\partial y(2, 1, z)$ (f) $\partial w/\partial z(2, 1, 0)$.

43–46 Find f_x, f_y, and f_z. ■

43. $f(x, y, z) = z \ln(x^2y \cos z)$

44. $f(x, y, z) = y^{-3/2} \sec\left(\dfrac{xz}{y}\right)$

45. $f(x, y, z) = \tan^{-1}\left(\dfrac{1}{xy^2z^3}\right)$

46. $f(x, y, z) = \cosh(\sqrt{z}) \sinh^2(x^2yz)$

47–50 Find $\partial w/\partial x$, $\partial w/\partial y$, and $\partial w/\partial z$. ■

47. $w = ye^z \sin xz$

48. $w = \dfrac{x^2 - y^2}{y^2 + z^2}$

49. $w = \sqrt{x^2 + y^2 + z^2}$

50. $w = y^3e^{2x+3z}$

51. Let $f(x, y, z) = y^2e^{xz}$. Find
(a) $\partial f/\partial x|_{(1,1,1)}$ (b) $\partial f/\partial y|_{(1,1,1)}$ (c) $\partial f/\partial z|_{(1,1,1)}$.

52. Let $w = \sqrt{x^2 + 4y^2 - z^2}$. Find
(a) $\partial w/\partial x|_{(2,1,-1)}$ (b) $\partial w/\partial y|_{(2,1,-1)}$
(c) $\partial w/\partial z|_{(2,1,-1)}$.

85–92 Confirm that the mixed second-order partial derivatives of f are the same. ■

85. $f(x, y) = 4x^2 - 8xy^4 + 7y^5 - 3$

86. $f(x, y) = \sqrt{x^2 + y^2}$

87. $f(x, y) = e^x \cos y$

88. $f(x, y) = e^{x-y^2}$

89. $f(x, y) = \ln(4x - 5y)$

90. $f(x, y) = \ln(x^2 + y^2)$

91. $f(x, y) = (x - y)/(x + y)$

92. $f(x, y) = (x^2 - y^2)/(x^2 + y^2)$

95. Given $f(x, y) = x^3y^5 - 2x^2y + x$, find
(a) f_{xxy} (b) f_{yxy} (c) f_{yyy}.

96. Given $z = (2x - y)^5$, find
(a) $\dfrac{\partial^3 z}{\partial y \partial x \partial y}$ (b) $\dfrac{\partial^3 z}{\partial x^2 \partial y}$ (c) $\dfrac{\partial^4 z}{\partial x^2 \partial y^2}$.

97. Given $f(x, y) = y^3 e^{-5x}$, find
(a) $f_{xyy}(0, 1)$ (b) $f_{xxx}(0, 1)$ (c) $f_{yyxx}(0, 1)$.

98. Given $w = e^y \cos x$, find
(a) $\left.\dfrac{\partial^3 w}{\partial y^2 \partial x}\right|_{(\pi/4,0)}$ (b) $\left.\dfrac{\partial^3 w}{\partial x^2 \partial y}\right|_{(\pi/4,0)}$

99. Let $f(x, y, z) = x^3y^5z^7 + xy^2 + y^3z$. Find
(a) f_{xy} (b) f_{yz} (c) f_{xz} (d) f_{zz}
(e) f_{zyy} (f) f_{xxy} (g) f_{zyx} (h) f_{xxyz}.

100. Let $w = (4x - 3y + 2z)^5$. Find
(a) $\dfrac{\partial^2 w}{\partial x \partial z}$ (b) $\dfrac{\partial^3 w}{\partial x \partial y \partial z}$ (c) $\dfrac{\partial^4 w}{\partial z^2 \partial y \partial x}$.

101. Show that the function satisfies ***Laplace's equation***

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

(a) $z = x^2 - y^2 + 2xy$
(b) $z = e^x \sin y + e^y \cos x$
(c) $z = \ln(x^2 + y^2) + 2\tan^{-1}(y/x)$

102. Show that the function satisfies the ***heat equation***

$$\frac{\partial z}{\partial t} = c^2 \frac{\partial^2 z}{\partial x^2} \quad (c > 0,\ \text{constant})$$

(a) $z = e^{-t} \sin(x/c)$ (b) $z = e^{-t} \cos(x/c)$

103. Show that the function $u(x, t) = \sin c\omega t \sin \omega x$ satisfies the wave equation [Equation (6)] for all real values of ω.

104. In each part, show that $u(x, y)$ and $v(x, y)$ satisfy the ***Cauchy–Riemann equations***

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

(a) $u = x^2 - y^2$, $v = 2xy$
(b) $u = e^x \cos y$, $v = e^x \sin y$
(c) $u = \ln(x^2 + y^2)$, $v = 2\tan^{-1}(y/x)$