Chapter 7: Normalization

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Database System Concepts, 7th Ed.

Recall: Database design

- Requirements analysis
  - User needs; what must database do?
- Conceptual design
  - High-level description; often using E/R model
- Logical design
  - Translate E/R model into relational schema
- Schema refinement
  - Check schema for redundancies and anomalies
- Physical design/tuning
  - Consider typical workloads, and further optimise

What is Normalization

- process of organizing data in a database to minimize redundancy and dependency

Advantages

- improving data integrity
- reducing data redundancy
- making the database more efficient in terms of storage and retrieval

over-normalization can also lead to performance issues

it's essential to strike a balance based on the specific requirements of the application

Anomalies

unexpected or undesirable outcomes that can occur when data is not properly organized or structured

- Insert anomaly
- Delete anomaly
- Update anomaly

Features of Good Relational Designs

- Suppose we combine *instructor* and *department* into *in_dep,* which represents the natural join on the relations *instructor* and *department*

ID	*name*	*salary*	*dept_name*	*building*	*budget*
22222	Einstein	95000	Physics	Watson	70000
12121	Wu	90000	Finance	Painter	120000
32343	El Said	60000	History	Painter	50000
45565	Katz	75000	Comp. Sci.	Taylor	100000
98345	Kim	80000	Elec. Eng.	Taylor	85000
76766	Crick	72000	Biology	Watson	90000
10101	Srinivasan	65000	Comp. Sci.	Taylor	100000
58583	Califieri	62000	History	Painter	50000
83821	Brandt	92000	Comp. Sci.	Taylor	100000
15151	Mozart	40000	Music	Packard	80000
33456	Gold	87000	Physics	Watson	70000
76543	Singh	80000	Finance	Painter	120000

- There is repetition of information
- Need to use null values (if we add a new department with no instructors)

A Normalisation Example

StudentID is the primary key.

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English Maths Info Tech	$50 $50 $100	B A B+

Is it 1NF?

(subject, subjectcost, grade)

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English Maths Info Tech	$50 $50 $100	B A B+

How can you make it 1NF?

Create new rows so each cell contains only one value

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English Maths Info Tech	$50 $50 $100	B A B+

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English	$50	B
19594332X	Mary Watson	10 Charles Street	Bob	Red	Maths	$50	A
19594332X	Mary Watson	10 Charles Street	Bob	Red	Info Tech	$100	B+

But now look – is the *studentID* primary key still valid?

No – the studentID no longer uniquely identifies each row

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English	$50	B
19594332X	Mary Watson	10 Charles Street	Bob	Red	Maths	$50	A
19594332X	Mary Watson	10 Charles Street	Bob	Red	Info Tech	$100	B+

You now need to declare *studentID* **and** *subject* **together** to uniquely identify each row.

So the new **key** is StudentID *and* Subject.

So. We now have 1NF.

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English	$50	B
19594332X	Mary Watson	10 Charles Street	Bob	Red	Maths	$50	A
19594332X	Mary Watson	10 Charles Street	Bob	Red	Info Tech	$100	B+

Is it 2NF?

StudentName and address are dependent on studentID (which is part of the key)
This is good.

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English	$50	B
19594332X	Mary Watson	10 Charles Street	Bob	Red	Maths	$50	A
19594332X	Mary Watson	10 Charles Street	Bob	Red	Info Tech	$100	B+

But they are **not** dependent on *Subject* (the *other* part of the key)

And 2NF requires…

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English	$50	B
19594332X	Mary Watson	10 Charles Street	Bob	Red	Maths	$50	A
19594332X	Mary Watson	10 Charles Street	Bob	Red	Info Tech	$100	B+

All non-key fields are dependent on the ENTIRE key (studentID + subject)

So it's not 2NF

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English	$50	B
19594332X	Mary Watson	10 Charles Street	Bob	Red	Maths	$50	A
19594332X	Mary Watson	10 Charles Street	Bob	Red	Info Tech	$100	B+

How can we fix it?

Make new tables

- Make a new table for each primary key field
- Give each new table its own primary key
- Move columns from the original table to the new table that matches their primary key…

Step 1

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English	$50	B
19594332X	Mary Watson	10 Charles Street	Bob	Red	Maths	$50	A
19594332X	Mary Watson	10 Charles Street	Bob	Red	Info Tech	$100	B+

STUDENT TABLE (key = StudentID)

Step 2

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English	$50	B
19594332X	Mary Watson	10 Charles Street	Bob	Red	Maths	$50	A
19594332X	Mary Watson	10 Charles Street	Bob	Red	Info Tech	$100	B+

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

SUBJECTS TABLE (key = Subject)

Step 3

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English	$50	B
19594332X	Mary Watson	10 Charles Street	Bob	Red	Maths	$50	A
19594332X	Mary Watson	10 Charles Street	Bob	Red	Info Tech	$100	B+

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

RESULTS TABLE (key = StudentID+Subject)

Step 3

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English	$50	B
19594332X	Mary Watson	10 Charles Street	Bob	Red	Maths	$50	A
19594332X	Mary Watson	10 Charles Street	Bob	Red	Info Tech	$100	B+

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

RESULTS TABLE (key = StudentID+Subject)

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

Step 4 - relationships

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

Step 4 - cardinality

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

1

Each student can only appear ONCE in the student table

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

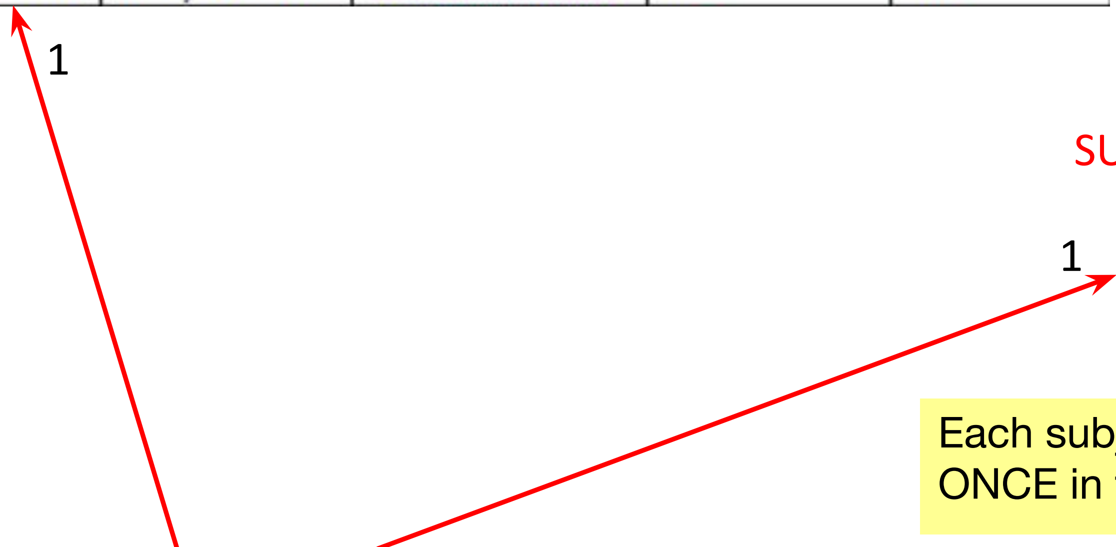
Step 4 - cardinality

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100



Each subject can only appear ONCE in the subjects table

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

Step 4 - cardinality

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

1

SUBJECTS TABLE (key = Subject)

1

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

A subject can be listed MANY times in the results table (for different students)

∞

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

Step 4 - cardinality

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

1

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

1

A student can be listed MANY times in the results table (for different subjects)

∞

∞

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

A 2NF check

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	\$50
Maths	\$50
Info Tech	\$100

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

SubjectCost is only dependent on the primary key, *Subject*

A 2NF check

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

Grade is only dependent on the primary key (*studentID + subject*)

A 2NF check

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

Name, Address are only dependent on the primary key (*StudentID*)

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

So it is 2NF!

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

But is it 3NF?

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

A 3NF check

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

1

Oh oh…

SUBJECTS TABLE (key = Subject)

1

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

∞ ∞

What?

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

A 3NF check

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

HouseName is dependent on both ***StudentID + HouseColour***

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

A 3NF check

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

1

***Or* HouseColour** is dependent on both **StudentID + HouseName**

SUBJECTS TABLE (key = Subject)

1

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

∞ ∞

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

A 3NF check

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

*But either way, non-key fields are dependent on MORE THAN THE PRIMARY KEY (*studentID*)*

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

A 3NF check

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

1

And 3NF says that non-key fields must depend on ***nothing but the key***

SUBJECTS TABLE (key = Subject)

1

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

∞ ∞

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

A 3NF check

STUDENT TABLE (key = StudentID)

StudentID	StudentName	Address	HouseName	HouseColor
19594332X	Mary Watson	10 Charles Street	Bob	Red

WHAT DO WE DO?

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	\$50
Maths	\$50
Info Tech	\$100

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

Again, carve off the offending fields

StudentTable

StudentID	StudentName	Address	HouseName
19594332X	Mary Watson	10 Charles Street	Bob

Primary key: StudentID

1

∞ ∞

1

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

A 3NF fix

StudentTable

StudentID	StudentName	Address
19594332X	Mary Watson	10 Charles Street

Primary key: StudentID

HouseTable

HouseName	HouseColor
Bob	Red

Primary key: HouseName

1

∞

∞

1

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

A 3NF fix

StudentTable

StudentID	StudentName	Address	HouseName
19594332X	Mary Watson	10 Charles Street	Bob

Primary key: StudentID

∞

1

1

HouseTable

HouseName	HouseColor
Bob	Red

Primary key: HouseName

SUBJECTS TABLE (key = Subject)

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

∞

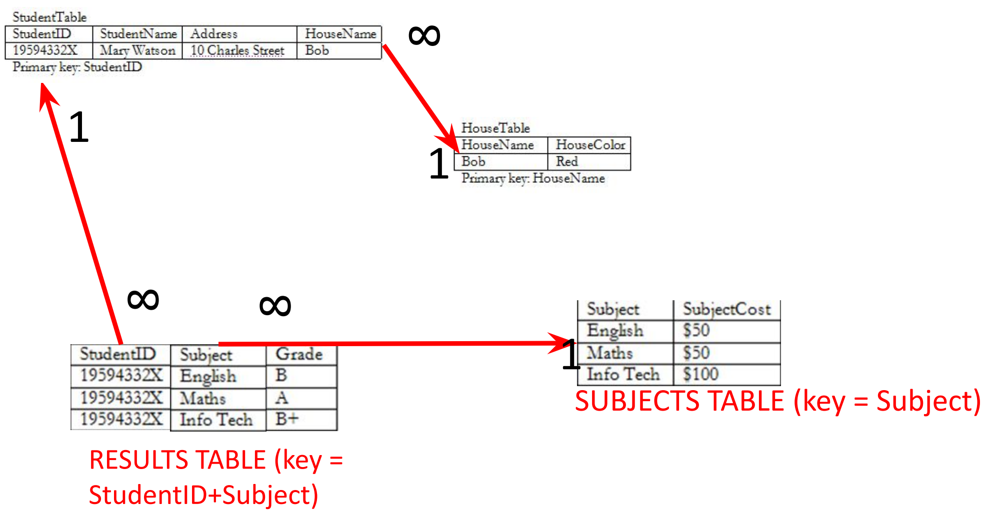
∞

1

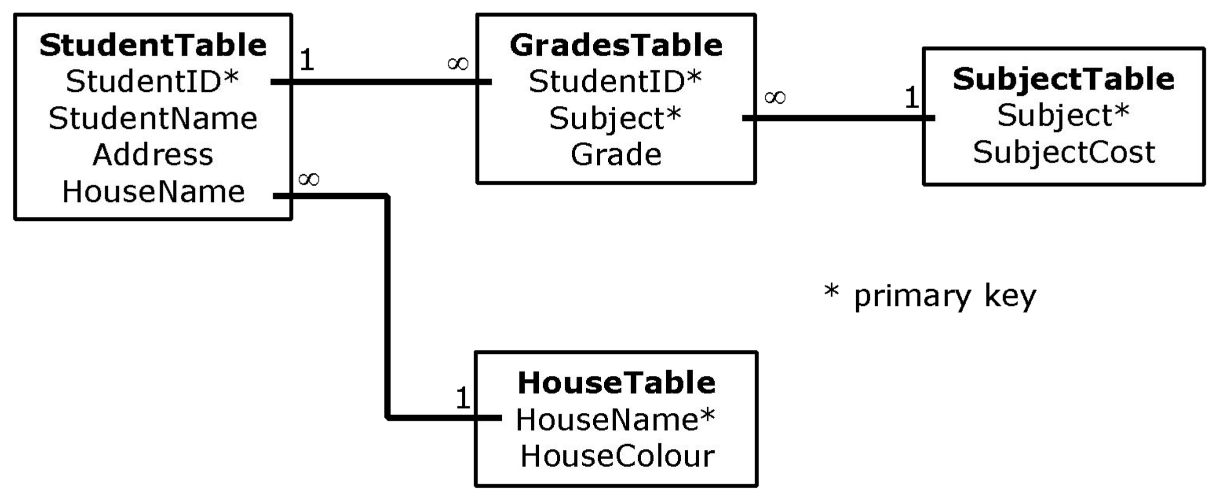
StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

A 3NF win!


StudentTable
StudentID | StudentName | Address | HouseName
19594332X | Mary Watson | 10 Charles Street | Bob
Primary key: StudentID
∞
1
HouseTable
HouseName | HouseColor
Bob | Red
Primary key: HouseName
1
∞
∞
StudentID | Subject | Grade
19594332X | English | B
19594332X | Maths | A
19594332X | Info Tech | B+
1
Subject | SubjectCost
English | $50
Maths | $50
Info Tech | $100
SUBJECTS TABLE (key = Subject)
RESULTS TABLE (key = StudentID+Subject)

Or…


StudentTable
StudentID*
StudentName
Address
HouseName
1
∞
GradesTable
StudentID*
Subject*
Grade
∞
1
SubjectTable
Subject*
SubjectCost
∞
1
HouseTable
HouseName*
HouseColour
* primary key

The Reveal

Before…

StudentID	StudentName	Address	HouseName	HouseColor	Subject	SubjectCost	Grade
19594332X	Mary Watson	10 Charles Street	Bob	Red	English Maths Info Tech	$50 $50 $100	B A B+

After…

HouseTable

HouseName	HouseColor
Bob	Red

Primary key: HouseName

StudentTable

StudentID	StudentName	Address	HouseName
19594332X	Mary Watson	10 Charles Street	Bob

Primary key: StudentID

Subject	SubjectCost
English	$50
Maths	$50
Info Tech	$100

SUBJECTS TABLE (key = Subject)

StudentID	Subject	Grade
19594332X	English	B
19594332X	Maths	A
19594332X	Info Tech	B+

RESULTS TABLE (key = StudentID+Subject)

Functional Dependencies

- There are usually a variety of constraints (rules) on the data in the real world.
- For example, some of the constraints that are expected to hold in a university database are:
  - Students and instructors are uniquely identified by their ID.
  - Each student and instructor has only one name.
  - Each instructor and student is (primarily) associated with only one department.
  - Each department has only one value for its budget, and only one associated building.

Functional Dependencies (Cont.)

- An instance of a relation that satisfies all such real-world constraints is called a **legal instance** of the relation;
- A legal instance of a database is one where all the relation instances are legal instances
- Constraints on the set of legal relations.
- Require that the value for a certain set of attributes determines uniquely the value for another set of attributes.
- A functional dependency is a generalization of the notion of a *key.*

Functional Dependencies (in simple term)

- relationship between two attributes in a relation (table) where the value of one attribute determines the value of another attribute.

means that given the value of the attribute(s) in *X*, we can uniquely determine the value of the attribute(s) in *Y*

Functional Dependencies Definition

- Let R be a relation schema

  $\alpha \subseteq R$ *and* $\beta \subseteq R$
- The **functional dependency**

  $\boldsymbol{\alpha \rightarrow \beta}$

  holds on R if and only if for any legal relations r(R), whenever any two tuples t_1 and t_2 of r agree on the attributes α, they also agree on the attributes β. That is,

  $$t_1[\alpha] = t_2[\alpha] \Rightarrow t_1[\beta] = t_2[\beta]$$
- Example: Consider $r(A, B)$ with the following instance of r.

1	4
1	5
3	7

- On this instance, $B \rightarrow A$ hold; $A \rightarrow B$ does **NOT** hold,

Keys and Functional Dependencies

- K is a superkey for relation schema R if and only if $K \rightarrow R$
- K is a candidate key for R if and only if
  - $K \rightarrow R$, and
  - for no $\alpha \subset K$, $\alpha \rightarrow R$
- Functional dependencies allow us to express constraints that cannot be expressed using superkeys. Consider the schema:

  in_dep (<u>*ID,*</u> *name, salary*<u>*, dept_name,*</u> *building, budget*).

  We expect these functional dependencies to hold:

  $dept\_name \rightarrow building$

  ID □ *building*

  but would not expect the following to hold:

  $dept\_name \rightarrow salary$

Use of Functional Dependencies

- We use functional dependencies to:
  - To test relations to see if they are legal under a given set of functional dependencies.
    - If a relation *r* is legal under a set *F* of functional dependencies, we say that *r* **satisfies** *F.*
  - To specify constraints on the set of legal relations
    - We say that *F* **holds on** *R* if all legal relations on *R* satisfy the set of functional dependencies *F.*
- Note: A specific instance of a relation schema may satisfy a functional dependency even if the functional dependency does not hold on all legal instances.
  - For example, a specific instance of *instructor* may, by chance, satisfy
    $name \rightarrow ID.$

Trivial Functional Dependencies

- *A* functional dependency is **trivial** if it is satisfied by all instances of a relation
- Example*:*
  - *ID, name* → *ID*
  - *name* → *name*
- In general, $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$

Armstrong's Axioms

- set of inference rules used in database normalization and the study of functional dependencies
  - **Reflexive rule:** if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
  - **Augmentation rule:** if $\alpha \rightarrow \beta$, then $\gamma \alpha \rightarrow \gamma \beta$
  - **Transitivity rule:** if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$
- These rules are
  - **Sound** -- generate only functional dependencies that actually hold, and
  - **Complete** -- generate all functional dependencies that hold.

Example

- $R = (A, B, C, G, H, I)$
  $F = \{ A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- Check some FD
  - $A \rightarrow H$
    - by transitivity from $A \rightarrow B$ *and* $B \rightarrow H$
  - $AG \rightarrow I$
    - by augmenting $A \rightarrow C$ with G, to get $AG \rightarrow CG$ and then transitivity with $CG \rightarrow I$
  - $CG \rightarrow HI$
    - by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$, and augmenting of $CG \rightarrow H$ to infer $CGI \rightarrow HI$, and then transitivity

Armstrong's Axioms

- Additional rules:
  - **Union rule**: If $\alpha \rightarrow \beta$ holds *and* $\alpha \rightarrow \gamma$ holds, then $\alpha \rightarrow \beta\gamma$ holds.
  - **Decomposition rule**: If $\alpha \rightarrow \beta\gamma$ holds, then $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds.
  - **Pseudotransitivity rule**:If $\alpha \rightarrow \beta$ holds *and* $\gamma\beta \rightarrow \delta$ holds, then $\alpha\gamma \rightarrow \delta$ holds.
- The above rules can be inferred from Armstrong's axioms.

Closure of a Set of Functional Dependencies

- Given a set *F* set of functional dependencies, there are certain other functional dependencies that are logically implied by *F*.
    - If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow \mathrm{C}$
    - etc.
- The set of **all** functional dependencies logically implied by *F* is the **closure** of *F*.
- We denote the *closure* of *F* by $\boldsymbol{F^+}$.

Procedure for Computing F^+

- To compute the closure of a set of functional dependencies F:

  $F^+ = F$
  repeat
  for each functional dependency f in F^+
  apply reflexivity and augmentation rules on f
  add the resulting functional dependencies to F^+
  for each pair of functional dependencies f_1 and f_2 in F^+
  if f_1 and f_2 can be combined using transitivity
  then add the resulting functional dependency to F^+
  until F^+ does not change any further

- **NOTE**: We shall see an alternative procedure for this task later

Closure of Attribute Sets

- Given a set of attributes α, define the ***closure*** of α **under** *F* (denoted by α^+) as the set of attributes that are functionally determined by α under *F*
- Algorithm to compute α^+, the closure of α under *F*

result := α;
while (changes to *result*) **do**
    for each $\beta \rightarrow \gamma$ **in** *F* **do**
        begin
            if $\beta \subseteq$ *result* **then** *result* := *result* $\cup\ \gamma$
        end

Example of Attribute Set Closure

- $R = (A, B, C, G, H, I)$
- $F = \{A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- $(AG)^+$
  1. $result = AG$
  2. $result = ABCG$ $(A \rightarrow C$ and $A \rightarrow B)$
  3. $result = ABCGH$ $(CG \rightarrow H$ and $CG \subseteq AGBC)$
  4. $result = ABCGHI$ $(CG \rightarrow I$ and $CG \subseteq AGBCH)$
- Is AG a candidate key?
  1. Is AG a super key?
     1. Does $AG \rightarrow R$? == Is R $\supseteq$ $(AG)^+$
  2. Is any subset of AG a superkey?
     1. Does $A \rightarrow R$? == Is R $\supseteq$ $(A)^+$
     2. Does $G \rightarrow R$? == Is R $\supseteq$ $(G)^+$
     3. In general: check for each subset of size $n-1$

Uses of Attribute Closure

There are several uses of the attribute closure algorithm:

- Testing for superkey:
  - To test if α is a superkey, we compute $\alpha^{+,}$ and check if α^+ contains all attributes of *R*.
- Testing functional dependencies
  - To check if a functional dependency $\alpha \rightarrow \beta$ holds (or, in other words, is in F^+), just check if $\beta \subseteq \alpha^+$.
  - That is, we compute α^+ by using attribute closure, and then check if it contains β.
  - Is a simple and cheap test, and very useful
- Computing closure of F
  - For each $\gamma \subseteq R$, we find the closure γ^+, and for each $S \subseteq \gamma^+$, we output a functional dependency $\gamma \rightarrow S$.

Determine all candidate keys

1. Consider a relation R(A, B, C, D, E, F) with the following functional dependencies:
$A \rightarrow BC$
$BD \rightarrow E$
$EF \rightarrow A$
$C \rightarrow F$
2. Let's say we have a relation R(A, B, C, D, E, F) with the following functional dependencies:
$A \rightarrow BC$
$AB \rightarrow D$
$DE \rightarrow F$
$C \rightarrow E$
3. Suppose we have a relation R(A, B, C, D, E, F) with the following functional dependencies:
$AB \rightarrow C$
$BC \rightarrow D$
$DE \rightarrow A$
$EF \rightarrow B$
4. Consider a relation R(A, B, C, D, E, F) with the following functional dependencies.
$AB \rightarrow C$
$C \rightarrow DE$
$EF \rightarrow A$
$A \rightarrow B$

Minimal Cover

- simplified and reduced version of the given set of functional dependencies.
  - Also known as **irreducible set** and **canonical cover**
  - There is a slight difference between minimal and canonical cover.

Steps to find Minimal Cover

- Split the right-hand attributes of all FDs
  - A->XY, then A->X, A->Y
- Remove all redundant FDs
  - { A->B, B->C, A->C }
  - A->C is redundant
- Find the Extraneous attribute and remove it
  - AB->C, either A or B or none can be extraneous.
    If A closure contains B then B is extraneous and it can be removed.
    If B closure contains A then A is extraneous and it can be removed.

Example

Minimize {A->C, AC->D, E->H, E->AD}

- **Step 1**: {A->C, AC->D, E->H, E->A, E->D}
- **Step 2**: {A->C, AC->D, E->H, E->A}
  Here Redundant FD : {E->D}
- **Step 3**: {AC->D}
  {A}+ = {A,C}
  Therefore C is extraneous and is removed.
  {A->D}
- Minimal Cover = {A->C, A->D, E->H, E->A}

How to check redundant FD

- If an FD can be inferred from the closure of other FDs, it's redundant and can be removed.
- Armstrong's Axioms and closure computation can be used

Step 2: {A->C, AC->D, E->H, E->A, E->D}

For each FD p->q, calculate the closure of p.

TWICE

one with all the given FDs. Another one excluding p->q

If closure of p remains same, then p->q is redundant.

Extraneous Attributes

- attributes of a functional dependency (FD) that are unnecessary for expressing the dependency

- Minimize {AB->CE, D->E, E->C}

Decomposition

- The only way to avoid the repetition-of-information problem in the i*n_dep* schema is to decompose it into two schemas – instructor and *department* schemas.
- Not all decompositions are good. Suppose we decompose

    employee(ID, name, street, city, salary)

  into

    employee1 (*ID*, *name*)

    employee2 (*name*, *street, city, salary*)

  The problem arises when we have two employees with the same name
- The next slide shows how we lose information -- we cannot reconstruct the original *employee* relation -- and so, this is a **lossy decomposition**.

A Lossy Decomposition

ID	name	street	city	salary
⋮				
57766	Kim	Main	Perryridge	75000
98776	Kim	North	Hampton	67000
⋮				

employee

ID	name
⋮	
57766	Kim
98776	Kim
⋮	

name	street	
⋮		
Kim	Main	Pe
Kim	North	H
⋮		

natural join

ID	name	street	city	salary
⋮				
57766	Kim	Main	Perryridge	75000
57766	Kim	North	Hampton	67000
98776	Kim	Main	Perryridge	75000
98776	Kim	North	Hampton	67000
⋮				

Lossless Decomposition

- Let R be a relation schema and let R_1 and R_2 form a decomposition of R . That is R = R_1 U R_2
- We say that the decomposition is a **lossless decomposition** if there is no loss of information by replacing R with the two relation schemas R_1 U R_2
- Formally,

$$\prod_{R1}(r) \bowtie \prod_{R2}(r) = r$$

- And, conversely a decomposition is lossy if

$$r \subset \prod_{R1}(r) \bowtie \prod_{R2}(r) = r$$

Example of Lossless Decomposition

- Decomposition of $R = (A, B, C)$
  $R_1 = (A, B)$ $R_2 = (B, C)$

A	B	C
α	1	A
β	2	B

r

A	B
α	1
β	2

$\Pi_{A,B}(r)$

B	C
1	A
2	B

$\Pi_{B,C}(r)$

$\Pi_A (r) \bowtie \Pi_B (r)$

A	B	C
α	1	A
β	2	B

Lossless Decomposition

- We can use functional dependencies to show when certain decomposition are lossless.
- For the case of $R = (R_1, R_2)$, we require that for all possible relations r on schema R

  $r = \prod_{R1}(r) \bowtie \prod_{R2}(r)$
- A decomposition of R into R_1 and R_2 is lossless decomposition if at least one of the following dependencies is in F^+:
  - $R_1 \cap R_2 \rightarrow R_1$
  - $R_1 \cap R_2 \rightarrow R_2$
- The above functional dependencies are a sufficient condition for lossless join decomposition; the dependencies are a necessary condition only if all constraints are functional dependencies

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B, B \rightarrow C)$
- $R_1 = (A, B)$, $R_2 = (B, C)$
  - Lossless decomposition:

    $R_1 \cap R_2 = \{B\}$ and $B \rightarrow BC$
- $R_1 = (A, B)$, $R_2 = (A, C)$
  - Lossless decomposition:

    $R_1 \cap R_2 = \{A\}$ and $A \rightarrow AB$
- *Note:*
  - $B \rightarrow BC$

    is a shorthand notation for
  - $B \rightarrow \{B, C\}$

Dependency Preservation

- Testing functional dependency constraints each time the database is updated can be costly
- It is useful to design the database in a way that constraints can be tested efficiently.
- If testing a functional dependency can be done by considering just one relation, then the cost of testing this constraint is low
- When decomposing a relation it is possible that it is no longer possible to do the testing without having to perform a Cartesian Produced.
- A decomposition that makes it computationally hard to enforce functional dependency is said to be NOT **dependency preserving**.

Dependency Preservation Example

- Consider a schema:

  dept_advisor(s_ID, i_ID, department_name)

- With function dependencies:

  $i\_ID \rightarrow dept\_name$

  $s\_ID, dept\_name \rightarrow i\_ID$

- In the above design we are forced to repeat the department name once for each time an instructor participates in a *dept_advisor* relationship.
- To fix this, we need to decompose *dept_advisor*
- Any decomposition will not include all the attributes in

  $s\_ID, dept\_name \rightarrow i\_ID$

- Thus, the composition NOT be dependency preserving

Dependency Preservation

- Let F_i be the set of dependencies F^+ that include only attributes in R_i.
  - A decomposition is **dependency preserving**, if

    $$(F_1 \cup F_2 \cup \ldots \cup F_n)^+ = F^+$$
- Using the above definition, testing for dependency preservation take exponential time.
- Not that if a decomposition is NOT dependency preserving then checking updates for violation of functional dependencies may require computing joins, which is expensive.

Dependency Preservation (Cont.)

- Let F be the set of dependencies on schema R and let $R_1, R_2, .., R_n$ be a decomposition of R.
- The restriction of F to R_i is the set F_i of all functional dependencies in F^+ that include **only** attributes of R_i.
- Since all functional dependencies in a restriction involve attributes of only one relation schema, it is possible to test such a dependency for satisfaction by checking only one relation.
- Note that the definition of restriction uses all dependencies in in F^+, not just those in F.
- The set of restrictions $F_1, F_2, .., F_n$ is the set of functional dependencies that can be checked efficiently.

Testing for Dependency Preservation

- To check if a dependency $\alpha \rightarrow \beta$ is preserved in a decomposition of R into $R_1, R_2, \ldots, R_n$, we apply the following test (with attribute closure done with respect to F)
  - *result* = α
    repeat
    for each R_i in the decomposition
    $t = (result \cap R_i)^+ \cap R_i$
    $result = result \cup t$
    until (*result* does not change)
  - If *result* contains all attributes in β, then the functional dependency $\alpha \rightarrow \beta$ is preserved.
- We apply the test on all dependencies in F to check if a decomposition is dependency preserving
- This procedure takes polynomial time, instead of the exponential time required to compute F^+ and $(F_1 \cup F_2 \cup \ldots \cup F_n)^+$

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B$
  $\quad B \rightarrow C\}$
  Key = $\{A\}$
- R is not in BCNF
- Decomposition $R_1 = (A, B)$, $R_2 = (B, C)$
  - R_1 and R_2 in BCNF
  - Lossless-join decomposition
  - Dependency preserving

- ***R(A,B,C,D)***

  F = {A → B , C → D}

  decomposition ***R1(AB) and R2(CD)***

- R(A,B,C,D)

  F = { A → B , A → C , C → D}

  R1(A,B,C) and R2(C,D)

- R(A,B,C,D,E)

  $F = \{ A \rightarrow BCD$

  $B \rightarrow AE$

  $BC \rightarrow AED$

  $D \rightarrow E$

  $C \rightarrow DE \}$

  decompose it into R1(A,B)

  R2(B,C)

  R3(C,D,E)