United International University

School of Science and Engineering

Assignment–01, Coordinate Geometry (Calculus–III)

Course Code : MAT 2109, Section–**A**, Spring-2024

Due: February 10, 2024, in Class

Solve all questions.

1. What surface in $\mathbb{R}^3$ is represented by each of the following equations? You have to write a precise description of your findings.

    (a) $x = -1$

    (b) $y = 3$

    (c) $z = -2$

    (d) $y = -2,\ z = -3$

    (e) $x = -2$, $y = 2$, $z = 2$

2. (a) Which points $(x,\, y,\, z)$ satisfy the equations $x^2 + y^2 = 25$ and $z = 3$.

    (b) What does this inequality $1 \leq y^2 + z^2 \leq 9$ represent in $\mathbb{R}^3$?

    (c) What solid region in $\mathbb{R}^3$ is represented by the inequalities $x^2 + z^2 = 1,\ 1 \leq y \leq 3$?

    (d) Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $y = -x$.

3. (a) What are the projections of the point $(1, 2, -3)$ on the $xy-$, $yz-$, and $xz-$ planes? Draw a rectangular box with the origin and $(1, 2, -3)$ as opposite vertices and with its faces parallel to the coordinate planes. Label all vertices of the box. Find the coordinate of the middle point of the diagonal of the box.

    (b) Find an equation of the sphere with center $(1, -2, 0)$ and radius 2. What is the intersection of this sphere with the $xy-$ and $yz-$ plane?

    (c) Write inequalities to describe the region.

        i. The solid lower hemisphere of the sphere of radius 5 centered at the origin.

        ii. The region between the yz-plane and the vertical plane $x = 5$.

        iii. The solid cylinder that lies on or below the plane $z = 5$ and on or above the disk in the xy-plane with center the origin and radius 1.

4. .

    (a) Find a vector **a** with representation given by the directed line segment $\overrightarrow{AB}$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

i. $A(1,1,1)$ and $B(-2,1,0)$, ii. $A(0,-2,-1)$ and $B(0,0,1)$.

(b) Find the vector that has the same direction as $< 1, 5, 2 >$ but has length 5.

5. (a) A street vendor sells a hamburgers, b hot dogs, and c bottles of water on a given day. He charges \$2 for a hamburger, \$3 for a hot dog, and \$4 for a bottle of water. If $\mathbf{A} = \begin{pmatrix} a & b & c \end{pmatrix}$ and $\mathbf{P} = \left\langle 2 \quad 3 \quad 4 \right\rangle$, what is the meaning of the dot product $A \cdot P$?

(b) Use vectors to determine whether the triangle with vertices $P(1,-3,-2)$, $Q(2,0,-4)$ and $R(6,-2,-5)$ is right-angled.

(c) If a vector has direction angles $\alpha = \frac{\pi}{4}$ and $\gamma = \frac{2\pi}{3}$, find the third direction angle β.

(d) Find the scalar and vector projections of $\mathbf{b} = \left\langle 3 \quad -3 \quad 1 \right\rangle$ onto $\mathbf{a} = \left\langle 2 \quad 4 \quad -1 \right\rangle$.

(e) Determine whether the vectors $\mathbf{a} = \left\langle 4 \quad 5 \quad -2 \right\rangle$ and $\mathbf{b} = \left\langle 3 \quad -1 \quad 5 \right\rangle$ are orthogonal, parallel, or neither.

6. (a) Find the work done by a force $\mathbf{F} = \left\langle 2 \quad -3 \quad 4 \right\rangle$ that moves an object from the point $(0,3,4)$ along a straight line to the point $(1,2,3)$. The distance is measured in meters and the force in newtons.

(b) A tow truck drags a stalled car along a road. The chain makes an angle of $45°$ with the road and the tension in the chain is 1200 N. How much work is done by the truck in pulling the car 1 km?

7. (a) Find the area of the parallelogram with vertices $P(1,0,1)$, $Q(1,3,1)$, $R(1,1,2)$, and $S(-1,2,0)$.

(b) Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = \left\langle 1 \quad 1 \quad 2 \right\rangle$, $\mathbf{b} = \left\langle -1 \quad 1 \quad 2 \right\rangle$, and $\mathbf{c} = \left\langle 2 \quad 1 \quad 4 \right\rangle$.

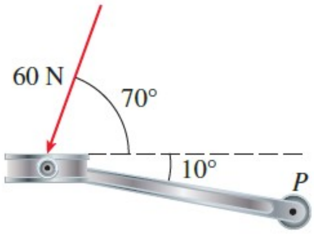
(c) Find two unit vectors orthogonal to both $\mathbf{a} = \left\langle 1 \quad -2 \quad 1 \right\rangle$ and $\mathbf{b} = \left\langle -1 \quad 1 \quad 0 \right\rangle$.

(d) Use the scalar triple product to determine whether the points $A(1,2,-1)$, $B(3,-1,6)$, $C(5,2,0)$ and $D(3,6,4)$ lie in the same plane.

(e) If $\mathbf{a} \cdot \mathbf{b} = \sqrt{5}$ and $\mathbf{a} \times \mathbf{b} = \left\langle 1 \quad 2 \quad -1 \right\rangle$, find the angle between $\mathbf{a}$ and $\mathbf{b}$.

8. (a) A bicycle pedal is pushed by a foot with a 60-N force as shown in Figure 1. The shaft of the pedal is 20 cm long. Find the magnitude of the torque about P.

Figure 1:



(b) A wrench 20 cm long lies along the positive y-axis and grips a bolt at the origin. A force is applied in the direction $\left\langle 0 \quad 1 \quad -2 \right\rangle$ at the end of the wrench. Find the magnitude of the force needed to supply 200 $N.m$ of torque to the bolt.