United International University
School of Science and Engineering
Assignment–04, Spring 2024
Coordinate Geometry (MAT 2109)
Due: May 04 in Class

Solve all problems.

1. (a) What do the equations represent in spherical systems?

   i. $\theta = \frac{\pi}{3}$ and $\rho = 1$

   ii. $\rho = 1$ and $\phi = \frac{\pi}{3}$

   iii. $\theta = \frac{\pi}{3}$ and $\rho = \frac{\pi}{3}$

   iv. $\rho = 1$ and $\theta = \frac{\pi}{3}$ and $\phi = \frac{\pi}{3}$

   (b) Evaluate $\int\int\int_E y^2 dV$, where E is the solid hemisphere $x^2 + y^2 + z^2 < 9$, $y > 0$.

   (c) Evaluate $\int\int\int_E xe^{x^2+y^2+z^2} dV$, where E is the portion of the unit ball $x^2 + y^2 + z^2 < 1$ that lies in the first octant.

   (d) Evaluate $\int\int\int_E \sqrt{x^2 + y^2 + z^2} dV$, where E lies above the cone $z = \sqrt{x^2 + y^2}$ and between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$.

2. (a) Evaluate the line integral $\int_C (x^2 y + \sin x) dy$, where C is the arc of the parabola $y = x^2$ from $(0, 0)$ to (π, π^2).

   (b) Evaluate the line integral $\int_C \mathbf{F}.\, d\mathbf{r}$, where $\mathbf{F}(x, y, z) = xz\mathbf{i} + z^3\mathbf{j} + y\mathbf{k}$ and C is given by the vector function $\mathbf{r}(t) = e^t\mathbf{i} + e^{2t}\mathbf{j} + e^{-t}\mathbf{k}$, $-1 \leq t \leq 1$.

3. (a) Determine whether or not $\mathbf{F}(x, y) = ye^x\mathbf{i} + (e^x + e^y)\mathbf{j}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

   (b) A vector field $\mathbf{F}(x, y) = (e^x + \sin y)\mathbf{i} + x \cos y\mathbf{j}$ and a curve $C :\ x = t,\ \ y = t(3 - t)$, $0 \leq t \leq 3$ are given.

   i. Show that $\mathbf{F}$ is conservative and find a potential function f.

   ii. Evaluate $\int_C \mathbf{F}.\, d\mathbf{r}$, using the Fundamental Theorem for Line Integrals.

4. (a) Evaluate the line integral $\oint_C xydx + x^2y^3dy$, where C is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 2)$ by two methods: (i) directly and (ii) using Green's Theorem.

   (b) Use Green's Theorem to evaluate the line integral $\int_C y^3dx - x^3dy$, along the given positively oriented curve C is the circle $x^2 + y^2 = 4$.

5. (a) Find the curl and the divergence of the vector field $\mathbf{F}(x, y, z) = \sin yz\,\mathbf{i} + \sin zx\,\mathbf{j} + \sin xy\,\mathbf{k}$.

   (b) Determine whether or not $\mathbf{F}(x, y, z) = yz \sin xy\,\mathbf{i} + zx \sin xy\,\mathbf{j} - \cos xy\,\mathbf{k}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.