United International University

School of Science and Engineering

Assignment–03, Spring- 2024

Coordinate Geometry and Vector Analysis (MAT 2109)

Due: April 20 in Class

Solve all problems.

1. (a) Find the equation of the tangent plane to the elliptic paraboloid $z = x^2+2y^2$ point $(1,\, 1,\, 1)$.

   (b) Sketch the paraboloid and the tangent plane at the point $(1,\, 1,\, 1)$.

   (c) Find the Linearization L of $f(x,\, y)$ at $(1,\, 1)$.

   (d) Use the L to approximate $f(0.95,\, 0.95)$ and then find the exact value of $f(1,\, 1)$. What can you conclude?

2. Consider the surface $x^2 + y^2 + z^2 = 4$ where $f(x, y, z) = x^2 + y^2 + z^2$.

   (a) Find the gradient field for this surface and evaluate it at $(2, 0, 0)$ and $(0, 2, 0)$.

   (b) What can you conclude about the directions of the gradient vector field from part (a)?

   (c) In which direction is the directional derivative at $(2, 0, 0)$ maximum? Find the maximum rate of change at $(2, 0, 0)$.

3. Find the area of the surface.

   (a) The part of the plane $6x+4y+2z = 1$ that lies inside the cylinder $x^2+y^2 = 25$.

   (b) The part of the plane $3x + 2y + z = 6$ that lies in the first octant.

   (c) The part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane $z = 1$.

4. (a) What do the equations represent in cylindrical systems?

   i. $\theta = \dfrac{\pi}{3}$ and $r \leq 1$

   ii. $r = 1$ and $z = 3$

   iii. $\theta = \dfrac{\pi}{3}$ and $z = 3$

   iv. $r = 1$ and $z = 3$ and $\theta = \dfrac{\pi}{3}$

   (b) Evaluate $\int\int\int_E x^2 dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$, and below the cone $z^2 = 4x^2 + 4y^2$.

(c) Find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.

(d) Find the volume of the solid that is enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 2$

(e) Find the volume of the solid that lies between the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 2$.