United International University

School of Science and Engineering

**Assignment**–2, Coordinate Geometry and Vector Analysis

Course Code : MAT 2109, Section–**A**, **Spring-2024**

Due: **February 27, 2024**, in Class

**Solve all questions.**

1. (a) Find parametric equations and symmetric equations for the line through $(2, 1, 0)$ and perpendicular to both $\mathbf{i} + \mathbf{k}$ and $\mathbf{j} + \mathbf{k}$.

   (b) Determine whether the lines $L_1$ and $L_2$ are parallel, skew, or intersecting. If they intersect, find the point of intersection.

   $$L_1 : \frac{x}{1} = \frac{y-1}{-1} = \frac{z-2}{3}; \quad L_2 : \frac{x-2}{2} = \frac{y-3}{-2} = \frac{z}{7}$$

   (c) Is the line through $(2, 4, 0)$ and $(1, 1, 1)$ perpendicular to the line through $(2, 3, 4)$ and $(3, 1, -8)$?

2. (a) Find an equation of the plane through the points $(2, 1, 2)$, $(3, -8, 6)$, and $(-2, -3, 1)$.

   (b) Use intercepts to help sketch the plane $3x + 2y - 3z = 6$.

   (c) The plane that passes through the point $(3, 1, 4)$ and contains the line of intersection of the planes $x + 2y + 3z = 1$ and $2x - y + z = -3$

   (d) The plane that passes through the point $(1, 5, 1)$ and is perpendicular to the planes $2x + y - 2z = 2$ and $x + 3z = 4$.

3. (a) Find the vertex, focus, and directrix of the parabola and sketch its graph $(x - 3)^2 = 8(y + 1)$.

   (b) Find the vertices, foci, and asymptotes of the hyperbola and sketch its graph $9y^2 - 4x^2 - 36y - 8x = 4$

   (c) Identify the type of conic section whose equation is $x^2 = 4y - 2y^2$ and find the vertices and foci.

   (d) Reduce the equation: $17x^2 + 18xy - 7y^2 - 16x - 32y - 18 = 0$; to one of the standard forms, where the origin is transferred to $(1, -1)$. Then, classify the surface, and sketch it.

4. Match the equation with its graph (Figure 1 (labeled I–VIII)). Give reasons for your choice, i. e. with the help of traces illustrate the graphs.

(a) $x^2 + 4y^2 + 9z^2 = 1$

(b) $x^2 - y^2 + z^2 = 1$

(c) $y = 2x^2 + z^2$

(d) $x^2 + 2z^2 = 1$

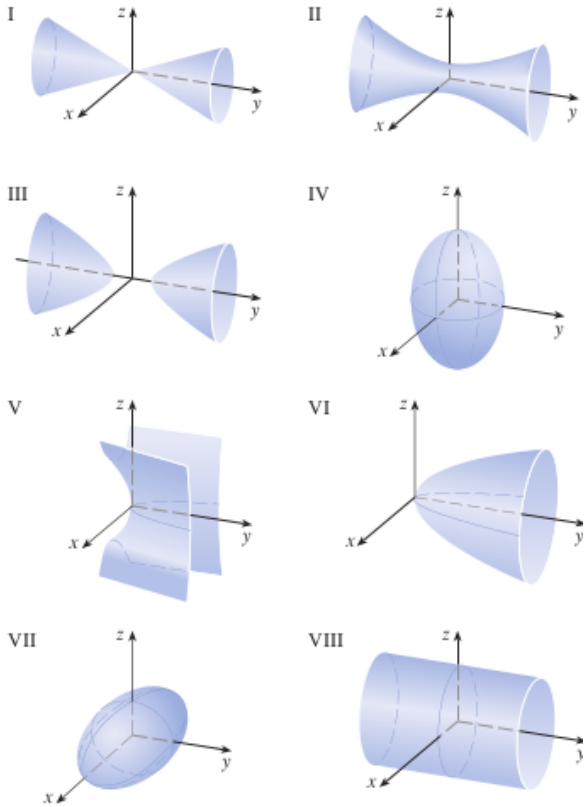
(e) $9x^2 + 4y^2 + z^2 = 1$

(f) $-x^2 + y^2 - z^2 = 1$

(g) $y^2 = x^2 + 2z^2$

(h) $y = x^2 - z^2$

Figure 1:




5. Use traces to sketch and identify the surface.

(a) $x = y^2 + 4z^2$

(b) $x^2 = 4y^2 + z^2$

(c) $z^2 - 4x^2 - y^2 = 4$

(d) $9y^2 + 4z^2 = x^2 + 36$

(e) $3x^2 - y^2 + 3z^2 = 0$

(f) $x = y^2 - z^2$

6. Following two figures are the satellite dish and cooling towers of nuclear reactors. Name these two figures with proper reasoning.

Figure 2: