

CSEN 1003: Compilers

Tutorial 6 - Bottom Up Parsing

Today's Plan

- 1 Bottom Up Parsing
- 2 Shift-Reduce Parsers
- 3 Recap

Bottom Up Parsing

- A bottom up parser constructs a parse tree from the leaves up to the root.

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Example

$$S \rightarrow SS+ \mid SS* \mid a$$

Input: $aa+a*$

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Example

$$S \rightarrow SS+ \mid SS* \mid a$$

Input: $aa+a*$

$$aa+a* \mapsto Sa+a*$$

Bottom Up Parsing

- A bottom up parser constructs a parse tree from the leaves up to the root.

Example

$$S \rightarrow SS+ \mid SS* \mid a$$

Input: aa+a*

$$aa+a* \mapsto S a+a* \mapsto SS+a*$$

Bottom Up Parsing

- A bottom up parser constructs a parse tree from the leaves up to the root.

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$$S \rightarrow SS+ \mid SS* \mid a$$

Input: $aa+a*$

$$aa+a* \mapsto Sa+a* \mapsto SS+a* \mapsto Sa*$$

Bottom Up Parsing

- A bottom up parser constructs a parse tree from the leaves up to the root.

Example

$$S \rightarrow SS+ \mid SS* \mid a$$

Input: $aa+a*$

$$aa+a* \mapsto Sa+a* \mapsto SS+a* \mapsto Sa* \mapsto SS*$$

Bottom Up Parsing

- A bottom up parser constructs a parse tree from the leaves up to the root.

Example

$$S \rightarrow SS+ \mid SS* \mid a$$

Input: $aa+a*$

$$aa+a* \mapsto Sa+a* \mapsto SS+a* \mapsto Sa* \mapsto SS* \mapsto S$$

Reductions and Handles

Example

$$S \rightarrow SS+ \mid SS* \mid a$$

Input: $aa+a*$

$aa+a* \mapsto Sa+a* \mapsto SS+a* \mapsto Sa* \mapsto SS* \mapsto S$

- Hence, basic bottom up parsing corresponds to searching for a **reduction** from the input to the start variable.

Reductions and Handles

Example

$$S \rightarrow SS+ \mid SS* \mid a$$

Input: $aa+a*$

$aa+a* \mapsto Sa+a* \mapsto SS+a* \mapsto Sa* \mapsto SS* \mapsto S$

- Hence, basic bottom up parsing corresponds to searching for a **reduction** from the input to the start variable.
- We are particularly interested in reductions that correspond to the **reverse of a right-most derivation**.

Reductions and Handles

Example

$$S \rightarrow SS+ \mid SS* \mid a$$

Input: $aa+a*$

$aa+a* \mapsto Sa+a* \mapsto SS+a* \mapsto Sa* \mapsto SS* \mapsto S$

- Hence, basic bottom up parsing corresponds to searching for a **reduction** from the input to the start variable.
- We are particularly interested in reductions that correspond to the **reverse of a right-most derivation**.
- The sentential form that is replaced at each reduction step with a variable is called a **handle**.

Handles

Example

$$S \rightarrow SS+ \mid SS* \mid a$$

Indicate the handle in each of the following.

① $SSS+a*+$

Handles

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Handles

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$$S \rightarrow SS+ \mid SS* \mid a$$

Indicate the handle in each of the following.

① $SSS+a*+$

$S \Rightarrow SS+$

Handles

Example

$$S \rightarrow SS+ \mid SS* \mid a$$

Indicate the handle in each of the following.

① $SSS+a*+$

$S \Rightarrow SS+ \Rightarrow SSS*+$

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Example

$$S \rightarrow SS+ \mid SS* \mid a$$

Indicate the handle in each of the following.

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$$S \Rightarrow SS+ \Rightarrow SSS*+ \Rightarrow SSa*+$$

Handles

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Indicate the handle in each of the following.

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② $SS+a*a+$

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Indicate the handle in each of the following.

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Handles

Example

$$S \rightarrow SS+ \mid SS* \mid a$$

Indicate the handle in each of the following.

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$$S \Rightarrow SS+ \Rightarrow SSS*+ \Rightarrow SSa*+ \Rightarrow SSS+a*+$$

② $SS+a*a+$

$$S \Rightarrow SS+$$

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Indicate the handle in each of the following.

① $SSS+a*+$

$$S \Rightarrow SS+ \Rightarrow SSS*+ \Rightarrow SSa*+ \Rightarrow SSS+a*+$$

② $SS+a*a+$

$$S \Rightarrow SS+ \Rightarrow Sa+$$

Handles

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$$S \rightarrow SS+ \mid SS* \mid a$$

Indicate the handle in each of the following.

① $SSS+a*+$

$$S \Rightarrow SS+ \Rightarrow SSS*+ \Rightarrow SSa*+ \Rightarrow SSS+a*+$$

② $SS+a*a+$

$$S \Rightarrow SS+ \Rightarrow Sa+ \Rightarrow SS*a+$$

Handles

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Indicate the handle in each of the following.

① $SSS+a*+$

$$S \Rightarrow SS+ \Rightarrow SSS*+ \Rightarrow SSa*+ \Rightarrow SSS+a*+$$

② $SS+a*a+$

$$S \Rightarrow SS+ \Rightarrow Sa+ \Rightarrow SS*a+ \Rightarrow Sa*a+$$

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Handles: Exercise 6-2

Example

$$S \rightarrow SS+ \mid SS* \mid a$$

Indicate the handle in each of the following.

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$$S \Rightarrow SS+ \Rightarrow Sa+ \Rightarrow SS*a+ \Rightarrow Sa*a+ \Rightarrow SS+a*a+$$

Handles: Exercise 6-2

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$$S \rightarrow SS+ \mid SS* \mid a$$

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② $SS+a*a+$

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③ $aaa*a++$

Handles: Exercise 6-2

Example

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② $SS+a*a+$

$$S \Rightarrow SS+ \Rightarrow Sa+ \Rightarrow SS*a+ \Rightarrow Sa*a+ \Rightarrow SS+a*a+$$

③ $aaa*a++$

$$S \Rightarrow SS+ \Rightarrow SSS++ \Rightarrow SSa++ \Rightarrow SSS*a++ \Rightarrow SSa*a++ \Rightarrow Saa*a++ \Rightarrow aaa*a++$$

Handles: Exercise 6-2

Example

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Indicate the handle in each of the following.

① $SSS+a*+$

$$S \Rightarrow SS+ \Rightarrow SSS*+ \Rightarrow SSa*+ \Rightarrow SSS+a*+$$

② $SS+a*a+$

$$S \Rightarrow SS+ \Rightarrow Sa+ \Rightarrow SS*a+ \Rightarrow Sa*a+ \Rightarrow SS+a*a+$$

③ $aaa*a++$

$$S \Rightarrow SS+ \Rightarrow SSS++ \Rightarrow SSa++ \Rightarrow SSS*a++ \Rightarrow SSa*a++ \Rightarrow Saa*a++ \Rightarrow aaa*a++$$

Handles: Exercise 6-3

Example

$$S \rightarrow 0S1 \mid 0$$

Indicate the handle in each of the following.

① 001

Handles: Exercise 6-3

Example

$$S \rightarrow 0S1 \mid 0$$

Indicate the handle in each of the following.

① 001

$$S \Rightarrow 0S1 \Rightarrow 001$$

Handles: Exercise 6-3

Example

$$S \rightarrow 0S1 \mid 0$$

Indicate the handle in each of the following.

① 001

$$S \Rightarrow 0S1 \Rightarrow 001$$

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Shift Reduce Parsers

- A **non-deterministic** bottom up parser that constructs the reverse of a right most derivation.

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- In q_s , push \$ to the stack and enter q_{loop} .

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Shift Reduce Parsers

- A **non-deterministic** bottom up parser that constructs the reverse of a right most derivation.
- Again, the parser is a PDA with three main states: q_s , q_{loop} , and q_a .
- In q_s , push \$ to the stack and enter q_{loop} .
- In q_{loop} non-deterministically choose one of the following:
 - ① **Shift**: an input symbol by reading it and pushing it to the stack.
 - ② **Reduce**: by popping zero or more stack symbols s_1, \dots, s_n (with s_n on the top of the stack) and $A \rightarrow s_1 \dots s_n$ is a non-deterministically chosen rule, and pushing A.

Shift Reduce Parsers

- A **non-deterministic** bottom up parser that constructs the reverse of a right most derivation.
- Again, the parser is a PDA with three main states: q_s , q_{loop} , and q_a .
- In q_s , push \$ to the stack and enter q_{loop} .
- In q_{loop} non-deterministically choose one of the following:
 - ① **Shift**: an input symbol by reading it and pushing it to the stack.
 - ② **Reduce**: by popping zero or more stack symbols s_1, \dots, s_n (with s_n on the top of the stack) and $A \rightarrow s_1 \dots s_n$ is a non-deterministically chosen rule, and pushing A.
 - ③ Pop the start variable and \$ and enter q_a .

Exercise 6-4

Example

Consider the following grammar:

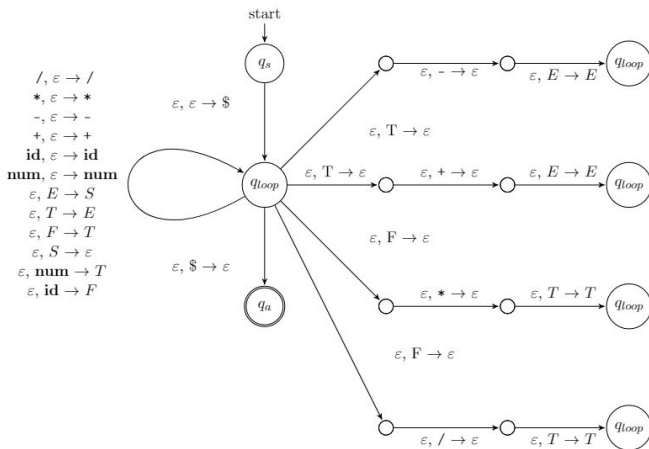
$$\begin{aligned} S &\rightarrow E \\ E &\rightarrow E + T \mid E - T \mid T \\ T &\rightarrow T * F \mid T / F \mid F \\ F &\rightarrow \text{num} \mid \text{id} \end{aligned}$$

and the string: **id - num * id**.

Construct the shift/reduce PDA and trace it on the input string.

Exercise 6-4

Example



Exercise 6-4

Example

	Stack	Input	Action
1	\$	id - num * id \$	

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * id \Rightarrow E - F * id \Rightarrow$
 $E - num * id \Rightarrow T - num * id \Rightarrow F - num * id \Rightarrow id - num * id$

Exercise 6-4

Example

	Stack	Input	Action
1	\$	id - num * id \$	Shift

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * id \Rightarrow E - F * id \Rightarrow$
 $E - num * id \Rightarrow T - num * id \Rightarrow F - num * id \Rightarrow id - num * id$

Exercise 6-4

Example

	Stack	Input	Action
1	\$	id - num * id \$	Shift
2	\$id	- num * id \$	

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * id \Rightarrow E - F * id \Rightarrow$
 $E - num * id \Rightarrow T - num * id \Rightarrow F - num * id \Rightarrow id - num * id$

Exercise 6-4

Example

	Stack	Input	Action
1	\$	id - num * id \$	Shift
2	\$ id	- num * id \$	Reduce $F \rightarrow \mathbf{id}$

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * id \Rightarrow E - F * id \Rightarrow$
 $E - num * id \Rightarrow T - num * id \Rightarrow F - num * id \Rightarrow id - num * id$

Exercise 6-4

Example

	Stack	Input	Action
1	\$	id - num * id \$	Shift
2	\$ id	- num * id \$	Reduce $F \rightarrow \mathbf{id}$
3	\$ F	- num * id \$	

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * id \Rightarrow E - F * id \Rightarrow$
 $E - num * id \Rightarrow T - num * id \Rightarrow F - num * id \Rightarrow id - num * id$

Exercise 6-4

Example

	Stack	Input	Action
1	\$	id - num * id \$	Shift
2	\$id	- num * id \$	Reduce $F \rightarrow \mathbf{id}$
3	\$F	- num * id \$	Reduce $T \rightarrow F$

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * id \Rightarrow E - F * id \Rightarrow$
 $E - num * id \Rightarrow T - num * id \Rightarrow F - num * id \Rightarrow id - num * id$

Exercise 6-4

Example

	Stack	Input	Action
1	\$	id - num * id \$	Shift
2	\$ id	- num * id \$	Reduce $F \rightarrow \mathbf{id}$
3	\$ F	- num * id \$	Reduce $T \rightarrow F$
4	\$ T	- num * id \$	

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * id \Rightarrow E - F * id \Rightarrow$
 $E - num * id \Rightarrow T - num * id \Rightarrow F - num * id \Rightarrow id - num * id$

Exercise 6-4

Example

	Stack	Input	Action
1	\$	id - num * id \$	Shift
2	\$ id	- num * id \$	Reduce $F \rightarrow \mathbf{id}$
3	\$ F	- num * id \$	Reduce $T \rightarrow F$
4	\$ T	- num * id \$	Reduce $E \rightarrow T$
5	\$ E	- num * id \$	

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * id \Rightarrow E - F * id \Rightarrow$
 $E - num * id \Rightarrow T - num * id \Rightarrow F - num * id \Rightarrow id - num * id$

Exercise 6-4

Example

	Stack	Input	Action
1	\$	id - num * id \$	Shift
2	\$ id	- num * id \$	Reduce $F \rightarrow \mathbf{id}$
3	\$ F	- num * id \$	Reduce $T \rightarrow F$
4	\$ T	- num * id \$	Reduce $E \rightarrow T$
5	\$ E	- num * id \$	Shift

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * id \Rightarrow E - F * id \Rightarrow$
 $E - num * id \Rightarrow T - num * id \Rightarrow F - num * id \Rightarrow id - num * id$

Exercise 6-4

Example

	Stack	Input	Action
1	\$	id - num * id \$	Shift
2	\$id	- num * id \$	Reduce $F \rightarrow \text{id}$
3	\$F	- num * id \$	Reduce $T \rightarrow F$
4	\$T	- num * id \$	Reduce $E \rightarrow T$
5	\$E	- num * id \$	Shift
6	\$E-	num * id \$	

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * \text{id} \Rightarrow E - F * \text{id} \Rightarrow$
 $E - \text{num} * \text{id} \Rightarrow T - \text{num} * \text{id} \Rightarrow F - \text{num} * \text{id} \Rightarrow \text{id} - \text{num} * \text{id}$

Exercise 6-4

Example

	Stack	Input	Action
1	\$	id - num * id \$	Shift
2	\$id	- num * id \$	Reduce $F \rightarrow \text{id}$
3	\$F	- num * id \$	Reduce $T \rightarrow F$
4	\$T	- num * id \$	Reduce $E \rightarrow T$
5	\$E	- num * id \$	Shift
6	\$E-	num * id \$	Shift

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * \text{id} \Rightarrow E - F * \text{id} \Rightarrow$
 $E - \text{num} * \text{id} \Rightarrow T - \text{num} * \text{id} \Rightarrow F - \text{num} * \text{id} \Rightarrow \text{id} - \text{num} * \text{id}$

Exercise 6-4

Example

	Stack	Input	Action
1	\$	id - num * id \$	Shift
2	\$id	- num * id \$	Reduce $F \rightarrow \text{id}$
3	\$F	- num * id \$	Reduce $T \rightarrow F$
4	\$T	- num * id \$	Reduce $E \rightarrow T$
5	\$E	- num * id \$	Shift
6	\$E-	num * id \$	Shift
7	\$E-num	* id \$	

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * \text{id} \Rightarrow E - F * \text{id} \Rightarrow$
 $E - \text{num} * \text{id} \Rightarrow T - \text{num} * \text{id} \Rightarrow F - \text{num} * \text{id} \Rightarrow \text{id} - \text{num} * \text{id}$

Exercise 6-4

Example

	Stack	Input	Action
1	\$	id - num * id \$	Shift
2	\$id	- num * id \$	Reduce $F \rightarrow \text{id}$
3	\$F	- num * id \$	Reduce $T \rightarrow F$
4	\$T	- num * id \$	Reduce $E \rightarrow T$
5	\$E	- num * id \$	Shift
6	\$E-	num * id \$	Shift
7	\$E-num	* id \$	Reduce $F \rightarrow \text{num}$

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * \text{id} \Rightarrow E - F * \text{id} \Rightarrow$
 $E - \text{num} * \text{id} \Rightarrow T - \text{num} * \text{id} \Rightarrow F - \text{num} * \text{id} \Rightarrow \text{id} - \text{num} * \text{id}$

Exercise 6-4

Example

	Stack	Input	Action
8	$\$E-F$	* id \$	Reduce $T \rightarrow F$
9	$\$E-T$	* id \$	Shift
10	$\$E-T*$	id \$	Shift
11	$\$E-T*\mathbf{id}$	\$	Reduce $F \rightarrow \mathbf{id}$
12	$\$E-T*F$	\$	Reduce $T \rightarrow T*F$
13	$\$E-T$	\$	Reduce $E \rightarrow E-T$
14	$\$E$	\$	Accept

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * id \Rightarrow E - F * id \Rightarrow$
 $E - num * id \Rightarrow T - num * id \Rightarrow F - num * id \Rightarrow id - num * id$

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Example

	Stack	Input	Action
8	$\$E-F$	* id \$	Reduce $T \rightarrow F$
9	$\$E-T$	* id \$	Shift
10	$\$E-T*$	id \$	Shift
11	$\$E-T*\mathbf{id}$	\$	Reduce $F \rightarrow \mathbf{id}$
12	$\$E-T*F$	\$	Reduce $T \rightarrow T*F$
13	$\$E-T$	\$	Reduce $E \rightarrow E-T$
14	$\$E$	\$	Accept

RMD: $E \Rightarrow E - T \Rightarrow E - T * F \Rightarrow E - T * id \Rightarrow E - F * id \Rightarrow$
 $E - num * id \Rightarrow T - num * id \Rightarrow F - num * id \Rightarrow id - num * id$

Can the very first step be a reduce?

Conflicts

- 1 **Shift-Reduce Conflicts:** when both shifting and reducing are possible.
- 2 **Reduce-Reduce Conflicts:** when two ways of reduction are possible.

Exercise 6-4: Failing Branch

Example

	Stack	Input	Action
	\$id-num*id	\$	Reduce $F \rightarrow id$
	\$id-num*F	\$	Reduce $T \rightarrow F$
	\$id-num*T	\$	Reduce $E \rightarrow T$
	\$id-num*E	\$	

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Covered Topics

- 1 Bottom Up Parsing.
- 2 Shift-Reduce Parsers.

Next Tutorial: SLR Parsing!