Lexical Analysis

Lecture 2

Objectives

By the end of this lecture you should be able to:

1. Identify the role of lexical analysis in a compiler.
2. Design regular definitions for regular languages.
3. Design action-augmented regular definitions for regular languages.
4. Design fallback DFA with actions for regular languages.

Outline

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1. The Role of Lexical Analysis
2. Regular Definitions
3. Digression: Lexical Ambiguity
4. The Problem of String Tokenization

What It Does

Main Function

1. Partition the input stream into lexemes.
2. Generate a token for each lexeme.

Auxiliary Function

Ignores substrings which are insignificant for the compiling process.

- e.g., comments and white spaces.

Other Possible Functions

- Macro expansion.
- Keeping track of line numbers for informative error messages.

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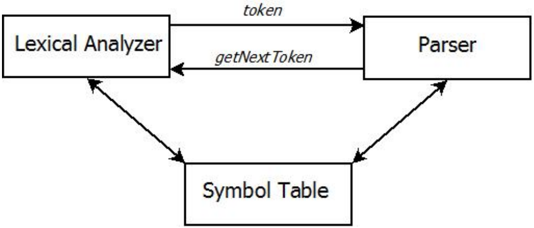
Connection to the Rest of the System



Associated Concepts: Tokens

Tokens

- A token is a tuple of the form $\langle L, A\rangle$ or $\langle L\rangle$, where
  - L is the name of a lexical category, and
  - A is an attribute.
- Lexical categories are terminals from the parser perspective.
- Attributes carry semantically-relevant information about the particular instance of L encountered.
- No attributes are needed if L is a single-instance category. (Hence, the second form)
- An instance of a lexical category is called a lexeme.

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Associated Concepts: Patterns

Patterns

- The set of instances of a lexical category constitute a language.
- Typically, this language is regular.
- A pattern for a particular lexical category is a description of the form of lexemes in that language.
- This description is sometimes a simple enumeration of the lexemes.
  - As in natural languages.
- May also be described by some kind of grammar; typically a regular grammar or, equivalently, a regular expression.

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Example

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Lexical Category	Pattern
if	{`if`, `If`, `iF`, `IF`}
else	{`else`, ..., `ELSE`}
comp	$\{>, <, >=, <=, ==, != \}$
id	any letter followed by letters or digits
num	any numeric literal
lit	any string between " and "
lp	{(}
rp	{)}

- Input: `if (x > 10) printf("Yes") else printf("No")`
- Output:
  [⟨**if**⟩, ⟨**lp**⟩, ⟨**id**, 1⟩, ⟨**comp**, >⟩, ⟨**num**, 10⟩, ⟨**rp**⟩,
  ⟨**id**, 2⟩, ⟨**lp**⟩, ⟨**lit**, `Yes`⟩, ⟨**rp**⟩,
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Regular Expressions

Definition

R is a regular expression over alphabet Σ if R is

1. a for some $a \in \Sigma$,
2. ε,
3. $\emptyset$,
4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions,
5. $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions, or
6. (R_1^*), where R_1 is a regular expression.

- Note:
  - $L(a) = \{a\}; L(\varepsilon) = \{\varepsilon\}; L(R^*) = (L(R))^*$.
  - $L(R_1 \otimes R_2) = L(R_1) \otimes L(R_2)$, for $\otimes \in \{\cup, \circ\}$.

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Extended Language of Regular Expressions

Some convenient abbreviations:

- $R_1|R_2 = R_1 \cup R_2$.
- $R_1R_2 = R_1 \circ R_2$.
- $R^+ = R \circ R^*$.
- $\Sigma = a_1|a_2|\ldots|a_n$, where $\Sigma = \{a_1, a_2, \ldots, a_n\}$.
- $R? = R|\varepsilon$.
- $[a_1a_2\ldots a_n] = a_1|a_2|\ldots|a_n$, where $\{a_1, a_2, \ldots, a_n\} \subseteq \Sigma$.
- $[a_1 - a_n] = [a_1, a_2, \ldots, a_n]$, provided that $\langle a_1, a_2, \ldots, a_n\rangle$ is a natural permutation of $\{a_1, a_2, \ldots, a_n\}$.

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Regular Definition

Definition

A regular definition $\mathfrak{R}(\Sigma, D)$ over alphabets Σ and D is a finite sequence $\langle P_1, \ldots, P_n \rangle$ of pairs $P_i = \langle D_i, R_i \rangle$, where

- $D = \{D_1, \ldots, D_n\}$.
- $D \cap \Sigma = \varnothing$.
- $|D| = n$.
- R_i is a regular expression over $\Sigma \cup \{D_1, \ldots, D_{i-1}\}$, for $1 \leq i \leq n$.

Note:

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Example 1

Example (C Identifiers)

$$\begin{array}{lcl} letter\_ & \longrightarrow & [A-Za-z] \mid \_ \\ digit & \longrightarrow & [0-9] \\ id & \longrightarrow & letter\_(letter\_|digit)^* \end{array}$$

Example 2

Example (Unsigned Numeric Literals)

$$
\begin{array}{lcl}
digit & \longrightarrow & [0-9] \\
digits & \longrightarrow & digit^{+} \\
opFrac & \longrightarrow & (.\ digits)? \\
opExp & \longrightarrow & (\texttt{E}\ [+-]?\ digits)? \\
number & \longrightarrow & digits\ opFrac\ opExp
\end{array}
$$

How to Build a Lexical Analyzer in Five Steps

1. Write a regular definition where, for each of the n lexical categories of our language L, there is a pair $\langle D_i, R_i \rangle$, where D_i is the category name and R_i is its pattern.
   - That this can be done is a non-trivial, but practically-valid, assumption.
2. For each D_i $(1 \leq i \leq n)$ compile a corresponding regular expression R_i.
   - How?
3. Construct the expression $R = (R_1|R_2|\cdots|R_n)^+$.
   - $L(R) \supseteq L$ is the set of *lexically-correct* strings of our language.
4. Convert R into an equivalent NFA.
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But . . .

- This can only *recognize* lexically-correct programs.
- It does not split the input into lexemes and does not generate a stream of tokens.
- Need to augment this procedure with mechanisms to do so.

Outline

Lexical Ambiguity

- When scanning a program from left to right, we do not know where a potential lexeme ends.
- This is caused by two types of *lexical ambiguity*:
  1. There is more than one way to split the program into lexemes.
  2. For a fixed splitting, there is more than one stream of tokens that can be generated.
- The first kind of ambiguity occurs when one lexeme is a proper prefix of another.
- The second occurs when a lexeme matches the patterns of more than one lexical category.

Examples

Example (Artificial Languages)

Let $R = (\texttt{a} \mid \texttt{abb} \mid \texttt{a}^*\texttt{b}^+)^*$.

1. The string `aabb` could be split as [`a`, `abb`] or as [`aabb`].
2. The string `abb` matches both `abb` and $\texttt{a}^*\texttt{b}^+$.

Example (Programming Languages)

- <= is either [⟨**comp**, <⟩, ⟨**comp**, =⟩] or [⟨**comp**, <=⟩].
- `if` is either [⟨**if**⟩] or [⟨**id**, ?⟩].

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- `if` is either $[\langle \textbf{if} \rangle]$ or $[\langle \textbf{id}, ? \rangle]$.

Common Disambiguation Strategies

- For the first type of ambiguity, opt for the splitting with the longest possible prefix lexeme:
    - If $[l_{11}, l_{21}, \ldots, l_{n1}]$ and $[l_{12}, l_{22}, \ldots, l_{m2}]$ are two splittings with $|l_{i1}| > |l_{i2}|$ and $l_{j1} = l_{j2}$ for $1 \leq j < i$, choose the first.
- For the second type, opt for the lexical category whose pattern appears earlier in the regular definition.
    - If the lexeme matches p_i and p_j, where $i < j$, choose $\langle D_i, p_i \rangle$.

Outline

Action-Augmented Regular Definitions

Definition

An action-augmented regular definition is a triple $\langle \mathfrak{R}(\Sigma, D), \mathcal{C}, \mathcal{A} \rangle$, where

- $\mathfrak{R}(\Sigma, D)$ is a regular definition,
- $\mathcal{C} \subseteq D$, and
- $\mathcal{A}$ is a function which maps every $c \in \mathcal{C}$ to some *action*.

- An action is an algorithm which possibly returns some value and side-affects some data structures (the symbol table, for example).

Example

Example

In the sequel, *lex* is the lexeme matching a pattern.

$$
\begin{array}{lcl}
ws & \longrightarrow & [\ \texttt{\textbackslash t\textbackslash n}]^{+} \\
 & & \mathcal{A}(ws) = \{\} \\
letter\_ & \longrightarrow & [\texttt{A} - \texttt{Za} - \texttt{z}] \mid \_ \\
digit & \longrightarrow & [\texttt{0} - \texttt{9}] \\
id & \longrightarrow & letter\_(letter\_|digit)^{*} \\
 & & \mathcal{A}(id) = \{return(\langle \mathbf{id}, consultTable(lex)\rangle)\} \\
number & \longrightarrow & (digit)^{+}\ (.\ digit^{+})?\ (\texttt{E}\ [+-]?\ digit^{+})? \\
 & & \mathcal{A}(number) = \{return\langle \mathbf{num}, lex\rangle\}
\end{array}
$$

Fallback DFA with Actions

Definition

A fallback DFA with actions is a 6-tuple $\langle Q, \Sigma, \delta, q_0, F, A \rangle$, where

- Q, Σ, δ, q_0, and F are as usual; and
- A maps every $q \in Q$ into an action.

Anatomy

- A fallback DFA with actions consists of a finite control, an infinite (to the right) tape, a stack, and two heads: L and R.
- Both heads are read-only heads.
- Initially, both heads point at the left-most tape cell, where the input starts.
- R can move only to the right.
- L can move to the right and to the left as explained in the sequel.

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Operation

- A fallback DFA with actions operates like the standard DFA, moving only L, and pushing every state it enters into the stack with every transition.
- This continues until the DFA runs out of input.
- If it runs out of input in state $q_a \in F$, it executes $A(q_a)$ and halts.
- If it runs out of input in $q_r \notin F$, it
  1. continues to simultaneously pop the stack and move L one step to the left until the stack gets empty or some $q_a \in F$ is popped.
  2. In the first case, the DFA executes $A(q_r)$ and halts.
  3. In the second case it does the following.
     1. Executes $A(q_a)$ (with *lex* being the string extending from R to L).
     2. Moves L one step to the right.
     3. Moves R to where L is.
     4. Empties the stack.
     5. Enters q_0.

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How to Build a Lexical Analyzer in Five Steps (I)

1. Write an action-augmented regular definition where, for each of the n lexical categories of our language L, there is a pair $\langle D_i, R_i \rangle$, where D_i is the category name and R_i is its pattern, with $D_i \in \mathcal{C}$.
   - Actions should produce the appropriate tokens and update the symbol table as needed.

How to Build a Lexical Analyzer in Five Steps (II)

2. For each $c \in \mathcal{C}$ compile a corresponding regular expression R_c.
   - Each R_c is associated with the pair $\langle c, \mathcal{A}(c) \rangle$.

How to Build a Lexical Analyzer in Five Steps (III)

3. Construct the regular expression $R = (R_{c_1}|R_{c_2}|\cdots|R_{c_m})$, where $\mathcal{C} = \{c_1, c_2, \ldots, c_m\}$.
   - Note the absence of $+$.

How to Build a Lexical Analyzer in Five Steps (IV)

4. Construct an NFA N which is equivalent to R, provided that
   1. In constructing an NFA N_i equivalent to R_i, make sure that N_i has a unique accept state.
   2. The unique accept state of N_i has the label c_i.

How to Build a Lexical Analyzer in Five Steps (V)

5. Construct a fallback DFA with actions, M, which is equivalent to N, provided that

1. For every $q_a \in F$, $A(q_a) = \mathcal{A}(c_i)$, where
   1. $c_i \in q_a$ and
   2. if $c_j \in q_a$, then $i \leq j$.
2. For every $q_r \notin F$, $A(q_a)$ is a suitable "error action."

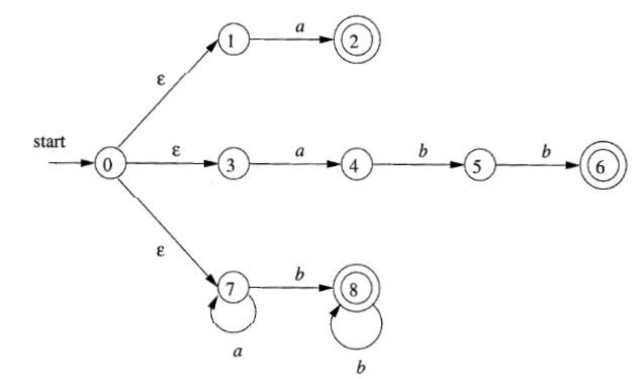
Example (I)

Example (Action-Augmented Regular Definition)

$$
\begin{aligned}
2 &\longrightarrow \mathtt{a} \\
& \quad \mathcal{A}(2) = \{return(\langle \mathbf{2}, lex \rangle)\} \\
6 &\longrightarrow \mathtt{abb} \\
& \quad \mathcal{A}(6) = \{return(\langle \mathbf{6}, lex \rangle)\} \\
8 &\longrightarrow \mathtt{a}^{*}\mathtt{b}^{+} \\
& \quad \mathcal{A}(8) = \{return(\langle \mathbf{8}, lex \rangle)\}
\end{aligned}
$$

Example (II)

Example (Equivalent NFA)



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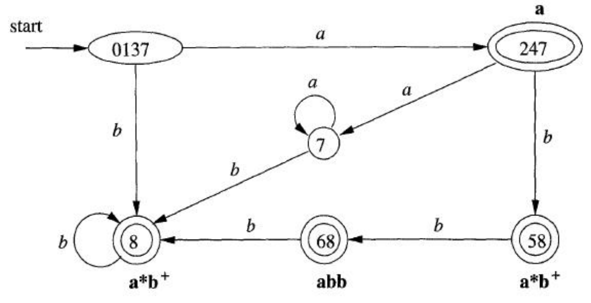
Example (III)

Example (Fallback DFA with Actions)



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What happens on input `abba`?