

Lecture (2)

Cryptographic Tools

Early History

- □ Egypt (Old Kingdom), ca. 1900BC. Use of non-standard hieroglyphs (probably not a serious attempt at secret communication)
- Mesopotamia, ca. 1500BC. Encrypted recipe for pottery glaze on clay tablet
- ☐ Hebrew, ca. 500BC. Monoalphabetic substitution cipher used by scholars

Not so Early History

- ■800AD. Early description of frequency analysis for breaking substitution ciphers (credited to Arab mathematician Al-Kindi, 801–873). Works about cryptanalysis of single- and polyalphabetic ciphers
- Ahmad al-Qalqashandi 1355–1418 List of ciphers in his encyclopedia contains a cipher with multiple substitutions for each letter; frequency tables for letters to aid cryptanalysis
- □1467: Leon Battista Alberti ("father of Western cryptology") describes the polyalphabetic cipher

Recent History

☐ WWII heavy use of rotor machines for complex polyalphabetic substitution ciphers

The "Enigma" machine:







Recent History

- ☐ The time of WWII brought massive advances in cryptography as well as cryptanalysis
- ☐ In the 20th century, mathematical cryptography was developed
 - Works by Claude Shannon. Any theoretically unbreakable cipher must have keys which are at least as long as the plaintext and used only once (one-time pad)
 - Publication of the Data Encryption Standard in 1970
- □ 1976 Ground-breaking paper: Whitfield Diffie and Martin Hellman. "New Directions in Cryptography" solves key exchange problem and sparks development of asymmetric key algorithms

Classical Substitution Ciphers

- Where letters of plaintext are replaced by other letters or by numbers or symbols
- Or if plaintext is viewed as a sequence of bits, then substitution involves replacing plaintext bit patterns with ciphertext bit patterns

Caesar Cipher

- Earliest known substitution cipher (invented by Julius Caesar)
- First attested use in military affairs
- Replaces each letter by 3rd letter down in the alphabet. Example:

```
Meet me after the toga party PHHW PH DIWHU WKH WRJD SDUWB
```

Can define transformation as:

```
abcdefghijklmnopqrstuvwxyz
DEFGHIJKLMNOPQRSTUVWXYZABC
```

Mathematically give each letter a number

```
abcdefghij k l m n o p q r s t u v w x y z 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
```

■ Then have Caesar cipher as:

$$c = E(p) = (p + k) \mod (26)$$

 $p = D(c) = (c - k) \mod (26)$

Breaking Caesar Cipher

- Only have 26 possible ciphers
 - A maps to A,B,..Z
- Could simply try each in turn
- a brute force search
- Given ciphertext, just try all shifts of letters
- Do need to recognize when have plaintext
- Compression reduces chance of breaking

```
~+Wµ"— \Omega-O)\leq 4{\infty‡, ë~\Omega%ràu·¯Í ^{\circ}Z- Ú\neq 2Ô\#Åæ^{\circ} œ«q7,\Omegan·®3N^{\circ}Ú Œz'Y-f\inftyÍ[\pmÛ_ è\Omega,<NO¬\pm«×xã Åä£èü3Å x}ö§k°Â _yÍ ^\DeltaÉ] ,¤ J/°iTê&ı 'c<u\Omega- ÄD(G WÄC~Y_ïÕÄW PÔı«Î܆ç],¤; Ĭ^ûÑ\pi~*L 90gflO &Œ\leq ¬\leq ØÔ§″: Č!SGqèvo^ ú\,S>h<-*6\phi‡%x′″|fiÓ\#~my%*\geqñP<,fi Áj Å^{\circ}¿″Zù- \Omega"Õ 6Œÿ{% "\OmegaÊÓ ,ï \pi÷Áî°úO2çSÿ´O- 2Äflßi /@^"\PiK°°PŒ\pi,úé^´3\Sigma~ö°ÔZÌ"Y¬Ÿ\Omega@Y> \Omega+eô/`<K£¿*÷~"\leqû~ B Z^{\circ}K°Qßÿüf,!ÒflÎzsS/]>ÈQ ü
```

```
PHHW PH DIWHU WKH WRJD SDUWB
KEY
          oggv og chvgt vjg vgic rctva
          nffu nf bgufs uif uphb qbsuz
          meet me after the toga party
          ldds ld zesdg sqd snfz ozgsx
          kccr kc ydrcp rfc rmey nyprw
          jbbq jb xcqbo qeb qldx mxoqv
          iaap ia wbpan pda pkcw lwnpu
          hzzo hz vaozm ocz ojbv kvmot
          gyyn gy uznyl nby niau julns
          fxxm fx tymxk max mhzt itkmr
   10
          ewwl ew sxlwj lzw lgys hsjlq
   11
          dvvk dv rwkvi kyv kfxr grikp
   12
          cuuj cu qvjuh jxu jewq fqhjo
   13
          btti bt puitg iwt idvp epgin
   14
          assh as othsf hvs houo dofhm
   15
          zrrg zr nsgre gur gbtn cnegl
   16
          yqqf yq mrfqd ftq fasm bmdfk
   17
          xppe xp lgepc esp ezrl alcej
   18
          wood wo kpdob dro dyqk zkbdi
   19
          vnnc vn jocna cqn cxpj yjach
   20
          ummb um inbmz bpm bwoi xizbg
   21
          tlla tl hmaly aol avnh whyaf
   22
          skkz sk glzkx znk zumg vgxze
   23
          rjjy rj fkyjw ymj ytlf ufwyd
   24
          qiix qi ejxiv xli xske tevxc
```

Monoalphabetic Cipher

- Rather than just shifting the alphabet, shuffle (jumble) the letters arbitrarily
- Each plaintext letter maps to a different random ciphertext letter
- Hence key is 26 letters long

Plain: abcdefghijklmnopqrstuvwxyz Cipher: DKVQFIBJWPESCXHTMYAUOLRGZN

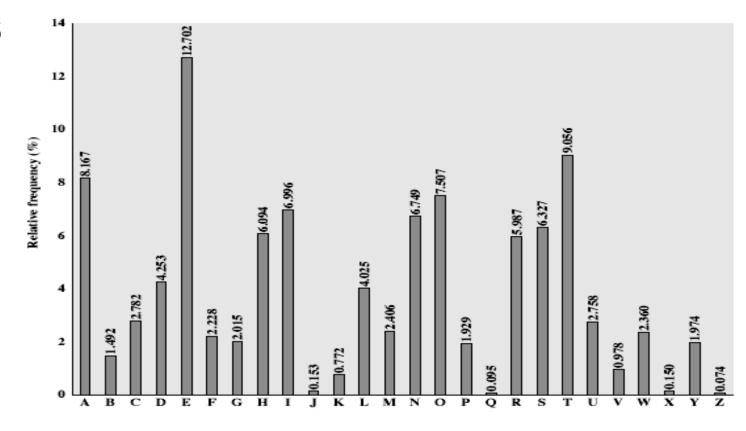
Plaintext: ifwewishtoreplaceletters Ciphertext: WIRFRWAJUHYFTSDVFSFUUFYA

- Now have a total of $26! = 4 \times 10^{26}$ keys
- With so many keys, might think is secure
- But would be !!!WRONG!!!

Given ciphertext:

UZQSOVUOHXMOPVGPOZPEVSGZWSZOPFPESXUDBMETSXAIZ
VUEPHZHMDZSHZOWSFPAPPDTSVPQUZWYMXUZUHSX
EPYEPOPDZSZUFPOMBZWPFUPZHMDJUDTMOHMQ

- Human languages are redundant, eg "secrty s awsm"
- letters are not equally commonly used
- In English E is by far the most common letter
 - followed by T,R,N,I,O,A,S
- Other letters likeZ,J,K,Q,X are fairlyrare
- Have tables of single, double & triple letter frequencies for various languages



- Key concept monoalphabetic substitution ciphers do not change relative letter frequencies
- Discovered by Arabian scientists in 9th century
- Calculate letter frequencies for ciphertext
- Compare counts/plots against known values
- If Caesar cipher look for common peaks/troughs
 - peaks at: A-E-I triple, NO pair, RST triple
 - troughs at: JK, X-Z
- For monoalphabetic must identify each letter
 - tables of common double/triple letters help

Given ciphertext:

UZQSOVUOHXMOPVGPOZPEVSGZWSZOPFPESXUDBMETSXAIZ VUEPHZHMDZSHZOWSFPAPPDTSVPQUZWYMXUZUHSX EPYEPOPDZSZUFPOMBZWPFUPZHMDJUDTMOHMO

Count relative letter frequencies

P 13.33	H 5.83	F 3.33	B 1.67	C 0.00
Z 11.67	D 5.00	W 3.33	G 1.67	K 0.00
S 8.33	E 5.00	Q 2.50	Y 1.67	L 0.00
U 8.33	V 4.17	T 2.50	I 0.83	N 0.00
O 7.50	X 4.17	A 1.67	J 0.83	R 0.00
M 6.67	A 4.17	A 1.07	J 0.83	K 0.00

- Guess P & Z are e and t
- Guess ZW is "th" and hence ZWP is "the". Frequency of two-letter combinations is known as digrams
- Proceeding with trial and error finally get:

```
it was disclosed yesterday that several informal but
direct contacts have been made with political
representatives of the viet cong in moscow
```

Polyalphabetic Cipher

- Improve security using multiple cipher alphabets
- Make cryptanalysis harder with more alphabets to guess and flatter frequency distribution
- Use a key to select which alphabet is used for each letter of the message
- Use each alphabet in turn
- Repeat from start after end of key is reached

Vigenère Cipher

- Simplest polyalphabetic substitution cipher
- Effectively multiple Caesar ciphers
- \square Key is multiple letters long $K = k_1 k_2 \dots k_d$, ith letter specifies ith alphabet to use
- Use each alphabet in turn
- Repeat from start after d letters in message
- Decryption simply works in reverse
- □ Write the plaintext out. Write the keyword repeated above it
- Use each key letter as a Caesar cipher key. Encrypt the corresponding plaintext letter

eg using keyword deceptive

<pre>key:</pre>

- plaintext:
- ciphertext:

d	e	С	e	р	t	i	>	е	d	е	С	е	р	t	i	V	е	d	е	С	е	р	t	i	>	е
w	e	a	r	е	d	ï	S	С	0	V	е	r	е	d	S	а	V	e	У	0	u	r	S	е		f
Z	ı	С	V	Т	W	Q	Ν	G	R	Z	G	V	Т	W	Α	V	Z	Ι	С	Q	Υ	G	L	М	G	J

	Α	В	C	D	Ш	F	G	Η	1	1	K	L	М	N	0	Р	Q	R	S	T	U	٧	W	Х	Υ	Z
A	Α	В	С	D	E	F	G	Н	ı	Ţ	K	L	M	N	Ó	P	Q	R	S	T	U	٧	W	Х	Υ	Z
В	В	C	D	Е	F	G	H	ł	J	K	L	M	N	0	P	Q	R	S	T	U	٧	W	Х	Υ	Z	Α
С	С	D	E	F	G	Н	I	j	K	L	M	N	0	P	Q	R	S	T	U	٧	W	Х	Y	Z	Α	В
D	D	Е	F	G	Н	ı	j	K	L	M	N	0	P	Q	R	S	Т	U	٧	W	Х	Y	Z	Α	В	С
Е	Е	F	G	Н	I	J	K	L	M	N	0	P	Q	R	S	Т	U	٧	W	Х	Y	Z	Α	В	С	D
F	F	G	Н	I	j	K	L	M	N	0	P	Q	R	S	T	U	٧	W	Х	Y	Z	Α	В	С	D	Ε
G	G	Н	I	j	K	L	M	N	0	Р	Q	R	S	Т	כ	٧	W	Х	Υ	Z	Α	В	С	D	E	F
H	Н	1	j	K	L	M	N	0	Р	ď	R	S	Т	U	٧	W	Х	Υ	Z	Α	В	С	D	E	F	G
- 1	I	j	K	L.	M	N	0	P	Q	R	S	T	U	٧	W	Х	Y	Z	Α	В	С	D	Е	F	G	Н
J	J	K	L	M	N	0	Р	Q	R	S	Т	U	٧	W	Х	Y	Z	A	В	С	D	E	F	G	Н	ı
K	K	L	M	N	О	P	ď	R	S	I	U	V	W	Х	Y	Z	Α	В	C	D	Е	F	G	Н	I	j
L	L	M	N	0	P	Q	R	S	T	U	٧	W	Х	Y	Z	Α	В	С	Ď	E	F	G	H	I	J	K
M	M	N	0	Р	Q	R	S	Т	D	٧	W	Х	Υ	Z	Α	В	С	D	Е	F	G	Н	I	j	K	L
N	N	0	Р	Q	R	S	T	U	٧	W	Х	Y	Z	A	В	C	D	Е	F	G	H	ı	1	K	L	M
O	O	P	ď	R	S	Т	۳	٧	٧	Χ	Y	Z	Ą	ф	O	۵	E	F	U	I	-	j	K	L	M	N
Р	P	ď	R	S	T	٦	>	W	Х	Y	Z	Α	В	U	۵	ш	F	ø	π	-	į	K	ш	M	N	0
Q	ø	Ŕ	S	 	U	٧	¥	Х	Y	Z	Ą	В	O	۵	ш	ш	G	I	1		K	L	M	N	0	P
R	R	S	T	D	٧	W	Х	Υ	Z	A	В	O	۵	Ш	Щ	ø	Η	-	₩,	K	ш	M	N	0	Р	Q
W	5	-	۳	>	W	Х	Y	Z	Ą	В	φ	۵	ш	Щ	G	I		j	K	ш	Δ	N	0	P	Q	R
T	T	Ç	<	W	Х	Υ	Z	Α	В	С	D	Е	F	G	Н	ı	J	K	L	M	N	0	P	Q	R	S
U	U	٧	W	Х	Y	Z	Α	В	O	D	Е	F	G	Τ	I	J	K	L	M	N	0	P	ø	R	S	Т
٧	٧	W	Х	Υ	Z	Α	В	С	۵	ш	H.	G	H	-	J	K	L	Μ	Z	0	P	Q	R	S	T	U
W	W	Х	Υ	Z	Α	В	O	D	Ш	щ	G	I	I	j	K	L	M	N	0	P	ď	R	S	Т	U	٧
Х	Х	Y	Z	Α	В	С	D	E	F	G	Н	I	j	K	ш	M	N	0	P	Q	R	S	Т	U	٧	W
Υ	Υ	Z	Α	В	С	D	Ε	F	G	Н	I	j	K	L	М	N	0	Р	Q	R	S	Т	U	٧	W	Х
Z	Z	A	В	С	D	Е	F	G	Н	1	J	K	L	М	N	0	P	Q	R	S	Т	U	٧	W	Х	Υ

Breaking the Vigenère Cipher

- Have multiple ciphertext letters for each plaintext letter
- Hence letter frequencies are obscured, but not totally lost
- Start with letter frequencies
 - see if look like monoalphabetic or not
- If not, then need to determine number of alphabets, since then can attach each
- Kasiski method developed by Babbage/Kasiski is a way of breaking Vigenere
- Repetitions in ciphertext give clues to period, so find same plaintext an exact period apart which results in the same ciphertext
- Of course, could also be random fluke
- eg repeated "VTW" in previous example suggests size of 3 or 9
- Then attack each monoalphabetic cipher individually using same techniques as before

One-time Pad

- □ A random key as long as the message is used to encrypt and decrypt a single message
- ☐ The key is then discarded, never to be used again
- ☐ The output bears no statistical relationship to the plaintext
- Given any plaintext of equal length to the ciphertext, there is a key that produces that plaintext. Therefore, if you did an exhaustive search of all possible keys, you would end up with many legible plaintexts, with no way of knowing which was the intended plaintext

ciphertext: ANKYODKYUREPFJBYOJDSPLREYIUNOFDOIUERFPLUYTS key: pxlmvmsydofuyrvzwc tnlebnecvgdupahfzzlmnyih plaintext: mr mustard with the candlestick in the hall ciphertext: ANKYODKYUREPFJBYOJDSPLREYIUNOFDOIUERFPLUYTS key: mfugpmiydgaxgoufhklllmhsqdqogtewbqfgyovuhwt plaintext: miss scarlet with the knife in the library

☐ Large number of keys need to be used. Key distribution is a big problem

Transposition Ciphers

- Now consider classical transposition or permutation ciphers
- These hide the message by rearranging the letter order without altering the actual letters used
- Can recognise these since have the same frequency distribution as the original text
- Simplest transposition cipher is the Rail Fence Cipher
- Write message letters out diagonally over a number of rows
- Then read off cipher row by row
- eg. write message out as:

```
m e m a t r h t g p r y
e t e f e t e o a a t
```

Giving ciphertext
 MEMATRHTGPRYETEFETEOAAT

Row Transposition Ciphers

- A more complex transposition
- Write letters of message out in rows over a specified number of columns, then reorder the columns according to some key before reading off the rows

Key:

Plaintext:

4	3	1	2	5	6	7
а	t	t	а	С	k	Р
0	S	t	Р	0	n	е
d	u	n	t	i	I	t
W	0	а	m	х	У	Z

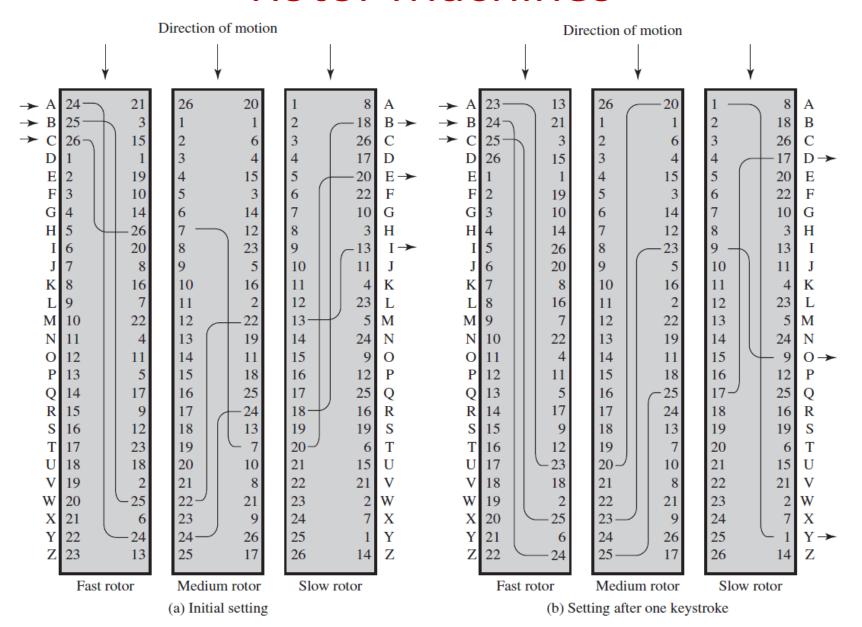
Ciphertext: TTNAAPTMTSUOAODWCOIXKNLYPETZ

- Ciphers using substitutions or transpositions are not secure because of language characteristics
- Hence consider using several ciphers in succession to make harder, but:
 - two substitutions make a more complex substitution
 - two transpositions make more complex transposition
 - but a substitution followed by a transposition makes a new much harder cipher

Rotor Machines

- Before modern ciphers, rotor machines were most common complex ciphers in use
- Widely used in WW2
 - German Enigma, Allied Hagelin, Japanese Purple
- Implemented a very complex, varying substitution cipher
- Used a series of cylinders, each giving one substitution, which rotated and changed after each letter was encrypted
- □ With 3 cylinders have 26³=17576 alphabets

Rotor Machines



Note

The material in this lecture can be found in Chapter 2, Cryptography and network security.