

CSEN 1003: Compilers

Tutorial 7 - Simple LR Parsing

Today's Plan

① LR Parsing

② SLR Parsing

③ Recap

LR Parsing

- $LR(k)$ Parsers are deterministic shift-reduce bottom up parsers.

LR Parsing

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 - Left to right input scanning.
 - Reverse of a **R**ight-most derivation.
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LR Parsing

- **LR(k)** Parsers are deterministic shift-reduce bottom up parsers.
 - Left to right input scanning.
 - Reverse of a **Right-most** derivation.
 - **k** symbols of lookahead.
- **LR grammars** are grammars for which deterministic LR parsers can be constructed.

Conflicts in Shift-Reduce Parsers

Example

Consider the following grammar:

$$\begin{aligned} E &\rightarrow E + T \mid E - T \mid T \\ T &\rightarrow T * F \mid T / F \mid F \\ F &\rightarrow \text{num} \mid \text{id} \end{aligned}$$

and the string: **id + num * id**.

| | Stack | Input | Action |
|----|--------------|-------------------------|----------------------------------|
| 1 | \$ | id - num * id \$ | Shift |
| 2 | \$ id | - num * id \$ | Reduce $F \rightarrow \text{id}$ |
| .. | .. | .. | .. |
| .. | .. | .. | .. |
| 13 | \$ $E-T$ | \$ | Reduce $E \rightarrow E-T$ |

LR(0) Items and LR(0) DFA

- An **LR(0) item** is a production rule with a dot somewhere on the RHS.
 - $A \rightarrow \alpha.\beta$ indicates the state of the parser attempting to parse the input using the rule $A \rightarrow \alpha\beta$ where α is already parsed, and the parser is expecting to parse β next (example: $E \rightarrow E.+T$).
 - Whenever, $A \rightarrow \alpha\beta.$, it might be suitable to reduce $\alpha\beta$ to A (example: $E \rightarrow E+T.$).

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 - Whenever, $A \rightarrow \alpha\beta.$, it might be suitable to reduce $\alpha\beta$ to A (example: $E \rightarrow E+T.$).
- The LR(0) DFA keeps track of the state of the parser regarding what we saw so far and what we need to do next.

Today's Plan

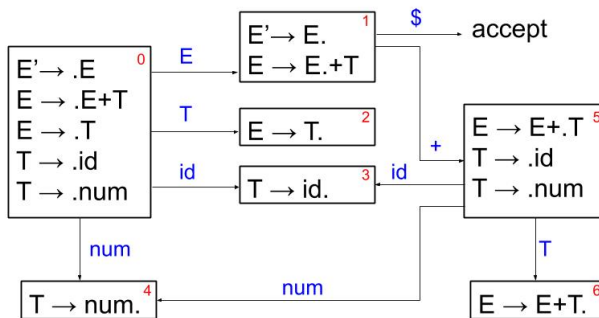
1 LR Parsing

2 SLR Parsing

3 Recap

Step 1: LR(0) DFA Construction

Example

$$\begin{aligned} E &\rightarrow (a) E + T \mid (b) T \\ T &\rightarrow (c) \text{num} \mid (d) \text{id} \end{aligned}$$


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- To fill the table:
 - 1 $\forall A \in V, \text{GOTO}(q, A) = \delta(q, A).$
 - 2 If $A \rightarrow \alpha.a\beta \in q, \text{ACTION}(q, a) = \text{shift}\delta(q, a).$

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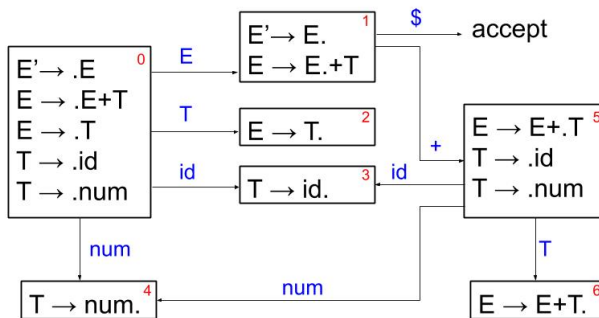
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- To fill the table:
 - ① $\forall A \in V, \text{GOTO}(q, A) = \delta(q, A)$.
 - ② If $A \rightarrow \alpha.a\beta \in q$, $\text{ACTION}(q, a) = \text{shift}\delta(q, a)$.
 - ③ If $A \neq S'$ and $A \rightarrow \alpha. \in q$, $\text{ACTION}(q, a) = \text{reduce } A \rightarrow \alpha$ where $a \in \text{Follow}(A)$.

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- To fill the table:
 - 1 $\forall A \in V, \text{GOTO}(q, A) = \delta(q, A).$
 - 2 If $A \rightarrow \alpha.a\beta \in q, \text{ACTION}(q, a) = \text{shift} \delta(q, a).$
 - 3 If $A \neq S'$ and $A \rightarrow \alpha. \in q, \text{ACTION}(q, a) = \text{reduce } A \rightarrow \alpha$ where $a \in \text{Follow}(A).$
 - 4 If $S' \rightarrow S. \in q, \text{ACTION}(q, \$) = \text{accept}.$
- If conflicts arise while filling the table, then the grammar is not SLR.

LR(0) Automaton

Example

$$\begin{aligned} E &\rightarrow (a) E + T \mid (b) T \\ T &\rightarrow (c) \text{num} \mid (d) \text{id} \end{aligned}$$


SLR Parsing Table Example

Example

$$E \rightarrow (a) E + T \mid (b) T$$

$$T \rightarrow (c) \text{num} \mid (d) \text{id}$$

$$\text{Follow}(E) = \{\$, +\}$$

$$\text{Follow}(T) = \{\$, +\}$$

| State | Action | | | | GOTO | |
|-------|--------|----|-----|-----|----------|----------|
| | + | id | num | \$ | <i>E</i> | <i>T</i> |
| 0 | s3 | s4 | | | 1 | 2 |
| 1 | s5 | | | acc | | |
| 2 | rb | | | rb | | |
| 3 | rc | | | rc | | |
| 4 | rd | | | rd | | |
| 5 | s3 | s4 | | | | 6 |
| 6 | ra | | | ra | | |

Step 3: The LR Parsing Algorithm

- 1 Push the start state of the LR(0) automaton to the stack.
- 2 Loop (S is the top the stack, a is the next input symbol):
 - a. If $ACTION[S, a] = \text{shift } i$, push i to the stack.
 - b. If $ACTION[S, a] = \text{reduce } A \rightarrow \alpha$, pop $|\alpha| = r$ states off the stack and push $GOTO(q_{n-r}, A)$.

SLR Parsing Example

Example

| | Stack | Symbols | Input | Action |
|---|-------|---------|-------------|--------|
| 1 | 0 | | id + num \$ | |

SLR Parsing Example

Example

| | Stack | Symbols | Input | Action |
|---|-------|---------|-------------|---------|
| 1 | 0 | | id + num \$ | Shift 3 |

SLR Parsing Example

Example

| | Stack | Symbols | Input | Action |
|---|-------|-----------|--------------------|----------------|
| 1 | 0 | | id + num \$ | Shift 3 |
| 2 | 03 | id | + num \$ | |

SLR Parsing Example

Example

| | Stack | Symbols | Input | Action |
|---|-------|-----------|--------------------|---------------------------|
| 1 | 0 | | id + num \$ | Shift 3 |
| 2 | 03 | id | + num \$ | Reduce $T \rightarrow id$ |

SLR Parsing Example

Example

| | Stack | Symbols | Input | Action |
|---|-------|-----------|--------------------|---------------------------|
| 1 | 0 | | id + num \$ | Shift 3 |
| 2 | 03 | id | + num \$ | Reduce $T \rightarrow id$ |
| 3 | 02 | T | + num \$ | |

SLR Parsing Example

Example

| | Stack | Symbols | Input | Action |
|---|-------|-----------|---------------------------|---------------------------|
| 1 | 0 | | id + num \$ | Shift 3 |
| 2 | 03 | id | + num \$ | Reduce $T \rightarrow id$ |
| 3 | 02 | T | + num \$ | Reduce $E \rightarrow T$ |
| 4 | 01 | E | + num \$ | |

SLR Parsing Example

Example

| | Stack | Symbols | Input | Action |
|---|-------|-----------|--------------------|---------------------------|
| 1 | 0 | | id + num \$ | Shift 3 |
| 2 | 03 | id | + num \$ | Reduce $T \rightarrow id$ |
| 3 | 02 | T | + num \$ | Reduce $E \rightarrow T$ |
| 4 | 01 | E | + num \$ | Shift 5 |

SLR Parsing Example

Example

| | Stack | Symbols | Input | Action |
|---|-------|-----------|--------------------|---------------------------|
| 1 | 0 | | id + num \$ | Shift 3 |
| 2 | 03 | id | + num \$ | Reduce $T \rightarrow id$ |
| 3 | 02 | T | + num \$ | Reduce $E \rightarrow T$ |
| 4 | 01 | E | + num \$ | Shift 5 |
| 5 | 015 | E+ | num\$ | |

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Example

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| 1 | 0 | | id + num \$ | Shift 3 |
| 2 | 03 | id | + num \$ | Reduce $T \rightarrow id$ |
| 3 | 02 | T | + num \$ | Reduce $E \rightarrow T$ |
| 4 | 01 | E | + num \$ | Shift 5 |
| 5 | 015 | E+ | num\$ | Shift 4 |
| 6 | 0154 | E+num | \$ | |

SLR Parsing Example

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Example

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| 5 | 015 | E+ | num\$ | Shift 4 |
| 6 | 0154 | E+num | \$ | Reduce $T \rightarrow num$ |
| 7 | 0156 | E+T | \$ | |

SLR Parsing Example

Example

| | Stack | Symbols | Input | Action |
|---|-------|--------------|--------------------|------------------------------|
| 1 | 0 | | id + num \$ | Shift 3 |
| 2 | 03 | id | + num \$ | Reduce $T \rightarrow id$ |
| 3 | 02 | T | + num \$ | Reduce $E \rightarrow T$ |
| 4 | 01 | E | + num \$ | Shift 5 |
| 5 | 015 | E+ | num\$ | Shift 4 |
| 6 | 0154 | E+num | \$ | Reduce $T \rightarrow num$ |
| 7 | 0156 | E+T | \$ | Reduce $E \rightarrow E + T$ |
| 8 | 01 | E | \$ | |

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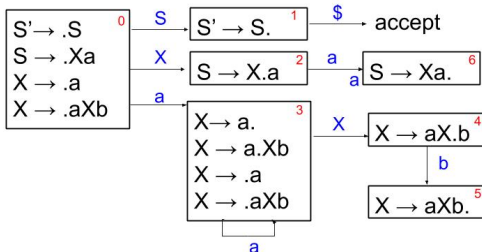
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| 6 | 0154 | E+num | \$ | Reduce $T \rightarrow num$ |
| 7 | 0156 | E+T | \$ | Reduce $E \rightarrow E + T$ |
| 8 | 01 | E | \$ | accept |

Conflicts Example - Exercise 7-3

Example

$$S \rightarrow Xa$$

$$X \rightarrow a \mid aXb$$


| State | a | b | \$ | S | X |
|-------|------------------------|--------------------|----|---|---|
| 3 | $s3, rX \rightarrow a$ | $rX \rightarrow a$ | | | 4 |

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Covered Topics

- 1 LR Parsing.
- 2 The LR(0) Automaton and SLR Parsing.

Next Session: LR(1) and LALR Parsing!