

## Lista 2

1- Dados os vetores  $\vec{u} = 2\vec{i} - 3\vec{j}$ ,  $\vec{v} = \vec{i} - \vec{j}$  e  $\vec{w} = -2\vec{i} + \vec{j}$ , determinar:

$$-\frac{3}{2} + 2 - 1 = \frac{-3+4-2}{2} = -\frac{1}{2}$$

a)  $2\vec{u} - \vec{v}$

$$2(2, -3) - (1, -1) = (3, -5)$$

b)  $\vec{v} - \vec{u} + 2\vec{w}$

$$(1, -1) - (2, -3) + 2(-2, 1) = (-5, 4)$$

c)  $\frac{1}{2}\vec{u} - 2\vec{v} - \vec{w}$

$$\frac{1}{2}(2, -3) - 2(1, -1) - (-2, 1) = (1, -\frac{1}{2})$$

2- Dados  $\vec{u} = (3, -1)$ ,  $\vec{v} = (-1, 2)$ , determine  $\vec{w}$ , sabendo que:

$\vec{w} = ?$

a)  $4(\vec{u} - \vec{v}) + \frac{1}{3}\vec{w} = 2\vec{u} - \vec{w}$

$$\Rightarrow 4((3, -1) - (-1, 2)) + \frac{1}{3}(m, y) = 2(3, -1) - (m, y)$$

$$\Rightarrow 4(4, -3) + \frac{1}{3}(m, y) = (6, -2) - (m, y)$$

$$\Rightarrow (16, -12) + \frac{1}{3}(m, y) = (6, -2) - (m, y)$$

$$\Rightarrow \frac{1}{3}(m, y) + (m, y) = (6, -2) - (16, -12)$$

$$\Rightarrow \frac{4}{3}(m, y) = (-10, 10) \Rightarrow (m, y) = \frac{3}{4}(-10, 10) = \frac{3}{2}(-5, 5)$$

$$= \left(-\frac{15}{2}, \frac{15}{2}\right)$$

$$b) 3\vec{m} - (2\vec{v} - \vec{u}) = 2(4\vec{m} - 3\vec{u})$$

$$\Rightarrow 3(m, y) - [2(-1, 2) - (3, -1)] = 2[4(m, y) - 3(3, -1)]$$

$$\Rightarrow (3m, 3y) - [(-2, -4) - (3, -1)] = 2[(4m, 4y) - (9, -3)] =$$

$$\Rightarrow (3m, 3y) - (-5, 5) = 2(4m, 9, 4y + 3)$$

$$\Rightarrow (3m + 5, 3y - 5) = (2(4m, 9), 2(4y + 3))$$

\* igualdade m

$$3m + 5 = 8m - 18$$

$$3m - 8m = -18 - 5$$

$$-5m = -23$$

$$m = \frac{23}{5}$$

\* igualdade y

$$3y - 5 = 8y + 6$$

$$3y - 8y = 6 + 5$$

$$-5y = 11$$

$$y = -\frac{11}{5}$$

$$m = \left( \frac{23}{5}, -\frac{11}{5} \right)$$

3- Dados os vetores  $\vec{u} = (2, -4)$ ,  $\vec{v} = (-5, 1)$  e  $\vec{w} = (-12, 6)$ , determine  $a_1$  e  $a_2$  tais que  $\vec{w} = a_1 \vec{u} + a_2 \vec{v}$ .

$$\vec{w} = a_1 \vec{u} + a_2 \vec{v}$$

$$\Rightarrow (-12, 6) = a_1(2, -4) + a_2(-5, 1)$$

$$\Rightarrow (-12, 6) = (2a_1, -4a_1) + (-5a_2, a_2)$$

$$\Rightarrow \begin{cases} -12 = 2a_1 - 5a_2 \\ 6 = -4a_1 + a_2 \end{cases} \cdot 2$$

$$-24 = 4a_1 - 10a_2$$

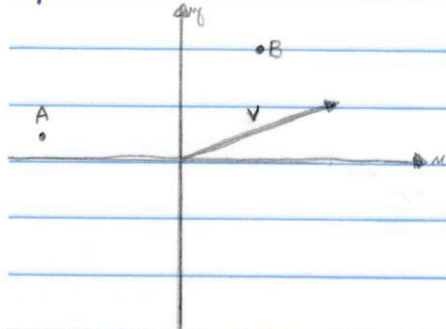
$$-18 = 0 - 9a_2 \Rightarrow a_2 = 2$$

$$6 = -4a_1 + 2 \Rightarrow 4 = -4a_1 \Rightarrow a_1 = -1$$



4- Dados os pontos  $A(-5, 1)$  e  $B(1, 3)$ . Determine o vetor  $\vec{v} = (a, b)$ , tal que

a)  $B = A + 2\vec{v}$



b)  $A = B + 3\vec{v}$

$$\Rightarrow (-5, 1) = (1, 3) + 3(m, y)$$

$$\Rightarrow (-5, 1) = (1, 3) + (3m, 3y)$$

$$\Rightarrow (-5, 1) = (1 + 3m, 3 + 3y)$$

$$* -5 = 1 + 3m \Rightarrow m = (-5, 1) / 3 = -2$$

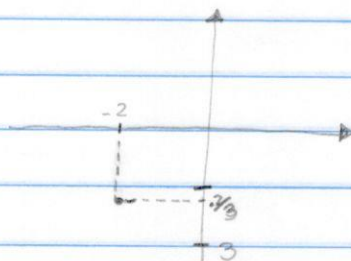
$$* 1 = 3 + 3y \Rightarrow y = (1 - 3) / 3 = -2/3$$

$$\Rightarrow (1, 3) = (-5, 1) + 2(m, y)$$

$$\Rightarrow (6, 2) = 2(m, y) \Rightarrow$$

$$\Rightarrow (m, y) = (3, 1)$$

$$\Rightarrow v = (3, 1)$$



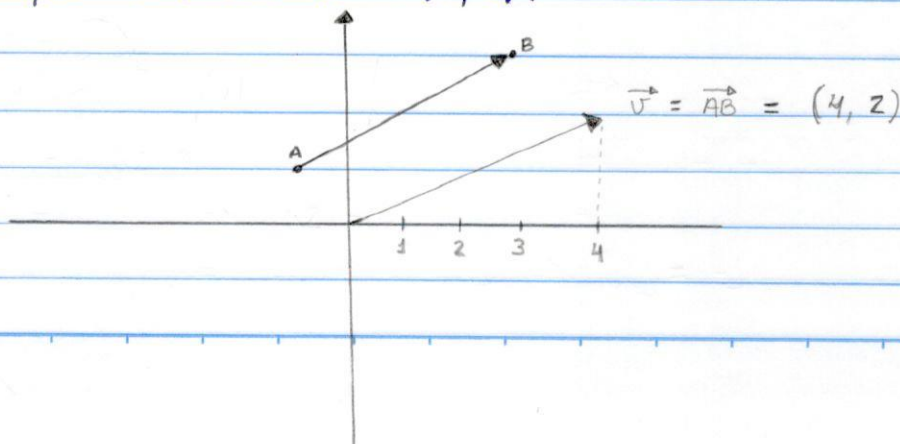
5- Qual o ponto inicial do segmento orientado que representa o vetor  $\vec{v} = (-1, 3)$ , sabendo que sua extremidade está em  $(3, 1)$ ? Representar graficamente este segmento.

→ Pela lógica, podemos fazer dessa maneira simples:

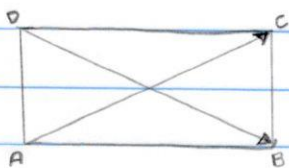
$$(3, 1) - (-1, 3) = (4, -2)$$

$$(3 - (-1), 1 - 3) = (4, -2)$$

6- Representar no gráfico o vetor  $\vec{AB}$  e o correspondente vetor posição, em que  $A(-1, 3)$  e  $B(3, 5)$ .



7 - Encontrar o vértice oposto a B, no paralelogramo ABCD, para  $A(-3, -1)$ ,  $B(4, 2)$  e  $C(5, 5)$ .



$$AB = DC \Rightarrow B - A = C - D$$

$$\Rightarrow (4, 2) - (-3, -1) = (5, 5) - (m, y)$$

$$\Rightarrow (7, 3) - (5, 5) = -(m, y) = (2, -2)$$

$$\Rightarrow (m, y) = (-2, 2)$$

8 - Dados os pontos  $A(-3, 2)$  e  $B(5, -2)$ , determinar os pontos M e N pertencentes ao segmento AB tais que  $\overrightarrow{AM} = \frac{1}{2} \overrightarrow{AB}$  e  $\overrightarrow{AN} = \frac{2}{3} \overrightarrow{AB}$ . Construir o gráfico, marcando os pontos A, B, M e N.

$$M = ? , N = ? \quad | \quad AM = \frac{1}{2} AB, \quad AN = \frac{2}{3} AB$$



$$M = \left( \frac{2}{2}, \frac{0}{2} \right) = (1, 0)$$

$$\begin{aligned} N = \frac{2}{3} AB \Rightarrow N - A &= \frac{2}{3} (B - A) = (m, y) - (-3, 2) \\ &= (m, y) - (-3, 2) \\ &= \frac{2}{3} ((5, -2) - (-3, 2)) \end{aligned}$$

$$\Rightarrow (m, y) - (-3, 2) = \frac{2}{3} (8, 4)$$

$$= \left( \frac{16}{3}, \frac{-8}{3} \right) \Rightarrow (m, y) = \left( \frac{16}{3}, \frac{-8}{3} \right) + (-3, 2)$$

$$(m, y) = \left( \frac{7}{3}, \frac{-2}{3} \right)$$



9- Dados os vetores  $\vec{u} = (1, -1)$ ,  $\vec{v} = (-3, 4)$ , calcular:

a)  $|\vec{u}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

b)  $|\vec{v}| = \sqrt{(-3)^2 + 4^2} \Rightarrow \sqrt{9 + 16} \Rightarrow \sqrt{25} \Rightarrow 5$

c)  $|\vec{u} + \vec{v}| = \sqrt{((-2)^2 + (-3)^2)} \Rightarrow \sqrt{4 + 9} \Rightarrow \sqrt{13}$

$\vec{u} + \vec{v} = (1, -1) + (-3, 4)$

$\vec{u} + \vec{v} = (-2, -3)$

d)  $\frac{|\vec{v}|}{|\vec{u}|} =$

e)  $\frac{|\vec{v}|}{|\vec{u}|} \Rightarrow \frac{\sqrt{(-3)^2 + 4^2}}{\sqrt{1^2 + (-1)^2}} \Rightarrow \frac{\sqrt{9 + 16}}{\sqrt{1 + 1}} \Rightarrow \frac{\sqrt{25}}{\sqrt{2}} \Rightarrow \frac{5}{\sqrt{2}} \Rightarrow \frac{5\sqrt{2}}{2}$

10- Calcular os valores de  $a$  para que o vetor  $\vec{u} = (a, 2)$  tenha módulo 4.

$|\vec{u}| = 4$

$|\vec{u}|^2 = a^2 + (-2)^2$

$4^2 = a^2 + 4 \Rightarrow a^2 = 12$

$a = \pm\sqrt{12}$

$= \pm 2\sqrt{3}$

11- Dados o vetor  $\vec{v} = (1, -3)$ , determinar o vetor paralelo a  $\vec{v}$  e que tenha:

a) sentido contrário a de  $\vec{v}$  e duas vezes o módulo de  $\vec{v}$ .

$$|\vec{v}| = 2$$

$$-2 \cdot (1, -3) = (-2, 6)$$

b) O mesmo sentido de  $\vec{v}$  e módulo 2;

$$\frac{2 \cdot (1, -3)}{\sqrt{1^2 + (-3)^2}} \Rightarrow \frac{2 \cdot (1, -3)}{\sqrt{10}} \Rightarrow$$

$$\frac{2 \cdot (1, -3)}{\sqrt{10}} =$$

$$\left( \frac{2}{\sqrt{10}}, -\frac{6}{\sqrt{10}} \right)$$

c) sentido contrário ao de  $\vec{v}$  e módulo 4.

$$\frac{-4 \cdot (1, -3)}{\sqrt{1^2 + (-3)^2}} =$$

$$\frac{-4 \cdot (1, -3)}{\sqrt{10}} =$$

$$\left( -\frac{4}{\sqrt{10}}, \frac{12}{\sqrt{10}} \right)$$