Lista 5

1- Se $\vec{u} = 3\vec{i} - \vec{j} - 2\vec{k}$, $\vec{v} = 2\vec{i} + 4\vec{j} - \vec{k}$ e $\vec{w} = -\vec{i} + \vec{k}$, determine

a) $|\vec{u} \cdot \vec{u}| =$

i	j	k
3	-1	-2
3	-1	-2

$|i \cdot (-1) \cdot (-2) + j \cdot (-2) \cdot (3) + k \cdot 3 \cdot (-1) - k \cdot (-1) \cdot 3 - i \cdot (-2) \cdot (-1) - j \cdot 3 \cdot (-2)|$

$\vec{u} \cdot \vec{v} = 2i - 6j - 3k - 2i + 6j$

$\vec{u} \cdot \vec{v} = 0$ //

b) $(2\vec{v}) \cdot (3\vec{v}) =$

$2\vec{v} = (4, 8, -2)$

$\vec{3v} = (6, 12, -3)$

$V =$

i	j	k
4	8	-12
6	12	-3

$|i \cdot 8 \cdot (-3) + j \cdot (-2) \cdot 6 + k \cdot 4 \cdot 12 - k \cdot 4 \cdot 12 - i \cdot (-2) \cdot 12 - j \cdot 4 \cdot (-3)|$

* Concluindo, que

$2\vec{v} \cdot 3\vec{v} = -24i - 12j + 48k + 24i + 12j$

$2\vec{v} \cdot 3\vec{v} = \vec{0}$.

d) $(\vec{u} \cdot \vec{v}) \cdot \vec{v} =$

i	j	k
3	-1	-2
2	4	-1

$(\vec{u} \cdot \vec{v} = 9i - j + 14k)$

$(9, -1, 14) \cdot (2, 4, -1) = 0$ //

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c) $(\vec{u} - \vec{v}) \cdot \vec{w}$ $(18, -4, 14)$

$(1, -5, -1) \cdot (-1, 0, 1)$

$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ 1 & -5 & -1 & 1 & -5 \\ -1 & 0 & 1 & -1 & 0 \end{array}\right|$$

$\Rightarrow -5k + 0i - 1j - 5i + 1j + 0k$

$\Rightarrow -5i + 0j - 5$

$\Rightarrow -5 + 0 - 5$

e) $(\vec{u} \cdot \vec{v}) \cdot \vec{w}$

2- Determinar um vetor simultaneamente ortogonal aos vetores $\vec{u} + 2\vec{v}$ e $\vec{v} - \vec{u}$, $\vec{u} = (-3, 2, 0)$ e $\vec{v}\ (0, -1, -2)$.

$\vec{u}\ (-3, 2, 0)$, $\vec{v}\ (0, -1, -2)$

$(u + 2v) \cdot (v - u)$

$u \cdot v - |u|^2 + 2\ |v|^2 - 2\ v \cdot u - v \cdot u$

$- u \cdot u \qquad 2\ v \cdot u$

$\downarrow \qquad \downarrow$

$0 \qquad 0 \qquad -3\ v \cdot u$

$$-3 \quad \begin{array}{ccc|cc} i & j & k & i & j \\ 0 & -1 & -2 & 0 & -1 \\ -3 & 2 & 0 & -3 & 2 \end{array}$$

$-3j + 4i + 0j + 0i + 6j + 0k$

$= 4, +6 -3$

$= -12, -18, 9$

3- Determinar um vetor de módulo 2 ortogonal a $\vec{u} = (3, 2, 2)$ e $\vec{v} = (0, 1, 1)$.

$$\begin{vmatrix} i & j & k \\ 3 & 2 & 2 \\ 0 & 1 & 1 \end{vmatrix}$$

$w = -3j + 3k$

$w = (0, -3, 3)$

$V(w) = \frac{w}{|w|}$

$|w| = \sqrt{[(-3)^2 + 3^2]}$

$|w| = \sqrt{(9 + 9)}$

$|w| = \sqrt{18} = 3\sqrt{2}$

$V(w) = (0, -3, 3) / (3\sqrt{2})$

$V(w) = \left(0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

$V(w) = \left(0, \frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$

* vetor que procuramos:

$\Rightarrow 2V(w) =$

$\Rightarrow 2 * \left(0, -\sqrt{2}/2, \frac{-\sqrt{2}}{2}\right)$

$\Rightarrow (0, -\sqrt{2}, -\sqrt{2})$

logo o vetor é $(0, -\sqrt{2}, -\sqrt{2})$

DOM SEG TER QUA QUI SEX SÁB
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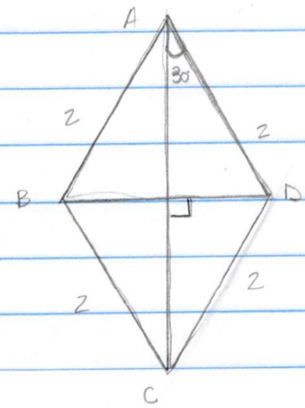
4 - Com base na figura, determine:



a) $|\vec{AB} \cdot \vec{AD}| = |\vec{AB}| \cdot |\vec{AD}| \cdot |\vec{Am}\,\theta| \quad (\theta = 60) = 2\sqrt{3}$

$2 \cdot 2 \cdot \frac{\sqrt{3}}{2}$

b) $|\vec{BA} \cdot BC| = |\vec{BA}| \cdot |\vec{BC}| \cdot |\text{sen } 120| = 2\sqrt{3}$

$2 \cdot 2 \cdot \frac{\sqrt{3}}{2}$

c) $|\vec{AB} \cdot \vec{DC}| = |\vec{AB}| \cdot |\vec{DC}| \text{ sen } 0$

$|\vec{AB}| \cdot |\vec{DC}| = 2 \cdot 2 \cdot \text{sen } 0°$

$|\vec{AB}| \cdot |\vec{DC}| = 4 \cdot 0$

$|\vec{AB}| \cdot |\vec{DC}| = 0$

d) $|\vec{AB} \cdot \vec{CD}| = |\vec{AB}| \cdot |\vec{CD}| \text{ sen } 0$

$= 2 \cdot 2 \text{ sen } 0$

$= 4 \cdot 0$

$= 0$

e) $|\vec{BD} \cdot \vec{AC}| = |\vec{BD}| \cdot |\vec{AC}| \text{ sen } \theta$

$= 2 \cdot 2\sqrt{3} \cdot \text{sen } 90°$

$= 4\sqrt{3}$ //

f) $|\vec{BD} \cdot \vec{CD}| = |\vec{BD}|\ |\vec{CD}| \text{ sen } \theta$

$= 2 \cdot 2 \cdot \text{sen } 60°$

$= 2 \cdot 2 \cdot \frac{\sqrt{3}}{2}$

$= 2\sqrt{3}$ //

5- Determinar $\vec{u} \cdot \vec{v}$, dado que $|\vec{u} \cdot \vec{v}| = 12$, $|\vec{u}| = 13$ e $\vec{v}$ é unitario.

$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cdot \operatorname{sen} \theta$

$12 = 13 \cdot 1 \cdot \operatorname{sen} \theta \Rightarrow \frac{12}{13}$

$\vec{u} \cdot \vec{v} = |u| \cdot |v| \cdot \cos \theta$

$\vec{u} \cdot \vec{v} = 13 \cdot 1 \cdot \cos 67°$

$\vec{u} \cdot \vec{v} = 5$

//

6- Mostrar que o quadrilátero ABCD de vertices $A(4,1,2)$, $B(5,0,1)$, $C(-1,2,-2)$ e $D(-2,3,-1)$ é um paralelogramo e calcular sua área.

$AB \Rightarrow B - A = (1,-1,-1)$

$CD \Rightarrow D - C = (-1,1,1) \Rightarrow AB = -1\,(CD)$, $\boxed{-1 = k} = AB \parallel CD$

$|AB| = \sqrt{3}$, $|CD| = \sqrt{3}$

$AC = C - A = (-5,1,-4)$

$$\Rightarrow |AB \cdot AC| = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 1 & -1 & -1 & 1 & -1 \\ 5 & 1 & 4 & -5 & 1 \end{array}\right|$$

$|-5k + 1i + 4j + 4i + 5j + 1k|$

$\Rightarrow |(5,9,-4)| = \sqrt{25 + 81 + 16} = \sqrt{122}$

//

7 - Calcular a distancia de p (4, 3, 3) à reta que passa por A (1, 2, -1) e B (3, 1, 1).

$$\vec{AB} \cdot \vec{AP} = \begin{vmatrix} i & j & k \\ 2 & -1 & 2 \\ 3 & 1 & 4 \end{vmatrix}$$

$$(4\vec{i} + 6\vec{j} + 2\vec{k}) - (2\vec{i} + 8\vec{j} + 3\vec{k}) = (-6\vec{i} - 2\vec{j} + 5k$$

$$|\vec{AB} \cdot \vec{AP}| = \sqrt{(-6)^2 + (-2)^2 + 5^2} = \sqrt{65}$$

* Como queremos descobrir a reta, devemos seguir esta regra

$d = \dfrac{|\vec{AB} \cdot \vec{AP}|}{|\vec{AB}|}$, temos que:

$$\vec{AB} = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{9} = 3$$

logo o resultante ficará

$$d = \frac{|\vec{AB} \cdot \vec{AP}|}{|\vec{AB}|} = \frac{\sqrt{65}}{3} \,//$$

8 - Calcular a área do triangulo ABC e a altura relativa ao lado BC, dados A(-4, 1, 1), B(1, 0, 1) e C(0, -1, 3).

AB = B - A

(1, 0, 1) - (-4, 1, 1)

AB = (1 - (-4)) i (0, 1) j (1 - 1)

AB = (5, -1, 0) //

$AC = C - A$

$AC = (0, -1, 3) - (-4, 1, 1)$

$\boxed{AC = (4, -2, 2)}$

$BC = (0, -1, 3) - (1, 0, 1)$

$BC = (-1, -1, 2)$

$$N = \begin{vmatrix} i & j & k \\ 5 & -1 & 0 \\ 4 & -2 & 2 \end{vmatrix} = (-2, -10, -6)$$

$A = \sqrt{-2^2 + -10^2 + (-6)^2}$

$A = \sqrt{4 + 100 + 36}$

$A = \sqrt{140}$

* Fatorando 140 *

$\sqrt{140} = 2\sqrt{5 \times 7} = 2\sqrt{35}$

$A = \dfrac{2\sqrt{35}}{3} = \sqrt{35}$ //

* Área relativa

$|\vec{BC}| = \sqrt{1^2 + 1^1 + 2^2}$

$|\vec{BC}| = \sqrt{6}$

9- Calcular z, sabendo-se que A(2,0,0), B(0,2,0) e C (0,0,z) são vertices de um triangulo de área 6.

$$V = \begin{vmatrix} i & j & k \\ -2 & 2 & 0 \\ -2 & 0 & z \end{vmatrix} \begin{matrix} i & j \\ -2 & 2 \\ -2 & 0 \end{matrix}$$

$\vec{u} \cdot \vec{v} = 2zi + 4k + 2zj = (2z, 2z, 4)$

$|\vec{u} \cdot \vec{v}| = \sqrt{((2z)^2 + (2z)^2 + (4)^2)} = \sqrt{(4z^2 + 4z^2 + 16)} = \sqrt{8z^2 + 16}$

* calculando a area

$$A = \frac{|\vec{u} \cdot \vec{v}|}{2} = 6$$

$$\frac{\sqrt{(8z^2 + 16)}}{2} = 6$$

$$\sqrt{8z^2 + 16} = 12$$

$$8z^2 + 16 = 144$$

$$8z^2 = 144 - 16 = 128$$

$$z^2 = \frac{128}{8} = 16$$

$$z^1 = 4, \quad z^2 = -4 \;//$$