ESERCIZIO 1.1

SET 1 HOMEWORK

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1) $f_1(n)\ \ n^{0,999999}\log n \sim O(\log n)$
$f_2(n) = 10000000n \sim O(n)$
$f_4(n) = n^2 \sim O(n^2)$
$f_3(n) = 1,000001^n \sim O(k^n)\ \ k>1$
$f_1 << f_2 << f_4 << f_3$

$n! >> 2^n >> n^3 >> n^2 >> n\log n >> n >> \log n >> 1$

$\binom{n}{x} = \frac{n!}{x!(n-x)!}$

2) $f_1(n) = 2^{2^{100000}} \sim O(1)$
$f_4(n) = n\sqrt{n} \sim O(n^{3/2})$
$f_3(n) = \binom{n}{2} \sim O(n^2) \longrightarrow \binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{n(n-1)\cancel{(n-2)!}}{2!\cancel{(n-2)!}} = O(n^2)$
$f_2(n) = 2^{10000n} \sim O(2^n)$
$f_1 << f_4 << f_3 << f_2$

3) $f_4(n) = \sum_{i=1}^{n}(i+1) \longrightarrow \sum_{i=1}^{n}(i+1) = \underbrace{\sum_{i=1}^{n} i}_{\frac{n(n+1)}{2}} + \underbrace{\sum_{i=1}^{n} 1}_{n-1+1} = O(n^2)$
$f_1(n) = n^{\sqrt{n}} \longrightarrow n^{\sqrt{n}} = 2^{\log_2 n^{\sqrt{n}}} = 2^{\sqrt{n}\log_2 n} = \sqrt{n}\log_2 n$
$f_3(n) = n^{10}\cdot 2^{n/2} \longrightarrow 2^{n/2}\cdot n^{10} = 2^{n/2}\cdot 2^{\log_2 n^{10}} = 2^{n/2}\cdot 2^{10\log_2 n} \Rightarrow \frac{n}{2} + 10\log_2 n$
$f_2(n) = 2^n \sim O(2^n)$
$f_4 << f_1 << f_3 << f_2$

ESERCIZIO 1.2

$n! >> 2^n >> n^3 >> n^2 >> n\log n >> n >> \log n >> 1$

1) $f(n) = (n^2-n)/2 \sim O(n^2)$ $\quad g(n) = 6n \sim O(n)$ $\quad \Rightarrow g(n) = O(f(n))$

2) $f(n) = n + 2\sqrt{n} \sim O(n)$ $\quad g(n) = n^2 \sim O(n^2)$ $\quad \Rightarrow f(n) = O(g(n))$

3) $f(n) = n\log n \sim O(n\log n)$ $\quad g(n) = \frac{n\sqrt{n}}{2} \sim O(n^{3/2})$ $\quad \Rightarrow f(n) = O(g(n))$

4) $f(n) = n + \log n \sim O(n)$ $\quad g(n) = \sqrt{n} \sim O(n^{1/2})$ $\quad \Rightarrow g(n) = O(f(n))$

5) $f(n) = 2(\log n)^2 \sim O((\log n)^2)$ $\quad g(n) = \log n + 1 \sim O(\log n)$ $\quad \Rightarrow g(n) = O(f(n))$

6) $f(n) = 4n\log n + n \sim O(n\log n)$ $\quad g(n) = \frac{(n^2-n)}{2} \sim O(n^2)$ $\quad \Rightarrow f(n) = O(g(n))$

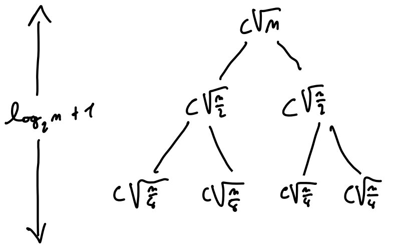
ESERCIZIO 1.3

1) $2^{n+1} = O(2^n) \Rightarrow 2^{n+1} = 2^n\cdot 2 = O(2^n)$ VERO ✓

2) $2^{2n} = O(2^n) \Rightarrow 2^{2n} = 2^n\cdot 2^n = O(2^{2n})$ FALSO DOBBIAMO CONSIDERARE LE COSTANTI MOLTIPLICATIVE ALL'ESPONENTE ✗

ESERCIZIO 1.4

1) $T(n) = 2T(n/2) + O(\sqrt{n})$



METODO DELL'ESPERTO

$a = 2,\ b = 2,\ n^{\log_b a} = n^{\log_2 2} = n$

$f(n) = O(n^{\log_b a - \varepsilon}) = O(n^{1-\varepsilon})$ $\quad \varepsilon = 1/2 \Rightarrow O(n^{1/2})$ 1° caso teorema dell'esperto

$\Rightarrow T(n) = \Theta(n^{\log_b a}) = \Theta(n)$

METODO DELL'ALBERO

Ogni nodo ha complessità $c \cdot \frac{n}{2^i} = 1 \Rightarrow n = 2^i \Rightarrow \log_2 n = \log_2 2^i \Rightarrow \log_2 n = i$ serie geometrica

$$\sum_{i=0}^{\log_2 n} 2^i c\sqrt{\frac{n}{2^i}} + \Theta(n) = c\sqrt{n} \cdot \sum_{i=0}^{\log_2 n} \frac{2^i}{2^{i/2}} = c\sqrt{n} \cdot \sum_{i=0}^{\log_2 n} \sqrt{2^i} = c\sqrt{n} \cdot \sum_{i=0}^{\log_2 n} (\sqrt{2})^i =$$

$$= c\sqrt{n} \cdot \frac{\sqrt{2}^{\log_2 n+1} - 1}{\sqrt{2} - 1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = c\sqrt{n}(\sqrt{2}+1)(\sqrt{2}^{\log_2 n+1}-1) = (c\sqrt{n}\sqrt{2} + c\sqrt{n})(\sqrt{2}^{\log_2 n+1} - 1) =$$

$$\sqrt{2}^{\log_2 n+1} = 2^{\frac{\log_2 n}{2}+\frac{1}{2}} = n^{\frac{1}{2}\log_2 2} \cdot 2^{\frac{1}{2}} = \sqrt{2}\cdot\sqrt{n}$$

$$= (c\sqrt{n}\sqrt{2} + c\sqrt{n})(\sqrt{2}\sqrt{n} - 1) = O(\sqrt{n}\cdot\sqrt{n}) = O(n)$$

2) $T(n) = T(\sqrt{n}) + \Theta(\log_2\log_2 n) =$ $\qquad \log_2 n = m \Rightarrow n = 2^m$

$= T(2^m) = T(2^{m/2}) + \Theta(\log_2 m) =$ $\qquad S(m) = T(2^m)$

$= S(m) = S(\frac{m}{2}) + \log m$

METODO DELL'ESPERTO

$a = 1,\ b = 2,\ f(m) = \log_2 m$

$f(m) = \Theta(m^{\log_b a} \cdot (\log m)^k) \Rightarrow T(m) = \Theta(m^{\log_b a}\cdot(\log m)^{k+1}) =$ CASO 2 TEOREMA DELL'ESPERTO

$= \Theta(m^0 \cdot (\log m)^1) \Rightarrow S(m) = \Theta(\log^2(m)) \Rightarrow T(n) = \Theta(\log^2(\log n))$

METODO DELL'ALBERO

$$1^{\log_2 m}\, T(1) = 2^{\log_2(1^{\log_2 m})} = 2^{\log_2 m \log_2 1} = 2^0 = 1$$

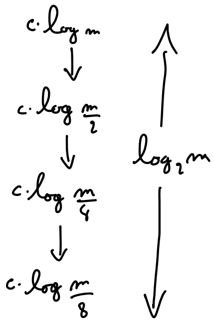
$S(m) = S(\frac{m}{2}) + \log m$

$c \cdot \log \frac{m}{2^i} = 1 \Rightarrow$

$\Rightarrow \log m - \log 2^i = 1 \Rightarrow$

$\Rightarrow \log m - i\log_2 2 = 1 \Rightarrow$

$\Rightarrow \log_2 m = i + 1 \Rightarrow i = \log_2 m - 1$



$$\sum_{i=0}^{\log_2 m-1} c\log_2\frac{m}{2^i} + \Theta(1) = \sum_{i=0}^{\log_2 m-1}(c\log_2 m - c\log_2 2^i) + \Theta(1) = c\log_2^2 m - \sum_{i=0}^{\log_2 m-1} c\, i\log_2 2 + \Theta(1) =$$

$$= c\log_2^2 m - \sum_{i=0}^{\log_2 m-1} c\cdot i + \Theta(1) = c\log_2^2 m - \sum_{i=1}^{\log_2 m} c\cdot i + \Theta(1) = c\log_2^2 m - c\left[\frac{\log_2 m(\log_2 m+1)}{2}\right] + \Theta(1) =$$

$$= O(\log^2 m) = O(\log^2(\log n))$$

3) $T(n) = 10T\left(\frac{n}{3}\right) + 17n^{1,2}$

METODO DELL' ESPERTO

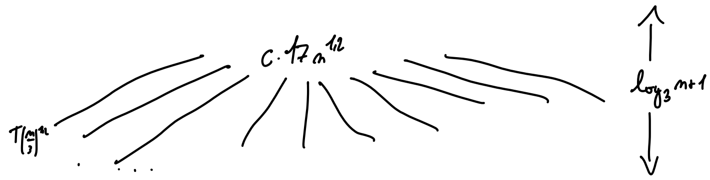
$a = 10 \quad b = 3 \quad f(n) = 17n^{1,2}$

$O\left(n^{\log_3 10 - \varepsilon}\right) = O \qquad \log_3 10 - \varepsilon = 1,2 \Rightarrow \varepsilon = \log_3 10 - 1,2 \simeq 0,9 > 0$

$n^{1,2} = O\left(n^{\log_3 10 - \log_3 10 + 1,2}\right) = O\left(n^{1,2}\right) \Rightarrow 1°$ CASO TEOREMA DELL' ESPERTO

$T(n) = \Theta\left(n^{\log_3 10}\right)$

METODO DELL' ALBERO



$10^{\log_3 n}\, T(1) = \Theta\left(n^{\log_3 10}\right)$

$c17\left(\frac{n}{3^i}\right)^{1,2} = 1 \Rightarrow c17 \cdot \frac{n^{1,2}}{3^{i\,1,2}} = 1 \Rightarrow c17n^{1,2} = 3^{1,2i} \qquad \log_3 c17n^{1,2} = 1,2i \Rightarrow i = \frac{\log_3 c17n^{1,2}}{1,2} = \frac{\cancel{1,2}}{\cancel{1,2}} \log_3 n \cdot 17 \cdot c \simeq \log_3 n$

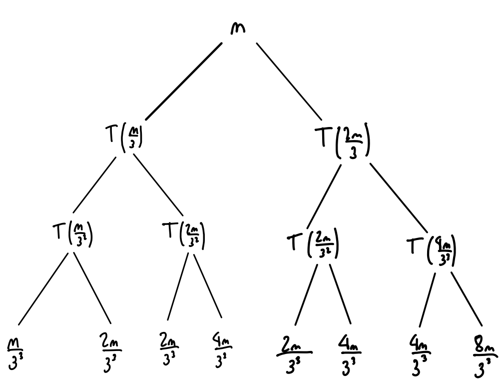
SERIE GEOMETRICA

$$17\sum_{i=0}^{\log_3 n} 10^i c\left(\frac{n}{3^i}\right)^{1,2} + \Theta\left(n^{\log_3 10}\right) = 17c\sum_{i=0}^{\log_3 n} 10^i \frac{n^{1,2}}{3^{1,2i}} + \Theta\left(n^{\log_3 10}\right) = 17cn^{1,2}\sum_{i=0}^{\log_3 n}\left(\frac{10}{3^{1,2}}\right)^i + \Theta\left(n^{\log_3 10}\right) = 17cn^{1,2}\left[\left(\frac{10}{3^{1,2}}\right)^{\log_3 n+1} - 1 \Big/ \left(\frac{10}{3^{1,2}}\right) - 1\right] + \Theta\left(n^{\log_3 10}\right) =$$

$$= \Theta\left(n^{1,2}\left(\frac{10}{3^{1,2}}\right)^{\log_3 n}\right) + \Theta\left(n^{\log_3 10}\right) = \Theta\left(n^{1,2} \cdot \frac{10^{\log_3 n}}{3^{\log_3 n \cdot 1,2}}\right) + \Theta\left(n^{\log_3 10}\right) = \Theta\left(\cancel{n^{1,2}} \cdot \frac{10^{\log_3 n}}{\cancel{n^{1,2}}}\right) + \Theta\left(n^{\log_3 10}\right) = \Theta\left(10^{\log_3 n}\right) + \Theta\left(n^{\log_3 10}\right) =$$

$$= \Theta\left(10^{\left(\frac{\log_{10} n}{\log_{10} 3}\right)}\right) + \Theta\left(n^{\log_3 10}\right) = \Theta\left(10^{\log_{10} n \cdot 1/\log_{10} 3}\right) + \Theta\left(n^{\log_3 10}\right) = \Theta\left(n^{1/\log_{10} 3}\right) + \Theta\left(n^{\log_3 10}\right) = \Theta\left(n^{\log_3 10}\right)$$

4) $T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + cn$



IL COSTO DI CIASCUN LIVELLO COMPLETO E' SEMPRE cn:

LIVELLO 1: $c\frac{n}{3} + c\frac{2n}{3} = cn$

LIVELLO 2: $c\frac{n}{3^2} + c\frac{2n}{3^2} + c\frac{2n}{3^2} + c\frac{4n}{3^2} = cn$

DATO CHE CONSIDERIAMO IL LIMITE INFERIORE, CONSIDERIAMO IL CASO OTTIMALE.

PER UN NODO AL LIVELLO K NEL PERCORSO OTTIMALE LA DIMENSIONE DEI SOTTOPROBLEMI E' $c \cdot \frac{n}{3^i}$, LA DIMENSIONE 1 SI HA QUANDO $c\frac{n}{3^i} = 1 \Rightarrow i = \log_3 cn$

PER CONFRONTO: $cn \log_3 n \gg n \log_3 n = \Omega(n \log n)$