

Department of Mathematics
National Institute of Technology, Srinagar.

B.Tech 4 th Semester	Subject	Examination	Session	Time allowed	Max. Marks
CSE/IT	Statistics & Prob.	Major	Spring-2018	3 Hours	60

NOTE: Attempt any four questions all questions carry equal marks.

Q.1 (a) A random variable X is said to have the Beta distribution with parameters α and β , prove that $\mu = \frac{\alpha}{\alpha + \beta}$ and $\sigma^2 = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$

(b) Two horses A and B were tested according to the time (in seconds) to run a particular track with the following results:

Horse A: 28 30 32 33 33 29 34

Horse B: 29 30 30 24 27 29

Test whether the two horses have the same running capacity.

(c) State and prove Bayes Theorem.

Q.2 (a) What is moment generating function. Find the moment generating function of a random variable having density function

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

determine the first four moments about the origin. Find $E(e^{2X/3})$.

(b) Assuming that the height distribution of a group of men is normal, find the mean and standard deviation, if 84% of the men have heights less than 65.2 inches and 68% have height lying between 65.2 and 62.8 inches.

Q.3 (a) For a chi-square distribution with n degrees of freedom, find its moments, β_1 and β_2 .

(b) A random variable X has density function

$$f(x) = \begin{cases} cx^2 & 1 \leq x \leq 2 \\ cx & 2 \leq x < 3 \\ 0 & \text{otherwise} \end{cases}$$

Find (i) the constant c, (ii) $P(X > 2)$ (iii) $P(1/2 < X < 3/2)$.

(9, 6) [CO4]

Q.4 (a) If X is a Poisson variate with mean m, show that $\frac{X-m}{\sqrt{m}}$ is a variable with mean zero and variance unity. Find the M.G.F for this variable and show that it approaches to $\frac{e^{t^2/2}}{e^{t^2/2}}$ as $m \rightarrow \infty$.

(b) A coin is tossed 5 times. What is probability of getting at least 2 tails? In four throws with a pair of dice, what is probability of throwing doublets: (i) twice (ii) at least twice?

(c) The arithmetic mean of a certain distribution is 5. The second and the third moments about the mean are 20 and 140 respectively. Find the third moment of the distribution about 10.

(6,6,3) [CO1]

Q.5 (a) The following table gives experimental values of the pressure P of a given mass of gas corresponding to various values of the volume V. According to thermodynamic principles, a relationship having the form $PV^\gamma = C$, where γ and C are constants, should exist between the variables. (i) Find the values of γ and C. (ii) Write the equation connecting P and V. (iii) Estimate P when $V=100.0 \text{ m}^3$.

(b) Prove that the coefficient of correlation lies between +1 and -1. Also prove that the it is the geometric mean of the coefficient of regression.

(8,7) [CO5,CO6]

Volume (m^3)	54.3	61.8	72.4	88.7	118.6	194
Pressure (N/m ²)	61.2	49.5	37.6	28.4	19.2	10.1

$$PV^\gamma = C$$

$$\log P + \gamma \log V = \log C$$

$$\log P = C - \gamma \log V$$