

NATIONAL INSTITUTE OF TECHNOLOGY, SRINAGAR
DEPARTMENT OF MATHEMATICS

Examination: Major Semester: B.Tech 4th Semester Branch: CSE/IT/Chemical
Course Title: Introduction to Probability Theory & Statistics Course No: MTH-403
Date: 18/06/2019 Max Marks: 60 Time: 3 hours

Note: Attempt any four questions. Marks are indicated against each question.

Q1: [C04] (a) If X is $b(2, p)$, Y is $b(4, p)$ and $\Pr(X \geq 1) = \frac{5}{9}$ then find $\Pr(Y \geq 1)$? (5)

(b) [C04] Derivation of mean, variance and moment generating function of a Normal Distribution? (5)

(c) [C03] State and prove Baye's Theorem? (5)

Q2: [C02] (a) Let the random variable X have the Pdf $f(x) = \frac{2}{\sqrt{2\pi}} e^{-x^2/2}$; $0 < x < \infty$ and $f(x) = 0$; elsewhere. Find the mean and variance of X ? (8)

(b) [C02] Find the constant c such that the function $f(x)$ is a Pdf of the random variable $f(x) = c \left(\frac{1}{2}\right)^x$; $x = 1, 2, 3, \dots$ and $f(x) = 0$; elsewhere. Also find $\Pr(|X| \leq 4)$; $\Pr(1 \leq X \leq 5)$? (7)

Q3: [C04] (a) If the random variable X has a Poisson distribution such that $\Pr(X = 1) = \Pr(X = 2)$, find $\Pr(X = 4)$? (5)

(b) [C04] If X is $b(n, p)$, show that $E\left(\frac{x}{n}\right) = p$, $E\left[\left(\frac{x}{n} - p\right)^2\right] = \frac{p(1-p)}{n}$ and $Cov\left(\frac{x}{n}, \frac{n-x}{n}\right) = -\frac{p(1-p)}{n}$? (5)

(c) [C03] Five cards are drawn at random and without replacement from an ordinary deck of playing cards. Relative to the hypothesis that there are atleast two cards of Spades. What is the conditional probability that there are atleast three cards of spades? (5)

Q4: (a) [C03] Let X have a Pdf $f(x)$ that is positive at $x = -1, 0, 1$ and is zero elsewhere

(i) If $f(0) = \frac{1}{2}$ find $E(X^2)$

(ii) If $f(0) = \frac{1}{2}$ and $E(X) = \frac{1}{6}$ then find $f(-1)$ and $f(1)$? (10)

(b) [C01] If the first, second and third moments of the distribution about the point 7 be 3, 11 and 15, respectively. Find the mean μ of X , then find the first, second and third moments of the distribution about the point μ ? (5)

Q5: (a) [C05] Define the term "line of best fit". Fit a parabola of second degree to the following data:

X:	1	2	3	4	5	6	7	8	9	
Y:	2	6	7	8	10	11	11	10	9	(6)

(b) [C01] Find the mode of the following frequency distribution:

Size (x):	1	2	3	4	5	6	7	8	9	10	11	12
Freq (f):	3	8	15	23	35	40	32	28	20	45	14	6

(9)
