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Note: Attempt all questions. All notations and symbols have their usual meanings.

1. (a) Define a binary relation on $\mathbf{R}^2$ as follows $\mathfrak{R} = \{((a,b),(c,d)) \in \mathbf{R}^2 \times \mathbf{R}^2 | a^2 + b^2 = c^2 + d^2\}$. Prove that $\mathfrak{R}$ is an equivalence relation. Find the equivalence classes of $\mathfrak{R}$. (5)

   (b) Show that $[d \to ((\sim a) \wedge b) \wedge c]$ and $\sim [(a \vee (\sim (b \wedge c))) \wedge d]$ are equivalent. (5)

   (c) Give an example of Distributive lattice, complemented lattice, bounded lattice, infinite lattice with finite length and normal subgroup. (5)

2. (a) Show by an example that the subgroups of a non-Abelian group are Abelian. (5)

   (b) Draw the undirected graph for the following adjacency matrix

   $$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$
   (5)

   (c) Draw the Hasse diagram of D_{36}. (5)

3. (a) Show that the graph K_5 is non-planar. (8)

   (b) Show that the chromatic number of a non-null graph is 2 if and only if the graph is bipartite. (7)

4. (a) State and prove Lagrange's theorem. Give an example, where converse is true. (8)

   (b) Define spanning tree. Find two different spanning trees for the below given graph

   (7)
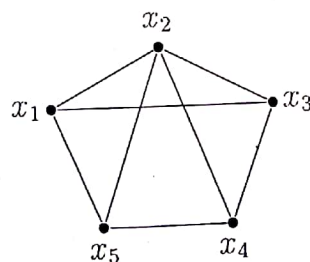
5. (a) Prove that $(Z_n, \oplus_n, \otimes_n)$ is a ring. (5)

   (b) If a connected planar graph G has n vertices, e edges and r regions, than prove that $n - e + r = 2$. (5)

   (c) Find all cyclic subgroups of $(z_{10}, \oplus_{10})$. (5)

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