IR MATRIX : A DEFORMATION-INVARIANT AND TRANSLATION-INVARIANT REPRESENTATION OF REAL-VALUED VECTORS

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ABSTRACT

In this paper, we introduce a new deformation-invariant and translation-invariant representation of real-valued vectors called the Inter-Relations Matrix or IR Matrix for short. By deformation-invariant, it is meant that the actual distances between the vector's components are ignored in the calculation of the matrix, and the matrix focuses on the ordering relationships between the different components of the vector. As a result, the matrix is robust at detecting features that do not change with changing scale, with deformation, or with translation.

PREFACE

The motivation behind this paper is to solve the problem of measuring the similarity and difference between representations, especially visual ones, in Machine Learning. This paper is intended to be exploratory work into the subject of vector representations, and to hopefully inspire more thorough work into the subject. As such, a lot of the work presented is not rigorously defined.

INTRODUCTION

The measure of similarity or difference between vectors is a very valuable metric. It is widely used in all fields of science. Some of the widely used measures of difference and similarity between vectors are: All types of norms of differences between vectors (Manhattan, Euclidean, etc.), Absolute Square Error, Absolute Mean Square Error, Cosine Similarity, etc. These metrics suffice for a lot of applications, but they only measure the relationships between the corresponding components of vectors, and they do not measure the relationships between the different components of the vectors. For example, the cosine similarity between the n-dimensional vectors $\vec{a}$ and $\vec{b}$ is can be described as the following:

$$cos(\theta) \propto \sum_{i=1}^{n} a_i \cdot b_i$$

It can be seen that such metric only calculates the relationships between the ith components of $\vec{a}$ and $\vec{b}$ (where $i = 1, 2, 3, .., n$), then sums them up. We would also like to incorporate relationships between the ith element of $\vec{a}$ and the jth element of $\vec{a}$, between the ith element of $\vec{b}$ and the jth element of $\vec{b}$, and between ith element of $\vec{a}$ and the jth element of $\vec{b}$. This will give us more insights about the relationship between different components of the same vector, and between the vectors $\vec{a}$ and $\vec{b}$. Deformation-invariance involves studying the relationships between the different components of the same vector. Before we continue, let us present a method for graphically describing an n-dimensional vector v:

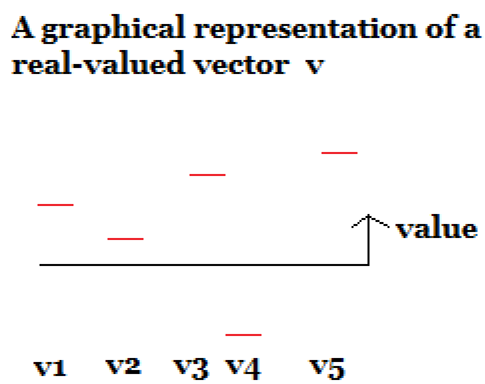

Figure 1: A graphical method for decribing an n dimensional vector v. Each component is displayed as a red horizontal bar. The height of the bar represents the value of the vector's component at that location. The method is similar to a bar chart for categorical data.

Now, let us take a look at the following problem:

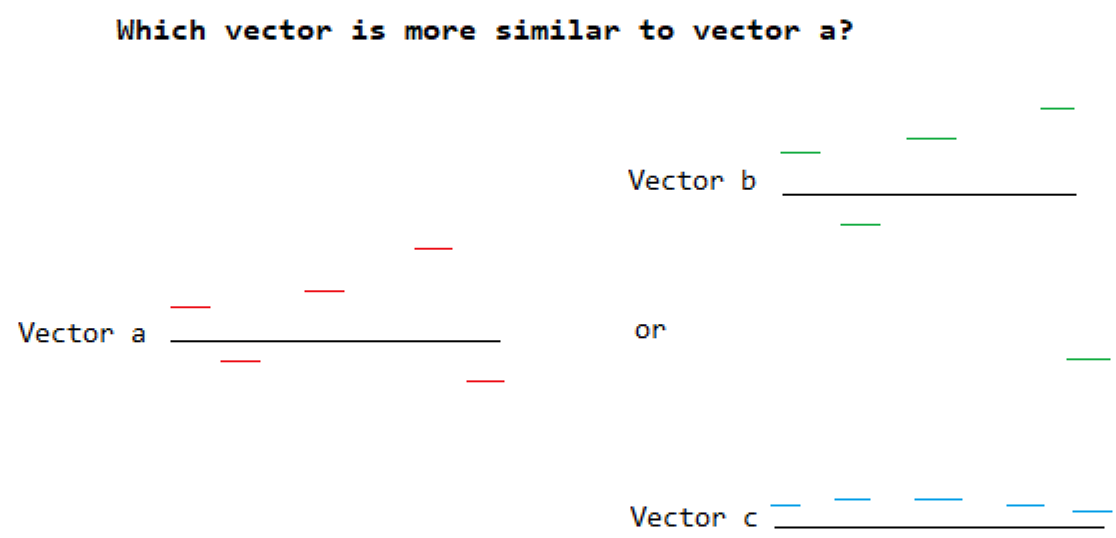

Figure 2: A diagram depicting the problem with deformation-variant metrics. Visually, vector b seems like it is more similar to vector $\vec{a}$ than vector $\vec{c}$ is. Many metrics will suggest that vector $\vec{c}$ is closer to vector a than vector $\vec{b}$ is.

While many distance or similarity metrics might determine that vector $\vec{c}$ is more similar than vector $\vec{b}$, there are still metrics that will pick vector $\vec{b}$ as the closer vector to $\vec{a}$. For example, the cosine similarity will be higher for vector $\vec{b}$ than vector $\vec{c}$, but the proposed representation measures something different than the cosine similarity. Below is a diagram to explain the difference:

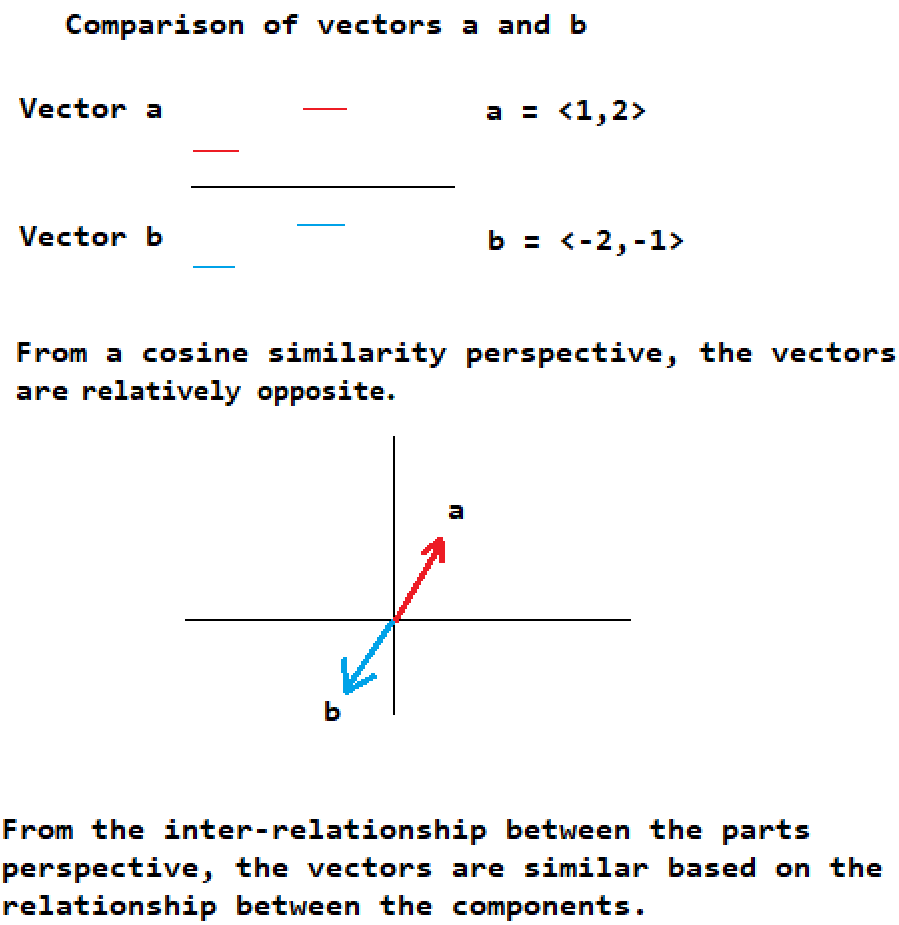

Figure 3: A diagram depicting two vectors, and the corresponding cosine measure along with the IR Matrix perspective. Using cosine similarity, the vectors are relatively opposite to each other. Using the IR Matrix, the vectors are the same.

To understand why the this representation will measure them to be the same, let us take a look at an example where we define an ordering on the components of a vector:

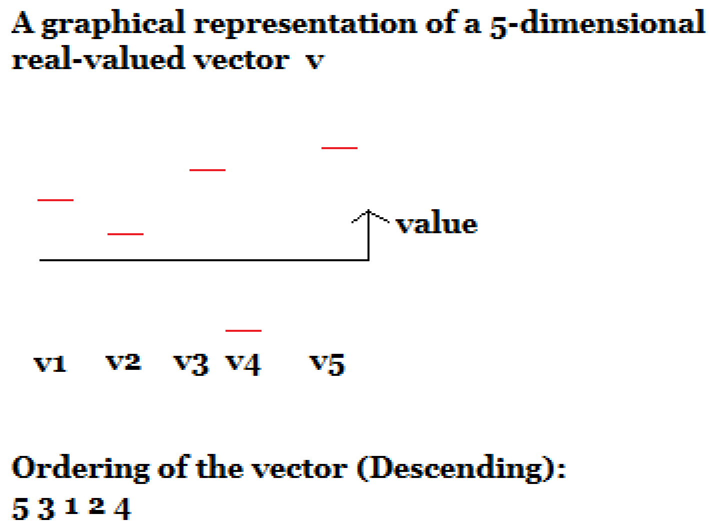

Figure 4: An ordering of the vector's components based on value (descending) can describe the vector.

Based on the ordering, the 5th component can be any value larger than its current value, but the ordering will not change. Similarly, the 4th component can be lower than its current value, and that will not change the ordering. As for the other components, they have a certain range where they can vary, and yet keep the ordering the same. The Ordering is not affected by actual distances. There are lots of vector values that will maintain the same ordering. We consider these vectors to belong to the same class. From this ordering representation, a matrix can be induced.

THE INTER-RELATIONS MATRIX

If we start with a vector $\vec{a}$ where $\vec{a} \in R^n$, we can define an Inter-relations matrix ($IRM(\vec{a})$). Each element of the IRM matrix is defined as follows:

$$IRM(\vec{a})_{ij} = \left\{ \begin{array}{ll} +1, & \text{if } a_i > a_j \\ 0, & \text{if } a_i = a_j \\ -1, & \text{if } a_i < a_j \end{array} \right\}$$

This matrix captures the relationships between each pair of components of the vector $\vec{a}$, and the relationships are not exact measures of the distances between the components. They are rather a measure of the relative ordering of the components of the vector. This allows the matrix to be insensitive to changes in values as long as the changes are not changing the ordering. This leads to the partitioning of the space R^n, where each partition is a class or category that is defined by the ordering of the components' values. Below is a depiction of a vector, along with all the possible vectors that are in its class:

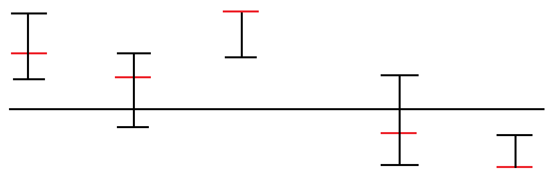

Figure 5: A diagram depicting a class of vectors. The value of each component can be changed within the whiskers without changing the entries of the IR Matrix, which means all of these possible vectors will produce the same IR matrix, and they will "collapse" to the same class.

The IR matrix can be extended to vectors with components of dimension higher than 1. An IR matrix can be created for each dimension of the components.

POSSIBLE APPLICATIONS

A possible application for this representation is for image classification. Imagine that we have an image of key pixels that determine a shape.

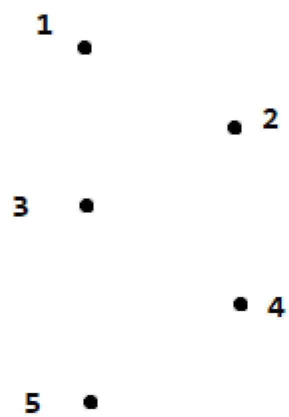

Figure 6: An image depicting key points to draw a version of the digit 3.

There are many possible configurations of the points that will lead to the same IR matrix, and therefore allow the correct classification of the shape, even with the presence of deformation.

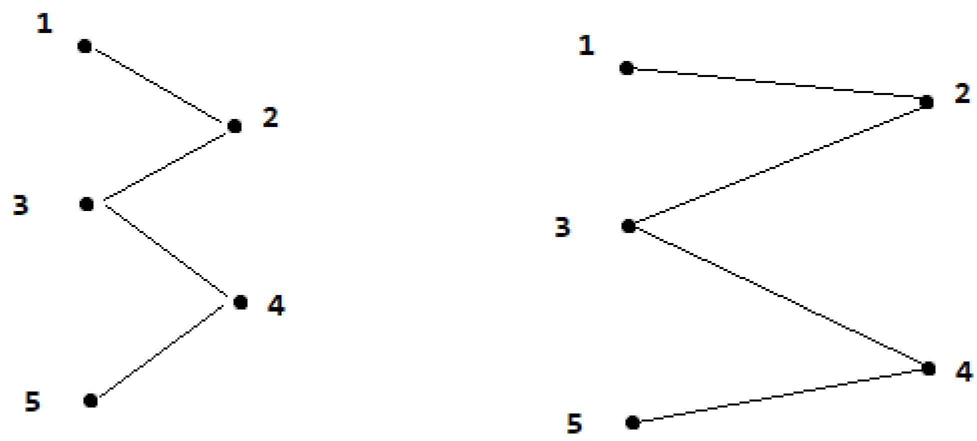

Figure 7: A diagram depicting two shapes with the same IR matrix. One is a deformation of the other.

There are many possible deformations/configurations that will lead to the same IR matrix. Many other deformations will lead to a very similar IR matrix with a high overlap between the matrices' entries. So using this representation, a machine learning algorithm like a neural network might need far fewer data points to capture the main relations that determine a visual symbol like the digit 3.

CONCLUSION AND DISCUSSION

In this paper, we discussed a deformation-invariant representation of real-valued vectors. The representation is the ordering of the vector's components by value. From this representation, we defined a matrix called the Inter-Relations Matrix or IR Matrix. The entries of the matrix represent the ordering relations between the different components of the vector that it represents. The IR Matrix can have applications in computer vision, but more research is required.