Exercise -

Solve, $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 1 + x + x^2$.

Solve, $(D^2 + 5D + 4)y = 3 - 2x$

If $R(x) = \sin ax$ or $\cos ax$

then, $y_p = \frac{1}{f(D^2)} R(x)$ | $D^2 = -a^2$
~~$D = a$~~

If $f(D^2) = 0$ then, $y_p = x \frac{1}{f'(D^2)} R(x)$

Example - 1:

Solve, $(D^2 + 9)y = \cos 4x$

Solⁿ: $y_c = c_1 \cos 3x + c_2 \sin 3x$

$\therefore y_p = \frac{1}{D^2 + 9} \cos 4x$

$= \frac{1}{-4^2 + 9} \cos 4x = \frac{1}{-16 + 9} \cos 4x$

$= \frac{-1}{7} \cos 4x$.

$\therefore y = y_c + y_p = c_1 \cos 3x + c_2 \sin 3x - \frac{1}{7} \cos 4x$.

Example - 2:

Solve, $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = \cos 2x$

Solⁿ:

$$y_c = e^{-x}\left[c_1 \cos\frac{\sqrt{6}}{2}x + c_2 \sin\frac{\sqrt{6}}{2}x\right]$$

$$\therefore y_p = \frac{1}{D^2+2D+2}\cos 2x$$

$$= \frac{1}{-2^2+2D+2}\cos 2x$$

$$= \frac{1}{-4+2D+2}\cos 2x$$

$$= \frac{1}{2D-2}\cos 2x$$

$$= \frac{1}{2(D-1)}\cos 2x$$

$$= \frac{1\cdot(D+1)}{2(D-1)(D+1)}\cos 2x$$

$$= \frac{(D+1)}{2(D^2-1)}\cos 2x$$

$$= \frac{(D+1)}{2(-2^2-1)}\cos 2x$$

$$= \frac{(D+1)}{2\times(-5)}\cos 2x = \frac{(D+1)}{-10}\cos 2x$$

$$= \frac{-1}{10}\left[D(\cos 2x) + \cos 2x\right]$$

$$= \frac{-1}{10}\left[-2\sin 2x + \cos 2x\right]$$

$$\therefore y = y_c + y_p$$

$$=$$

Example -3:

Solve, $(D^2 - 3D + 4)y = \cos(4x+7)$.

Soln:
$$y_c = e^{\frac{3x}{2}}\left(c_1 \cos \frac{\sqrt{7}}{2}x + c_2 \sin \frac{\sqrt{7}}{2}x\right).$$

$$\therefore y_p = \frac{1}{D^2 - 3D + 4}\cos(4x+7)$$

$$= \frac{1}{-4^2 - 3D + 4}\cos(4x+7).$$

$$= \frac{1}{-3D - 12}\cos(4x+7)$$

$$= \frac{1}{-3(D+4)}\cos(4x+7)$$

$$= \frac{1\cdot(D-4)}{-3(D+4)(D-4)}\cos(4x+7)$$

$$= \frac{(D-4)}{-3(D^2-16)}\cos(4x+7)$$

$$= \frac{(D-4)}{-3(-4^2-16)}\cos(4x+7)$$

$$= \frac{(D-4)}{(-3)(-32)}\cos(4x+7)$$

$$= \frac{1}{96}\left[D(\cos(4x+7)) - 4\cos(4x+7)\right]$$

$$= \frac{1}{96}\left[-4\sin(4x+7) - 4\cos(4x+7)\right]$$

$$= \frac{-4}{96}\left[\sin(4x+7) + \cos(4x+7)\right]$$

$$= \frac{-1}{24}\left[\sin(4x+7) + \cos(4x+7)\right].$$

$$\therefore y = y_c + y_p$$

$$=$$

Example - 4:

Solve, $\frac{d^2y}{dx^2} + y = \sin x$

Sol^n:

$$y_c = c_1 \cos x + c_2 \sin x$$

$$\therefore y_p = \frac{1}{D^2+1} \sin x$$

$$= \frac{1}{-1^2+1} \sin x$$

$$= x \cdot \frac{1}{2D} \sin x$$

$$= x \cdot \frac{D}{2D^2} \sin x$$

$$= x \frac{D}{2\cdot(-1^2)} \sin x$$

$$= \frac{x}{-2} D(\sin x)$$

$$= \frac{-x}{2} \cos x$$

$$\therefore y = y_c + y_p =$$

25. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0, \qquad y(0) = 3, \qquad y'(0) = 5.$

26. $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 10y = 0, \qquad y(0) = -4, \qquad y'(0) = 2.$

27. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0, \qquad y(0) = 1, \qquad y'(0) = 6.$

28. $3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 4y = 0, \qquad y(0) = 2, \qquad y'(0) = -4.$

29. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0, \qquad y(0) = 2, \qquad y'(0) = -3.$

30. $4\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 9y = 0, \qquad y(0) = 4, \qquad y'(0) = 9.$

31. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0, \qquad y(0) = 3, \qquad y'(0) = 7.$

32. $9\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + y = 0, \qquad y(0) = 3, \qquad y'(0) = -1.$

33. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 29y = 0, \qquad y(0) = 0, \qquad y'(0) = 5.$

If $R(x) = x\sin ax$ or $x\cos ax$ then

$$y_p = \frac{1}{f(D)} x\sin ax / x\cos ax$$

$$= x\frac{1}{f(D)} \sin ax/\cos ax - \frac{f'(D)}{[f(D)]^2} \sin ax/\cos ax$$

<u>Example-1:</u>

Solve, $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\sin x$

<u>Sol^n:</u>

For complementary function,

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

Let $y = e^{mx}$

$$\frac{dy}{dx} = me^{mx}$$

$$\frac{d^2y}{dx^2} = m^2e^{mx}$$

$\therefore$ $m^2e^{mx} - 2me^{mx} + e^{mx} = 0$

$\Rightarrow$ $m^2 - 2m + 1 = 0$

$\Rightarrow (m-1)^2 = 0$

$\Rightarrow (m-1)(m-1) = 0$ $\quad\therefore m = 1, 1$

$\therefore y_c = (c_1 + c_2x)e^x.$

$$\therefore y_p = \frac{1}{D^2 - 2D + 1} x \sin x$$

$$= x \frac{1}{D^2 - 2D + 1} \sin x - \frac{2D - 2}{[D^2 - 2D + 1]^2} \sin x$$

$$= x \frac{1}{-1^2 - 2D + 1} \sin x - \frac{2(D-1)}{[-1^2 - 2D + 1]^2} \sin x$$

$$= x \frac{1}{-2D} \sin x - \frac{2(D-1)}{[-2D]^2} \sin x$$

$$= \frac{-x}{2} \frac{1}{D} \sin x - \frac{2(D-1)}{4D^2} \sin x$$

$$= \frac{-x}{2} \cdot (-\cos x) - \frac{2(D-1)}{4 \cdot (-1^2)} \sin x$$

[$D \rightarrow$ Differentiation

$\frac{1}{D} \rightarrow$ Integration

$$= \frac{x}{2} \cos x + \frac{2(D-1)}{4} \sin x$$

$$= \frac{x}{2} \cos x + \frac{1}{2} [D \cdot (\sin x) - \sin x]$$

$$= \frac{x}{2} \cos x + \frac{1}{2} [\cos x - \sin x]$$

$$\therefore y = y_c + y_p$$

$$= (C_1 + C_2 x) e^x + \frac{x}{2} \cos x + \frac{1}{2} [\cos x - \sin x]$$

Ans.

Example - 2:

Solve, $(D^2+9)y = x\cos x$

Sol^n: For complementary function,

$$(D^2+9)y=0$$

Let $y = e^{mx}$

$$\frac{dy}{dx} = me^{mx}$$

$$\frac{d^2y}{dx^2} = m^2e^{mx}$$

$\therefore \quad m^2e^{mx} + 9e^{mx} = 0$

$\Rightarrow m^2+9=0 \qquad \Rightarrow m^2 = -9$

$\Rightarrow m^2 = 9i^2 \qquad \Rightarrow m = \pm 3i$

$\therefore y_c = e^{0x}(c_1\cos 3x + c_2\sin 3x)$

$= c_1\cos 3x + c_2\sin 3x$

$\therefore y_p = \frac{1}{D^2+9} x\cos x$

$= x\frac{1}{D^2+9}\cos x - \frac{2D}{[D^2+9]^2}\cos x$

$= x\frac{1}{-1^2+9}\cos x - \frac{2D}{[-1^2+9]^2}\cos x$

$$= x \frac{1}{8} \cos x - \frac{2D}{8^2} \cos x$$

$$= \frac{x}{8} \cos x - \frac{2}{64} D(\cos x)$$

$$= \frac{x}{8} \cos x - \frac{1}{32} (-\sin x)$$

$$= \frac{x}{8} \cos x + \frac{1}{32} \sin x$$

$$\therefore y = y_c + y_p$$

$$= c_1 \cos 3x + c_2 \sin 3x + \frac{x}{8} \cos x + \frac{1}{32} \sin x.$$

Ans

Example - 1:

Solve, $\frac{d^2y}{dx^2} + y = \sin 2x \cos x$.

Sol^n:

For complementary sol^n,

$$\frac{d^2y}{dx^2} + y = 0$$

Let $y = e^{mx}$

$$\frac{dy}{dx} = me^{mx}$$

$$\frac{d^2y}{dx^2} = m^2 e^{mx}$$

$\therefore$ $m^2 e^{mx} + e^{mx} = 0$

$\Rightarrow m^2 + 1 = 0$

$\Rightarrow m^2 = -1 = i^2$

$\therefore m = \pm i$

$\therefore y_c = e^{0 \cdot x}(c_1 \cos x + c_2 \sin x)$

$= c_1 \cos x + c_2 \sin x$

$\therefore y_p = \frac{1}{D^2+1} \sin 2x \cos x$

$$* \ 2\sin A\cos B = \sin(A+B)+\sin(A-B)$$

$$\Rightarrow \sin A\cos B = \frac{1}{2}\{\sin(A+B)+\sin(A-B)\}$$

$$\therefore y_p = \frac{1}{D^2+1}\cdot\frac{1}{2}\{\sin(2x+x)+\sin(2x-x)\}$$

$$= \frac{1}{2(D^2+1)}\sin 3x + \frac{1}{2(D^2+1)}\sin x$$

$$= \frac{1}{2(-3^2+1)}\sin 3x + \frac{1}{2(-1^2+1)}\sin x$$

$$= \frac{1}{2(-8)}\sin 3x + \frac{1\cdot x}{2\cdot 2D}\sin x$$

$$= \frac{1}{-16}\sin 3x + \frac{x}{4}\frac{1}{D}\sin x$$

$$= \frac{-1}{16}\sin 3x + \frac{x}{4}(-\cos x)$$

$$= \frac{-1}{16}\sin 3x - \frac{x}{4}\cos x$$

$$\therefore y = y_c + y_p$$

$$= c_1\cos x + c_2\sin x - \frac{1}{16}\sin 3x - \frac{x}{4}\cos x.$$

<u>Ans.</u>

H.W

Solve, $(D^2+4)y = \sin 2x \sin x$.

If $R(x) = e^{ax}\sin bx / e^{ax}\cos bx$ then

$$y_p = \frac{1}{f(D)} e^{ax} \sin bx / e^{x} \cos bx$$

$$= \frac{e^{ax}}{f(D+a)} \sin bx / \cos bx$$

Example - 2:

Solve, $(D^2 - 2D + 1)y = e^x \sin x$.

Soln:

$$y_c = (c_1 + c_2 x)e^x$$

$$\therefore y_p = \frac{1}{D^2 - 2D + 1} e^x \sin x$$

$$= \frac{e^x}{(D+1)^2 - 2(D+1) + 1} \sin x$$

$$= \frac{e^x}{D^2 + 2D + 1 - 2D - 2 + 1} \sin x$$

$$= \frac{e^x}{D^2} \sin x$$

$$= e^x \frac{1}{D^2} \sin x$$

$$= e^x \frac{1}{-1^2} \sin x$$

$$= -e^x \sin x$$

$$\therefore y_p = -e^x \sin x$$

$$\therefore y = y_c + y_p$$

$$=$$

Example - 3:

Solve, $(D^2 - 9)y = e^{3x} \cos x$.

Sol: $m = 3, -3$

$$y_c = c_1 e^{3x} + c_2 e^{-3x}$$

$$y_p = \frac{1}{D^2 - 9} e^{3x} \cos x$$

$$= e^{3x} \frac{1}{(D+3)^2 - 9} \cos x$$

$$= e^{3x} \frac{1}{D^2 + 6D + 9 - 9} \cos x$$

$$= e^{3x} \frac{1}{D^2 + 6D} \cos x$$

$$= e^{3x} \frac{1}{-1^2 + 6D} \cos x$$

$$= e^{3x} \frac{1}{6D - 1} \cos x$$

$$= e^{3x} \frac{(6D+1)}{(6D-1)(6D+1)} \cos x$$

$$= e^{3x} \frac{(6D+1)}{36D^2 - 1} \cos x$$

$$= e^{3x} \frac{(6D+1)}{36(-1^2) - 1} \cos x$$

$$= e^{3x} \frac{(6D \cdot \cos x + \cos x)}{-36 - 1}$$

$$= \frac{-e^{3x}}{37} (-6 \sin x + \cos x)$$

$$\therefore y = y_c + y_p =$$

H.W

i) $(D^2 - 1) y = e^x \sin \frac{x}{2}$

ii) $(D^2 - 4) y = e^x \sin x$