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(25) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0$

Solution:

Let $y = e^{mx} \Rightarrow \frac{dy}{dx} = me^{mx} \Rightarrow \frac{d^2y}{dx^2} = m^2e^{mx}$

Substituting these values

$\Rightarrow m^2e^{mx} - me^{mx} - 12e^{mx} = 0$

$\Rightarrow e^{mx}(m^2 - m - 12) = 0$

$\Rightarrow m^2 - m - 12 = 0$

$\Rightarrow m^2 - 4m + 3m - 12 = 0$

$\Rightarrow m(m-4) + 3(m-4) = 0$

$\Rightarrow (m-4)(m+3) = 0$

So, $m_1 = 4$; $m_2 = 3$

m_1 and m_2 are real number and $m_1 \neq m_2$

So the general equation is:

$y(x) = c_1e^{4x} + c_2e^{-3x}$

Now

$\Rightarrow y(0) = 2$

$\Rightarrow c_1e^{4.0} + c_2e^{-3.0} = 3 \Rightarrow c_1 + c_2 = 3$ ——— (i)

and

$y'(0) = 5$

$\Rightarrow y'(x) = 4c_1e^{4x} - 3c_2e^{-3x}$

$\Rightarrow y'(0) = 4c_1e^0 - 3c_2e^0 = 5$

$\Rightarrow 4c_1 - 3c_2 = 5$ ——— (ii)

from equation — (i)

$\Rightarrow c_1 + c_2 = 3$

$\Rightarrow c_1 = 3 - c_2$ ——— (iii)

putting c_1 equation on equation — (ii)

$\Rightarrow 4(3 - c_2) - 3c_2 = 5$

$\Rightarrow 12 - 4c_2 - 3c_2 = 5$

$\Rightarrow 12 - 7c_2 = 5$

$\Rightarrow -7c_2 = 5 - 12$

$\Rightarrow -7c_2 = -7$

$\therefore c_2 = 1$

So, $c_1 = 3 - 1 = 2$

$\therefore y(x) = 2e^{4x} + e^{-3x}$ Ans: