



Figure 1: **NNETARCH model architecture.** This hybrid model combines two neural network modules: the top branch estimates the conditional mean $f(y_{t-1}, \dots, y_{t-p})$ using an autoregressive neural network, and the bottom branch models the time-varying volatility function $g(e_{t-1}, \dots, e_{t-q})$, where $e_t = y_t - f(\cdot)$. The two outputs are synthesized via a stochastic innovation $\varepsilon_t \sim \mathcal{N}(0, 1)$, resulting in a heteroskedastic forecast \hat{y}_t that adapts to nonlinear dynamics and volatility clustering.